Analysis of a Poiseuille Flow of an Incompressible Fluid Between a Concentric Circular Cylinder

A Thesis Submitted to the Department of Mathematics, Jimma University in Partial Fulfillment for the Requirements of the Degree of Masters of Sciences (MSC.) in Mathematics

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DECLARATION

I, the undersigned declare that thesis entitled as "**Analysis of a Poiseuille Flow of an Incompressible Fluid between a concentric Circular Cylinders"** is my own original work and it has not been submitted to any institution or University elsewhere for the award of an academic degree or like.

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ABSTRACT

The aim of this thesis is to analyze the the solution of analytical solution of a Poiseulle flow of incompressible fluid between two fixed concentric circular cylinders with radius R_1 and R_2 subjact to slip boundary conditions with the some appropriate assumptions were established by considering three distinct cases. That is,those are the parallel flow to the wall with both slip boundary condition, slip and no-slip and viceversal boundary conditions on the inner and outer surface of the two concentric cylinders.

In each case the steady state velocity and volume flow rate distribution in the field were determined at diferent boundary condition and the result indicates that both of them are affected by the radius of the gap of concentric circular cylinder and slip length were discussed. Moreover, the fluid velocity of distribution decrease and maximum velocity distribution at the center line of parallel flow of fluid, when the radius of the gap of concentric circular cylinders increases in the vertical fluid motion.

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1.Introduction

1.1. Background of the Study

According to Bansal (2005), Fluid mechanics is the branch of science which deals with the behavior of the fluids (liquids or gases) at rest as well as in motion. Thus, this branch of science deals with the static, kinematics and dynamics aspects of fluids. The study of fluids at rest is called fluid statics. The study of fluids in motion where the pressure forces are not considered is called fluid kinematics and if the pressure forces are also considered for the fluids in motion, that branch of science is called fluid dynamics.

Fluids are usually classified as liquids and gases. A liquid has intermolecular forces which hold it together so that it has a volume but no definite shape. A gas on the other hand has molecules in motion which collide with each other and tending to disperse so that a gas has no definite shape or volume.

According to Dylan *et al*.,(2012)The Navier-Stokes equations are a set of partial differential equations describing the flow of a viscous, incompressible fluid. They represent one of the most physically motivated models in the field of computational fluid dynamics (CFD) and are widely used to model both liquids and gases in various regimes. It's use widespread in such research fields as astrophysics and geophysics as well as in industries including aeronautical, biomedical, chemical, and mechanical.The Navier-Stokes equations are developed in several commercial software packages which are then used to design mechanical machines such as airplanes, boats, bicycles, and cars. In this context the model has been shown to reproduce accurate models of real world fluid problems of practical importance.

Solutions to the Navier-Stokes equations are in general difficult to obtain. Exact analytical solutions exist typically only for problems where the non-linear terms vanish, such as Poiseuille and Couette flow. Mathematical theory of general solutions to Navier-Stokes equations in 3D is an open problem and one of the Clay Millennium Prize Problems (Dylan *et al*., 2012)

Incompressible flow is an approximation of flow where flow speed is insignificant everywhere compared to the speed of sound of the medium. If incompressible flow is defined in this way, the majority of the fluid and associated flow we encounter in our daily live belong to the incompressible category. For example most of the flow associated with air and water (such as flow related to automobiles, ships, submarines, the water supply through pipes and channels, hydraulic turbines, pumps, low speed airplanes, and trout swimming in mountain streams) and flow of bio fluid (such as blood) are all in the incompressible flow domain.

One of the earliest mathematical models of incompressible flow is the famous equation by Bernoulli, who in 1730 developed the model equation while investigating blood flow (Quartertone, 2000). It is not surprising that scientists have been investigating incompressible flow analytically, experimentally, and computationally ever since. As flow devices become increasingly compact and efficient-pushing the conventional operating envelope-requirements on incompressible CFD tools have become more demanding, just as aerodynamic performance prediction tools require quantitative prediction capability. This trend is reflected in the development of various incompressible flow solution methods and tools, especially, in conjunction with high-fidelity computations using high-end computing facilities.

Owing to the nonlinear nature of the Navier-Stokes equations, their exact solutions are far and few in number. Importance of the exact solutions lies in the fact that they serve as standards for validating the corresponding solutions obtained by numerical methods and other approximate techniques. The inverse or the indirect method is often used to compute these exact solutions (Nem´enyi, 1951).

Finding exact solution using the inverse method consists of making an assumption on the general form of the stream function ψ, involving certain unknown functions, without considering the shape of boundaries of the solution domain occupied by the fluid. We then substitute this assumed form of ψ in the compatibility equation for the stream function to find the unknown functions involved in ψ. This provides the stream function ψ, and subsequently, the fluid velocity components. Once the fluid velocity components are available, then the second step is to compute the fluid pressure field using the component form of the Navier-Stokes equations.This kind of methods with applications in various fields of continuum mechanics is given by (Nem´enyi, 1951). Moreover, a number of reviews on the exact solutions for Navier-Stokes equations have been published,by (Dryden *et al*.,1932; Nem´enyi,1951; Dochan *et al*.,2010; Laglois *et al.*, 2014).Several investigations indicate the existence of slip at the solid boundary (Basset,1961; Nieille *et al.*,1986). As a matter of fact,long back proposed a general boundary condition that permits the possibility of fluid at a solid boundary (Navier,1823). As Chen and Zhu (2008) obtained the analytical solution of Couette- Poiseuille flow of Bingham fluids between two porous parallel plates with slip conditions, analytical solutions of some fully developed flows of couple stress fluid between concentric cylinder with slip boundary condition (Devakar *et al*., 2014).As Song and Chen (2008) investigated the Poiseuille flow of simple fluids in cylindrical nanochannels.

As Hron *et al.*,(2008) established closed form analytical solution for the flows of incompressible non-Newtonian fluids with Navier's slip conditions at the boundary. As it is presented in Master's degree thesis on analysis of circular Couttee flow of incompressible fluid over circular cylinder, generated due to constant density and viscosity using no-slip boundary conditions. Three distinct cases have been identified in Couette flow of incompressible fluid between concentric rotating cylinders.(Temesgen Dagu *et al.*, 2016).

A ccording to Ferr´as *et al*., (2012)presented analytical solutions for both Newtonian and inelastic non-Newtonian fluids with slip boundary conditions in Couette and Poiseuille flows using the Navier linear and non-linear slip laws.While for the linear slip model it was always possible to obtain closed form analytical solutions, for the remaining non-linear models it is always necessary to obtain the numerical solution of a transcendent equation.The Solutions are included with different slip laws or different slip coefficients at different walls. Without assumption, it would not be possible to obtain a simple solution for Navier–Stoke equations (McDonough, J.M ,2009).

This motivates the researcher to establishe analytical solutions of the Poiseuille flow of an incompressible fluid between a concentric circular cylinder by applying appropriate assumptions subject to slip boundary conditions. The boundary conditions were applied on the boundaries of the outer and inner cylinder. And it was assumed that both inner and outer cylinders are at rest and the flow is driven by constant pressure gradient.

The study was aimed to establish an analytical solution of a Poiseulle flow of incompressible fluid between concentric circular cylinders of Navier-Stokes equation of the form:-

a) The continuity equation expresses conservation of mass which is given as

$$\frac{\partial\rho}{\partial t}+\nabla.(\rho u)=0 \tag{1}$$

b) For incompressible flow this equation reduced to $\nabla.\mathrm{u}=0$, where u is velocity component of a fluid which is a function of (t, x, y, z) and ρ fluid density (John,1996).

c) The momentum equation is a vector equation obtained by applying Newton's law of motion to a fluid element which is given as:

$$\rho\frac{Du}{Dt}=\rho\mathrm{g}-\nabla p+\mu\nabla^2 u \tag{2}$$

Subject to the slip boundary condition and appropriate assumptions.Where, g is gravity, p is fluid pressure and μ is dynamic viscosity (John,1996).

1.2. Statement of the problem

Due to its vast applications in engineering and industry, pressure driven flow or the Poiseuille flow has attracted attention of various researchers. Although a pressure driven flows are unidirectional and have been studied earlier for Newtonian and some non-Newtonian fluids, they still attract special attention in a number of emerging problems (Tang, 2012).

From the above list of literature review, many authors tried to investigated closed analytical solutions of Newtonian and non- Newtonian fluid with slip and no slip condition on horizontal plan poiseuille flow, channel with differant dimenstional. geometry and cross section with linear and non-linear slip laws. But,they are not consider an incompressible fluid between vertical concentric circular cylinders governed by a Navier-Stokes equation with three dimensional geometry. And also Poisseuille flow occurs in the gap between two fixed parallel flow to the walls of concentric circular cylinders subject to slip boundary conditions by applying some appropriate assumptions with constants pressure gradient.

That is ,the aim of this study is to construct analytical solutions of a Poiseuille flow of an incompressible fluid between a concentric circular cylinder by applying some appropriate assumptions.

More specifically, The study was focused on the following problem to:

a) apply some assumptions such as: a flow is parallel flow to the wall ,axisymmetric flow, steady flow, fully developed and Newtonian, gravity and constant pressure gradient those used to in a Poiseuille flow.

b) analyze the relationship between radius and slip length with velocity and volume flow rate profiles in a Poiseuille flow.

c) demonstrate the velocity profile using specific numerical example with MATLB in a Poiseuille flow.

1.3. Objectives of the study

1.3.1. General objective

The general objective of this thesis is to establish an analytical solutions to the governing equations of a Poiseuille flow of incompressible fluid between a concentric circular cylinders.

1.3.2. Specific objectives

The specific objectives of the study are to:

a) apply some assumptions such as: a flow is parallel flow to the wall, steady flow, axisymmetric flow, fully developed and Newtonian , gravity and constant pressure gradient those used to in a Poiseuille flow.
b) analyze the necessary relationship between the volume flow rate and velocities profile in a Poiseuille flow.
c) demonstrate the velocity profile using specific numerical value with MATLAB in a Poiseuille flow.

1.4. Significance of the study

This study may have the following importances :

a) It may provide further understanding for researchers who are interested to make further investigation on the characteristics of the Navier-Stokes equation.
b) It develop the research skill of the researcher in the area of applied mathematics.

1.5. Delimitation of the Study

The study is delimited to the governing partial differential conservation equations for natural convection and focus only on the some assumptions to constructed analytical solutions for a poiseuille flow of an incompressible fluid between two concentric circular cylinders.

1.6. Definition of Important Terms:-

Navier-Stokes equations are a set of partial differential equations describing the flow of a viscous, incompressible fluid.

Incompressible fluid :**Incompressible fluid** is one in which the fluid is constant danesity when it is subjected to pressure-gradients.

Fully-developed flow refers to the flow in a region far enough from the entrance that the flow is purely axial. As a result, the velocity distribution in the tube is fixed .

Real fluid: A fluid which possesses viscosity. All the fluids, in actual practice, are real fluids..

Newtonian fluids: A real fluids in which shear stress is directly proportional to the rate of shear strain .

Boundary condition: Conditions for the velocity components of a fluid when it makes contact with a solid surface.

Non-Newtonian fluids: A real fluids in which the shear stress is not proportional to the rate of shear strain.

Inviscid flow: A flow in which the effect of viscosity is negligible.

Stream line: A path in a steady flow field along which a given fluid particle travels.

Laminar flow: An organized flow field that can be described with stream line.

Compressible fluid: A compressible fluid is one in which the fluid density changes when it is subjected to high pressure-gradients.

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CHAPTER TWO

2. LITERATURE REVIEW

2. 1. Plane Poiseuille Flow (channel flow)

As McDonough (2009) the exact solution to the Navier–Stoke equations we consider is plane Poiseuille flow.This is a pressure-driven flow in a duct over a finite length L, but of infinite extent in the z direction, as depicted in Fig.1.For the flow as shown we assume $p_1 > p_2$ with p_1 and p_2 given, and that pressure is constant in the z direction at each x location

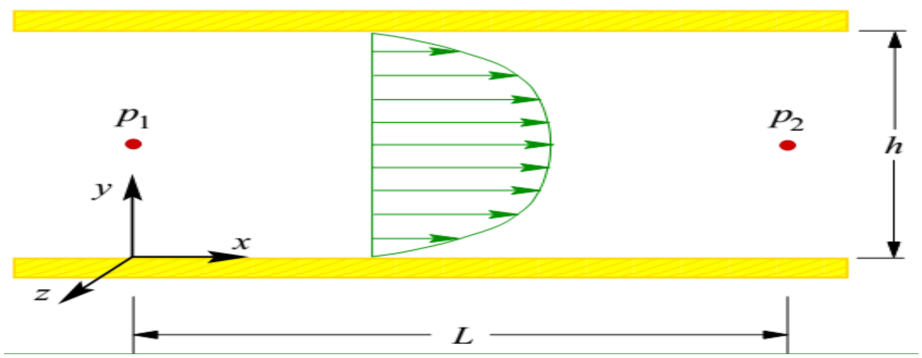


Fig. 1.Coordinate system for plane poiseuille flow

Assume the flow to be steady, that body forces of gravity are negligible, and velocity does not change in the x direction. It is not obvious that this last assumption should hold because pressure is changing in the x direction; but it will be apparent that it leads to no physical or mathematical inconsistencies, and without the assumption it would not be possible to obtain a simple solution as we will do.

As McDonough (2009).In particular,since the flow varies only in the y direction, the continuity equation collapses to $\boldsymbol{v}_y = 0$ from which it follows that $v \equiv 0$ must hold. The y-momentum equation becomes simply $p_y = 0$,which implies that the pressure **p** does not depend on y. Similarly, w is stand for z – momentum direction is zero at both the top and bottom, plates, and the z-momentum equation can again be reduced to $\boldsymbol{w}_{zz} = 0$ by utilizing the preceding assumptions and results, it follows that $\boldsymbol{w} \equiv 0$.

Now applying the steady-state assumption with $\boldsymbol{v}$ = 0 and $\boldsymbol{w}$ = 0 and with the assumption that the flow velocity does not vary in the x direction, we have $\boldsymbol{u}_x = 0$ and $\boldsymbol{u}_{xx} = 0$. Next we note, since $\boldsymbol{u}$ is independent of x and z, that $\boldsymbol{u} = \boldsymbol{u}$ (y) only. This in turn implies that p_x cannot depend on x and must therefore be constant. We can express this constant as

$$P_x = \frac{\Delta P}{L} = \frac{P_1 - P_2}{L} \tag{3}$$

$$\boldsymbol{u}_{yy} = \frac{1}{\boldsymbol{\mu}}\frac{\Delta P}{L} \tag{4}$$

The boundary conditions to be provided for this equation arise from the no-slip condition on the upper and lower plates. Hence, $\boldsymbol{u}(0) = 0$ and $\boldsymbol{u}(h) = 0$.

One integration of equation (4) yields $u_y = \frac{1}{\boldsymbol{\mu}}\frac{\Delta P}{L} y + C_1,$

a second integration gives $u(y) = \frac{1}{2\boldsymbol{\mu}}\frac{\Delta P}{L} y^2 + C_1 y + C_2.$

Application of the first and second boundary condition we obtain $C_2 = 0$ and $C_1 = -\frac{1}{2\boldsymbol{\mu}}\frac{\Delta P h}{L}.$

Substituting of this into the above expression for $u(y)$ result in $u(y) = \frac{1}{2\boldsymbol{\mu}}\frac{\Delta P}{L} y(y-h).$

We can also provide further analysis of the pressure. We noted earlier that P_x could not be a function of x. But this does not imply that p, itself, is independent of x. Indeed, the fact that $P_1 \neq P_2$ requires x dependence. We can integrate Eq. (3) to obtain; $P(x) = \frac{\Delta P}{L} x + C$ and from the fact that $P(0) = P_1 = C$, we see that $P(x) = \frac{\Delta P}{L} x + P_1.$

This is simply the pressure equation between P_1 and P_2 over the distance L.

2.2. Incompressible Flow

As Victor (1962), Mathematically the incompressible flow formulation poses unique issues not present incompressible equations because of the incompressibility requirement.Physically, information travels at infinite speed in an incompressible medium,which imposes stringent requirements on computational algorithms for satisfying incompressibility as well as difficulties in designing downstream boundary conditions.

Differences in various methods of solving the incompressible flow equations originate from differences in strategies of satisfying incompressibility.

Generally,two different approaches are used in solving viscous incompressible flow equations that is incompressible Navier-Stokes equations.The first is based on satisfying the incompressibility directly. If primitive variables, that is, pressure and velocity are chosen, pressure is used as a mapping parameter to satisfy incompressibility. This class of methods is generally known as the “pressure-based” method. Instead of pressure and velocity derived quantities such as stream function-vortices and vortices-velocity can be used, resulting in different sets of governing equations. For general three-dimensional applications, however, the primitive variable formulation poses the least difficulty in geometry modeling and in setting the boundary conditions. The second approach is based on compressible flow formulation where momentum and continuity equations are coupled through the use of density. Incompressibility is recovered as a limiting case of this formulation.This class of methods is known as the “density-based” method.

According to Dochan *et al.*(2010) the major difference between the incompressible and the compressible Navier-Stokes formulations is in the continuity equation.The incompressible formulation can be viewed as a singular limit of the compressible one, Satisfying the mass conservation equation, therefore, is the primary issue in solving the above equation. Physically, incompressible flow is characterized by elliptic behavior of the pressure waves, the speed of which in a truly incompressible flow is infinite

In realistic 3-D problems, these derived quantities are difficult to define or impractical to use.The primitive 'variable formulation, namely, using pressure and velocities as dependent variables, then becomes very convenient and flexible in 3-D applications. However, in this formulation, mass conservation and its relation to pressure must be properly handled while achieving computational efficiency. Although various techniques have been developed in the past, none have proven to be universally better than the others.

2.3. Poiseuille Flow (circular Pipe Flow)

Consider steady, incompressible, fully developed laminar and viscous flow through a straight circular tube of constant cross-section. This type of flow is called Hagen-Poiseuille flow, or simply Poiseuille flow named in honor of J.L.Poiseuille (1799-1869), Probably the most important exact solution in applied viscous hydrodynamics is the Poiseuille solution for pressure flow through a straight circular pipe of uniform diameter.

Let a cylindrical polar coordinate system be defined with Z-axis along the axis of the pipe, in view of the axial symmetry of the situation, the specific orientation of the $\theta = 0$ direction is unimportant. If the radius of the pipe is denoted by R, the no-slip condition requires;

$$V_r = V_\theta = V_Z = O, \quad at \quad r = R \tag{5}$$

$$V_r = V_\theta = O, V_Z = U(r), P = P(Z) \tag{6}$$

The continuity equationis automatically satisfied, as the momentum equation which reduces to:

$$\mu\left(\frac{\partial}{r\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{\partial v_z}{\partial r}\right) = \frac{\partial p}{\partial z} \tag{7}$$

As for the flow between plates, both sides must be equal to the same constant,say G.With the boundary conditions (5) and the restriction that the velocity be finite at the tube axis for unsteady flow of motion.(Laglois and Deville ,2014).

Poiseuille flow is pressure-induced flow (Channel Flow) in a long duct, usually a pipe. Specifically, it is assumed that there is Laminar Flow of an incompressible Newtonian fluid of viscosity (μ) induced by a constant positive pressure difference or pressure drop Δp in a pipe of length L and radius R << L.By a pipe is meant a right circular cylindrical duct that is a duct with a circular cross section normal to its axis or generator.Because of the geometry, Poiseuille flow is analyzed using cylindrical polar coordinates (t,r,θ,z)with origin on the center-line of the pipe entrance and z-direction aligned with the centerline(see Fig 2).

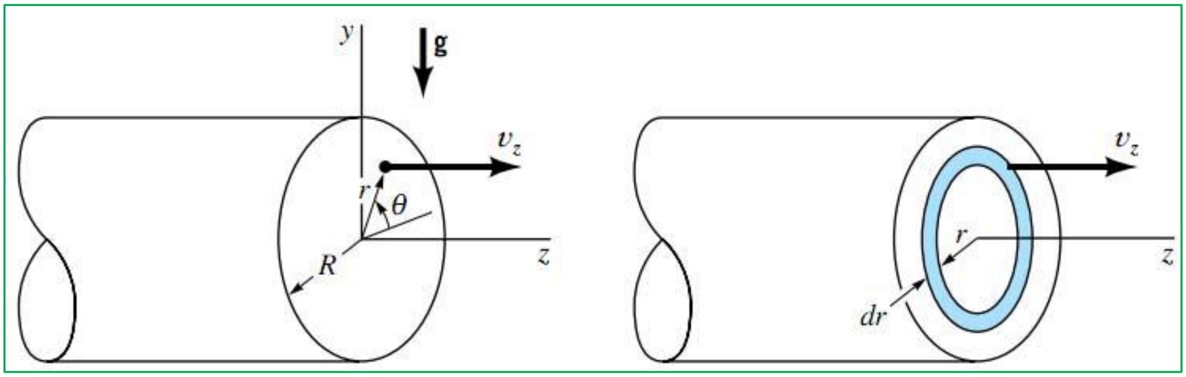


Fig.2 Coordinate system for poiseuille flows   Fig3. Flow through differential annular ring

Thus the axial velocity profile is parabolic (fig.3).

$$U_Z = \frac{\Delta P(R^2 - r^2)}{4\mu L} \tag{8}$$

The volume rate of flow through the pipe is given by

$$Q = \frac{\pi(R^4)}{4\mu}\frac{\partial P}{\partial Z} \tag{9}$$

We can superimpose a swirl without disturbing the parabolic velocity profile (8). Let us suppose that we rotate the pipe about its own axis, not necessarily with constant angular velocity. The boundary conditions (5) must then be modified:

$$v_r = v_z = 0 \;\; at \;\; r = R \tag{10}$$

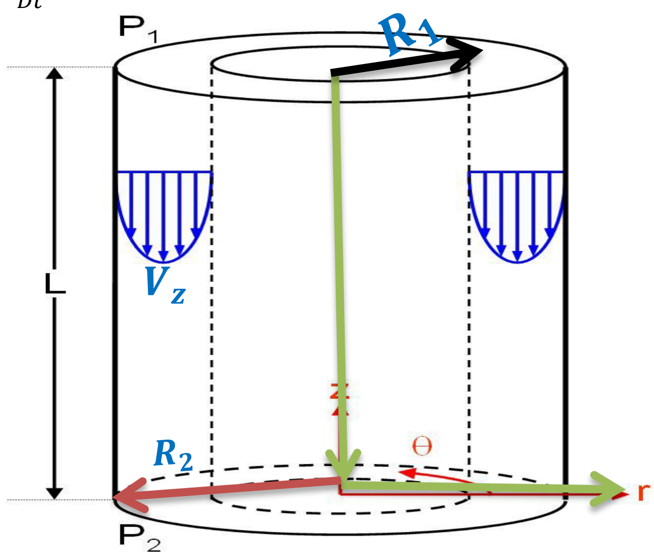
$$V_\theta = R\omega \;\; (t) \;\;\; at \;\;\; r = R. \tag{11}$$

If we set $V_r = 0, V_\theta = u(r,t), v_z = u(r), where\ u(r)$ is the Poiseuiie solutions in equation (8) the continuity equation (1) is automatically satisfied. The pressure field (P= C$-$ G_Z) will not quite suffice. where C is a constant of integration (Laglois and Deville ,2014).

Based on from the above literature review exist related to this study Poisseuille flow occurs in the gap between two fixed parallel flow to the walls of concentric circular cylinders is untouched titel. So that this thesis was conducted to full fill this gap by appliying approperite assumptions subject to slip at boundary condition governed by Navier stokes equations. The inner cylinder of radius R_1 is at rest and the outer cylinder of radius R_2 is stationary. The apparatus has a height L which is much larger than the radius of either cylinder so that the apparatus height is supposed finite. The cylindrical coordinates system (r, θ, z) in the steady state velocity field is such that:

This v_Z (velocity field) is then determined from the integration of the momentum equation:

$$\rho \frac{Du}{Dt} = \rho g - \nabla p + \mu \nabla^2 u \tag{12}$$



Figur.4.a Poiseuille flow between concentric cylinder

According to Laglois and Deville (2014), the motion of fluid contained between two concentric circular pipes of constant radii R_1 and R_2 with $R_1 < R_2$ is as indicated in Fig.4. The pipes rotate about their common axis with constant angular velocities Ω_o and Ω_I respectively. In addition the pipes may translate steadily, parallel to their common axis; let us say that the outer pipe stationary with velocity U relative to the inner. We define a cylindrical coordinate system (r,θ,z) which fixed with the inner pipe but does not rotate with it. The z-axis lies along the common axis of the pipes; because of the axial symmetry, the orientation of the $\theta = 0$ axis is unimportant. Since the cylindrical coordinate system is not accelerated, the fluid motion is governed by the continuity equation and momentum equation. The no-slip condition requires that:

$$V_r = V_Z = 0; V_\theta = R_1 \Omega \;\; at \;\; r = R_1 \;\; and$$

$$v_r = 0, v_\theta = R_1 \Omega_o, v_z = U \text{ at r} = R_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(13)}$$

CHAPTER THREE

3. METHODOLOGY

3.1. Study Site and Period

The study was conducted in Jimma University College of Natural Sciences Department of Mathematics from November 2016 G.C to October 2017 G.C.

3.2. Study Design

This research is conducted by using both analytical and experimental approaches .

3.3. Sources of Information

To complete this study, the researcher was used related reference books, internet , Journals or published article as source of necessary information.

3.4. Procedure of the Study

The study was conducted based on the general procedure for solving each problem involves the following steps ;

a) Reasonable assumptions are made to simplification; That is,assumed to be the parallel flow to the wall with the slip on boundary condition at 0 = $k_o < k_1 < k_2$; $R_1 \leq r \leq R_2$; $\frac{R_2-R_1}{L} \ll 1$ and k ≤ 2, steady flow, fully developed laminar and Newtonian with constant density, viscosity and pressure gradient those used in Poiseuille flow.(where k_i is represents the slip length,R_1, R_2 $and\ r\ is\ radius\ and\ L\ is\ length\ of\ a\ cylindes$)

b)The equations of motion both mass (continuity) and momentum balances have been written and simplified according to the assumptions

c) The simplified equation has been integrated in order to obtain expressions for the dependent variables such as velocities and the volume flow rate those used in Poiseuille flow.

d) The constants appearing in the previous step have been evaluated useing to the boundary conditions.

e)Analytical solutions for the volume flow rate and velocities profile have been constructed.

f)A sketch is produced for each using MATLAB, with different numerical values.

CHAPTER FOUR

4. RESULTS AND DISCUSSIONS

4.1. Discussions

Finding analytical solutions of the Navier-Stokes equations is difficult mathematically due to the nonlinear character of the equation. However, it is possible to find analytical solutions in a certain particular cases, generally when the nonlinear convective terms vanish naturally.

In order to construct analytical solution of a Poiseuille flow of incompressible fluid between two concentric circular cylinder subject to slip boundary condition by employing (using) Navier-stoke equation with constant pressure gradient,the following assumptions were considered.

1. The flow is parallel to the walls so that there is only one non-zero velocity component, namely in the direction of flow or the axial, v_z. Thus, $v_r = v_\theta = 0$.

2.Steady flow is no time dependency

$$\text{Implies that} \quad \frac{\partial(V_\gamma)}{\partial t} = \frac{\partial(V_\theta)}{\partial t} = \frac{\partial(v_Z)}{\partial t} = \frac{\partial(\rho)}{\partial t} = 0$$

3. The flow is axisymmetric flow: no change in the tangential direction or in θ direction.

$$\text{Implies that} \quad \frac{\partial(V_\gamma)}{\partial \theta} = \frac{\partial(V_\theta)}{\partial \theta} = \frac{\partial(v_Z)}{\partial \theta} = 0$$

4. Fully developed flow in the Z - direction or no change in the axial

$$\text{Implies that} \quad \frac{\partial(V_\gamma)}{\partial Z} = \frac{\partial(V_\theta)}{\partial Z} = \frac{\partial(v_Z)}{\partial Z} = 0$$

5. Gravity acts vertically down wards in the Z-direction is constant, so that,

Implies that $g_Z = g,\ g_\gamma = 0, g_\theta = 0$, where, g_Z is force gravity in the Z-direction.

6. The *z*-component of the pressure gradient, $\frac{\partial p}{\partial z}$ is not a function of *r* or θ, so that

$$\Longrightarrow -\frac{\partial p}{\partial z} + \frac{\partial p}{\partial r} = p\ ; \quad \text{where p is a constant pressure gradient}$$

Beased on the above assumption, a fluid flow analysis was made to determine the relationship between radius, slip of length with volume flow rate and velocity by solving the Navier-Stokes equation subject to a geometric boundary condition, which is the interface surface where a fluid contacts a solid object.

Governing Equations

The governing equations are equations of viscose, incompressible fluid flow, known as the Navier-Stokes (N-S) equation. The poiseuille flow in the gap between parallel to the wall of two fixed concentric cylinders was computed by solving the Navier-Stokes equations for incompressible fluid in a three dimensional geometry.

The continuity equation expresses conservation of mass, which is given by;

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho u) = 0 \tag{14}$$

For incompressible flow this equation reduces to $\nabla . u = 0$

Where, u is velocity component of a fluid which is a function of (t,x,y,z) and ρ fluid density. The continuity equation can be written as:-

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0 \tag{15}$$

The momentum equation is a vector equation obtained by applying Newton's law of motion to a fluid element which is given by:-

$$\rho \frac{\partial U}{\partial t} = \rho g - \nabla p + \mu \nabla^2 u \tag{16}$$

Subject to the boundary condition and appropriate assumptions, where, g is gravity, p is fluid pressure, ρ fluid density μ is viscosity and ∇ is operate .

Even though the above equations are complicated to solve, for flow of interest with constant density and constant viscosity some simplification, however, possible. there are three momentum equations for incompressible, Newtonian fluid (Navier- Stokes equation) have three components in cylindrical polar coordinates. One for each of the r,θ,z directions.

As (Bird et al.,1987)equation of motion for incompressible, Newtonian fluid (Navier-Stokes equation) three components in Cartesian coordinates.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \rho g_x$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial P}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + \rho g_y$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial P}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) + \rho g_z\ ,$$

where **u**, **v** and **w** are velocity components in the directions of x, y and z respectively. P is the pressure, $\boldsymbol{\mu}$ is the coefficient of viscosity, ρ is the density of the fluid.

Momentum Equations in cylindrical polar coordinate :-

1)r-Momentum Equation:

$$\rho\left(\frac{\partial(v_r)}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta} - \frac{{v_\theta}^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) =$$

$$P\boldsymbol{g}_r - \frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rV_r)\right)_{\frac{v_r}{r^2}} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2} + \frac{\delta^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}\right] \quad (17)$$

2) θ -Momentum Equation:

$$\rho\left(\frac{\partial(v_\theta)}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) =$$

$$\rho\boldsymbol{g}_\theta - \frac{1}{r}\frac{\partial p}{\partial\theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rV_\theta)\right)_- \frac{v_\theta}{r^2} + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2} + \frac{\partial {v_\theta}^2}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial\theta}\right] \quad (18)$$

3) z-Momentum Equation:

$$\rho\left(\frac{\partial(v_z)}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta} + v_z\frac{\partial v_z}{\partial z}\right) =$$

$$\rho\boldsymbol{g}_z - \frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_Z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2} + \frac{\partial {v_z}^2}{\partial z^2}\right] \quad (19)$$

4.2. Main Results

4.2.1.Analysis of a Poiseuille flow of incompressible fluid between concentric circular Cylinders

Consider Newtonian fluid of constant density ρ and viscosity μ contained between two fixed concentric cylinders with fixed radii R_1 and $R_{2.}$ assum the flow *is* parallel to the wall with slip on boundary condition at $o = k_0 < k_1 < k_2; R_1 \le r \le R_2; \frac{R_1 - R_2}{L} \ll 1, K \le 2$ and the two fixed concentric cylinders are considered long compared to the gap between them which is filled with an incompressible fluid that can be modeled as a Newtonian fluid. We made the following determinations about the flow field in the gap:

(a) The velocity distribution of the fluid in the gap between the two concentric cylinders.
(b) The volume flow rate distribution in the gap between the two concentric cylinders.
(c) The velocity and volume flow rate profiles on the effect of r.

4.2.2. Steady state velocity distribution

In laminar flow, the fluid is expected to travel parallel to the wall between two fixed concentric cylinder with slip at boundary condition. only the Z-components exist.we were verified this by using the following.

Flow conditions; No change in the axial or in z direction is assumed.

In r-direction $\boldsymbol{v}_r = 0$,

In z-direction $\boldsymbol{v}_z =$ function of (t, r, θ, z),

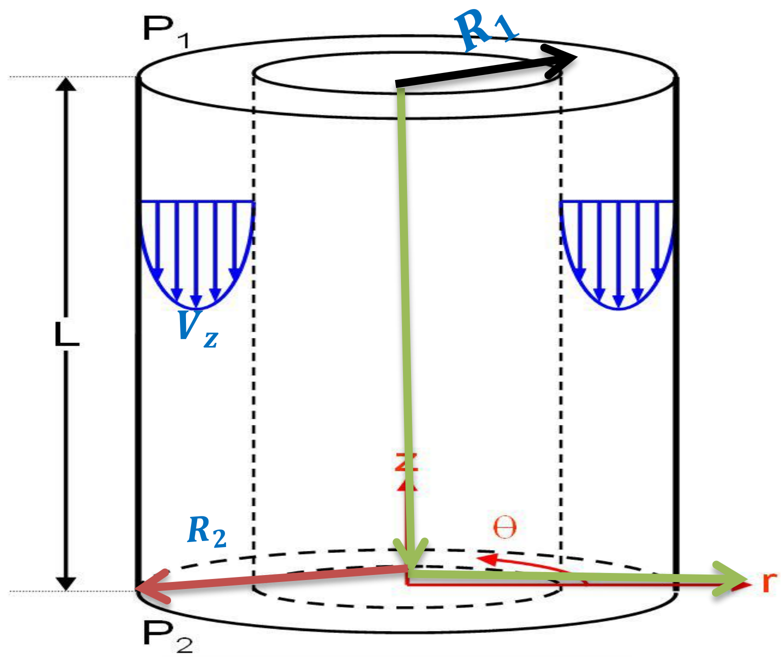
In θdirection $\boldsymbol{v}_\theta = 0$; From continuity equation we have;

$$\frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0 \tag{20}$$

Since,The fluid is Newtonian and incompressible; for which density is a constant and by assumptions $\frac{\partial \boldsymbol{v}_\theta}{\partial \theta}$ =0 and $\frac{\partial \boldsymbol{v}_r}{\partial r}$ =0. Thus,equation (20) become reduced to

$$\frac{\partial v_z}{\partial z} = 0 \tag{21}$$

So, we verified that v_z is independent of distance from the inlet and that the velocity profile $v_z = v_z$ (r) and P= P (z) are appear the same for all value of Z.



Figur.5.a Poiseuille flow between concentric cylinder

Now we are going to use the momentum equation .

Radial component:

$$\rho\left(\frac{\partial(v_r)}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{{v_\theta}^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=$$

$$\rho \boldsymbol{g}_r-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rV_r)\right))_-\frac{v_r}{r^2}+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}+\frac{\delta^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] \quad (22)$$

Based on the above assumptions 1-6, equation (22) is reduced to :

$$\rho \boldsymbol{g}_r-\frac{\partial p}{\partial r}=0 \;\text{;since}\; \boldsymbol{g}_r=0 \quad \Rightarrow \quad \frac{\partial p}{\partial r}=0 \quad (23)$$

From this we observe that p is not a function of r.

Tangential component :

$$\rho\left(\frac{\partial(v_\theta)}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+v_z\frac{\partial v_\theta}{\partial z}+\frac{v_r v_\theta}{r}\right)=$$

$$\rho g_\theta-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rV_\theta)\right)-\frac{v_\theta}{r^2}+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{\partial {v_\theta}^2}{\partial z^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] \quad (24)$$

Based on the above assumptions 1-6 , equation (24) is reduced to:-

$$\rho g_{\theta} - \frac{1}{r}\frac{\partial p}{\partial \theta} += 0 \quad , \text{ since } g_{\theta} = 0 \text{ ,we have } \frac{\partial p}{\partial \theta} = 0 \tag{25}$$

From this we observe that p is not a function of θ.

Axial component:

$$\rho\left(\frac{\partial(v_z)}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = \rho \boldsymbol{g}_z - \frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_Z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial {v_z}^2}{\partial z^2}\right] \tag{26}$$

Based on the above assumptions 1-6, the Axial component of the momentum equation is reduced to :

$$0= \rho \boldsymbol{g}_z + p + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_Z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial {v_z}^2}{\partial z^2}\right]$$

$$\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_Z}{\partial r}\right)\right) =- p - \rho g_z \tag{27}$$

In which total derivatives are used because V_Z dependes only on r, From the assumptions we have made, it is clear that equation (27) can be expressed $-\frac{\partial p}{\partial z} + \frac{\partial p}{\partial r} = p$, since $\frac{\partial p}{\partial r} = 0; -\frac{\partial p}{\partial z} = p$ where p is a constant pressure gradient.

In which both sides of the equation are,two successive integration of equation (27):

$$\int\left(\frac{\partial}{\partial r}\left(r\frac{\partial V_Z}{\partial r}\right)\right)dr = \int \frac{(-P-\rho g_z) r dr}{\mu} \qquad \frac{r dV_Z}{dr} = \frac{-(P+\rho g_z) r^2}{2\mu} + A \tag{28}$$

Once again , Integrating equation (28)with represent to r , we obtain ,

$$\Rightarrow V_Z = \frac{-(P+\rho g_Z)}{4\mu} r^2 + A\ln r + B \tag{29}$$

where A and B are unknown constants.

Case I:

Consider two fixed long concentric cylinders with the inner cylinder radius R_1 and the outer cylinder radius R_2 . Determine the steady state velocity distribution and the volume flow rate of distribution in the field. The geometry suggests the solution to this flow has to be deal with in cylindrical coordinates.

The boundary conditions (BCs)are the fluid slip at the surface of the two cylindrical surfaces.When both the outer and the inner cylinder are fixed,then the boundary conditions are: at $r = R_1$, $\boldsymbol{v}_Z = U$,

at $r = R_2$, $\boldsymbol{v}_Z = U$

The two constants may be evaluated by apply the boundary condition of non-zero velocity at the surface of inner cylinder and the outer cylinder. To determine A and B wit$h$ both slip boundary condition and from equation (29) we can find $\boldsymbol{v}_Z$;

$$\text{At } r = R_1 \;,\; V_Z = U \;. \qquad U = \frac{-(P+\rho g_z)}{4\mu} R_1^{\;2} + A \ln R_1 + B \tag{30}$$

$$\text{At } r = R_2 \;,\; V_Z = U \;. \qquad U = \frac{-(P+\rho g_z)}{4\mu} R_2^{\;2} + A \ln R_2 + B \tag{31}$$

By subtracting equation (30) from equation (31), we get. $A = \frac{(P+\rho g_z)(R_2^{\;2} - R_1^{\;2})}{4\mu \ln\frac{R_2}{R_1}}$ and also

substitute $the\ value\ of\ A$ in to equation (30) ; we get :-

$$B = U + \frac{(P+\rho g_z)R_1^{\;2}}{4\mu} - \frac{(P+\rho g_z)(R_2^{\;2} - R_1^{\;2})}{4\mu \ln\frac{R_2}{R_1}} \ln R_1$$

substitution of those volues for the constants of integration into equation (30)gives the final expression for the velocity profile;

$$V_Z = \frac{-(P+\rho g_z)r^2}{4\mu} + \frac{(P+\rho g_z)(R_2^{\;2} - R_1^{\;2})}{4\mu \ln\frac{R_2}{R_1}} \ln r + U + \frac{(P+\rho g_z)R_1^{\;2}}{4\mu} - \frac{(P+\rho g_z)(R_2^{\;2} - R_1^{\;2})}{4\mu \ln\frac{R_2}{R_1}} \ln R_1$$

$$\Leftrightarrow V_Z = \frac{(P+\rho g_z)}{4\mu}\left[\frac{(R_2^{\;2} - R_1^{\;2})\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}} - (r^2 - R_1^{\;2})\right] + U$$

$$\Leftrightarrow V_Z = \frac{(P+\rho g)}{4\mu}\left[\frac{\left(R_2^2 - R_1^2\right)\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}} - \left(r^2 - R_1^2\right)\right] + k\left(1-\left(\frac{r}{R_1}\right)^2\right) \tag{32}$$

Where $k\,(1-(\frac{r}{R_1})^2) = U$ is velocity specifed ,k is slip length or friction coefficient.

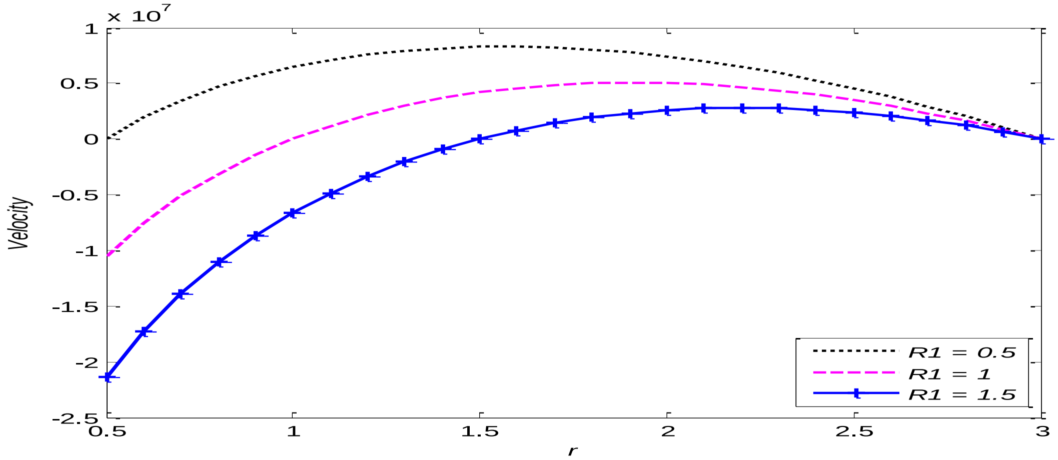


Figure 6:velocity profiles

From Figure (6.), we observed that as $r \to R_I$, $0 \leq k \leq 2$, there is a increases in the velocity and it can also be seen that velocity of the fluid is maximum when the parallel flow of fluid at the centerline their near to slip boundary condition; r $\in$ [1.25 ,2]. On the other hand , as $r \to R_2 and\ 0 \leq k \leq 2$ that the velocity of the fluid is decreases near the slip boundary condition when the radius increases; r $\in$ [2.25 , 3] or the radius decreases; r $\in$ [0.5 ,1]. Also indicate that , decreasing of the gap between concentric cylinders has a decreases effect on the fluid velocity and the decreasing slip boundary has a decreasing effect on the velocity. That is, the more close the fluid slips at the boundary, the less its velocity is affected by the slip length of boundary condition.

Observe that the flow rate through concentric circular cylinders of internal radius *r* and external radius *r* + *dr* is $dQ = V_Z 2\pi r dr$.From the above equation (32) , we get the volume flow rate by integrating the velocity profile using $Q = \int_{R_1}^{R_2} V_Z\, 2\pi r dr$

Then, substituting the velocity profile and integrating gives

$$Q = \int_{R_1}^{R_2} \left[\frac{(\mathbf{p+\rho g_z})}{\mathbf{4\mu}} \left[\frac{(R_2{}^2 - R_1{}^2)\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}} - (r^2 - R_1{}^2) \right] + \boldsymbol{U} \right] 2\pi r dr$$

$$Q = \frac{2\pi(p+\rho g_z(R_2{}^2 - R_1{}^2)}{4\mu\ \ln\frac{R_2}{R_1}} \left[\int_{R_1}^{R_2} \left(\ln\frac{r}{R_2}\right) r dr - \int_{R_1}^{R_2} r^3\, dr + R_1{}^2 \int_{R_1}^{R_2} r dr \right] + \int_{R_1}^{R_2} k\left(1 - \left(\frac{r}{R_1}\right)^2\right) 2\pi r dr$$

After integrating each term by using integration by part ,we can obtain the volume flow rate distribution,which is equals to :-

$$Q = \left(\frac{\pi\left((R_2{}^2 - R_1{}^2\right)}{8}\left[\frac{(p+\rho g_z)}{\mu}\left[(R_2{}^2 + R_1{}^2 - \frac{(R_2{}^2 - R_1{}^2)}{\ln\frac{R_2}{R_1}}\right] + \frac{4k(R_2{}^2 - R_1{}^2)}{R_2{}^2}\right]\right) \tag{33}$$

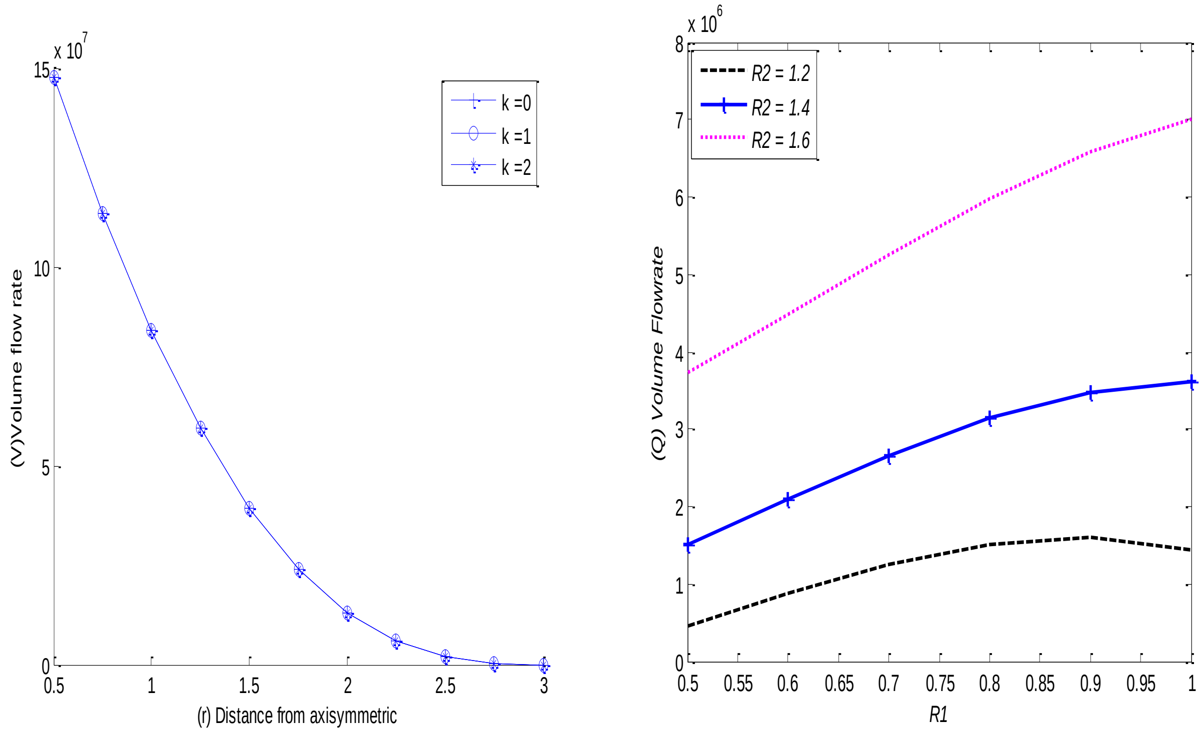


Figure 7: volume flow rate profiles

From Figure 7. it can be observed that the graphs decreasing of the values of r between concentric cylinders has a decreasing effect on the volume flow rate of fluid that near to slip boundary condition from both surface of concentric cylinder at the r $\in$ [2.5 ,3] or r $\in$ [0.5 ,0.75]. It can also be seen from the above figures that the volume flow rate of maximum when the parallel flow of fluid at the centerline of fluid that near to both slip boundary condition at surface of concentric cylinder at r = 1.25,r= 2.25,and at r=1boundary condition.

Case II:

Consider two fixed long concentric circular cylinders with the inner cylinder radius R_1 and the outer cylinder radius R_2. Determine the steady state velocity distribution and the volume rate of flow distribution in the field.

The boundary conditions (BCs) are that the fluid does not slip (no-slip) at the surface of the inner cylinder and the fluid slip at the surface of the outer cylinder. When the outer cylinder and the inner cylinder is fixed, then the boundary condition are:

$$\text{at } r = R_1 , \qquad \boldsymbol{v}_Z = 0.$$

$$\text{at } r = R_2, \qquad \boldsymbol{v}_Z = U$$

The two constants may be evaluated by apply the following boundary condition of zero velocity at the surface of the inner cylinder and the non-zero velocity at the surface of outer cylinder ; From the above equation (29) we can find $\boldsymbol{v}_Z$;

$$\text{no slip BC at r= } R_1 \ , \ \mathrm{V_Z} = 0 \ , \ 0 = \frac{-(\mathrm{p}+\rho\mathrm{g}_z)}{4\mu} {R_1}^{2} + \mathrm{A} \ln R_1 + \mathrm{B} \qquad (34)$$

$$\text{slip BC at r} = R_2 \qquad \mathrm{V_Z} = U \ , \quad U = \frac{-(\mathrm{p}+\mathrm{g}_z)}{4\mu} {R_2}^{2} + \mathrm{A} \ln R_2 + \mathrm{B} \qquad (35)$$

we can obtain the value of A and B; when equation (34) is substracted from equation (35); we can get

; $\mathrm{A} = \frac{(U4\mu+(\mathrm{p}+\rho\mathrm{g}_z)({R_2}^{2} - {R_1}^{2})}{4\mu \ \ln\frac{R_2}{R_1}}$

and also substitute $the\ value\ of\ A$ in to equation (35); we get $the\ value\ of\ B: -$

$$\mathrm{B} = \mathrm{U} - \frac{U \ln R_2}{\ln\frac{R_2}{R_1}} + \frac{(\mathrm{p}+\rho\mathrm{g}_z){R_2}^{2}}{4\mu} - \frac{(\mathrm{p}+\rho\mathrm{g}_z)({R_2}^{2} - {R_1}^{2})}{4\mu \ \ln\frac{R_2}{R_1}} \ln R_2$$

substituting those volues of the constants into equation (29) we get the final expression for the velocity profile:

$$\mathrm{V_Z} = \frac{-(\mathrm{p}+\rho\mathrm{g}_z)\, r^2}{4\mu} + \frac{(U4\mu+(\mathrm{p}+\rho\mathrm{g}_z)({R_2}^{2} - {R_1}^{2})}{4\mu \ \ln\frac{R_2}{R_1}} \ln r + U - \frac{U \ln R_2}{\ln\frac{R_2}{R_1}} + \frac{(\mathrm{p}+\rho\mathrm{g}_z){R_2}^{2}}{4\mu} - \frac{(\mathrm{p}+\rho\mathrm{g}_z)({R_2}^{2} - {R_1}^{2})}{4\mu \ \ln\frac{R_2}{R_1}} \ln R_2$$

$$. \Leftrightarrow \mathbf{V_Z} = \frac{(\mathrm{p}+\boldsymbol{\rho}\mathrm{g}_z)}{\mathbf{4\mu}} \left[\frac{({R_2}^{2} - {R_1}^{2}) \ln\frac{r}{R_2}}{\ln\frac{R_2}{R_1}} - (r^2 - {R_2}^{2}) \right] + \boldsymbol{U} \frac{\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}}$$

$$\Leftrightarrow \mathbf{V_Z} = \frac{(\mathrm{p}+\boldsymbol{\rho}\mathrm{g}_z)}{\mathbf{4\mu}} \left[\frac{({R_2}^{2} - {R_1}^{2}) \ln\frac{r}{R_2}}{\ln\frac{R_2}{R_1}} - (r^2 - {R_2}^{2}) \right] + \left(\mathbf{k}\, (1 - (\tfrac{r}{R_2})^2) \right) \frac{\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}} \qquad (36)$$

where $\boldsymbol{U} = \mathbf{k}\, (1-(\frac{r}{R_2})^2)$, $\boldsymbol{U}$ is aconstant or velocity specifed and **k** is slip length or friction coefficient.

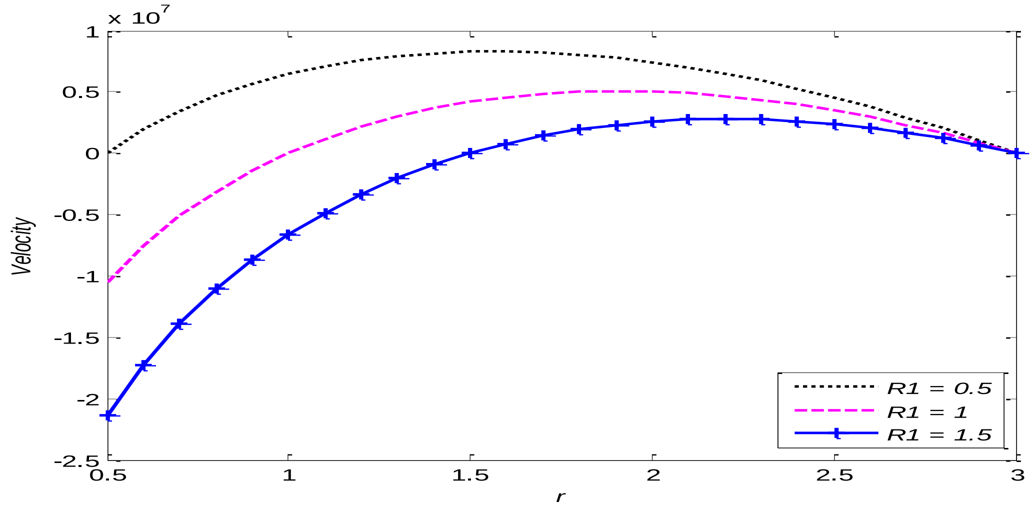


Figure 8: velocity profiles

From Figure 8; we observed that as $r \to R_I$, $r \to R_2$ and $0 \leq k \leq 2$,there is a increases in the velocity and it can also be seen from the figures that velocity of the fluid is maximum at the centerline of the parallel flow of fluid to no-slip and slip boundary condition of the inner and outer surface of concentric cylinder respectively when r $\in$ [1.25 , 2].

On the other hand ,the velocity of the fluid decreases near to slip and no-slip boundary condition of the inner and outer surface of concentric cylinder respectively.That is when r $\in$ [2.25 ,3] or r $\in$ [0.5 ,1] ; The graphs also indicate that decreasing of the gap between concentric cylinders has a decreasing effect on the fluid velocity.

The decreasing slip length of boundary condition in motion of parallel flow of fluid has a decreasing effect on the velocity.That is, the more close the fluid slip and no-slip boundary condition at the surface of concentric cylinder. the less its velocity is affected by the slip length of boundary condition.

Next, we obtain the total volume flow rate distribution is from equation(36) That is

$$\boldsymbol{Q} = \int_{R_1}^{R_2} V_Z \, dA = \int_{R_1}^{R_2} V_Z \, (2\pi r) dr$$

$$Q = \int_{R_1}^{R_2} \left[\frac{(p+\rho g_z)}{4\mu} \left[\frac{(R_2{}^2 - R_1{}^2) \ln\frac{r}{R_2}}{\ln\frac{R_2}{R_1}} - \left(r^2 - R_2{}^2\right) \right] + \mathbf{k}\,(1-(\frac{r}{R_2})^2) \frac{\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}} \right] 2\pi r dr$$

Now, integrating each terms by using integration by parts

$$Q = \frac{2\pi(p+\rho g_z(R_2{}^2 - R_1{}^2)}{4\mu \ln\frac{R_2}{R_1}} [\int_{R_1}^{R_2} (\ln\frac{r}{R_2}) r dr - \int_{R_1}^{R_2} r^3 \, dr + R_2{}^2 \int_{R_1}^{R_2} r dr] + k \int_{R_1}^{R_2} (1-(\frac{r}{R_2})^2) \frac{\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}} 2\pi r dr$$

$$\Leftrightarrow Q=\left[\frac{\left(\pi(p+\rho g_z)({R_2}^2-{R_1}^2)\right)}{8\mu}\left[\left({R_2}^2+{R_1}^2-\frac{({R_2}^2-{R_1}^2)}{\ln\frac{R_2}{R_1}}\right]\right]+\frac{K{R_2}^2\pi}{2}-\frac{-\pi({R_2}^2-{R_1}^2)(3{R_2}^2-{R_1}^2)}{8({R_2}^2\ln\frac{R_2}{R_1}}\right)$$

$$Q=\left(\frac{(\pi)({R_2}^2-{R_1}^2)}{8}\right)\left(\left(\frac{(p+\rho g_z)}{\mu}\right)\left[\frac{({R_2}^2-{R_1}^2)}{\ln\frac{R_2}{R_1}}+{R_2}^2+{R_1}^2\right]+\left[k\frac{3{R_2}^2-{R_1}^2}{{R_2}^2\ln\frac{R_2}{R_1}}\right]\right)+\frac{K{R_2}^2\pi}{2} \qquad (37)$$

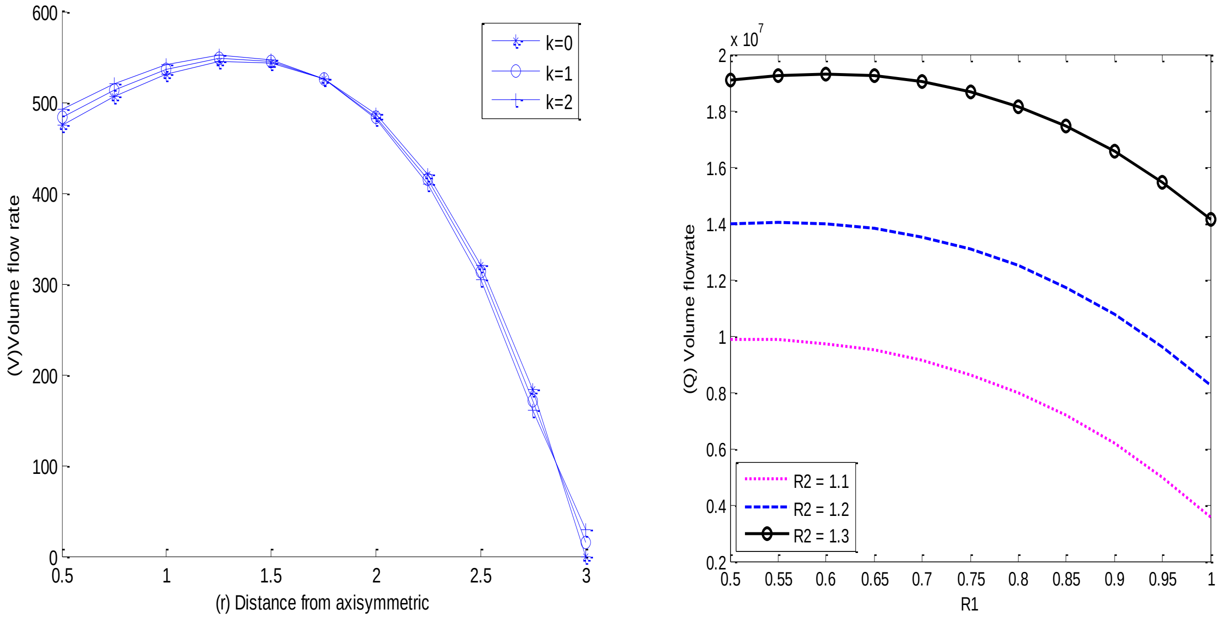


Figure 9: volume flow rate of profiles

From Figure 9; it can be observed that the graphs increases as the values of r between concentric cylinders has a decreasing effect on the volume flow rate of fluid that near to no-slip and slip boundary condition at surface of concentric cylinder r ∈ [0.5, 0.75) or r ∈ [2, 3]; It can also be seen from the above figures that the volume flow rate of fluid maximum (increases) when the parallel flow of fluid at the centerline of fluid that near to no-slip and slip boundary condition at surface of concentric cylinder r ∈ [0.75,1.75]

Case III:

Consider two fixed long concentric circular cylinders with the inner cylinder radius R_1 and the outer cylinder radius R_2 . Determine the steady state velocity distribution and the volume rate of flow distribution in the field.

The boundary conditions (BCs) are that the fluid slip at the surface of the inner cylinder surfaces and the fluid does not slip (no-slip) at the surface of the outer cylinder.When the outer cylinder and the inner cylinder is fixed, then the boundary condition are:

$$\text{at } r = R_1, \qquad \boldsymbol{v}_Z = U,$$

$$\text{at } r = R_2, \qquad \boldsymbol{v}_Z = 0$$

The two constants may be evaluated by apply the boundary condition of non-zero velocity at the inner cylinder and the zero velocity at the outer cylinder From the velocity profile result of equation (29) and we can find $\boldsymbol{v}_Z$

slip BC at r=R_1 ,$V_Z = U$. $$U = \frac{-(p+\rho g_z)}{4\mu} {R_1}^2 + A \ln R_1 + B \qquad (38)$$

no slip BC at $r = R_2$,$V_Z = 0$. $$0 = \frac{-(p+\rho g_z)}{4\mu} {R_2}^2 + A \ln R_2 + B \qquad (39)$$

by substructing equation (38) from equation (39), we can obtain $A = \frac{-(U4\mu - (p+\rho g_z)({R_2}^2 - {R_1}^2)}{4\mu \ln\frac{R_2}{R_1}}$ and

also substitute $the\ value\ of\ A$ in to equation (38) ; we get $the\ value\ of\ B$:

$$B = U + \frac{(p+\rho g_z){R_1}^2}{4\mu} + \frac{U4\mu-(p+\rho g_z)({R_2}^2 - {R_1}^2)}{4\mu \ln\frac{R_2}{R_1}} \ln R_1$$

by substitution those values of the constants of into equation (29),the final expression for the velocity profile

$$V_Z = \frac{-(p+\rho g_z)r^2}{4\mu} - \frac{(U4\mu-(p+\rho g_z)({R_2}^2 - {R_1}^2)}{4\mu \ln\frac{R_2}{R_1}} \ln r + \frac{(p+\rho g_z){R_1}^2}{4\mu} + U + \frac{U4\mu-(p+\rho g_z)({R_2}^2 - {R_1}^2)}{4\mu \ln\frac{R_2}{R_1}} \ln R_1$$

$$. \Leftrightarrow V_Z = \frac{(p+\rho g_z)}{4\mu}\left[\frac{({R_2}^2 - {R_1}^2)\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}} - (r^2 - {R_1}^2)\right] - U\frac{\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}}$$

$$\Leftrightarrow \mathbf{V_Z} = \frac{(\mathrm{p}+\boldsymbol{\rho} \mathrm{g}_Z)}{\mathbf{4\mu}}\left[\frac{({R_2}^2 - {R_1}^2)\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}} - (r^2 - {R_1}^2)\right] - \frac{\boldsymbol{K}({R_1}^2 - r^2)}{{R_1}^2}\frac{\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}} \qquad (40)$$

Where $\boldsymbol{U}$ = **k** $(1-(\frac{r}{{R_1}^2})^2)$ is aconstant or velocity specifed and **k** is slip length or friction coefficient.

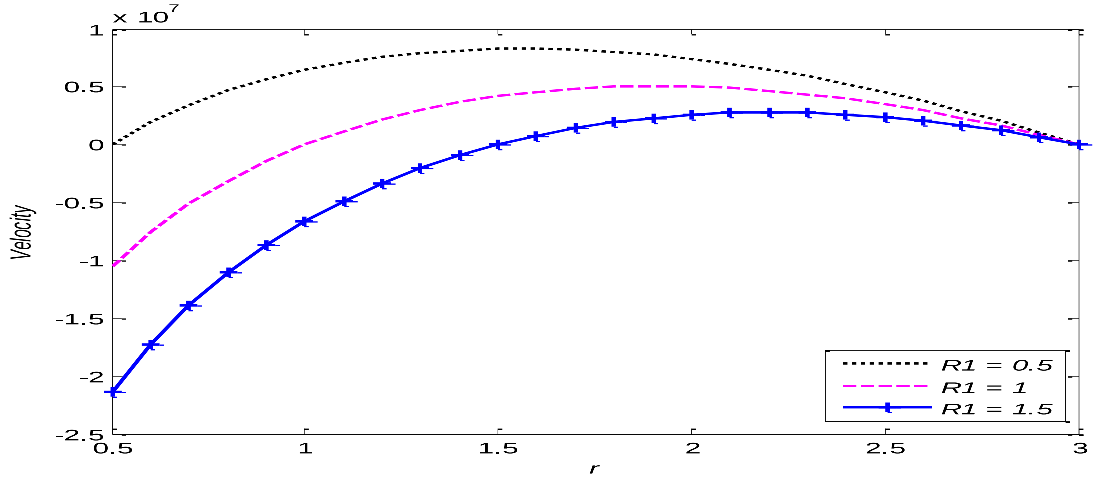


Figure10: velocity profiles

From Figure 10, we observed that as $r \to R_I$, $r \to R_2$ and $0 \leq k \leq 2$,there is a increases in the velocity and it can also be seen that velocity of the fluid is maximum at the centerline of the parallel flow of fluid to slip and no- slip boundary condition of the inner and outer surface of concentric cylinder respectively when r $\in [1.25 , 2]$.

On the other hand ,the velocity of the fluid is decrease near to slip boundary condition and no-slip boundary condition of the inner and outer surface of concentric cylinder respectively,when the radius r $\in [2.25 ,3]$ or the radius r $\in [0.5 ,1]$.

Also indicate that, decreasing of the gap between concentric cylinders has a decreasing effect on the fluid velocity. The decreasing slip length of boundary condition in motion of parallel flow of fluid has a decreasing effect on the velocity. That is, the more close the fluid slip and no-slip boundary condition at the surface of concentric cylinder, the less its velocity is affected by the slip length of boundary condition.

Frome the above velocity profile equation (40),the total volume flow rate distribution between the two fixed concentric circular cylinders :

$$\boldsymbol{Q} = \int_{R_1}^{R_2} \mathrm{V_Z}\, dA = \int_{R_1}^{R_2} \mathrm{V_Z}(2\pi) r dr$$

Now, integrating each terms by using integration by parts

$$Q=\int_{R_1}^{R_2}\left[\frac{(\mathrm{p}+\boldsymbol{\rho}\mathrm{g}_z)}{\mathbf{4\mu}}\left[\frac{(R_2{}^2-R_1{}^2)\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}}-(r^2-R_1{}^2)\right]-\frac{\boldsymbol{K}(R_1{}^2-r^2)}{R_1{}^2}\frac{\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}}\right]2\pi r dr$$

$$\Leftrightarrow Q=\frac{2\pi(\mathrm{P}+\rho\mathrm{g_z}(\mathrm{R_2}{}^2-\mathrm{R_1}{}^2)}{4\mu\ \ln\frac{\mathrm{R_2}}{\mathrm{R_1}}}\left[\int_{\mathrm{R_1}}^{\mathrm{R_2}}(\ln\frac{\mathrm{r}}{\mathrm{R_1}})\mathrm{rdr}-\int_{\mathrm{R_1}}^{\mathrm{R_2}}\mathrm{r}^3\,\mathrm{dr}+\mathrm{R_1}{}^2\int_{\mathrm{R_1}}^{\mathrm{R_2}}\mathrm{rdr}\right]-\int_{\mathrm{R_1}}^{\mathrm{R_2}}\frac{\mathbf{K}(\mathrm{R_1}{}^2-\mathrm{r}^2)}{\mathrm{R_1}{}^2}\frac{\ln\frac{\mathrm{r}}{\mathrm{R_1}}}{\ln\frac{\mathrm{R_2}}{\mathrm{R_1}}}2\pi\mathrm{rdr}$$

After integration and some algebraic manuplations, we get the volume flow rate distribution between the two fixed concentric circular cylinders.

$$Q=\left(\frac{\pi(\mathrm{p}+\rho\mathrm{g}_z)(R_2{}^2-R_1{}^2)}{8\mu}\left[R_2{}^2+R_1{}^2-\frac{(R_2{}^2-R_1{}^2)}{\ln\frac{R_2}{R_1}}\right]+\frac{k\pi(R_2{}^2-R_1{}^2)}{2\ln\frac{R_2}{R_1}}\left(\frac{(3R_1{}^2-R_2{}^2)}{4R_1{}^2}\right]-\frac{KR_2{}^2\pi}{2}\right)$$

$$Q=\frac{\pi(R_2{}^2-R_1{}^2)}{8}\left[\frac{(\mathrm{p}+\boldsymbol{\rho}\mathrm{g}_z)}{\boldsymbol{\mu}}\left[R_2{}^2+R_1{}^2-\frac{(R_2{}^2-R_1{}^2)}{\ln\frac{R_2}{R_1}}\right]+\left(k\frac{(3R_1{}^2-R_2{}^2)}{R_1{}^2\ln\frac{R_2}{R_1}}\right]-\frac{KR_2{}^2\pi}{2} \qquad (41)$$

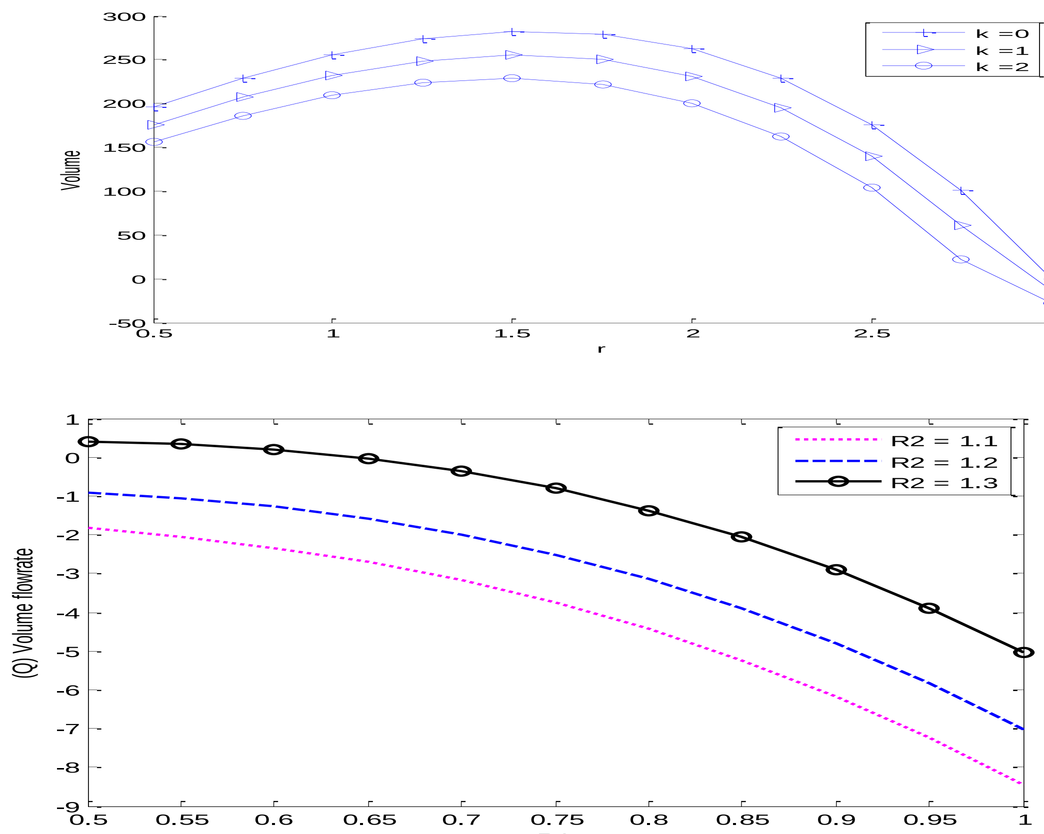


Figure 11: Volume flow rate profiles

From Figure 11: we can observed that for all values of k $\in$ [0 , 2] ,decreasing from the middle value of R_1 and R_2 with $R_1 < R_2$ toR_1 or increasesing of the values of r to R_2 decreases the volume flow rate of fluid in concentric cylinder with stated boundary condition. The figure also illustrates the volume flow rate of fluid is maximum when r is at near to the center line of the parallel flow of fluid that is near to the mid point R_1 and R_2 it when $r \in [1, 2]$

CHAPTER FIVE

5. CONCLUSIONS AND FUTURE SCOPE

5.1. Conclusion

In this study three distinct cases have been considered to establish analytical solution a Poiseuille flow of incompressible fluid between two vertical fixed concentric cylinders subjact to slip boundary conditions. In each case the velocity and volume flow rate of the fluid were analyzed by considering different slip length. Moreover,the effect of r velocity and volume flow rate the fluid between two concentric cylinders were discussed.

From the result obtained we observed that is.

In a Poiseuille flow of incompressible fluid, the velocity field involves only v_z and its magnitude is a function of the axial coordinate. Both the slip and no-slip boundary condition have an effect the velocity and volume flow rate of the fluid in the concentric circular cylinders. This implies the velocity and volume flow rate of the fluid depenes on the value of r and the value of slip length k.

It can also be seen from the discussion that velocity of the fluid is maximum at the centerline of the parallel flow of fluid to near slip boundary condition and no- slip boundary condition of the inner and outer surface of concentric cylinder respectively.

The decreasing slip length of boundary condition in motion of parallel flow of fluid has a decreasing effect on the velocity. That is, the more close the fluid slip and no-slip boundary condition to the surface of concentric cylinder, the less its velocity is affected by the slip length of boundary condition. In general in the three cases ,we observed from the figure as the velocity of the fluid decrease between the two concentric cylinder , the volume flow rate also decreases depending on different Slip length and the radius between two two concentric cylinder.

In all of the three cases, the governing equations give rise to the Navier-Stokes differential equations. The analytical solutions are obtained based on the stated assumptions and are simplified using slip boundary condition. The volume flow rate and velocity profiles for different slip length and interval of numerical values of r are presented.

5.2. Future Scope

Incompressible fluid flow between concentric circular cylinders has received notable attention in fluid mechanics, applied mathematics and chemical engineering.But in this thesis we study the volume flow rate and velocity of the fluid between concentric circular cylinders with constant pressure gradiante. So, it is recommended that the subsequent researcher can provide additional information on a Poiseuille flow of incompressible fluid between two fixed concentric circular cylinders driven by different pressure gradiante and Visicosity in three dimensional geometry .

References

Badur J., Karcz M., Lemanski M. and Nastałek L.(2011).Foundations of the Navier-Stokes boundary conditions in fluid mechanics,Transactions of the Institute of Fluid-Flow Machinery no.123, 2011, pp.3–55

Bansal,Dr.R.K.(2005).ATextBook of Fluid Mechanics and Hydraulic Machines Scanned by Fahid PDF created by AAZSwapnil.

Basset, A.B. (1961). Treatise on Hydrodynamics, Dover, New York, Ny,USA, vol 2. Bulletin National Research Council. vol. 84, pp. 155–332.

Bird, R.,Armstrong, C. &Hassager,O. (1987). *Dynamics of Polymeric Fluids, Fluid Mechanics, Wiley: NY.* Volume 1

Chen, Y.-L&Zhu, K. Q. (2008). "Couette-Poiseuille flow of Bingham fluid between two porous parallel plates with slip condtions,"Journal of Non-Newtonian Fluid Mechanics,vol.153, no.1,pp.1-11.

Devakar M., Sreenivasu D., and Shankar B.(2014)."Analytical solutions of some fully developed flows of couple stress fluid between concentric cylinders with slip boundary conditions," Int. J. Eng. Math., 2014, pp. 1-13.

Dochan, K.,Cetin, K.& Chang, S.K.nd (2010). "Computational Challenges of Viscous Incompressible Flow,"NASA–Ames Research Center,Moffett Field.

Dryden, F., Murnaghan, D.& Bateman, F.D. (1932).Report of the committee on Hydrodynamics.

Dylan Nelson, Ariana Minot, Philip Mocz, Pengfei Liu ,(2012). AM274 Final Project: Continuous Galerkin Navier-Stokes in 2D. pp.1.

Ferr´as, L. L.,Oberga, M.N& Pinho,F.T. (2012). "Analytical solutions for Newtonian and inelastic non- Newtonian flows with wall slip,"Journal of Non-Newtonian Fluid Mechanics, vol.175-176, pp.76-88.

Hron,J.,Le-Roux,C.,Malek,J & Rajagopal,K.R.(2008). "Flows of incompressible fluids Subject to Navier's slipon the boundary,"Computers andMathematics with Applications,vol.56,no.8, pp. 2128-2143.

John, K.H. (1996). Nonlinear Evolution Equations1.Pp. 3-6

K Karunakar Reddy ,Prof. Sanjay Mittal:Analytical and Numerical Solutions of FlowProblems, IIT Kanpur Indian Institute of Technology, Guwahati9th Indo-German Winter Academy Dec 11-17, Pune .

Laglois, W.E, & Deville, M.O.(2014). Slow viscous flow http://www.springer.com/978-3-319- 03834-6, pp. 116-126.

McDonough, J.M. (2009).Lectures in Elementary Fluid Dynamics. pp. 122-126

Navier, C.L.M.H.(1823).“Memoirsdel’Academie,”Royale des sciences de l’Intitut de France,

vol.1, pp. 414–416.

Navier,C.L.M.H. (1827) Memoire sur les lois du mouvement des fluids, Mem. Acad. Roy. Sci. Inst. de France, vol. 6, pp. 389–440.

Nem´enyi, (1951). Recent developments in inverse and semi inverse methods in the Mechanics of Continua Advances in Applied Mechanics, vol. 2, pp. 123–151.

Nieille,M.E.,Ranger,O.K.&Brenner,H.B.(1986).”Slip at the surface of a translating-rotating Spherical bisected by a free surface bounding a semi-infinite viscous: removal of the Contact line singularity,” physics of Fluids, vol.29, pp.913-924.

Quarteroni, A.(2000). "Modeling the Cardiovascular System: A Mathematical Adventure –Part I," SIAM News, Vol. 34, No. 5.

Robert P. Hesketh, (2008). Flow Between Parallel Plates is Modified from the COMSOL ChE Library module Chemical Engineering, Rowan University rev 10/13/08

Song, X. and Chen, J.K.(2008).“A comparative study on poiseuille flow of simple fluids through cylindrical and slit-like nanochannels,”International Journal of Heat and Mass Transfer,vol.51, no.7-8, pp.177 0–1779.

Tang, H. S.(2012). “Analysis on creeping channel flows of compressible fluids subject to wall slip,” Rheologica Acta, vol. 51,no. 5, pp. 421–439.

Temesgen Dagu,Mitiku Daba,Kefyalew Hailu.(2016).Analysis of acircular couette flow of incompressible fluid over acircular cylinder.(unpublished)

Victor, L.S.(1962) .Fluid Mechanics, McGRAW-HILL BOOK COMPANY, INC, pp. 174-177