ANALYSIS OF INTERSTELLAR MAGNETIC FIELD IN EARTH'S MAGNETOSPHERE BY LINEARIZED MHD EQUATIONS

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To the Lord of Lords

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Abstract

Life on earth without sun is impossible. So always the study of earth sun coupling remains forever as long as humans exist. Fortunately, the technological advancement we have at hand tody has helped to advance research in this field. However, a number of theoretical principles need to be worked out. For example, the solar magnetic-storm on earth that causes electronic interruption etc need to be solved. One of the difficulties is that the complex nature of the MHD equations involving non-linear differential equations. If there is any, most works assume the collision-free system with simplify boundary conditions. Motivated by this scientific background, we did study the solar wind - earth's magnetosphere interactions through the interstellar magnetic field dynamics in the linearized MHD equations.The results we derived here is in conformity to conclude the existing models indeed work at large but has limitations to include very fine local effects like magnetic storms and the observed relativistic particle anomalies in ionosphere. So there is a need of advanced analytical and computational works to extract more accurate data from the full MHD equations.medskip Key words: IMF, Ionosphere, Magnetosphere, MHD, magnetopause, Sun,magnetic reconnection.

Key words: IMF, Ionosphere, Magnetosphere, MHD, Space Weather, Sun.

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General Introduction

I. Background

Studying the Sun is important to understand the various ways in which it affects our terrestrial environment. For more than a century, scholars studying the Sun and Sun-Earth connection have identified different regions within the sun, the solar wind and its mechanism through interplanetary space to the Earth's magnetosphere and regions of the Earth's atmosphere. Today, we have more sophisticated ground-based and space-based observations to study solar activity and its effect on surrounding environment. And on the other hand, there is a great deal of theoretical works going on with a plethora of models to fit the observations[1]. Yet, there are debates over the theoretical works in matching the observations. One of the key points contributing to the debates is the difficulty of working out the full Magnetohydrodynamic (MHD) equations that includes all effects like collisions and relativistic effects. So motivated with this scientific rationale, we are interested to work on MHD interactions of solar-terrestrial environment from the sun through to the earth's ionosphere which is crucial for how to control, adjust and get ready to live when it comes to life matter on earth. Accordingly we have dynamical equations such as the Interrestrial Magnetic Field (IMF) from MHD system that includes collision effect. Then, the physical contents and implications of the analytically derived equations are being analyzed and discussed with some numerical values setting appropriate boundary conditions.

The outline of the work is organized as follows: IN chapter one we give the introductory, the review literature, statement of the problem,general and specific objectives and the methodology we used. In chapter two we introduce the basic physics and principles of plasma fluid. Here, we do derive the fundamental plasma equations from Boltzmann's Transport Equation (BTE), the MHD equations where it is being used in our system of the solar wind and earth's magnetosphere interaction. In the third chapter we present the solar wind-earth's magnetosphere interaction. Here, the structure and the geometry of the magnetosphere is provided and described. In chapter four we discuss on the the physical contents and implications of the derived MHD equations on the solar wind - magnetosphere coupling with and without collision by setting appropriate boundary conditions. In the fifth final chapter we give the summary and conclusions of our results.

II. Literature Review

According to current understanding, the earth has an internal dipole magnetic moment created by a magnetic dynamo deep inside the earth in the fluid, electrically conducting core. The sun emits magnetized plasma consisting of mainly protons and electrons to its surrounding terrestrial environment (the solar wind). The pressure exerted by this flowing plasma is counteracted by the earths magnetic field just at the end of the upper atmosphere (ionosphere)[8]. This coupling layer with the solar wind is commonly known as the magnetosphere.
Currently, ground-based and space-borne solar observations reveal that a geomagnetic storm can be regarded as an event in which disturbances are triggered by solar eruptions. These features, that have their origin in the magnetic activity of the sun, propagate through interplanetary space and interact with the terrestrial magnetosphere subsequently affecting the near-earth space environment and the upper atmosphere.

Consequently, space weather discipline involves different physical scenarios, which are characterized by very different physical conditions, ranging from the sun to the terrestrial magnetosphere and ionosphere[12],[16]. Today, great modeling efforts made during the last years that a few sun-to-ionosphere physics-based numerical codes have been developed. However, the success of the prediction is still far from achieving the desirable results and much more progress is needed. Some aspects involved in this progress concern both the technical progress (developing and validating tools to forecast, selecting the optimal parameters as inputs for the tools, improving accuracy in prediction with short lead time) and the scientific development, i.e., deeper understanding of the energy transfer process from the solar wind to the coupled magnetosphere-ionosphere- system[19].

Generally, in order to understand how the solar wind interacts with the magnetosphere and in turn the magnetosphere reacts needs examining the motion of charged particles in magnetic fields and the creation of the electric and magnetic fields by these same particles. The charged particles moving in a magnetic field experiences Lorentz's forces with complex motion pertaining to the interactions where the particle gyrates around the magnetic fields. On the other hand if the magnetic field varies it causes to induce currents, electric fields and further interactions in the environment[20],[21].

The basic laws of the charges, currents, electromagnetic fields in plasma together with the conservation of mass, momentum and energy are being used to derive the governing equations in the form of waves (in the plasma). Accordingly, to the current understanding there are three propagation speed modes in the magnetized solar wind plasma. The fast mode wave compresses the magnetic field and plasma; the intermediate mode wave bends the flow and magnetic field, but does not compress it; and the slow mode wave rarefies the field while it compresses the plasma and vice versa. The solar wind travels faster than the propagation speed of all three of these waves so when it reaches the Earth's magnetosphere the pressure waves needed to deflect the solar wind plasma cannot propagate upstream into

the solar wind without creating a shock front. The geometry of this shock, the deflected flow, called the magnetosheath and the magnetopause, the boundary between the magnetosheath and the magnetosphere can flow. So the interaction of the solar wind with a dipole magnetic field is somewhat complicated. The solar-terrestrial environment comprises fully ionized plasmas, partially ionized plasma (the ionosphere) and Earth's neutral atmosphere. The sun-Earth interaction depends on chains of coupling processes involving the solar interior, the solar atmosphere, and solar wind-magnetosphere, magnetosphere-ionosphere, and ionosphere atmosphere interactions. These interactions occur through radiative, dynamic and electromagnetic processes. Within the chain of relations, the sun is the primary driver of change as it continuously emits charged particles and radiation in the solar-terrestrial environment[18].

The Sun and the Earth represent a coupled dynamic system. The space between the Earth and the sun is filled with the solar wind plasma. This solar wind also carries with it a magnetic field, which allows it to interact strongly with the Earth's magnetosphere due to the so called magnetic field reconnection. The interaction of the solar wind magnetic field with the Earth's magnetosphere has a great effect on the near-Earth space environment. The effect of this interaction is called space weather. Space weather is an important topic of current research. Space weather can pose a danger to trans-polar flights, while geomagnetically induced current can flow through power lines and pipelines near the surface and damage them as a result of auroral disturbances[9].

Extreme space weather situations are called geomagnetic storms, and are defined as periods of intense geomagnetic activity. Geomagnetic activity is primarily driven by magnetic reconnection between the Interplanetary Magnetic Field (IMF) and the terrestrial magnetic field. Coherent solar wind structures containing magnetic fields and high velocities are most efficient drivers of space weather events. We can observe the geomagnetic activity with ground-based magnetometers. Geomagnetic storms have been related to the interaction of the solar wind with the Earth's magnetosphere. Several fundamental points in the problem of magnetic reconnection still remain to be clarified. For example, even if theoretically the main features distinguishing a resistive or a collisionless reconnection layer are fairly well understood, the debate concerning which micro-process drives reconnection is still open. Another outstanding problem of magnetic reconnection is the interplay between the micro and macrophysics during the full evolution of the large scale system. On the other hand, magnetic reconnection has been the focus of extended studies since its first introduction by[18],[19]. To explain the sudden release of energy in solar flares. Nowadays it is considered as the key ingredient for theories of coronal heating, solar flares and jets, and coronal mass ejections in the Sun, of magnetic storms and sub-storms in the Earth magnetosphere.Such attributes made it attractive for high-energy astrophysics to explain, radiation and flares in active galactic nuclei (AGNs) jets or in gamma-ray bursts, the heating of Active Galactic Nuclei (AGN)and micro quasar coronae and associated flares, the flat radio spectra from galactic nuclei and AGNs[22].

III.Statement of the Problem

Life on earth without sun is impossible. So always the study of earth sun coupling remains forever as long as humans exist. Fortunately, the technological advancement we have at hand tody has helped to advance research in this field. However, a number of theoretical principles need to be worked out. For example, the solar magnetic storm on earth that causes electronic interruption etc ned to be solved. One of the difficulties is that the complex nature of the MHD equations involving non linear differential equations.

Research Questions

- How does the earth protects us from harmful solar wind?
- How a Solar wind affect atmosphere of the earth?

- How Solar wind and Interplanetary Magnetic Field(IMF) couple with earth's magnetosphere and produce disturbances on earth?
- What is the significance of collision effect in the coupling shock waves between the solar wind and the magnetosphere?

IV. Objectives

General objective

To study solar-wind-earth's magnetosphere interaction through the interstellar magnetic field dynamics in the linearized MHD equations.

Specific Objectives

- To derive MHD equations that govern the interaction of solar-wind earth's magnetosphere interaction dynamics.
- To linearize the MHD equations that describes the IMF dynamics of the solar-wind and earth's magnetosphere interaction.
- To study the implications of the linearized MHD equations of the IMF in the solar-wind-earth's magnetosphere interaction.

Methodology

The general method is to derive dynamical equations from which relevant dynamical parameters such as conservation of mass, energy and momentum are being derived from appropriate MHD equations in plasma state by using the techniques of linearized MHD equations. The analytically derived equations are used to generate numerical values computationally with MATHEMATICA. The results had been discussed and summarized to remark.

ch1

Chapter 1

Fundamentals of Plasma Physics and Magnetohydrodynamic Equations

The interplanetary space is not empty, it is filled with fields, matter, energy, and activity often invisible to the eye. The charged particles and the magnetic field originating from the sun form the solar wind (SW) and interplanetary magnetic field (IMF). A plasma is a quasi-neutral gas of charged (and possibly neutral) particles which exhibit collective behaviour. A wind of these charged particles, the so-called solar wind, continuously blows from our sun towards the Earth's atmosphere carrying along the solar magnetic field. Therefore, one can not understand the physics in the regions of the Sun, Solar wind- Magnetosphere interaction and Interplanetary magnetic field without the knowledge of Plasma Physics.[1]

1.1 Basic definitions of Plasma

Stellar interiors and atmospheres, gaseous nebulae, and the interstellar medium are mostly plasmas. They can be found in near-Earth space(Earth's ionosphere and magnetosphere) and in the solar system (interplanetary space and planetary magnetospheres), too. More than 99 percent of all known basic states of matter in the observable universe are plasmas,that is, matter is in the form of ionized gas, since conditions such as high temperature and low density are favorable for maintaining gas ionization. Plasma is a quasi-neutral gas consisting of positively and negatively charged particles (usually ions and electrons) which are subject to electric, magnetic and other forces, and which exhibit collective behavior,i.e. Plasma is a gas of electrically charged particles in which the number of negatively charged particles is roughly equal to the number of positively charged particles.[2] [3]

The positive and negative charge carriers are evenly distributed throughout a plasma resulting in average charge neutrality. This equality between the density of the negative and positive charge carriers on a large scale, along with possible smaller regions of charge imbalance, is known as quasineutrality. Plasmas can contain some neutral particles which interact with charged particles via collisions or ionization. Examples include the Earth's ionosphere, upper atmosphere, interstellar medium, molecular clouds. Ionization of atomic hydrogen forms the simplest plasma of equal numbers of (low mass) electrons and (heavier) protons. Hence, any ionized gas cannot be called plasma, of course; there is always some small degree of ionization in any gas. In most plasmas the average kinetic energy of a particle is much larger than the average potential energy between the particle and its neighbours. This is the definition of a free particle. Plasma is characterized by the following three basic parameters.[3][6]

1. The particle density n (measured in particles per cubic meter)

2. The temperature T of each species (usually measured in eV, where (1 eV=11,605 K)

3. The steady state magnetic field B (measured in Tesla).

From the above three fundamental parameters, a host of susidiary parameters like Debye length, Larmor radius, plasma frequency, cyclotron frequency and thermal velocity can be derived.

For the existence of plasma state,the following fundamental criteria must be satisfied.

1. At length's much greater than the Debye length ($L \gg \lambda_D$), the electrostatic force from individual charges is essentially zero and so the plasma is quasi-neutral.

2. The plasma parameter of the ionized gas must be large, i.e $\Lambda \gg 1$.

3. For the plasma state to remain ,the time average between two charged-neutral particle collisions (τ_n) must be larger than the reciprocal of the plasma frequency ,i.e $\omega_p \tau_n \gg 1$

1.2 Boltzmann transport phenomena and Vlasov Equation

1.2.1 Boltzmann and Vlasov Equation

Lorentz force is the most important force in the plasma physics.

$$F_i = q_i(E + V_i X B) \tag{1.2.1}$$

where q_i is the charge of particle i, E the electric field intensity, and B the magnetic flux density, must be evaluated at the time and location of the particle.

$$m_i \frac{dv_i}{dt} = F_i \text{ and}$$

$$\frac{dx_i}{dt} = v_i \tag{1.2.2}$$

As the star shine there is inflow and outflow of energy to/from the interior of the stars. We now turn to an extremely simple description of how this flow can be quantified; this treatment is due to Ludwig Boltzmann and should not be confused with the Boltzmann formula of Maxwell-Boltzmann statistics. The Boltzmann transport equation basically expresses the change in the phase density within a differential volume, in terms of the flow through these faces, and the creation or destruction of particles within that volume. considering the in flow and out flow to and from a differential phase volume dV ,we have:

$$\text{particle density } XdV = \tfrac{N}{V}Adx_i$$
$$\tfrac{N}{V}A\tfrac{dx_i}{dt}dt = v\tfrac{N}{V}Adt$$

where v is the flow velocity.

Then the inflow of particles across a given volume in time dt is given by the equation

$$Inflow = [\frac{dx_i}{dt}f(x,t)\frac{dv}{dx_i}]dt \tag{1.2.3}$$

Similarly, the number of particles flowing out of the opposite face located dx_i away is

$$Outflow = \frac{dx_i}{dt}(f(x_i + dx_i, t)\frac{dv}{dx_i})dt \tag{1.2.4}$$

The net change due to flow in and out of the six-dimensional volume is obtained by calculating the difference between the inflow and outflow and summing over all faces of the volume yields:

The net inflow and outflow is

$$= \sum_i \frac{dx_i}{dt}[\frac{f(x_i + dx_i - f(x_i,t)}{dx_i}]dvdt \tag{1.2.5}$$
$$= \textstyle\sum_i \tfrac{dx_i}{dt}\tfrac{\partial f}{\partial x_i}dvdt$$

The distribution function $f_\alpha(x,v,t)$ describes the density of particles phase space (x, v), it is the solution of the kinetic Boltzmann Vlasov equation and by taking the partial differential of the function.

$$f_\alpha = f_\alpha(x_i, v, t) \tag{1.2.6}$$
$$\frac{df_\alpha}{dt} = \frac{\partial f_\alpha}{\partial x_i}\frac{dx_i}{dt} + \frac{\partial f_\alpha}{\partial v}\frac{dv}{dt} + \frac{\partial f}{\partial t}\frac{dt}{dt}$$
$$\frac{df_\alpha}{dt} = \sum_i v_i\frac{\partial f}{\partial x_i} + a_i\frac{\partial f}{\partial v_i} + \frac{\partial f}{\partial t} \tag{1.2.7}$$
$$\frac{df_\alpha}{dt} = \frac{\partial f_\alpha}{\partial t} + v.\nabla f_\alpha + a.\nabla_v f_\alpha = \frac{df_{\alpha(x,v,t)}}{dt}|_{collision} \tag{1.2.8}$$

where the total time derivative is

$$|\frac{df_{\alpha(x,v,t)}}{dt}|_{collision} = \frac{\partial f_\alpha}{\partial t} + v.\nabla f_\alpha + a.\nabla_v f_\alpha,$$

Where from Lorentz force we have,

$$a = \frac{q_\alpha}{m\alpha}(E + VXB) \tag{1.2.9}$$

In the absence of collision, the collision integral is zero, and

$$\frac{\partial f_\alpha}{\partial t} + v.\nabla f_\alpha + \frac{q_\alpha}{m_\alpha}(E + VXB).\nabla_v f_\alpha = 0 \tag{1.2.10}$$

The Boltzmann equation with the collisional term equal to zero is called collisionless Boltzmann equation or Vlasov equation. Therefore,equation (1.2.7) becomes,

$$\sum_i \frac{\partial f_\alpha}{\partial x_i}\frac{dx_i}{dt} = \frac{df_\alpha}{dt} - \frac{\partial f_\alpha}{\partial t}$$

$$v_i\frac{\partial f_\alpha}{\partial x_i} = -(\frac{\partial f_\alpha}{\partial t} - \frac{df_\alpha}{dt}) \tag{1.2.11}$$

The net outflow is given by

$$\sum_i v_i\frac{\partial f_\alpha}{\partial x_i}dvdt = -(\frac{\partial f_\alpha}{\partial t} - \frac{df_\alpha}{dt})dvdt \tag{1.2.12}$$

Note that x_i represents the spatial and momentum coordinates.

$$\frac{df_\alpha}{dt} = \sum_i \dot{x}_i\frac{\partial f_\alpha}{\partial x_i} + \dot{p}_i\frac{\partial f_\alpha}{\partial p_i} + \frac{\partial f_\alpha}{\partial t} = \Im \tag{1.2.13}$$

The action considered as the creation rate therefore,

$$\sum_i \frac{dx_i}{dt}\frac{\partial f_\alpha}{\partial x_i}dvdt = -(\frac{\partial f_\alpha}{\partial t} - (\frac{\partial f_\alpha}{\partial t} - \Im)dvdt \tag{1.2.14}$$

$$\Im = \sum_i[(\dot{x}_i\frac{\partial f\alpha}{\partial x_i}) + (\dot{p}_i\frac{\partial f_i}{\partial t}) + \frac{\partial f_i}{\partial t}] \tag{1.2.15}$$

This is known as the Boltzmann transport equation and can be written in several different ways. In vector notation we get,

$$\Im = \frac{\partial f_\alpha}{\partial t} + \vec{v}.\vec{\nabla} f + \dot{F}.\frac{1}{m}\vec{\nabla}_v f \tag{1.2.16}$$

But, $\vec{F} = -\vec{\nabla}\Phi$

$$\Im = \tfrac{\partial f_\alpha}{\partial t} + \vec{v}.\vec{\nabla} f - \tfrac{1}{m}\vec{\nabla}\Phi.\vec{\nabla}_v f$$

Here the potential gradient $\nabla\Phi$ has replaced the momentum time derivative while v is a gradient with respect to velocity. The quantity m is the mass of a typical particle. It is also not unusual to find the Boltzmann transport equation written in terms of the total Stoke's time derivative

$$\frac{Df}{Dt} = \frac{\partial}{\partial t} + \vec{v}.\vec{\nabla} \tag{1.2.17}$$

Equation 1.2.17 is total Stoke's time derivation

$$\frac{Df}{Dt} = \Im \tag{1.2.18}$$

If $\Im = 0$, no particle creation and destruction and $\frac{Df}{Dt} = 0$ is called the homogeneous Boltzmann Transport equation.[23]

1.2.2 Magnetohydrodynamic Equations

Magnetohydrodynamics (MHD)is the dynamics of an electrically conducting fluid (a fully or partially ionized gas or a liquid metal) containing a magnetic field. It is a fusion of fluid dynamics and electromagnetism.

Magnetohydrodynamics (MHD) describes the "slow" evolution of an electrically conducting fluid most often a plasma consisting of electrons and protons (perhaps seasoned sparingly with other positive ions). In MHD "slow" means evolution on time scales longer than those on which individual particles are important, or on which the electrons and ions might evolve independently of one another.

Particle motion in the two-fluid system is described by the individual species mean velocities u_e,u_i and by the pressure P_e,P_i which give information on the random deviation of the velocity from its mean value. Magnetohydrodynamics is an alternate description of the plasma where instead of using u_e,u_i are used.[3]

The current density is given by

$$J = \sum_{\alpha} n_\alpha u_\alpha q_\alpha \tag{1.2.19}$$

Where the center of mass velocity is

$$u = \frac{1}{\rho} \sum_\alpha m_\alpha n_\alpha u_\alpha \ ,$$

where

$$\rho = \sum_{\alpha} m_\alpha n_\alpha \tag{1.2.20}$$

1.2.3 Continuity Equation and the Zeroth velocity moment

Here there is an expectation of conservation of mass. The n^{th} moment of the Boltzmann equation can be represented by

$$\int V^n [\frac{\partial f_\alpha}{\partial t} + V.\nabla f_\alpha + a.\nabla_v f_\alpha] dv = \int V^n \frac{\partial f \alpha}{\partial t}|_{collision} dv \tag{1.2.21}$$

Taking the zeroth moment of Boltzmann equation $n = 0$, multiplying equation (1.2.21) by V raised to the power zero and then integrating over all velocity space, then by multiplying by the mass of the species m_α and sum over species to obtain the following.

$$\sum_\alpha m_\alpha \int [\frac{\partial f_\alpha}{\partial t} + v.\nabla f_\alpha + a.\nabla_v f_\alpha] dv = \sum_\alpha m_\alpha \int \frac{\partial f_\alpha}{\partial t}|_{collision} dv \tag{1.2.22}$$

The account for the collision term in Boltzmann equation is:

$$Q^\rho(x,t) = m \int_{-\infty}^{\infty} \frac{\partial f_\alpha}{\partial t}|_c dv \tag{1.2.23}$$

$$Q^p(x,t) = m \int_{-\infty}^{\infty} (V-U) \frac{\partial f_\alpha}{\partial t}|_c dv \tag{1.2.24}$$

$$Q^E(x,t) = \frac{1}{2} m \int_{-\infty}^{\infty} (V-U)^2 \frac{\partial f_\alpha}{\partial t}|_{collision} dv \tag{1.2.25}$$

$$\sum_\alpha m_\alpha \int \frac{\partial f_\alpha}{\partial t} dv + \sum_\alpha m_\alpha \int (v.\nabla f_\alpha) dv$$
$$+ \sum_\alpha m_\alpha \int (a.\nabla_v f_\alpha) dv = \sum_\alpha m_\alpha \int \frac{\partial f_\alpha}{\partial t}|_c dv \quad (1.2.26)$$

Using the definition in equation (1.2.2) the first term of this equation gives

$$\sum_\alpha m_\alpha \int \frac{\partial f_\alpha}{\partial t} dv = m_\alpha \frac{\partial}{t} \sum_\alpha \int f_\alpha dv = \frac{\partial}{\partial t} \sum_\alpha m_\alpha n_\alpha \quad (1.2.27)$$

The time and velocity are independent variables. Here equation (1.2.27) can be given by

$$\sum_\alpha m_\alpha \int \frac{\partial f_\alpha}{\partial t} dv = \frac{\partial \rho}{\partial t} \quad (1.2.28)$$

The zeroth moment of the second term of Eq.(1.2.26) gives

$$\sum_\alpha m_\alpha \int (v.\nabla f_\alpha) dv + = \sum_\alpha m_\alpha \int \nabla.(f_\alpha v) dv \quad (1.2.29)$$

Using the identity

$$\nabla.(AB) = A\nabla.B + B.\nabla A \quad (1.2.30)$$

We can write Equation (1.2.29)

$$\nabla.(f_\alpha v) = f_\alpha \nabla.v + v.\nabla f_\alpha \quad (1.2.31)$$

since V is independent of the position X in phase space, the first term on the right hand side of Equation (1.2.31)vanishes.

$$\sum_\alpha m_\alpha \int (v.\nabla f_\alpha) dv = \sum_\alpha m_\alpha \int \nabla.(v f_\alpha) dv = \nabla. \sum_\alpha m_\alpha \int (v f_\alpha) dv \quad (1.2.32)$$

We define mean velocity flow as

$$\vec{U} = \vec{V} = \frac{\int_{-\infty}^{\infty} v f_\alpha \vec{dv}}{\int_{-\infty}^{\infty} f_\alpha \vec{dv}} \quad (1.2.33)$$

Using the definition in Eq.(1.2.19) and Eq.(1.2.2)

$$\sum_\alpha m_\alpha \int (v.\nabla f_\alpha) dv = \nabla. \sum_\alpha m_\alpha \int u_\alpha (f_\alpha) dv$$

$$= \nabla . \sum_{\alpha} m_\alpha n_\alpha u_\alpha$$
$$= \nabla . (\rho U) \tag{1.2.34}$$

Similarly using the identity given in equation (1.2.30)

$$\nabla_v . (a f_\alpha) = f_\alpha \nabla_v . a + a . \nabla_v f_\alpha \tag{1.2.35}$$

since a is independent of V in phase space, the first term on the right hand side of equation (1.2.35) vanishes. Then

$$\nabla_v . (a f_\alpha) = a . \nabla_v f_\alpha \tag{1.2.36}$$

Thus, using the result of Eq.(1.2.36) , we get

$$\int_v \nabla_v . (a f_\alpha) dv = \int_s (a f_\alpha) ds$$

However, the distribution function, f_α, vanish as $V \to \infty$, and this surface integral in velocity space therefore vanishes, as there is no particle with infinite velocity. The collision term of the Boltzmann equation reduces to Q^ρ such that in summary the zeroth moment of the Boltzmann equation reduces to:

$$\frac{\partial \rho}{\partial t} + \nabla . (\rho u) = Q^\rho \tag{1.2.37}$$

Thus, the above equation is the continuity equation with collision term Q^ρ. This equation is noting but the mass conservation equation. If collision is neglected, the above continuity equation can be given by:

$$\frac{\partial \rho}{\partial t} + \nabla . (\rho u) = 0$$

1.2.4 Equation of motion

Multiplying Boltzmann equation by $\vec{v}$ and integrating over all velocity space will produce momentum like moments. To obtain an equation of motion we take the first moment of

Boltzmann equation then multiplying by V , m_α and sum over species to obtain:

$$\sum_\alpha m_\alpha \int v[\frac{\partial f_\alpha}{\partial t} + v.\nabla f_\alpha + a.\nabla_v f_\alpha]dv = \sum_\alpha m_\alpha \int |v\frac{\partial f_\alpha}{\partial t}|_c dv \tag{1.2.38}$$

We now define the velocity V relative to the mean velocity of the species U as $V = V^{'} + U$ where $V^{'}$ is the random velocity of the species. Therefore, Eq.(1.2.38) can be written as

$$\sum_\alpha m_\alpha \int (U + V^{'})[\frac{\partial f_\alpha}{\partial t} + v.\nabla f_\alpha + a.\nabla_v f_\alpha]dv = \sum_\alpha m_\alpha \int |(V^{'} + U)\frac{\partial f_\alpha}{\partial t}dv|_c$$

$$= \sum_\alpha m_\alpha \int (U + V^{'})\frac{\partial f_\alpha}{\partial t}dv + \sum_\alpha m_\alpha \int (V^{'} + U)(v.\nabla f_\alpha) + \sum_\alpha m_\alpha \int (V^{'} + U)(a.\nabla_v f_\alpha)dv$$

$$= \sum_\alpha m_\alpha \int |(V^{'} + U)\frac{\partial f_\alpha}{\partial t}dv|_c$$

$$= \frac{\partial}{\partial t}\sum_\alpha m_\alpha \int (U + V^{'})^2 f_\alpha dv + \frac{\partial}{\partial x}.\sum_\alpha m_\alpha \int (V^{'} + U)(V^{'} + U) f_\alpha dv$$

$$+ \sum_\alpha m_\alpha \int V\frac{\partial}{\partial v}.(a) f_\alpha dv = \sum_\alpha m_\alpha \int |V\frac{\partial f_\alpha}{\partial t}dv|_c$$

$$= \frac{\partial}{\partial t}\sum_\alpha m_\alpha \int (V^{'} + U) f_\alpha dv + \frac{\partial}{\partial x}.\sum_\alpha m_\alpha \int (VV^{'} + V^{'}U + UU) f_\alpha dv$$

$$+ \sum_\alpha m_\alpha \int V\frac{\partial}{\partial v}.(a) f_\alpha dv = \sum_\alpha m_\alpha \int |V\frac{\partial f_\alpha}{\partial t}dv|_c \tag{1.2.39}$$

Expanding equation (1.2.39)

$$\frac{\partial}{\partial t}\sum_\alpha m_\alpha \int V^{'} f_\alpha dV + \frac{\partial}{\partial t}\sum_\alpha m_\alpha \int U f_\alpha dv + \frac{\partial}{\partial x}.\sum_\alpha m_\alpha \int V^{'}V^{'} f_\alpha dV$$

$$+ \frac{\partial}{\partial x}.\sum_\alpha m_\alpha \int V^{'}U f_\alpha dV + \frac{\partial}{\partial x}.\sum_\alpha m_\alpha \int UV^{'} f_\alpha dV + \frac{\partial}{\partial x}.\sum_\alpha m_\alpha \int UU f_\alpha dV$$

$$+ \sum_\alpha m_\alpha \int V\frac{\partial}{\partial V}.(a) f_\alpha dv = \sum_\alpha m_\alpha \int |V\frac{\partial f_\alpha}{\partial t}dv|_c \tag{1.2.40}$$

The average of the fluctuation vanishes. That is,

$$< V^{'} > = \int V^{'} f_\alpha dV = 0 \tag{1.2.41}$$

making use of Eq.(1.2.41) the first,fourth,and fifth terms on the left hand side of Eq.(1.2.40) vanishes. Therefore, Eq.(1.2.40) reduces to

$$\frac{\partial}{\partial t}\sum_{\alpha} m_{\alpha}\int u f_{\alpha} dv + \frac{\partial}{\partial x}\cdot\sum_{\alpha} m_{\alpha}\int V^{'}V^{'} f_{\alpha} dV$$

$$+\frac{\partial}{\partial x}\cdot\sum_{\alpha} m_{\alpha}\int UU f_{\alpha} dV$$

$$+\sum_{\alpha} m_{\alpha}\int V\frac{\partial}{\partial V}.(a) f_{\alpha} dv = \sum_{\alpha} m_{\alpha}\int |V\frac{\partial f_{\alpha}}{\partial t} dv|_{c} \tag{1.2.42}$$

Using Eq.(1.2.2), the first on the left hand side of Eq.(1.2.42) can be written as

$$\frac{\partial}{\partial t}\sum_{\alpha} m_{\alpha}\int U f_{\alpha} dv = \frac{\partial}{\partial t}(\rho U) \tag{1.2.43}$$

The second integral can be evaluated by using the definition of the pressure tensor. The MHD pressure tensor is now defined in terms of the random velocities relative to u as

$$P = \sum_{\alpha} m_{\alpha}\int V^{'}V^{'} f_{\alpha} dv \tag{1.2.44}$$

Using this definition, the second term on the left hand-side of Eq.(1.2.42) can be written as

$$\frac{\partial}{\partial X}\cdot\sum_{\alpha} m_{\alpha}\int V^{'}V^{'} f_{\alpha} dv = \frac{\partial}{\partial x}P \tag{1.2.45}$$

similarly the third term on the left hand side of Eq.(1.2.42)

$$\frac{\partial}{\partial X}\cdot\sum_{\alpha} m_{\alpha}\int UU f_{\alpha} dv = \frac{\partial}{\partial x}(\rho UU) \tag{1.2.46}$$

In order to simplify the acceleration first let us solve the first integral.

$$\sum_{\alpha} m_{\alpha}\int V\frac{\partial}{\partial V}.(a) f_{\alpha} dv = 0 \tag{1.2.47}$$

In order to simplify the equation first let us reduce the integral to its lowest form,i.e.,

$$\sum_{\alpha} m_{\alpha}\int V\frac{\partial}{\partial V}.(a f_{\alpha}) dv = \sum_{\alpha} m_{\alpha}[\int \frac{\partial}{\partial v}.(aV) f_{\alpha} dv - \int a f_{\alpha}\frac{dv}{dv} dv] \tag{1.2.48}$$

Since $f_\alpha \to 0$ as $V \to \infty$, this integral in phase space vanishes. Therefore, the integral becomes

$$\sum_\alpha m_\alpha \int v \frac{\partial}{\partial V}.(af_\alpha)dv = -\sum_\alpha m_\alpha \int af_\alpha \frac{dv}{dv}dv \tag{1.2.49}$$

Thus , using the definition of tensor $\frac{\partial v}{\partial v} = 1$ and substituting the value of Eq.(1.2.4) in to Eq.(1.2.48) becomes

$$\sum_\alpha m_\alpha \int v \frac{\partial}{\partial V}.(af_\alpha)dv = -\sum_\alpha m_\alpha \int q_\alpha(E + VxB)f_\alpha dv$$

$$\sum_\alpha m_\alpha \int v \frac{\partial}{\partial V}.(af_\alpha)dv = -\sum_\alpha m_\alpha q_\alpha(E + VxB) \int f_\alpha dv$$

$$= -\sum_\alpha m_\alpha q_\alpha E \int f_\alpha dv - \sum_\alpha m_\alpha \int q_\alpha(VxB)f_\alpha dv$$

Using the definition of $n_\alpha = \int f_\alpha dV$ this equation becomes

$$-\sum_\alpha q_\alpha n_\alpha E - \sum_\alpha q_\alpha n_\alpha(VxB) \tag{1.2.50}$$

Making use of Eq.1.2.2 we can use J instead of $\sum_\alpha q_\alpha n_\alpha V$. Therefore, Eq.1.2.50 becomes

$$-\sum_\alpha q_\alpha n_\alpha E - \sum_\alpha q_\alpha n_\alpha(VxB) = -\sum_\alpha q_\alpha n_\alpha E - JXB \tag{1.2.51}$$

Substituting Eq.(1.2.2),(1.2.45),(1.2.46) and (1.2.50) in to Eq.(1.2.42) gives

$$\frac{\partial \rho U}{\partial t} + \nabla.(\rho UU) - \sum_\alpha q_\alpha n_\alpha E - JXB + \nabla.P = \sum_\alpha m_\alpha \int |V \frac{\partial f_\alpha}{\partial t} dv|_c \tag{1.2.52}$$

MHD is typically used to describe phenomena with large special scales where the plasma is essentially neutral,so that $\sum_\alpha q_\alpha n_\alpha \approx 0$. Thus, Eq.(1.2.52) reduces to

$$\frac{\partial \rho U}{\partial t} + \nabla.(\rho UU) - JXB + \nabla.P = \sum_\alpha m_\alpha \int |V \frac{\partial f_\alpha}{\partial t} dv|_c \tag{1.2.53}$$

The first two terms of left hand side of Eq.(1.2.53) can be simplified as

$$\frac{\partial\rho U}{\partial t} + \nabla.(\rho UU) = \rho\frac{\partial U}{\partial t} + \rho U.\nabla U + U\nabla.\rho U$$

Or

$$\frac{\partial\rho U}{\partial t} + \nabla.(\rho UU) = \rho(\frac{\partial}{\partial t} + U.\nabla)U + U[\frac{\partial\rho}{\partial t} + \nabla.(\rho U)] \tag{1.2.54}$$

But the second term inside the bracket of right hand side of Eq.(1.2.54) is a continuity equation which has the value of $Q^{\rho} = m\int \frac{\partial f_{\alpha}}{\partial t}|_c$. Hence Eq.(1.2.53) reduces to

$$\rho(\frac{\partial}{\partial t} + U.\nabla)U - JXB + \nabla.P = \sum_{\alpha} m_{\alpha}\int |V\frac{\partial f_{\alpha}}{\partial t}dv|_c \tag{1.2.55}$$

Then by comparing Eqs.(1.2.55) and (1.2.54),

$$\rho(\frac{\partial}{\partial t} + U.\rho)U + m\int U\frac{\partial f}{\partial t}dv = JXB - \nabla.P + m\int V\frac{\partial f}{\partial t}dv$$

$$\rho(\frac{\partial}{\partial t} + U.\rho)U - JXB + \nabla.P = m\int(V - U)\frac{\partial f_{\alpha}}{\partial t}dv \tag{1.2.56}$$

$$\rho(\frac{\partial}{\partial t} + U.\rho)U - JXB + \nabla.P = Q^p \tag{1.2.57}$$

$$\rho(\frac{\partial}{\partial t} + U.\rho)U = JXB - \nabla.P + Q^p$$

Or

$$\rho\frac{DU}{Dt} = JXB - \nabla.P + Q^p \tag{1.2.58}$$

Where

$$\frac{DU}{Dt} = (\frac{\partial}{\partial t} + U.\rho)U$$

This equation (1.2.58) is the standard form of the magnetohydrodynamic equation of motion with the collision term Q^p

1.2.5 Energy Equation

Taking the second moment of the Boltzmann equation we derive the energy equation for homogeneous collision plasma as follows. We multiply the Boltzmann equation by $(\frac{m_\alpha V^2}{2})$ and integrating it by parts with respect to the volume in space velocity, dv to get

$$\sum_\alpha \frac{m_\alpha}{2}\int V^2[\frac{\partial f}{\partial t}+v.\nabla f_\alpha+\vec{a}.\nabla_v f]dv+=\sum_\alpha \frac{m_\alpha}{2}\int V^2|\frac{\partial f_\alpha}{\partial t}dv|_c \tag{1.2.59}$$

$$\sum_\alpha \frac{m_\alpha}{2}\int V^2\frac{\partial f}{\partial t}dv+\sum_\alpha \frac{m_\alpha}{2}\int V^2(v.\nabla f_\alpha)dv+\sum_\alpha \frac{m_\alpha}{2}\int V^2(a.\nabla_v f_\alpha]dv+=\sum_\alpha \frac{m_\alpha}{2}\int V^2|\frac{\partial f_\alpha}{\partial t}dv|_c$$

$$\frac{\partial}{\partial t}\int \frac{m_\alpha}{2}V^2 f_\alpha dv+\frac{\partial}{\partial x}\int \frac{m_\alpha}{2}V^2(vf_\alpha)dv+\int \frac{m_\alpha}{2}V^2\frac{\partial}{\partial v}(af_\alpha)dv=\int \frac{m_\alpha}{2}V^2|\frac{\partial f_\alpha}{\partial t}dv|_c \tag{1.2.60}$$

By using the definition $V = V^{'} + U$ the more general definition of the pressure P of N diminishes as

$$P=\frac{m_\alpha}{N}\int V^{'}V^{'}dV \tag{1.2.61}$$

Each term of Eq.(1.2.60) can be simplified separately as follow. The first time derivative term becomes

$$\frac{\partial}{\partial t}\int \frac{m_\alpha}{2}V^2 f_\alpha dv=\frac{\partial}{\partial t}\int \frac{m_\alpha}{2}(V'+U)^2 f_\alpha dv$$

$$=\frac{\partial}{\partial t}\int \frac{m_\alpha}{2}(V^{'}V^{'}+2V^{'}U+UU)f_\alpha dv$$

$$=\frac{\partial}{\partial t}\int \frac{m_\alpha}{2}V^{'}V^{'}f_\alpha dv+\frac{\partial}{\partial t}\int \frac{m_\alpha}{2}U^2 f_\alpha dv+\frac{\partial}{\partial t}\int m_\alpha V^{'}Uf_\alpha dv \tag{1.2.62}$$

The third term of this equation can be ignored

$$\frac{\partial}{\partial t}\int \frac{m_\alpha}{2}V^{'}V^{'}f_\alpha dv+\frac{\partial}{\partial t}\int \frac{m_\alpha}{2}U^2 f_\alpha dv$$

$$\frac{\partial}{\partial t}\int \frac{m_\alpha}{2}V^2 f_\alpha dv = \frac{\partial}{\partial t}(\frac{NP}{2} + \frac{m_\alpha n_\alpha U_\alpha^2}{2}) \tag{1.2.63}$$

Again using $V = V^{'} + U$ the space derivative term become

$$\frac{\partial}{\partial x}\int \frac{m_\alpha}{2}V^2(Vf_\alpha)dv = \frac{\partial}{\partial x}\int \frac{m_\alpha}{2}(V^{'} + U)^2(V^{'} + U)f_\alpha dv \tag{1.2.64}$$

$$= \frac{\partial}{\partial x}\int [\frac{m_\alpha}{2}(V^{'}V^{'} + 2V^{'}U + UU)(V^{'} + U)f_\alpha dv]$$

$$\frac{\partial}{\partial x}\int \frac{m_\alpha}{2}[V^{'}V^{'}V^{'} + 2V^{'}V^{'}U + U^2V^{'} + 2V^{'}U^2 + V^{'}V^{'}U + U^2U]f_\alpha dv$$
$$= \frac{\partial}{\partial x}\int \frac{m_\alpha}{2}V^{'}V^{'}V^{'}f_\alpha dv + \frac{\partial}{\partial x}\int \frac{m_\alpha}{2}2V^{'}V^{'}Uf_\alpha dv + \frac{\partial}{\partial x}\int \frac{m_\alpha}{2}U^2V^{'}f_\alpha dv$$
$$+ \frac{\partial}{\partial x}\int \frac{m_\alpha}{2}2V^{'}U^2 f_\alpha dv + \frac{\partial}{\partial x}\int \frac{m_\alpha}{2}V^{'}V^{'}Uf_\alpha dv + \frac{\partial}{\partial x}\int \frac{m_\alpha}{2}U^2Uf_\alpha dv \tag{1.2.65}$$

From Eq.(1.2.65) the 3rd and 4th terms from the right hand side vanishes, because of the average of fluctuation is zero. Then defining the heat flux as:[3]

$$Q = \frac{m_\alpha}{2}\int V^{'}V^{'}V^{'}f_\alpha dv$$

Eq. (1.2.64) yields

$$\frac{\partial}{\partial x}\int \frac{1}{2}m_\alpha v^2(Vf_\alpha)dv = \frac{\partial}{\partial x}[Q + \frac{2+N}{2}PU + \frac{m_\alpha n_\alpha u^2}{2}U] \tag{1.2.66}$$

Lastly,from Eq.(1.2.60) the third term gives

$$m_\alpha\int \frac{1}{2}v^2.a\frac{\partial}{\partial v}f_\alpha dV = m_\alpha\int \frac{\partial}{\partial v}(\frac{1}{2}aV^2 f_\alpha dv) - \int \frac{1}{2}f_\alpha v^2\frac{\partial}{\partial v}.adV - \int \frac{1}{2}f_\alpha a.\frac{\partial V^2}{\partial v}dv \tag{1.2.67}$$

But the first integral on the right hand-side of Eq.(1.2.67) is the volume integral of a divergence in velocity space. Therefore Gausss theorem gives f_α evaluated on a surface at $V \to \infty$ because $f_\alpha \to 0$ as $V \to \infty$ this surface integral in velocity space vanishes. Thus, using the expression for the acceleration and simplifying it this equation reduces to

$$\int \frac{m_\alpha}{2} V^2 a \frac{\partial}{\partial v} f_\alpha dv = -m_\alpha \int \frac{1}{2} f_\alpha a . \frac{\partial V^2}{\partial v} dv$$
$$= -m_\alpha \int a f_\alpha v dv$$

$$= - \int q(E + UXB) f_\alpha v dv$$
$$\int \frac{m_\alpha}{2} V^2 a \frac{\partial}{\partial v} f_\alpha dv = -q n_\alpha E.U \qquad (1.2.68)$$

now combining Eq.(1.2.63), and (1.2.68), we have

$$\frac{\partial}{\partial t}[\frac{N}{2}P + \frac{m_\alpha n_\alpha u^2}{2}] + \nabla.[Q + \frac{2+N}{2}PU + \frac{m_\alpha n_\alpha u^2}{2}U] - q n_\alpha E.U = \int \frac{m_\alpha}{2} V^2 \frac{\partial f_\alpha}{\partial t}|_c dv \quad (1.2.69)$$

The plasma is assumed to be perfect conductor and the heat flux, Q is neglected and taking some mathematical identities on Eq.(1.2.69),we have

$$\frac{\partial}{\partial t}\frac{N}{2}P + \nabla.[\frac{2+N}{2}PU] + \frac{\partial}{\partial t}(\frac{m_\alpha n_\alpha u^2}{2}) + \nabla.(\frac{m_\alpha n_\alpha u^2}{2}U) - q n_\alpha E.U = \int |\frac{mV^2}{2}\frac{\partial f}{\partial t} dv|_c$$

$$(\frac{\partial}{\partial t} + U.\nabla)\frac{N}{2}P + \frac{N}{2}P\nabla.U + \nabla.(PU) + (\frac{\partial}{\partial t} + U.\nabla)\frac{m_\alpha n_\alpha u^2}{2} - q n_\alpha E.U = \int |\frac{mV^2}{2}\frac{\partial f}{\partial t} dv|_c$$

$$\frac{D}{Dt}(\frac{N}{2}P) + \frac{N}{2}P\nabla.U + \nabla.(PU) + n_\alpha m_\alpha \frac{D}{Dt}(\frac{U^2}{2}) - q n_\alpha E.U = \int |\frac{mV^2}{2}\frac{\partial f}{\partial t} dv|_c \quad (1.2.70)$$

To simplify Eq. (1.2.70) further, let us take the dot product of Eq.(1.2.57) with the velocity u we get

$$\frac{\partial}{\partial t}(\rho U).U + \nabla.(\rho UU).U + (\nabla.P).U - [\sum_\alpha q_\alpha n_\alpha E - JxB].U = \int U(V-U)\frac{\partial f_\alpha}{\partial t} \quad (1.2.71)$$

Now using some identities as before we arrive at

$$\frac{n_\alpha m_\alpha}{2}\frac{D}{Dt}U^2 + U.\nabla P = q n_\alpha E.U + \frac{1}{2} m_\alpha \int (V-U)^2 \frac{\partial f_\alpha}{\partial t} dv \qquad (1.2.72)$$

Then substituting Eq.(1.2.72) into Eq. (1.2.70) yields

$$\frac{D}{Dt}(\frac{N}{2}P) + \frac{N}{2}P\nabla.U + \nabla.(PU) + n_\alpha m_\alpha \frac{D}{Dt}(\frac{u^2}{2}) - n_\alpha m_\alpha \frac{D}{Dt}(\frac{u^2}{2}) + U.\nabla P$$
$$= \frac{1}{2}m_\alpha \int (V-U)^2 \frac{\partial f_\alpha}{\partial t} dv$$

$$\frac{D}{Dt}(\frac{N}{2}P) + \frac{N}{2}P\nabla.U + \nabla.(PU) - U.\nabla P = \frac{1}{2}m_\alpha \int (V-U)^2 \frac{\partial f_\alpha}{\partial t} dv \tag{1.2.73}$$

For

$$\nabla.(PU) - u.(\nabla P) = (P\nabla).U$$

But here, P is the tensorial pressure [1]

$$(P\nabla).U = (P.\nabla)U \tag{1.2.74}$$

Finally from the preceeding two equations Eq.(1.2.74) is written as

$$\frac{D}{Dt}(\frac{N}{2}P) + \frac{N}{2}P\nabla.U + (P.\nabla)U = Q^E$$

For N=3

$$\frac{D}{Dt}(\frac{3}{2}P) + \frac{3}{2}P\nabla.U + (P.\nabla)U = Q^E \tag{1.2.75}$$

This is the MHD energy equation with collision term Q^E.

1.2.6 Linearizing Magnetohydrodynamics

The magnetohydrodynamic equations are:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho V) = 0$$
$$\rho(\frac{\partial}{\partial t} + V.\nabla)V + \nabla P - JXB = 0$$

$$\frac{\partial P}{\partial t} + V.\nabla P + \gamma P \nabla.V = 0$$

To linearize the MHD equations, we have to use the dispersion relationship for sound wave representing the variables as the sum of the bachground component (denoted 'o') and a small perturbed component (denoted '1').

Now, to linearize the first equation we substitute $\rho(x,t) = \rho_0 + \rho_1(x,t)$ and $V(x,t) = V_1(x,t)$ in to the above continuity equation.

$$\frac{\partial \rho_0}{\partial t} + \frac{\partial \rho_1}{\partial t} + \nabla.(\rho_0 V_1) + \nabla.(\rho_1 V_1) = 0$$
$$\frac{\partial \rho_1}{\partial t} + \nabla.(\rho_0 V_1) = 0 \tag{1.2.76}$$

This is the linearized continuity equation.

The next is linearizing equation of motion.We want to insert $B(x,t) = B_0 + B_1$ in to the motion equation.

$$\rho(\frac{\partial}{\partial t} + V.\nabla)V + \nabla P - \frac{(\nabla XB)XB}{4\pi} = 0$$

$$(\rho_0 + \rho_1)(\frac{\partial}{\partial t} + V.\nabla)V + \nabla(P_0 + P_1) - \frac{(\nabla XB_1)XB_0}{4\pi} = 0$$
$$\rho_0 \frac{\partial}{\partial t} V_1 + \rho_1 \frac{\partial}{\partial t} V_1 + \rho_0 V_1.\nabla V_1 + \rho_1 V_1.\nabla V_1 + \nabla P_0 + \nabla P_1 - \frac{(\nabla XB_1)XB_0}{4\pi} = 0$$

By neglecting the 2nd, the third ,the forth and the fifth terms of the above equation, we have

$$\rho_0 \frac{\partial}{\partial t} V_1 + \nabla P_1 - \frac{(\nabla XB_1)XB_0}{4\pi} = 0 \tag{1.2.77}$$

This is the linearized equation of motion.

Finally,we have to insert $P(x, t = P_0 + P_1$ in to MHD energy equation.

$$\frac{\partial}{\partial t}P + V.\nabla P + \gamma P \nabla.V = 0$$

$$\frac{\partial}{\partial t}(P_0 + P_1) + V_1.\nabla(P_0 + P_1) + \gamma(P_0 + P_1)\nabla.V_1 = 0$$

$$\frac{\partial}{\partial t}P_0 + \frac{\partial}{\partial t}P_1 + V_1.\nabla P_0 + V_1.\nabla P_1 + \gamma P_0 \nabla.V_1 + \gamma P_1 \nabla.V_1 = 0$$

By neglecting the second and higher order terms, we get

$$\frac{\partial}{\partial t}P_1 + \gamma P_0 \nabla.V_1 = 0 \tag{1.2.78}$$

This is the linearized energy equation.

1.2.7 Magnetohydrodynamic Waves

Waves are ubiquitous in magnetized plasmas, just as sound waves are ubiquitous in air and are the simplest way that a system responds to disturbances and applied forces.THey propagate information and energy through a system and are closely related to shocks, instabilities, and turbulence. Plasma displays a rich variety of waves within and beyond MHD. Whenever a plasma is disturbed, there will be waves.

[24] The one dimensional wave equation for U is given by

$$\frac{\partial^2 U}{\partial t^2} = c^2 \frac{\partial^2 U}{\partial x^2} \tag{1.2.79}$$

Where c is a real constant that represents the wave speed.The solutions are waves traveling at velocities of $\pm c$. The wave equation is a hyperbolic partial differential equation.

Let us define two new variables to find the algebraic solution to the one dimensional wave equation.

$$\xi(x,t) = x - ct, \eta(x,t) = x + ct \tag{1.2.80}$$

By rewriting the wave equation as

$$\frac{\partial^2 U}{\partial \xi \partial \eta} = 0, \tag{1.2.81}$$

, the solutions are:

$$U(\xi,\eta) = R(\xi) + L(\eta) \tag{1.2.82}$$

$$U(x,t) = R(x - ct) + L(x + ct) \tag{1.2.83}$$

Where R and L are arbitrary functions traveling at velocities $\pm c$ (to the right and to the left).

Eigenmode decomposition of the 1-D wave equation

Let us Use separation of variables and look for solutions of the form:

$$U_w(x,t) = e^{i\omega t} f(x) \tag{1.2.84}$$

We plug this solution in to the wave equation

$$\frac{\partial^2}{\partial t^2}(e^{-i\omega t} f(x)) = c^2 \frac{\partial^2}{\partial x^2}[e^{i\omega t} f(x)] \tag{1.2.85}$$

$$-\omega^2 e^{i\omega t} f(x) = c^2 e^{i\omega t} \frac{d^2}{dx^2} f(x) \tag{1.2.86}$$

$$\frac{-\omega^2}{c^2} f(x) = \frac{d^2}{dx^2} f(x)$$

$$-k^2 f(x) = \frac{d^2}{dx^2} f(x) \tag{1.2.87}$$

where $k = \frac{\omega}{c}$. This is an eigenvalue equation for f(x).

Let's look for solutions of the form:

$$f(x) = Ae^{\pm ikx} \tag{1.2.88}$$

The solution to the wave equation for this eigenmode is

$$U_w(x,t) = Ae^{-ikx - i\omega t} + Be^{ikx - i\omega t} \tag{1.2.89}$$

From Euler's formula ,we have

$$e^{ix} = \cos x + isinx \tag{1.2.90}$$

By taking the real part of Eq. (1.2.89) we get

$$U_w(x,t) = A\cos(kx + \omega t) + B\cos(kx - \omega t) \tag{1.2.91}$$

The solutions are waves propagating in the $\pm x$ directions.

We note that the lines in the u-x plane on which x -ct or x + ct are constant and are called characteristics and the wave vector k points in the direction of wave propagation and has a magnitude of $k = \frac{2\pi}{\lambda}$ where λ is the wavelength .

The phase velocity is the rate at which the phase of a wave propagates through space.

$$V_p = \frac{\omega}{k} \tag{1.2.92}$$

Noting that the group velocity is the rate at which the overall shape of the waves' amplitudes propagates through space.

$$Vg = \frac{\partial \omega}{\partial k} \tag{1.2.93}$$

To find the dispersion relationship for sound waves, let's represent variables as the sum of a background component (denoted '0') and a small perturbed component (denoted '1')

$$\rho(x,t) = \rho_0 + \rho 1(x,t) \tag{1.2.94}$$

$$P(x,t) = P_0 + P_1(x,t) \tag{1.2.95}$$

$$V(x,t) = V_1(x,t) \tag{1.2.96}$$

Assume the background is homogeneous, time-independent, and static ($V_0 = 0$).

Linearizing the equations of hydrodynamics

The equations of hydrodynamics are:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho V) = 0 \tag{1.2.97}$$

$$\rho(\frac{\partial}{\partial t} + V.\nabla)V + \nabla P = 0 \tag{1.2.98}$$

$$\frac{\partial P}{\partial t} + V.\nabla P + \gamma P \nabla.V = 0 \tag{1.2.99}$$

Let's linearize these equations by dropping higher order terms and taking the background is constant.

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \nabla . V_1 = 0 \tag{1.2.100}$$

$$\rho_0 (\frac{\partial V_1}{\partial t} + \nabla P_1 = 0 \tag{1.2.101}$$

$$\frac{\partial P_1}{\partial t} + \gamma P_0 \nabla . V_1 = 0 \tag{1.2.102}$$

Now, to linearize the first equation we substitute $\rho(x,t) = \rho_0 + \rho_1(x,t)$ and $V(x,t) = V_1(x,t)$ in to Equ.(1.2.97)

$$\frac{\partial \rho_0}{\partial t} + \frac{\partial \rho_1}{\partial t} + \nabla . (\rho_0 V_1) + \nabla . (\rho_1 V_1) \tag{1.2.103}$$

$$\frac{\partial \rho_1}{\partial t} + \nabla . (\rho_0 V_1) = 0 \tag{1.2.104}$$

Deriving a wave equation for hydrodynamics

By taking the time derivative of Eq.(1.2.101),

$$\frac{\partial}{\partial t} [\rho_0 \frac{\partial V_1}{\partial t} + \nabla P_1] = 0$$

$$\rho_0 \frac{\partial^2 V_1}{\partial t^2} + \nabla \frac{\partial}{\partial t} P_1 = 0 \tag{1.2.105}$$

Then substitute $\frac{\partial P_1}{\partial t} = -\gamma P_0 \nabla . V_1$ from Eq.(1.2.102) to get a wave equation.

$$\rho_0 \frac{\partial^2 V_1}{\partial t^2} + \nabla \frac{\partial}{\partial t} P_1 = 0$$

$$\rho_0 \frac{\partial^2 V_1}{\partial t^2} + \nabla(-\gamma P_0 \nabla . V_1) = 0$$

$$\rho_0 \frac{\partial^2 V_1}{\partial t^2} - \gamma P_0 \nabla(\nabla . V_1) = 0$$

$$\frac{\partial^2 V_1}{\partial t^2} - \frac{\gamma P_0}{\rho_0} \nabla(\nabla . V_1) = 0$$

$$\frac{\partial^2 V_1}{\partial t^2} - C_s^2 \nabla(\nabla . V_1) = 0 \tag{1.2.106}$$

Where C_s is the sound speed and it is given by the equation

$$C_s = \sqrt{\frac{\gamma P_0}{\rho_0}} \tag{1.2.107}$$

Assuming the solution is a superposition of plane waves and the plane wave solutions of the form

$$V_1(x,t) = \sum_k \hat{V}_k e^{i(k.x-\omega t)} \tag{1.2.108}$$

Differential operators turn in to multiplications with algebraic factors

$$\nabla \rightarrow ik, \frac{\partial}{\partial t} \rightarrow -i\omega \tag{1.2.109}$$

The problem is linear and homogeneous, so we consider each component separately. The wave equation then becomes

$$\frac{\partial^2}{\partial t^2} V_1 - C_s^2 \nabla(\nabla . V_1) = 0 \tag{1.2.110}$$

$$(-i\omega)^2 V_1 - C_s^2 (ik(ik.V_1)) = 0$$

$$\omega^2 V_1 - C_s^2 k(k.V_1) = 0 \tag{1.2.111}$$

The dispersion relationship for sound waves

By choosing coordinates $k = k_z \hat{z}$, which then implies that $V_1 = V_{1z}\hat{z}$. Eq.(1.2.111) becomes

$$\omega^2 V_1 - C_s^2 k(k.V_1) = 0$$

$$\omega^2 (V_{1z}\hat{z}) - C_s^2 (k_z \hat{z})(k_z \hat{z}.V_{1z}\hat{z}) = 0$$

$$\omega^2 - C_s^2 k^{2z} V_{1z} = 0$$

$$(\omega^2 - C_s^2 k_z^2) V_{1z} = 0 \tag{1.2.112}$$

The non-trivial solutions are

$$\omega = \pm k_z C_s \tag{1.2.113}$$

The phase velocity and group velocities are:

$$V_p = \frac{\omega}{k_z} = \frac{\pm k_z C_s}{k_z} = C_s$$

$$V_p = \pm C_s \tag{1.2.114}$$

$$V_g = \frac{\partial}{\partial k_z}\omega = \frac{\partial}{\partial k_z}(\pm k_z C_s) = \pm C_s$$

$$V_g = \pm C_s \tag{1.2.115}$$

Note

Sound waves are compressional because $\nabla.V_1 \neq 0$ and are longitudinal because V_1 and k are parallel.

1.2.8 The dispersion relation for MHD waves

The continuity, momentum, induction, and adiabatic energy equations are linearized to become

$$\frac{\partial}{\partial t}\rho_1 = -V_1.\nabla\rho_0 - \rho_0\nabla.V_1 \tag{1.2.116}$$

$$\rho_0\frac{\partial}{\partial t}V_1 = \frac{(\nabla X B_1) X B_0}{4\pi} - \nabla P_1 \tag{1.2.117}$$

$$\frac{\partial}{\partial t}B_1 = \nabla X(V_1 X B_0) \tag{1.2.118}$$

$$\frac{\partial}{\partial t}\rho_1 = -V_1.\nabla P_0 - \gamma P_0\nabla.V_1 \tag{1.2.119}$$

These are obtained by ignoring the second and higher order terms and uses Ampere's law.The terms $-V_1.\nabla\rho_0$ and $-\rho_0\nabla.V_1$ vanish if we assume the background is uniform. The displacement vector, ξ, describes how much the plasma is displaced from the equilibrium state. If $\xi(r, t = 0) = 0$, then the displacement vector is

$$\xi(r,t) = \int_0^t V_1(r,t')dt' \tag{1.2.120}$$

Its time derivative is the perturbed velocity,

$$\frac{\partial \xi}{\partial t} = V_1(r,t) \tag{1.2.121}$$

$$\frac{\partial}{\partial t}\rho_1 = -V_1.\nabla\rho_0 - \rho_0\nabla.V_1 \tag{1.2.122}$$

$$= \frac{\partial \xi}{\partial t}.\nabla\rho_0 - \rho_0\nabla.\frac{\partial \xi}{\partial t} \tag{1.2.123}$$

Integrating Eq.1.2.123 with respect to time

$$\int_0^t \frac{\partial}{\partial t'}\rho_1 dt' = \int_0^t [-\frac{\partial \xi}{\partial t'}.\nabla\rho_0 - \rho_0\nabla.\frac{\partial \xi}{\partial t'}]dt' \tag{1.2.124}$$

which leads to a solution for ρ_1 in terms of just ξ.

$$\rho_1(r,t) = -\xi(r,t).\nabla\rho_0 - \rho_0\nabla.\xi(r,t) \tag{1.2.125}$$

Similarly, the linearized induction and energy equations are put in terms of ξ. Integrating the linearized equations with respect to time yields solutions for the perturbed density, magnetic field, and plasma pressure:

$$\rho_1(r,t) = -\xi(r,t).\nabla\rho_0 - \rho_0\nabla.\xi(r,t) \tag{1.2.126}$$

$$B_1(r,t) = \nabla X\frac{[\xi(r,t)XB_0(r)}{c}] \tag{1.2.127}$$

$$P_1(r,t) = -\xi(r,t).\nabla P_0(r) - \gamma P_0(r)\nabla.\xi(r,t) \tag{1.2.128}$$

The perturbed density ρ_1 doesn't appear in the other equations, which form a closed set. The linearized momentum equation can be given in terms of ξ and$F[\xi]$ using the solutions for ρ_1, B and P_1 we arrive at

$$\rho_0 \frac{\partial^2 \xi}{\partial t^2} = F[\xi(r,t)] \tag{1.2.129}$$

which is reminiscent of Newton's second law while the ideal MHD force operator is

$$F(\xi) = \nabla\xi.\nabla P_0 + \gamma P_0 \nabla.\xi + \frac{1}{4\pi}(\nabla X B_0) X [\nabla X (\xi X B_0)] + \frac{1}{4\pi}[\nabla X \nabla X (\xi X B_0)] X B_0 \tag{1.2.130}$$

which is a function of the displacement vector ξ and equilibrium fields, but not of $V_1 = \frac{\partial \xi}{\partial t}$. **Note**

The displacement vector ξ gives the direction and distance a parcel of plasma is displaced from the equilibrium state.

The force operator $F(\xi)$ gives the direction and magnitude of the force on a parcel of plasma when it is displaced by ξ. **Deriving the dispersion relation for MHD waves**

By assuming that the plasma is uniform and infinite, we Perform a Fourier analysis by assuming solutions of the form:

$$\xi(r,t) = \sum_{k,\omega} \xi(k,\omega) e^{-(k.r-\omega t)} \tag{1.2.131}$$

The linearized momentum equation is,

$$\rho_0 \frac{\partial^2}{\partial t^2} \xi = F(\xi(r,t)), \tag{1.2.132}$$

then becomes

$$\rho_0 \omega^2 \xi = k\gamma p_0 (k.\xi) + \frac{kX[kX(\xi X B_0)]XB_0}{4\pi} \tag{1.2.133}$$

Choosing cartesian axes such that

$$K = k_\perp \hat{y} + k_\parallel \hat{z} \tag{1.2.134}$$

Expanding the vector products yields

$$(\omega^2 - k_\perp^2 V_A^2)\xi_x = 0 \qquad (1.2.135)$$

$$(\omega^2 - k_\perp^2 C_s^2 - k^2 V_A^2)\xi_y - k_\perp k_\parallel C_s^2 \xi_z = 0 \qquad (1.2.136)$$

$$- k_\perp k_\parallel C_s^2 \xi_y + (\omega^2 - k_\parallel^2 C_s^2)\xi_z = 0 \qquad (1.2.137)$$

where c_s is the sound speed.

By considering a simple shear flow $V = V_0 sin(ky)\hat{x}$, for the magnetic field lying in the x-y plane, $\mathbf{B} = B_x\hat{x} + B_y\hat{y}$, the ideal induction equation.

$$\frac{\partial B}{\partial t} - \nabla X(VXB) = 0 = \frac{\partial B}{\partial t} + (V.\nabla)B + (B.\nabla)V - B(\nabla.V)$$

, becomes

$$\frac{\partial B_x}{\partial t} = V_0 sin(ky)\frac{\partial B_x}{\partial x} + V_0 k cos(ky) B_y$$

$$\frac{\partial B_y}{\partial t} = V_0 sin(ky)\frac{\partial B_y}{\partial x}$$

If the initial configuration is a uniform vertical field $B(x, 0) = B_0\hat{y}$, then the solution to the induction equation is

$$B(x, y, t) = B_0 k v_0 t cos(ky)\hat{x} + B_0\hat{y} \qquad (1.2.138)$$

Since B_y is always positive the y coordinate is just such a parameter and we can write field lines x(y). The y-parameterized field line satisfies the equation

$$\frac{dx}{dy} = \frac{B_x[x(y), y]}{B_y[x(y), y]} = k v_0 t cos(ky) \qquad (1.2.139)$$

Beginning at $x(0) = x_0$ this equation has the solution,

$$x(y) = v_0 t sin(ky) + x_0 \qquad (1.2.140)$$

The magnetic field from the induction equation is then

$$B(x,y,t) = B_0 k v_0 t cos(ky)\hat{x} + B_0\hat{y}$$

The force-density may be readily found by the equation,

$$\frac{1}{4\pi}(\nabla X B)XB = \frac{1}{4\pi}(B_0V_0t)^2k^3 sin(ky)cos(ky)\hat{y} - \frac{1}{4\pi}B_0^2k^2V_0 t sin(ky)\hat{x} \tag{1.2.141}$$

The first term is a vertical force directed away from regions of stronger magnetic field: $ky = m\pi$ there is the pressure force. The second term acts horizontally, and is strongest where the field is most curved $ky = \frac{\pi}{2}, \frac{3\pi}{2}$

By re-writing the above equ.1.2.141, we have

$$\frac{1}{4\pi}(\nabla X B)XB = -\frac{1}{8\pi}\hat{y}\frac{\partial}{\partial y}[B_0V_0ktcos(ky)]^2 + \frac{1}{4\pi}B_0\frac{\partial}{\partial y}[B_0V_0ktcos(ky)\hat{x}]$$

$$= -\frac{1}{8\pi}\nabla|B|^2 + \frac{1}{4\pi}(B.\nabla)B \tag{1.2.142}$$

To work self-consistently, we replace $v_0 t sin(ky) = x(y,t)$ and discard terms proportional to x^2. This leaves only the tension force

$$\frac{1}{4\pi}(\nabla X B)XB \simeq -\frac{B_0^2k^2}{4\pi}x(y)\hat{x} = \frac{B_0^2}{4\pi}\frac{\partial^2 x}{\partial y^2}\hat{x} \tag{1.2.143}$$

The horizontal velocity is $v = \hat{x}\frac{\partial x}{\partial t}$ and the horizontal momentum equation becomes

$$\rho\frac{\partial^2 x}{\partial t^2} = \frac{B_0^2}{4\pi}\frac{\partial^2 x}{\partial y^2} \tag{1.2.144}$$

Equation 1.2.144 is a wave equation and has a periodic solutions.It is a standing Alfven wave of wave length $\frac{2\pi}{k}$.

Finally,

$$\frac{\partial^2 x}{\partial t^2} = \frac{B_0^2}{4\pi\rho}\frac{\partial^2 x}{\partial y^2} \tag{1.2.145}$$

where

$\frac{B_0^2}{4\pi\rho} = V_A^2$ is the Alfven speed and it is simplified as

$$V_A = \sqrt{\frac{B_0^2}{4\pi\rho}} = \frac{B_0}{\sqrt{4\pi\rho}} \tag{1.2.146}$$

Chapter 2

Interstellar Magnetic field Dynamics in Earth's Magnetosphere

2.1 The Solar Wind

The region between the Sun and its planets is filled by a tenuous magnetized plasma,the solar wind ,which is a mixture of ions and electrons flowing away from the Sun. The Sun's outer atmosphere is so hot that not even the Suns enormous gravity can prevent it from continually evaporating. The solar wind is a high-speed plasma out flow which originates from the Suns corona and terminates at the boundary of interstellar space (some 160 AU from the Sun),the heliopause.As the solar wind flows through the heliosphere its properties, such as speed and temperature, change. The medium which permeates the heliosphere is known as the solar wind consists of a gas of charged particles i.e plasma flowing radially out ward from the Sun. These escaping plasma carries the solar magnetic field along, out to the border of the heliosphere where its dominance finally ends. The solar wind being under the control of the magnetic field of the sun is ejected from the sun in all directions. That is, being constraints to move along the magnetic lines of force, the solar wind has no fixed directions. It moves through space in a most complex manner filling up the whole interplanetary space with the interplanetary magnetic field (IMF).

2.1.1 The origin of the solar wind

The Sun's magnetic field can be considered as having two distinct components:- Open magnetic flux in which the field lines remain attached to the Sun and are dragged outward into the heliosphere with the solar wind and Closed magnetic flux in which the field remains entirely attached to the Sun, and forms loops and active regions in the solar corona. The solar wind is in fact the outward extension of the million degree hot upper atmosphere of the Sun, called the corona because of its crown-like shape which can be seen during eclipses. Close to the Sun, this atmosphere is strongly bound since the mean gravitational energy per ion is roughly ten times the thermal energy. However, because this medium is ionized and very hot, it conducts heat very efficiently hence the temperature decreases very slowly with altitude so that the thermal energy becomes greater than the gravitational energy beyond about ten solar radii. In static fluid equilibrium, the pressure would not decrease very much beyond this point, and since it is many times higher than that of the tenuous interstellar medium, the corona expands away into space, i.e solar wind blows from this region.

2.1.2 Types of Solar wind and compositions

The solar wind is a stream of charged particles released from the upper atmosphere of the Sun and is a collection of streams of energetic particles and escapes through the coronal holes at supersonic speeds. This plasma consists of mostly electrons, protons and alpha particles with thermal energies between 1.5 and 10 keV. Embedded within the solar-wind plasma is the interplanetary magnetic field. The composition of the solar wind is a mixture of materials found in the solar plasma, composed of ionized hydrogen (electrons and protons) with an eight percent component of helium (alpha particles) and trace amount of heavy ions and atomic nuclei: Carbon, Nitrogen, Neon, magnesium, Silicon, Sulphur and Iron ripped apart by heating of the suns outer atmosphere, that is the corona [14]. Although the solar wind is electrically balanced, and consists almost exclusively of charged particles and is an excellent electrical conductor, i.e plasma. By exploring the sun in a distance of 1 AU, at present there are known three different kinds of solar wind, each having unique properties respectively termed the slow solar wind, fast solar wind and transient wind.[13]. The slow solar wind has an average velocity of about 400 km/s, a temperature of about $1.2-1.6X10^6$K and a very high density of $11X10^6kg/m^3$ at a distance of r=1 AU from the sun. Its composition closely matches that of the corona and it represents an unsteady flow and strongly depends on the solar cycle. The origin of the slow solar wind involves magnetic reconnection, which leads to transient openings of coronal loops and feeds plasma to the slow wind. By contrast, The source of the fast solar wind is located at the coronal holes. It is stable for long periods and is therefore accredited to the quiet sun. It has a typical velocity of about 800 Km/s (varying between 600-800 km/s), a temperature of $8X10^5$ K, the density in a distance of 1 AU is very low $\approx 3X10^6kg/m^3$ and the fraction of helium is about 3-4 percent. The third type-the transient wind is related primarily to big flares and coronal mass ejections (CMEs) that may in the interplanetary space later evolve into magnetic clouds.

2.2 Earth's Magnetic field

The earth's magnetic field would resemble a simple magnetic dipole, much like a big bar magnet, except that the solar wind distorts its shape. The solar wind stretches the earth's magnetic field into a bullet shape, forming the large cavity known as the magnetosphere. The magnetosphere is that region dominated by the Earth's magnetic field. Close to the earth's surface the magnetic field approximates a dipole field, but further out the field becomes increasingly distorted. The Earth's magnetic field or geomagnetic field is produced by a dynamo process in the Earth's liquid outer core at a depth of 3000-5000 km. The magnetic field can roughly be approximated as a dipole, with its magnetic north pole located close to the geographic south pole and the magnetic south pole close to the geographic north pole. The angle between the Earth's rotation axis and the dipole axis is about 11^0 and the magnitude of the field ranges from about 30 T at the equator to 60 T at the poles. The offset between the actual observed magnetic pole and the geographic pole is 11.5^0 in the northern and 14.5^0 in the southern hemisphere

2.2.1 Properties and Origin of the Earth's Magnetic field

As a result of the motion of molten iron inside the Earth, a relatively strong magnetic field surrounds Earth. Like the magnetic field in sunspot pairs or magnets, Earth's magnetic field emerges from one hemisphere with a certain direction and points towards the opposite hemisphere[15]. Earth's magnetic field, also known as the geomagnetic field is the magnetic field that extends from the earth's interior out into space and it is a super position of magnetic fields generated by different sources. The field generated by a magnetic dynamo in the Earth's liquid core by a geodynamo mechanism is called the main field and is by far the most dominant one. It represents more than 90 percent of the geomagnetic field measured at the earth's surface. It ranges in magnitude from some 30,000nT at the equator up to more than 60,000nT in polar regions. The crustal field or Lithosphere, is also an associated magnetic field. This field is some 400 times smaller that the core contributions and generally ranges from $0to \pm$ 1000nT. The physical processes originating the lithospheric field are completely different from those generating the core field since generated by magnetized rocks in the Earth's crust. The external field, produced by electric currents flowing in the ionosphere and in the magnetosphere, owing to the interaction of the solar electromagnetic radiation and the solar wind with the Earth's magnetic field. Because ultraviolet light from the sun ionizes the atoms in the upper atmosphere, the sunlit hemisphere and strong electric currents circulate in the sunlit hemisphere generating their own magnetic fields, with intensities up to 80nT. There is also a magnetic field resulting from an electromagnetic induction process generated by electric currents induced in the crust and the upper mantle by the external magnetic field time variations which adds to the total geomagnetic field. Generally, the geomagnetic field components which make up the total geomagnetic field are: The main field, the external field and the induced field.[11]

2.3 The Magnetosphere

The terrestrial magnetosphere is a vast plasma cavity around the Earth, which is created by electrodynamic interaction between the solar wind and the Earth's magnetic field. The Earth has an internal dipole magnetic moment of $8X10^{22}Tm^3$ that produces on the Earth's surface a magnetic field of about 30,000 nT at the equator and about 60,000 nT at the poles. The solar wind compresses the sunward side of the magnetosphere to a distance of typically 10 Earth radii (RE) and it drags the night-side magnetosphere out to some 1000 Earth radii. The magnetosphere is a complex, dynamic and fluctuating system and the physical mechanisms and processes driving the magnetosphere are not fully known. However, the variable properties of the solar wind are known to play an essential role in the dynamics of the magnetosphere. Due to the huge amounts of molten iron in the Earth's outer core acting as a geodynamo, the Earth has its own geomagnetic field.The magnetosphere is that region dominated by the Earth's magnetic field. The magnetosphere is a large plasma cavity generated by the interaction of Earth's magnetic field and the solar wind plasma. Close to the earth's surface the magnetic field approximates a dipole field, but further out the field becomes increasingly distorted. The magnetosphere is blunt on the sunward side, extending out 10 to 12 RE, earth radii (64,000-75,000 km), toward the sun and also has a long tail on the anti-sunward side, extending thousands of Earth radii (millions of kilometers) in the direction away from the sun.The terrestrial magnetosphere is defined as the region of space in which electrically charged particles are influenced by the Earth's magnetic field. Its size is variable, depending upon the impinging solar wind conditions, and its shape is distorted from the dipole approximation. If the Earth were positioned in an isolated system, free of any outside magnetic interference, the configuration of its magnetic field would be approximated by a magnetic dipole field . Due to interactions with the magnetised solar wind, the dipole approximation is only valid for a couple of RE from the Earth's surface. At distances further than this, complex interactions between the IMF and the terrestrial magnetic field create structures not found in a dipole field.

2.3.1 The Structure of the Magnetosphere

The earth's magnetic field would resemble a simple magnetic dipole, much like a big bar magnet, except that the solar wind distorts its shape. The solar wind stretches the earth's magnetic field into a bullet shape, forming the large cavity known as the magnetosphere. The magnetosphere is that region dominated by the Earth's magnetic field. The magnetosphere is a large plasma cavity generated by the interaction of Earth's magnetic field and the solar wind plasma. Close to the earth's surface the magnetic field approximates a dipole field, but further out the field becomes increasingly distorted. The magnetosphere is blunt on the sunward side, extending out 10 to 12 RE, earth radii (64,000-75,000 km), toward the sun and also has a long tail on the anti-sunward side, extending thousands of Earth radii (millions of kilometers) in the direction away from the sun. This portion of the magnetosphere is called the magnetotail . Although much of the magnetosphere is nearly a perfect vacuum, its huge size allows energy in the solar wind to drive electric currents and set plasma in motion within the magnetosphere and earth's upper atmosphere. Since all space operations occur in the magnetosphere, it is important to recognize how the magnetospheric activity impacts various operations in space. The structure of the magnetosphere is best described in the frame of reference with a fixed Sun-Earth axis as shown in the figure 3.1 Adapted from NASA data system.The inner magnetosphere is composed of three populations of charged particles that are trapped in the Earth's magnetic field. These particles move in circular motions-or gyrate-around the field lines but rarely interact with each other.

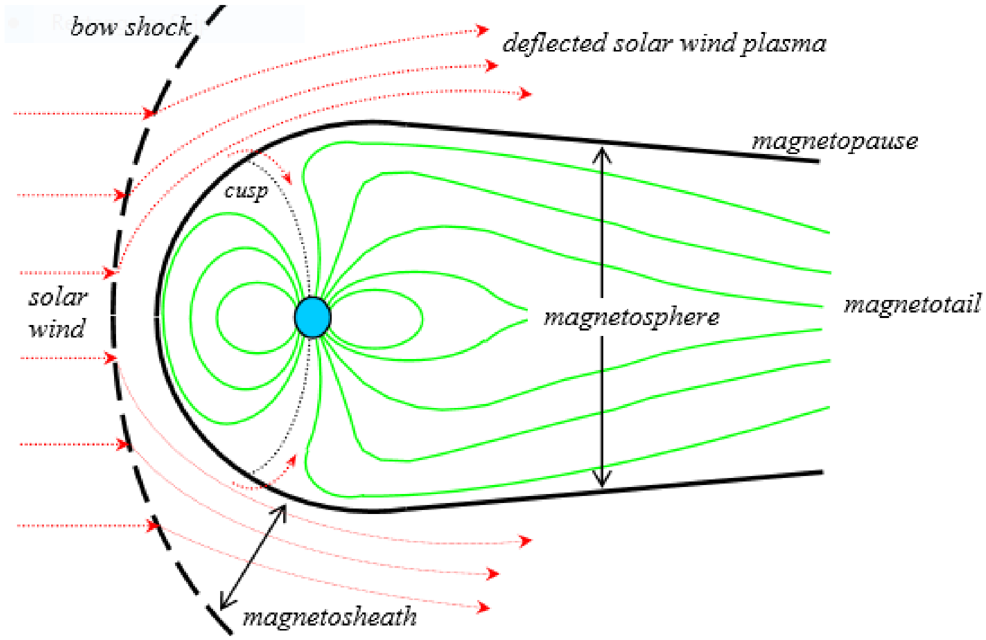

Figure 2.1: The structure of the Magnetospher: Adopted from NASA data system

The magnetosphere then stays fixed in space while the Earth rotates inside it. This system divides the magnetosphere into two parts, a day side directed towards the Sun and a night side facing the magnetotail.

Generally,the Earth's magnetosphere is a large low density plasma cavity generated by the interaction of geomagnetic field and the solar wind plasma.

2.4 Solar wind Interaction with the Earth's Magnetosphere

The size of the terrestrial magnetosphere is determined by the balance between the solar wind dynamic pressure and the pressure exerted by the magnetosphere, principally that of its magnetic field. The shape of the magnetosphere is additionally influenced by the drag of the solar wind, or tangential stress, on the magnetosphere. This drag is predominantly caused by the mechanism known as reconnection in which the magnetic field of the solar wind links with the magnetic field of the magnetosphere. Within the magnetosphere the dynamics of charged particles (the plasma) are determined by the configuration of Earth's magnetic field, which looks like a dipole farther from Earth because of its interaction with the magnetized solar wind.

2.4.1 The Magnetopause

The magnetopause is the actual boundary between the shocked solar wind and the magnetospheric plasma and it is defined by an equilibrium between the solar wind kinetic pressure and the pressure of the terrestrial magnetic field.The magnetospheric cavity is separated from the interplanetary magnetic field by the magnetopause.It is a discontinuity separating both fields forming a cavity in the solar wind. However, the magnetosphere is not closed in terms of the magnetic field but there is considerable magnetic flux crossing the magnetopause. Also the boundary does permit a certain amount of solar wind plasma entry and entry is easier along magnetic field lines. The magnetopause is a highly important region because the physical processes at this boundary control the entry of plasma, momentum, energy and the redistribution of geomagnetic flux. The position of magnetopause depends on the strength of the solar wind pressure, which is primarily due to solar wind density and velocity and balance between the solar wind pressure and the magnetic pressure of the magnetosphere. As solar wind pressure increases, it moves the magnetopause earthward. When solar wind pressure decreases, the entire magnetosphere expands.

2.4.2 The Bow Shock and the Magnetosheath

Since the solar wind is a magnetised, supersonic and superalfvenic plasma flow, a standing shock wave is formed when it encounters a magnetised obstacle such as the Earth's magnetic field. This standing shock wave known as the bow shock forms around the magnetosphere. The bow shock is prominent feature in front of the magnetosphere where the supersonic solar wind speed is slowed down to subsonic speed. The solar wind plasma and magnetic field carries with it, i.e, interplanetary magnetic field cannot cross into the earth's magnetic field because of this bow shock, but it can distort it. The bow shock slows the solar wind and begins to divert it around the magnetosphere. So that, the size, shape and location of the bow shock are not constant and are affected by changes in the solar wind. Sometimes the supersonic solar wind passes through the bow shock where it is then slowed and this passed plasma is the shocked solar wind plasma. The bow shock is the shock in front of the magnetosphere and the magnetosheath is the shocked solar wind plasma. This shocked plasma is diverted around the magnetosphere due to the frozen in magnetic field not allowing it pass through the Earth's magnetic field. Therefore, it is not directly the solar wind plasma which constitutes the boundary of the magnetosphere but the strongly heated and compressed plasma behind the bow shock. The region is rich in various wave phenomena, boundaries and shocks are often treated as discontinuities.

2.4.3 Cusps and Mantle

The cusp and mantle regions are directly adjacent and inward of the magnetopause. Are two small regions where the earth's magnetic field is perpendicular to the magnetopause. The cusp is a region where highly energetic particle can be produced and it is very active in terms of turbulence and wave energy because the boundary field lines converge in the cusp and all waves which travel along the magnetic field are channeled into this region. Cusps exists above the magnetic poles of the earth. This geometry allows to go slowly or gradually of charged solar wind protons and alpha particles to directly enter the magnetosphere. Therefore, these cusp regions are one of the main routes for solar wind plasma to enter into the Earth's magnetosphere and hence contain both magnetosheath plasma and magnetospheric plasma. The magnetospheric plasma is a mixed distribution since the magnetic field lines in the cusps connect to all regions of the magnetosphere. The solar wind particles that enter the polar cusps are funneled along magnetic field lines straight to the earth's atmosphere where they ionize atoms and emit light. This steady stream of particles causes a weak, diffuse glow extending over a large part of the polar cap of the aurora. The mantle region represents a boundary to the magnetotail usually filled with solar wind plasma but with a stretched magnetospheric magnetic field.

2.4.4 The Magnetotail

The interaction of Earth's magnetic field with the magnetized solar wind produces a long magnetotail. The magnetotail is the long tail-like extension of the magnetosphere on anti-sunward side of the magnetosphere. The magnetotail is divided into two lobes. Magnetic field lines in the northern lobe are directed toward the earth, and those in the southern lobe are directed away from the earth. Since the magnetic field points toward the Earth in the northern lobe and away in the southern lobe, there is a current in the westward direction. Because of its structure there is considerable energy stored in the magnetic field in the magnetotail. During magnetically quiet times convection is typically low and energy in the plasma flow is only a tiny fraction of the overall energy density. In the magnetotail the earth's magnetic field lines are drawn back in the antisolar direction by the motion of the solar plasma attempting to pass around the magnetopause. The solar plasma draws the tail backward for millions of kilometers where it eventually becomes indistinguishable from the interplanetary magnetic field. Beyond about 10 RE the magnetic field lines of the earth's field are essentially parallel to those of the IMF.

2.4.5 The plasma sheet and Radiation belts

Since the magnetic field resembles a dipole close to Earth, the dipole region of Earth's magnetosphere is called the inner magnetosphere. The plasma sheet is a region of concentrated hot plasma that extends from down the magnetotail. Plasma sheet has two regions:- The distant plasma sheet (neutral sheet) and the inner plasma sheet. At a time of interaction plasma concentration which stretches down the magnetotail from about 30 RE, is called the neutral sheet. Protons and electrons from the solar wind diffuse across the magnetopause in the magnetotail, drift toward the plasma sheet, and accelerate earthward. Electric current flows from dawn to dusk in the neutral sheet in order to keep the magnetic lobes separated. The energy of electrons in the neutral sheet range from 200 eV to more than 12 keV. The inner plasma sheet is the plasma sheet region extending inward from about 30 RE to about 8 RE (in the antisolar direction) and further inward as it tapers north and south toward the geomagnetic poles. The magnetic field lines of the inner plasma sheet are closed. Within the earthward edge of the inner plasma sheet (1-8 RE), magnetic field lines are less distended and more dipole-shaped in the radiation belts. Here high-energy charged particles are trapped. Charged particles spiral around magnetic field lines, reflecting from the magnetic mirror points near the magnetic poles. The radiation belt consist of two distinct regions of energetic particles. The outer belt, composed mostly of energetic electrons, has its inner edge around 3 RE and its highly variable outer edge. It contains low-energy protons (200 keV to 1 MeV) of solar wind origin. Protons and electrons from the solar wind diffuse across the magnetopause in the magnetotail, drift toward the plasma sheet and on arrival from the plasmasheet, these particles are ejected into the outer radiation belt. The protons are deflected westward by the earth's magnetic field, and the electrons are deflected eastward. The inner belt, which consists of energetic electrons and protons, extends out to about 2.5 RE. It is composed of high-energy protons (up to hundreds of MeV) primarily of terrestrial origin. The inner belt is more dipole-shaped than the outer belt and particles can be trapped for long time. The region between the belts (called the slot) is generally kept clear of energetic particles by mechanisms that enhance the loss of the particles into the ionosphere. The radiation belts contain intense radiation that can kill astronauts and damage or destroy sensitive electronics on spacecraft.[12],[15]

2.4.6 The Ring Current

In the Earth's magnetosphere, the solar wind energy is dissipated into various sinks during magnetic storms and substorms. The main energy dissipation channels are the ring current encircling the Earth at the equatorial plane, the ionospheric Joule heating, auroral precipitation, plasma sheet heating in the nightside magnetosphere, and release of plasmoids from the magnetospheric tail. About half, perhaps even a larger part, of the energy ends up in the ionosphere, of which the Joule heating consumes the main part. The ring current is an equatorial westward current flowing around the Earth at altitudes 3 to 8 RE. It results from the differential gradient, curvature, and magnetization drifts of electrons and protons in the near-Earth region. Earth's dipole magnetic field region, energetic ions flow from midnight to the dusk side, and energetic electrons flow in the opposite direction. This difference in flow directions of positively charged ions and negatively charged electrons gives rise to an electric current, a ring current that circles Earth or the dominant magnetospheric energy sinks are ring current injections, plasmoid formation and plasma sheet heating. The ring current consists of the current due to the eastward (electron) and westward (proton) drift in the radiation belts and causes a net decrease in the magnetic field on the surface of the Earth. This ring current in turn gives rise to a magnetic field that points in the opposite direction to the dipole field at Earth's surface. Therefore, the ring current decreases the strength of Earth's magnetic field as measured on the surface. When the ring current some times suddenly changes its intensity, there is a rapid decrease in magnetic field strength of the earth .

Generally,the ring current is a population of medium-energy particles that drift around the Earth, with protons drifting in one direction and electrons drifting in the opposite direction.

2.5 Magnetic reconnection

Magnetic reconnection is a fundamental process which violates the concept of frozen in flow. A direct consequence of the frozen-in theorem is that when two different plasma regimes meet (such as the solar wind and magnetosphere), a boundary layer must be formed between them (e.g. the magnetopause). This is because plasma can only flow along field lines (not across them) and so the two distinct plasma regimes cannot mix. The magnetic field topology on either side of the boundary layer can be completely different, with differing field direction and strength, but, close to the boundary, the fields will be antiparallel and tangential to the boundary. Solar wind plasma is magnetized fluid, which makes interaction between the solar wind flow and Earth's magnetosphere. When two magnetic fields are brought together, the fields combine. So if we bring two magnets close to each other and measure the strength of the field at some point, we would measure contributions from both. When this occurs in a magnetized plasma, such as in the solar wind and magnetosphere, the fields can interact in a new way to form new field lines. In this process, called magnetic reconnection, energy is taken from the magnetic field and put into particle motion (magnetic energy is converted to particle kinetic energy). Magnetic reconnection is reconfiguration of two different magnetic field topologies in which plasma elements that are initially connected to one magnetic field become attached to another magnetic field. Note that there are two distinct field lines when solar wind plasma interact with the earth's magnetosphere, one that has both ends in the solar wind and one connected to both poles of Earth. When they come together, they can reconnect, and in addition to converting some of the magnetic energy into particle kinetic energy, the two original field lines topologies are converted into two new field line topologies. Field lines with both ends connected to Earth are called closed, and those with one end connected to Earth and the other in the solar wind are called open.[15][16]

2.6 Magnetosphere-Ionosphere Coupling

The ionosphere is the region where the atmosphere is partially ionized plasma and neutrals strongly interact and often is described as the base of the magnetosphere. This interaction exerts a drag on the plasma. The plasma density can be very high but also strongly variable such that the ionospheric conductance can vary by orders of magnitude. Magnetospheric plasma motion is transmitted into the ionosphere and forces ionospheric convection. This also implies the existence of strong currents along magnetic field lines which close through the ionosphere. In particular at high latitudes these currents lead to magnetic perturbations during times of strong magnetospheric activity (fast convection and changes of the magnetospheric configuration). Currents and solar wind plasma flows couple the ionosphere and the magnetosphere. The motion of particles, plasmas, and magnetic fields gives rise to currents and currents in the magnetosphere associated with its large-scale structure current inside the magnetopause and the tail current separating the southern and northern lobes. In addition, the drift of charged particles trapped in the radiation belts gives rise to a ring current. These currents are perpendicular to the magnetic field and another kind of current flows parallel to the magnetic field and therefore can provide coupling between the ionosphere and the magnetosphere. The entire configuration then can be interpreted as a circuit with the solar wind-magnetosphere interaction working as a dynamo and the ionosphere being a load with dissipative losses.[20]

2.7 Solar Wind-Magnetosphere-Ionosphere Coupling

As discussed in the previous sections, the solar wind is a plasma out flow which constantly bombards the near-Earth system. Protecting the Earth from this bombardment is the terrestrial magnetic field which, due to the frozen-in acts as a shield preventing the plasma from stripping away the Earth's atmosphere. However, the frozen-in can, and does, break down. In cases where the magnetic field approaches unity, such as at the boundary between the magnetosheath and the magnetosphere, the frozen-in no longer holds and magnetic reconnection takes place. Magnetic reconnection couples field lines from the interplanetary magnetic field to the terrestrial magnetic field and allows solar wind particles, which would not normally be able to cross magnetic field lines, to enter into the near-Earth environment. Reconnection at the dayside magnetosphere causes open magnetic field lines, traveled at the Earth's pole, to stretch out into interplanetary space. The solar wind flow drags these open field lines anti-sunward towards the magnetotail region of the magnetosphere. In this region open field lines reconnect with those from the opposite pole to form a new geomagnetic field line and a new interplanetary field line. The geomagnetic field line travels earthward whilst the interplanetary field line flows off into interplanetary space. In addition to creating plasma flow, reconnection at the magnetopause causes an inward flux of solar wind particles to populate the terrestrial magnetic field lines. This increase in charged particles enhances magnetospheric currents, such as the ring current, and can modify the magnetic field strength on the Earth's surface. Additionally, Energetic particles like electrons, protons and alpha particles from the solar wind and the magnetosphere can penetrate the Ionosphere and contribute to an extraordinary production of ions and electrons in the ionosphere. Such particle precipitation is closely related to the northern light phenomenon, and the layers of the ionosphere vary, therefore, often very irregularly, in the auroral zone.

2.8 Geomagnetic Activity

2.8.1 Magnetic storms and substorms

Magnetic storms and substorms are the two main appearances of geomagnetic activity. The magnetic storms are the strongest geomagnetic variations caused by the Sun, producing 50-300 nT changes in the Earth's magnetic field at the equator. Magnetic storms often begin with a sudden global increase in the value of geomagnetic field, which is followed by a rapid decrease of the horizontal component during the storm main phase. Storms end with a slow recovery phase which may last from one to several days. During stormy days the energy content of the ring current increases to unusually large values, which is the main reason for the equatorial field suppression.

A substorm can be defined as a transient process initiated on the nightside of the Earth, in which a significant amount of energy, derived from the solar wind-magnetosphere interaction, is deposited in the auroral ionosphere and in the magnetosphere . Some phenomena accompanied with substorms:- (1) increase in auroral luminosity in the midnight sector, (2) Pi 2 pulsation bursts, (3) intensification of the westward electrojet, and (4) a westward traveling surge. Substorms are defined as intervals of increased energy dissipation into the auroral ionosphere, which is determined on the basis of the westward auroral electrojet. Substorms play an important role in magnetospheric energetics, in yearly averages even more important than the magnetic storms, although the latter are much more intense. Substorms are known to cause 100-2500 nT changes in the Earth's magnetic field at high latitudes, which is 0.2-4 percent of the normal magnetic field.

At the beginning of the substorm research era, substorms were thought to be building blocks of magnetic storms. However, a substorm can occur either during magnetic storms (to be called stormtime substorms, SS), or independent of storms (isolated substorms, IS). These two classes of substorms are quite different from each other interms of numbers, intensities and energies. Magnetic substorms take place when the IMF is slightly southward (about 5 nT) fore an hour or so, whereas magnetic storms develop when the IMF is strongly southward for a prolonged period, for several hours. The only visible signs of magnetic storms and substorms, the aurora, are seen in the northern(Aurora Borealis) and southern (Aurora Australis) auroral regions.[19][21]

Chapter 3

Result and Discussion

The set of summarized resistive (collision inclusive) MHD equations used in the solar wind-earth's magnetosphere interaction are:

The continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho u) = 0; \tag{3.0.1}$$

The motion equation is given by

$$\frac{\partial \rho u}{\partial t} + \nabla.(\rho u u) = -\nabla p + JXB; \tag{3.0.2}$$

The Energy equation is given by

$$\frac{1}{\gamma - 1}(\frac{\partial p}{\partial t} + \nabla.(pu)) = -p\nabla.u + \eta J^2; \tag{3.0.3}$$

The Dynamo equation is given by

$$\frac{\partial B}{\partial t} = \nabla X(uXB) + \eta \nabla^2 B; \tag{3.0.4}$$

The set of Linearized Magnetohydrodynamic equations are:

Linearized continuity equation.

$$\frac{\partial \rho_1}{\partial t} + \nabla.(\rho_0 V_1) = 0 \tag{3.0.5}$$

Linearized equation of motion

$$\rho_0 \frac{\partial}{\partial t} V_1 + \nabla P_1 - \frac{(\nabla X B_1) X B_0}{4\pi} = 0 \tag{3.0.6}$$

Linearized energy equation

$$\frac{\partial}{\partial t} P_1 + \gamma P_0 \nabla . V_1 = 0 \tag{3.0.7}$$

Linearized dynamo equation

$$\frac{\partial B_1}{\partial t} = \nabla X (V_1 X B_0) \tag{3.0.8}$$

The speed of sound $C_s$is

$$C_s = \sqrt{\frac{\gamma P_0}{\rho_0}} \tag{3.0.9}$$

The Alfven speed $V_A$is

$$V_A = \sqrt{\frac{B_0^2}{4\pi\rho}} \tag{3.0.10}$$

The coupling of the flowing plasma (the solar wind plasma), earth's magnetic field including its ionosphere particle distribution play key role in the earth's magnetosphere geometrical structure where the expected dipole configuration is being disturbed with the like of drifts, drags, diffusion, etc. So we discuss on how the summarized MHD equations inform the IMF dynamics in magnetosphere during the solar wind interaction with the earths atmosphere.

Note that Desta & Tolu (2016), did work on the implications of the decoupled MHD equations in the interaction of the system. But it lacks the how these equations do so. Here we re-introduce it in terms of the linerized ones.

1. The dynamo equation

The ratio of the second term (commonly known as the inertial force) to the first term (known as the viscous force) of the right hand side of equation 3.0.4 called the Lundquist of magnetic Reynolds number is used to measure and analyse magnetic diffusion through the fluid. Considering small changes to apply linearization, the Reynolds number R is given by

$$R = \frac{uL}{\eta} or R = \mu_0 \sigma u L \tag{3.0.11}$$

where L is the characteristic length over which the field change. Equation 3.0.4 implies that in a highly conducting ($\sigma \to \infty$ or $\eta \to 0$) fluid the diffusion is negligible and thus the second term vanishes. Then, the total magnetic flux crossing a surface S bounded by a closed curve at some initial time, will remain constant in time as the plasma fluid moves through the system and the location of S and or its shape change in agreement with the work of Alfven and Falthammar (1963). All plasma elements initially connected by a magnetic flux tube of cross section S then remain linked by the same flux tube in time as the plasma drifts through space, a consequence of the frozen-in magnetic field. Here, we observe that the plasma regimes of different properties can mix easily along B, but not perpendicularly. The magnetic field is then tangential on either side of the boundary, but will in general be of different magnitude and direction with a current sheet at the boundary according to Ampres law. On the other hand if the plasma flow on either side is negligible, the dynamo equation is dominated by the diffusion component. In this case, in particular if B is oppositely directed on either side of an infinite plane, but of same magnitude and locally parallel to the plane the magnetic flux will then diffuse toward the current sheet where it annihilates and magnetic field energy is converted into heat. In time, the gradient in the field decreases, results to reduce diffusion rate as the local plasma pressure builds up; a self-limiting process of balancing pressure. Finally, when the magnetic Reynolds number $\simeq 1$ the current sheet then equals unity, which means that the MHD frozen-in flux condition breaks down. However, the continuity equation requires that there must be an outflow as long as there is a plasma inflow and a second dimension gets introduced that sets a limit to the extension of the diffusion region. where the magnetic reconnection come into picture. But it is difficult to describe when and where exactly the reconnection will occur. However, literature reviews point out the possible factors believed to influence the probability of the reconnection onset and the rate by which reconnection takes place.

2.The conservation law of MHD equations, discontinuity layers, shocks and structures of the magnetosphere

According to the current understanding, beyond being the source of energy for life on earth, the sun streams out hot charged particle (plasma) called the solar wind. The Earth is to a large extent protected against this plasma flow by its magnetic field, which is generated by the thermal convection of its liquid interior. The geomagnetic field deflects most of the solar wind around the Earth, and creates a cavity in the solar wind that we call the magnetosphere. Because of the magnetic field geometry of the earth at higher and lower latitudes, due to day-side and night-side temperature distributions, atmospheric particle density distribution variation with altitude, orientation (facing) of the earth with respect to the sun and the solar wind distributions and variations there are set of magnetosphere layers classified based on mean characteristic parameters. For example, the first boundary layer that first interacts with the solar wind is characterized by collisionless plasma by the similar reason as described under case (1). At this boundary, conservation of mass, conservation of momentum, conservation of energy are all being considered to derive the appropriate dynamical parameters like B both along the direction of the wind and tangential to the shock bow at the interface of the solar wind and the magnetosphere.

Chapter 4

Summary and Conclusion

The solar wind carries a frozen-in magnetic field. Its convection to the earth's atmosphere provides inside into configurational changes in the magnetic topology, distribution of magnetic energy, mixing of plasma and connection of different region of the magnetosphere. This interaction has a great effect on the near-Earth space environment. This space weather interaction further affects human life activities and life on earth. As a result, today there are enormous studies and interest in solar-terrestrial environments and interactions due to the availability of satellite observations of space, particularly in the near-Earth environment. However, though, there is an overall progress in the observational works the theories that fit the observations lack correlation and need further works pertaining to the difficulty of exploiting the full Magnetohydrodynamic (MHD) equations. If there is any, most works assume the collision-free system with simplify boundary conditions. In this thesis we did work on the MHD equations with simple boundary conditions and were able to conclude that they indeed work within limitations such as over mean global characteristic parameters including the steady solar wind etc. But has limitations to determine exactly eventful observations like coronary mass ejections and their interaction with the magnetosphere and the local magnetic storms. We also observe that it is important to develop more advanced computational works to extract more accurate data from the full MHD equations.

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