A Coherently Driven Superposed N Two-Level Laser with Vacuum Reservoir

A Thesis Submitted to the Department of Physics

Jimma University

In Partial Fulfilment of the Requirement of the Degree of

Master of science in Physics(Quantum Optics and Information)

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Jimma, Ethiopia

January 2019

DECLARATION

I hereby declare that this Msc thesis is my original work and has not been presented for a degree in any other university, and that all sources of material used for the thesis have been duly acknowledged.

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Place and date of submission:

Department of Physics

Jimma University

January 2019

JIMMA UNIVERSITY

DEPARTMENT OF PHYSICS

January, 2019

Author: **Tamiru Deressa**

Title: **A Coherently Driven Superposed** N **Two Level Laser With Vacuum Reservoir**

.

Degree:**M.SC**

Convocation: **January** Year: **2019**

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Acknowledgement

First of all, I would like to thanks almighty God for letting me accomplished this thesis. Secondly, I would like to express my deepest heartfelt gratitude and respect to my advisor and instructor, Dr. Tamirat Abebe, for formulating this Msc thesis and for inspirational guidance, his unreserved support and effort, critical comments and helpful discussion through out my research work. I would like to thank Mr. Milkessa Gebeyehu, my co-advisor, who give me moral and materials support. Finally, I am deeply indebted to my families especially to my brother Mr. Sori Deressa, who give me materials support and my wife Dirribe Haile for her sincere encouragement and moral support, as well as, all member of Department of physics of Jimma university.

Abstract

The dynamics of a coherently driven two level laser with vacuum reservoir is analyzed. The combination of interaction Hamiltonian and Langevin equation is presented to study the quantum properties of light. By using these equations, we have determined the time evolution of the expectation values of the cavity mode and atomic operators. With aid of this results, the correlation properties of the noise operators and the large-time approximation scheme, we calculate the mean photon number, the quadrature variance and the power spectrum for the cavity light.

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Introduction

A two level laser is a quantum optical system in which light is generated by two level atoms inside a cavity and coupled to vacuum reservoir. A two level laser is a source of coherent or chaotic light emitted by two level atoms inside a closed cavity and coupled to vacuum reservoir via single-port mirror[1]. In other model the two-level atoms available in the cavity are pumped at constant rate to the upper level by coherent light[2-7]. Some authors are interest to study the properties of light generated by two level laser [8-13]. In addition, Fesseha [1] has studied the squeezing and the statistical properties of the light produced by a degenerate two-level laser with the atoms in a closed cavity and pumped by electron bombardment.

There has been a considerable interest in the analysis of the squeezing and statistical properties of the light generated by two-level lasers [14-28]. The squeezing and statistical properties of the light produced by two-level lasers when the atoms are initially prepared in a coherent superposition of the top and bottom levels or when these levels are coupled by a strong coherent light have been studied by several authors [29-31]. Other authors have found that the quantum optical systems can generate squeezed light under certain conditions, the properties of the fluorescent light emitted by two level atom in a cavity driven by coherent light and coupled to squeezed vacuum have been studied by several authors [5, 9, 13, 15].

A coherently driven two level atom in closed cavity operating below threshold chaotic [5, 9, 10, 21, 22, 25]. Moreover, some authors have studied the squeezing and the statistical properties of the light produced by a two-level laser with the atoms in a closed cavity and pumped by electron bombardment[1-10]. The maximum quadrature squeezing of the light generated by the laser operating below threshold, is found to be 50% below the vacuum-state level [10,13, 15, 29, 30].
In this study we will consider a closed cavity and a two level- laser coupled with vacuum reservoir.

This thesis have two parts. In the first part, we wish to study the squeezing and statistical properties of the light generated by a coherently driven two-level laser in a closed cavity and coupled to a vacuum reservoir via a single-port mirror. We carry out our calculation by putting the noise operators associated with the vacuum reservoir in normal order. We thus first determine the interaction Hamiltonian for a coherently driven two-level laser in a closed cavity and coupled to a vacuum reservoir and the quantum Langevin equations for the cavity mode operators. In addition, employing the interaction Hamiltonian Langevin equations, Heisenberg equation and the large-time approximation scheme, we obtain equations of evolution of the expectation values of atomic operators. Moreover, we determine the solutions of the equations of evolution of the expectation values of the atomic operators and cavity mode operators. Then applying the resulting solutions, we calculate the photon statistics and the quadrature variances of the single-mode cavity light beams. Furthermore, applying the same solutions, we determine the quadrature squeezing of the single-mode cavity light.

In the second part of this thesis, we seek to analyze the squeezing and statistical properties of a pair of superposed two-level laser light beams produced by a coherently driven two-level atom with a closed cavities and coupled to a vacuum reservoirs. We thus first determine the Q function for two-level laser light beam. Then using the resulting Q function we obtain the density operator for a pair of superposed two-level laser light beams. Applying this density operator, we calculate the photon statistics and the quadrature squeezing of the superposed two-level laser light beams.

2

Operator Dynamics and Cavity Mode Operator

In this chapter, we consider a two-level laser driven by coherent light and with the cavity modes coupled to a single-mode vacuum reservoir via a single-port mirror.

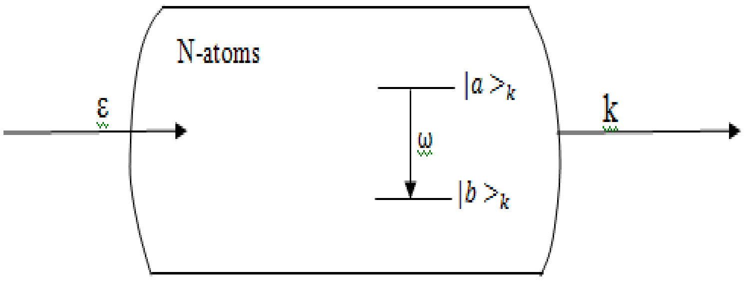

Figure 2.1: schematic representation of a coherently driven two level laser in a closed cavity and coupled to a vacuum reservoir .

As clear indicated, in figure. (1), a two level atom with upper and lower energy eigenstate represented by $|a\rangle$ and $|b\rangle$ respectively.

2.1 The Interaction Hamiltonian

We consider here the case in which N two- level atoms are available in a cavity. Then the interaction of the cavity mode with one of the atom can be described at resonance by Hamiltonian

$$\hat{H} = ig(\hat{\sigma}_k^\dagger \hat{b} - \hat{b}^\dagger \hat{\sigma}_k) + \frac{i\Omega}{2}(\hat{\sigma}_k^\dagger - \hat{\sigma}_k), \tag{2.1}$$

where

$$\hat{\sigma}_k = |b\rangle_{kk}\langle a| \tag{2.2}$$

is the lowering atomic operator, $\hat{b}$ is the annihilation operator for the cavity mode and g is the coupling constant between the atom and the cavity mode and $\Omega = 2\epsilon\lambda$ in which ε is considered to be real and constant in the amplitude of the driving coherent light and λ is coupling constant between the driving coherent light and atomic operator.

2.2 Quantum Lagevin Equation

We recall that the laser light is coupled to vacuum reservoir via a single-port mirror. In addition, we carry out our calculation by putting the noise operator associated with vacuum reservoir in normal order. Thus the noise operator will not have any effect on the dynamics of the cavity mode operators [8]. We can then drop the noise operators and write the quantum Langevin equation for the operator $\hat{b}$ as

$$\frac{d\hat{b}}{dt} = -\frac{\kappa}{2}\hat{b} - i[\hat{b}, \hat{H}], \tag{2.3}$$

where κ is cavity damping constant, then in view of Eq. (2.1), we see the quantum Langevin equation

$$\frac{d\hat{b}}{dt} = -\frac{\kappa}{2}\hat{b} - g\hat{\sigma}_k. \tag{2.4}$$

The procedure of normal ordering the noise operators renders the vacuum reservoir to be in noiseless physical entity. We uphold the view point that the predication mode base on the would turn out to be in agreement with observation, Furthermore, employing the Heisenberg equation

$$\frac{d}{dt}\langle\hat{A}\rangle = -i\langle[\hat{A},\hat{H}]\rangle. \tag{2.5}$$

With aid of Eq. (2.1), one can readily obtain

$$\frac{d}{dt}\langle\hat{\sigma}_k\rangle = -i\Big\langle\Big[\hat{\sigma}_k, ig(\hat{\sigma}_k^\dagger\hat{b} - \hat{b}^\dagger\hat{\sigma}_k) + \frac{i\Omega}{2}(\hat{\sigma}_k^\dagger - \hat{\sigma}_k)\Big]\Big\rangle.$$

$$= A + B, \tag{2.6}$$

where

$$A = g\Big([\hat{\sigma}_k, \hat{\sigma}_k^\dagger\hat{b}] - \hat{b}^\dagger[\hat{\sigma}_k, \hat{\sigma}_k]\Big), \tag{2.7}$$

$$B = i\Big\langle[\hat{\sigma}_k, \frac{i\Omega}{2}(\hat{\sigma}_k^\dagger - \hat{\sigma}_k)]\Big\rangle, \tag{2.8}$$

$$[\hat{\sigma}_k, \hat{\sigma}_k] = 0, \tag{2.9}$$

$$\hat{\sigma}_k\hat{\sigma}_k^\dagger = \hat{\sigma}_b^k, \tag{2.10}$$

$$\hat{\sigma}_k^\dagger\hat{\sigma}_k = \hat{\sigma}_a^k. \tag{2.11}$$

In view of these results, we have

$$A = \Big(\langle\hat{\sigma}_b^k\hat{b}\rangle - \langle\hat{\sigma}_a^k\hat{b}\rangle\Big) \tag{2.12}$$

and

$$B = \frac{\Omega}{2}\Big(\langle\hat{\sigma}_b^k\rangle - \langle\hat{\sigma}_a^k\rangle\Big). \tag{2.13}$$

Therefore, by adding Eqs. (2.12) and (2.13), one can get

$$\frac{d}{dt}\langle\hat{\sigma}_k\rangle = g(\langle\hat{\sigma}_b^k\hat{b}\rangle - \langle\hat{\sigma}_a^k\hat{b}\rangle) + \frac{\Omega}{2}(\langle\hat{\sigma}_b^k\rangle - \langle\hat{\sigma}_a^k\rangle). \tag{2.14}$$

Following the same procedure

$$\frac{d}{dt}\langle\hat{\sigma}_a^k\rangle = g(\langle\hat{\sigma}_k^\dagger\hat{b}\rangle + \langle\hat{b}^\dagger\hat{\sigma}_k\rangle) + \frac{\Omega}{2}(\langle\hat{\sigma}_k^\dagger\rangle + \langle\hat{\sigma}_k\rangle), \tag{2.15}$$

$$\frac{d}{dt}\langle\hat{\sigma}_b^k\rangle = -g(\langle\hat{\sigma}_k^\dagger\hat{b}\rangle + \langle\hat{b}^\dagger\hat{\sigma}_k\rangle) - \frac{\Omega}{2}(\langle\hat{\sigma}_k^\dagger\rangle + \langle\hat{\sigma}_k\rangle). \tag{2.16}$$

Employing large time-approximation of Eq. (2.4), one can obtain

$$\hat{b} = \frac{-2g}{\kappa}\hat{\sigma}_k \tag{2.17}$$

and adjoint of this equation is

$$\hat{b}^\dagger = \frac{-2g}{\kappa}\hat{\sigma}_k^\dagger. \tag{2.18}$$

Therefore, when substitute Eqs. (2.17) and (2.18) into Eq. (2.14), we find

$$\frac{d}{dt}\langle\hat{\sigma}_k\rangle = -\frac{\gamma_c}{2}\langle\hat{\sigma}_b\rangle + \frac{\Omega}{2}(\langle\hat{\sigma}_b^k\rangle - \langle\hat{\sigma}_a^k\rangle), \tag{2.19}$$

$$\frac{d}{dt}\langle\hat{\sigma}_a^k\rangle = -\gamma_c\langle\hat{\sigma}_a^k\rangle + \frac{\Omega}{2}(\langle\hat{\sigma}_k^\dagger\rangle + \langle\hat{\sigma}_k\rangle), \tag{2.20}$$

$$\frac{d}{dt}\langle\hat{\sigma}_b^k\rangle \;=\; \gamma_c\langle\hat{\sigma}_a^k\rangle - \frac{\Omega}{2}(\langle\hat{\sigma}_k^\dagger\rangle + \hat{\sigma}_k\rangle), \tag{2.21}$$

where

$$\gamma_c \;=\; \frac{4g^2}{\kappa} \tag{2.22}$$

is stimulated emission decay constant.

We next Eqs. (2.19)-(2.21) sum over the N two-two level atoms, so that

$$\frac{d}{dt}\langle\hat{m}\rangle = -\frac{\gamma_c}{2}\langle\hat{m}\rangle + \frac{\Omega}{2}\Big[\langle N_b\rangle - \langle N_a\rangle\Big], \tag{2.23}$$

$$\frac{d}{dt}\langle N_a\rangle = -\gamma_c\langle N_a\rangle + \frac{\Omega}{2}\Big[\langle\hat{m}^\dagger\rangle + \langle\hat{m}\rangle\Big], \tag{2.24}$$

$$\frac{d}{dt}\langle N_b\rangle = \gamma_c\langle N_a\rangle - \frac{\Omega}{2}\Big[\langle\hat{m}^\dagger\rangle + \langle\hat{m}\rangle\Big], \tag{2.25}$$

where

$$\hat{m}_a \;=\; \sum_{k=1}^{N}\hat{\sigma}_a^k, \tag{2.26}$$

$$\hat{m}_b \;=\; \sum_{k=1}^{N}\hat{\sigma}_b^k, \tag{2.27}$$

$$\hat{m} = \sum_{k=1}^{N}|b\rangle_{kk}\langle a| = N|b\rangle\langle a|, \tag{2.28}$$

$$\hat{N}_a \;=\; \sum_{k=1}^{N}\hat{\eta}_a, \tag{2.29}$$

$$\hat{N}_b \;=\; \sum_{k=1}^{N}\hat{\eta}_b. \tag{2.30}$$

with the operator $\hat{N}_a$ and $\hat{N}_b$ representing the number of atom in the top and bottom levels respectively. In addition, employing the completeness relation.

$$\hat{\eta}_a + \hat{\eta}_b = I. \tag{2.31}$$

Sum over N of the above equation results in

$$\langle \hat{N}_a \rangle + \langle \hat{N}_b \rangle = N. \tag{2.32}$$

Furthermore, Using the definition given by Eq. (2.2) and adjoint, setting for any k

$$\hat{\sigma}_k = |b\rangle\langle a|, \tag{2.33}$$

from a which follows

$$\hat{m} = N|b\rangle\langle a|. \tag{2.34}$$

Following the same procedure, one can find

$$\hat{N}_a = N|a\rangle\langle a|, \tag{2.35}$$

$$\hat{N}_b = N|b\rangle\langle b|, \tag{2.36}$$

$$\hat{m}^\dagger \hat{m} = N\hat{N}_a, \tag{2.37}$$

$$\hat{m}\hat{m}^\dagger = N\hat{N}_b, \tag{2.38}$$

$$\hat{m}^2 = 0. \tag{2.39}$$

In the presence of N two-level atoms, we can rewrite Eq. (2.4) as

$$\frac{d}{dt}\hat{b} = -\frac{\kappa}{2}\hat{b} + \lambda\hat{m}. \tag{2.40}$$

where λ is constant and whose value remains to be fixed. Employing Eq. (2.17) and (2.18), the commutation relation for the cavity mode operator are found to be

$$[\hat{b}, \hat{b}^\dagger] = \frac{4g^2}{\kappa^2}(\hat{\sigma}_k\hat{\sigma}_k^\dagger - \hat{\sigma}_k^\dagger\hat{\sigma}_k). \tag{2.41}$$

With aid of Eq. (2.10) and (2.11), one can readily obtain

$$[\hat{b}, \hat{b}^\dagger] = \frac{\gamma_c}{\kappa}(\hat{\sigma}_b^k - \hat{\sigma}_a^k). \tag{2.42}$$

and on summing over all atoms, we have

$$[\hat{b}, \hat{b}^\dagger] = \frac{\gamma_c}{\kappa}(N_b - N_a), \tag{2.43}$$

where

$$[\hat{b}, \hat{b}^\dagger] = \sum_{k=1}^{N}[\hat{b}, \hat{b}^\dagger]_k \tag{2.44}$$

stands for the commutation of $\hat{b}$ and $\hat{b}^\dagger$, when the cavity mode is interacting with all the N two level atoms. On the other hand, using the steady-state solution of Eq. (2.40), one can easily verify that.

$$[\hat{b}, \hat{b}^\dagger] = N\left(\frac{2\lambda}{\kappa}\right)^2\left(\langle\hat{N}_b\rangle - \langle\hat{N}_a\rangle\right). \tag{2.45}$$

Thus on account of Eqs. (2.43) and (2.45), we see that

$$\lambda = \pm\frac{g}{\sqrt{N}}. \tag{2.46}$$

Hence in veiw of this result the equation of evolution of the cavity operator given by Eq. (2.40), can be rewritten as

$$\frac{d}{dt}\hat{b} = -\frac{\kappa}{2}\hat{b} + \frac{g}{\sqrt{N}}\hat{m}. \tag{2.47}$$

2.3 Solution of the expectation value of the cavity and atomic mode operators

In order to determine the mean photon number and the variance of the photon number and the quadrature squeezing of the single mode cavity light in any frequency interval at steady state, we first need to calculate the solution of the equation of evolution of the expectation value of the atomic and the cavity mode operator. To this end the expectation value of the solution of Eq. (2.47) is expressible as

$$\langle\hat{b}(t)\rangle = \langle\hat{b}(0)\rangle e^{\frac{-\kappa t}{2}} + \frac{g}{\sqrt{N}} e^{\frac{-\kappa t}{2}} \int_0^t dt' e^{\frac{-\kappa t'}{2}} \langle\hat{m}(t')\rangle. \tag{2.48}$$

We next wish to obtain the expectation value of expression of $\hat{m}(t)$ that in Eq. (2.48). Thus applying the large time approximation scheme of Eqs. (2.23) and (2.24) to find

$$\frac{d}{dt}\langle\hat{m}\rangle = -\frac{1}{2}(\frac{\gamma_c^2 + 2\Omega^2}{\gamma_c})\langle\hat{m}(t)\rangle + \frac{\Omega}{2}N. \tag{2.49}$$

Upon setting , $\mu = \frac{\gamma_c^2+2\Omega^2}{\gamma_c}$, Eq. (2.49) reduce to

$$\frac{d}{dt}\langle\hat{m}\rangle = -\frac{\mu}{2}\hat{m}(t) + \frac{\Omega}{2}N. \tag{2.50}$$

Following that from steady state solution of Eq. (2.50), we obtain

$$\langle\hat{m}(t)\rangle = \frac{\Omega}{\mu}N. \tag{2.51}$$

Now substituting Eq. (2.51) into Eq. (2.48), One can get

$$\langle\hat{b}(t)\rangle = \langle\hat{b}(0)\rangle e^{\frac{-\kappa t}{2}} + \frac{g}{\sqrt{N}} e^{\frac{-\kappa t}{2}} \int_0^t dt' e^{\frac{-\kappa t'}{2}} (\frac{\Omega}{\mu}N)$$

$$= \langle\hat{b}(0)\rangle e^{\frac{-\kappa t}{2}} - \frac{g}{\sqrt{N}} e^{\frac{-\kappa t}{2}} (\frac{\Omega}{\mu} N) \int_0^t dt' e^{\frac{-\kappa t'}{2}}$$

$$= \langle\hat{b}(0)\rangle e^{\frac{-\kappa t}{2}} - \frac{g}{\sqrt{N}} e^{\frac{-\kappa t}{2}} (\frac{\Omega}{\mu} N)(-\frac{1}{\kappa t})[e^{\frac{-\kappa t'}{2}}]|_0^t$$

$$= \langle\hat{b}(0)\rangle e^{\frac{-\kappa t}{2}} + \frac{g}{\sqrt{N}} e^{\frac{-\kappa t}{2}} (\frac{2\Omega}{\mu\kappa} N)[e^{\frac{-\kappa t}{2}} - 1]$$

$$= \langle\hat{b}(0)\rangle e^{\frac{-\kappa t}{2}} + \frac{g}{\sqrt{N}} (\frac{2\Omega}{\mu\kappa} N)[e^{-\kappa t} - e^{\frac{-\kappa t}{2}}].$$

and at steady-state

$$\langle\hat{b}(t)\rangle = 0. \tag{2.52}$$

Therefore, in view of the linear equation described by expressions of Eq. (2.47) with Eq. (2.52), we claim that $\langle\hat{b}(t)\rangle$ is Gaussian variable with zero mean. We finally seek to determine the solution of expectation value of the atomic operator at steady - state. Moreover, the steady state solution of Eqs. (2.23)- (2.25), yield

$$\langle\hat{m}\rangle = \left[\frac{\Omega\gamma_c}{\gamma_c^2 + 2\Omega^2}\right] N, \tag{2.53}$$

$$\langle\hat{N}_a\rangle = \left[\frac{\Omega^2}{\gamma_c^2 + 2\Omega^2}\right] N, \tag{2.54}$$

$$\langle\hat{N}_b\rangle = \left[\frac{\gamma_c^2 + \Omega^2}{\gamma_c^2 + 2\Omega^2}\right] N, \tag{2.55}$$

From commutation relation of the cavity operator, one can get

$$\lambda = \left(\frac{\gamma_c N}{\kappa}\right)\left(\frac{\gamma_c^2}{\gamma_c^2 + 2\Omega^2}\right). \tag{2.56}$$

Upon setting $\eta = \frac{\Omega}{\gamma_c}$ into Eqs. (2.53) (2.54), (2.55) and (2.56), one can obtain

$$\langle\hat{m}\rangle = \left[\frac{\eta}{1 + 2\eta^2}\right] N, \tag{2.57}$$

$$\langle \hat{N}_a \rangle = \left[\frac{\eta^2}{1+2\eta^2}\right] N, \tag{2.58}$$

$$\langle \hat{N}_b \rangle = \left[\frac{1+\eta^2}{1+2\eta^2}\right] N, \tag{2.59}$$

$$\lambda = \frac{\gamma_c}{\kappa} N \left[\frac{1}{1+2\eta^2}\right]. \tag{2.60}$$

Intially (when $\eta = 0$), all the atom are on the lower level i.e $\langle N_b \rangle$=N while the number of atoms on the upper energy level is zero and if $\eta \to \infty$, $\langle \hat{N}_a \rangle = \frac{1}{2}$N and $\langle \hat{N}_b \rangle = \frac{1}{2}$N

3

Photon statistics

In this chapter, we seek to study the statistical properties of the light produced by a coherently driven two level laser in closed cavity and coupled to a vacuum reservoir. Applying the solution of the equation of evaluation of the expectation value of the atomic operator and the quantum Langavian equation for cavity mode operator, we obtain the global and local for mean and variance of photon numbers statistics of cavity light.

3.1 Mean photon number

In this section we seek to obtain the global mean and variance of photon numbers, as well as the local mean and variance of the photon number.

3.1.1 Global mean photon number

Here we seek to determine the mean photon number of cavity light in the entire frequency interval produced by the system under consideration. The mean photon number represented by the operators $\hat{b}$ and $\hat{b}^\dagger$ is defined by

$$\overline{n} = \langle \hat{b}^\dagger \hat{b} \rangle. \tag{3.1}$$

We note that the steady-state solution of Eq. (2.47) is

$$\hat{b} = \frac{2g}{k\sqrt{N}}\hat{m}, \tag{3.2}$$

so that introducing Eq. (3.2) and its adjoint into Eq. (3.1), we get

$$\overline{n} = \frac{\gamma_c}{\kappa}\langle N_a\rangle. \tag{3.3}$$

Now substitute Eq. (2.55) in to (3.3), one can get

$$\overline{n} \;=\; \frac{\gamma_c}{\kappa}\left[\frac{\Omega^2}{\gamma_c^2+2\Omega^2}\right]N. \tag{3.4}$$

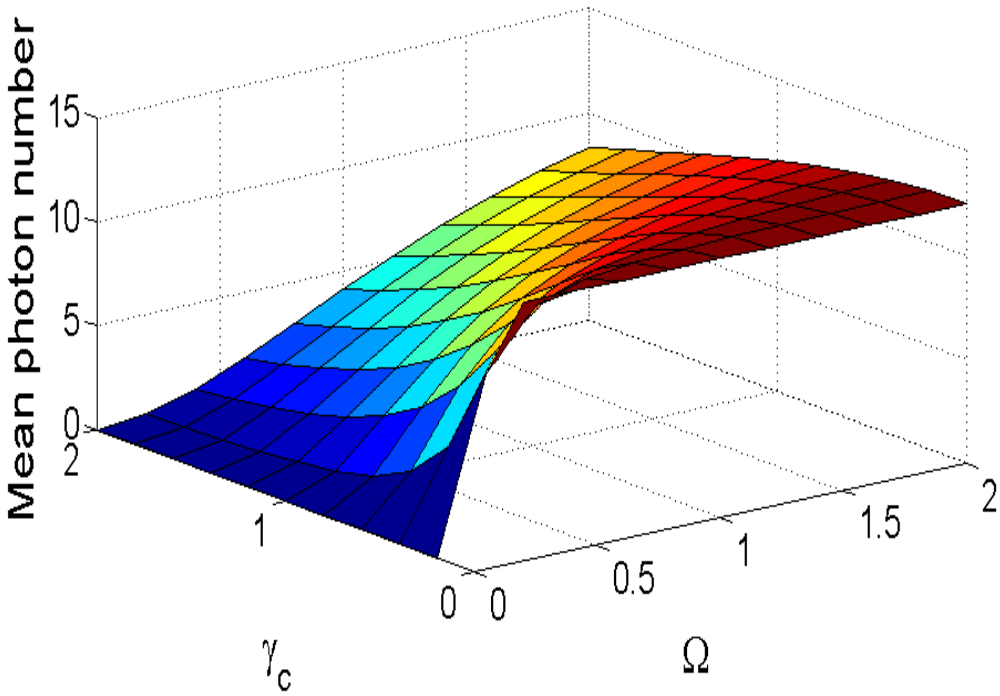

Figure 3.1: The 3D plots of Eq.(3.4) Mean photon number v_s Ω and γ_c, for $\gamma_c = 0.4$, $\Omega = 2$, $\kappa = 0.8$ and N=50

This is the steady - state mean photon number produced by a coherently driven two level laser in closed cavity and coupled to a vacuum reservoir and when multiplying Eq. (3.4) by $\frac{1}{\gamma_c^2}$ on both nominator and denominator.

$$\overline{n} = \frac{\gamma_c}{\kappa}\left[\frac{\frac{\Omega^2}{\gamma_c^2}}{\frac{\gamma_c^2}{\gamma_c^2} + 2(\frac{\Omega}{\gamma_c})^2}\right]N, \tag{3.5}$$

Setting $\frac{\Omega}{\gamma_c} = \eta$

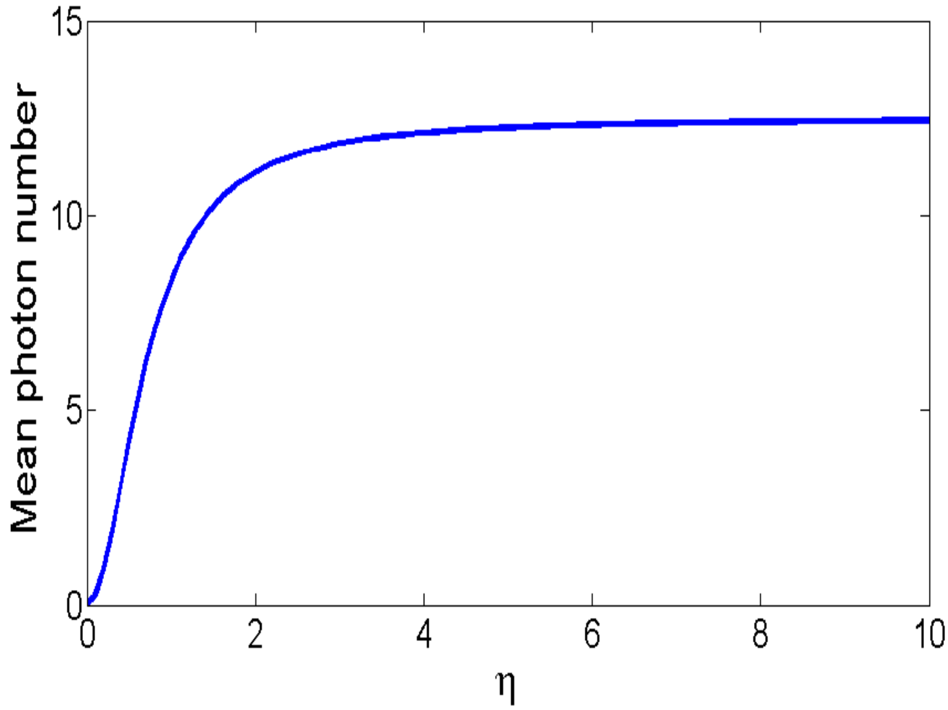

Figure 3.2: The plots of Eq.(3.6) Mean photon number v_s η for $\gamma_c = 0.4$, $\kappa = 0.8$ and N=50

$$\overline{n} = \frac{\gamma_c}{\kappa}\left[\frac{\eta^2}{1 + 2\eta^2}\right]N. \tag{3.6}$$

It is not difficult to see, for $\eta \gg 1$, that

$$\overline{n} = \frac{\gamma_c}{2\kappa}N. \tag{3.7}$$

3.1.2 Local mean photon number

We seek to determine the mean photon number in a given frequency interval. Employing the power spectrum of single mode cavity light with central common frequency ω_o defined as

$$P(\omega) \;=\; \frac{1}{\pi}Re\int_0^\infty d\tau e^{i(\omega-\omega_o)\tau}\langle\hat{b}^\dagger(t)\hat{b}(t+\tau)\rangle_{ss}. \tag{3.8}$$

Next we seek to calculate the two- time correlation function for single mode cavity light. To this end, we realize that the solution of Eq. (2.48) can written as

$$\hat{b}(t+\tau) \;=\; \langle\hat{b}(t)\rangle e^{\frac{-\kappa\tau}{2}} + \frac{g}{\sqrt{N}}e^{\frac{-\kappa\tau}{2}}\int_0^\tau d\tau' e^{\frac{-\kappa\tau'}{2}}\hat{m}(t+\tau'). \tag{3.9}$$

On other hand, one can put Eq. (2.50) in the form

$$\frac{d}{dt}\langle\hat{m}\rangle \;=\; -\frac{\mu}{2}\hat{m}(t) + \frac{\Omega}{2}N + \hat{F}_m(t), \tag{3.10}$$

in which $\hat{F}_m$ is the noise operator with zero mean,the solution is expressible as

$$\begin{aligned}\hat{m}(t+\tau) \;&=\; \hat{m}(t)e^{-\frac{\mu\tau}{2}} + e^{-\frac{\mu\tau'}{2}}\int_0^\tau d\tau'(\frac{\Omega}{\mu})N(t'+\tau') \\ &+e^{-\frac{\mu\tau}{2}}\int_0^\tau d\tau' e^{-\frac{\mu\tau}{2}}\hat{F}_m(t'+\tau'). \end{aligned}\tag{3.11}$$

Substituting Eq. (3.12) into (3.9), one can obtain

$$\begin{aligned}\hat{b}(t+\tau) \;&=\; \hat{b}(t)e^{-\frac{\kappa\tau}{2}} + \frac{g}{\sqrt{N}}e^{-\frac{\kappa\tau}{2}}\int_0^\tau d\tau' e^{-\frac{\kappa\tau'}{2}}\hat{m}(t)e^{-\frac{\mu\tau'}{2}} \\ &+e^{-\frac{\mu\tau'}{2}}\int_0^{\tau'} d\tau''\frac{\Omega}{\mu}N(t+\tau'') \\ &+e^{-\frac{\mu\tau'}{2}}\int_0^{\tau'} +e^{-\frac{\mu\tau'\tau}{2}}\hat{F}_m(t+\tau\prime\prime). \end{aligned}\tag{3.12}$$

On multiplying both sides on the left by $\hat{b}^\dagger(t)$ and taking expectation value of the resulting equation, we get

$$\langle\hat{b}^\dagger(t)\hat{b}(t+\tau)\rangle = \langle\hat{b}^\dagger(t)\hat{b}(t)\rangle_{ss}e^{-\frac{\kappa\tau}{2}} + \frac{g}{\sqrt{N}}e^{+\frac{\kappa\tau}{\mu}}\int_0^\tau d\tau' e^{-\frac{(\kappa-\mu)\tau'}{2}}\langle\hat{b}^\dagger(t)\hat{m}(t)\rangle$$
$$+\frac{g}{\sqrt{N}}e^{-\frac{\kappa\tau}{2}}\int_0^\tau d\tau' e^{\frac{(\kappa-\mu)\tau'}{2}}\int_0^{\tau'} d\tau''(\frac{\Omega}{\mu})$$
$$\times\Big[\langle\hat{b}^\dagger(t)(t+\tau'')\rangle + \langle\hat{b}^\dagger(t)\hat{F}_m(t+\tau\prime\prime)\rangle\Big]. \tag{3.13}$$

Moreover, applying the large-time approximation scheme to Eq. (2.48), we can obtain

$$\hat{m}(t) = \frac{\kappa\sqrt{N}}{2g}\hat{b}(t). \tag{3.14}$$

Since the cavity mode operator and noise operator of atomic mode are not correlated, we see that

$$\langle\hat{b}(t)\hat{F}_m(t+\tau\prime\prime)\rangle = 0. \tag{3.15}$$

Hence introducing Eq. (3.14) and Eq. (3.17) into Eq. (3.13), one can readily find

$$\langle\hat{b}^\dagger(t)\hat{b}(t+\tau)\rangle_{ss} = \overline{n}\Big[\frac{\kappa}{\kappa-\mu}e^{-\frac{\mu\tau}{2}} - \frac{\mu}{\kappa-\mu}e^{-\frac{\mu\tau}{2}}\Big], \tag{3.16}$$

where

$$\overline{n} = \langle\hat{b}^\dagger(t)\hat{b}(t)\rangle_{ss}. \tag{3.17}$$

On introducing Eq. (3.16) into Eq. (3.8) and carrying out the integration, we see that

$$P(w) = \overline{n}\Big\{\frac{\kappa}{\kappa-\mu}\Big[\frac{\frac{\mu}{2\pi}}{(\frac{\mu}{2})^2+(\omega-\omega_o)^2}\Big] - \frac{\mu}{\kappa-\mu}\Big[\frac{\frac{\kappa}{2\pi}}{(\frac{\kappa}{2})^2+(\omega-\omega_o)^2}\Big]\Big\}. \tag{3.18}$$

The mean photon number in the frequency interval between $\omega' = -\lambda$ and $\omega' = +\lambda$ is expressible as

$$n_{\pm\lambda} = \int_{-\lambda}^{+\lambda} P(\omega')d\omega', \tag{3.19}$$

in which $\omega' = \omega - \omega_o$. Thus upon substituting Eq. (3.18) into Eq. (3.19) we find

$$n_{\pm\lambda} = \left[\frac{\kappa\overline{n}}{\kappa-\mu}\right]\int_{-\lambda}^{+\lambda}\left[\frac{\frac{\mu}{2\pi}}{(\omega-\omega_o)^2+(\frac{\mu}{2})^2}\right]d\omega'$$

$$-\left[\frac{\mu\overline{n}}{\kappa-\mu}\right]\int_{-\lambda}^{\lambda}\left[\frac{\frac{\kappa}{2\pi}}{(\omega-\omega_o)^2+(\frac{\kappa}{2})^2}\right]d\omega'. \tag{3.20}$$

And on carrying out the integration over ω' by applying the relation

$$\int_{-\lambda}^{+\lambda}\frac{dx}{x^2+a^2} = \frac{2}{a}\tan^{-1}\left(\frac{\lambda}{a}\right) \tag{3.21}$$

we arrive at

$$\overline{n}_{\pm\lambda} = z(\lambda)\overline{n}, \tag{3.22}$$

where

$$z(\lambda) = \left[\frac{\frac{2\kappa}{\pi}}{\kappa-\mu}\right]\tan^{-1}\left(\frac{2\lambda}{\mu}\right) - \left[\frac{\frac{2\mu}{\pi}}{\kappa-\mu}\right]\tan^{-1}\left(\frac{2\lambda}{\kappa}\right). \tag{3.23}$$

We can see from of Eq. (3.23) that z(0.5)=0.9611 z(1)=0.9764 and z(2)=0.9833, then from combination of these result with Eq.(3.23), yields $\overline{n}_{\pm 0.5} = 0.9611\overline{n}$, $\overline{n}_{\pm 1} = 0.9764\overline{n}$ and $\overline{n}_{\pm 2} = 0.9833\overline{n}$. From these results one can easily observe that large part of the total mean photon number is confined in relative small frequency interval.

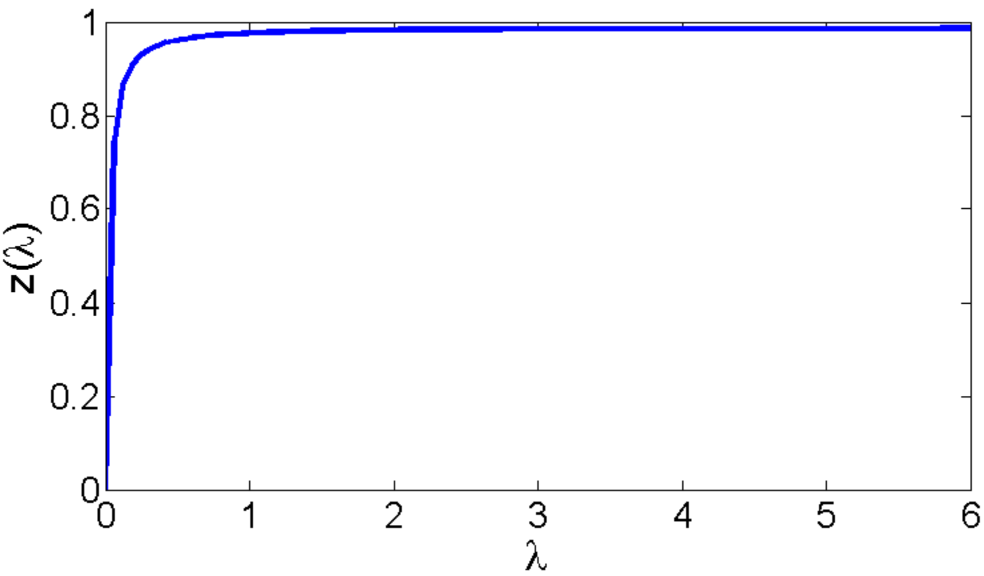

Figure 3.3: plot of Eq.(3.23) $z(\lambda)\ v_s\ \lambda$, for $\gamma_c = 0.8$, $\Omega = 8$, and $\kappa = 0.05$.

3.2 variance of photon number

Here we seek to determine the global and Local variance of photon number of the cavity produced by a coherently driven two-level laser in closed cavity and coupled to vacuum reservoir.

3.2.1 Global photon number variance

Here we seek to obtain the variance of photon number in the entire frequency mode interval. The variance of photon number is defined as

$$(\Delta n)^2 \quad = \quad \langle (\hat{b}^\dagger \hat{b})^2 \rangle - \langle \hat{b}^\dagger \hat{b} \rangle^2. \tag{3.24}$$

Applying the fact that $\hat{b}$ is a Gaussian variable with zero mean, we arrive at

$$(\Delta n)^2 \quad = \quad \langle \hat{b}^\dagger \hat{b} \rangle \langle \hat{b}^\dagger \hat{b} \rangle + \langle \hat{b} \hat{b}^\dagger \rangle \langle \hat{b}^\dagger \hat{b} \rangle - \langle \hat{b}^\dagger \hat{b} \rangle^2 + \langle \hat{b}^{\dagger 2} \rangle \langle \hat{b}^2 \rangle, \tag{3.25}$$

from which follows

$$(\Delta n)^2 = \langle \hat{b}^\dagger \hat{b} \rangle \langle \hat{b} \hat{b}^\dagger \rangle + \langle \hat{b}^{\dagger 2} \rangle \langle \hat{b}^2 \rangle. \tag{3.26}$$

In view of the steady state solution of Eq. (2.47) along Eqs, (2.37)-(2.40), we see that

$$\langle \hat{b}^2 \rangle = 0, \tag{3.27}$$

$$\langle \hat{b}^\dagger \hat{b} \rangle = \frac{\gamma_c}{\kappa} \langle N_a \rangle, \tag{3.28}$$

$$\langle \hat{b} \hat{b}^\dagger \rangle = \frac{\gamma_c}{\kappa} \langle N_b \rangle. \tag{3.29}$$

Introducing Eqs. (3.27)-(3.29) into Eq. (3.26), results in

$$(\Delta n)^2 = \left(\frac{\gamma_c}{\kappa} \right)^2 \langle N_a \rangle \langle N_b \rangle. \tag{3.30}$$

Since with aid of Eq. (3.3) and (3.25), we get

$$(\Delta n)^2 = \overline{n} \lambda + \overline{n}^2. \tag{3.31}$$

and by substitute Eqs. (2.56) and (3.4) into (3.31), we can find

$$(\Delta n)^2 = \left[\frac{\gamma_c}{\kappa} N \right]^2 \left\{ \left[\frac{\Omega \gamma_c}{\gamma_c^2 + 2\Omega^2} \right]^2 + \left[\frac{\Omega^2}{\gamma_c^2 + 2\Omega^2} \right]^2 \right\}, \tag{3.32}$$

By setting $\eta = \frac{\Omega}{\gamma_c}$, we have

$$(\Delta n)^2 = \left[\frac{\gamma_c}{\kappa} N \right]^2 \left\{ \left[\frac{\eta}{1 + 2\eta^2} \right]^2 + \left[\frac{\eta^2}{1 + 2\eta^2} \right]^2 \right\}. \tag{3.33}$$

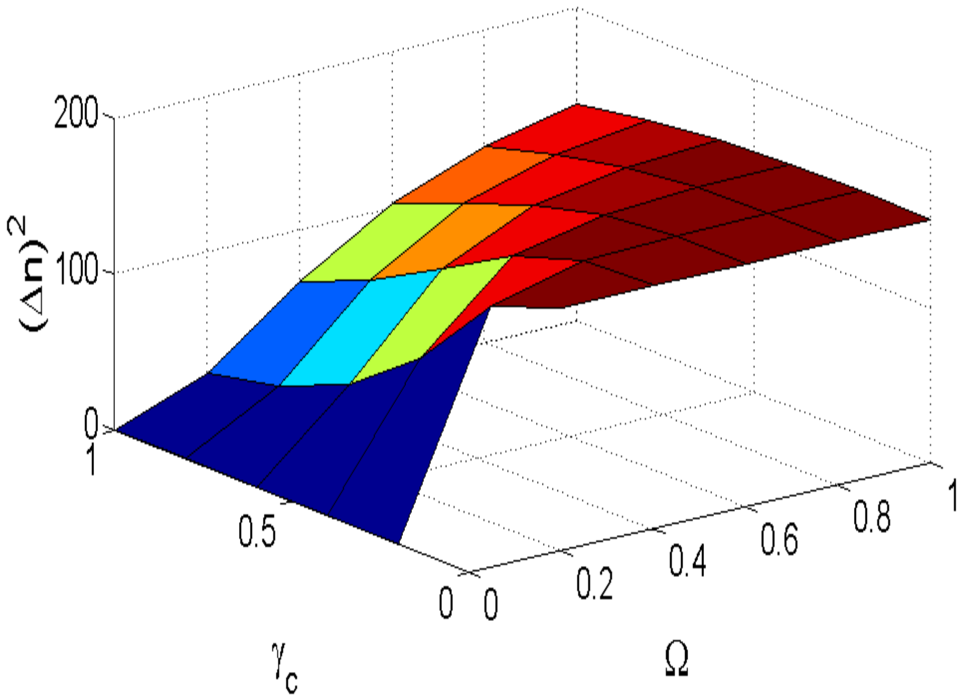

Figure 3.4: The 3D plots of Eq.(3.32) Δn^2 v_s Ω and γ_c, for $\kappa = 0.8$ and N=50

This is is the global photon number variance of cavity light produced by the coherently driven two-level laser with a closed cavity and coupled to vacuum reservoir. For $(\eta \gg 1)$, we see that

$$(\Delta n)^2 = \frac{1}{4}\left(\frac{\gamma_c}{\kappa}N\right)^2 = \overline{n}^2, \tag{3.34}$$

where $\overline{n}$ is given by (3.7).

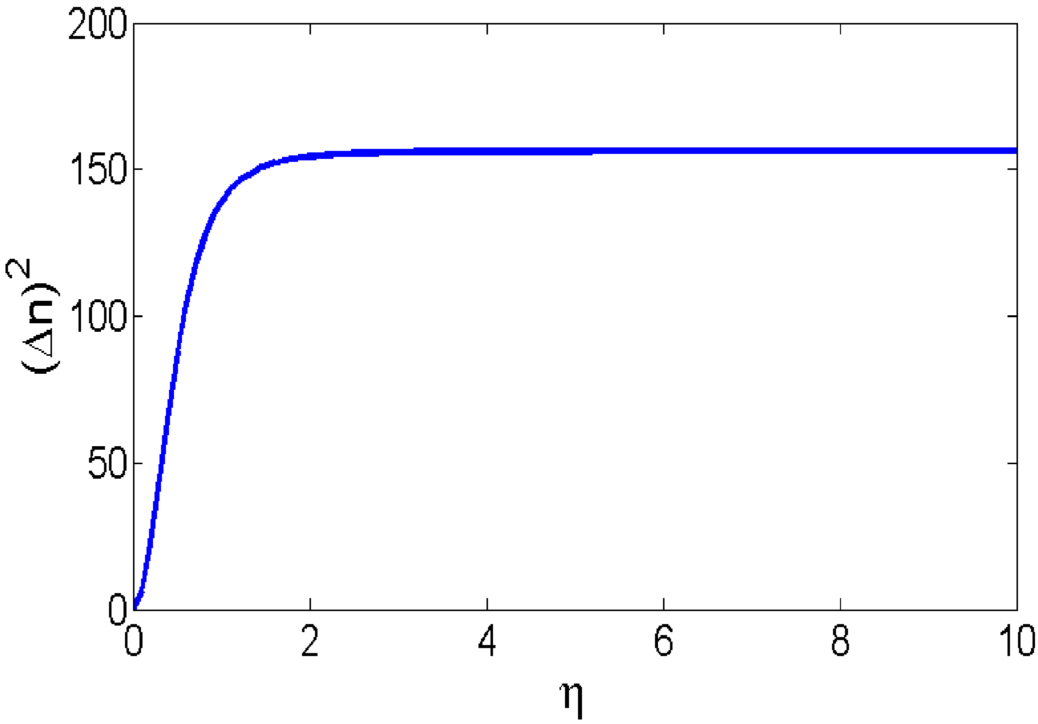

Figure 3.5: plots of Eq.(3.33) $(\Delta n)^2$ v_s η, for $\gamma_c = 0.4$, $\kappa = 0.8$ and N=50

3.2.2 The local variance of photon number

Here we wish to obtain the variance of photon number in a given frequency interval. Employing the spectrum of photon number fluctuation for the superposition of light of central common frequency of ω_o, this spectrum of the photon number fluctuation can be expressed as

$$\Gamma(\omega) = \frac{1}{\pi} Re \int_0^{\infty} d\tau e^{i(\omega-\omega_o)\tau} \langle \hat{n}(t), \hat{n}(t+\tau) \rangle_{ss}, \tag{3.35}$$

where

$$\hat{n}(t) = \hat{b}^{\dagger}(t)\hat{b}(t), \tag{3.36}$$

and

$$\hat{n}(t+\tau) = \hat{b}^{\dagger}(t+\tau)\hat{b}(t+\tau). \tag{3.37}$$

Applying the relation [11],

$$\langle\hat{n}(t),\hat{n}(t+\tau)\rangle = \langle\hat{n}(t)\hat{n}(t+\tau)\rangle - \langle\hat{n}(t)\rangle\langle\hat{n}(t+\tau)\rangle, \tag{3.38}$$

and with aid of Eqs. (3.36) and (3.37) we obtain

$$\begin{aligned}\langle\hat{n}(t),\hat{n}(t+\tau)\rangle &= \langle\hat{b}^{\dagger}(t)\hat{b}^{\dagger}(t+\tau)\rangle\langle\hat{b}(t)\hat{b}(t+\tau)\rangle \\ &+\langle\hat{b}(t)\hat{b}^{\dagger}(t+\tau)\rangle\langle\hat{b}^{\dagger}(t+\tau)\rangle. \end{aligned}\tag{3.39}$$

Introducing Eq. (3.39) into Eq. (3.35), the photon number fluctuation can be expressed as

$$\begin{aligned}\Gamma(\omega) &= \frac{1}{\pi}Re\int_0^{\infty} d\tau e^{i(\omega-\omega_o)\tau}\Big[\langle\hat{b}^{\dagger}(t)\hat{b}^{\dagger}(t+\tau)\rangle\langle\hat{b}(t)\hat{b}(t+\tau)\rangle \\ &+\langle\hat{b}(t)\hat{b}^{\dagger}(t+\tau)\rangle\langle\hat{b}(t)\hat{b}^{\dagger}(t+\tau)\rangle\Big]. \end{aligned}\tag{3.40}$$

By using Eq.(3.12) and multiply $\hat{b}(t)$ from the left on both side, one can obtain

$$\begin{aligned}\langle\hat{b}(t)\hat{b}(t+\tau)\rangle &= \langle\hat{b}^2\rangle e^{-\frac{\kappa\tau}{2}} + \frac{g}{\sqrt{N}}e^{-\frac{\kappa\tau}{2}}\int_0^{\infty} d\tau e^{\frac{\kappa\tau'}{2}}\langle\hat{b}(t)\hat{m}(t)\rangle e^{-\frac{\mu\tau'}{2}} \\ &+e^{-\frac{\mu\tau'}{2}}\int_0^{\infty} d\tau''(-\frac{\Omega}{2})\langle\hat{b}(t)N(t+\tau'')\rangle \\ &+e^{-\frac{\mu\tau'}{2}}\int_0^{\infty} d\tau'' e^{-\frac{\mu\tau''}{2}}\langle\hat{b}(t)\hat{F}_m(t+\tau'')\rangle. \end{aligned}\tag{3.41}$$

Since the cavity mode operator at earlier time does not affect the noise operator and atomic operator at later time, we have

$$\langle\hat{b}(t)N(t+\tau')\rangle = \langle\hat{b}(t)\hat{F}_m(t+\tau'')\rangle = 0. \tag{3.42}$$

Using Eq. (3.14) and Eq. (3.41), we have

$$\langle\hat{b}(t)\hat{b}(t+\tau)\rangle_{ss} = \langle\hat{b}^2(t)\rangle e^{-\frac{\kappa\tau}{2}} + \frac{g}{\sqrt{N}}e^{-\frac{\kappa\tau}{2}}\int_0^\infty d\tau e^{\frac{\kappa\tau'}{2}}\langle\hat{b}(t)\hat{b}(t)\rangle\frac{\kappa\sqrt{N}}{2g}e^{-\frac{\mu\tau'}{2}}, \tag{3.43}$$

from which follows

$$\langle\hat{b}(t)\hat{b}(t+\tau)\rangle_{ss} = \langle\hat{b}^2(t)\rangle\left[\frac{\kappa}{\kappa-\mu}e^{-\frac{\mu\tau}{2}} - \frac{\mu}{\kappa-\mu}e^{-\frac{\kappa\tau}{2}}\right]. \tag{3.44}$$

Following a similar procedure, we see that

$$\langle\hat{b}^\dagger(t)\hat{b}(t+\tau)\rangle_{ss} = \langle\hat{b}(t)\hat{b}(t)\rangle\left[\frac{\kappa}{\kappa-\mu}e^{-\frac{\mu\tau}{2}} - \frac{\mu}{\kappa-\mu}e^{-\frac{\kappa\tau}{2}}\right], \tag{3.45}$$

$$\langle\hat{b}^\dagger(t)\hat{b}^\dagger(t+\tau)\rangle_{ss} = \langle\hat{b}^{\dagger^2}(t)\rangle\left[\frac{\kappa}{\kappa-\mu}e^{-\frac{\mu\tau}{2}} - \frac{\mu}{\kappa-\mu}e^{-\frac{\kappa\tau}{2}}\right], \tag{3.46}$$

$$\langle\hat{b}(t)\hat{b}^\dagger(t+\tau)\rangle_{ss} = \langle\hat{b}(t)\hat{b}^\dagger(t)\rangle_{ss}\left[\frac{\kappa}{\kappa-\mu}e^{-\frac{\mu\tau}{2}} - \frac{\mu}{\kappa-\mu}e^{-\frac{\kappa\tau}{2}}\right]. \tag{3.47}$$

Upon introducing Eqs. (3.44)- (3.47) into (3.40) and on carrying out the integration over τ, the spectrum of the photon number fluctuation for single mode cavity light is found to be

$$\Gamma(\omega) = \frac{1}{\pi}Re\int_0^\infty d\tau e^{i(\omega-\omega_o)\tau}\left[\langle\hat{b}^\dagger\hat{b}\rangle\langle\hat{b}\hat{b}^\dagger\rangle + \langle\hat{b}^2\rangle\langle\hat{b}^{\dagger^2}\rangle\right] \times\left[\frac{\kappa}{\kappa-\mu}e^{-\frac{\mu\tau}{2}} - \frac{\mu}{\kappa-\mu}e^{-\frac{\kappa\tau}{2}}\right]. \tag{3.48}$$

Now introducing Eq. (3.26) into (3.48), one can obtain

$$\Gamma(\omega) = (\Delta n)^2 \Big\{ \Big[\frac{\kappa^2}{(\kappa-\mu)}\Big]\Big[\frac{\frac{\mu}{2\pi}}{(\omega-\omega_o)^2+(\frac{\mu}{2})^2}\Big] + \Big[\frac{\mu^2}{(\kappa-\mu)}\Big]\Big[\frac{\frac{\kappa}{\pi}}{(\omega-\omega_o)^2+(\frac{\kappa}{2})^2}\Big] - \Big[\frac{2\kappa\mu}{(\kappa-\mu)^2}\Big]\Big[\frac{\frac{\kappa+\mu}{2\pi}}{(\omega-\omega_o)^2+(\frac{\kappa+\mu}{2})^2}\Big]\Big\}. \quad (3.49)$$

Upon integrating both sides of Eq. (3.49) over ω, we can easily that

$$\int_{-\infty}^{\infty} \Gamma(\omega) d\omega = (\Delta n)^2_{ss}. \quad (3.50)$$

On the basis of Eq. (3.50), we observe that $\Gamma(\omega)d\omega$ represent the steady-state variance of the photon number for the single- mode cavity light in the interval between ω and $\omega + d\omega$, we thus realize that the photon number variance in the interval between $\omega' = -\lambda$ and $\omega' = +\lambda$

$$(\Delta n)^2_{\pm\lambda} = \int_{-\lambda}^{+\lambda} \Gamma(\omega) d\omega, \quad (3.51)$$

in which $\omega' = \omega - \omega$. Thus on substituting Eq. (3.49) into Eq. (3.51) and on carrying out the integration over ω' by applying the relation described in Eq. (3.21) we readily get

$$(\Delta n)^2_{\pm\lambda} = (\Delta n)^2 z'(\lambda), \quad (3.52)$$

where

$$z'(\lambda) = \Big[\frac{\frac{2\kappa^2}{\pi}}{(\kappa-\mu)^2}\Big]\tan^{-1}\Big(\frac{\lambda}{\mu}\Big) + \Big[\frac{\frac{2\mu^2}{\pi}}{(\kappa-\mu)^2}\Big]\tan^{-1}\Big(\frac{\lambda}{\kappa}\Big) - \Big[\frac{\frac{4\kappa\mu}{\pi}}{(\kappa-\mu)^2}\Big]\tan^{-1}\Big(\frac{2\lambda}{\kappa+\mu}\Big). \quad (3.53)$$

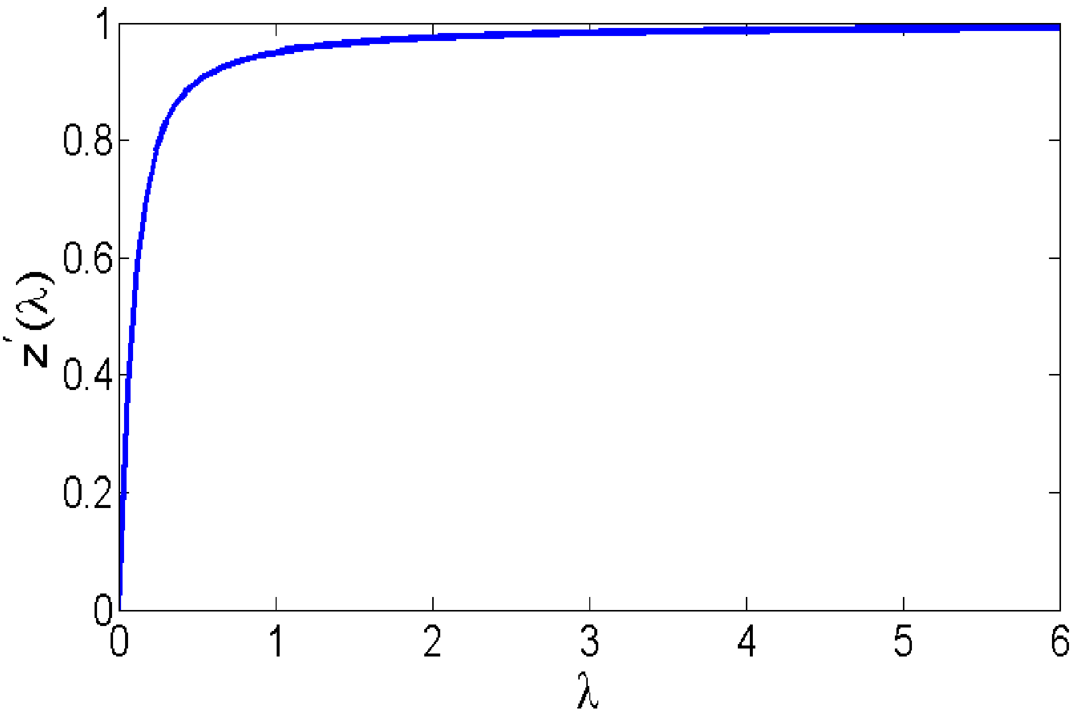

Figure 3.6: plot of Eq.(3.53)$z'(\lambda)v_s\lambda$,for $\gamma_c = 0.4, \Omega = 6$ and $\kappa = 0.08.$

One can readily observe from figure. (3.4) that $z'(0.5) = 0.8991$ $z'(1) = 0.9495$ and $z'(2) = 0.9746$. Then combination of these results with Eq.(3.53) yields $(\Delta n)^2_{\pm 0.5} = 0.8991(\Delta n)^2$, $(\Delta n)^2_{\pm 1} = 0.9495(\Delta n)^2$ and $(\Delta n)^2_{\pm 2} = 0.9746(\Delta n)^2$. Therefore, a large part of total variance of photon number is confined in relative small frequency interval.

4

Quadrature Squeezing

Applying the steady-state solutions of the equations of evolution of the expectation values of the atomic operators and the quantum Langevin equations for the cavity mode operators, we obtain the global quadrature variance of cavity light. In addition, we determine the Local quadrature squeezing of the single-mode cavity light.

4.1 Quadrature variance

In this section, we obtain the quadrature variance of light in the entire frequency interval produced by the system under consideration.

4.1.1 Global quadrature variance

Here we wish to calculate the quadrature variance of the cavity light in the entire frequency interval. The squeezing properties of the cavity light are described by two quadrature operators

$$\hat{b}_+ = \hat{b}^\dagger + \hat{b} \tag{4.1}$$

and

$$\hat{b}_- = i(\hat{b}^\dagger - \hat{b}), \tag{4.2}$$

where $\hat{b}_+$ and $\hat{b}_-$ are hermitian operators representing physical quantities called plus and minus quadrature respectively while $\hat{b}^\dagger$ and $\hat{b}$ are the creation and annihilation operators for light mode. With help of Eq. (4.1) and Eq. (4.2), we can show that the two quadrature operators satisfy commutation relation.

$$[\hat{b}_-, \hat{b}_+] = \hat{b}_-\hat{b}_+ - \hat{b}_+\hat{b}_- \tag{4.3}$$

and we note that

$$\hat{b}_-\hat{b}_+ = i(\hat{b}^{\dagger^2} + \hat{b}^\dagger\hat{b} - \hat{b}\hat{b}^\dagger - \hat{b^2}), \tag{4.4}$$

$$\hat{b}_+\hat{b}_- = i(\hat{b}^{\dagger^2} - \hat{b}^\dagger\hat{b} + \hat{b}\hat{b}^\dagger - \hat{b^2}). \tag{4.5}$$

On account of these results, we see that

$$[\hat{b}_-, \hat{b}_+] = -2\lambda i. \tag{4.6}$$

It then follows that

$$[\hat{b}_-, \hat{b}_+] = 2i\frac{\gamma_c}{\kappa}(\langle N_b\rangle - \langle N_a\rangle). \tag{4.7}$$

In view of this result and using the relation

$$\Delta A\Delta B \geq \frac{1}{2}|\langle[A, B]\rangle|,$$

the uncertainty relation for the plus and minus quadrature operator is expressible as

$$\Delta b_+\Delta b_- \geq \frac{1}{2}|\langle[b_+, b_-]\rangle|$$
$$\geq \frac{1}{2}|2i\langle\lambda\rangle|$$

$$\geq |\langle\lambda\rangle|.$$

With aid of Eq. (2.43), we have

$$\Delta b_{+}\Delta b_{-} \geq \frac{\gamma_c}{\kappa}|\langle N_b\rangle - \langle N_a\rangle|. \tag{4.8}$$

Now introducing Eqs. (2.54) and (2.55) into (4.8), one can get

$$\Delta b_{+}\Delta b_{-} \geq \frac{\gamma_c}{\kappa}N\left|\frac{\gamma_c^2}{\gamma_c^2 + 2\Omega^2}\right|, \tag{4.9}$$

Moreover, we consider the case in which the driving coherent light is absent, thus upon setting $\Omega = 0$ in Eq. (4.9), we readily get

$$\Delta b_{+}\Delta b_{-} \geq \frac{\gamma_c N}{\kappa}. \tag{4.10}$$

Therefore, we notice that the product of the uncertainties in the two quadratures satisfies the minimum uncertainty relation in a vacuum state.

Next we produced to calculate quadrature variance of the plus and minus quadrature operator are defined by

$$(\Delta b_{\pm})^2 \quad = \quad \langle\hat{b}_{\pm}^2\rangle - \langle\hat{b}_{\pm}\rangle^2. \tag{4.11}$$

One can also write

$$(\Delta b_{+})^2 \quad = \quad \langle\hat{b}_{+}^2\rangle - \langle\hat{b}_{+}\rangle^2 \tag{4.12}$$

and

$$(\Delta b_{-})^2 \quad = \quad \langle\hat{b}_{-}^2\rangle - \langle\hat{b}_{-}\rangle^2. \tag{4.13}$$

With aid of Eqs. (4.1) and (4.12), can be expressed in terms of creation and annihilation operator

$$(\Delta b_{+})^2 \quad = \quad \lambda + 2\langle\hat{b}^{\dagger}\hat{b}\rangle + \langle\hat{b}^{\dagger^2}\rangle + \langle\hat{b}^2\rangle - \langle\hat{b}\rangle^2 - \langle\hat{b}^{\dagger}\rangle^2 - 2\langle\hat{b}\rangle\langle\hat{b}^{\dagger}\rangle. \tag{4.14}$$

In addition, on account of Eqs. (4.2) and (4.13), we get

$$(\Delta b_-)^2 = \lambda + 2\langle \hat{b}^\dagger \hat{b}\rangle - \langle \hat{b}^{\dagger 2}\rangle - \langle \hat{b}^2\rangle + \langle \hat{b}\rangle^2 + \langle \hat{b}^\dagger\rangle^2 - 2\langle \hat{b}\rangle\langle \hat{b}^\dagger\rangle. \qquad (4.15)$$

The combination of Eqs. (4.14) and (4.15) gives

$$(\Delta b_\pm)^2 = \lambda + 2\langle \hat{b}^\dagger \hat{b}\rangle \pm \langle \hat{b}^{\dagger 2}\rangle \pm \langle \hat{b}^2\rangle \mp \langle \hat{b}\rangle^2 \mp \langle \hat{b}^\dagger\rangle^2 - 2\langle \hat{b}\rangle\langle \hat{b}^\dagger\rangle. \qquad (4.16)$$

Employing Eqs. (3.29)- (3.31) into (4.16), one can readily obtain

$$(\Delta b_\pm)^2 = \left(\frac{\gamma_c}{\kappa}N\right)\left(\frac{2\Omega^2 + \gamma_c^2}{\gamma_c^2 + 2\Omega^2}\right). \qquad (4.17)$$

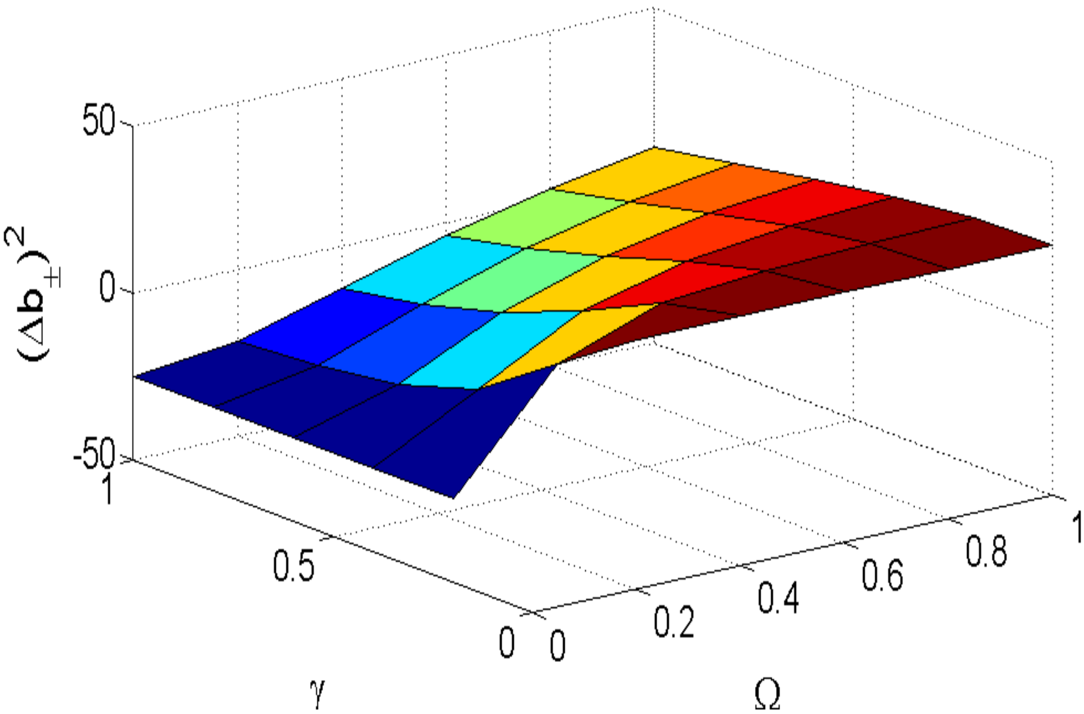

Figure 4.1: The plots of Eq.(4.17) $\overline{n}$ v_s Ω and γ_c, for $\kappa = 0.8$ and N=50

Thus upon Setting $\eta = \frac{\Omega}{\gamma_c}$ on Eq. (4.17), we see that

$$(\Delta b_{\pm})^2 = \left(\frac{\gamma_c}{\kappa}N\right)\left(\frac{2\eta^2+1}{1+2\eta^2}\right). \tag{4.18}$$

From this result, we can conclude that,

$$(\Delta b_{+})^2 = (\Delta b_{-})^2 = \left(\frac{\gamma_c}{\kappa}N\right)\left(\frac{2\eta^2+1}{1+2\eta^2}\right). \tag{4.19}$$

In addition, we note that for $\Omega \gg \gamma_c$ Eq. (4.17) reduce

$$(\Delta b_{\pm})^2 = \frac{\gamma_c}{\kappa}N. \tag{4.20}$$

With aid Eq. (3.7) this can be expressed as

$$(\Delta b_{\pm})^2 = 2\overline{n}, \tag{4.21}$$

where $\overline{n}$ is the mean photon number given by Eq. (3.7).

We see that Eq. (4.20) Moreover, we consider the case in which the driving coherent light is absent or vacuum state. Thus upon setting $\Omega = 0$ and Eq. (4.17) become

$$(\Delta b_{+})^2_v = (\Delta b_{-})^2_v = (\frac{\gamma_c}{\kappa}N), \tag{4.22}$$

which is normally ordered quadrature variance of single mode cavity vacuum state. We note that for $\Omega = 0$ the uncertainty in plus and minus quadratures are equal and satisfy the minimum uncertainties relation.

4.2 Quadrature squeezing

In this section, we seek to study the local quadrature squeezing for the light produced by the system under consideration.

4.2.1 Local quadrature squeezing

Here we wish to obtain the quadrature squeezing of single mode cavity light in a given frequency interval. To this end, we first obtain the spectrum fluctuation of the superposition of light mode. We define this spectrum fluctuation of the single mode cavity light,

$$S_{\pm}(\omega) = \frac{1}{\pi} Re \int_0^{\infty} d\tau e^{i(\omega-\omega_o)\tau} \langle \hat{b}_{\pm}(t), \hat{b}_{\pm}(t+\tau) \rangle, \tag{4.23}$$

where

$$\hat{b}_{+}(t+\tau) = \hat{b}^{\dagger}(t+\tau) + \hat{b}(t+\tau), \tag{4.24}$$

$$\hat{b}_{-}(t+\tau) = i(\hat{b}^{\dagger}(t+\tau) - \hat{b}(t+\tau)), \tag{4.25}$$

and ω_o is the central frequency. In view of Eq. (2.52), we obtain

$$\langle \hat{b}_{\pm}(t), \hat{b}_{\pm}(t+\tau) \rangle = \langle \hat{b}_{\pm}(t) \hat{b}_{\pm}(t+\tau) \rangle. \tag{4.26}$$

On account of Eqs. (4.1), (4.2), (4.24) and (4.25), one can write Eq. (4.26), as

$$\langle \hat{b}_{\pm}(t), \hat{b}_{\pm}(t+\tau) \rangle = \langle \hat{b}^{\dagger}(t) \hat{b}(t+\tau) \rangle + \langle \hat{b}(t) \hat{b}^{\dagger}(t+\tau) \rangle \pm \langle \hat{b}^{\dagger}(t) \hat{b}^{\dagger}(t+\tau) \rangle$$
$$\pm \langle \hat{b}(t) \hat{b}(t+\tau) \rangle. \tag{4.27}$$

Upon substituting of Eqs. (3.44)-(3.47) into Eq. (4.26), we arrive at

$$\langle \hat{b}_{\pm}(t), \hat{b}_{\pm}(t+\tau) \rangle = \Big[\langle \langle \hat{b}^{\dagger}(t) \hat{b}(t+\tau) \rangle + \langle \hat{b}(t) \hat{b}^{\dagger}(t+\tau) \rangle \pm \langle \hat{b}^{\dagger}(t) \hat{b}^{\dagger}(t+\tau) \rangle$$
$$\pm \langle \hat{b}(t) \hat{b}(t+\tau) \rangle \Big] \left[\frac{\kappa}{\kappa-\mu} e^{-\frac{\mu\tau}{2}} - \frac{\mu}{\kappa-\mu} e^{-\frac{\kappa\tau}{2}} \right], \tag{4.28}$$

from which follows

$$\langle \hat{b}_{\pm}(t), \hat{b}_{\pm}(t+\tau)\rangle = (\Delta b_{\pm})^2\left[\frac{\kappa}{\kappa-\mu}e^{-\frac{\mu\tau}{2}} - \frac{\mu}{\kappa-\mu}e^{-\frac{\kappa\tau}{2}}\right], \quad (4.29)$$

where

$$(\Delta b_{\pm})^2 = \langle \hat{b}^\dagger(t)\hat{b}(t+\tau)\rangle + \langle \hat{b}(t)\hat{b}^\dagger(t+\tau)\rangle \pm \langle \hat{b}^\dagger(t)\hat{b}^\dagger(t+\tau)\rangle$$
$$\pm\langle \hat{b}(t)\hat{b}(t+\tau)\rangle. \quad (4.30)$$

It then follows that

$$\langle \hat{b}_{+}(t), \hat{b}_{+}(t+\tau)\rangle = (\Delta b_{+})^2\left[\frac{\kappa}{\kappa-\mu}e^{-\frac{\mu\tau}{2}} - \frac{\mu}{\kappa-\mu}e^{-\frac{\kappa\tau}{2}}\right] \quad (4.31)$$

and

$$\langle \hat{b}_{-}(t), \hat{b}_{-}(t+\tau)\rangle = (\Delta b_{-})^2\left[\frac{\kappa}{\kappa-\mu}e^{-\frac{\mu\tau}{2}} - \frac{\mu}{\kappa-\mu}e^{-\frac{\kappa\tau}{2}}\right]. \quad (4.32)$$

Now introducing Eq. (4.32) into (4.23) and carrying out integration over τ, we find the spectrum of the minus quadrature fluctuation for a single mode cavity light as

$$S_{-}(\omega) = (\Delta b_{-})^2_{ss}\left\{\frac{\kappa}{\kappa-\mu}\left[\frac{\frac{\mu}{2\pi}}{(\frac{\mu}{2})^2+(\omega-\omega_o)^2}\right] - \frac{\mu}{\kappa-\mu}\left[\frac{\frac{\kappa}{2\pi}}{(\frac{\kappa}{2})^2+(\omega-\omega_o)^2}\right]\right\} \quad (4.33)$$

Upon integration both side of Eq. (4.33), over ω we get

$$\int_{-\infty}^{\infty} S_{-}(\omega)d\omega = (\Delta b_{-})^2. \quad (4.34)$$

On the basis of Eq. (4.34), we observe that $S_{-}(\omega)d\omega$ is the steady-state quadrature variance in the interval between ω and $\omega + d\omega$. Thus, we realize that the variance of the minus quadrature in the interval between $\omega' = -\lambda$ and $\omega' = +\lambda$ is expressible as

$$(\Delta b_{\pm})^2_{\pm\lambda} = \int_{-\lambda}^{+\lambda} S_{-}(\omega')d\omega', \quad (4.35)$$

in which $\omega - \omega_o = \omega'$. On introducing Eq. (4.33) into Eq. (4.35) and carrying out the integration over ω', employing relation described by Eq. (3.21), we find

$$(\Delta b_-)^2_{\pm\lambda} \quad = \quad (\Delta b_-)^2 z(\lambda), \tag{4.36}$$

where $z(\lambda)$ is give by Eq. (3.23), we define the quadrature squeezing of a cavity in the interval of $\lambda_\pm$ by

$$S_{\pm\lambda} \quad = \quad 1 - \frac{(\Delta b_-)^2_{\pm\lambda}}{(\Delta b_-)^2_{v\pm\lambda}}. \tag{4.37}$$

Furthermore, upon setting $\eta = 0$, in Eq. (4.36), we seek that the local quadrature variance of of single mode cavity in vacuum state in the same frequency is found to be

$$(\Delta b_-)^2_{v\pm\lambda} \quad = \quad (\Delta b)^b_v z_v(\lambda), \tag{4.38}$$

by setting $\mu = \frac{\gamma_c^2 + 2\Omega^2}{\gamma_c}$, in Eq.(3.23) and consider in vacuum state, one can get

$$z_v(\lambda) \quad = \quad \left[\frac{\frac{2\kappa}{\pi}}{\kappa - \gamma_c}\right] \tan^{-1}\left(\frac{2\lambda}{\gamma_c}\right) - \left[\frac{\frac{2\gamma_c}{\pi}}{\kappa - \gamma_c}\right] \tan^{-1}\left(\frac{2\lambda}{\kappa}\right). \tag{4.39}$$

BY introducing Eqs. (4.36) and and (4.38) into (4.37), one can readily obtain

$$S_{\pm(\lambda)} = \frac{[z_v(\lambda) - z(\lambda)]}{z_v(\lambda)}, \tag{4.40}$$

now in the view of Eqs. (3.23) and (4.39) into (4.40), one can readily get

$$S_{\pm(\lambda)} \quad = \quad \left\{1 - \frac{\left[\frac{2\kappa/\pi}{\kappa - \mu}\right] \tan^{-1}(\frac{2\lambda}{\mu}) - \left[\frac{2\mu/\pi}{\kappa - \mu}\right] \tan^{-1}(\frac{2\lambda}{\kappa})}{\left[\frac{2\kappa/\pi}{\kappa - \gamma_c}\right] \tan^{-1}(\frac{2\lambda}{\gamma_c}) - \left[\frac{2\gamma_c/\pi}{\kappa - \gamma_c}\right] \tan^{-1}(\frac{2\lambda}{\kappa})}\right\}. \tag{4.41}$$

This is the local quadrature squeezing.

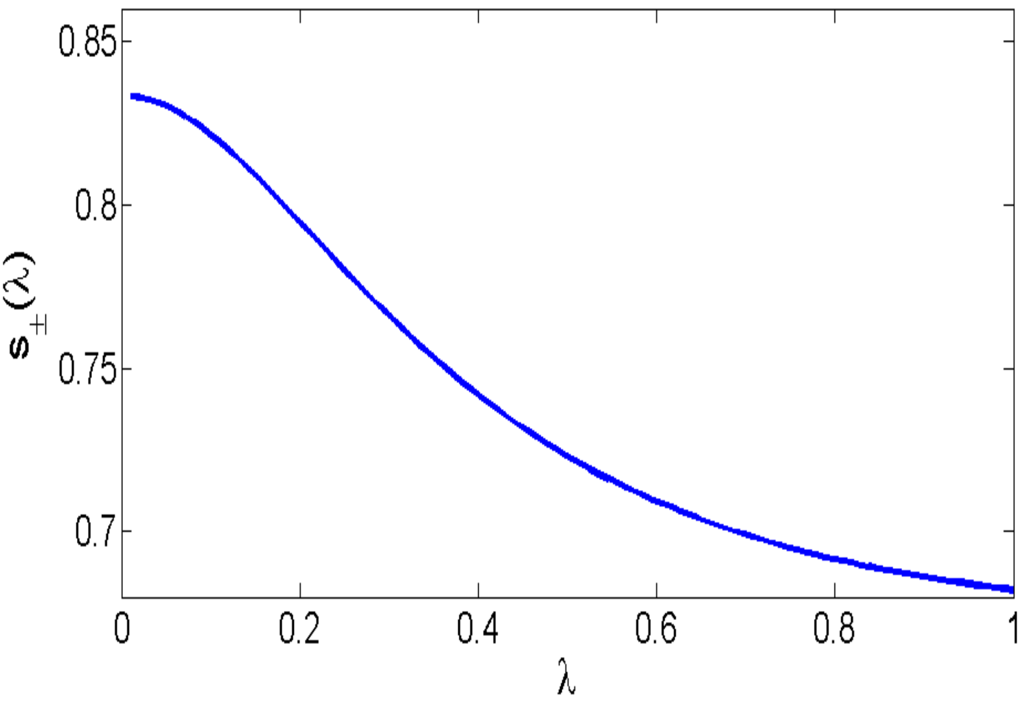

Figure 4.2: A plots of Eq.(4.41) Local quadrature squeezing versus λ for $\gamma_c = 0.4$, $\Omega = 2$ and $\kappa = 0.8$.

The the plots in figure 4.2 shows that the maximum local quadrature squeezing is 83.33% below vacuum level. This occurs in the frequency interval $\lambda_{\pm} = 0.01$. In additions, we note that the local quadrature squeezing increase as λ increase.

5

Superposed Two Level Laser Light Beams

In this chapter we seek to study the squeezing and statistical properties of a pair of superposed laser light beams produced by a coherently driven superposed two level laser and coupled to a vacuum reservoir via a single-port mirror. We thus first obtain the Q function, with the aid of the antinormally-ordered characteristic function defined in the Heisenberg picture for laser light beams. Then using the resulting Q function, we determine the density operator for the pair of superposed laser light beams. Applying this density operator, we calculate the global photon statistics and quadrature squeezing.

5.1 The number and coherent state

We seek here to determine various relation, involving the number and coherent state, which hold for arbitrary commutation relation of the annihilation and creation operators, we shall later see some application of these relation in the quantum analysis of laser light. We now consider a light mode represented by the operators $\hat{b}$ and $\hat{b}^{\dagger}$ subject to the commutation relation,

$$[\hat{b}, \hat{b}^{\dagger}] = \lambda, \tag{5.1}$$

where λ is constant c-number using this commutation relation, one can readily verify that

$$\hat{b}|n\rangle = \sqrt{n}|n-\lambda\rangle. \tag{5.2}$$

and

$$\hat{b}^{\dagger}|n\rangle = \sqrt{n+\lambda}|n+\lambda\rangle. \tag{5.3}$$

With aid of Eq. (5.3), we can also easily established that

$$\hat{b}^{\dagger n}|0\rangle = \sqrt{\lambda^{n} n!|\lambda n\rangle}. \tag{5.4}$$

We next proceed to obtain an expression for the coherent state $|\alpha\rangle$, in terms of number state. Thus applying Eq. (5.1), one can put this coherent states in the form

$$|\alpha\rangle = e^{-(\lambda\alpha^{*}\alpha)/2}e^{\alpha\hat{b}^{\dagger}}|0\rangle, \tag{5.5}$$

so that expanding in power series the second exponential function function and taking into account Eq. (5.4), we get

$$|\alpha\rangle = e^{-(\lambda\alpha^{*}\alpha)}\sum_{n=0}^{n=\infty}\frac{\alpha^{n}}{\sqrt{n!}}\sqrt{\lambda^{n}}|\lambda n\rangle. \tag{5.6}$$

Now with aid of Eq. (5.6), one can write

$$\langle\alpha|\alpha\rangle = e^{-(\lambda\alpha^{*}\alpha)}\sum_{m,n}\frac{\alpha^{*m}}{\sqrt{m!}}\sqrt{\lambda^{m+n}}\langle\lambda m|\lambda n\rangle. \tag{5.7}$$

On assuming that

$$\langle\lambda m|\lambda n\rangle = \delta_{mn}, \tag{5.8}$$

we readily arrive at $\langle\alpha|\alpha\rangle = 1$. Since a set of orthonormal eigenstates is complete, we can express the identity operator in the form

$$I = \sum_{n=0}^{n=\infty} |\lambda n\rangle\langle\lambda n|. \tag{5.9}$$

In addition, employing Eqs. (5.6) and (5.8), one can verify that

$$\langle\lambda n|\alpha\rangle = e^{-(\lambda\alpha^*\alpha)/2}\frac{\alpha^n}{\sqrt{n!}}\sqrt{\lambda^n} \tag{5.10}$$

and

$$\langle\alpha|\beta\rangle = exp[\lambda(\alpha^*\beta - \frac{1}{2}\alpha^*\alpha - \frac{1}{2}\beta^*\beta)]. \tag{5.11}$$

Using Eq. (5.1), we also easily established that

$$D(\alpha)|\beta\rangle = exp[\frac{1}{2}\lambda(\alpha\beta^* - \alpha^*\beta)]|\alpha+\beta\rangle. \tag{5.12}$$

We now proceed to obtain the completeness relationes for coherent states. To this end,we note that

$$\begin{aligned}\int d^2\beta\rangle\langle\beta &= \sum_{n,m}\int d^2\beta|\lambda n\rangle\langle\lambda n|\beta\langle\beta|\lambda m\rangle\langle\lambda m| \\ &= \sum_{n,m}\frac{|\lambda n\rangle\langle\lambda m|\sqrt{\lambda^{n+m}}}{\sqrt{n!m!}}\int d^2\beta e^{-\lambda\beta^*\beta}\beta^n\beta^{*m}.\end{aligned} \tag{5.13}$$

One can write

$$\int d^2\beta e^{-\lambda\beta^*\beta}\beta^n\beta^{*m} = \frac{\partial^n}{\partial a^n}\frac{\partial^m}{\partial b^m}\int d^2\beta exp[-\lambda\beta^*\beta + a\beta + b\beta^*]_{a=b=0}. \tag{5.14}$$

It then follows that

$$\int d^2\beta e^{-\lambda\beta^*\beta}\beta^n\beta^{*m} = \frac{\pi}{\lambda}\frac{\partial^n}{\partial a^n}\frac{\partial^m}{\partial b^m}e^{ab/\lambda}\Big|_{a=b=0}$$

$$= \frac{\pi}{\lambda}\sum_l \frac{1}{l!}\frac{\partial^n}{\partial a^n}\frac{\partial^m}{\partial b^m}\left[\frac{a^l b^l}{\lambda^l}\right]_{a=b=0}. \tag{5.15}$$

Moreover, using the identity given by Eq. (5.11) and applying the condition $a = b = 0$, one finds

$$\begin{aligned}\int d^2\beta e^{-\lambda\beta^*\beta}\beta^n\beta^{*m} &= \frac{\pi}{\lambda}\sum \frac{1}{l!}\frac{1}{\lambda^l}\frac{1}{(1-n)!}\frac{1}{(1-m)!}\delta_{ln}\delta_{lm} \\ &= \frac{\pi}{\lambda}\frac{1}{\lambda^n}\frac{n!}{(n-m)!}\delta_{nm}.\end{aligned} \tag{5.16}$$

Now in the view of Eq. (5.13) and (5.16), there follows

$$\begin{aligned}\int d^2\beta|\beta\rangle\langle\beta| &= \frac{\pi}{\lambda}\sum_{n,m}|\lambda n\rangle\langle\lambda m|\frac{\sqrt{\lambda^{n+m}}}{\sqrt{n!m!}}\frac{n!}{(n-m)!\lambda^n}\delta_{nm}, \\ &= \frac{\pi}{\lambda}\sum_{n=0}^{\infty}|\lambda n\rangle\langle\lambda m|, \\ &= \frac{\pi}{\lambda}\hat{I}.\end{aligned} \tag{5.17}$$

We then see that

$$\hat{I} = \frac{\lambda}{\pi}\int d^2\beta|\beta\rangle\langle\beta|. \tag{5.18}$$

This represent the completeness relation for coherent states subject to the commutation relation given by Eq. (5.1).

We next seek to obtain some useful commutation relations. To this end, expanding the operator function $f(\hat{b}, \hat{b}^\dagger)$ in the antinormal order, one can write

$$\left[\hat{b}, f(\hat{b}, \hat{b}^\dagger)\right] = \sum_{lm} c_{lm}\hat{b}^l\left[\hat{b}, \hat{b}^{+m}\right], \tag{5.19}$$

so that in view of the identity

$$\left[\hat{b}, \hat{b}^{+m}\right] = \lambda\frac{\partial}{\partial\hat{b}^\dagger}\hat{b}^{\dagger m}, \tag{5.20}$$

we have

$$\left[\hat{b}, f(\hat{b}, \hat{b}^\dagger)\right] \;=\; \lambda \frac{\partial}{\partial \hat{b}^\dagger} f(\hat{b}, \hat{b}^\dagger). \tag{5.21}$$

One also show in similar manner that

$$\left[\hat{b}^\dagger, f(\hat{b}, \hat{b}^\dagger)\right] \;=\; -\lambda \frac{\partial}{\partial \hat{b}} f(\hat{b}, \hat{b}^\dagger). \tag{5.22}$$

Now using Eq. (5.21), we easily find

$$[\hat{b}, D(\alpha)] \;=\; \lambda \alpha D(\alpha), \tag{5.23}$$

where

$$D(\alpha) \;=\; exp(\alpha \hat{b}^\dagger - \alpha^* \hat{b}) \tag{5.24}$$

is the usual displacement operator. Applying Eq.(5.23), we get

$$\hat{b}\hat{D}(\alpha) \;=\; \hat{D}(\alpha)(\hat{b} + \lambda\alpha), \tag{5.25}$$

and up on multiplying on the right by $|0\rangle$, we have

$$\hat{b}\hat{D}(\alpha)|0\rangle \;=\; D(\alpha)(\hat{b} + \lambda\alpha)|0\rangle. \tag{5.26}$$

It then follows that

$$\hat{b}|\alpha\rangle \;=\; \lambda\alpha|\alpha\rangle. \tag{5.27}$$

Furthermore, we that

$$\frac{\partial}{\partial \beta^*}\Big(|\beta\rangle\langle\beta|\Big) \;=\; \Big(\frac{\partial}{\partial \beta^*}\hat{D}(\beta)|0\rangle\Big)\langle\beta| + |\beta\rangle\Big(\langle 0|\frac{\partial}{\partial \beta^*}\hat{D}(-\beta)\Big). \tag{5.28}$$

Applying displacement operator $\hat{D}(\beta)$ in the antinormal order, one can readily verify that

$$\frac{\partial}{\partial\beta^*}\hat{D}(\beta) = \hat{D}\left(\frac{1}{2}\lambda\beta - \hat{b}\right)\hat{D}(\beta). \tag{5.29}$$

In addition, employing the displacement operator $\hat{D}(-\beta)$ in the normal order, we easily get

$$\frac{\partial}{\partial\beta^*}\hat{D}(-\beta) = \hat{D}\left(-\frac{1}{2}\lambda\beta + \hat{b}\right). \tag{5.30}$$

Hence on substituting Eq. (5.29) and (5.30) into Eq. (5.28), we have

$$\frac{\partial}{\partial\beta^*}\left(|\beta\rangle\langle\beta|\right) = \left(\frac{1}{2}\lambda\beta - \hat{b}\right)|\beta\rangle\langle\beta| + |\beta\rangle\langle\beta|\left(-\frac{1}{2}\lambda\beta + \hat{b}\right). \tag{5.31}$$

from which follows

$$|\beta\rangle\langle\beta|\hat{b} = \left(\lambda\beta + \frac{\partial}{\partial\beta^*}\right)|\beta\rangle\langle\beta|. \tag{5.32}$$

5.2 The Q function

With aid of the completeness relation given by Eq. (5.18), the antinormally ordered characters function,

$$\Phi_a(z) = Tr\left(\hat{\rho}e^{-z^*\hat{b}}e^{z\hat{b}^\dagger}\right), \tag{5.33}$$

can be rewrite as

$$\Phi_a(z) = \frac{\lambda}{\pi}\int d^2\beta Tr\left(\hat{\rho}e^{-z^*\hat{b}}|\beta\rangle\langle\beta|\hat{\rho}e^{+z^*\hat{b}}\right), \tag{5.34}$$

so that on account of Eq. (5.27), we get

$$\Phi_a(z) = \int d^2\beta\lambda Q(\lambda\beta)exp(z\lambda\beta^* - z^*\lambda\beta), \tag{5.35}$$

where $Q(\lambda\beta)$ is the Q function. Introducing the variable $\alpha = \lambda\beta$, we easily find

$$\Phi_a(z) = \int d^2\alpha \frac{Q(\alpha)}{\lambda} exp(z\alpha^* - z^*\alpha), \tag{5.36}$$

since $\frac{Q(\alpha)}{\lambda}$ is the inverse fourier transform of the characteristic function, we see that

$$Q(\alpha) = \frac{\lambda}{\pi^2} \int d^2\alpha \Phi_a(z) exp(z\alpha^* - z^*\alpha). \tag{5.37}$$

Upon integrating both side of Eq. (5.37), over α and taking into account the fact that

$$\frac{1}{\pi^2} \int d^2\alpha exp(z\alpha^* - z^*\alpha), = \delta^2(z), \tag{5.38}$$

we arrive at

$$\int d^2\alpha Q(\alpha) = \lambda \int d^2 z Tr(\hat{\rho} e^{z\hat{b}} e^{-z\hat{b}^\dagger}) \delta^2(z), \tag{5.39}$$

from which follows

$$\int d^2\alpha Q(\alpha) = \lambda. \tag{5.40}$$

This shows that the Q function is normalized to λ.

We now proceed to obtain the explicit from the antinormally ordered characteristic function of a two level laser. Upon replacing the atomic operator that pear in Eq. (2.45), by the expectation values of the commutation relation for the light generated by the two level laser can be write as

$$[\hat{b}, \hat{b}^\dagger] = \lambda, \tag{5.41}$$

in which

$$\lambda = \frac{\gamma_c}{\kappa} \Bigg[\langle N_b \rangle - \langle N_a \rangle \Bigg]. \tag{5.42}$$

Thus applying the Baker-Haudroff identity along with Eq. (5.41), one can put Eq. (5.33), in the form

$$\Phi_a(z,t) = e^{-(\lambda z^* z)/2}\left\langle e^{z\hat{b}^\dagger(t)-z^*\hat{b}(t)}\right\rangle, \tag{5.43}$$

since $\hat{b}(t)$ is a Gaussian variable with zero mean, we can rewrite Eq. (5.43) as

$$\Phi_a(z,t) = e^{-(\lambda z^* z)/2} exp\left[\frac{1}{2}\langle(z\hat{b}^\dagger(t) - z^*\hat{b}(t))^2\rangle\right]. \tag{5.44}$$

It then follows that

$$\begin{aligned}\Phi_a(z,t) = e^{-(\lambda z^* z)/2} exp\Big[\frac{1}{2}\Big\langle & z^2\hat{b}^{\dagger^2}(t) + z^{*^2}\hat{b}^2(t) \\ & -z^* z\hat{b}^\dagger(t)\hat{b}(t) - z^* z\hat{b}(t)\hat{b}^\dagger(t)\Big\rangle\Big].\end{aligned} \tag{5.45}$$

On account of

$$\langle\hat{b}^\dagger\hat{b}\rangle = \frac{\gamma_c}{\kappa}\langle N_a\rangle \tag{5.46}$$

and

$$\langle\hat{b}\hat{b}^\dagger\rangle = \frac{\gamma_c}{\kappa}\langle N_b\rangle, \tag{5.47}$$

the antinormally ordered characteristic function can be put in the form

$$\Phi_a(z,t) = exp\left[-\frac{1}{2}z^* z[\lambda + \frac{\gamma_c}{\kappa}(\langle N_a\rangle + \langle N_b\rangle)]\right], \tag{5.48}$$

so that with aid of Eq. (5.42), we have

$$\Phi_a(z,t) = e^{-az^* z}, \tag{5.49}$$

where

$$a = \frac{\gamma_c}{\kappa}\langle N_b\rangle. \tag{5.50}$$

Finally, upon the introducing Eqs. (5.49) and (5.37) and carrying out integration, we see that

$$Q(\alpha) = \frac{\lambda}{\pi a} exp\left(\frac{-\alpha^*\alpha}{a}\right). \tag{5.51}$$

Our interest is to calculate the photon statistics and quadrature variance for a pair of superposed light beams produced by a coherently driven two level atom.

5.3 Density operator

We now seek to drive the density operator for a pair of superposed two level laser light beams, we consider the case in which the commutation relation for the annihilation and creation operator for the two light beams is given by Eq. (5.1). Let $\hat{\rho}(\hat{b}^\dagger, \hat{b})$ be the density operator for one of the light beams. The upon expanding this density operator in the normal order and employing the completeness relation for coherent states given by Eq. (5.18), one easily finds

$$\hat{\rho}' = \frac{\lambda}{\pi} \int d^2\beta \sum_{kl} C_{kl}(\lambda\beta^*)^k |\beta\rangle\langle\beta|\hat{b}^l \tag{5.52}$$

and on account of the relation

$$|\beta\rangle\langle\beta|\hat{b}^l = (\lambda\beta + \frac{\partial}{\partial\beta^*})^l |\beta\rangle\langle\beta|, \tag{5.53}$$

there follows

$$\hat{\rho}' = \lambda \int d^2\beta Q(\lambda\beta^*, \lambda\beta + \frac{\partial}{\partial\beta^*})|\beta\rangle\langle\beta|. \tag{5.54}$$

This expression for density operator can be put in the form

$$\hat{\rho}' = \lambda \int d^2\beta Q(\lambda\beta^*, \lambda\beta + \frac{\partial}{\partial\beta^*})\hat{D}(\beta)|0\rangle\langle 0|\hat{D}(-\beta). \tag{5.55}$$

We now realize that the density operator for the superposition of the first light beams and another one is expressible as

$$\hat{\rho} \;=\; \lambda \int d^2\gamma Q(\lambda\gamma^*, \lambda\gamma + \frac{\partial}{\partial\gamma^*})\hat{D}(\gamma)\hat{\gamma}'\hat{D}(-\gamma), \tag{5.56}$$

on that in view of Eq. (5.56), we have

$$\hat{\rho} \;=\; \lambda \int d^2\beta d^2\gamma Q(\lambda\beta^*, \lambda\beta + \frac{\partial}{\partial\beta^*}) \times Q(\lambda\gamma^*, \lambda\gamma + \frac{\partial}{\partial\gamma^*})|\beta + \gamma\rangle\langle\gamma + \beta|. \tag{5.57}$$

This is density operator for the superposed two laser light beams.

5.4 Photon statics

In this section, we wish to calculate the mean and variance of the photon number for the superposed laser light beams under the consideration. Employing Eq. (5.54), the expectation value of a given operator function $A(\hat{b}^\dagger, \hat{b})$ can be written as

$$\langle A\rangle \;=\; \int \frac{d^2\alpha}{\lambda} Q(\alpha^*, \alpha + \lambda\frac{\partial}{\partial\alpha^*})A_n(\alpha^*, \alpha), \tag{5.58}$$

in which

$$Q(\alpha^*, \alpha + \lambda\frac{\partial}{\partial\alpha^*}) \;=\; \frac{1}{\pi}\sum_{lm} C_{lm}(\alpha^*)^l(\alpha + \lambda\frac{\partial}{\partial\alpha^*})^m, \tag{5.59}$$

$A_n(\alpha^*, \alpha)$ is the c-number function corresponding to $\hat{A}$ in the normal ordered and $\alpha = \lambda\beta$.

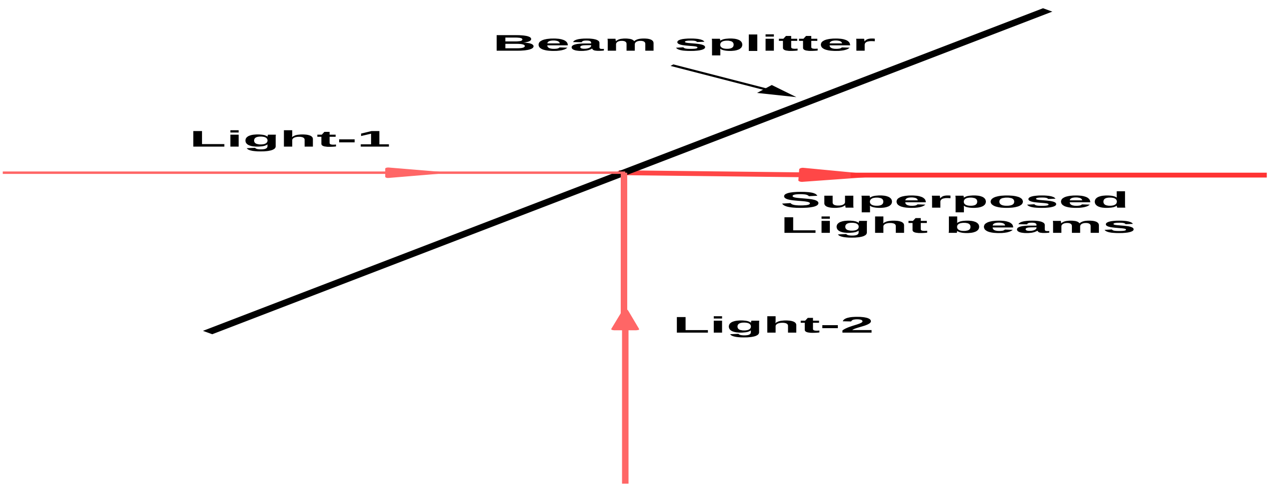

Figure 5.1: The superposed laser light beams, with $\kappa = 1$ and $\kappa = 0$ for the upper and lower surfaces of the beam splitter.

5.4.1 The mean photon number

Suppose $\hat{a}$ and $\hat{a}^\dagger$ represent the superposed laser beams. Then applying the density operator by given by Eq. (5.57), then mean photon number of the superposed laser beams can be put in the form

$$\overline{n}_s = \lambda^4 \int d^2\beta d^2\gamma Q(\lambda\beta^*, \lambda\beta + \frac{\partial}{\partial\beta^*})Q(\lambda\gamma^*, \lambda\gamma + \frac{\partial}{\partial\gamma^*}) \times(\beta^*\beta + \gamma^*\gamma + \beta^*\gamma + \beta\gamma^*). \tag{5.60}$$

Now introducing the variable $\alpha_1 = \lambda\beta$ and $\alpha_2 = \lambda\gamma$, Eq. (5.59) can be rewrite as

$$\overline{n}_s = \frac{1}{\lambda^2}\int d^2\alpha_1 d^2\alpha_2 Q(\alpha_1^*, \alpha_1 + \lambda\frac{\partial}{\partial\alpha_1^*})Q(\alpha_2^*, \alpha_2 + \lambda\frac{\partial}{\partial\alpha_2^*})$$

$$\times(\alpha_1^*\alpha_1 + \alpha_2^*\alpha_2 + \alpha_1^*\alpha_2 + \alpha_1\alpha_2^*), \tag{5.61}$$

from which follows

$$\begin{aligned}\overline{n}_s = &\frac{1}{\lambda}\int d^2\alpha_1 Q(\alpha_1^*, \alpha_1 + \lambda\frac{\partial}{\partial\alpha_1^*})\alpha_1^*\alpha_1 \\ &+\frac{1}{\lambda}\int d^2\alpha_2 Q(\alpha_2^*, \alpha_2 + \lambda\frac{\partial}{\partial\alpha_2^*})\alpha_2^*\alpha_2 \\ &+\frac{1}{\lambda}\int d^2\alpha_1 Q(\alpha_1^*, \alpha_1 + \lambda\frac{\partial}{\partial\alpha_1^*})\alpha_1^* \\ &\times\frac{1}{\lambda}\int d^2\alpha_2 Q(\alpha_2^*, \alpha_2 + \lambda\frac{\partial}{\partial\alpha_2^*})\alpha_2 \\ &+\frac{1}{\lambda}\int d^2\alpha_1 Q(\alpha_1^*, \alpha_1 + \lambda\frac{\partial}{\partial\alpha_1^*})\alpha_1 \\ &\times\frac{1}{\lambda}\int d^2\alpha_2 Q(\alpha_2^*, \alpha_2 + \lambda\frac{\partial}{\partial\alpha_2^*})\alpha_2^*.\end{aligned} \tag{5.62}$$

Now on account of Eq. (5.58), we see that

$$\overline{n}_s = \langle\hat{a}_1^\dagger\hat{a}_1\rangle + \langle\hat{a}_2^\dagger\hat{a}_2\rangle + \langle\hat{a}_1\rangle\langle\hat{a}_2^\dagger\rangle + \langle\hat{a}_1^\dagger\rangle\langle\hat{a}_2\rangle \tag{5.63}$$

in which

$$[\hat{a}_1, \hat{a}_1^\dagger] = [\hat{a}_2, \hat{a}_2^\dagger] = \lambda. \tag{5.64}$$

In view of the fact that the Q function of the identical laser light beams have exactly the same form, we observe that

$$\langle\hat{a}_2^\dagger\hat{a}_2\rangle = \langle\hat{a}_1^\dagger\hat{a}_1\rangle \tag{5.65}$$

and

$$\langle \hat{a}_2 \rangle = \langle \hat{a}_1 \rangle. \tag{5.66}$$

Hence the mean photon number of the superposed laser light beam is expressible as

$$\overline{n}_s = 2\langle \hat{a}_1^\dagger \hat{a}_1 \rangle + 2\langle \hat{a}_1^\dagger \rangle \langle \hat{a}_1 \rangle. \tag{5.67}$$

We next proceed to evaluate the expectation value involved in the expression. Thus with aid of Eq. (5.58), along with Eq. (5.51) one can write

$$\langle \hat{a}_1 \rangle = \frac{1}{\pi a} \int d^2\alpha_1 e^{\frac{-1}{a}\alpha_1^*\alpha_1} e^{\frac{-\lambda}{a}\alpha_1^*\frac{\partial}{\alpha_1^*}} \alpha_1. \tag{5.68}$$

On taking into account the fact that α_1 and α_1^* are independent variable, we easily find

$$\langle \hat{a}_1 \rangle = 0. \tag{5.69}$$

Moreover, applying Eq. (5.58) together with Eq. (5.51), the expectation value of $\hat{a}_1^\dagger \hat{a}_1$ can be put in the form

$$\langle \hat{a}_1^\dagger \hat{a}_1 \rangle = \frac{1}{\pi a} \int d^2\alpha_1 e^{\frac{-1}{a}\alpha_1^*\alpha_1} e^{\frac{-\lambda}{a}\alpha_1^*\frac{\partial}{\alpha_1^*}} \alpha_1^*\alpha_1. \tag{5.70}$$

Hence in view of identity

$$e^{\frac{a\partial}{\partial z}} f(z) = f(z+a), \tag{5.71}$$

we have

$$\langle \hat{a}_1^\dagger \hat{a}_1 \rangle = \frac{a-\lambda}{\pi a^2} \int d^2\alpha_1 e^{\frac{-1}{a}\alpha_1^*\alpha_1} \alpha_1^*\alpha_1, \tag{5.72}$$

so that on carrying the integration, we get

$$\langle \hat{a}_1^\dagger \hat{a}_1 \rangle = a - \lambda. \tag{5.73}$$

Now in view of Eq. (5.68) and Eq. (5.72), substitute into Eq.(5.66) takes the form

$$\overline{n}_s = 2(a - \lambda). \tag{5.74}$$

Finally, with aid of Eq. (5.42) and Eq. (5.50), the mean photon number is expressible as

$$\overline{n}_s = \frac{2\gamma_c}{\kappa}\langle Na \rangle. \tag{5.75}$$

5.4.2 Variance of photon number

Here we calculate the variance of photon number for the superposed two-level laser light beams. We can put an arbitrary function of $\hat{a}$ and $\hat{a}^\dagger$ in the normal order by making use of the commutation relation

$$[\hat{a}, \hat{a}^\dagger] = 2\lambda, \tag{5.76}$$

which hold for the superposed laser light beams. Applying Eq. (5.76), the photon number variance can be expressed as

$$(\Delta n)_s^2 = \langle \hat{a}^{\dagger^2} \hat{a}^2 \rangle + 2\lambda\overline{n} - \overline{n}^2. \tag{5.77}$$

Thus employing density operator described by Eq. (5.57), we readily get

$$\langle \hat{a}^{\dagger^2} \hat{a}^2 \rangle = \lambda^6 \int d^2\beta d^2\gamma Q(\lambda\beta^*, \lambda\beta + \frac{\partial}{\partial\beta^*}) Q(\lambda\gamma^*, \lambda\gamma + \frac{\partial}{\partial\gamma^*})$$
$$\times\Big(\beta^{*^2}\beta^2 + \beta^{*^2}\gamma^2 + \beta^2\gamma^{*^2} + 2\beta^{*^2}\beta\gamma + 2\beta\gamma^{*^2}\gamma$$

$$+2\beta^*\beta^2\gamma^* + 2\beta^*\gamma^*\gamma^2 + 4\beta^*\beta\gamma^*\gamma + \gamma^{*^2}\gamma^2\Big). \tag{5.78}$$

Using the variable $\alpha_1 = \lambda\beta$ and $\alpha_2 = \lambda\gamma$, Eq. (5.78) can put the form

$$\begin{aligned}\langle\hat{a}^{\dagger^2}\hat{a}^2\rangle &= \frac{1}{\lambda^2}\int d^2\alpha_1 d^2\alpha_2 Q(\alpha_1^*, \alpha_1 + \lambda\frac{\partial}{\partial\alpha_1^*})Q(\alpha_2^*, \alpha_2 + \lambda\frac{\partial}{\partial\alpha_2^*}) \\ &\times\Big(\alpha_1^{*^2}\alpha_1^2 + \alpha_1^{*^2}\alpha_2^2 + \alpha_1^2\alpha_1^{*^2} + 2\alpha_1^{*^2}\alpha_1\alpha_2 + 2\alpha_1\alpha_2^{*^2}\alpha_2 \\ &+2\alpha_1^*\alpha_1^2\alpha_2^* + \alpha_1^*\alpha_2^*\alpha_2^2 + 4\alpha_1^*\alpha_1\alpha_2^*\alpha_2 + \alpha_2^{*^2}\alpha_2^2\Big).\end{aligned} \tag{5.79}$$

From which follows

$$\begin{aligned}\langle\hat{a}^{\dagger^2}\hat{a}^2\rangle = \frac{1}{\lambda}\int d^2\alpha_1 Q(\alpha_1^*, \alpha_1 + \partial\lambda/\partial\alpha_1^*)\alpha_1^{*^2}\alpha_1^2 \\ +\frac{1}{\lambda}\int d^2\alpha_2 Q(\alpha_2^*, \alpha_2 + \partial\lambda/\partial\alpha_2^*)\alpha_1^{*^2}\alpha_2^2 \\ +\frac{1}{\lambda}\int d^2\alpha_1 Q(\alpha_1^*, \alpha_1 + \partial\lambda/\partial\alpha_1^*)\alpha_1^2 \\ +\frac{1}{\lambda}\int d^2\alpha_1 Q(\alpha_1^*, \alpha_1 + \partial\lambda/\partial\alpha_1^*)\alpha_2^{*^2} \\ +\frac{2}{\lambda}\int d^2\alpha_1 Q(\alpha_1^*, \alpha_1 + \partial\lambda/\partial\alpha_1^*)\alpha_1^{*^2} \\ \times\frac{1}{\lambda}\int d^2\alpha_1 Q(\alpha_1^*, \alpha_1 + \partial\lambda/\partial\alpha_1^*)\alpha_1 \\ \times\frac{1}{\lambda}\int d^2\alpha_2 Q(\alpha_2^*, \alpha_2 + \partial\lambda/\partial\alpha_2^*)\alpha_2 \\ +\frac{2}{\lambda}\int d^2\alpha_1 Q(\alpha_1^*, \alpha_1 + \partial\lambda/\partial\alpha_1^*)\alpha_1 \\ \times\lambda\int d^2\alpha_2 Q(\alpha_2^*, \alpha_2 + \partial\lambda/\partial\alpha_2^*)\alpha_2^{*2} \\ \times\frac{1}{\lambda}\int d^2\alpha_2 Q(\alpha_2^*, \alpha_2 + \partial\lambda/\partial\alpha_2^*)\alpha_2 \\ +\frac{2}{\lambda}\int d^2\alpha_1 Q(\alpha_1^*, \alpha_1 + \partial\lambda/\partial\alpha_1^*)\alpha_1^* \\ \times\frac{2}{\lambda}\int d^2\alpha_1 Q(\alpha_1^*, \alpha_1 + \partial\lambda/\partial\alpha_1^*)\alpha_1^2\end{aligned}$$

$$
\begin{aligned}
&\times\frac{2}{\lambda}\int d^2\alpha_2 Q(\alpha_2^*,\alpha_2+\partial\lambda/\partial\alpha_2^*)\alpha_2^* \\
&+\frac{2}{\lambda}\int d^2\alpha_1 Q(\alpha_1^*,\alpha_1+\partial\lambda/\partial\alpha_1^*)\alpha_1^* \\
&\times\frac{1}{\lambda}\int d^2\alpha_2 Q(\alpha_2^*,\alpha_2+\partial\lambda/\partial\alpha_2^*)\alpha_2^* \\
&\times\frac{1}{\lambda}\int d^2\alpha_2 Q(\alpha_2^*,\alpha_2+\partial\lambda/\partial\alpha_2^*)\alpha_2^2 \\
&+\frac{4}{\lambda}\int d^2\alpha_1 Q(\alpha_1^*,\alpha_1+\partial\lambda/\partial\alpha_1^*)\alpha_1^* \\
&\times\frac{1}{\lambda}\int d^2\alpha_1 Q(\alpha_1^*,\alpha_1+\partial\lambda/\partial\alpha_1^*)\alpha_1 \\
&\times\frac{1}{\lambda}\int d^2\alpha_2 Q(\alpha_2^*,\alpha_2+\partial\lambda/\partial\alpha_2^*)\alpha_2^* \\
&\times\frac{1}{\lambda}\int d^2\alpha_2 Q(\alpha_2^*,\alpha_2+\partial\lambda/\partial\alpha_2^*)\alpha_2 \\
&+\frac{1}{\lambda}\int d^2\alpha_2 Q(\alpha_2^*,\alpha_2+\partial\lambda/\partial\alpha_2^*)\alpha_2^{*2} \\
&\times\frac{1}{\lambda}\int d^2\alpha_2 Q(\alpha_2^*,\alpha_2+\partial\lambda/\partial\alpha_2^*)\alpha_2 \qquad (5.80)
\end{aligned}
$$

Now in the basis of Eq. (5.59), we can put Eq. (5.79), in the form

$$
\begin{aligned}
\langle\hat{a}^{\dagger^2}\hat{a}^2\rangle &= \langle\hat{a}_1^{\dagger^2}\hat{a}_1^2\rangle+\langle\hat{a}_1^{\dagger^2}\rangle\langle\hat{a}_2^2\rangle+\langle\hat{a}_1^2\rangle\langle\hat{a}_2^{\dagger^2}\rangle \\
&+2\langle\hat{a}_1^{\dagger^2}\hat{a}_1\rangle\langle\hat{a}_2\rangle+2\langle\hat{a}_1\rangle\langle\hat{a}_2^{\dagger^2}\hat{a}_2\rangle+2\langle\hat{a}_1^{\dagger}\hat{a}_1^2\rangle\langle\hat{a}_2^{\dagger}\rangle \\
&+2\langle\hat{a}_1^{\dagger}\rangle\langle\hat{a}_2^{\dagger^2}\hat{a}_2^2\rangle+4\langle\hat{a}_1^{\dagger}\hat{a}_1\rangle\langle\hat{a}_2^{\dagger}\hat{a}_2\rangle+\langle\hat{a}_2^{\dagger^2}\hat{a}_2^2\rangle. \qquad (5.81)
\end{aligned}
$$

We note that

$$\langle\hat{a}_2^{\dagger^2}\hat{a}_2^2\rangle = \langle\hat{a}_1^{\dagger^2}\hat{a}_1^2\rangle \qquad (5.82)$$

and

$$\langle\hat{a}_2^2\rangle = \langle\hat{a}_1^2\rangle. \qquad (5.83)$$

Hence on account of Eqs. (5.65), (5.82) and (5.83) along with Eq. (5.69), find

$$\langle \hat{a}^{\dagger 2}\hat{a}^2\rangle = 2\langle \hat{a}_1^{\dagger 2}\hat{a}_1^2\rangle + 2\langle \hat{a}_1^{\dagger 2}\rangle\langle \hat{a}_1^2\rangle + 4\langle \hat{a}_1^{\dagger}\hat{a}_1\rangle^2. \tag{5.84}$$

Furthermore, applying Eq. (5.58) together with Eq. (5.51), we easily find

$$\langle \hat{a}_1^2\rangle = 0. \tag{5.85}$$

In addition, with the aid of Eq. (5.58) along with (5.51) and (5.71), one can readily verify that

$$\langle \hat{a}_1^{\dagger 2}\hat{a}_1^2\rangle = \frac{1}{2}\overline{n}_s^2, \tag{5.86}$$

where $\overline{n}$ is given by Eq. (5.75). Consequently we see that

$$\langle \hat{a}^{\dagger 2}\hat{a}^2\rangle_s = 2\overline{n}^2, \tag{5.87}$$

one can easily check that

$$\lambda\overline{n}_s = \frac{2\gamma_c\overline{n}}{\kappa}\langle N_b\rangle - \frac{\gamma_c}{\kappa}N\overline{n}. \tag{5.88}$$

Finally, in view of Eqs. (5.77), (5.87) and (5.88) the variance of the photon number for superposed cavity light beams is expressible laser light beams can be write as

$$(\Delta n)_s^2 = 2\overline{n}^2 + 2\lambda\overline{n} - \overline{n}^2,$$

from which follows

$$(\Delta n)_s^2 = \overline{n}^2 + 2\lambda\overline{n}. \tag{5.89}$$

5.5 Quadrature squeezing

In this section, we obtain the global quadrature variance and the local quadrature squeezing for the superposed laser light beams.

5.5.1 Quadrature variance

We finally wish to established the connection under which the superposed laser light beams are in a coherent state. To this end, we first determine the uncertainty relation for superposed laser light beams. We define the quadrature operator for superposed laser light beams by

$$\hat{a}_{+} = \hat{a}^{\dagger} + \hat{a} \tag{5.90}$$

$$\hat{a}_{-} = i(\hat{a}^{\dagger} - \hat{a}). \tag{5.91}$$

Then using Eq. (5.79) together with (5.42), one can readily verify that

$$[\hat{a}_{-}, \hat{a}_{+}] = 4i\frac{\gamma_c}{\kappa}(\langle N_a\rangle\langle N_b\rangle). \tag{5.92}$$

Now in view of this result, we see that

$$\Delta a_{+}\Delta a_{-} = \frac{2\gamma_c}{\kappa}\left|\langle N_a\rangle\langle N_b\rangle\right|. \tag{5.93}$$

Employing the commutation relation given by Eq. (5.76), the quadrature variance of the superposed cavity light beams is expressible as

$$(\Delta a_{\pm})^2 = 2\lambda + 2\langle\hat{a}^{\dagger}\hat{a}\rangle \pm \langle\hat{a}^{\dagger^2}\rangle \pm \langle\hat{a}^2\rangle \mp \langle\hat{a}\rangle^2 \mp \langle\hat{a}^{\dagger}\rangle^2 - 2\langle\hat{a}^{\dagger}\rangle\langle\hat{a}\rangle. \tag{5.94}$$

We can established that

$$\langle\hat{a}\rangle = \langle\hat{a}_1\rangle + \langle\hat{a}_2\rangle. \tag{5.95}$$

Then in view of Eq. (5.66) along with (5.69), follows

$$\langle\hat{a}\rangle = 0. \tag{5.96}$$

One can also check that

$$\langle \hat{a}^2 \rangle = \langle \hat{a}_1^2 \rangle + \langle \hat{a}_1 \rangle^2. \tag{5.97}$$

Hence combination of Eqs. (5.69), (5.85) and (5.95) leads to

$$\langle \hat{a}^2 \rangle = 0. \tag{5.98}$$

Now account of Eq. (5.96) and (5.98), into (5.94) takes the form

$$(\Delta a_\pm)_s^2 = 2\lambda + 2\overline{n}_s \tag{5.99}$$

by compering Eq. (4.16) with Eq. (5.94) and with aid of Eq.(5.75) one can readily get

$$(\Delta a_\pm)_s^2 = 2(\Delta b_\pm)^2. \tag{5.100}$$

5.5.2 Local superposed quadrature squeezing

In this section, we define the local quadrature squeezing for superposed laser light beams by.

$$(S_{\pm\lambda})_s = \left[1 - \frac{(\Delta a_-)^2_{\pm\lambda}}{(\Delta a_-)^2_{v\pm\lambda}}\right]_s. \tag{5.101}$$

From Eq.(5.100) and (4.16) we have

$$(\Delta a_\pm)_s^2 = 2\frac{\gamma_c}{\kappa}\bigg(\langle N_b \rangle + \langle N_a \rangle\bigg) \tag{5.102}$$

and

$$(\Delta b_\pm)^2 = \frac{\gamma_c}{\kappa}\bigg(\langle N_b \rangle + \langle N_a \rangle\bigg). \tag{5.103}$$

From Eq. (5.101) and (5.102), it shows

$$(\Delta a_{\pm})^2 = 2(\Delta b_{\pm})^2. \tag{5.104}$$

With aid of Eq. (5.101) and (5.104) we get

$$(S_{\pm\lambda})_s = \left[1 - \frac{(\Delta b_-)^2_{\pm\lambda}}{(\Delta b_-)^2_{v\pm\lambda}}\right]_s. \tag{5.105}$$

Now with account of Eq. (4.36) and (4.38), we get

$$(S_{\pm\lambda})_s = \left[1 - \frac{z(\lambda)}{z_v(\lambda)}\right]_s, \tag{5.106}$$

follows Eq. (3.23) and (4.39), one can find

$$(S_{\pm\lambda})_s = \left\{1 - \frac{\left[\frac{2\kappa/\pi}{\kappa-\mu}\right]\tan^{-1}(\frac{2\lambda}{\mu}) - \left[\frac{2\mu/\pi}{\kappa-\mu}\right]\tan^{-1}(\frac{2\lambda}{\kappa})}{\left[\frac{2\kappa/\pi}{\kappa-\gamma_c}\right]\tan^{-1}(\frac{2\lambda}{\gamma_c}) - \left[\frac{2\gamma_c/\pi}{\kappa-\gamma_c}\right]\tan^{-1}(\frac{2\lambda}{\kappa})}\right\}_s. \tag{5.107}$$

This show that local quadrature squeezing of single mode cavity light is equal to local quadrature squeezing superposed light beams.

6

Conclusion

In this thesis we have studied the squeezing and statistical properties of the light produced by a coherently driven superposed two-level laser and coupled to vacuum reservoir via a single-port. The steady-state analysis of the squeezing and statistical properties of the light produced by two-level laser with closed cavity and coupled to vacuum reservoir is presented.

We have carried out our calculation by putting the noise operator associated with the vacuum reservoir in normal order. Applying the interaction Hamiltonian, Langavin and Heisenberg equation we have obtained the solution of the equation of expectation value of atomic operator and cavity mode operator, using these equation, we have determined the global and Local mean photon number and the variance of photon number, as well as, the quadrature squeezing.

We observe that the variance of photon number is the greater than the mean of photon number and hence the light produced by two-level laser has super-poissonian photon statistics.

Our result show that a large part of the total mean photon number and variance of photon number is confined in relatively small frequency interval.

We have found from fig 4.2, that the maximum local quadrature squeezing is 83.33% below vacuum level for $\gamma_c = 0.4$, $\Omega = 2$ and $\kappa = 0.8$. This occurs in the frequency interval $\lambda_{\pm} = 0.01$. In additions, we note that the local quadrature squeezing increase as λ increase. Furthermore, we put out that unlike photon number and variance of photon number, the local quadrature squeezing does not depends on the number of atoms. This implies that quadrature squeezing of cavity light is independent number of atoms.

On the other hand, employing the density operator for a pair of superposed two level laser light beams together with the Q functions, we have calculated the mean and variance of the photon number as well as the quadrature variance and the quadrature squeezing. We have found that both the mean photon number and the quadrature variance for the pair of superposed two level laser light beams is the sum of the mean photon numbers and the quadrature variances of the constituent two level laser light beams. However, the variance of the photon number of the pair of superposed two-level light beams is not the sum of the variances of the photon numbers of the constituent two level laser light beams.

We observe that the local quadrature squeezing of a pair of superposed light beams is exactly equal to that of quadrature squeezing of the separate light beams.

Reference

[1] Fesseha Kassahun, Optics. commun. 284 (2011) 1357-1363.

[2] N. A. Ansari, (1992) phys. Rev. A **46**, 1560.

[3] N. A. Ansari, J. Gea-Banacloche, and M. S. Zubairy, Phys. Rev. A **41**, 5179 (1990).

[4] Fesseha Kassahun, arXiv: 1209.4723v1 [quant-ph] 2012.

[5] E. Alebachew, J. Mod. Optics. **55**, 1159 (2008).

[6] Fesseha Kassahun, Fundamental of Quantum optics (Lulu pres Inc, North Carolina, 2010).

[7] M. O. Scully, M. S. Zubairy. Quantum Optics, Cambridge, University press, Cambridge, 1997.

[8] Fesseha Kassahun, *Refined Quantum Analysis of Light*(Create Space Independent Publishing Platform, 2016 Revised edition).

[9] T. Abebe, N. GEMECHU. Ukr. J. phys. 2018. Vol. **63**. No. 7

[10] Solomon Getahun, Global journal of science Frontire Research, Vol. **14**, 11-25 (2014).

[11] R.Loudon, The Quantum Theory of light (Oxford University press, New York, 2000).

[12] D. F. Walls and J. G. Milburn, *Quantum Optics*(Springer-Verlag, Berlin,1994).

[13] Eyob Alebachew and K. Fesseha, arXiv: [quant-ph] 06123v2. 2006.

[14] Sintayehu Tesfa. Journal of Modern optics, Vol. **55** (2008)1683-1696.

[15] M. O. Scully. W. E. Lamb Jr, Rev. 208(1967)159.

[16] Solomon Getaun, Fundamental Journal Modern Physics,Vol. **8**, 14, 11-25 (2014).

[17] Y.K. Wang. W. E. Lamb. Jr, phys. Rev. A **8**(1973)866.

[18] L. Davidovich, Rev. Mod. Phys. **68**, 127 (1997).

[19] F. T. Arecchi, M. Asdente and A. M. Ricca, Phys. Rev. A **14**, 383 (1976).

[20] C. Benkert and O.M. Scully, Phys. Rev. A **41**, 2756 (1990).

[21] R. pike, S Sarkar, Quantum Theory of Radiation, London, Cambridge, 1996.

[22] K. J. McNeil, D. F. Walls, J. phys. A **8** (1975) 104.

[23] C.W. Gardiner. phys. Rev. Lett. **56**, 1917 (1986).

[24] H. J. Carmichael, A. S. Lane, D. F. Walls. phys, Rev. Lett. **58**, 2539 (1987).

[25] H. J. Carmichael, A. S. Lane, D. F. Walls. J. Mod. Opt. **34**, 821(1987).

[26] A. S. parkins, phys. Rev. A **42**, 4352 (1990)

[27] J. I. Cirac, phys. Rev. A **46**, 4354 (1992).

[28] P. R. Rice, C. A. Baird. phys, Rev. A **53**, 3633 (1996).

[29] D. Erenso, R. Vyas. Phys. Rev. A **65**, 06388(2002).

[30] Fesseha Kassahun , Fundamental of Quantum Optics (Lulu, 2008).

[31] Fesseha Kassahun , arXiv: 1611. 01003 V2 [quant-ph] 2018.