Global Journal of Science Frontier Research: A
Physics and Space Science
Volume 15 Issue 4 Version 1.0 Year 2015
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Cv Bipartite Entanglement of Non-Degenerate Three-Level Laser with Squeezed Modes Pumped by Coherent Light

By Getachew A. Gebru & Solomon Getahun

Jimma University, Ethiopia

Abstract- We analyze the CV bipartite entanglement of the light generated by a coheretly pumped non degenerate three-level laser with a two-mode subharmonic generator coupled to a two-mode vacuum reservoir via a single-port mirror, whose open cavity contains N non-degenerate three- level cascade atoms. We carry out our analysis by putting the noise operators associated with a vacuum reservoir in normal order. It is found that the photon-state of the system is strongly entangled at steady state where as the atom state of the system is not entangled. We have also shown that as the stimulated decay constant increases, the degree of entanglement increases. In addition, we have established that the photons in the laser cavity are highly correlated and the photon correlation and entanglement increases as the amplitude of the coherent light driving the pump mode increases. Moreover, we have realized that the presence of the subharmonic generator leads to an increase in the degree of entanglement and correlations.

Keywords: continuous variables, bipartite, entanglement, quadrature squeezing, correlations, two-mode subharmonic generator.

GJSFR-A Classification : FOR Code: 020502

Cv Bipartite Entanglement of Non-Degenerate Three-Level Laser with Squeezed Modes Pumped by Coherent Light

Getachew A. Gebru [α] & Solomon Getahun [σ]

Abstract- We analyze the CV bipartite entanglement of the light generated by a coherently pumped non degenerate three-level laser with a two-mode subharmonic generator coupled to a two-mode vacuum reservoir via a single-port mirror, whose open cavity contains N non-degenerate three- level cascade atoms. We carry out our analysis by putting the noise operators associated with a vacuum reservoir in normal order. It is found that the photon-state of the system is strongly entangled at steady state where as the atom state of the system is not entangled. We have also shown that as the stimulated decay constant increases, the degree of entanglement increases. In addition, we have established that the photons in the laser cavity are highly correlated and the photon correlation and entanglement increases as the amplitude of the coherent light driving the pump mode increases. Moreover, we have realized that the presence of the subharmonic generator leads to an increase in the degree of entanglement and correlations.

Keywords: continuous variables, bipartite, entanglement, quadrature squeezing, correlations, two-mode subharmonic generator.

I. Introduction

One of the most fundamentally interesting and intriguing phenomena associated with the composite quantum system is entanglement. In recent years, the topic of continuous-variable entanglement has received a significant amount of attention as it plays an important role in all branches of quantum information processing [1]. The efficiency of quantum information schemes highly depends on the degree of entanglement. A two-mode subharmonic generator at and above threshold has been theoretically predicted to be a source of light in an entangled state [2,3]. Recently, the experimental realization of the entanglement in two-mode subharmonic generator has been demonstared by Zhang et al [4]. On the other hand, Xiong et al [5] have recently proposed a scheme for an entanglement based on a non-degenerate three level laser when the three-level atoms are injected at the lower level and the top and bottom levels are coupled by a strong coherent light. They have found that a non-degenerate three-level laser can generate light in an entangled state employing the entanglement criteria for Bipartite continuous-variable state [5].

Moreover, Tan et al [6] extended the work of xiong et al. and examined and the generation and evolution of the entangled ligth in the Wigner representation using the sufficient and necessary inseparability criteria for a two-mode Gaussian state proposed by Duan et al [5] and simon [7]. Tesfa [8] have considered a similar system when the atomic coherence is induced by superposition of atomic states and analyzed the entanglement at steady state. Furthermore, Ooi [9] has studied the steady-state entanglement in a two-mode Λ laser.

More recently, Eyob [10] has studied continuous-variable entanglement in non-degenerate three-level laser with a parametric amplifier. In this model the injected atomic coherence introduced by initially preparing the atoms in a coherent superposition of the top and bottom levels. in addition, to exhibiting a two-mode squeezed light, this combined system produces light in an entangled state. In one model of such a laser, three-level atoms initially in the upper level are injected at a constant rate into the cavity and removed after they have decayed due to spontaneous emission. It appears to be quite difficult to prepare the atoms in a coherent superpositions of the top and bottom levels before they are injected into the laser cavity. Beside, it should certainly be hard to find out that the atoms have decayed spontaneously before they are removed from the cavity.

In order to avoid the aforementioned problems, Fesseha [11] have considered that N two-level atoms available in a closed cavity are pumped to the top level by means of electron bombardment. He has shown that the light generated by this laser operating well above threshold is coherent and the light generated by the same laser operating below threshold is chaotic. In addition, Fesseha [12,13] has studied the squeezing and the statistical properties of the light produced by a degenerate three- level laser with the atoms in a closed cavity and pumped by electron bombardment. He has shown that the maximum quadrature squeezing of the light generated by the laser, operating far below threshold, is 50% below the coherent-state level. Alternatively, the three-level atoms available in a closed cavity and pumped by coherent light also generated

Author α: Department of Physics, Addis Ababa University, P. O. Box 1176, Addis Ababa, Ethiopia. e-mail: getachew.asmelash@gmail.com
Author σ: Department of Physics, Jimma University, P. O. Box 378, Jimma, Ethiopia. e-mail: solgett@yahoo.com

squeezed light under certain conditions, with the maximum quadrature squeezing being 43% below the coherent-state level. In view of these results, better squeezing is found from the laser, in which the atoms are pumped by electron bombardment than by coherent light.

In this paper, we seek to study CV bipartite entanglement for the light generated by a coherently pumped non-degenerate three-level laser with a two-mode subharmonic generator coupled to a two-mode vacuum reservoir via a single-port mirror whose open cavity contains N non-degenerate three-level cascade atoms. In order to carry out our analysis, we put the noise operators associated with the vacuum reservoir in the normal order and by considering the interaction of the three-level atoms with a two-mode vacuum reservoir outside the cavity. We then first drive the quantum Langevin equations for the cavity mode operators. We next determine the equations of evolution of the expectation values of atomic operators employing the pertinent master equation. Applying the steady-state solutions of the equations of evolution of atomic and cavity mode operators, we analyze the CV atomic and photon state entanglement as well as atom and photon state correlations.

II. Model and Dynamics of Atomic and Cavity Mode Operators

We consider a coherently pumped non-degenerate three-level laser with two-mode subharmonic generator coupled to a two-mode vacuum reservoir whose cavity contains N non-degenerate three-level atoms in cascade configuration as depicted in Fig 1. For the sake of convenience, we denote the top, middle, and bottom levels of these atoms by $|2\rangle_j$, $|1\rangle_j$ and $|0\rangle_j$ respectively. We seek to represent the light emitted from the top level by $\hat{a}_2$ and the light emitted from the middle by $\hat{a}_1$. In addition, in order to expedite the cascading process, it is assumed that the parity of energy levels $|2\rangle_j$ and $|0\rangle_j$ is the same, where as that of $|1\rangle_j$, is different. This entails that direct transition between energy level $|2\rangle_j$ and $|0\rangle_j$ are electric dipole forbidden but due to parity difference, the transition between $|2\rangle_j \rightarrow |1\rangle_j$, and $|1\rangle_j \rightarrow |0\rangle_j$ are allowed.

The interaction of one of the three-level atoms with light modes a_1 and a_2 can be described at resonance by the Hamiltonian

$$\hat{H}_1(t) = ig\Big[\hat{\sigma}_1^{\dagger j}(t)\hat{a}_1(t) - \hat{a}_1^{\dagger}(t)\hat{\sigma}_1^{j}(t) + \hat{\sigma}_2^{\dagger j}(t)\hat{a}_2(t) - \hat{a}_2^{\dagger}(t)\hat{\sigma}_2^{j}(t)\Big], \qquad (1)$$

where

$$\hat{\sigma}_1^{j} = |0\rangle_{jj}\langle 1| \qquad (2)$$

and

$$\hat{\sigma}_2^{j} = |1\rangle_{jj}\langle 2| \qquad (3)$$

are lowering atomic operators, $\hat{a}_1(t)$ and $\hat{a}_2(t)$ is the annihilation operators for light modes a_1 and a_2, and g is the coupling constant between the atom and the cavity modes.

On the other hand, a pump mode photon of frequency (ω_3) directly interacts with the nonlinear crystal (NLC) to produce the signal-idler photon pairs having different frequencies as the two cavity modes. Furthermore, we consider the case for which the pump mode emerging from the NLC (or two-mode subharmonic) does not couple the top and bottom levels. This could be realized by putting on the right-hand side of the NLC a screen which absorbs the pump mode. The top and bottom levels of the three-level atoms are coupled by a strong driving coherent light with the frequency (ω_4). The coupling of the top and bottom levels of a three-level atom by coherent light can be described at resonance by the Hamiltonian

$$\hat{H}_2(t) = \frac{i\Omega}{2}\left(\hat{\sigma}_0^{\dagger j}(t) - \hat{\sigma}_0^{j}(t)\right), \qquad (4)$$

in which

$$\hat{\sigma}_0^{j} = |0\rangle_{jj}\langle 2| \qquad (5)$$

and

$$\Omega = 2\mu_0\lambda_0. \qquad (6)$$

Here, μ_0 is the amplitude of the driving coherent light and λ_0 is the coupling constant between the driving coherent light and the three-level atom. Moreover, in a two-mode subharmonic generator, a pump photon of frequency ω_3 is down converted into highly correlated signal and idler photons with frequencies ω_1 and ω_2 such as $\omega_3 = \omega_1 + \omega_2$ [13]. This quantum optical process leads to the generation of squeezed light. With the pump mode treated classically represented by a real and constant c-number μ, the process of two-mode subharmonic generation can be described by the Hamiltonian

$$\hat{H}_3(t) = i\varepsilon\left(\hat{a}_1^{\dagger}\hat{a}_2^{\dagger} - \hat{a}_1\hat{a}_2\right), \qquad (7)$$

in which $\varepsilon = 2\eta_0\mu$, with η_0 is the coupling constant between the pump mode and nonlinear crystal and μ is proportional to the amplitude of the coherent light driving the pump mode. Thus upon combining Eqs. (1), (4), and (7), the interaction of the three-level atoms with the cavity modes and the driving coherent light, and the parametric down-conversion can be described by the Hamiltonian

Global Journal of Science Frontier Research (A) Volume XV Issue IV Version I Year 2015

$$\hat{H}_s(t) = ig\Big[\hat{\sigma}_1^{\dagger j}(t)\hat{a}_1(t) - \hat{a}_1^\dagger(t)\hat{\sigma}_1^j(t) + \hat{\sigma}_2^{\dagger j}(t)\hat{a}_2(t) - \hat{a}_2^\dagger(t)\hat{\sigma}_2^j(t)\Big] + i\varepsilon\Big(\hat{a}_1^\dagger\hat{a}_2^\dagger - \hat{a}_1\hat{a}_2\Big) + \frac{i\Omega}{2}\Big(\hat{\sigma}_0^{\dagger j}(t) - \hat{\sigma}_0^j(t)\Big). \tag{8}$$

The master equation for a pair of cavity modes coupled to a two-mode vacuum reservoir has the form [14]

$$\frac{d}{dt}\hat{\rho}(t) = -i\Big[\hat{H}_s(t), \hat{\rho}(t)\Big] + \frac{\beta}{2}\Big[2\hat{\sigma}_1^j\hat{\rho}\hat{\sigma}_1^{\dagger j} - \hat{\sigma}_1^{\dagger j}\hat{\sigma}_1^j\hat{\rho} - \hat{\rho}\hat{\sigma}_1^{\dagger j}\hat{\sigma}_1^j\Big] + \frac{\beta}{2}\Big[2\hat{\sigma}_0^j\hat{\rho}\hat{\sigma}_0^{\dagger j} - \hat{\sigma}_0^{\dagger j}\hat{\sigma}_0^j\hat{\rho} - \hat{\rho}\hat{\sigma}_0^{\dagger j}\hat{\sigma}_0^j\Big], \tag{9}$$

where γ is the spontaneous emission decay constant. Now with the aid of Eq. (8), one can put Eq. (9) in the form

$$\frac{d}{dt}\hat{\rho}(t) = g\left[\hat{\sigma}_1^{\dagger j}\hat{a}_1\hat{\rho} - \hat{a}_1^\dagger\hat{\sigma}_1^j\hat{\rho} + \hat{\sigma}_2^{\dagger j}\hat{a}_2\hat{\rho} - \hat{a}_2^\dagger\hat{\sigma}_2^j\hat{\rho}\right] - g\left[\hat{\rho}\hat{\sigma}_1^{\dagger j}\hat{a}_1 - \hat{\rho}\hat{a}_1^\dagger\hat{\sigma}_1^j + \hat{\rho}\hat{\sigma}_2^{\dagger j}\hat{a}_2 - \hat{\rho}\hat{a}_2^\dagger\hat{\sigma}_2^j\right] + \varepsilon\left[\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{\rho} - \hat{a}_1\hat{a}_2\hat{\rho} - \hat{\rho}\hat{a}_1^\dagger\hat{a}_2^\dagger + \hat{\rho}\hat{a}_1\hat{a}_2\right] + \frac{\Omega}{2}\left[\hat{\sigma}_0^{\dagger j}\hat{\rho} - \hat{\sigma}_0^j\hat{\rho} + \hat{\rho}\hat{\sigma}_0^j - \hat{\rho}\hat{\sigma}_0^{\dagger j}\right] + \frac{\beta}{2}\left[2\hat{\sigma}_1^j\hat{\rho}\hat{\sigma}_1^{\dagger j} - \hat{\sigma}_1^{\dagger j}\hat{\sigma}_1^j\hat{\rho} - \hat{\rho}\hat{\sigma}_1^{\dagger j}\hat{\sigma}_1^j\right] + \frac{\beta}{2}\left[2\hat{\sigma}_0^j\hat{\rho}\hat{\sigma}_0^{\dagger j} - \hat{\sigma}_0^{\dagger j}\hat{\sigma}_0^j\hat{\rho} - \hat{\rho}\hat{\sigma}_0^{\dagger j}\hat{\sigma}_0^j\right] \tag{10}$$

We recall that the laser cavity is coupled to a two-mode vacuum reservoir via a single-port mirror. In addition, we carry out our analysis by putting the noise operators associated with the vacuum reservoir in normal order. Thus the noise operators will not have any effect on the dynamics of the cavity mode operators [13]. In view of this, we can drop the noise operators and write the quantum Langevin equation for the operators $\hat{a}_1$ and $\hat{a}_2$ as

$$\frac{d}{dt}\hat{a}_1(t) = -\frac{k}{2}\hat{a}_1(t) - i\Big[\hat{a}_1(t), \hat{H}_s(t)\Big] \tag{11}$$

and

$$\frac{d}{dt}\hat{a}_2(t) = -\frac{k}{2}\hat{a}_2(t) - i\Big[\hat{a}_2(t), \hat{H}_s(t)\Big], \tag{12}$$

where k is the cavity damping constant for the light modes a_1 and a_2. Then with the aid of Eqs. (8), (11), and (12), we easily

$$\frac{d}{dt}\hat{a}_2(t) = -\frac{k}{2}\hat{a}_1(t) + \varepsilon\hat{a}_2^\dagger(t) - g\hat{\sigma}_1^j, \tag{13}$$

$$\frac{d}{dt}\hat{a}_2(t) = -\frac{k}{2}\hat{a}_2(t) + \varepsilon\hat{a}_1^\dagger(t) - g\hat{\sigma}_2^j. \tag{14}$$

Making use of the pertinent master equation and the fact that $\frac{d}{dt}\langle\hat{A}\rangle = Tr\left(\frac{d\hat{\rho}(t)}{dt}\hat{A}\right)$ where ($\hat{A}$ is an operator), it is not difficult to verify that

$$\frac{d}{dt}\left\langle\hat{\sigma}_1^j\right\rangle = g\Big[\left\langle\hat{n}_0^j\hat{a}_1\right\rangle - \left\langle\hat{n}_1^j\hat{a}_1\right\rangle - \langle\hat{a}_2^\dagger\hat{\sigma}_0^j\rangle\Big] - \frac{\Omega}{2}\langle\hat{\sigma}_2^{\dagger j}\rangle - \frac{\beta}{2}\langle\hat{\sigma}_1^j\rangle, \tag{15}$$

$$\frac{d}{dt}\left\langle\hat{\sigma}_2^j\right\rangle = g\Big[\left\langle\hat{n}_1^j\hat{a}_2\right\rangle - \left\langle\hat{n}_2^j\hat{a}_2\right\rangle + \langle\hat{a}_1^\dagger\hat{\sigma}_0^j\rangle\Big] + \frac{\Omega}{2}\langle\hat{\sigma}_1^{\dagger j}\rangle - \frac{\beta}{2}\langle\hat{\sigma}_2^j\rangle, \tag{16}$$

$$\frac{d}{dt}\left\langle\hat{\sigma}_0^j\right\rangle = g\Big[\left\langle\hat{\sigma}_1^j\hat{a}_2\right\rangle - \left\langle\hat{\sigma}_2^j\hat{a}_1\right\rangle\Big] + \frac{\Omega}{2}\Big[\left\langle\hat{n}_0^j\right\rangle - \left\langle\hat{n}_2^j\right\rangle\Big] - \frac{\beta}{2}\left\langle\hat{\sigma}_0^j\right\rangle, \qquad (17)$$

$$\frac{d}{dt}\left\langle\hat{n}_1^j\right\rangle = g\Big[\langle\hat{\sigma}_1^{\dagger j}\hat{a}_1\rangle + \langle\hat{a}_1^\dagger\hat{\sigma}_1^j\rangle - \langle\hat{\sigma}_2^{\dagger j}\hat{a}_2\rangle - \langle\hat{a}_2^\dagger\hat{\sigma}_2^j\rangle\Big] + \beta\Big[\langle\hat{n}_2^j - \langle\hat{n}_1^j\rangle\Big], \qquad (18)$$

$$\frac{d}{dt}\left\langle\hat{n}_2^j\right\rangle = g\Big[\langle\hat{\sigma}_2^{\dagger j}\hat{a}_2\rangle + \langle\hat{a}_2^\dagger\hat{\sigma}_2^j\rangle\Big] + \frac{\Omega}{2}\Big[\langle\hat{\sigma}_0^{\dagger j}\rangle + \langle\hat{\sigma}_0^j\rangle\Big] - \beta\left\langle\hat{n}_2^j\right\rangle, \qquad (19)$$

$$\frac{d}{dt}\left\langle\hat{n}_0^j\right\rangle = -g\Big[\langle\hat{\sigma}_1^{\dagger j}\hat{a}_1\rangle + \langle\hat{a}_1^\dagger\hat{\sigma}_1^j\rangle\Big] - \frac{\Omega}{2}\Big[\langle\hat{\sigma}_0^{\dagger j}\rangle + \langle\hat{\sigma}_0^j\rangle\Big] + \beta\Big[\langle\hat{n}_1^j\rangle + \langle\hat{n}_2^j\rangle\Big], \qquad (20)$$

where

$$\hat{n}_0^j = |0\rangle_{jj}\langle 0|, \qquad (21)$$

$$\hat{n}_1^j = |1\rangle_{jj}\langle 1|, \qquad (22)$$

$$\hat{n}_2^j = |2\rangle_{jj}\langle 2|. \qquad (23)$$

We see that Eqs. (15)-(20) are nonlinear and coupled differential equations. Therefore, it is not possible to obtain the exact time-dependent solutions. We intend to overcome this problem by applying the large-time approximation [13]. Then using this approximation scheme, we get from Eqs. (13) and (14) the approximately valid relations

$$\hat{a}_1 = \frac{2\varepsilon}{k}\hat{a}_2^\dagger - \frac{2g}{k}\hat{\sigma}_1^j \qquad (24)$$

and

$$\hat{a}_2 = \frac{2\varepsilon}{k}\hat{a}_1^\dagger - \frac{2g}{k}\hat{\sigma}_2^j. \qquad (25)$$

Evidently, these turn out to be exact relations at steady state. Solving these equation simultaneously, one easily verify that

$$\hat{a}_1 = -\frac{4\varepsilon g}{(k^2-4\varepsilon^2)}\hat{\sigma}_2^{\dagger j} - \frac{2gk}{(k^2-4\varepsilon^2)}\hat{\sigma}_1^j \qquad (26)$$

and

$$\hat{a}_2 = -\frac{4\varepsilon g}{(k^2-4\varepsilon^2)}\hat{\sigma}_1^{\dagger j} - \frac{2gk}{(k^2-4\varepsilon^2)}\hat{\sigma}_2^j. \qquad (27)$$

Now introducing (26) and (27), into Eqs. (15)-(20), we get

$$\frac{d}{dt}\left\langle\hat{\sigma}_1^j\right\rangle = -\frac{1}{2}\Big[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\Big]\left\langle\hat{\sigma}_1^j\right\rangle - \frac{\Omega}{2}\langle\hat{\sigma}_2^{\dagger j}\rangle, \qquad (28)$$

$$\frac{d}{dt}\left\langle\hat{\sigma}_2^j\right\rangle = -\frac{1}{2}\Big[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\Big]\left\langle\hat{\sigma}_2^j\right\rangle - \frac{1}{2}\Big[\frac{2\gamma_c\varepsilon k}{k^2-4\varepsilon^2} - \Omega\Big]\langle\hat{\sigma}_1^{\dagger j}\rangle, \qquad (29)$$

$$\frac{d}{dt}\left\langle\hat{\sigma}_0^j\right\rangle = -\frac{1}{2}\Big[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\Big]\left\langle\hat{\sigma}_0^j\right\rangle + \frac{\gamma_c\varepsilon k}{k^2-4\varepsilon^2}\Big[\left\langle\hat{n}_1^j\right\rangle - \left\langle\hat{n}_0^j\right\rangle\Big] + \frac{\Omega}{2}\Big[\left\langle\hat{n}_0^j\right\rangle - \left\langle\hat{n}_2^j\right\rangle\Big], \qquad (30)$$

$$\frac{d}{dt}\left\langle\hat{n}_1^j\right\rangle = -\Big[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\Big] \times \Big[\left\langle\hat{n}_1^j\right\rangle - \left\langle\hat{n}_2^j\right\rangle\Big] + \frac{\gamma_c\varepsilon k}{k^2-4\varepsilon^2}\Big[\langle\hat{\sigma}_0^{\dagger j}\rangle + \langle\hat{\sigma}_0^j\rangle\Big], \qquad (31)$$

$$\frac{d}{dt}\left\langle\hat{n}_2^j\right\rangle = -\Big[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\Big]\left\langle\hat{n}_2^j\right\rangle - \frac{1}{2}\Big[\frac{2\gamma_c\varepsilon k}{k^2-4\varepsilon^2} - \Omega\Big] \times \Big[\langle\hat{\sigma}_0^{\dagger j}\rangle + \langle\hat{\sigma}_0^j\rangle\Big], \qquad (32)$$

$$\frac{d}{dt}\langle\hat{n}_0^j\rangle = \left[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\right]\langle\hat{n}_1^j\rangle - \frac{\Omega}{2}\left[\langle\hat{\sigma}_0^{\dagger j}\rangle + \langle\hat{\sigma}_0^j\rangle\right] + \beta\langle\hat{n}_2^j\rangle, \quad (33)$$

where

$$\gamma_c = \frac{4g^2}{k} \quad (34)$$

is the stimulated emission decay constant.

We next sum Eqs. (29)-(33) over the N three-level atoms, so that

$$\frac{d}{dt}\langle\hat{\Sigma}_1\rangle = -\frac{1}{2}\left[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\right]\langle\hat{\Sigma}_1\rangle - \frac{\Omega}{2}\langle\hat{\Sigma}_2^\dagger\rangle, \quad (35)$$

$$\frac{d}{dt}\langle\hat{\Sigma}_2\rangle = -\frac{1}{2}\left[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\right]\langle\hat{\Sigma}_2\rangle - \frac{1}{2}\left[\frac{2\gamma_c\varepsilon k}{k^2-4\varepsilon^2} - \Omega\right]\langle\hat{\Sigma}_1^\dagger\rangle, \quad (36)$$

$$\frac{d}{dt}\langle\hat{\Sigma}_0\rangle = -\frac{1}{2}\left[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\right]\langle\hat{\Sigma}_0\rangle + \frac{\gamma_c\varepsilon k}{k^2-4\varepsilon^2}\left[\langle\hat{N}_1\rangle - \langle\hat{N}_0\rangle\right] + \frac{\Omega}{2}\left[\langle\hat{N}_0\rangle - \langle\hat{N}_2\rangle\right], \quad (37)$$

$$\frac{d}{dt}\langle\hat{N}_1\rangle = -\left[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\right] \times \left[\langle\hat{N}_1\rangle - \langle\hat{N}_2\rangle\right] + \frac{\gamma_c\varepsilon k}{k^2-4\varepsilon^2}\left[\langle\hat{\Sigma}_0^\dagger\rangle + \langle\hat{\Sigma}_0\rangle\right], \quad (38)$$

$$\frac{d}{dt}\langle\hat{N}_2\rangle = -\left[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\right]\langle\hat{n}_2^j\rangle - \frac{1}{2}\left[\frac{2\gamma_c\varepsilon k}{k^2-4\varepsilon^2} - \Omega\right] \times \left[\langle\hat{\Sigma}_0^\dagger\rangle + \langle\hat{\Sigma}_0\rangle\right], \quad (39)$$

$$\frac{d}{dt}\langle\hat{N}_0\rangle = \left[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\right]\langle\hat{N}_1\rangle - \frac{\Omega}{2}\left[\langle\hat{\Sigma}_0^\dagger\rangle + \langle\hat{\Sigma}_0\rangle\right] + \beta\langle\hat{N}_2\rangle, \quad (40)$$

in which

$$\hat{\Sigma}_1 = \sum_{j=1}^{N}\hat{\sigma}_1^j, \quad (41)$$

$$\hat{\Sigma}_2 = \sum_{j=1}^{N}\hat{\sigma}_2^j, \quad (42)$$

$$\hat{\Sigma}_0 = \sum_{j=1}^{N}\hat{\sigma}_0^j, \quad (43)$$

$$\hat{N}_0 = \sum_{j=1}^{N}\hat{n}_0^j, \quad (44)$$

$$\hat{N}_1 = \sum_{j=1}^{N}\hat{n}_1^j, \quad (45)$$

$$\hat{N}_2 = \sum_{j=1}^{N}\hat{n}_2^j, \quad (46)$$

with the operators $\hat{N}_2$, $\hat{N}_1$, and $\hat{N}_0$ representing the number of atoms in the top, middle, and bottom levels. In addition, employing the completeness relation

$$\hat{n}_0^j + \hat{n}_1^j + \hat{n}_2^j = \hat{I}, \quad (47)$$

we easily arrive at

$$\langle\hat{N}_0\rangle + \langle\hat{N}_1\rangle + \langle\hat{N}_2\rangle = N. \quad (48)$$

Furthermore, applying the definition given by Eq. (2) and setting for any j

$$\hat{\sigma}_1^j = |0\rangle\langle 1|, \quad (49)$$

we have

$$\hat{\Sigma}_1 = N|0\rangle\langle 1|. \quad (50)$$

Following the same procedure, one can easily find

$$\hat{\Sigma}_2 = N|1\rangle\langle 2|, \quad (51)$$

$$\hat{\Sigma}_0 = N|0\rangle\langle 2|, \quad (52)$$

$$\hat{N}_0 = N|0\rangle\langle 0|, \quad (53)$$

$$\hat{N}_1 = N|1\rangle\langle 1|, \quad (54)$$

$$\hat{N}_2 = N|2\rangle\langle 2|. \tag{55}$$

Moreover, using the definition

$$\hat{\Sigma} = \hat{\Sigma}_1 + \hat{\Sigma}_2 \tag{56}$$

and taking into account Eqs. (50)-(55), it can be readily established that

$$\hat{\Sigma}^\dagger\hat{\Sigma} = N(\hat{N}_1 + \hat{N}_2), \tag{57}$$

$$\hat{\Sigma}\hat{\Sigma}^\dagger = N(\hat{N}_0 + \hat{N}_1), \tag{58}$$

$$\hat{\Sigma}^2 = N\hat{\Sigma}_0. \tag{59}$$

We next seek to calculate the expectation value of the atomic operators $\hat{\Sigma}_0$, $\hat{\Sigma}_1$, and $\hat{\Sigma}_2$, To this end, applying the large time approximation scheme to Eqs. (36) and (37), we easily get

$$\langle\hat{\Sigma}_0\rangle = \frac{1}{(\gamma_c k^2 + \beta k^2 - 4\beta\varepsilon^2)} \Big\{\Omega\left[k^2 - 4\varepsilon^2\right] \times \left[N - \langle\hat{N}_1\rangle - 2\langle\hat{N}_2\rangle\right] -2\gamma_c\varepsilon k\left[N - \langle\hat{N}_2\rangle - 2\langle\hat{N}_1\rangle\right]\Big\}, \tag{60}$$

$$\langle\hat{\Sigma}_2\rangle = \frac{1}{2}\left[\frac{\Omega k^2 - 4\Omega\varepsilon^2 - 2\gamma_c\varepsilon k}{\gamma_c k^2 + \beta k^2 - 4\beta\varepsilon^2}\right] \times\langle\hat{\Sigma}_1^\dagger\rangle \tag{61}$$

and in view of the adjoint of (61), Eq. (35) takes the form

$$\frac{d}{dt}\langle\hat{\Sigma}_1(t)\rangle = -\frac{1}{2}\eta\langle\hat{\Sigma}_1(t)\rangle, \tag{62}$$

where

$$\eta = \beta + \frac{\gamma_c k^2}{k^2 - 4\varepsilon^2} + \frac{\Omega}{2}\left[\frac{\Omega k^2 - 4\Omega\varepsilon^2 - 2\gamma_c\varepsilon k}{\gamma_c k^2 + \beta k^2 - 4\beta\varepsilon^2}\right]. \tag{63}$$

We notice that the steady-state solution of Eq. (62) for η different from zero is

$$\langle\hat{\Sigma}_1(t)\rangle = 0. \tag{64}$$

Now on account of (64), one can write (61) in the form

$$\langle\hat{\Sigma}_2(t)\rangle = 0. \tag{65}$$

III. Correlations

In this section we seek to analyze the degree of photon number and atomic-number correlation. In addition, we wish to study the entanglement of photon-state and atomic states in the laser cavity. In the cascading transition from energy level $|2\rangle_j$ to $|0\rangle_j$ via $|1\rangle_j$, a correlation between the two emitted photons a_1 and a_2 can readily be established. Hence the photon number correlation for the cavity modes can be defined as

$$g\left(\hat{n}_1, \hat{n}_2\right)_p = \frac{\langle\hat{a}_1^\dagger(t)\hat{a}_1(t)\hat{a}_2^\dagger(t)\hat{a}_2(t)\rangle}{\langle\hat{a}_1^\dagger(t)\hat{a}_1(t)\rangle\langle\hat{a}_2^\dagger(t)\hat{a}_2(t)\rangle}. \tag{66}$$

On the other hand, using Eqs. (13) and (14) together with (26) and (27), the equation of evolution of cavity mode operators $\hat{a}_1$ and $\hat{a}_2$ can be rewritten as

$$\frac{d}{dt}\hat{a}_1(t) = -\frac{1}{2}\left[\frac{k^2 - 4\varepsilon^2}{k}\right]\hat{a}_1(t) - g\hat{\sigma}_1^j - \frac{2g\varepsilon}{k}\hat{\sigma}_2^{\dagger j}, \tag{67}$$

$$\frac{d}{dt}\hat{a}_2(t) = -\frac{1}{2}\left[\frac{k^2 - 4\varepsilon^2}{k}\right]\hat{a}_2(t) - g\hat{\sigma}_2^j - \frac{2g\varepsilon}{k}\hat{\sigma}_1^{\dagger j}. \tag{68}$$

Applying the steady state solution of Eqs. (67) and (68), one readily established the commutation relation of thecavity mode operator $\hat{a}_1$ and $\hat{a}_1^\dagger$ as well as $\hat{a}_2$ and $\hat{a}_2^\dagger$.

Hence, we notice that

$$\left[\hat{a}_1, \hat{a}_1^\dagger\right]_j = \frac{\gamma_c k}{(k^2 - 4\varepsilon^2)^2}\Big[k^2\left(\hat{n}_0^j - \hat{n}_1^j\right) + 4\varepsilon^2\left(\hat{n}_2^j - \hat{n}_1^j\right) + 2\varepsilon k\left(\hat{\sigma}_0^{\dagger j} + \hat{\sigma}_0^{\dagger j}\right)\Big], \tag{69}$$

$$\left[\hat{a}_2, \hat{a}_2^\dagger\right]_j = \frac{\gamma_c k}{(k^2 - 4\varepsilon^2)^2}\Big[k^2\left(\hat{n}_1^j - \hat{n}_2^j\right) + 4\varepsilon^2\left(\hat{n}_1^j - \hat{n}_0^j\right) - 2\varepsilon k\left(\hat{\sigma}_0^{\dagger j} + \hat{\sigma}_0^{\dagger j}\right)\Big], \tag{70}$$

and summing over all atoms, we obtain

$$\left[\hat{a}_1, \hat{a}_1^\dagger\right] = \frac{\gamma_c k}{(k^2 - 4\varepsilon^2)^2}\Big[k^2\left(\hat{N}_0 - \hat{N}_1\right) + 4\varepsilon^2\left(\hat{N}_2 - \hat{N}_1\right) + 2\varepsilon k\left(\hat{\Sigma}_0^\dagger + \hat{\Sigma}_0\right)\Big], \quad (71)$$

$$\left[\hat{a}_2, \hat{a}_2^\dagger\right] = \frac{\gamma_c k}{(k^2 - 4\varepsilon^2)^2}\Big[k^2\left(\hat{N}_1 - \hat{N}_2\right) + 4\varepsilon^2\left(\hat{N}_1 - \hat{N}_0\right) - 2\varepsilon k\left(\hat{\Sigma}_0^\dagger + \hat{\Sigma}_0\right)\Big], \quad (72)$$

where

$$\left[\hat{a}_i, \hat{a}_k^\dagger\right] = \delta_{ik}\sum_{j=1}^{N}\left[\hat{a}_i, \hat{a}_k^\dagger\right]_j \quad (73)$$

stands for the commutators of $\left(\hat{a}_1, \hat{a}_1^\dagger\right)$ and $\left(\hat{a}_2, \hat{a}_2^\dagger\right)$ when the cavity light modes α_1 and α_2 is interacting with all the N three-level atoms.

In the presence of N three-level atoms, we rewrite Eqs. (67) and (68) as

$$\frac{d}{dt}\hat{a}_1(t) = -\frac{1}{2}\left[\frac{k^2 - 4\varepsilon^2}{k}\right]\hat{a}_1(t) + \lambda_1'\hat{\Sigma}_1 + \lambda_1''\hat{\Sigma}_2^\dagger \quad , \quad (74)$$

$$\frac{d}{dt}\hat{a}_2(t) = -\frac{1}{2}\left[\frac{k^2 - 4\varepsilon^2}{k}\right]\hat{a}_2(t) + \lambda_2'\hat{\Sigma}_2 + \lambda_2''\hat{\Sigma}_1^\dagger \quad , \quad (75)$$

in which λ_1', λ_1'', λ_2' and λ_2'' are a constant whose values remains to be fixed. The steady-state solution of Eqs. (74) and (75) is

$$\hat{a}_1 = \frac{2\lambda_1' k}{(k^2 - 4\varepsilon^2)}\hat{\Sigma}_1 + \frac{2\lambda_1'' k}{(k^2 - 4\varepsilon^2)}\hat{\Sigma}_2^\dagger, \quad (76)$$

$$\hat{a}_2 = \frac{2\lambda_2' k}{(k^2 - 4\varepsilon^2)}\hat{\Sigma}_2 + \frac{2\lambda_2'' k}{(k^2 - 4\varepsilon^2)}\hat{\Sigma}_1^\dagger. \quad (77)$$

On account of Eq. (76) and (77), the commutation relation for the cavity mode operators is

$$\left[\hat{a}_1, \hat{a}_1^\dagger\right] = \frac{4Nk^2}{(k^2 - 4\varepsilon^2)^2}\Big[\lambda_1'^2\left(\hat{N}_0 - \hat{N}_1\right) + \lambda_1''^2\left(\hat{N}_2 - \hat{N}_1\right) + \lambda_1'\lambda_1''\left(\hat{\Sigma}_0^\dagger + \hat{\Sigma}_0\right)\Big], \quad (78)$$

$$\left[\hat{a}_2, \hat{a}_2^\dagger\right] = \frac{4Nk^2}{(k^2 - 4\varepsilon^2)^2}\Big[\lambda_2'^2\left(\hat{N}_1 - \hat{N}_2\right) + \lambda_2''^2\left(\hat{N}_1 - \hat{N}_0\right) - \lambda_1'\lambda_1''\left(\hat{\Sigma}_0^\dagger + \hat{\Sigma}_0\right)\Big]. \quad (79)$$

On comparing Eqs. (71) and (78) together with (72) and (79), shows that

$$\lambda_1' = \lambda_2' = \frac{g}{\sqrt{N}} \quad (80)$$

and

$$\lambda_1'' = \lambda_2'' = \frac{2g\varepsilon}{k\sqrt{N}}. \quad (81)$$

Then Eqs. (76) and (77) can be written as

$$\hat{a}_1 = \frac{2kg}{\sqrt{N}\,(k^2 - 4\varepsilon^2)}\hat{\Sigma}_1 + \frac{4g\varepsilon}{\sqrt{N}\,(k^2 - 4\varepsilon^2)}\hat{\Sigma}_2^\dagger \quad (82)$$

and

$$\hat{a}_2 = \frac{2kg}{\sqrt{N}\,(k^2 - 4\varepsilon^2)}\hat{\Sigma}_2 + \frac{4g\varepsilon}{\sqrt{N}\,(k^2 - 4\varepsilon^2)}\hat{\Sigma}_1^\dagger. \quad (83)$$

Furthermore, the expectation value of the solution of Eqs. (74) and (75) together with (80) and (81) is expressible as

$$\langle\hat{a}_1(t)\rangle = \langle\hat{a}_1(0)\rangle e^{-\frac{1}{2}\eta_0 t} + \frac{g}{\sqrt{N}}e^{-\frac{1}{2}\eta_0 t} \times \int_o^t e^{\frac{1}{2}\eta_0 t'}\langle\hat{\Sigma}_1(t')\rangle + \frac{2g\varepsilon}{k\sqrt{N}}e^{-\frac{1}{2}\eta_0 t}\int_o^t e^{\frac{1}{2}\eta_0 t'}\langle\hat{\Sigma}_2^\dagger(t')\rangle \quad (84)$$

and

$$\langle\hat{a}_2(t)\rangle = \langle\hat{a}_2(0)\rangle e^{-\frac{1}{2}\eta_0 t} + \frac{g}{\sqrt{N}} e^{-\frac{1}{2}\eta_0 t} \int_o^t e^{\frac{1}{2}\eta_0 t'}\langle\hat{\Sigma}_2(t')\rangle + \frac{2g\varepsilon}{k\sqrt{N}} e^{-\frac{1}{2}\eta_0 t} \int_o^t e^{\frac{1}{2}\eta_0 t'}\langle\hat{\Sigma}_1^\dagger(t')\rangle, \quad (85)$$

where

$$\eta_o = \frac{k^2 - 4\varepsilon^2}{k}. \quad (86)$$

Now in view of Eqs. (64) and (65) with the assumption that the cavity light is initially in a vacuum state, Eqs. (84) and (85) goes over into

$$\langle\hat{a}_1(t)\rangle = \langle\hat{a}_2(t)\rangle = 0. \quad (87)$$

On account of this result as well as Eqs. (74) and (75) that $\hat{a}_1(t)$ and $\hat{a}_2(t)$ are Gaussian variables with zero mean. Then Eq. (66) can be rewritten as

$$g\left(\hat{n}_1, \hat{n}_2\right)_p = 1 + \frac{\langle\hat{a}_1^\dagger(t)\hat{a}_2^\dagger(t)\rangle\langle\hat{a}_1(t)\hat{a}_2(t)\rangle}{\langle\hat{a}_1^\dagger(t)\hat{a}_1(t)\rangle\langle\hat{a}_2^\dagger(t)\hat{a}_2(t)\rangle} + \frac{\langle\hat{a}_1^\dagger(t)\hat{a}_2(t)\rangle\langle\hat{a}_2^\dagger(t)\hat{a}_1(t)\rangle}{\langle\hat{a}_1^\dagger(t)\hat{a}_1(t)\rangle\langle\hat{a}_2^\dagger(t)\hat{a}_2(t)\rangle}. \quad (88)$$

Thus employing Eqs. (82) and (83) together with (50) and (51) along with (88), the photon-number correlation turns out to be

$$g\left(\hat{n}_1, \hat{n}_2\right)_p = 1 + \frac{W_1}{W_2}, \quad (89)$$

where

$$W_1 = 4\varepsilon k\Big[\left(k^2 + 4\varepsilon^2\right)\langle\hat{\Sigma}_0\rangle + 2\varepsilon k\left(\langle\hat{N}_2\rangle + \langle\hat{N}_0\rangle\right)\Big], \quad (90)$$

$$W_2 = \left[k^2 + 4\varepsilon^2\right]\Big[k^2\langle\hat{N}_2\rangle + 4\varepsilon^2\langle\hat{N}_0\rangle + 4\varepsilon k\langle\hat{\Sigma}_0\rangle\Big], \quad (91)$$

in which $\langle\hat{\Sigma}_0\rangle$ is given by Eq. (60). Moreover, the atom-number correlation is defined by

$$g\left(\hat{n}_1, \hat{n}_2\right)_a = \frac{\langle\hat{\Sigma}_1^\dagger\hat{\Sigma}_1\hat{\Sigma}_2^\dagger\hat{\Sigma}_2\rangle}{\langle\hat{\Sigma}_1^\dagger\hat{\Sigma}_1\rangle\langle\hat{\Sigma}_2^\dagger\hat{\Sigma}_2\rangle}. \quad (92)$$

We recall that the atomic operators $\hat{\Sigma}_1$ and $\hat{\Sigma}_2$ are Gaussian variables with zero mean. Hence Eq. (92) can be rewritten as

$$g\left(\hat{n}_1, \hat{n}_2\right)_a = 1 + \frac{\langle\hat{\Sigma}_1^\dagger\hat{\Sigma}_2^\dagger\rangle\langle\hat{\Sigma}_1\hat{\Sigma}_2\rangle}{\langle\hat{\Sigma}_1^\dagger\hat{\Sigma}_1\rangle\langle\hat{\Sigma}_2^\dagger\hat{\Sigma}_2\rangle} + \frac{\langle\hat{\Sigma}_1^\dagger\hat{\Sigma}_2\rangle\langle\hat{\Sigma}_2^\dagger\hat{\Sigma}_1\rangle}{\langle\hat{\Sigma}_1^\dagger\hat{\Sigma}_1\rangle\langle\hat{\Sigma}_2^\dagger\hat{\Sigma}_2\rangle}. \quad (93)$$

Thus in view of Eqs. (50) and (51), we obtain

$$g\left(\hat{n}_1, \hat{n}_2\right)_a = 1. \quad (94)$$

We immediately see that the maximum degree of photon number correlation observed when more atoms in the lower energy level than on the upper level. This occurs when the three-level laser is operating below threshold. On the other hand, we note that Eq. (94) that unlike the photon-number correlation, the atoms in the laser cavity are not correlated. Moreover, we point out that in the absence of subharmonic generator, one can never realize correlated photons in the laser cavity.

IV. Entanglement Quantification

Here, we seek to analyze the entanglement of photon-states and atomic states in the laser cavity. Quantum entanglement is a physical phenomenon that occurs when pairs or groups of particles cannot be described independently instead, a quantum state may be given for the system as a whole. Measurements of physical properties such as position, momentum, spin polariza- tion, etc performed on entangled particles are found to be appropriately correlated.

A pair of particles is taken to be entangled in quantum theory, if its states cannot be expressed as a product of the states of its individual constituents. The preparation and manipulation of these entangled states that have non-classical and non-local properties lead to better understanding of the basic quantum principles. It is in this spirit that this section is devoted to the analysis of the entanglement of the two modes (photon-states). In other words, it is a well-known fact that a quantum system is said to be entangled, if it is not separable. That is, if the density operator for the combined state cannot be described as a combination of the product density operators of the constituents,

$$\hat{\rho} \neq \sum_k P_k \hat{\rho}_k^{(1)} \otimes \hat{\rho}_k^{(2)}, \quad (95)$$

in which $P_k \gg 0$ and $\sum_k P_k = 1$ to verify the normalization of the combined density states. On the other hand, on entangled continuous variable (CV) state can be expressed as a common eigenstate of a pair of EPR-type operators [14] such as $\hat{x}_2 - \hat{x}_1$ and $\hat{p}_2 + \hat{p}_1$. The total variance of these two operators reduces to zero for maximally entangled CV states. According to the inseparable criteria given by Duan et al [5], cavity photon-states of a system are entangled, if the sum of the variance of a pair of EPR-like operators,

$$\hat{s} = \hat{x}_2 - \hat{x}_1 \tag{96}$$

and

$$\hat{t} = \hat{p}_2 + \hat{p}_1, \tag{97}$$

where

$$\hat{x}_1 = \frac{1}{\sqrt{2}}\left(\hat{a}_1 + \hat{a}_1^\dagger\right), \tag{98}$$

$$\hat{x}_2 = \frac{1}{\sqrt{2}}\left(\hat{a}_2 + \hat{a}_2^\dagger\right), \tag{99}$$

$$\hat{p}_1 = \frac{i}{\sqrt{2}}\left(\hat{a}_1^\dagger - \hat{a}_1\right), \tag{100}$$

$$\hat{p}_2 = \frac{i}{\sqrt{2}}\left(\hat{a}_2^\dagger - \hat{a}_2\right), \tag{101}$$

are quadrature operators for modes α_1 and α_2, satisfy

$$(\Delta s)^2 + (\Delta t)^2 < 2N \tag{102}$$

and recalling the cavity mode operators $\hat{a}_1$ and $\hat{a}_2$ are Gaussian variables with zero mean, we readily get

$$(\Delta s)^2 + (\Delta t)^2 = \Big[\langle \hat{a}_1^\dagger \hat{a}_1\rangle + \langle \hat{a}_1 \hat{a}_1^\dagger\rangle + \langle \hat{a}_2^\dagger \hat{a}_2\rangle + \langle \hat{a}_2 \hat{a}_2^\dagger\rangle\Big] - \Big[\langle \hat{a}_1 \hat{a}_2\rangle + \langle \hat{a}_1^\dagger \hat{a}_2^\dagger\rangle + \langle \hat{a}_2 \hat{a}_1\rangle + \langle \hat{a}_2^\dagger \hat{a}_1^\dagger\rangle\Big]. \tag{103}$$

Thus with the aid of Eqs. (82) and (83) along with (50) and (51), we arrive at

$$(\Delta s)^2 + (\Delta t)^2 = \frac{\gamma_c k}{(k^2 - 4\varepsilon^2)^2} \times \Big[k^2 + 4\varepsilon^2 - 4\varepsilon k\Big] \times \Big[2N - \langle \hat{N}_0\rangle - \langle \hat{N}_2\rangle - 2\langle \hat{\Sigma}_0\rangle\Big]. \tag{104}$$

Upon setting $\varepsilon = 0$, we see that

$$(\Delta s)^2 + (\Delta t)^2 = \frac{\gamma_c}{k}\Big[2N - \langle \hat{N}_0\rangle - \langle \hat{N}_2\rangle - 2\langle \hat{\Sigma}_0\rangle_0\Big], \tag{105}$$

where

$$\langle \hat{\Sigma}_0\rangle_0 = \frac{\Omega}{(\gamma_c + \beta)}\Big[N - \langle \hat{N}_1\rangle - 2\langle \hat{N}_2\rangle\Big] \tag{106}$$

On the other hand, cavity atomic-states of a system are entangled, if the sum of the variance of a pair of EPR-like operators,

$$\hat{u} = \hat{x}_2' - \hat{x}_1' \tag{107}$$

and

$$\hat{v} = \hat{p}_2' + \hat{p}_1', \tag{108}$$

where

$$\hat{x}_1' = \frac{1}{\sqrt{2}}\left(\hat{\Sigma}_1 + \hat{\Sigma}_1^\dagger\right), \tag{109}$$

$$\hat{x}_2' = \frac{1}{\sqrt{2}}\left(\hat{\Sigma}_2 + \hat{\Sigma}_2^\dagger\right), \tag{110}$$

$$\hat{p}_1' = \frac{i}{\sqrt{2}}\left(\hat{\Sigma}_1^\dagger - \hat{\Sigma}_1\right), \tag{111}$$

$$\hat{p}_2' = \frac{i}{\sqrt{2}}\left(\hat{\Sigma}_2^\dagger - \hat{\Sigma}_2\right), \tag{112}$$

are quadrature operators for the cavity atoms, satisfy

$$(\Delta u)^2 + (\Delta v)^2 < 2N^2. \tag{113}$$

Since $\hat{\Sigma}_1$ and $\hat{\Sigma}_2$ are Gaussian variables with zero mean, so one can easily verify that

$$(\Delta u)^2 + (\Delta v)^2 = \Big[\langle \hat{\Sigma}_1^\dagger \hat{\Sigma}_1\rangle + \langle \hat{\Sigma}_1 \hat{\Sigma}_1^\dagger\rangle + \langle \hat{\Sigma}_2^\dagger \hat{\Sigma}_2\rangle + \langle \hat{\Sigma}_2 \hat{\Sigma}_2^\dagger\rangle\Big] - \Big[\langle \hat{\Sigma}_2^\dagger \hat{\Sigma}_1^\dagger\rangle + \langle \hat{\Sigma}_1 \hat{\Sigma}_2\rangle\Big]. \tag{114}$$

Now with the aid of (50) and (51), Eq. (114) takes the form

$$(\Delta u)^2 + (\Delta v)^2 = N\Big[2N - \langle \hat{N}_0\rangle - \langle \hat{N}_2\rangle - 2\langle \hat{\Sigma}_0\rangle\Big]. \tag{115}$$

V. Conclusion

In this paper we have studied a coherently driven non-degenerate three-level laser with, two-mode subharmonic generator, coupled to a two-mode vacuum reservoir via a single-port mirror whose open cavity contains N non-degenerate three-level atoms. We carried out our analysis by putting the noise operators associated with the vacuum reservoir in normal order and by considering the interaction of the three-level atoms with the vacuum reservoir outside the cavity. Results show that the presence of parametric ampli_er is to increase the squeezing and the mean photon number of the two-mode cavity light significantly. It is found that the photon-states of the system is strongly entangled at steady state where as the atomic state of the system is not entangled. We have also shown that as the stimulated decay constant increases, the degree of entanglement decreases. In addition, we have established that the photons in the laser cavity are highly correlated and the degree of photon number correlation and entanglement increases as the amplitude of the coherent light driving the pump mode increases. Moreover, we have shown that the presence of the subharmonic generator leads to an increase in the degree of entanglement and correlation. Moreover, we point out that in the absence of subharmonic generator, one can never realize correlated photons.

References Références Referencias

1. Quantum Information Theory with Continuous Variables, edited by S. L. Braunstein and A. K. Pati (Klawer, Dordrecht, 2003); C. H. Bennet, Phys. Today 48 (10), 24 (1995); D.P. Di-Vicenzo, Science 270, 255 (1995); A. Furusawa et al. ibid. 282, 706 (1998).
2. M. D. Ried and P. D. Drummond, Phys. Rev. Lett. 60, 2731 (1988); P.Grangier, M. J. Potasek, and B. Yurke, Phys. Rev. A 38, 3132 (1988); B.J.Obliver and C. R. Stroud, Phys. Lett. A 135, 407 (1989).
3. K. Dechoum, P. D. Drummond, S. Chaturvedi, and M.D.Reid, Phys. Rev. A 70, 053807 (2004).
4. Y. Zhang, H. Wang, X. Li, J. Jing, C. Xie, and K. Peng, Phys. Rev. A 62, 023813 (2000).
5. H. Xiong, M. O. Scully, and M. S. Zubairy, Phys. Rev. Lett. 94, 023601 (2005).
6. L. M. Duan, G. Giedke, J. J. Cirac, and P. Zoller, Phys. Rev. Lett. 84, 2722 (2000).
7. H. T. Tan, S. Y. Zhu, and M. S. Zubairy, Phys. Rev. A 72, 022305 (2005).
8. R. Simon, Phys. Rev. Lett. 84, 2726 (2000).
9. S.Tesfa, Phys. Rev. A74, 043816 (2006).
10. C. H. Raymond Ooi, eprintar Xiv:quant-ph/0703084.
11. E. Alebachew, Phys. Rev. A 76, 023808 (2007).
12. Fesseha Kassahun, Opt. Commun. 284, 1357 (2011).
13. Fesseha Kassahun, eprintarXiv:1105.1438v3 quant-ph (2012).
14. Fesseha Kassahun, Refined Quantum Analysis of Light, [Create Space Independent Publishing Platform, 2014].
15. A. Einstein, B. Podolsky, and R. Rosen, Phys. Rev. 47, 777 (1935).14