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CV Entanglement Analysis in the Superposition of Subharmonic and Second Harmonic Generation of Light with Injected Squeezed Laser Beams

By Solomon Getahun, Gelana Chibssa, Sisay Abedella, Melkamu Mossisa & Mohammed Nuri

Jimma University

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I. Introduction

Second harmonic generation (SHG) is a well known non-linear optical phenomena which can be observed only in non-centrosymmetric crystals due to non-zero hyperpolarizability. The SHG signals measured from these crystals were as large as potassium dihydrogen phosphate crystals, KH2PO4 (KDP) [5].

Fesseha considered the case for which the nonlinear crystal is placed inside a cavity driven by coherent light and coupled to two independent vacuum reservoirs via a single-port mirror. By employing the linearization scheme of approximation, he obtained a closed form expressions for the quadrature variance, the mean photon number and the squeezing spectrum for the fundamental mode as well as the second harmonic mode. In his study, he observed once more that the fundamental and second harmonic modes are in a squeezed state and the squeezing in each case occurs in the plus quadrature. It is perhaps worth mentioning that for $\omega_\alpha = \omega_b$ there is a fifty-fifty percent squeezing in the fundamental mode as well as the second harmonic mode [8].

Interaction of three-level atoms with a radiation has attracted a great deal of interest in recent years [20-26]. It is believed that an atomic coherence is found to be responsible for various important quantum features of the emitted light. In general, the atomic coherence can be induced in a three-level atom by coupling the levels between which a direct transition is dipole forbidden by an external radiation or by preparing the atom initially in a coherent superposition of these two levels [27]. It is found that the cavity radiation exhibits squeezing under certain conditions for both cases [28]. In a cascade three-level atom the top, intermediate and bottom levels are conveniently denoted by $|a\rangle$, $|b\rangle$ and $|c\rangle$ in which a direct transition between levels $|a\rangle$ and $|c\rangle$ is dipole forbidden. When the three-level cascade atom decays from $|a\rangle$ to $|c\rangle$ via the level $|b\rangle$ two photons are generated. If the two photons have identical frequency, then the three-level atom is referred to as a degenerate.

Some authors have already studied such a scheme in which the atomic coherence is induced by an external radiation and when initially the atoms are prepared in the top level and bottom level[29,30]. They found that the three-level laser in these cases resemble the parametric oscillator for a strong radiation. Moreover, recently Saavedra et al. studied the three-level laser when the atoms are initially prepared in a coherent superposition and the forbidden transition is induced by driving with strong external radiation. They found that there is lasing without population inversion with the favorable noise reduction occurs for equal population of the two levels and when the initial coherence is maximum. A degenerate three-level laser is in a squeezed state when the probability for the injected atom to be in the bottom level is greater than that of the upper level and the degree of squeezing increases with the linear gain coefficient.

On the other hand, a theoretical analysis of the quantum fluctuations and photon statistics of the signal-mode produced by a sub-harmonic generator has been

Author α σ ρ Ѡ: Department of Physics, Jimma University, P. O. Box 378, Jimma, Ethiopia. e-mail: solgett@yahoo.com
Author ¥: Department of Physics, Defence Engineering College, Bishoftu, Ethiopia.

made by number of authors [4-10]. A maximum of 50% squeezing of the signal mode produced by the subharmonic generator has been predicted by a number of authors [3, 4, 5, 6]. Among other things, it has been predicted that the signal mode has a maximum squeezing of 50% below the vacuum-state level [4-6].

In this article, we seek to investigate the squeezing, entanglement and statistical properties of the superposition of subharmonic and second harmonic light with injected degenerate three level squeezed laser beams applying the superposed density operator.

II. Second Harmonic Light Beams

Second harmonic generation (also, called frequency doubling or SHG) is a nonlinear optical process in which photons with the same frequency interacting with a nonlinear material are effectively" combined" to generate new photon with twice the energy and therefore twice the frequency and half the wave length of the initial photons. It is special case of sum frequency generation and inverse of half-harmonic generation. We consider the case in which the nonlinear crystals (non centrosymmetric KDP crystal) is placed inside the cavity mode driven by coherent light and coupled to vacuum reservoir via a single port mirror.

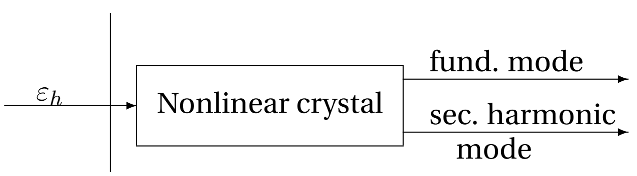


Figure 1: Second harmonic generator

In frequency doubling generation a light mode of frequency ω (fundamental mode) interacts with a nonlinear crystal and is up converted into a light mode of frequency 2ω (second harmonic mode).

Generating the second harmonic, often called frequency doubling, is also a process in radio communication, it was developed in early in the 20th century and has been used with frequencies in the Mhz range. On the other hand, second-harmonic light scattering from colloidal particles has been developed into a powerful and versatile technique for characterizing particle surface. At present, second harmonic light scattering from the particle surface can be quantitatively described by theoretical models and used to measure the adsorption kinetics, molecular structure and reaction ratio at surfaces of micrometer to nanometer sized particles, including biological cells. That is why, we are interested in this paper to present analytical predictions of photon statistics and quadrature squeezing of the second harmonic light superposed with sub-harmonic light in which the cavity driven by degenerate three level squeezed laser. We perform the analysis of second harmonic generation using c-number Langevin equations associated with the normal ordering. Employing the linearization scheme of approximation, we find solutions of cavity mode variables with the aid of which the anti-normally ordered characteristic function and the Q function are calculated. The resulting Q function is then used to determine the expression for the mean photon number, the variance of the photon number, the quadrature variance and squeezing of second harmonic light beam [2,9].

The process of second harmonic generation is described by the Hamiltonian as[8]

$$\hat{H} = i\varepsilon(\hat{a}^{\dagger} - \hat{a}) + \frac{i\lambda}{2}\Big(\hat{b}^{\dagger}\hat{a}^{2} - \hat{b}\hat{a}^{\dagger 2}\Big), \tag{1}$$

where $\hat{a}(\hat{b})$ is the annihilation operator for the fundamental(second harmonic) mode, λ is the coupling constant and ε is proportional to the amplitude of the driving coherent light. Applying Eq.(1) and taking into account the interaction of the fundamental mode and second harmonic mode with the independent vacuum reservoir, the master equation for the cavity mode can be written as [8,11]

$$\begin{aligned}\frac{d\hat{\rho}}{dt} &= \varepsilon\Big(\hat{a}^{\dagger}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger} + \rho\hat{a} - \hat{a}\rho\Big) + \frac{\lambda}{2}\Big(\hat{b}^{\dagger}\hat{a}^{2}\hat{\rho} - \hat{\rho}\hat{b}^{\dagger}\hat{a}^{2} + \hat{\rho}\hat{b}^{\dagger}\hat{a}^{2} - \hat{b}\hat{a}^{\dagger 2}\hat{\rho}\Big)\\ &+\frac{\kappa_a}{2}\Big(2\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger}\hat{a}\Big) + \frac{\kappa_b}{2}\Big(2\hat{b}\hat{\rho}\hat{b}^{\dagger} - \hat{b}^{\dagger}\hat{b}\hat{\rho} - \hat{\rho}\hat{b}^{\dagger}\hat{b}\Big),\end{aligned} \tag{2}$$

in which κ_a and κ_b are the damping constants.

Then the Q function for second harmonic mode is given as

$$Q(\beta^*, \beta, t) = \frac{1}{\pi^2}\int d^2\eta\,\phi_a(\eta^*, \eta, t)e^{\eta^*\beta - \eta\beta^*}, \tag{3}$$

where the anti-normally ordered characteristic function ϕ_a is defined by

$$\phi_a(\eta^*, \eta, t) = Tr\Big(\hat{\rho}e^{-\eta^*\hat{b}}e^{\eta\hat{b}^{\dagger}}\Big). \tag{4}$$

Then applying the Bakers Housdorff identity

$$e^{\hat{A}}e^{\hat{B}} = e^{\hat{A}+\hat{B}+\frac{1}{2}[\hat{A},\hat{B}]}$$

and replacing the operators $\hat{b}$ and $\hat{b}^\dagger$ by the c- number variables β and β^*, we find

$$\phi_a(\eta^*,\eta,t) = exp\bigg(-(1+\langle\beta^*\beta\rangle)\eta^*\eta + \frac{1}{2}(\eta^{*2}\langle\beta^2\rangle + \eta^2\langle\beta^{*2}\rangle)\bigg). \quad (5)$$

The characteristic function can be rewritten as

$$\phi_a(\eta^*,\eta,t) = exp\bigg(-c\eta^*\eta + \frac{d}{2}(\eta^2+\eta^{*2})\bigg), \quad (6)$$

where

$$c \;=\; 1+\frac{\varepsilon_2}{\kappa_b}[\frac{\varepsilon}{\frac{\kappa_a}{2}+\varepsilon_2}]^2 + \frac{\varepsilon_2^2}{2}\bigg[\frac{1}{(\frac{\kappa_a+\kappa_b}{2}-\varepsilon_2)(\frac{\kappa_a}{2}+\varepsilon_2)} - \frac{1}{(\frac{\kappa_a+\kappa_b}{2}+\varepsilon_2)(\frac{\kappa_a}{2}+3\varepsilon_2)}\bigg] \quad (7)$$

and

$$d \;=\; \frac{\varepsilon_2}{\kappa_b}[\frac{\varepsilon}{\frac{\kappa_a}{2}+\varepsilon_2}]^2 - \frac{\varepsilon_2^2}{2}\bigg[\frac{1}{(\frac{\kappa_a+\kappa_b}{2}-\varepsilon_2)(\frac{\kappa_a}{2}+\varepsilon_2)} + \frac{1}{(\frac{\kappa_a+\kappa_b}{2}+\varepsilon_2)(\frac{\kappa_a}{2}+3\varepsilon_2)}\bigg]. \quad (8)$$

Substituting Eq.(6) into Eq.(3) and upon carrying out the integration over the variable η employing the identity

$$\int \frac{d^2\eta}{\pi} exp\bigg(-a\eta^*\eta + b\eta + c\eta^* + A\eta^2 + B\eta^{*2}\bigg) = \frac{1}{\sqrt{a^2-4AB}} exp\bigg(\frac{abc+Ac^2+Bb^2}{a^2-4AB}\bigg), a>0$$

we see that

$$Q(\beta^*,\beta,t) = \frac{1}{\pi}\sqrt{k^2-l^2} exp\bigg(-k\beta^*\beta + \frac{l}{2}(\beta^2+\beta^{*2})\bigg), \quad (9)$$

where

$$k = \frac{c}{c^2-d^2} \quad (10)$$

and

$$l = \frac{d}{c^2-d^2}. \quad (11)$$

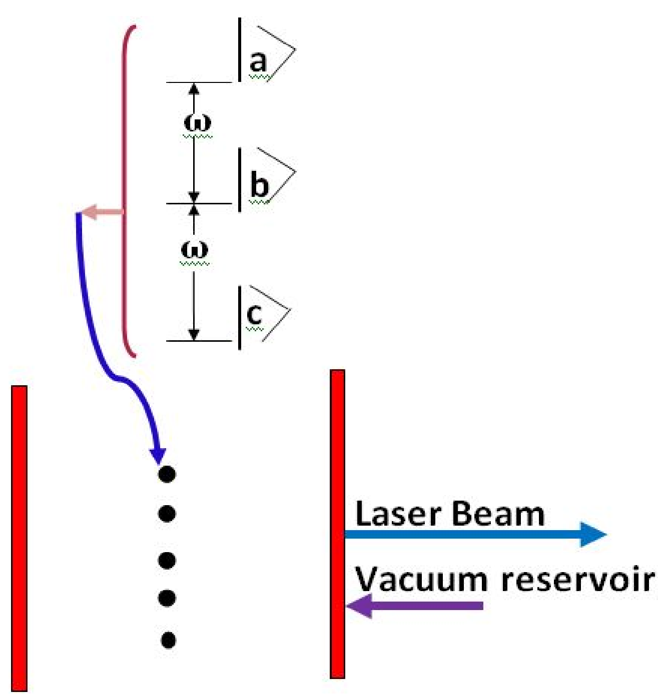


Figure 2: Schematic diagram for degenerate squeezed laser beam

III. Degenerate Three Level Squeezed Laser

A three-level laser is a quantum optical device in which light is generated by three-level atoms in a cavity usually coupled to a vacuum reservoir via a single-port mirror. The statistical and squeezing properties of the light generated by such a three-level laser have been investigated by several authors. It is found that three level laser generates squeezed light under certain conditions. When a three-level atoms in a cascade configuration makes a transition from the top to the bottom level via the intermediate level, photons are generated. In a cascade three-level atom the top, intermediate and bottom levels are conveniently denoted by $|a\rangle$, $|b\rangle$ and $|c\rangle$ in which a direct transition between levels jai and jci is dipole forbidden.

We define a degenerate three-level laser as a quantum optical system in which three level atoms in a cascade configuration and initially prepared in a coherent superposition of the top and bottom levels are injected at a constant rate into a cavity and decays from $|a\rangle$ to $|c\rangle$ via the level $|b\rangle$, photons having the same frequency are generated due to spontaneous emission. These atoms are removed from the cavity after some time.

We seek here to analyze the quantum properties of the light generated by a degenerate three-level laser. To this end, we first derive the equation of evolution of the density operator employing approximation scheme for the cavity mode of the three-level laser. With the aid of this equation, we obtain c-number Langevin equations associated with the normal ordering. The steady-state solution of the resulting equations are then used to determine the anti-normally ordered characteristic function with the aid of which the Q function is obtained. Finally, the Q function is used to calculate the mean photon number, the variance of the photon number and the quadrature squeezing. The interaction of three level atom with the cavity mode can be described by the interaction Hamiltonian [12]

$$\hat{H}_I = ig\Big[(|a\rangle\langle b| + |b\rangle\langle c|)\hat{a} - \hat{a}^{\dagger}(|b\rangle\langle a| + |c\rangle\langle b|)\Big], \quad (12)$$

where g-is the coupling constant and $\hat{a}$ is annihilation operator for cavity mode. The master equation for the cavity mode can be put in the form

$$\frac{d\rho}{dt} = \frac{1}{2}A\rho_{aa}^{(0)}\Big(2\hat{a}^{\dagger}\hat{\rho}\hat{a} - \hat{\rho}\hat{a}\hat{a}^{\dagger} - \hat{a}\hat{a}^{\dagger}\hat{\rho}\Big) + \frac{1}{2}(A\rho_{cc}^{(0)} + \kappa)\Big(2\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{\rho}\hat{a}^{\dagger}\hat{a} - \hat{a}^{\dagger}\hat{a}\hat{\rho}\Big)$$
$$+\frac{1}{2}A\rho_{ac}^{(0)}\Big(\hat{\rho}\hat{a}^{\dagger 2} - \hat{a}^{\dagger 2}\hat{\rho} - 2\hat{a}^{\dagger}\hat{\rho}\hat{a}^{\dagger} + \frac{1}{2}A\rho_{ca}^{(0)}\Big(\hat{\rho}\hat{a}^{2} - \hat{a}^{2}\hat{\rho} - 2\hat{a}\hat{\rho}\hat{a}\Big), \quad (13)$$

in which κ is the cavity damping constant and $A = \frac{2r_a g}{\gamma^2}$ is linear gain coefficient with r_a is the rate at which atoms are injected into the cavity as well as γ is atomic decay constant. It is worth mentioning that the quantum properties of the light generated by the three level laser are determined utilizing the above master equation. It is easy to observe that with $\rho_{aa}^{(0)} = 1$ and $\rho_{ac}^{(0)} = \rho_{cc}^{(0)} = 0$, this equation reduces to the master equation for a two-level laser operating below threshold. Now we seek to obtain the Q function for a light produced by degenerate three level laser. The Q function can be expressed by using the anti-normally ordered characteristic function as [12]

$$Q(\alpha^*, \alpha, t) = \frac{1}{\pi^2}\int d^2z\phi_a(z^*, z, t)e^{(z^*\alpha - z\alpha^*)}. \quad (14)$$

The anti-normally ordered characteristics function turns out to be [12]

$$\phi_a(z^*, z, t) = exp\Big(-ez^*z + (\frac{z^2f^* + z^{*2}f}{2})\Big), \quad (15)$$

where

$$e = 1 + \frac{A(1-\eta)}{2(A\eta + k)} \quad (16)$$

and

$$f = \frac{A(1-\eta^2)^{\frac{1}{2}}}{2(A\eta + k)}e^{i\theta}, \quad (17)$$

where η to be defined as $\eta = \rho_{cc}^{(0)} - \rho_{aa}^{(0)}$. Hence introducing Eq.(15) into (14) and upon carrying out the integration over the variable z, the Q function found, at steady state, to be

$$Q\Big(\alpha^*, \alpha, t\Big) = \frac{1}{\pi}\Big[u^2 - vv^*\Big]^{\frac{1}{2}} exp\Big(-u\alpha^*\alpha + \frac{v^*\alpha^2 + v\alpha^{*2})}{2}\Big), \quad (18)$$

where

$$u = \frac{e}{e^2 - ff^*} \quad (19)$$

and

$$v = \frac{f}{e^2 - ff^*}. \quad (20)$$

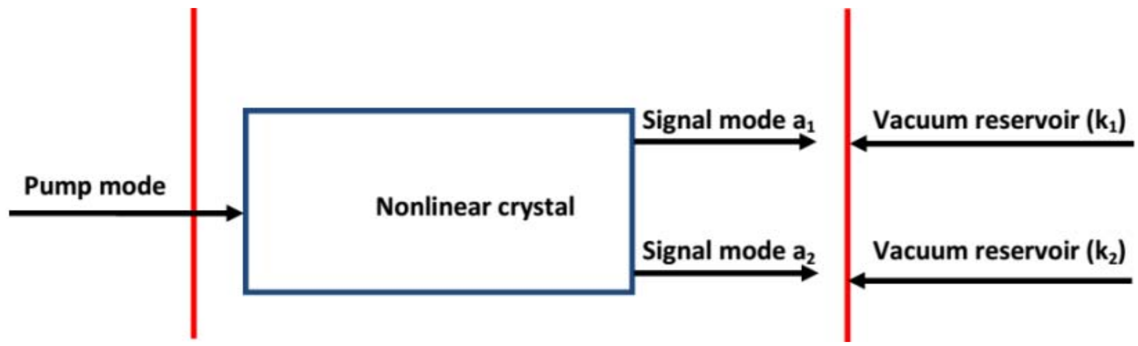


Figure 3: Schematic diagram for subharmonic generator

IV. Subharmonic Light Beams

Sub-harmonic generator is one of the most interesting and well characterized optical devices in quantum optics. In this device, a pump photon interacts with a nonlinear crystal inside a cavity and is down-converted into two highly correlated photons. If these photons have the same frequency, the device is called a one-mode sub-harmonic generator, otherwise it is called a two mode sub-harmonic generator. Then using the master equation, we obtain operator dynamics. The process of subharmonic generation leading to the creation of twin light modes with the same or different frequencies can be described by the Hamiltonian

$$\hat{H} = i\mu(\hat{b}^\dagger - \hat{b}) + i\lambda(\hat{b}^\dagger\hat{a}_1\hat{a}_2 - \hat{b}\hat{a}_1^\dagger\hat{a}_2^\dagger), \quad (21)$$

where $\hat{a}_1$ and $\hat{a}_2$ are the annihilation operators for the light modes, $\hat{b}$ is the annihilation operator for the pump mode, λ is the coupling constant and μ is proportional to the amplitude of the coherent light deriving the pump mode. With the pump mode represented by a real and constant c-number γ, the process of two-mode subharmonic generation can be described by the Hamiltonian

$$\hat{H}_S = i\Gamma(\hat{a}_1\hat{a}_2 - \hat{a}_1^\dagger\hat{a}_2^\dagger), \quad (22)$$

where $\Gamma = \lambda\gamma$. We note that the master equation for a cavity mode coupled to vacuum reservoir can be written

$$\frac{d}{dt}\hat{\rho} = \Gamma(\hat{a}_1\hat{a}_2\hat{\rho} - \hat{\rho}\hat{a}_1\hat{a}_2 + \hat{\rho}\hat{a}_1^\dagger\hat{a}_2^\dagger - \hat{a}_1^\dagger\hat{a}_2^\dagger\hat{\rho}) + \frac{\kappa}{2}(2\hat{a}_1\hat{\rho}\hat{a}_1^\dagger - \hat{a}_1^\dagger\hat{a}_1\hat{\rho} - \hat{\rho}\hat{a}_1^\dagger\hat{a}_1) + \frac{\kappa}{2}(2\hat{a}_2\hat{\rho}\hat{a}_2^\dagger - \hat{a}_2^\dagger\hat{a}_2\hat{\rho} - \hat{\rho}\hat{a}_2^\dagger\hat{a}_2), \quad (23)$$

in which κ is the cavity damping constant for light modes $\hat{a}_1$ and $\hat{a}_2$. Finally, the Q function for the subharmonic light beams found to be

$$Q(\gamma_1, \gamma_2) = \frac{1}{\pi^2}(p^2 - q^2)exp[-p(\gamma_1^*\gamma_1 + \gamma_2^*\gamma_2) - q(\gamma_1\gamma_1 + \gamma_1^*\gamma_2^*)], \quad (24)$$

in which

$$p = \frac{r}{(r^2 - s^2)} \quad (25)$$

and

$$q = \frac{s}{(r^2 - s^2)}, \quad (26)$$

with

$$r = \frac{(\kappa^2 - 2\Gamma^2)}{(\kappa^2 - 4\Gamma^2)}, \quad (27)$$

$$s = \frac{\kappa\Gamma}{(\kappa^2 - 4\Gamma^2)}. \quad (28)$$

V. Superposition of Subharmonic and Second Harmonic Light with Squeezed Laser Beams

In this section, we wish to study the statistical and squeezing properties of the light generated by the superposition of subharmonic and second harmonic light with degenerate three level squeezed laser beams. We first determine the density operator for the superposed light beams. Then employing this density operator, we calculate the mean photon number, the quadrature variance, quadrature squeezing and continuous variable entanglement.

a) *Density operator*

Here we seek to determine the density operator for the superposed subharmonic and the second harmonic light with degenerate three level squeezed laser beams. The density operator for first light beam is expressible as

$$\hat{\rho}'(\hat{b}^\dagger, \hat{b}, t) = \int d^2\beta Q_1\left(\beta^*, \beta + \frac{\partial}{\partial\beta^*}, t\right)|\beta\rangle\langle\beta|. \quad (29)$$

In terms of displacement operator, this expression can be put in the form

$$\hat{\rho}'(\hat{b}^{\dagger},\hat{b},t)=\int d^2\beta Q_1\Big(\beta^*,\beta+\frac{\partial}{\partial\beta^*},t\Big)\hat{D}(\beta)\hat{\rho}_0\hat{D}(-\beta), \tag{30}$$

in which

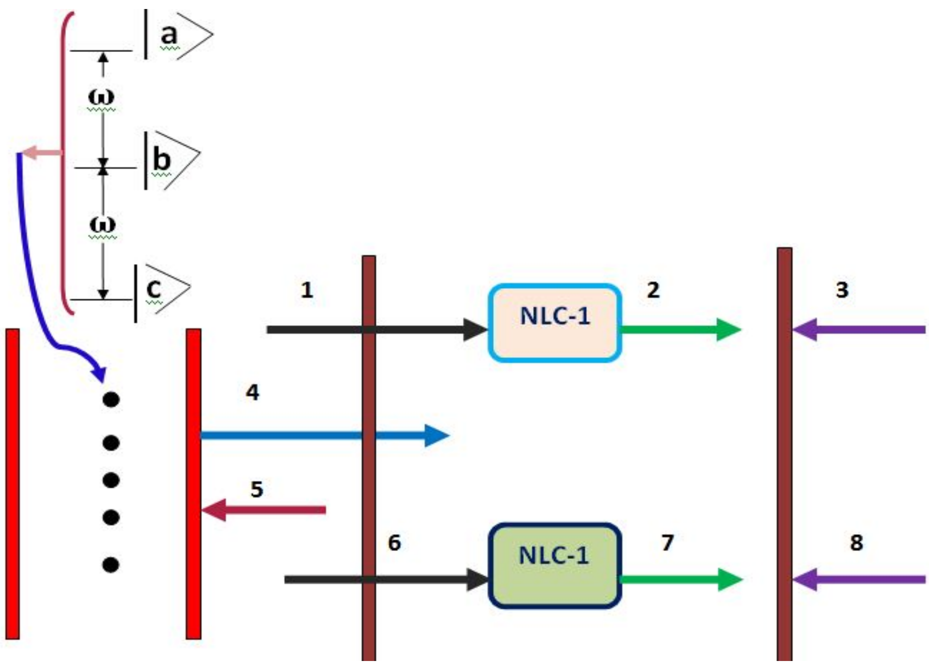
$$|\beta\rangle\langle\beta|=\hat{D}(\beta)\hat{\rho}_0\hat{D}(-\beta), \tag{31}$$

with $\hat{\rho}_0 = |0\rangle\langle 0|$. Now we realize that the density operator for the superposition of second harmonic light and degenerate three level squeezed laser leads to

$$\hat{\rho}(\hat{a}^{\dagger},\hat{a},t)=\int d^2\alpha Q_2\Big(\alpha^*,\alpha+\frac{\partial}{\partial\alpha^*},t\Big)\hat{D}(\alpha)\hat{\rho}'(t)\hat{D}(-\alpha), \tag{32}$$

so that in view of Eq. (30), the density operator for the superposed light beams turns out to be

$$\hat{\rho}(\hat{c}^{\dagger},\hat{c},t)=\int d^2\beta d^2\alpha Q_1\Big(\beta^*,\beta+\frac{\partial}{\partial\beta^*},t\Big)Q_2\Big(\alpha^*,\alpha+\frac{\partial}{\partial\alpha^*},t\Big)|\beta+\alpha\rangle\langle\alpha+\beta|. \tag{33}$$



1.) Pump mode-1. 2.) Second harmonic light.
3.) Vacuum reservoir-1. 4.) Laser beam.
5.) Vacuum reservoir-3. 6.) Pump mode-2.
7.) Subharmonic light. 8.) Vacuum reservoir-2.

Figure 4: Schematic diagram for superposition of subharmonic and second harmonic generation of light with injected squeezed laser beams

Following a similar procedure, one can readily establish the density operator for the superposition of subharmonic and second harmonic generation of light with squeezed laser beams as

$$\begin{aligned}\hat{\rho}_{sup.}(\hat{c}^{\dagger},\hat{c},t) &= \int d^2\beta d^2\alpha d^2\gamma_1 d^2\gamma_2 Q_1\Big(\beta^*,\beta+\frac{\partial}{\partial\beta^*},t\Big)Q_2\Big(\alpha^*,\alpha+\frac{\partial}{\partial\alpha^*},t\Big)\\ &\times Q_3\Big(\gamma_1^*,\gamma_2^*,\gamma_1+\frac{\partial}{\partial\gamma_1^*},\gamma_2+\frac{\partial}{\partial\gamma_2^*},t\Big)|\beta+\alpha+\gamma_2+\gamma_1\rangle\langle\alpha+\beta+\gamma_2+\gamma_1|. \end{aligned} \tag{34}$$

It is worth noting that the expectation value of operator $\hat{A}(\hat{c}^{\dagger},\hat{c},t)$ in terms of the density operator can be put in the form

$$\langle\hat{A}(\hat{c}^{\dagger},\hat{c},t)\rangle=Tr(\hat{\rho}_{sup.}(t)\hat{A}(0)). \tag{35}$$

Introducing Eq.(34) into Eq.(35), we find

$$\langle \hat{A}(\hat{c}^\dagger,\hat{c},t)\rangle = \int d^2\beta d^2\alpha d^2\gamma_1 d^2\gamma_2 Q_1\left(\beta^*,\beta+\frac{\partial}{\partial\beta^*},t\right)Q_2\left(\alpha^*,\alpha+\frac{\partial}{\partial\alpha^*},t\right)$$
$$\times Q_3\left(\gamma_1^*,\gamma_2^*,\gamma_1+\frac{\partial}{\partial\gamma_1^*},\gamma_2+\frac{\partial}{\partial\gamma_2^*},t\right)\hat{A}_n(\beta,\alpha,\gamma_1,\gamma_2), \tag{36}$$

in which $\hat{A}_n(\beta,\alpha,\gamma_1,\gamma_2)=\langle\beta+\alpha+\gamma_2+\gamma_1|A_0|\gamma_1+\gamma_2+\alpha+\beta\rangle$ is the c- number function associated with the operators $\hat{A}(\hat{c}^\dagger,\hat{c},t)$ in the normal ordering [11,12].

b) Photon statistics

The statistical properties of the light beams produced by the superposed light beams can also be studied employing the density operator. Here we calculate the mean photon number and the variance of photon number of the light produced by the superposed subharmonic and second harmonic light with degenerate three level squeezed laser beams.

i. *The mean photon number*

The mean photon number of the superposed light beams can be expressed in terms of density operator as

$$\bar{n} = Tr(\hat{\rho}(t)\hat{c}^\dagger(0)\hat{c}(0)), \tag{37}$$

where $\hat{c}$ represents the annihilation operator for the superposed light beams and

$$\hat{c} = \hat{b}+\hat{a}+\hat{a}_1+\hat{a}_2. \tag{38}$$

Upon substituting Eq.(38) and its dagger into Eq. (37), we see that

$$\bar{n} = Tr\Big[\hat{\rho}(t)\Big(\hat{b}^\dagger\hat{b}+\hat{b}^\dagger\hat{a}+\hat{a}^\dagger\hat{a}_1+\hat{b}^\dagger\hat{a}_2$$
$$+\hat{a}^\dagger\hat{b}+\hat{a}^\dagger\hat{a}+\hat{a}^\dagger\hat{a}_1+\hat{a}^\dagger\hat{a}_2+$$
$$+\hat{a}_1^\dagger\hat{b}+\hat{a}_1^\dagger\hat{a}+\hat{a}_1^\dagger\hat{a}_1+\hat{a}_1^\dagger\hat{a}_2+$$
$$+\hat{a}_2^\dagger\hat{b}+\hat{a}_2^\dagger\hat{a}+\hat{a}_2^\dagger\hat{a}_1+\hat{a}_2^\dagger\hat{a}_2\Big)\Big]. \tag{39}$$

Thus, introducing the superposed density operator into Eq.(37) and with the aid of Eq. (36) as well as applying the cyclic property of the trace, we find

$$\bar{n} = \langle\hat{b}^\dagger(t)\hat{b}(t)\rangle + \langle\hat{a}^\dagger(t)\hat{a}(t)\rangle + \langle\hat{a}_1^\dagger(t)\hat{a}_1(t)\rangle + \langle\hat{a}_2^\dagger(t)\hat{a}_2(t)\rangle. \tag{40}$$

It then follow that

$$\bar{n}_{ss} = \frac{\varepsilon_2^2}{2}\left(\frac{1}{(\frac{\kappa_a+\kappa_b}{2}-\varepsilon_2)(\frac{\kappa_a}{2}+\varepsilon_2)}-\frac{1}{(\frac{\kappa_a+\kappa_b}{2}+\varepsilon_2)(\frac{\kappa_a}{2}+3\varepsilon_2)}\right)$$
$$+\frac{\varepsilon_2}{\kappa_b}\left(\frac{\varepsilon}{\frac{\kappa_a}{2}+\varepsilon_2}\right)^2+\frac{A(1-\eta)}{2(A\eta+\kappa)}+\frac{4\Gamma^2}{\kappa^2-4\Gamma^2}. \tag{41}$$

This result immediately indicates that the mean photon number of the superposed subharmonic and second harmonic light with degenerate three level squeezed laser is the sum of the mean photon number of the individual light beams.

ii. *The variance of the photon number*

The variance of the photon number, for the superposed light beams, can be defined as

$$(\Delta n)^2 = \langle(\hat{c}^\dagger(t)\hat{c}(t))^2\rangle - \langle\hat{c}^\dagger(t)\hat{c}(t)\rangle^2. \tag{42}$$

Since $\hat{c}(t)$ is Gaussian variable with zero mean and applying the commutation relation of the superposed light beams

$$[\hat{c}\,,\,\hat{c}^\dagger] = 4, \tag{43}$$

we see that

$$(\Delta n)^2 = \bar{n}^2+4\bar{n}+\langle\hat{a}^2(t)\rangle^2+\langle\hat{b}^2(t)\rangle^2+\langle\hat{a}_1(t)\hat{a}_2(t)\rangle^2+\langle\hat{a}_2(t)\hat{a}_1(t)\rangle^2$$

$$+2\Bigg[\langle\hat{a}^2(t)\rangle\langle\hat{b}^2(t)\rangle+\langle\hat{a}^2(t)\rangle\langle\hat{a}_1(t)\hat{a}_2(t)\rangle+\langle\hat{a}^2(t)\rangle\langle\hat{a}_2(t)\hat{a}_1(t)\rangle$$

$$+\langle\hat{b}^2(t)\rangle\langle\hat{a}_1(t)\hat{a}_2(t)\rangle+\langle\hat{b}^2(t)\rangle\langle\hat{a}_2(t)\hat{a}_1(t)\rangle+\langle\hat{a}_1(t)\hat{a}_2(t)\rangle\langle\hat{a}_2(t)\hat{a}_1(t)\rangle\Bigg]. \quad (44)$$

Now making use of the Q functions of the separate light beams, one can readily establish the expectation value of the operators described by Eq.(44). Hence, the variance of the photon number for the superposed subharmonic and second harmonic light with degenerate three level squeezed laser beams turns out to be

$$(\Delta n)^2 = 4\bar{n}+\bar{n}^2++d^2+\frac{A^2(1-\eta^2)}{4(A\eta+\kappa)^2}+\frac{Ad(1-\eta^2)^{\frac{1}{2}}}{(A\eta+\kappa)}$$

$$+\frac{4\kappa^2\Gamma^2}{(\kappa^2-4\Gamma^2)^2}+\frac{4d\kappa\Gamma}{\kappa^2-4\Gamma^2}+\left(\frac{A(1-\eta^2)^{\frac{1}{2}}}{(A\eta+\kappa)}\right)\left(\frac{\kappa\Gamma}{\kappa^2-4\Gamma^2}\right). \quad (45)$$

This calculation shows that unlike the mean photon number, the variance of the photon number of the light produced by the superposed sub-harmonic and second-harmonic light with three level squeezed laser beams is not the sum of the variance of the photon number of the constituent light beams. However, by setting $\varepsilon_2=0$ and $\Gamma=0$, we easily get the variance of the photon number for the degenerate three level laser. On the other hand, by setting A=0 and $\Gamma=0$, we immediately notice the variance of the photon number for the second-harmonic light. Similarly, if switch off the second-harmonic light and the laser beam, we clearly notice the variance of the photon number of the sub-harmonic light.

c) Quadrature fluctuations

Here we wish to study the quadrature squeezing of the system under consideration. First we evaluate the quadrature variance of the superposed light beams and then we determine the degree of quadrature squeezing.

i. *Quadrature variance*

We define the quadrature variance of the superposed light beams as

$$(\Delta c_\pm)^2=\langle\hat{c}_\pm^2(t)\rangle-\langle\hat{c}_\pm(t)\rangle^2, \quad (46)$$

where

$$\hat{c}_+(t)=\hat{c}^\dagger(t)+\hat{c}(t) \quad (47)$$

and

$$\hat{c}_-(t)=i\Big(\hat{c}^\dagger(t)-\hat{c}(t)\Big), \quad (48)$$

are the plus and minus quadratures for the superposed light beams. Using Eq.(47) and (48) along with the commutation relation given by Eq.(43), Eq.(46) can be rewritten as

$$\Big(\Delta c_\pm\Big)^2 = \Big[1+2\langle\hat{b}^\dagger(t)\hat{b}(t)\rangle\pm 2\langle\hat{b}^2(t)\rangle\Big]+\Big[1+2\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle\pm 2\langle\hat{a}^2(t)\rangle\Big].$$

$$+\Big[1+2\langle\hat{a}_1^\dagger(t)\hat{a}_1(t)\rangle\pm 2\langle\hat{a}_1^2(t)\rangle\Big]+\Big[1+2\langle\hat{a}_2^\dagger(t)\hat{a}_2(t)\rangle\pm 2\langle\hat{a}_2^2(t)\rangle\Big]. \quad (49)$$

This expression leads to

$$\Big(\Delta\hat{c}_\pm\Big)^2=\Big(\Delta\hat{b}_\pm\Big)^2+\Big(\Delta\hat{a}_\pm\Big)^2+\Big(\Delta\hat{a}_{\pm1}\Big)^2++\Big(\Delta\hat{a}_{\pm2}\Big)^2. \quad (50)$$

We easily observe that the the quadrature variance of the superposed light beams is the sum of that of the individual light beams. It then follows that

$$\Big(\Delta\hat{c}_\pm\Big)^2 = \left[1\mp\frac{2\varepsilon_2^2}{(\frac{\kappa_a+\kappa_b}{2}\pm\varepsilon_2)(\frac{\kappa_a}{2}+(2\pm1)\varepsilon_2)}\right]$$

$$+\left[\frac{\kappa+A\Big(1\pm(1-\eta^2)^{\frac{1}{2}}cos\theta\Big)}{A\eta+\kappa}\right]$$

$$+\left[2 \mp \frac{4\Gamma}{k+2\Gamma}\right]. \tag{51}$$

We notice that squeezing occurs in the minus quadrature for the degenerate three level laser, but in the plus quadrature for the subharmonic and second harmonic light beams. It is worth noting that by setting $\varepsilon_2 = 0$ as well as $\Gamma = 0$, one can easily check that

$$\left(\Delta\hat{c}_{\pm}\right)^2 = \left(\Delta\Omega_{\pm}\right)^2 + \left[\frac{\kappa + A\left(1 \pm (1-\eta^2)^{\frac{1}{2}} cos\theta\right)}{A\eta + \kappa}\right], \tag{52}$$

in which $\left(\Omega_{\pm}\right)^2 = 3$ being the quadrature variance of the superposed coherent light beams. On the other hand, by setting A = 0 as well as $\Gamma = 0$ and A= 0 as well as $\varepsilon_2 = 0$, respectively, we easily verify that

$$\left(\Delta\hat{c}_{\pm}\right)^2 = \left(\Delta\Omega_{\pm}\right)^2 \mp \frac{2\varepsilon_2^2}{(\frac{\kappa_a+\kappa_b}{2} \pm \varepsilon_2)(\frac{\kappa_a}{2} + (2 \pm 1)\varepsilon_2)} \tag{53}$$

and

$$\left(\Delta\hat{c}_{\pm}\right)^2 = \left(\Delta\Omega_{\pm}\right)^2 \mp \frac{4\Gamma}{\kappa+2\Gamma}, \tag{54}$$

in which $\left(\Delta\Omega_{\pm}\right)^2 = 4$ being the quadrature variance of the superposed coherent light beams. It is essential to point out that the quadrature variance of the coherent light indeed affects the quadrature variance of the superposed light beams.

ii. *Quadrature squeezing*

The degree of quadrature squeezing for the superposition of subharmonic and second-harmonic light with degenerate three level squeezed laser beams can be defined as

$$S = \frac{\left[1 - (\Delta\hat{b}_+)^2 + 1 - (\Delta\hat{a}_-)^2 + 1 - (\Delta\hat{a}_{+1})^2 + 1 - (\Delta\hat{a}_{+2})^2\right]}{4}. \tag{55}$$

Then Eq(55) leads to

$$S = \frac{1}{4}\left[1 - \left(1 - \frac{2\varepsilon_2^2}{(\frac{\kappa_a+\kappa_b}{2} + \varepsilon_2)(\frac{\kappa_a}{2} + 3\varepsilon_2)}\right)\right.$$

$$\left. +1 - \left(\frac{\kappa + A\left(1-(1-\eta^2)^{\frac{1}{2}}\right)}{A\eta+\kappa}\right) + \left(2 - \frac{4\Gamma}{\kappa+2\Gamma}\right)\right]. \tag{56}$$

This result immediately indicates that the maximum degree of squeezing for the superposed light beams is the average of that of the separate light beams and it is approximately 54.18% below the coherent state level for the values $\kappa = \kappa_a = \kappa_b$ and $\kappa = 2\Gamma$. Furthermore, we notice that the degree of squeezing of the superposed light beams certainly reduced by 16.68% if upon setting $\varepsilon_2 = 0$ in the second harmonic generation. On the contrary, upon setting $A = 0$ in the three-level laser, the degree of squeezing decreases by 13%. But, the degree of squeezing of the superposed light beams amplified by 25% if $\Gamma = 0$.

d) Continuous variable (CV) entanglement

Encoding quantum information in continuous variables, as the quadrature of light beams, is a power full method to quantum information science and technology. Continuous-variable entanglement, is nothing but light beams on Einstein-Podolsky-Rosen (EPR) states, is a key resource for quantum information protocols and enables hybridization between continuous-variable and single-photon discrete-variable quantum systems. However, continuous variable systems are currently limited by their implementation in free-space optical networks and demand for increased complexity, it gives an alternative approach to low loss, high-precision alignment and stability as well as hybridization [30]. In this section we wish to verify the degree of photonic continuous variable entanglement in the superposition of subharmonic and second harmonic light with degenerate three-level squeezed laser beams that might be help full in quantum information processing.

It is known that the density operator of superposed light beams ρ of two modes a and b is said to be entangled or not separable if it is not possible to express in the form

$$\rho = \sum_i P_i \rho_i^{(a)} \otimes \rho_i^{(b)}, \tag{57}$$

where $\rho_i^{(a)}$ and $\rho_i^{(b)}$ being the normalized density operators of mode-a and mode-b, respectively with $P_i \geqslant 0$ and $\sum_i P_i = 1$. A maximally entangled continuous variable state can be expressed as a co-eigenstate of a pair of Einstein- Podolsky-Rosen EPR-type operators [31] such as $\hat{x}_a - \hat{x}_b$ and $\hat{p}_a + \hat{p}_b$. Thus the sum of the variances of these operators is reduced to zero for the maximally entangled continuous variable state [32].

In order to check the entanglement condition of the photons between the superposition of subharmonic and second harmonic light with degenerate three-level laser beams, we apply the criterion presented in Ref. [32]. On the basis of this criterion, a quantum state of a system is said to be entangled if the sum of the variances of the two EPR-like operators $\hat{r}$ and $\hat{s}$ of the four modes satisfy the inequality

$$(\Delta\hat{r})^2 + (\Delta\hat{s})^2 < 4, \tag{58}$$

in which

$$\hat{r} = \hat{x}_a - \hat{x}_b - \hat{x}_{a_1} - \hat{x}_{a_2}, \tag{59}$$

$$\hat{s} = \hat{p}_a + \hat{p}_b + \hat{p}_{a_1} + \hat{p}_{a_2}, \tag{60}$$

with

$$\hat{x}_a = \frac{1}{2}(\hat{a}^\dagger + \hat{a}), \tag{61}$$

$$\hat{x}_b = \frac{1}{2}(\hat{b}^\dagger + \hat{b}), \tag{62}$$

$$\hat{x}_{a_1} = \frac{1}{2}(\hat{a}_1^\dagger + \hat{a}_1), \tag{63}$$

$$\hat{x}_{a_2} = \frac{1}{2}(\hat{a}_2^\dagger + \hat{a}_2), \tag{64}$$

$$\hat{p}_a = \frac{i}{2}(\hat{a}^\dagger - \hat{a}), \tag{65}$$

$$\hat{p}_b = \frac{i}{2}(\hat{b}^\dagger - \hat{b}), \tag{66}$$

$$\hat{p}_{a_1} = \frac{i}{2}(\hat{a}_1^\dagger - \hat{a}_1) \tag{67}$$

and

$$\hat{p}_{a_2} = \frac{i}{2}(\hat{a}_2^\dagger - \hat{a}_2), \tag{68}$$

are the quadrature operators for cavity light modes in the superposition of subharmonic and second harmonic light with squeezed laser beams. The variance of the operators $\hat{r}$ and $\hat{s}$ takes the form

$$(\Delta\hat{r})^2 + (\Delta\hat{s})^2 = 2 + \langle\hat{a}^\dagger\hat{a}\rangle + \langle\hat{b}^\dagger\hat{b}\rangle + \langle\hat{a}_1^\dagger\hat{a}_1\rangle + \langle\hat{a}_2^\dagger\hat{a}_2\rangle. \tag{69}$$

Thus, on account of Eq. (41), the sum of the variance of the operators $\hat{r}$ and $\hat{s}$ turns out to be

$$(\Delta\hat{r})^2 + (\Delta\hat{s})^2 = 2 + \bar{n}. \tag{70}$$

Now setting $\varepsilon = 0$ and $\kappa_a = \kappa_b = \kappa$ in Eq. (70), we obtain

$$(\Delta\hat{r})^2 + (\Delta\hat{s})^2 = 2 + \frac{\varepsilon_2^2}{2}\left[\frac{3\kappa\varepsilon_2 + 4\varepsilon_2^2}{\left(\frac{\kappa^2}{2} + \frac{1}{2}\kappa\varepsilon_2 - \varepsilon_2^2\right)\left(\frac{\kappa^2}{2} + \frac{7}{2}\kappa\varepsilon_2 + 3\varepsilon_2^2\right)}\right] + \frac{A(1-\eta)}{2(A\eta + \kappa)} + \frac{4\Gamma^2}{\kappa^2 - 4\Gamma^2}, \tag{71}$$

so that taking numerical values $\varepsilon_2 = \Gamma = 0.25$, $\kappa = 0.8$ and $A = 75$, we find

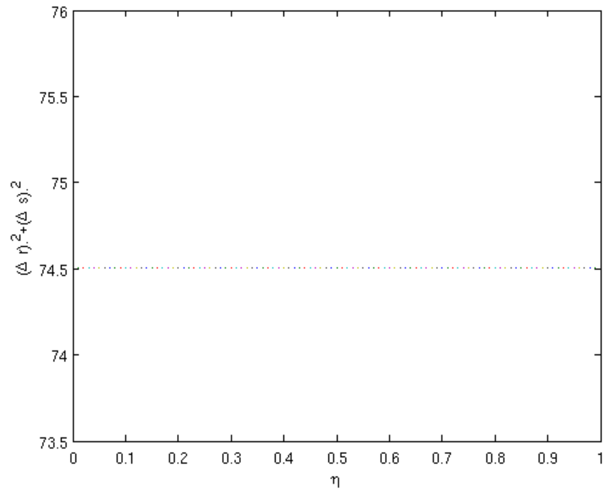


Figure 5: A plot of $(\Delta r)^2 + (\Delta s)^2$ [Eq. 72] versus η

$$(\Delta \hat{r})^2 + (\Delta \hat{s})^2 \;=\; 2.7 + \frac{75(1-\eta)}{(150\eta + 0.64)}. \tag{72}$$

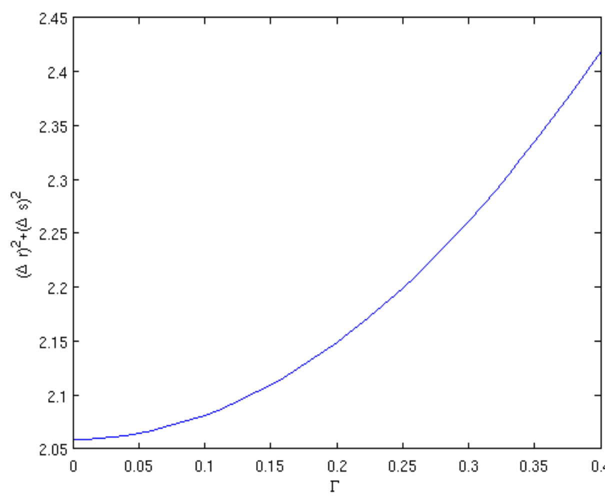


Figure 6: A plot of $(\Delta r)^2 + (\Delta s)^2$ [Eq. 73] versus Γ for $\kappa = 0.8$, $\varepsilon_2 = 0.25$ and $A = 0$

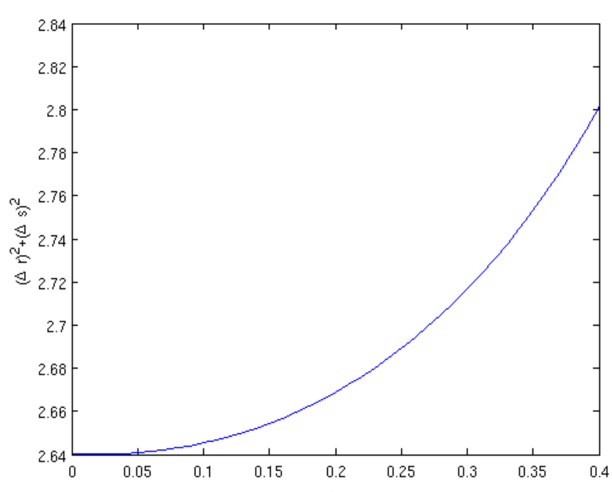


Figure 7: A plot of $(\Delta r)^2 + (\Delta s)^2$ [Eq. 74] versus ε_2 for $\kappa = 0.8$, $\Gamma = 0.25$ and $A = 0$

We easily see from the plot in Fig. 5 that $(\Delta\hat{r})^2 + (\Delta\hat{s})^2 = 74.5$. Hence, the entanglement criterion described by Eq. (58) is not satisfied. This shows that the superposition of subharmonic and second harmonic light with injected squeezed laser beams are not entangled photons at steady state.

On the contrary, taking numerical values $\varepsilon_2 = 0.25$, $\kappa = 0.8$ and $A = 0$, we immediately note that

$$(\Delta\hat{r})^2 + (\Delta\hat{s})^2 = 2.059 + \frac{4\Gamma^2}{\kappa^2 - 4\Gamma^2} \tag{73}$$

and setting $\Gamma = 0.25$, $\kappa = 0.8$ and $A = 0$ in Eq. (71), we check that

$$(\Delta\hat{r})^2 + (\Delta\hat{s})^2 = 2.64 + \frac{1.2\varepsilon_2^3 + 2\varepsilon_2^4}{0.1 + 0.99\varepsilon_2 + 2.96\varepsilon_2^2 - 2.8\varepsilon_2^3 - 3\varepsilon_2^4}, \tag{74}$$

which are less than 4 as shown in the plot at Figure-6 and Figure-7, respectively. These expressions reveal that the superposition of subharmonic and second harmonic light beams in the absence of the laser beam satisfy the entanglement condition of the photons. Furthermore, setting $\varepsilon = \kappa_a = \kappa_b = \kappa = 0$ in Eq. (71), we see that

$$(\Delta\hat{r})^2 + (\Delta\hat{s})^2 = \frac{1-\eta}{2\eta} - 0.33. \tag{75}$$

We observe from the plot in Fig. 8 that $(\Delta\hat{r})^2 + (\Delta\hat{s})^2 = 0.5$. Therefore, the entanglement

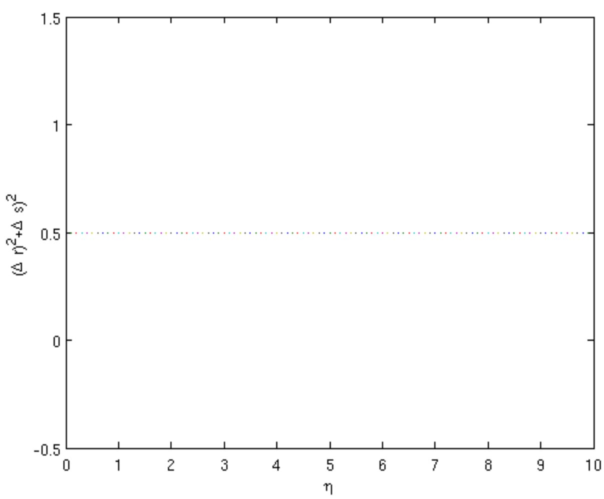


Figure 8: A plot of $(\Delta r)^2 + (\Delta s)^2$ [Eq. 71] versus η criterion is satisfied

VI. Conclusion

In this paper, we have studied the squeezing and statistical properties of the light generated by the superposed subharmonic and second harmonic light with injected degenerate three-level squeezed laser beams. Employing the superposed density operator, we have found that the mean photon number of the superposed light beams to be the sum of that of the mean photon number of the constituent light beams. However, the photon number variance of the superposed light beams does not happen to be the sum of the photon number variances of the separate light beams.

Furthermore, the squeezing occurs in the minus quadrature for the degenerate three level laser but, for the subharmonic and second harmonic light occurs in the plus quadrature. We have also obtained that the quadrature variance of the superposed light beams is the sum of the quadrature variance of segregate light beams and this leads to the degree of squeezing is the average of the constituent light beams and is approximately 53.95% below the coherent state level.

Even though, we have proven that the superposition of subharmonic and second harmonic light with injected degenerate three-level squeezed laser beams do not show continuous variable entanglement, the absence of the laser beam in the superposition state give rise entangled photons.

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