JIMMA UNIVERSITY
COLLEGE OF NATURAL SCIENCES
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DYNAMICS OF ACCRETION PROCESS AROUND AGN IN SCHWARZSCHILD-DE SITTER GEOMETRY

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To my family

Table of Contents

Table of Contents iv

List of Figures vi

Acknowledgements vii

Abstract viii

1 Introduction 1
1.1 Background 1
1.2 Literature Review 3
1.3 Statement of the problem 6
1.4 Research Questions 6
1.5 Objectives 7
1.5.1 General Objective 7
1.5.2 Specific Objectives 7
1.6 Methodology 7

2 Einstein Theory of General Relativity 9
2.1 Introduction to Einstein General Relativity 9
2.1.1 The Metric Tensor 10
2.1.2 Christoffel Tensor 10
2.1.3 The Riemann-Christoffel Curvature Tensor 12
2.2 Einstein Field Equation 13
2.3 Introduction of Cosmological Constant into Einstein Field Equations 19
2.4 Schwarzschild de Sitter Metric 20

3 AGN and Dynamic Accretion Process Around AGN in sds Geometry 24
3.1 Historical Discovery and Research Development about AGN 24
3.2 Dynamic of Accretion Process Around AGN in sds Geometry 27

4 Particle Dynamics in Schwarzschild De -Sitter Geometry **29**
4.1 Lagrangian and Hamiltonian Dynamic of Particles Motion 29
4.2 Derive Dynamic parameters by Corresponding Principle in sds Geometry . 32
4.3 Orbital Dynamics Around sds Spacetime 35

5 Result and Discussion **37**
5.1 Particle dynamics along circular orbit 40

6 Conclusion **43**

Bibliography **44**

List of Figures

5.1 angular momentum of λ_{sw} and λ_{sd} 38

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Abstract

More recent literature reviews point out that most galaxies, especially early type galaxies with Active Galactic Nuclei (AGNs), contain Massive Black Holes (MBHs) considered to be comparable to the masses of high redshift quasars to the evolution of early galaxies. Some of these sources seem to accrete matter at a very high rate as reported. As a result it is believed that Electromagnetic (EM) spectrum observations are required to provide information on black holes in the centers of active galaxies. On the other hand, Gravitational Wave (GW) observations are considered to provide the complementary information about the capture of particles including compact objects like Black Holes (BHs) that are mostly invisible to EM observations. Thus, the astrophysical study of AGNs in its full relativistic effect is active and fresh research. For example, the efficient mechanisms to describe the energy - momentum and particles flow of the accretion system that could be exploited to match observations are important and open to research. Motivated by this, we were proposing to study on the dynamics of accretion flow around AGNs. The method we used to derive relevant dynamical parameters from the Lagrangian and Hamiltonian of general relativistic (GR) particle geodesy in the Schwarzschild - de Sitter (SdS) geometry where the classical analogy is adopted by the correspondence principle (CP). The analytically derived equations were used to generate numerical values computationally, where the results discussed and summarized to remark for observational validity.

Key words: Accretion, AGN, BH, CP, GR, SdS.

Chapter 1

Introduction

1.1 Background

In 1915 Einstein developed the general theory of relativity in which he considered objects accelerated with respect to one another. He developed this theory to explain apparent conflicts between the laws of relativity and the law of gravity. To resolve these conflicts he developed an entirely new approach to the concept of gravity, based on the principle of equivalence [37].

Likewise, after completing his theory of GR, Einstein was interested to find a static solution of his field equations with the idea of incorporating Mach's principle [37]. But Einstein soon noticed that his original field equations yield a non - static solution. As the consequence, Einstein himself after a year, in 1917 introduced a positive cosmological constant, Λ with the belief of constructing a static solution. But at the same year that Einstein introduced the cosmological term, de Sitter presented solutions to static "Einstein universe", which had both static and dynamic features. On the other hand, in 1922, 1924 Freidmann constructed a matter dominated expanding universe without a cosmological constant. Moreover, the discovery of expanding universe by Hubble and contemporaries led Einstein to abandon the idea of a static universe misfortunately including the cosmological constant. However, a number of researchers were entailed to construct models with cosmological constant to explain measurements of the spectra of spiral nebulae that showed redshifted to construct an expanding model which originated from such an asymptotically static state (”static Einstein universe”) in the distant past. Since then, the cosmological constant has remained with debate where it was being cast out at a time and reintroduced at other time. However, a firm considerate of Λ is triggered in the 1960's when an excess quasi-stellar objects (QSOs) near the redshift $z \approxeq 1.95$ were observed. Then a number of authors emerged with models containing the cosmological constant in explaining the observed QSOs that was in agreement with the predicted inflationary scenario of the early universe. But the general perception is that owing to its tiny value,Λ does not lead to any significant observable effects in a local gravitational phenomenon. However, the recent discoveries on astrophysical phenomena favor a Cold Dark Matter with positive cosmological constant (ΛCDM) model that is consistent with observations.

Furthermore, the presence of a repulsive cosmological constant (positive) the spacetime geometry exterior to a static spherically symmetric gravitating system is Schwarzschild-de Sitter (SdS), in a spatially inflated Universe, rather than Schwarzschild. Motivated by this scientific rationale, we are interested to study the dynamics of accretion process around Active Galactic Nuclei (AGN) in Schwarzschild de Sitter Geometry.The paper is organized as follow: the Einstein GR ,Einstein field equation with cosmological constant and the solution of this field equation(in SdS space)of spherically symmetric are introduced. In section 2, the historical discovery and research development of AGN, and dynamic accretion around AGN in the case of high energy source are given. In section 3, the Newtonian analogue of SdS spacetime of a dynamic parameters from the Lagrangian and Hamiltonian of GR of particles motion is derived.In section 4,

1.2 Literature Review

The success of general theory of relativity (GR) in the observation of deflection of light[11], radar echo delay [27],precession of planetary motion[9] and gravitational redshift[20] by gravity are the manifestation of progress in astronomy and astrophysical studies. The discovery of the expanding universe at an accelerating phase[22],[23] and the direct confirmation of gravitational wave detection[5] are other astounding progresses in astronomy and astrophysics. There are great deals of progress in the subject both theoretically and observationally with direct and indirect detections. Whilst, there are encouraging past success of GR and the hopes ahead there is an outstanding debates on GR field equations dated back to their origin. After completing his theory of GR, Einstein was interested to find a static solution of his field equations with the idea of incorporating Mach's principle[37]. But Einstein noticed that his original field equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = kT_{\mu\nu} \tag{1.2.1}$$

yields a non - static solution. Where, $R_{\mu\nu}$ is the Ricci curvature tensor, R is the scalar curvature, $g_{\mu\nu}$ is the metric tensor, k scalar constant and $T_{\mu\nu}$ is the energy - momentum source tensor.

As the consequence, Einstein himself introduced a positive cosmological constant Λ with the belief of constructing a static solution. The idea is that, the constant introduces a repulsive force which can counterbalance the attractive force of gravity leading to the "static Einstein universe". The modified Einstein's field equations with the cosmological constant is

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = kT_{\mu\nu} \tag{1.2.2}$$

But at the same year that Einstein introduced the cosmological term, de Sitter presented solutions to static Einstein universe, with $T_{\mu\nu} = 0$ and $\Lambda > 0$, which had both static and dynamic features, that allows a redshift-distance relation. The de Sitter's prediction is considered as the first step towards the theoretical discovery of expanding universe.

On the other hand, in 1922 Freidman constructed a matter dominated expanding universe without a cosmological constant. Then, the possibility that the universe is expanding led Einstein to abandon the idea of a static universe including the cosmological constant. However, a number of researchers were entailed to construct models with cosmological constant. For example, Lemaitre constructed an expanding model which originated from such an asymptotically static state ("static Einstein universe") in the distant past. Since then, the cosmological constant has remained with debate where it was being cast out at a time and reintroduced at other time. A firm considerate of Λ is triggered in the 1960's when an excess quasi-stellar objects (QSO's) near the redshift $Z \approxeq 1.95$ were observed. Then a number of authors, for instance [13] emerged with Lemaitre's model in explaining the observed QSO's that was in agreement with the predicted inflationary scenario of the early universe.

In general, according to current understanding a flat low density Cold Dark Matter with dark energy in the form of cosmological constant (CDM $+\Lambda$) universe with $\Omega_m = 0.3$(m= dark matter) and $\Omega_\Lambda = 0.7$(dark energy), with an approximately flat metric is favored over a wide range of observational data ranging from large and intermediate angle Cosmic Microwave Background Radiation (CMBR) anisotropies to observations of galaxy clustering on large scales [5],[18],[22]and[23].

In the presence of a repulsive cosmological constant (positive) the spacetime geometry exterior to a static spherically symmetric gravitating system in a spatially inflated Universe is Schwarzschild-de Sitter (SdS) (rather than Schwarzschild, contrary to previous perceptions), whose line element is first derived by Kottler [19] given as :

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2 \tag{1.2.3}$$

where

$$f(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2 \tag{1.2.4}$$

M is source of mass and $d\Omega^2$ is the spherical solid angle element. But the general perception is that owing to its tiny value, cosmological constant does not lead to any significant observable effects in a local gravitational phenomenon. However, the contribution of repulsive Λ could be significant (larger than the second order term) even in a local gravitational phenomenon when kiloparsecs to megaparsecs-scale distances are involved, such as the gravitational bending of light by cluster of galaxies [17].

Probably, a local effect of cosmological constant is claimed to be observable from relativistic accretion phenomena around massive BHs which involve distance-scale of the order of hundreds of parsecs or even more [16] and the references therein. However, a few studies have been carried out so far to investigate the effect of Λ in astrophysical jet/accretion flow paradigm [12],[34]. So the effect of Λ on the dynamics of kiloparsecs to megaparsecs scale astrophysical objects including jets need investigations. So far all the works on the effect of Λ on accreting systems were carried out under some restricted conditions. The obvious reason is to avoid the complex general relativistic magnetohydrodynamic (MHD) equations in a strong gravitational field regime. Owing to the complex and nonlinear character of the equations in GR regime, analytical/quasi numerical treatment of the problem is virtually discarded; see for example [35] and the references therein. Several early works on accretion related phenomena were based on pure Newtonian gravity. So the current standard ΛCDM model that is consistent with observation shall be exploited to study the effect of cosmological constant on dynamical systems including MHD instabilities around massive objects like BHs where they are mostly hosted by AGNs.

1.3 Statement of the problem

The ΛCDM model is more or less consistent with all the current cosmological observations[3]. The effect of cosmological constant at large scale is well considered. However, its local effect like perihelion shift of the orbits of gravitationally bound systems [18],etc are at debate. On the other hand, there is a plethora of evidences that claim its effect at the local size. Probably, a local effect of cosmological constant is claimed to be observable from relativistic jets dynamics and accretion phenomena around massive BHs hosted by AGN (or QSO) which involve distance-scale of the order of hundreds of parsecs or even more. However, a few studies have been carried out so far to investigate the effect of Λ in astrophysical jet/accretion flow paradigm [12],[34]. The obvious reason is to avoid the complex general relativistic (GR) magnetohydrodynamic (MHD) equations in a strong gravitational field regime. Owing to the complex and nonlinear character of the equations in GR regime, analytical/quasi numerical treatment of the problem is virtually discarded; see for example [35] and the references therein. To this end several early works on motion of objects (including accretion process) and related phenomena were based on pure Newtonian gravity or alternative GR theories. However, on the other hand, the recently confirmed gravitational wave presence shines on the matter to study high precision astrophysical phenomena at small scale level including cosmological effect. So the current standard ΛCDM model that is consistent with observation shall be exploited to study the dynamics accretion process around massive objects like BHs hosted by AGN.

1.4 Research Questions

- How a massive compact object curves spacetime around? And how the geometry influences the dynamics of objects around it?

- What is the significance of cosmological constant in the dynamics and observables of accretion system around massive compact objects?

- How the ΛCDM model incorporates accretion process near AGN?

1.5 Objectives

1.5.1 General Objective

To study the dynamics of accretion process around Active Galactic Nuclei in Schwarzschild de Sitter Geometry

1.5.2 Specific Objectives

- To derive dynamical equations from the GR equations in the SdS background.
- To derive dynamical observable parameters, like momentum, energy of accretion flow around AGN with the SdS metric.
- To study the contents of the dynamical observable parameters of accretion flow around AGN.

1.6 Methodology

The general method is to derive relevant dynamical parameters such as energy and momentum from the Lagrangian and Hamiltonian of general relativistic (GR) particle geodesy from Einstein static field equations in the Schwarzschild-de Sitter (SdS) geometry where the classical analogy is adopted by the correspondence principle (CP). The analytically derived equations are used to generate numerical values computationally with MATHEMATICA. Then, the results would be discussed and summarized to remark energy and momentum.

The steps are:

- Provide preliminary boundary conditions to derive the relevant set of dynamical equations from the GR equations in the SdS background.

- Study and examine the effects of the relevant parameters derived from the equations.
- Numerically generate some theoretical data from the formalism using computation.
- Discussion of the generates data.
- Summary and conclusion.

Chapter 2

Einstein Theory of General Relativity

2.1 Introduction to Einstein General Relativity

After many years of development Einstein presented his general theory of relativity in 1915,it was then published the following year in [2]. General relativity is an extension of special relativity which includes a modification of Newton's law of gravity. It provides a relativistic description of the gravitational field exerted by a massive object and its effects on the geometric structure of the surrounding spacetime. The theory states that the gravitational interaction due to the presence of matter causes spacetime to curve hence distorting the path of a nearby object. This differs from the original foundations of Newton's laws of gravitation, where gravity is an attractive force between two massive objects which interacts instantaneously. In this description, planetary orbits are a consequence of this gravitational pull emanating from the sun, therefore in this theory the suns gravitational field interacts directly with the planet as opposed to the surrounding spacetime. However given certain circumstances Newtonian theory provides an accurate description of the gravitational interaction, this includes a weaker gravitational field. This is known as the Newtonian limit in which spacetime is asymptotically flat and the field equations can be approximated with Newton's laws of motion. General relativity is required for a more significant gravitational field, when Newtonian gravity no longer agrees with observation. For instance, the observation of the precession of the perihelion of Mercury deviated slightly from the predictions of Newton's equations, whereas solutions in general relativity describe this orbit correctly

2.1.1 The Metric Tensor

The geometry of curved space was studied in 19^{th} century by Gauss, Riemann and other. Riemann realized that Euclidean geometry was just a particular choice suited to flat space and Mach realized that one had to abandon the concept of absolute space altogether . Einstein learned about tensors from his friend Marcel Grossmann, and used these key quantities to go from flat space Euclidean three-dimension space to curved Minkowskian four-dimension space in which physical quantities are described by invariants. Tensors are quantities which provide generally valid relations between different four-vectors. The quantities that consists contravariants indices with corresponding Lorenz transformation are tensor. In tensor notation the Minkowski metric includes the coordinate $dx^0 = cdt$ and so that the invariant line element can be:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{2.1.1}$$

where,ds^2 is the Minkowski line element, and $g_{\mu\nu}$ is the metric tensor used to determine the proper time interval between two events with a given infinitesimal coordinate separation and the gravitation potential, given by:

$$g_{\mu\nu} = \eta_{\alpha\beta} \frac{\partial \xi^\alpha}{\partial x^\mu} \frac{\partial \xi^\beta}{\partial x^\nu} \tag{2.1.2}$$

Where $\eta_{\alpha\beta}$ special relativistic metric, $\xi^\alpha(x)$ Minkowskian coordinates and x^μ general coordinate.

2.1.2 Christoffel Tensor

One of the invariant ranks three tensor derived from the metric tensor $g_{\mu\nu}(x)$ its first derivative is the so called Christoffel tensor which plays the role of gravitation. Affine connection($\Gamma^{\lambda}_{\mu\nu}$) is the field that determines the gravitational force and used to represent the gravitational field. It also call as christoffel second symbol which denoted as $\{\mu\nu, \lambda\}$ or $\Gamma^{\lambda}_{\mu\nu}$.

The mathematical definition of $\Gamma^{\lambda}_{\mu\nu}$ as,

$$\Gamma^{\lambda}_{\mu\nu} = \frac{\partial x^{\lambda}}{\partial \xi^{\alpha}} \frac{\partial^2 \xi^{\alpha}}{\partial x^{\mu} \partial x^{\nu}}$$

where ξ^{α} and ξ^{β} are local coordinates. The infinitesimal line element and the motion of particle in a gravitational field can be written as,

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho} (g_{\rho\mu,\nu} + g_{\rho\nu,\mu} - g_{\mu\nu,\rho}) \tag{2.1.3}$$

Now differentiating metric tensor equ(1.1.2) in gravitational field with respect to x^{λ},

$$\frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} = \frac{\partial}{\partial x^{\lambda}} \left[\frac{\partial \xi^{\alpha}}{\partial x^{\mu}} \frac{\partial \xi^{\beta}}{\partial x^{\nu}} \eta_{\alpha\beta} \right]$$

$$\frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} = \frac{\partial^2 \xi^{\alpha}}{\partial x^{\lambda} \partial x^{\mu}} \frac{\partial \xi^{\beta}}{\partial x^{\nu}} \eta_{\alpha\beta} + \frac{\partial \xi^{\alpha}}{\partial x^{\mu}} \frac{\partial^2 \xi^{\beta}}{\partial x^{\lambda} \partial x^{\nu}} \eta_{\alpha\beta} \tag{2.1.4}$$

The above equation can be derived as,

$$\frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} = \frac{\partial x^{\rho}}{\partial \xi^{\alpha}} \frac{\partial \xi^{\alpha}}{\partial x^{\rho}} \frac{\partial^2 \xi^{\alpha}}{\partial x^{\lambda} \partial x^{\mu}} \frac{\partial \xi^{\beta}}{\partial x^{\nu}} \eta_{\alpha\beta} + \frac{\partial x^{\rho}}{\partial \xi^{\beta}} \frac{\partial \xi^{\beta}}{\partial x^{\rho}} \frac{\partial \xi^{\alpha}}{\partial x^{\mu}} \frac{\partial^2 \xi^{\beta}}{\partial x^{\lambda} \partial x^{\nu}} \eta_{\alpha\beta}$$

After rearranging,

$$\frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} = g_{\rho\nu} \Gamma^{\rho}_{\lambda\mu} + g_{\mu\rho} \Gamma^{\rho}_{\lambda\nu} \tag{2.1.5}$$

The two $\Gamma^{\rho}_{\lambda\mu}$ and $\Gamma^{\rho}_{\lambda\nu}$ are the affine connections. If we are considering a freely falling particle of affine connection is the field that determine the gravitational force. Now using the symmetry property of affine connection with the exchange of lower indices ,i.e $\Gamma^{\rho}_{\lambda\mu} = \Gamma^{\rho}_{\mu\lambda}$. To solve for the affine connection ,it is a matter of adding to equation (1.1.5) the same equation with μ and λ inter changed and subtract the same equation with ν and λ interchanged,it shows

$$\frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} + \frac{\partial g_{\lambda\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\lambda}}{\partial x^{\nu}} = \Gamma^{\rho}_{\lambda\mu} g_{\rho\nu} + \Gamma^{\rho}_{\lambda\nu} g_{\rho\mu} + \Gamma^{\rho}_{\mu\lambda} g_{\rho\nu} + \Gamma^{\rho}_{\mu\nu} g_{\rho\lambda} - \Gamma^{\rho}_{\nu\mu} g_{\rho\lambda} - \Gamma^{\rho}_{\nu\lambda} g_{\rho\mu} \tag{2.1.6}$$

From the symmetry property of affine connection $\Gamma^{\rho}_{\mu\nu}$ and the metric tensor, $g_{\mu\nu}$, then

$$\frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} + \frac{\partial g_{\lambda\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\lambda}}{\partial x^{\nu}} = 2\Gamma^{\rho}_{\lambda\mu} g_{\rho\nu} \tag{2.1.7}$$

Now let us define metric $g_{\rho\sigma}$ as the inverse of $g^{\rho\nu}$,

$$g_{\rho\sigma} g^{\rho\nu} = \delta^{\sigma}_{\nu} = 1$$

for $\sigma = \nu$ and else zero. Therefore,

$$\Gamma^{\rho}_{\lambda\mu} = \frac{1}{2} g^{\rho\nu} \left(\frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} + \frac{\partial g_{\lambda\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\lambda}}{\partial x^{\nu}} \right) \tag{2.1.8}$$

Equation (1.1.8) is the relation developed between the metric tensor and affine connection in a gravitational field. Here both of them represent the presence of gravitational effect.

2.1.3 The Riemann-Christoffel Curvature Tensor

One of the other invariant tensor derived from the metric $g_{\mu\nu}$ is the four rank tensor called the Riemann curvature tensor from the metric itself, first and second derivative. The Riemann curvature tensor plays an important role in specifying the geometrical properties of spacetime. The spacetime is considered flat, if the tensor vanishes everywhere. It is also possible to denote the Riemann curvature tensor in its fully covariant form as:

$$R^{\lambda}_{\mu\lambda\kappa} = \Gamma^{\lambda}_{\mu\kappa,\lambda} - \Gamma^{\lambda}_{\mu\lambda,\kappa} + \Gamma^{\eta}_{\mu\kappa}\Gamma^{\lambda}_{\eta\lambda} - \Gamma^{\eta}_{\mu\lambda}\Gamma^{\lambda}_{\eta\kappa} \tag{2.1.9}$$

Ricci Tensor

An important tool related to curvature, the second rank Ricci tensor,$R_{\mu\nu}$ obtained from Riemann tensor by a summing operation over repeated Indies, called contraction

$$R_{\mu\kappa} = g_{\alpha\beta} g^{\lambda\alpha} R^{\beta}_{\mu\lambda\kappa} \tag{2.1.10}$$

Ricci scalar

By further contracting the Ricci tensor with contracting components of the metric, one can express curvature scalar as;

$$R = g^{\mu\kappa} R_{\mu\kappa} = R^{\mu}_{\mu} = R^{\kappa}_{\kappa} \tag{2.1.11}$$

2.2 Einstein Field Equation

The Einstein tensor, $G_{\mu\nu}$ is a measure of the curvature of spacetime. Mass is merely a form of energy and, as such, we denote the stress-energy tensor,$T_{\mu\nu}$, containing all of the information of the energy of a system. Thus, these two tensors must be in balance, which is represented in the Einstein field equations (EFE)

$$G_{\mu\nu} = 8\pi \frac{G}{c^2} T_{\mu\nu} \tag{2.2.1}$$

where we include the constants c,G to present the EFE in their usual form. Recall that we are using units such that c = G = 1. In Newtonian theory, gravity can only exist where there exists matter. However Einstein showed that matter and energy are only different faces of the same coin[10]. This encouraged him to make the conclusion that gravity is not only created by the presence of matter, it is in fact the product of the presence of energy. General relativity must present appropriate analogues of the two parts of the dynamics,one how particles move in response to gravity,and secondly,how particles generate gravitational effects [32]. The analogue of the poisson equation of the second idea can be,

$$\nabla^2 \phi = 4\pi G \rho(x) \tag{2.2.2}$$

Now we start to derive Einstein field equation under the approximation of a weak static field produced by a non-relativistic mass density ρ [24][33]. Therefore,the energy density for non-relativistic matter is,

$$T^{00} = \rho = T_{00}$$

One can write the poisson equation as,

$$\nabla^2\phi = 4\pi G T_{00} \tag{2.2.3}$$

When the metric to be close to the Minkowski metric $\eta_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

This equation for 00 components will be

$$g_{00} = -1 + h_{00}$$

And we get

$$\nabla^2\phi = \frac{1}{2}\nabla^2 g_{00} \tag{2.2.4}$$

Therefore poison equation result,

$$\frac{1}{2}\nabla^2 g_{00} = 4\pi G T_{00}$$

$$\nabla^2 g_{00} = 8\pi G T_{00} \tag{2.2.5}$$

From this fact the weak field equation for a general distribution of energy and momentum $T_{\alpha\beta}$ will take the form,

$$G_{\alpha\beta} = 8\pi G T_{00} \tag{2.2.6}$$

Where,$G_{\alpha\beta}$ is a linear combination of the metric tensor and its first and second derivatives. The principle of equivalence that the equation which govern gravitational fields of arbitrary strength must take the form,

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{2.2.7}$$

Therefore, equ(2.2.6)is the approximated form of equ(2.2.7) in a weak static gravitational field as equivalence principle states. Here is a tensor which reduce to $G_{\alpha\beta}$ for a weak fields and since $T_{\mu\nu}$ is symmetric, $G_{\mu\nu}$ also. To go further consider the nature of $G\mu\nu$;

1. By definition $G_{\mu\nu}$ is a tensor

2. By assumption $G_{\mu\nu}$ contain terms that are linear in the second derivative of the metric tensor or quadratic in the first derivative of the metric

3. Since $T_{\mu\nu}$ is symmetric so does $G_{\mu\nu}$

4. Since $T_{\mu\nu}$ is conserved in the absence of external forces,so does $G_{\mu\nu}$.

5. For a weak stationary field produced by non-relativistic matter ,the 00 component must satisfy

$$G_{00} \approx \nabla^2 g_{00} \tag{2.2.8}$$

Hence(1) and (2) required $G_{\mu\nu}$ to take the form

$$G_{00} = C_1 R_{\mu\nu} + C_2 g_{\mu\nu} R \tag{2.2.9}$$

where,C_1 and C_2 are constants.Since this is symmetric condition(3) is automatically satisfied. It follows from the above relation that.

$$g^{\sigma\mu} G_{\mu\nu} = C_1 g^{\sigma\mu} R_{\mu\nu} + C_2 g^{\sigma\mu} g_{\mu\nu} R \tag{2.2.10}$$

Equivalent to,

$$G^{\sigma}_{\nu} = C_1 R^{\sigma}_{\nu} + C_2 \delta^{\sigma}_{\nu} R \tag{2.2.11}$$

This follow as

$$G^{\sigma}_{\nu;\sigma} = C_1 R^{\sigma}_{\nu;\sigma} + C_2 \delta^{\sigma}_{\nu} R_{;\sigma} \tag{2.2.12}$$

Using this result, $R^{\sigma}_{\nu;\sigma} = \frac{1}{2}\delta^{\sigma}_{\nu} R_{;\sigma}$ into the above equation and it follows,

$$G^{\sigma}_{\nu;\sigma} = \frac{1}{2} C_1 \delta^{\sigma}_{\nu} R_{;\sigma} + C_2 \delta^{\sigma}_{\nu} R_{;\sigma} \tag{2.2.13}$$

If $\nu = \sigma$

$$G^{\sigma}_{\nu;\sigma} = \frac{1}{2}C_1 R_{;\sigma} + C_2 R_{;\sigma}$$

$$G^{\sigma}_{\nu;\sigma} = \left(\frac{C_1}{2} + C_2\right) R_{;\sigma} \qquad (2.2.14)$$

By the conservation of $G_{\mu\nu}$ we have $G^{\sigma}_{\nu;\sigma} = 0$ and this yield the relation,

$$\left(\frac{C_1}{2} + C_2\right) R_{;\sigma} = 0$$

$$\frac{C_1}{2} + C_2 = 0$$

$$\frac{C_1}{2} = -C_2 \qquad (2.2.15)$$

Therefore, we can write $G_{\mu\nu}$ as,

$$G_{\mu\nu} = C_1 R_{\mu\nu} - \frac{C_1}{2} g_{\mu\nu} R$$

$$G_{\mu\nu} = C_1(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) \qquad (2.2.16)$$

To fix the constant C_1,use the property [15] . A non relativistic system always has $\|T_{ij}\| \ll \|T_{00}\|$ and here look the case where $\|G_{ij}\| \ll \|G_{00}\|$ thus,

$$G_{ij} \approx 0 \qquad (2.2.17)$$

From equ(1.2.16) we can write as

$$R_{ij} - \frac{1}{2} g_{ij} R = 0$$

$$R_{ij} = \frac{1}{2} g_{ij} R \qquad (2.2.18)$$

Since we deal here with a weak field approximation (i.e $g_{\alpha\beta} \approx \eta_{\alpha\beta}$) as well as $g_{ij} \approx \eta_{ij}$. Therefore, this lead to write as,

$$R_{ij} \approx \frac{1}{2} \eta_{ij} R \qquad (2.2.19)$$

By applying the property of metric tensor $\eta_{ij} = 1$; for i = j = 1; 2; 3 and taking the sum over each indices,

$$R_{ij} = \sum_{i,j=1}^{3} \frac{1}{2}\eta_{ij} R \approx \frac{3}{2}R$$

$$R_{kk} = \frac{3}{2}R \tag{2.2.20}$$

The curvature scalar is therefore given by,

$$R \approx R_{kk} - R_{00} = \frac{3}{2} - R_{00}$$

$$R \approx 2R_{00} \tag{2.2.21}$$

Thus in the weak field approximation we have the following information,

$$R \approx 2R_{00}$$

$$g_{\alpha\beta} \approx \eta_{\alpha\beta}$$

$$G_{\mu\nu} = C_1(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R)$$

For the 00 component of $G_{\mu\nu}$ equals to,

$$G_{00} = C_1(R_{00} - \frac{1}{2}g_{00}R = C_1(R_{00} - \frac{1}{2}\eta_{00}R)$$

$$G_{00} = 2C_1 R_{00} \tag{2.2.22}$$

Now the task is to calculate R_{00}. Recall the expression given by the Riemann curvature tensor $R_{\lambda\mu\nu\kappa}$ that is,

$$R_{\lambda\mu\nu\kappa} = \frac{1}{2}\left[\frac{\partial^2 g_{\lambda\nu}}{\partial x^\kappa \partial x^\mu} - \frac{\partial^2 g_{\mu\nu}}{\partial x^\kappa \partial x^\lambda} - \frac{\partial^2 g_{\lambda\kappa}}{\partial x^\nu \partial x^\mu} + \frac{\partial^2 g_{\mu\kappa}}{\partial x^\nu \partial x^\lambda}\right] + g_{\eta\sigma}\left[\Gamma^\eta_{\nu\lambda}\Gamma^\sigma_{\mu\kappa} - \Gamma^\eta_{\kappa\lambda}\Gamma^\sigma_{\mu\nu}\right]$$

Since we are looking for a weak field approximation,it is better to use the linear part of $R_{\lambda\mu\nu\kappa}$,given by

$$R_{\lambda\mu\nu\kappa} = \frac{1}{2}\left[\frac{\partial^2 g_{\lambda\nu}}{\partial x^\kappa \partial x^\mu} - \frac{\partial^2 g_{\mu\nu}}{\partial x^\kappa \partial x^\lambda} - \frac{\partial^2 g_{\lambda\kappa}}{\partial x^\nu \partial x^\mu} + \frac{\partial^2 g_{\mu\kappa}}{\partial x^\nu \partial x^\lambda}\right] \tag{2.2.23}$$

When the field is static all the time derivatives vanish,and the components that we need are,

$$R_{0000} \approx 0$$

$$R_{i0j0} \approx \frac{1}{2}\frac{\partial^2 g_{00}}{\partial x^i \partial x^j} = \frac{1}{2}\nabla^2 g_{00} \tag{2.2.24}$$

where $\frac{\partial^2 g_{00}}{\partial x^i \partial x^j} = \nabla^2 g_{00}$ From the contraction of curvature tensor over the two indices

$$R_{00} = g^{\lambda\nu} R_{\lambda 0 \nu 0}$$

$$R_{00} = R_{i0j0} - R_{0000} \tag{2.2.25}$$

By using this relation in the equation into equ(2.2.22) for $G_{\mu\nu}$,

$$G_{00} = 2C_1(R_{i0j0} - R_{0000})$$

$$G_{00} = 2C_1(\frac{1}{2}\nabla^2 g_{00} - 0) = C_1\nabla^2 g_{00} \tag{2.2.26}$$

Comparing equation (2.2.8) and (2.2.26),

$$G_{00} = C_1\nabla^2 g_{00} = \nabla^2 g_{00} \tag{2.2.27}$$

The value of $C_1 = 1$, and therefore, we can write the equation for $G_{\mu\nu}$ as,

$$G_{\mu\nu} = (R\mu\nu - \frac{1}{2}g_{\mu\nu}R) = 8\pi G T_{\mu\nu}$$

$$(R\mu\nu - \frac{1}{2}g_{\mu\nu}R) = 8\pi G T_{\mu\nu} \tag{2.2.28}$$

Equation (1.2.28) is Einstein field equation. In this EFE, the expression on the left represents the curvature of spacetime as determined by the metric and the expression on the right represents the matter/energy content of spacetime. The EFE can then be interpreted as a set of equations dictating how the curvature of spacetime is related to the matter/energy content of the universe.

2.3 Introduction of Cosmological Constant into Einstein Field Equations

After completing his theory of GR, Einstein was interested to find a static solution of his field equations with the idea of incorporating Mach's principle, for details see [1]. But Einstein soon noticed that his original field equations yield a non - static solution. As the consequence, Einstein himself after a year, in 1917 introduced a positive cosmological constant with the belief of constructing a static solution,Eq (2.3.1). But at the same year that Einstein introduced the cosmological term, de Sitter presented solutions to static Einstein universe , which had both static and dynamic features. The de Sitter's prediction is considered as the first step towards the theoretical discovery of expanding universe. On the other hand, in 1922 Freidmann constructed a matter dominated expanding universe without a cosmological constant.Then, the possibility that the universe may be expanding led Einstein to abandon the idea of a static universe and, along with it, the cosmological constant. However, other groups sustained supporting a model with cosmological constant. For example, Weyl in 1923 recommended de Sitter's model to explain measurements of the spectra of spiral nebulae that showed redshifted; Lemaitre constructed an expanding model which originated from such an asymptotically static state (static Einstein universe) in the distant past. Since then, the cosmological constant has remained with debate where it is being cast out at a time and reintroduced at other time. Einstein field equation with cosmological constantΛ became,

$$(R\mu\nu - \frac{1}{2}g_{\mu\nu}R) + g_{\mu\nu}\Lambda = kT_{\mu\nu} \tag{2.3.1}$$

Where $k = 8\pi G$ and G is gravitational constantRecent observational data and results in modern cosmology revealed that the dark energy which is described in majority by the cosmological constant is of dominant importance in the dynamics of our Universe. Measurements conducted by Wilkinson Microwave Anisotropic Probe (WMAP) indicate that almost three fourth of total mass-energy in the Universe is Dark Energy and the leading theory of dark energy is based on the cosmological constant characterized by repulsive pressure which was introduced by Einstein in 1917 to obtain a static cosmological model. Later on Zeldovich [30] interpreted this quantity physically as a vacuum energy of quantum fluctuation whose size is of the order of $\sim 3 * 10^{-56} cm^{-2}$

2.4 Schwarzschild de Sitter Metric

A de Sitter universe is a cosmological solution to the Einstein field equation of general relativity named after Willian de Sitter. If a model of the universe as special flat and neglects ordinary matter. So the dynamic of the universe is dominated by the cosmological constant(Λ). And it also describes the specially symmetric solution to Einstein vacuum equation with a positive cosmological constant, as shown in eq(2.3.1)can be again defined as, by setting $T_{\mu\nu=0}$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$

But

$$R = g^{\mu\nu} R_{\mu\nu}$$

Substituting instead of Ricci the solution will be

$$R_{\mu\nu} - \frac{1}{2} g^{\mu\nu} g_{\mu\nu} R_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$

hence,

$$R_{\mu\nu} - 2R_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$

$$R_{\mu\nu} = \Lambda g_{\mu\nu} \tag{2.4.1}$$

Now Schwarzschild metric,

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

Now for spherical coordinate system (t,r,θ, ϕ)for vacuum EFE,

$$R_{tt} = \Lambda g_{tt}, R_{rr} = \Lambda g_{rr}, R_{\theta\theta} = \Lambda g_{\theta\theta}, R_{\phi\phi} = \Lambda g_{\phi\phi}$$

And using the definition of Ricci tensor from contraction of Reimann curvature,

$$R^{\sigma}_{\mu\sigma\nu} = R_{\mu\nu} = \Gamma^{\lambda}_{\mu\lambda,\nu} - \Gamma^{\lambda}_{\mu\nu,\lambda} + \Gamma^{\eta}_{\mu\lambda}\Gamma^{\lambda}_{\eta\nu} - \Gamma^{\eta}_{\mu\nu}\Gamma^{\lambda}_{\eta\lambda}$$

$$R_{rr} = \frac{B"r}{2Br} - \frac{1}{4}\left(\frac{B'(r)}{B(r)}\right)\left(\frac{A'(r)}{A(r)} + \frac{B'(r)}{B(r)}\right) - \frac{1}{r}\left(\frac{A'(r)}{A(r)}\right)$$

$$R_{\theta\theta} = -1 + \frac{r}{2A(r)}\left(\frac{-A'(r)}{A(r)} + \frac{B'(r)}{B(r)}\right) + \frac{1}{A(r)}$$

$$R_{\phi\phi} = \sin^2\theta R_{\theta\theta}$$

$$R_{tt} = -\frac{B"(r)}{2A(r} + \frac{1}{4}\frac{B'(r)}{A(r)}\left(\frac{A'(r)}{A(r)} + \frac{B'(r)}{B(r)}\right) - \frac{1}{r}\left(\frac{B'(r)}{A(r)}\right)$$

$R_{\mu\nu} = 0$ for $\mu \neq \nu$

If we follow through the original derivation of the Schwarzschild metric with,

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{2.4.2}$$

Where, $A(r) = g_{rr}$, $B(r) = g_{tt}$, $r^2 = g_{\theta\theta}$, $r^2 \sin^2\theta = g_{\phi\phi}$ and as a starting point ,then we get the equation

$$\frac{\partial A(r)}{\partial t} = 0 \tag{2.4.3}$$

Because A(r) is not expressed explicitly with time using the component of Ricci tensor (non-vanishing), it become

$$\frac{A(r)}{B(r)} + R_{rr} = \frac{2B'(r)}{r} + \frac{2B(r)\partial A(r)}{rA(r)} \tag{2.4.4}$$

Hence,

$$R_{\theta\theta} = r^2\Lambda \tag{2.4.5}$$

where,$r^2 = g_{\theta\theta}$

$$R_{\theta\theta} = \frac{-rB'(r)}{2A(r)B(r)} + \frac{rA'(r)}{2A^2} + 1 - \frac{1}{A} \tag{2.4.6}$$

Substituting equ(2.4.2) into equ(2.4.5) we get

$$\frac{A}{B}(\Lambda g_{tt}) + \Lambda g_{rr} = \frac{2B'(r)}{r} + \frac{2B(r)\partial A(r)}{rA(r)}$$

$$\frac{A}{B}(-B\Lambda) + A\Lambda = \frac{2B'(r)}{r} + \frac{2B(r)\partial A(r)}{rA(r)} \tag{2.4.7}$$

It yield,

$$0 = \frac{2B'(r)}{r} + \frac{2B(r)A'(r)}{rA(r)} \tag{2.4.8}$$

Multiply both side by $\frac{r}{2B(r)}$,

$$\frac{B'(r)}{B(r)} = -\frac{A'(r)}{A(r)} \tag{2.4.9}$$

Thus non-zero Λ doesn't change this equation. The only difference come from substituting equ(2.4.9) into equ(2.4.7) and rearranging terms,

$$R_{\theta\theta} = \frac{rA'}{2A^2} + 1 - \frac{1}{A}$$

$$r^2\Lambda - 1 = \frac{rA'}{2A^2} - \frac{1}{A}$$

$$\frac{d}{dr}(\frac{r}{A}) = 1 - r^2\Lambda \tag{2.4.10}$$

Integrate this equation the result will be,

$$\frac{r}{A(r)} = r - \frac{r^3\Lambda}{3} + C \tag{2.4.11}$$

where C is some constant and is called " mass" parameter.

$$\frac{1}{A(r)} = 1 - \frac{r^2\Lambda}{3} + \frac{C}{r}$$

And also

$$\frac{1}{A(r)} = B(r)$$

$$B(r) = 1 - \frac{r^2\Lambda}{3} + \frac{C}{r} \tag{2.4.12}$$

$$A(r) = \left(1 - \frac{r^2\Lambda}{3} + \frac{C}{r}\right)^{-1} \tag{2.4.13}$$

Note that Λ is the cosmological constant which using the value of $|\Lambda| \leqslant 10^{-56}m^{-2}$ is so very small that $\frac{C}{r} \gg \frac{\Lambda}{3}r^2$ for the value of r on the scale of intergalactic distance (million of light years), then for this distances the cosmological constant term can be neglected and the requirement that we reclaim Newton's law of gravity for this distances give us the condition $C = -2GM$

$$\frac{c}{r} = \frac{-2GM}{r}$$

Equations (1.4.12) and (1.4.13) will be ,

$$B = 1 - \frac{2GM}{r} - \frac{r^2\Lambda}{3} \tag{2.4.14}$$

and

$$A = \left(1 - \frac{2GM}{r} - \frac{r^2\Lambda}{3}\right)^{-1} \tag{2.4.15}$$

And metric,

$$g_{tt} = -\left(1 - \frac{2GM}{r} - \frac{r^2\Lambda}{3}\right)$$

$$g_{rr} = \left(1 - \frac{2GM}{r} - \frac{r^2\Lambda}{3}\right)^{-1}$$

Now the Schwarzschild -de Sitter metric with cosmological constant in EFE becomes

$$ds^2 = \left[1 - \frac{2GM}{r} - \frac{r^2\Lambda}{3}\right] dt^2 + \left[1 - \frac{2GM}{r} - \frac{r^2\Lambda}{3}\right]^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{2.4.16}$$

Chapter 3

AGN and Dynamic Accretion Process Around AGN in sds Geometry

3.1 Historical Discovery and Research Development about AGN

Early photographic observations of nearby galaxies detected some characteristic signatures of AGN emission, although there was not yet a physical understanding of the nature of the AGN phenomenon. Some early observations included the first spectroscopic detection of emission lines from the nuclei of NGC 1068 and Messier 81 by Edward Fath (published in 1909),[14] and the discovery of the jet in Messier 87 by Heber Curtis (published in 1918)[7]. After this first spectrum, more such emission line galaxies were discovered. In 1943, Carl Seyfert published a paper in which he described observations of nearby galaxies having bright nuclei that were sources of unusually broad emission lines[8]. Galaxies observed as part of this study included NGC 1068, NGC 4151, NGC 3516, and NGC 7469. Active galaxies such as these are known as Seyfert galaxies in honor of Seyfert's pioneering work.Seyfert galaxies are essentially normal spiral galaxies, with strong nuclear emission. The development of radio astronomy was a major catalyst to understanding AGN. Some galaxies with active nuclei are also strong radio emitters. Almost all of these radio galaxies are elliptical galaxies[4]. The radio emission is not constrained to the nucleus alone but also appears in extended jets which emanate from the nucleus. In photographic images, some of these objects were nearly point-like or quasi-stellar in appearance, and were classified as quasi-stellar radio sources (later abbreviated as "quasars"). Schmidt noted that if this object was extragalactic (outside the Milky Way, at a cosmological distance) then its large redshift of 0.158 implied that it was the nuclear region of a galaxy about 100 times more powerful than other radio galaxies that had been identified. Shortly afterward, optical spectra were used to measured the redshifts of a growing number of quasars including 3C 48, even more distant at redshift 0.37[28]. The enormous luminosities of these quasars as well as their unusual spectral properties indicated that their power source could not be ordinary stars. Accretion of gas onto a supermassive black hole was suggested as the source of quasars' power in papers by Edwin Salpeter and Yakov Zel'Dovich in 1964.[29] In 1969 Donald Lynden-Bell proposed that nearby galaxies contain supermassive black holes at their centers as relics of "dead" quasars, and that black hole accretion was the power source for the non-stellar emission in nearby Seyfert galaxies[21]. In the 1960s and 1970s, early X-ray astronomy observations demonstrated that Seyfert galaxies and quasars are powerful sources of X-ray emission, which originates from the inner regions of black hole accretion disks.

The study of Active Galactic Nuclei (AGN) has become a major astrophysical topic in the last decade, both observationally and theoretically[6]. AGN, in particular quasars, are the most luminous objects in the Universe and can thus be seen to the highest redshifts. Thus, they represent ideal probes for testing directly the physical conditions at large look back times. Although a general qualitative understanding exists about the nature of the central machines, namely accretion onto supermassive black holes, a detailed knowledge about the mechanisms of the actual physical emission processes and about the cosmological evolution of the objects is still missing[6].

However,the ROSAT instrument provided, for the first time, the opportunity to study a very large number of AGN in X-rays and more than 20000 AGN are expected to be detected in the Survey. Using recently available large scale sensitive radio surveys for a correlation with the RASS source catalogue, we obtained a list of several thousand radio/X-ray objects of which more than two thirds are currently optically unidentified.

Considerable effort has been put into the construction of these large AGN - samples, into the discussion of their class - specific statistical properties, and into the optical identification of hundreds of these sources.

In addition,AGN are important because they are a source of what we call "feedback". "Feedback" in a galaxy is any process that heats or disrupts the gas.

Moreover, all the works on the effect of Λ on accreting systems are carried out under some restricted conditions. This is because the study of accreting BH systems involves solving general relativistic (GR) MHD equations in a strong gravitational field regime. Owing to the complex and nonlinear character of the underlying equations in GR regime, analytical quasi numerical treatment of the problem is virtually discarded. Even numerical simulation is complicated by several issues such as different characteristic time scales for propagating modes of general relativity and relativistic hydrodynamics. Several early works on these accretion related phenomena were based on pure Newtonian gravity. After the seminal work of Paczy'nski and Witta[25], most of the authors treated accretion and its related processes around BHs using MHD equations in the Newtonian framework by using some PNPs which are essentially modification of Newtonian gravitational potential developed with the objective to reproduce (certain) features of relativistic gravitation. This is to avoid GR gas dynamical equations, which in most occasions become inconceivable in practice in describing a complex physical system like accreting plasma. Consequently, adopting PNPs, one can comprehensively construct more realistic accretion flow models in simple Newtonian paradigm, while the corresponding PNP would capture the essential GR effects in the vicinity of the compact objects.

3.2 Dynamic of Accretion Process Around AGN in sds Geometry

In astronomy, accretion assumes the increase in the mass of a celestial object by collecting of the surrounding gas and objects(of a small size)by gravity. Accretion is served as a source of energy in many astrophysical objects, including different types of binary stars, binary X-ray sources, most probably quasars and active galactic nuclei (AGN). While first development of accretion theory started long time ago, the intensive development of this theory began after discovery of first X-ray sources and quasars[31]. We may be sure in this case, that all gravitational energy of the falling matter will be transformed into heat and radiated outward. Situation is quite different for sources containing black holes, which are discovered in some binary X-ray sources in the galaxy, as well as in many AGN. Accretion flow surrounding a black hole was the subject of many theoretical studies in Astrophysics for last half a century[25]. Particularly after emergence of AGN model, where black hole accretion is supposed to be the central power house, as the unifying scheme [25] to explain the behaviour of exotic astronomical objects like Seyfart galaxies, Quasars, Blazars etc. According to this idea accretion dynamics around black hole should bear the signature of the surrounding space-time structure and hence properties of the black hole itself. Schwarzschild de Sitter (SdS) space-time with positive cosmological constant may be considered to be a plausible model of accelerating Universe with a centrally located spherical mass distribution [36]. Hence it is expected that for a sufficiently massive black hole, accretion dynamics over a large length scale should bear the signature of SdS space and hence the effect of the cosmological constant at large length scale too.

The accretion dynamics in general relativistic (GR) set up is quite complex to handle analytically. An alternative approach [25] is to handle the problem in Newtonian framework with an effective potential (pseudo-potential) which may mimic the general relativistic motion at least in a region much away from the event horizon. For SdS space such pseudo potential has recently been prescribed [38] in literature. In this work using the prescribed potential [38]) we have explored the accretion dynamics around a centrally located non-rotating black hole in the accelerating Universe model and compared it with the same in the non-accelerating one. In the present context, the cosmological horizon is obviously far away from the region where accretion flow takes place but the signature of the non-zero cosmological constant remains present, however small, in the space-time curvature of the accretion region.

Chapter 4

Particle Dynamics in Schwarzschild De -Sitter Geometry

4.1 Lagrangian and Hamiltonian Dynamic of Particles Motion

The main difference between particle theory and field theory is that the variables q_i no longer describe the motion of anything, that is, they are no longer functions of time. Rather, they become fixed labels for points in space. The position variables q_i become independent variables in the same way that the time (t) is independent. Taken together, they label points in spacetime. A field is some quantity that has a value for each point in spacetime, and it is this quantity that can change as we move from place to place or forward in time. For a scalar field such as temperature or density, the field consists of a single value $\phi\left(q^{\mu}\right)$ attached to each point in spacetime, where we now use the notation q^{μ} to represent the space components together with time. (That is, q^{μ} is a four-vector in special relativity, with $q^0 = t$, $q^1 = x$ and so on.) A vector field, such as the electric field $\mathbf{E}$, is actually composed of three separate fields, one for each spatial coordinate. Each of these fields again has a single value for each point q^{μ}. To work out the Euler-Lagrange equations for classical field theory, we need to think about what is meant by a path that the system follows. Because the spacetime coordinates q^{μ} are no longer dynamical variables, it doesnt make sense to ask how q^μ changes with time. What does change is the value of the field ϕ (or ϕ^r if we have more than one field, as with the electric field, in which case the index r ranges over all the fields), so it is the field ϕ that is the dynamical variable. As such, the path followed is determined by a function of the field values. By analogy with the Lagrangian in the particle case, we define the Lagrangian density $\mathcal{L}\left(\phi^r, \phi^r_{,\mu}, q^\mu\right)$. The notation $\phi^r_{,\mu}$ is defined as

$$\phi^r_{,\mu} \equiv \frac{\partial \phi^r}{\partial q^\mu} \tag{4.1.1}$$

The Lagrangian density is the Lagrangian per unit volume, and each infinitesimal volume element $d^3x = dq^1 dq^2 dq^3$ follows a path through time, so the action element of this volume element between times t_1 and t_2 is

$$dS = \int_{t_1}^{t_2} \mathcal{L}\left(\phi^r, \phi^r_{,\mu}, q^\mu\right) dt \tag{4.1.2}$$

The total action of the entire system is the integral of this over some spacetime volume Ω that encloses the entire system spatially during the time interval, so

$$S = \int_\Omega \mathcal{L}\left(\phi^r, \phi^r_{,\mu}, q^\mu\right) d^4q \tag{4.1.3}$$

The idea now is to apply the calculus of variations to this integral and require $\delta S = 0$ as in the particle case. Remember that were varying the fields at each spacetime point and not the coordinates q^μ. Therefore (Ill drop the superscript r to avoid confusion with the summation convention, so the following should be taken to apply to each field ϕ^r separately. A summation over μ is implied):

$$\delta S = \int_\Omega \left[\frac{\partial \mathcal{L}}{\partial \phi}\delta\phi + \frac{\partial \mathcal{L}}{\partial \phi_{,\mu}}\delta\phi_{,\mu}\right] d^4q \tag{4.1.4}$$

To work out the second term, we write out the derivative explicitly:

$$\frac{\partial \mathcal{L}}{\partial \phi_{,\mu}}\delta\phi_{,\mu} = \frac{\partial \mathcal{L}}{\partial \phi_{,\mu}} \frac{\partial\left(\delta\phi\right)}{\partial q^\mu} \tag{4.1.5}$$

$$= \frac{\partial}{\partial q^\mu}\left[\frac{\partial \mathcal{L}}{\partial \phi_{,\mu}}\delta\phi\right] - \frac{\partial}{\partial q^\mu}\left[\frac{\partial \mathcal{L}}{\partial \phi_{,\mu}}\right]\delta\phi \tag{4.1.6}$$

where the last line follows from the product rule. We therefore get

$$\delta S = \int_{\Omega} \left[\frac{\partial \mathcal{L}}{\partial \phi} - \frac{\partial}{\partial q^{\mu}} \left(\frac{\partial \mathcal{L}}{\partial \phi_{,\mu}} \right) \right] \delta\phi d^4 q + \int_{\Omega} \frac{\partial}{\partial q^{\mu}} \left[\frac{\partial \mathcal{L}}{\partial \phi_{,\mu}} \delta\phi \right] d^4 q \tag{4.1.7}$$

The last term is the integral of a 4-d divergence over a 4-d volume and we can use a 4-d analog of Gausss theorem to convert this to a surface integral over a 3-d surface Σ that bounds Ω. Making the usual assumption that this surface can be removed to infinity and that our system is finite so that $\mathcal{L} \to 0$ at infinity, this integral goes to zero. Were left with

$$\delta S = \int_{\Omega} \left[\frac{\partial \mathcal{L}}{\partial \phi} - \frac{\partial}{\partial q^{\mu}} \left(\frac{\partial \mathcal{L}}{\partial \phi_{,\mu}} \right) \right] \delta\phi d^4 q = 0 \tag{4.1.8}$$

The requirement that this is valid for all variations $\delta\phi$ in the field gives us the field theory version of the Euler-Lagrange equations (where Ive restored the index r indicating which field were talking about; note that μ is still summed):

$$\frac{\partial \mathcal{L}}{\partial \phi^r} - \frac{\partial}{\partial q^{\mu}} \left(\frac{\partial \mathcal{L}}{\partial \phi^r_{,\mu}} \right) = 0 \tag{4.1.9}$$

Because test particles follow geodesics in a fixed metric, the orbits of those particles may be determined using the calculus of variations, also called lagrangian approach. Geodesics in spacetime are defined as curves for which small local variations in their coordinates(while holding the endpoints events fixed)make no significant change in their overall lengths. This may be expressed mathematically using the calculus of variations by adopting the terminology of classical mechanics,

$$0 = \delta \int ds = \delta \int \sqrt{g_{\mu\nu} dx^{\mu} dx^{\nu}} d\tau = \delta \int \sqrt{2L} d\tau$$

Where τ is the proper time , $s = c\tau$ is the arc length in space-time and τ is defined as

$$2L = c^2 = \left(\frac{ds}{d\tau} \right)^2 = g_{\mu\nu} \frac{dx^{\mu}}{d\tau} \frac{dx^{\nu}}{d\tau} \tag{4.1.10}$$

Hence

$$2L = \left(\frac{ds}{d\tau} \right)^2 \tag{4.1.11}$$

the simpler Lagrangian

$$L = \frac{1}{2} g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} \tag{4.1.12}$$

the canonical momentum conjugate to x^μ equals the momentum one-form of the particle:

$$p_\mu = \frac{\partial L}{\partial \left(\frac{dx^\mu}{d\tau}\right)} = g_{\mu\nu} \frac{dx^\nu}{d\tau} \tag{4.1.13}$$

4.2 Derive Dynamic parameters by Corresponding Principle in sds Geometry

For a general class of static spherically symmetric spacetimes of the form (in the standard coordinates system)

$$ds^2 = -f(r)c^2dt^2 + f(r)^{-1}dr^2 + r^2d\Omega^2 \tag{4.2.1}$$

Where, $d\Omega^2 = r^2d\theta^2 + r^2sin^2\theta d\phi^2$ and f(r) is the generic metric function, the Lagrangian density of a particle of mass m is given by

$$2\mathcal{L} = -f(r)c^2 \left(\frac{dt}{d\tau}\right)^2 + f(r)^{-1} \left(\frac{dr}{d\tau}\right)^2 + r^2 \left(\frac{d\Omega}{d\tau}\right)^2 \tag{4.2.2}$$

From the symmetries, one obtains two constants of motion corresponding to two ignorable coordinates t and Ω as given by

$$p_t = \frac{\partial \mathcal{L}}{\partial \tilde{t}} = constant = -\epsilon \tag{4.2.3}$$

and

$$p_\Omega = \frac{\partial \mathcal{L}}{\partial \tilde{\Omega}} = r^2 \frac{d\Omega}{d\tau} = constant = \lambda \tag{4.2.4}$$

Where, ϵ and λ are specific energy and generalized specific angular momentum of the orbiting particle, respectively. Here, $\tilde{t}$ and $\tilde{\phi}$ represents the derivative of 't' and $'\phi'$ with respect to proper time τ. It needs to be mentioned that from now onwards, throughout the paper, the terms related to momentum, energy/Hamiltonian, potential and frequency, all of which are in fact their specific quantities, would be addressed without the using of word 'specific'. Using Eq.(4.2.3) we can derive

$$\frac{dt}{d\tau} = \frac{\epsilon}{c^2}\frac{1}{f(r)} \tag{4.2.5}$$

by using eq (4.1.12) $2\mathcal{L} = g_{\alpha\beta}p^\alpha p^\beta = -m^2c^2$ and substituting eq (4.2.3) and eq(4.2.4) in eq(4.2.2) we obtain

$$\left(\frac{dr}{d\tau}\right)^2 = \left(\frac{\epsilon^2}{c^2} - c^2\right) - c^2[f(r) - 1] - f(r)\frac{\lambda^2}{r^2} \tag{4.2.6}$$

By considering a locally inertial frame for a test particle motion, we write $E_{GN} = (\frac{\epsilon^2 - c^4}{2c^2})$ ('GN' symbolizes 'GR-Newtonian'). Second term in the above definition of E_{GN} is the rest mass energy of the particle which is subtracted from relativistic energy owing to low energy limit. from eq(4.2.5) and (4.2.6) and using eq(4.2.4) we get

$$d\tau = \frac{c^2}{\epsilon}f(r)dt$$

and also

$$\lambda^2 = \frac{\epsilon^2 r^4}{c^4}\frac{\dot{\Omega}^2}{f(r)^2}$$

Then,

$$\frac{dr}{d\tau} = \frac{\epsilon}{c^2 f(r)}\frac{dr}{dt}$$

By using these above equations ,$\frac{dr}{d\tau}$ will be

$$\frac{dr}{dt} = f(r)\frac{c^2}{\epsilon}\sqrt{2E_{GN} - c^2[f(r) - 1] - \dot{\Omega}^2\frac{r^2}{f(r)}} \tag{4.2.7}$$

Where, $\dot{\Omega}$ is the derivative with respect to coordinate time 't'. Using the condition for low energy limit $\frac{\epsilon}{c^2} \sim 1$ as we prefer to, E_{GN} is given by

$$E_{GN} = \frac{1}{2}\left(\frac{dr}{dt}\right)^2\frac{1}{f(r)^2} + \frac{r^2\dot{\Omega}^2}{2f(r)} + \frac{c^2}{2}[f(r) - 1] \tag{4.2.8}$$

In the asymptotic non-relativistic limit E_{GN} reduces to the Newtonian mechanical energy (= Hamiltonian of the motion). The generalized Hamiltonian E_{GN} in the low energy limit should then be equivalent to the Hamiltonian in Newtonian regime. The Hamiltonian in the Newtonian regime with the generalized analogous potential in spherical polar geometry will then be equivalent to E_{GN} in Eq. (4.2.8). Thus

$$E_{GN} \equiv \frac{1}{2}(\dot{r}^2 + r^2\dot{\Omega}^2) + V_{GN} - \dot{r}\frac{\partial V_{GN}}{\partial \dot{r}} - \dot{\Omega}\frac{\partial V_{GN}}{\partial \dot{\Omega}} \tag{4.2.9}$$

Where, $T = \frac{1}{2}(\dot{r^2} + r^2\dot{\Omega}^2)$ is the non-relativistic kinetic energy of the test particle. $\dot{r}$ is the derivative with respect to coordinate time 't'. V_{GN} is the analogous potential which would then be given by

$$V_{GN} = \frac{c^2[f(r)-1]}{2} - \frac{[1-f(r)]}{2f(r)}\left[\frac{1+f(r)}{f(r)}\dot{r}^2 + r^2\dot{\Omega}^2\right] \tag{4.2.10}$$

V_{GN}, thus, is the generalized three dimensional potential in spherical geometry in Newtonian analogue, corresponding to any generalized static GR metric given in eq.(4.2.1), with test particle motion in the low energy limit. Note that , the first term on the right hand side of the potential contains the explicit information of the source. For a purely spherically symmetric gravitational mass with zero charge and without any external effects, the classical Newtonian gravitational potential $\frac{-GM}{r}$ will be recovered from this term . The second term is the explicit velocity dependent and contains information of the test particle motion , thus contributing to the modification of Newtonian gravity. For sds metric the metric function $f(r) = 1 - \frac{2r_s}{r} - \frac{\Lambda}{3}r^2$, where Λ is the cosmological constant. $\Lambda > 0$ represents the sds metric with spatially inflated Universe,where $\Lambda < 0$ represents a Schwarzschild anti-de Sitter metric corresponding to negative vacuum energy density for contracting universe. $r_s = \frac{GM}{c^2}$. From Eq. (4.2.10) we then obtain the three dimensional generalized Newtonian analogous potential in spherical geometry, corresponding to sds geometry in the low energy limit,

$$V_{ds} = -\left(\frac{GM}{r} + \frac{\Lambda c^2 r^2}{6}\right) - \left(\frac{2r_s + \frac{\Lambda r^3}{3}}{r - 2r_s - \frac{\Lambda r^3}{3}}\right)\left(\frac{r - r_s - \frac{\Lambda r^3}{6}}{r - 2r_s - \frac{\Lambda r^3}{3}}\dot{r}^2 + \frac{r^2\dot{\Omega}^2}{2}\right) \tag{4.2.11}$$

where, subscript 'ds' denotes Schwarzschild de-Sitter.With $\Lambda = 0$ the above potential reduces to the potential corresponding to the simplest static Schwarzschild geometry [26].$M \equiv$

M_{BH} is the mass of the BH/central object. The denominator in the second term of the potential contains the exact metric function f(r) of sds geometry, and hence the potential V_{ds} would reproduce the exact location of the event horizon and cosmological horizon and other horizon properties, as that in full general relativity.Introducing a dimensional parameter or cosmological parameter $\zeta = \frac{\Lambda r_s^2}{3}$, the vanishing cubic polynomial f(r) with repulsive cosmological constant ($\Lambda > 0$) would give two real positive roots representing the locations of two horizons, namely the BH horizon and the cosmological horizon. The locations of these two horizons are then given by $r_H = \frac{2}{\sqrt{3\zeta}} cos\left[\frac{\pi}{3} + \frac{cos^{-1}(3\sqrt{3\zeta})}{3}\right]$ and $r_{CM} = \frac{2}{\sqrt{3\zeta}} cos\left[\frac{\pi}{3} - \frac{cos^{-1}(3\sqrt{3\zeta})}{3}\right]$, respectively.

4.3 Orbital Dynamics Around sds Spacetime

In the Newtonian framework, the Lagrangian of a particle in the presece of the sds analogous potential V_{ds} per unit mass using eq(4.2.8) is given by,

Lagrangian function

$$\mathcal{L}_{ds} = T - V_{ds}$$

Lagrangial density will be

$$\mathcal{L}_{ds} = \frac{1}{2}\frac{\dot{r}^2}{\left(1 - \frac{2r_s}{r} - \frac{\Lambda r^2}{3}\right)^2} + \frac{r^2\dot{\Omega}^2}{2\left(1 - \frac{2r_s}{r} - \frac{\Lambda r^2}{3}\right)} - \frac{c^2}{2}\left[-\frac{2r_s}{r} - \frac{\Lambda r^2}{3}\right]$$

Hence

$$\mathcal{L}_{ds} = \frac{1}{2}\left[\frac{r^2\dot{r}^2}{\left(r - 2r_s - \frac{\Lambda r^3}{3}\right)^2} + \frac{r^3\dot{\Omega}^2}{r - 2r_s - \frac{\Lambda r^3}{3}}\right] + \frac{GM}{r} + \frac{\Lambda c^2 r^2}{6} \tag{4.3.1}$$

Where, $\dot{\Omega}^2 = \dot{\theta}^2 + sin^2\theta\dot{\phi}^2$. Here over dots show that, the derivative with respect to coordinate time't'. we compute the conserved angular momentum and Hamiltonian using V_{ds} , given by

$$\lambda_{ds} = \frac{r^3\dot{\Omega}}{r - 2r_s - \frac{\Lambda r^3}{3}} \tag{4.3.2}$$

and

$$E_{ds} = \frac{1}{2}\left[\frac{r^2\dot{r}^2}{\left(r - 2r_s - \frac{\Lambda r^3}{3}\right)^2} + \frac{r^3\dot{\Omega}^2}{r - 2r_s - \frac{\Lambda r^3}{3}}\right] - \frac{GM}{r} - \frac{\Lambda c^2 r^2}{6} \tag{4.3.3}$$

respectively.

Using Eqs. (4.3.2) and (4.3.3), we obtain $\dot{r}$ that uniquely describes the test particle motion, which is given by

$$\frac{dr}{dt} = \frac{r - 2r_s - \frac{\Lambda}{3}r^2}{r}\sqrt{2E_{ds} + \frac{2GM}{r} + \frac{\Lambda c^2 r^2}{3} - \left(r - 2r_s - \frac{\Lambda r^3}{3}\right)\frac{\lambda_{ds}^2}{r^3}} \tag{4.3.4}$$

which is exactly equivalent to $\dot{r}$ in general relativity in low energy limit. Replacing $\dot{\Omega}$ and $\dot{r}$ in Eq (4.2.11) using Eqs (4.3.2) and (4.3.4) respectively, sds analogous potential can be written in terms of conserved Hamiltonian E_{ds} and angular momentum λ_{ds}, given by

$$\begin{aligned} V_{ds} \quad = \quad & \left(\frac{GM}{r} + \frac{\Lambda c^2 r^2}{6}\right) - \left(2r_s + \frac{\Lambda r^3}{3}\right)\left[\left(r - 2r_s - \frac{\Lambda r^3}{3}\right)\frac{\lambda_{ds}^2}{r^4}\left(\frac{1}{2} - \frac{r - r_s - \frac{\Lambda r^3}{6}}{r}\right)\right] \\ & - \left(2r_s + \frac{\Lambda r^3}{3}\right)\left[\frac{1}{r^2}\left(r - r_s - \frac{\Lambda r^3}{6}\right)\left(2E_{ds} + \frac{GM}{r} + \frac{\Lambda c^2 r^2}{3}\right)\right] \end{aligned} \tag{4.3.5}$$

Chapter 5

Result and Discussion

Table 5.1: Comparing angular momentum numerical data in sds and sw geometry in below table

$\lambda_{sw}(10^{25}Hz)$	0.707107	12.5743	2.23607	39.7635
$\lambda_{sds}(10^{25}Hz)$	0.7071	12.5704	22.1359	24.1068

We show the variation of V_{ds} in the form given in eq(3.3.5)with radial r, corresponding to $\Lambda > 0$ with $\Lambda r_s^2 = 1 * 10^{-27}$ in the low energy limit of test particle motion. The stated value of Λr_s^2 corresponding to $\Lambda = 10^{-56} cm^{-2}$ for $M_{BH} \sim 10^9 M_{\bigodot}$ .The profile of V_{ds} clearly shows both the BH event horizon as well as the cosmological horizon .With $\Lambda r_s^2 \sim 1 * 10^{-27}$, cosmological horizon extends up to $\sim 5.5 * 10^{13} r_s$. For a BH of $\sim 10^9 M_{\bigodot}$, it gives a radius of $5.5 * 10^3$ megaparsec.Although the locations of BH event horizon as well as cosmological horizon remain unaltered with the increase in test particle energy, nevertheless, there is a noticeable change in the magnitude of V_{ds} at both horizon radii. It is found that λ_{ds} has no effect on the nature of potential just beyond $\sim 100 r_s$. These indicate that with the increase in the value λ_{ds}, inner horizon shifts to larger radii. The variation of effective potential $V_{ds}^{eff} = V_{ds} + \frac{\lambda_{ds}^2}{2r^4}\left(r - 2r_s - \frac{\Lambda r^3}{3}\right)$ for the same value of Λ corresponding to sds spacetime. However V_{ds}^{eff} attains a higher peak as compared to V_{ds} in the vicinity of inner horizon for values of $\lambda_{ds} \gtrsim 3.5$. For any arbitrary physical quantity $\mathcal{F}$, the relative deviation (ξ_i) is defined as $\xi_i = 2\left|\frac{\mathcal{F}_{ds} - \mathcal{F}_{SW}}{\mathcal{F}_{ds} + \mathcal{F}_{SW}}\right|$ which essentially implies deviation between

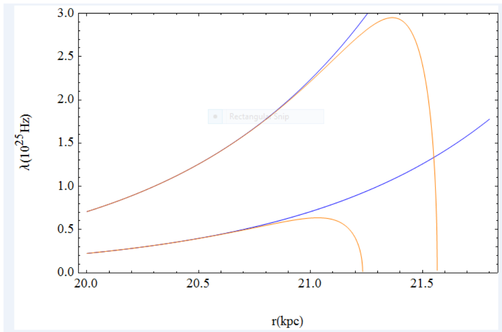

Figure 5.1: angular momentum of λ_{sw} and λ_{sd}

the analogous potential for sds and Schwarzschild geometry relative to the their average value.The subscript SW represents corresponding quantities in Schwarzschild geometry.At smaller distances ξ_i is very small, and detecting such small deviations experimentally does not seem to be possible at present or in near future.[26]The relative deviation may be considered to be non-negligible with the corresponding distance $r \sim 8.4 * 10^7 r_s$. With this consideration, this then implies that beyond such radius the influence of Λ cannot be neglected. At $r \gtrsim 8.4 * 10^8 r_s$ the relative deviation becomes substantial with ($\xi_i \gtrsim 10$) percent. ξ_i increases rapidly up to certain radius $r \sim 9 * 10^9 r_s$, beyond which it mostly remains constant.Around this particular radius, the Λ effects become too dominating, which renders V_{SW} to become negligible in compare to that of V_{ds}. We describe this radius $r \sim 9 * 10^9 r_s$ as some upper bound ($x_m ax$). For $M_{BH} \sim 10^9 M_{\bigodot}$, in accordance with the BH massin many AGNs/quasars or in massive galaxies, the lower bound in $r(x_{min})$ gives a radius of ~ 8 kiloparsec, whereas the upper bound x_{max} gives a radius of ~ 900 kiloparsec. Thus, the region between x_{min} and x_{max} or a region approximately existing between few kiloparsecs to a few 100 kiloparsecs would be strongly affected by cosmological constant Λ, thereby, directly influencing the kiloparsecs-scale structure in massive galaxies, in the local observable Universe. With a similar BH mass, the region beyond 80 kiloparsec would have most significant Λ effects,where ξ_i would be greater than $\sim$ 10percent. Next we obtain the equation of the orbital trajectory using Eqs. (3.3.2) and (3.3.4), as given by

$$\left(\frac{dr}{d\Omega}\right)^2 = \frac{r^4}{\lambda_{ds}^2}\left[2E_{ds} + \frac{2GM}{r} + \frac{\Lambda c^2 r^2}{3} - \left(r - 2r_s - \frac{\Lambda r^2}{3}\right)\frac{\lambda_{DS}^2}{r^3}\right] \tag{5.0.1}$$

Although Eq. (3.3.6) is derived with the condition of low energy limit, yet it is exactly the same as that in full general relativity which one can easily obtain using Eqs. (3.2.4) and (3.2.7) with relevant f(r). To furnish a complete behavior of the particle dynamics in sds background, we obtain the equations of motion of the test particle in the presence of V_{ds} from the Euler-Lagrange equations in spherical geometry which are given by

$$\ddot{r} = \left(-\frac{GM}{r^2} + \frac{\Lambda c^2 r}{3}\right)\left(\frac{r - 2r_s - \frac{\Lambda r^3}{3}}{r}\right)^2 + \frac{2\left(r_s - \frac{\Lambda r^3}{3}\right)}{r\left(r - 2r_s - \frac{\Lambda r^3}{3}\right)}\dot{r}^2 + (r - 3r_s)(\dot{\theta}^2 + sin^2\theta\dot{\phi}^2) \tag{5.0.2}$$

$$\ddot{\phi} = \frac{2\dot{r}\dot{\phi}}{r}\left(\frac{r - 3r_s}{r - 2r_s - \frac{\Lambda r^3}{3}}\right) - 2cot\theta\dot{\phi}\dot{\theta} \tag{5.0.3}$$

and

$$\ddot{\theta} = \frac{2\dot{r}\dot{\theta}}{r}\left(\frac{r - 3r_s}{r - 2r_s - \frac{\Lambda r^3}{3}}\right) + sin\theta cos\theta\dot{\phi}^2 \tag{5.0.4}$$

respectively.$\dot{\phi}$ and $\dot{\theta}$ equations are exactly the same to that in general relativity. Whereas $\ddot{r}$ eq(3.3.7) corresponds to that in general relativity in the low energy limit. The corresponding $\ddot{r}$ equation in general relativity given by

$$\ddot{r} = \left(-\frac{GM}{r^2} + \frac{\Lambda c^2 r}{3}\right)\left(\frac{r - 2r_s - \frac{\Lambda r^2}{3}}{r}\right)^2 \frac{c^4}{\epsilon^2} + \frac{2\left(r_s - \frac{\Lambda r^3}{3}\right)}{r(r - 2r_s - \frac{\Lambda r^2}{3})}\dot{r}^2 + (r - 3r_s)(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2) \tag{5.0.5}$$

It needs to be mentioned that previously a PNP has been prescribed in [26] corresponding to sds geometry based on a method adopted by [30-32], which has been formulated by considering a Keplerian rotation profile of the test particle motion. The form of the PNP is given by

$$\Phi(r) = \frac{r^3\frac{\Lambda}{3} - 3r(\frac{\Lambda}{3})^{1/3} + 2}{2\left[1 - 3(\frac{\Lambda}{3})^{1/3}\right](2 - r + r^3\frac{\Lambda}{3})} \tag{5.0.6}$$

which is derived on the premise that $\Phi(r) = 0$ at the static radius, preserving the analogy that the gravitational potential tends to zero in asymptotically flat spacetime. This potential does not have any dependence on test particle velocity. With $\Lambda = 0$, the PNP in Eq. (4.0.6) reduces to that of Paczy'nski-Witta potential corresponding to Schwarzschild geometry. The corresponding GR behavior is mimicked through this PNP by intending only to reproduce the marginally stable and bound orbits for circular orbital trajectory. This is in sharp contrast with the velocity dependent potential V_{ds}, which, a priori, focused on to replicate the general relativity by resembling the geodesic equations of motion. This ensures that most of the GR features could be reproduced accurately.

5.1 Particle dynamics along circular orbit

In order to compare the behavior of the particle motion in presence of V_{ds} and those in general relativity, we compute the dynamical variables for the simplest circular orbit trajectory. With the conditions for the circular orbits $\dot{r} = 0$ and $\ddot{r} = 0$, we obtain corresponding angular momentum λ^c_{ds}, we obtain corresponding angular momentum E^c_{ds}, and the orbital angular velocity $\dot{\Omega}^c_{ds}$ with V_{ds} using eqs(3.3.2), (3.3.4) and (3.3.7) given by

$$\lambda^c_{ds} = r\sqrt{\frac{GM - \Lambda c^2 r^3}{r - 3r_s}} \tag{5.1.1}$$

then,

$$E^c_{ds} = -\frac{GM}{2r}\left(\frac{r - 4r_s}{r - 3r_s}\right) + \left[\frac{\Lambda c^2 r^3}{6(r - 3r_s)}\right]\left(\frac{\Lambda r^2}{3} + \frac{4r_s}{r} - 2\right) \tag{5.1.2}$$

and

$$\dot{\Omega}^c_{ds} = \frac{r - 2r_s - \frac{\Lambda r^3}{3}}{r^2}\sqrt{\frac{GM - \Lambda c^2 r^3}{r - 3r_s}} \tag{5.1.3}$$

When $\Lambda = 0$ all above equation reduced into Schwarzschild geometry . To compute with sds geometry we use corresponding GR effective potential ; given by

$$V^{GR}_{eff}(r) = \left(1 - \frac{2r_s}{r} - \frac{\Lambda r^3}{3}\right)\left(c^2 + \frac{\lambda^2}{r^2}\right) \tag{5.1.4}$$

In GR, circular orbit of $\frac{dr}{d\tau} = 0$ and $\frac{\partial V_{eff}^{GR}}{\partial r} = 0$. We found energy ϵ for the particle motion in circular orbit,

$$\frac{\epsilon}{c^2} = \frac{\left(r - 2r_s - \frac{\Lambda r^3}{3}\right)}{\sqrt{r(r - 3r_s)}} \tag{5.1.5}$$

Angular momentum (λ^C) and the equivalent Hamiltonian $E^c = \frac{\epsilon^2 - c^4}{2c^2}$ for circular orbits in general relativity then exactly resemble the corresponding values obtained with V_{ds}, given by Eqs. (3.4.1) and (3.4.2) respectively. The orbital angular velocity in general relativity is then given by

$$\dot{\Omega}^2 = \sqrt{\frac{GM}{r^3} - \frac{\Lambda c^2}{3}} \tag{5.1.6}$$

whose analytical expression is not exactly equivalent to in Eq. (3.4.3). Note that the circular orbit corresponding to sds metric exists down to $3r_s$ in similarity to that in Schwarzschild geometry which represents the null hypersurface.

The appropriate comparison of the nature of potential V_{ds} in Eq. (3.2.11) and that of PNP in Eq.(4.0.6), corresponding to test particle motion in circular orbit. In the very inner and outer regions near to both the BH and cosmological horizons, the behavior of these potentials differ significantly. One of the major distinctive features of the PNP in Eq. (4.0.6) is that it becomes zero at the static radius ($\sim 1.4 * 10^9 r_s$) , whereas V_{ds} attains a value of $\sim -10^{-9}c^2$. In the intermediate region, the nature of the potentials remain mostly similar. It is seen that at $r \gtrsim 10^8 r_s$, where the effect of Λ is prominent, the PNP in Eq. (4.0.6) differs significantly as compared to V_{ds}. This radius approximately resembles x_{min}.

The variation of λ_{ds}^c corresponding to V_{ds} which exactly coincides with that in general relativity, for the entire spatial regime. we compare the nature of λ_{ds}^c with that corresponding to the PNP in Eq. (4.0.6) at the outer radii. For sds spacetime, there is a clear static radius at the outer radii where angular momentum abruptly falls to zero value.For BH of $\sim 10^9 M_{\odot}$, with $\Lambda = 10^{-56} cm^{-2}$, the static radius will be located at ~ 140 kiloparsec.Although the angular momentum for circular orbit trajectory corresponding to V_{ds} as well as for the PNP in Eq. (4.0.6) behaves quite similarly at the outer radii, and coincides near the static radius. However, due to the profound effect of Λ on the outer radii, the nature of λ_{ds}^{c} at the outer radii differs significantly from that corresponding to V_{SW}, especially beyond $4 * 10^{8} r_s$. In the inner radii, where the effect of Λ is negligible, λ_{ds}^{c} show appreciable deviation from the angular momentum profile corresponding to the PNP in Eq. (4.0.6).

The variation of conserved Hamiltonian E_{ds}^{c} with r, which is compared with the Hamiltonian corresponding to the PNP in Eq. (4.0.6) and the Hamiltonian corresponding to V_{SW}. It needs to be noted that E_{ds}^{c} corresponding to V_{ds} exactly resembles the corresponding expression in general relativity. In the outer region of the test particle motion in circular orbit, where the effect of repulsive Λ is prominent, there is a marked difference between the profile of conserved Hamiltonian corresponding to V_{ds} and that corresponding to the PNP in Eq. (4.0.6), especially at $r \gtrsim 5 * 10^{7} r_s$.

Chapter 6

Conclusion

General theory of relativity is the theory of gravitation and geometry of spacetime.It generalizes the spacial theory of relativity and Newton's law of universal gravitation. The matter and geometry of spacetime are related by the Einstein field equations ($G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$), where $G\mu\nu$ is Einstein field tensor that tells geometry of spacetime and $T_{\mu\nu}$ is energy-momentum tensor that is matter.

Generally, the spacetime geometry and gravitation described by tensors specially second rank like metric,Ricci tensor,Ricci scalar,Einstein field tensor and energy-momentum tensor . But energy-momentum tensor is vanished in vacuum Einstein field equation with cosmological constant. Although for this equation derive a solution such as Schwarzschild -de Sitter metric that is known as a line element which contain repulsive(positive cosmological constant).Farthermore, by corresponding principle of classical analogous used to Lagrangian and Hamiltonian of general relativistic particle from a line element in Schwarzschild geometry, drive symmetrically two constant of motion like specific energy and angular momentum. Finally, from that point of view general angular momentum,$\lambda_{ds} = \frac{r^3\dot{\Omega}}{r-2r_s-\frac{\Lambda r^3}{3}}$ and energy/Hamiltonian $E_{ds} = \frac{1}{2}\left[\frac{r^2\dot{r}^2}{\left(r-2r_s-\frac{\Lambda r^3}{3}\right)^2} + \frac{r^3\dot{\Omega}^2}{r-2r_s-\frac{\Lambda r^3}{3}}\right] - \frac{GM}{r} - \frac{\Lambda c^2 r^2}{6}$ analytical derived from Lagrangian equation in sds geometry. And also for this dynamic analytical equation found numerical values

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Jimma University
College of Natural Sciences
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We the undersigned, number of the Board of Examiners of the final open defense by **Yaecob Ayele Ashemo** have read and evaluated his/her thesis entitled

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Course Code	Course Title	Cr. hr	Number Grade	Rank **	Remark
Phys699	**MSc. Thesis**	**6**	**85.33**	**Excellent**	

** Excellent, Very Good, Good, Satisfactory, Fail.

Thesis Title

Dynamics of Accretion Process Around AGN in Schwarzschild de-Sitter Geometry

2. Board of Examiners decision Mark **X** in one of the boxes. Pass [**X**] Failed [ ]

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