DYNAMICS OF RELATIVISTIC JETS AROUND ACTIVE GALACTIC NUCLEI IN SCHWARZSCHILD DE SITTER GEOMETRY

By

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Table of Contents

Table of figure

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Abstract

More recent literature reviews point out that most galaxies, especially early type galaxies with Active Galactic Nuclei (AGN), contain Massive Black Holes (MBHs)considered to be comparable to the masses of high redshift quasars to the evolution of early galaxies. Some of these sources seem to accrete and eject matter-energy at a very high rate as reported. As a result it was believed that Electromagnetic(EM) spectrum observations are required to provide information on black holes in the centers of active galaxies. On the other hand, Gravitational Wave(GW) observations are considered to provide the complementary information about the capture of particles including compact objects like Black Holes(BHs) that are mostly invisible to EM observations. Thus, the astrophysical study of AGNs in its full relativistic effect is active and fresh research. For example, the efficient mechanisms to describe the energy - momentum and jets flow that could be exploited to match observations are important and open to research. Motivated by this scientific background, We worked on the dynamics of relativistic jets flow around AGNs by considering full General Relativity equations of charged fluid in Schwarzschild-de sitter geometry. The relevant dynamical parameters like pressure were being derived by setting simplifying boundary conditions. Finally we had work out on numerical analysis of the derived observable quantities to check their significance. In particular the pressure p_B and P_Λ in terms of radial distance from AGN was in agreement with the existing theory. The pressure due to magnetic field was, the dominant pressure at the surface of black hole horizon, is important for the formation of relativistic jets around AGN and far from black hole horizon pressure due to cosmology constant becomes dominant because of the effects of cosmology constant. Finally cosmology constant was a significant effects in local gravitating objects geometric structure and pressure. .

Key words: Accretion, AGN, BH, GR, MHD, SdS.

Introduction

General relativity has a great success in the observation of deflection of light, radar echo delay,precession of planetary motion and gravitational redshift by gravity are the manifestation of progress in astronomy and astrophysical studies. The discovery of the expanding universe at an accelerating phase, and the direct confirmation of gravitational wave detection are other astounding progresses in astronomy and astrophysics [19]. There are great deals of progress in the subject both theoretically and observationally with direct and indirect detections. It is in this context that one must appreciate Einstein's GR prediction in agreement with experimental observation, so that, in 1916 Einstein was interested to find a static solution of his field equations with the idea of incorporating Mach's principle, [8]. But Einstein soon noticed that his original field equations yield a non - static solution. As the consequence, Einstein himself after a year, in 1917 introduced a positive cosmological constant (Λ) with the belief of constructing a static solution. But at the same year that Einstein introduced the cosmological term, de Sitter presented solutions to static "Einstein universe",with $T_{\mu\nu} = 0$ and $\Lambda > 0$, which had both static and dynamic features,that allows a redshift-distance relation [23]. The de Sitter's prediction is considered as the first step towards the theoretical discovery of expanding universe. On the other hand, in 1922, 1924 Freidmann constructed a matter dominated expanding universe without a cosmological constant. Moreover, the discovery of expanding universe by Hubble and contemporaries led Einstein to abandon the idea of a static universe importunately including the cosmological constant. However, a number of researchers were entailed to construct models with cosmological constant to explain measurements of the spectra of spiral nebulae that showed redshifted to construct an expanding model which originated from such an asymptotically static state ("static Einstein universe") in the distant past [17]. Since then, the cosmological constant has remained with debate where it was being cast out at a time and reintroduced at other time. However, a firm considerate of Λ is triggered in the 1960's when an excess quasi-stellar objects (QSO's) near the redshift z $\cong$ 1.95 were observed [12]. Then a number of authors for instance [14] emerged with Lemaitre's models containing the cosmological constant in explaining the observed QSO's that was in agreement with the predicted inflationary scenario of the early universe. But the general perception is that owing to its tiny value, cosmological constant does not lead to any significant observable effects in a local gravitational phenomenon. However, the contribution of repulsive Λ could be significant (larger than the second order term) even in a local gravitational phenomenon when kiloparsecs to megaparsecs-scale distances are involved, such as the gravitational bending of light by cluster of galaxies [15] Moreover, the recently confirmed gravitational wave presence shines on the matter to study high precision astrophysical phenomena at small scale level. Probably, a local effect of cosmological constant is claimed to be observable from relativistic phenomena around massive BHs which involve distance-scale of the order of hundreds of parsecs or even more [17] and the references therein. However, the recent discoveries on astrophysical phenomena favor a Cold Dark Matter with positive cosmological constant (ΛCDM) model that is consistent with observations. Furthermore, the presence of a repulsive cosmological constant (positive) the spacetime geometry exterior to a static spherically symmetric gravitating system is Schwarzschild-de Sitter (SdS), in a spatially inflated Universe, rather than Schwarzschild. Motivated by this scientific rationale, we are interested to study the dynamics of jets around Active Galactic Nuclei (AGN) in Schwarzschild de Sitter Geometry. The description of important areas of modern astronomy, such as high-energy astrophysics or gravitational wave astronomy, requires general relativity. Einsteins theory of gravitation plays a major role in astrophysical scenarios involving compact objects such as neutron stars and black holes. High-energy radiation is often emitted in regions of strong gravitational fields near such compact objects [19]. The production of relativistic jets in active galactic nuclei, explained by either of hydrodynamic or electromagnetic mechanisms, involves rotating supermassive black holes. We adopt the conventions Throughout Greek indices run from 0 to 3 and Latin indices from 1 to 3; geometrized units G = c = 1 are used; G is Newtons gravitational constant and c is the speed of light is used. The outline of the research is:- Unit one contains introduction about general theory of relativity and introducing of cosmology constant in Einstein field equation. Unit two contain explanation about GRMHD background and derivative of basic equations of GRMHD to describe how energy extract from SMBH host by AGN during accretion, the pressure and differential equations of matter conservation. In this unit also we discuss the formation of jets ejected from AGN. Unit three and four includes result discussion and Summary respectively.

Statement of the problem

The ΛCDM model is more or less consistent with all the current cosmological observations. Its effect at large scale is well considered. However, its local effect like perihelion shift of the orbits of gravitationally bound systems, etc. are at debate [16]. On the other hand, there is a plethora of evidences that claim its effect at the local size. Probably, a local effect of cosmological constant is claimed to be observable from relativistic jets dynamics and accretion phenomena around massive BHs hosted by AGN (or QSO) which involve distance-scale of the order of hundreds of parsecs or even more [10]. However, a few studies have been carried out so far to investigate the effect of Λ in astrophysical jet flow paradigm [13, 14]. The obvious reason is to avoid the complex general relativistic (GR) magnetohydrodynamic (MHD) equations in a strong gravitational field regime. Owing to the complex and nonlinear character of the equations in GR regime, analytical/quasi numerical treatment of the problem is virtually discarded [15]. To this end several early works on motion of objects (including jet flows) and related phenomena were based on pure Newtonian gravity or alternative GR theories. However, on the other hand, the recently confirmed gravitational wave presence shines on the matter to study high precision astrophysical phenomena at small scale level including cosmological effect. So the current standard ΛCDM model that is consistent with observation shall be exploited to study the dynamics jets around massive objects like BHs hosted by AGN.

Research Questions

- How a massive compact object curves space-time around? And how the geometry influences the dynamics of objects around it?
- What is the significance of cosmological constant in the dynamics and observable of jets system around massive compact objects?

- How the ΛCDM model incorporates motion of jets near AGN?

Objectives

General Objective

To study the dynamics of relativistic jets around Active Galactic Nuclei in Schwarzschild-de Sitter Geometry.

Specific Objectives

- To derive dynamical equations from the GR equations in the SdS background.
- To derive dynamical observable parameters, like momentum, energy of relativistic jets around AGN with the SdS metric.
- To study the contents of the dynamical observable parameters of relativistic jets around AGN.

Methodology

The general method is to derive dynamical equations from which relevant dynamical parameters such as energy and momentum are being derived from Einstein static field equations. The analytically derived equations was converted to generate numerical values computationally with MATHEMATICA. Then, the results discussed and summarized.
The steps are:

- Provide preliminary boundary conditions to derive the relevant set of dynamical equations from the GR equations in the SdS background.
- Study and examine the effects of the relevant parameters derived from the equations.
- We had numerically generate some theoretical data from the formalism using computation.
- The result was discussed and Summarized.

Scope of the study

The scope of the study is limited to theoretical study and analysis where possible observable parameters are expected to be contained in the analytical equations. The derived equations are probably to be converted to EMWs or GWs or both spectrums for observational testing.

Literature Review

The success of general theory of relativity (GR) in the observation of deflection of light, radar echo delay, precession of planetary motion and gravitational redshift by gravity are the manifestation of progress in astronomy and astrophysical studies. The discovery of the expanding universe at an accelerating phase, and the direct confirmation of gravitational wave detection are other astounding progresses in astronomy and astrophysics. There are great deals of progress in the subject both theoretically and observationally with direct and indirect detections [19]. Whilst, there are encouraging past success of GR and the hopes ahead there is an outstanding debates on GR field equations dated back to their origin. After completing his theory of GR, Einstein was interested to find a static solution of his field equations with the idea of incorporating Mach's principle [8]. But Einstein noticed that his original field equations:-

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = KT_{\mu\nu}$$

yields a non - static solution. Where, $R_{\mu\nu}$ is the Ricci curvature tensor, R is the scalar curvature, $g_{\mu\nu}$ is the metric tensor, scalar constant and $T_{\mu\nu}$ is the energy - momentum source tensor. As the consequence, Einstein himself introduced a positive cosmological constant Λ with the belief of constructing a static solution. The idea is that, the constant introduces a repulsive force which can counterbalance the attractive force of gravity leading to the "static Einstein universe". The modified Einstein's field equations with the cosmological constant is,

$$R_{\mu\nu} - 1/2g_{\mu\nu}R + \Lambda g_{\mu\nu} = KT_{\mu\nu}$$

But at the same year that Einstein introduced the cosmological term, de Sitter (1917) presented solutions to static Einstein universe", with $T_{\mu\nu}$=0 and, $\Lambda > 0$ which had both static and dynamic features, that allows a redshift-distance relation [10]. The de Sitter's prediction is considered as the first step towards the theoretical discovery of expanding universe. On the other hand, in 1922 Freidmann constructed a matter dominated expanding universe without a cosmological constant. Then, the possibility that the universe is expanding led Einstein to abandon the idea of a static universe including the cosmological constant [19]. However, a number of researchers were entailed to construct models with cosmological constant. For example, Lemaitre constructed an expanding model which originated from such an asymptotically static state ("static Einstein universe") in the distant past. Since then, the cosmological constant has remained with debate where it was being cast out at a time and reintroduced at other time [3]. A firm considerate of Λ is triggered in the 1960's when an excess quasi-stellar objects (QSO's) near the redshift $z \approx 1.95$ were observed. Then a number of authors, for instance [9]emerged with Lemaitre's model in explaining the observed QSO's that was in agreement with the predicted inflationary scenario of the early universe.

In general, according to current understanding a flat low density Cold Dark Matter with dark energy in the form of cosmological constant (CDM $+\Lambda$) universe with $\Omega_m = 0.3$ and $\Omega\Lambda = 0.7$, with an approximately flat metric is favored over a wide range of observational data ranging from large and intermediate angle Cosmic Microwave Background Radiation (CMBR) anisotropies to observations of galaxy clustering on large scales, for instance see [6, 10]. In the presence of a repulsive cosmological constant (positive) the spacetime geometry exterior to a static spherically symmetric gravitating system is Schwarzschild-de Sitter (SdS), in a spatially inflated Universe, rather than Schwarzschild. But the general perception is that owing to its tiny value, cosmological constant does not lead to any significant observable effects in a local gravitational phenomenon. However, the contribution of repulsive Λ could be significant (larger than the second order term) even in a local gravitational phenomenon when kiloparsecs to megaparsecs-scale distances are involved, such as the gravitational bending of light by cluster of galaxies [11].

Probably, a local effect of cosmological constant is claimed to be observable from relativistic accretion phenomena around massive BHs which involve distance-scale of the order of hundreds of parsecs or even more [12] and the references therein. However, a few studies have been carried out so far to investigate the effect of Λ in astrophysical jet/accretion flow paradigm [13, 14]. So the effect of Λ on the dynamics of kiloparsecs to megaparsecs scale astrophysical objects including jets need investigations. So far all the works on the effect of Λ on accreting systems were carried out under some restricted conditions. The obvious reason is to avoid the complex general relativistic GR magnetohydrodynamic (MHD) equations in a strong gravitational field regime. Owing to the complex and nonlinear character of the equations in GR regime, analytical/quasi numerical treatment of the problem is virtually discarded; see for example [15] and the references therein. Several early works on accretion related phenomena were based on pure Newtonian gravity. So the current standard Λ CDM model that is consistent with observation shall be exploited to study the effect of cosmological constant on dynamical systems including MHD instabilities around massive objects like BHs where they are mostly hosted by AGNs.

Chapter 1

GENERAL THEORY OF RELATIVITY

1.1 Einstein General Theory Of Relativity

Einstein's theory of general relativity is a cornerstone of modern physics. After many years of investigation Einstein develop his general theory of relativity in 1915, it was published the following year [7]. The origin of GR can be traced to the conceptual revolutionary that followed Einstein's introduction of special theory of relatively in 1905. Newton's centuries old gravitational force law is inconsistent with general theory of relativity [19]. It Provides a relativistic description of the gravitational field exerted by massive objects and its effects on the geometric structure of the surrounding spacetime. Einstein introducing of relativistic theory of gravity resulted is not a new force law or a new theory of relativistic gravitational field, but profound conceptual revolution in our views of space and time. He saws that the experimental fact that all bodies fall with the same acceleration in a gravitational field led naturally to an understanding of gravity in terms of the curvature of the four dimensional union of space and time-spacetime [12]. Mass curves spacetime in its vicinity,and the trajectories along which all masses fall are the straight paths in this curved spacetime. This differs from the original foundations of Newton's law of gravitation, where gravity is an attractive force between two massive objects which interact instantaneously. In this description, planetary orbits are a consequence of this gravitational pull emanate from the sun.For example, the sun exerts a gravitational force on the earth and the earth moves around the sun in response to the gravitational field interaction. However,in GR point of view the mass of the sun curves the surrounded spacetime, and the earth moves on straight path in that curved spacetime. Since gravity is one of the four fundamental interaction and determine the geometry of spacetime [21]. However, given a certain circumstances Newtonian theory provides an accurate description of the gravitational interaction, this include a weaker gravitational field. This is known as the Newtonian limit in which spacetime is asymptotically flat. Einstein generalized his theory of general relativity by,

Spacetime tells matter how to move
Matter tells spacetime how to curve

General theory of gravitational is great acceptable theory than Newton theory of gravitational, the later has no longer in agreement with observation.For instance the deflection of light by the sun and also other applications of GR deviated slight from the prediction of Newton equation, where as solution in GR had been described successfully in the present modern astrophysics and astronomy [19].

1.2 Space-Time Geometry in General Relatively

After Riemannaian manifold, Lorentzian manifold from the most important subclass of pseudo-Riemannaian manifold M,this is equipped with the metric $g_{\varphi\nu}$, which can be used to determine local geometry such as angry and length. They are important in application of GR. A principal basis of GR is that spacetime can be modeled as 4-dimensional lorentzian manifold of signature (3,1) or equivalently (1,3) depending on our signs convention. For the purposes of this research we will consider a metric tensor with signature of (-+++) for the discussion of GR unless otherwise stated[19]. The metric $g_{\varphi\nu}$ is symmetric and its inverse will be $g^{\varphi\nu}$, so that

$$g_{\varphi\nu} = g_{\nu\varphi}$$

and

$$g^{\varphi\nu} = g^{\nu\varphi}$$

where, $g_{\nu\varphi} = dig(-1, 1, 1, 1)$, $g^{\varphi\mu} g_{\mu\nu} = \delta^{\varphi}_{\nu}$

$$g^{\varphi\mu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The line element can be described as ,

$$ds^2 = g_{\varphi\nu} dx^{\varphi} dx^{\nu}$$

the quantity ds is an invariant under reasonable arbitrary transformation, i.e all observers in any arbitrary frame of reference will agreed on the value of ds, is called diffeomorphisms.

1.3 Tensor Analysis In GR

The method is based on an alternative version of the principle of equivalence, known as the principle of general covariance. It state that a physical equation holds in an arbitrary gravitational fields. If two necessary conditions are met,

1. The equation holds in the absence of gravity ,i.e.

$$g_{\mu\theta} = \eta_{\mu\theta} \quad and \quad \Gamma^{\alpha}_{\tau\nu} = 0$$

2. The equation is generally covariant,that is preserve its form under arbitrary general coordinate transformation $x \to x'$. So that we need to develop the rules that the "good "(covariant) objects will obey when we pass from one coordinate system to an other.Such an objects are said to be a tensor. A tensor is 'something' that transforms like a vectors.

1.3.1 Metric Tensor

A metric tensor is such an important object in curved space, that it is given a new symbol $g_{\mu\nu}$ (while $\eta_{\mu\nu}$ is reserved specially for minkowski metric) There are few restrictions on the components of $g_{\mu\nu}$, other than it be a symmetric tensor. It is usually taken to be non-degenerate,meaning that the determinant $g = |g_{\mu\nu}|$ doesn't vanished. This allow to define the inverse metric as $g^{\mu\nu}$ via

$$g^{\mu\lambda} g_{\lambda\nu} = \delta^{\mu}_{\nu}$$

The metric tensor is defined as,

$$g_{\mu\nu} = \frac{\partial \xi^{\alpha}}{\partial x^{\mu}} \frac{\partial \xi^{\beta}}{\partial x^{\nu}} \eta_{\alpha\beta} \tag{1.3.1}$$

The symmetry of $g_{\mu\nu}$ implies that $g^{\mu\nu}$ is also symmetric metric. It help us to determine the proper distance/time/ interval between two event with a given infinitesimal coordinate separation. Proper distance between two event can defined as,

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}$$

1.3.2 Affain Connection

An invariant third order partial differential equation of metric tensor ($g_{\mu\nu}$). It is a field that determine the gravitational force and used to representant gravitational potential in GR. It can be called as christoffel symbol, transform as a tensor only under affain coordinate changes, which is denoted by either of the three $\{\mu\nu, \sigma\}$ or $\{^{\sigma}_{\mu\nu}\}$ or $\{\Gamma^{\sigma}_{\mu\nu}\}$.
The mathematical definition for affain connection as,

$$\Gamma^{\sigma}_{\mu\nu} = \frac{\partial x^{\sigma}}{\partial \xi^{\beta}} \frac{\partial^2 \xi^{\beta}}{\partial x^{\mu} \partial x^{\nu}} \tag{1.3.2}$$

where ξ^{α} and ξ^{β} are the local inertial coordinate. Now be taking partial differentiation of the general metric tensor in equ(1.3.1) at a local gravitational field with respect to an arbitrary general coordinate system x^{ρ}. It shows as,

$$\frac{\partial g_{\mu\nu}}{\partial x^{\rho}} = \frac{\partial}{\partial x \rho} [\frac{\partial \xi^{\alpha}}{\partial x^{\mu}} \frac{\partial \xi^{\beta}}{\partial x \nu} \eta_{\alpha\beta}]$$

Now here take differentiation by part,we have

$$\frac{\partial g_{\mu\nu}}{\partial x^{\rho}} = \frac{\partial^2 \xi^{\alpha}}{\partial x^{\rho}} \frac{\partial \xi^{\beta}}{\partial x^{\mu} \partial x^{\nu}} \eta_{\alpha\beta} + \frac{\partial \xi^{\alpha}}{\partial x^{\mu}} \frac{\partial^2 \xi^{\beta}}{\partial x^{\rho} \partial x^{\nu}} \eta_{\alpha\beta} \tag{1.3.3}$$

Now eqn.(1.3.3) can be written as using(1.3.1)

$$\frac{\partial g_{\mu\nu}}{\partial x^{\rho}} = \Gamma^{\lambda}_{\rho\mu}\frac{\partial \xi^{\alpha}}{\partial x^{\lambda}}\frac{\partial \xi^{\beta}}{\partial x^{\nu}}\eta_{\alpha\beta} + \Gamma^{\lambda}_{\rho\nu}\frac{\partial \xi^{\alpha}}{\partial x^{\mu}}\frac{\partial \xi^{\beta}}{\partial x^{\lambda}}\eta_{\alpha\beta} \tag{1.3.4}$$

Where,

$$\Gamma^{\lambda}_{\rho\mu}\frac{\partial \xi^{\alpha}}{\partial x^{\lambda}} = \frac{\partial^2 \xi^{\alpha}}{\partial x^{\rho}\partial x^{\mu}}$$

and

$$\Gamma^{\lambda}_{\rho\nu}\frac{\partial \xi^{\beta}}{\partial x^{\lambda}} = \frac{\partial^2 \xi^{\beta}}{\partial x^{\rho}\partial x^{\nu}}$$

Than eqn.(1.3.4) can be set as,

$$\frac{\partial g_{\mu\nu}}{\partial x^{\rho}} = \Gamma^{\lambda}_{\rho\mu}g_{\lambda\nu} + \Gamma^{\lambda}_{\rho\nu}g_{\mu\lambda} \tag{1.3.5}$$

Where,

$$g_{\lambda\nu} = \frac{\partial \xi^{\alpha}}{\partial x^{\lambda}}\frac{\partial \xi^{\beta}}{\partial x^{\nu}}\eta_{\alpha\beta}$$

and

$$g_{\mu\lambda} = \frac{\partial \xi^{\alpha}}{\partial x^{\mu}}\frac{\partial \xi^{\beta}}{\partial x^{\lambda}}\eta_{\alpha\beta}$$

The two $\Gamma^{\lambda}_{\rho\nu}$ and $\Gamma^{\lambda}_{\rho\mu}$ are the affain connections.If we considering free falling particles the affain connection is a field that determines the gravitational force. Now using the symmetric property of the affain connection with the exchange of lower indices i.e. $\Gamma^{\lambda}_{\rho\mu} = \Gamma^{\lambda}_{\mu\rho}$. To solve eqn.(1.3.5) for the affain connections it is a matter of adding to eqn.(1.3.5) the same equation with ρ and μ by interchange and subtract the same equation with ρ and ν by interchanging. It becomes,

$$\frac{\partial g_{\mu\nu}}{\partial x^{\rho}}+\frac{\partial g_{\rho\nu}}{\partial x^{\mu}}-\frac{\partial g_{\mu\rho}}{\partial x^{\nu}} = \Gamma^{\lambda}_{\mu\rho}g_{\lambda\nu}+\Gamma^{\lambda}_{\rho\nu}g_{\lambda\mu}+\Gamma^{\lambda}_{\mu\rho}g_{\lambda\nu}+\Gamma^{\lambda}_{\mu\nu}g_{\lambda\rho}-\Gamma^{\lambda}_{\mu\nu}g_{\rho\lambda}-\Gamma^{\lambda}_{\nu\rho}g_{\lambda\mu} = 2\Gamma^{\lambda}_{\mu\rho}g_{\lambda\nu} \tag{1.3.6}$$

From the symmetry property of affain connection, $\Gamma^{\lambda}_{\mu\rho}$ and metric tensor, $g_{\lambda\nu}$, then

$$\Gamma^{\lambda}_{\mu\rho}g_{\lambda\nu} = \frac{1}{2}(\frac{\partial g_{\mu\nu}}{\partial x^{\rho}} + \frac{\partial g_{\rho\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\rho}}{\partial x^{\nu}}) \tag{1.3.7}$$

Now let us define a metric $g_{\tau\nu}$ as the inverse of $g^{\tau\nu}$ so that,

$$g^{\tau\nu}g_{\lambda\nu} = \delta^{\tau}_{\lambda} \tag{1.3.8}$$

Where δ^{τ}_{λ} is called the kroneker delta defined as $\delta^{\tau}_{\lambda} = 1$, for $\tau = \lambda$ and for else. Therefore applying δ^{τ}_{λ}. And multiply both side by $g^{\tau\nu}$, it shows

$$\Gamma^{\lambda}_{\mu\rho}g^{\tau\nu}g_{\lambda\nu} = \frac{1}{2}g^{\tau\nu}\left(\frac{\partial g_{\mu\nu}}{\partial x^{\rho}} + \frac{\partial g_{\rho\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\rho}}{\partial x^{\nu}}\right)$$

$$\Gamma^{\lambda}_{\mu\rho}\delta^{\tau}_{\lambda} = \frac{1}{2}g^{\tau\nu}\left(\frac{\partial g_{\mu\nu}}{\partial x^{\rho}} + \frac{\partial g_{\rho\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\rho}}{\partial x^{\nu}}\right)$$

Now equation(1.3.7) becomes,

$$\Gamma^{\tau}_{\mu\rho} = \frac{1}{2}g^{\tau\nu}\left(\frac{\partial g_{\mu\nu}}{\partial x^{\rho}} + \frac{\partial g_{\rho\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\rho}}{\partial x^{\nu}}\right) \tag{1.3.9}$$

This equation is tell us about affain connection to determine gravitational field.

1.3.3 Curvature Tensor

We want to construct a tensor out of the metric and its derivatives. If we use only $g_{\mu\nu}$, and its first derivative,then no new tensor can be constructed. For any point we can find a coordinate system in which the first derivatives of the metric vanished,so in this coordinate system the desired tensor must be equal to one of those that can be constructed out of the metric tensor along ($g_{\mu\nu}$ or, g^{ν} and so on) and since this equality between it must be true in all coordinate system [19]. The Simplest possibilities to construct a tensor out of the metric tensor and its first and second derivatives. To this,let us recall the transformation rule for the affain connection, passing from x^{μ} to a different system x'^{μ}, we defined that

$$\Gamma'^{\lambda}_{\mu\nu} = \frac{\partial x'^{\lambda}}{\partial \xi^{\alpha}}\frac{\partial^2 \xi^{\alpha}}{\partial x'^{\mu}\partial x'^{\nu}}$$

It can be written as follow,

$$\Gamma'^{\lambda}_{\mu\nu} = \frac{\partial x'^{\lambda}}{\partial x^{\rho}}\frac{\partial x^{\rho}}{\partial \xi^{\alpha}}\frac{\partial}{\partial x'^{\mu}}\left[\frac{\partial x^{\sigma}}{\partial x'^{\nu}}\frac{\partial \xi^{\alpha}}{\partial x^{\sigma}}\right] \tag{1.3.10}$$

Now taking partial differentiation for bracket terms, it shows

$$\Gamma'^{\lambda}_{\mu\nu} = \frac{\partial x'^{\lambda}}{\partial x^{\rho}}\frac{\partial x^{\rho}}{\partial \xi^{\alpha}}\left[\frac{\partial x^{\sigma}}{\partial x'^{\nu}}\frac{\partial x^{\tau}}{\partial x'^{\mu}}\frac{\partial^2 \xi^{\alpha}}{\partial x^{\tau}\partial x^{\sigma}} + \frac{\partial^2 x^{\sigma}}{\partial x'^{\mu}\partial x'^{\nu}}\frac{\partial \xi^{\alpha}}{\partial x^{\sigma}}\right]$$

and referring back to the notation for affain connection in equation (1.3.2) and kronecker delta,

$$\frac{\partial x^{\rho}}{\partial \xi^{\alpha}}\frac{\partial \xi^{\alpha}}{\partial x^{\sigma}} = \delta^{\rho}_{\sigma}$$

where, $\delta^{\rho}_{\sigma} = 1$, for, $\rho = \sigma$, otherwise zero

$$\Gamma'^{\lambda}_{\mu\nu} = \frac{\partial x'^{\lambda}}{\partial x^{\rho}}\frac{\partial x^{\tau}}{\partial x'^{\mu}}\frac{\partial x^{\sigma}}{\partial x'^{\nu}}\left[\frac{\partial x^{\rho}}{\partial \xi^{\alpha}}\frac{\partial^2 \xi^{\alpha}}{\partial x^{\tau}\partial x^{\sigma}}\right] + \frac{\partial x'^{\lambda}}{\partial x^{\rho}}\left[\frac{\partial x^{\rho}}{\partial \xi^{\alpha}}\frac{\partial \xi^{\alpha}}{\partial x^{\sigma}}\right]\left[\frac{\partial^2 x^{\sigma}}{\partial x'^{\mu}\partial x'^{\nu}}\right] \tag{1.3.11}$$

The transformation equation for affain connection becomes,

$$\Gamma'^{\lambda}_{\mu\nu} = \frac{\partial x'^{\lambda}}{\partial x^{\rho}}\frac{\partial x^{\tau}}{\partial x'^{\mu}}\frac{\partial x^{\sigma}}{\partial x'^{\nu}}\Gamma^{\rho}_{\tau\sigma} + \frac{\partial x'\lambda}{\partial x^{\rho}}\frac{\partial^2 x^{\rho}}{\partial x'^{\nu} x'^{\mu}} \tag{1.3.12}$$

The first term on the right side in equation (1.3.12) is what we would expect if $\Gamma^{\lambda}_{\mu\nu}$ were a tensor according to general coordinate transformation; the second term is inhomogeneous, and makes it a non-tensor. Now invert and inter change the primed and unprimed coordinate terms of equation (1.3.12). It shows,

$$\Gamma^{\lambda}_{\mu\nu} = \frac{\partial x^{\lambda}}{\partial x'\tau}\frac{\partial x'^{\rho}}{\partial x^{\mu}}\frac{\partial x'^{\sigma}}{\partial x^{\nu}}\Gamma'^{\tau}_{\rho\sigma} + \frac{\partial x^{\lambda}}{\partial x'^{\tau}}\frac{\partial^2 x'^{\tau}}{\partial x^{\nu}\partial x^{\mu}}$$

It is the inhomogeneous term on the right side that keeps $\Gamma^{\lambda}_{\mu\nu}$ from being a tensor, so let us isolate this term,

$$\frac{\partial^2 x'^{\tau}}{\partial x^{\nu}\partial x^{\mu}} = \frac{\partial x'\tau}{\partial x^{\lambda}}\Gamma^{\lambda}_{\mu\nu} - \frac{\partial x'^{\rho}}{\partial x^{\mu}}\frac{\partial x'^{\sigma}}{\partial x^{\nu}}\Gamma'^{\tau}_{\rho\sigma} \tag{1.3.13}$$

Now, take partial differentiation on both side of the above equation (1.3.13) with respect to x^k gives,

$$\frac{\partial}{\partial x^k}\left(\frac{\partial^2 x'^{\tau}}{\partial x^{\nu}\partial x^{\mu}}\right) = \frac{\partial}{\partial x^k}\left(\frac{\partial x'\tau}{\partial x^{\lambda}}\Gamma^{\lambda}_{\mu\nu} - \frac{\partial x'^{\rho}}{\partial x^{\mu}}\frac{\partial x'^{\sigma}}{\partial x^{\nu}}\Gamma'^{\tau}_{\rho\sigma}\right)$$

Take differentiation by part for right side,

$$\frac{\partial^3 x'^{\tau}}{\partial x^k\partial x^{\nu}\partial x^{\mu}} = \frac{\partial^2 x'^{\tau}}{\partial x^k\partial x^{\lambda}}\Gamma^{\lambda}_{\mu\nu} + \frac{\partial x'^{\tau}}{\partial x^{\lambda}}\frac{\partial \Gamma^{\lambda}_{\mu\nu}}{\partial x^k} - \frac{\partial^2 x'^{\rho}}{\partial x^k\partial x^{\mu}}\frac{\partial x'^{\sigma}}{\partial x^{\nu}}\Gamma'^{\tau}_{\rho\sigma} - \frac{\partial x'^{\rho}}{\partial x^{\mu}}\frac{\partial^2 x'^{\sigma}}{\partial x^k\partial x^{\nu}}\Gamma'^{\tau}_{\rho\sigma} - \frac{\partial x'^{\rho}}{\partial x^{\mu}}\frac{\partial x'^{\sigma}}{\partial x^{\nu}}\frac{\partial \Gamma'^{\tau}_{\rho\sigma}}{\partial x^k} \tag{1.3.14}$$

Now here, using the relations we develop in equation (1.3.13) one can be write the following possibilities,

$$\frac{\partial^2 x'^{\tau}}{\partial x^k \partial x^\lambda} = \frac{\partial x'^{\tau}}{\partial x^\eta}\Gamma^{\eta}_{k\lambda} - \frac{\partial x'^{\rho}}{\partial x^k}\frac{\partial x'^{\sigma}}{\partial x^\lambda}\Gamma'^{\tau}_{\rho\sigma}$$
$$\frac{\partial^2 x'^{\rho}}{\partial x^k \partial x^\mu} = \frac{\partial x'^{\rho}}{\partial x^\eta}\Gamma^{\eta}_{k\mu} - \frac{\partial x'^{\eta}}{\partial x^k}\frac{\partial x'^{\xi}}{\partial x^\mu}\Gamma'^{\rho}_{\eta\xi}$$
$$\frac{\partial^2 x'^{\sigma}}{\partial x^k \partial x^\nu} = \frac{\partial x'^{\sigma}}{\partial x^\eta}\Gamma^{\eta}_{k\nu} - \frac{\partial x'^{\eta}}{\partial x^k}\frac{\partial x'^{\xi}}{\partial x^\nu}\Gamma'^{\sigma}_{\eta\xi}$$

Now, substitute the above possible equation into equation (1.3.14). It shows,

$$\frac{\partial^3 x'^{\tau}}{\partial x^k \partial x^\nu \partial x^\mu} = \tag{1.3.15}$$

$$\left(\frac{\partial x'^{\tau}}{\partial x^\eta}\Gamma^{\eta}_{k\lambda} - \frac{\partial x'^{\rho}}{\partial x^k}\frac{\partial x'^{\sigma}}{\partial x^\lambda}\Gamma'^{\tau}_{\rho\sigma}\right)\Gamma^{\lambda}_{\mu\nu} + \frac{\partial x'^{\tau}}{\partial x^\lambda}\frac{\partial \Gamma^{\lambda}_{\mu\nu}}{\partial x^k}$$
$$-\frac{\partial x'^{\sigma}}{\partial x^\nu}\left(\frac{\partial x'^{\rho}}{\partial x^\eta}\Gamma^{\eta}_{k\mu} - \frac{\partial x'^{\eta}}{\partial x^k}\frac{\partial x'^{\xi}}{\partial x^\mu}\Gamma'^{\rho}_{\eta\xi}\right)\Gamma'^{\tau}_{\rho\sigma}$$
$$-\frac{\partial x'^{\rho}}{\partial x^\mu}\left(\frac{\partial x'^{\sigma}}{\partial x^\eta}\Gamma^{\eta}_{k\nu} - \frac{\partial x'^{\eta}}{\partial x^k}\frac{\partial x'^{\xi}}{\partial x^\nu}\Gamma'^{\sigma}_{\eta\xi}\right)\Gamma'^{\tau}_{\rho\sigma} - \frac{\partial x'^{\rho}}{\partial x^\mu}\frac{\partial x'^{\sigma}}{\partial x^\nu}\frac{\partial \Gamma'^{\tau}_{\rho\sigma}}{\partial x^k}$$

Now, collect similar terms and juggling indices a bit,

$$\frac{\partial^3 x'^{\tau}}{\partial x^k \partial x^\nu \partial x^\mu} = \frac{\partial x'^{\tau}}{\partial x^\lambda}\left(\frac{\partial \Gamma^{\lambda}_{\mu\nu}}{\partial x^k} + \Gamma^{\eta}_{\mu\nu}\Gamma^{\lambda}_{k\eta}\right) \tag{1.3.16}$$

$$-\frac{\partial x'^{\rho}}{\partial x^\mu}\frac{\partial x'^{\sigma}}{\partial x^\nu}\frac{\partial x'^{\eta}}{\partial x^k}\left(\frac{\partial \Gamma'^{\tau}_{\rho\sigma}}{\partial x'^{\eta}} - \Gamma'^{\tau}_{\rho\lambda}\Gamma'^{\lambda}_{\eta\sigma} - \Gamma'^{\tau}_{\lambda\sigma}\Gamma'^{\lambda}_{\eta\rho}\right)$$
$$-\Gamma'^{\tau}_{\rho\sigma}\frac{\partial x'^{\sigma}}{\partial x^\lambda}\left(\Gamma^{\lambda}_{\mu\nu}\frac{\partial x'^{\rho}}{\partial x^k} + \Gamma^{\lambda}_{k\nu}\frac{\partial x'^{\rho}}{\partial x^\mu} + \Gamma^{\lambda}_{k\mu}\frac{\partial x'^{\rho}}{\partial x^\nu}\right)$$

Now, after subtracting the same equation with ν and κ inter changing (i.e $\nu \to \kappa$), we find that all term involving the product of Γ and Γ' are drop out, leaving equation (1.3.16) as,

$$0 = \frac{\partial x'^{\tau}}{\partial x^\lambda}\left(\frac{\partial \Gamma^{\lambda}_{\mu\nu}}{\partial x^k} - \frac{\partial \Gamma^{\lambda}_{k\mu}}{\partial x^\nu}\Gamma^{\eta}_{\mu\nu}\Gamma^{\lambda}_{k\eta} - \Gamma^{\eta}_{k\mu}\Gamma^{\lambda}_{\mu\eta}\right) - \frac{\partial x'^{\rho}}{\partial x^\mu}\frac{\partial x'^{\sigma}}{\partial x^\nu}\frac{\partial x'^{\eta}}{\partial x^k}\left(\frac{\partial \Gamma'^{\tau}_{\rho\sigma}}{\partial x'^{\eta}} - \frac{\partial \Gamma'^{\tau}_{\rho\eta}}{\partial x'^{\sigma}} - \Gamma'^{\tau}_{\sigma\lambda}\Gamma'^{\lambda}_{\eta\rho} + \Gamma'^{\tau}_{\lambda\eta}\Gamma'^{\lambda}_{\sigma\rho}\right)$$

$$R'^{\tau}_{\rho\sigma\eta} = \frac{\partial x'^{\tau}}{\partial x^\lambda}\frac{\partial x^{\mu}}{\partial x'^{\rho}}\frac{\partial x^{\nu}}{\partial x'^{\sigma}}\frac{\partial x^{k}}{\partial x'^{\eta}}R^{\lambda}_{\mu\nu\kappa} \tag{1.3.17}$$

Now, let us define the term in the left side of in the bracket in equation(1.3.16) using the transformation rule of the curvature tensor notion in equation (1.3.17).

$$R^{\lambda}_{\mu\nu\kappa} = \Gamma^{\lambda}_{\mu\nu,\kappa} - \Gamma^{\lambda}_{\mu\kappa,\nu} + \Gamma^{\eta}_{\mu\nu}\Gamma^{\lambda}_{\kappa\eta} - \Gamma^{\eta}_{\mu\kappa}\Gamma^{\lambda}_{\nu\eta} \tag{1.3.18}$$

From equation (1.3.17) we can say that $R^{\lambda}_{\mu\nu\kappa}$ is a tensor, and it is called Riemann-christoffel curvature

The Riemann curvature measure how much the space would have been curved. It is derived directly from second order partial derivative of the metric tensor and plays an important role in determining the geometry of spacetime in GR and also it is physical significant to determine gravitational tidal force. In minkowski(flat) spacetime geometry genuinely the Riemann curvature vanished everywhere.

Its fully covariant form of Riemann curvature tensor can be written instead of $R^{\lambda}_{\mu\nu\kappa}$ as,

$$R_{\lambda\mu\nu\kappa} = g_{\lambda\sigma}R^{\sigma}_{\mu\nu\kappa}$$

Here, by referring back to equation (1.3.9) for affain connection, the Riemann curvature tensor can be written us directly in terms of general spacetime metric tensor,

$$R_{\lambda\mu\nu\kappa} = g_{\lambda\sigma}(\Gamma^{\sigma}_{\mu\nu,\kappa} - \Gamma^{\sigma}_{\mu\kappa,\nu} + \Gamma^{\eta}_{\mu\nu}\Gamma^{\sigma}_{\kappa\eta} - \Gamma^{\eta}_{\mu\kappa}\Gamma^{\sigma}_{\nu\eta})$$

$$R_{\lambda\mu\nu\kappa} = \frac{1}{2}g_{\lambda\sigma}\frac{\partial}{\partial x^{\kappa}}g^{\sigma\rho}\left(g_{\rho\mu,\nu} + g_{\rho\nu,\mu} - g_{\mu\nu,\rho}\right) - \tag{1.3.19}$$
$$\frac{1}{2}g_{\lambda\sigma}\frac{\partial}{\partial x^{\nu}}g^{\sigma\rho}\left(g_{\rho\mu,\kappa} + g_{\rho\kappa,\mu} - g_{\mu\kappa,\rho}\right) + g_{\lambda\sigma}\left(\Gamma^{\eta}_{\mu\nu}\Gamma^{\sigma}_{\kappa\eta} - \Gamma^{\eta}_{\mu\kappa}\Gamma^{\sigma}_{\nu\eta}\right)$$

Again referring back to equation (1.3.8) for kronicker delta notation,
$\delta^{\eta}_{\lambda} = 1, for \eta = \lambda$, else zero.and,

$$g_{\lambda\sigma}\frac{\partial}{\partial x^{\kappa}}g^{\sigma\rho} = -g^{\sigma\rho}\frac{\partial}{\partial x^{\kappa}}g_{\lambda\sigma}$$
$$= -g^{\sigma\rho}(\Gamma^{\eta}_{\kappa\lambda}g_{\eta\sigma} + \Gamma^{\eta}_{\kappa\sigma}g_{\eta\lambda})$$

After certain manipulation using kronicker delta notation we will obtain Riemann curvature tensor by canceling the most $\Gamma\Gamma$ terms appearing in the equation(1.3.19), so that,

$$R_{\lambda\mu\nu\kappa} = \frac{1}{2}\left[g_{\lambda\nu,\kappa\mu} - g_{\mu\nu,\kappa\lambda} - g_{\lambda\kappa,\nu\mu} + g_{\mu\kappa,\nu\lambda}\right] + g_{\eta\sigma}\left[\Gamma^{\eta}_{\nu\lambda}\Gamma^{\sigma}_{\mu\kappa} - \Gamma^{\eta}_{\kappa\lambda}\Gamma^{\sigma}_{\mu\nu}\right] \tag{1.3.20}$$

Therefore, equation (1.3.20) is called covariant form of Riemann curvature tensor.

The algebraic property of the covariant curvature tensor are;

1. symmetry in the first two pair of indices

$$R_{\lambda\mu\nu\kappa} = R_{\nu\kappa\lambda\mu}$$

2. Antisymmetry in first pair of indices

$$R_{\lambda\mu\nu\kappa} = -R_{\mu\lambda\nu\kappa}$$

3. Antisymmetry in second pair of indices

$$R_{\lambda\mu\nu\kappa} = -R_{\lambda\mu\kappa\nu}$$

4. Cyclic permutation symmetry(first Bianchi identity)

$$R_{\lambda\mu\nu\kappa} + R_{\lambda\kappa\mu\nu} + R_{\lambda\nu\kappa\mu} = 0$$

Therefore, Riemann curvature tensor will have 20 total number of independent components for situation having a 4-dimensional spacetime geometry[19]. So that a general formulation for computing the total number of independent component in a given N-dimensional spacetime geometry will have;

$$\frac{N^2(N^2-1)}{12}$$

Ricci Tensor and Ricci Scalar

Ricci tensor: is a component of Einstein field equation obtained from a Riemann curvature tensor by contracting over two indices.

$$R_{\mu\kappa} = R^{\lambda}_{\mu\lambda\kappa} = g^{\lambda\nu} R_{\lambda\mu\nu\kappa} \tag{1.3.21}$$

Using equation (1.3.18) and equation (1.3.21), shows that,

$$R_{\mu\kappa} = \Gamma^{\lambda}_{\mu\lambda,\kappa} - \Gamma^{\lambda}_{\mu\kappa,\lambda} + \Gamma^{\eta}_{\mu\lambda}\Gamma^{\lambda}_{\kappa\eta} - \Gamma^{\eta}_{\mu\kappa}\Gamma^{\lambda}_{\lambda\eta}$$

And also one can be write in other form as, using equation (1.3.20) and equation (1.3.21) as,

$$R_{\mu\kappa} = \frac{1}{2}g^{\lambda\nu}\left[g_{\lambda\nu,\kappa\mu} - g_{\mu\nu,\kappa\lambda} - g_{\lambda\kappa,\nu\mu} + g_{\mu\kappa,\nu\lambda}\right] + g^{\lambda\nu}\left[\Gamma^{\eta}_{\nu\lambda}\Gamma^{\sigma}_{\mu\kappa} + \Gamma^{\eta}_{\kappa\lambda}\Gamma^{\sigma}_{\mu\nu}\right]$$

Using the property of Riemann curvature tensor, we can find the property of Ricci tensor its symmetry property,

$$R_{\mu\kappa} = R_{\kappa\mu}$$

Ricci Scalar:- is obtained by further contraction of the remaining two indices of the Ricci tensor with the contra-variant components of the metric, is also called curvature scaler.

$$R = g^{\mu\kappa}R_{\mu\kappa}$$

$$= g^{\lambda\nu}g^{\mu\kappa}R_{\lambda\mu\nu\kappa}$$

$$= R^{\mu}_{\mu}$$

Einstein Field tensor:- can be construct from Riemann curvature tensor and metric

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$$

Where $G_{\mu\nu}$ is a linear combinations of first and second order partial differential equation of metric. Since Ricci tensor and metric tensor are symmetric, so that Einstein field tensor also symmetric.

$$G_{\mu\nu} = G_{\nu\mu}$$

1.3.4 The Bianchi Identities

The Riemann curvature tensor obeys important differential identities, in addition to the algebraic identities. These can be most easily derived at a given point x, by adopting a locally inertial coordinate system x^{η}, in which a term containing Γ, in the Riemann curvature tensor, vanished but not its derivative.
Than at a given point x equation (1.3.20) becomes

$$R_{\lambda\mu\nu\kappa,\eta} = \frac{1}{2}\frac{\partial}{\partial x^{\eta}}\left(g_{\lambda\nu;\kappa\mu} - g_{\mu\nu;\kappa\lambda} - g_{\lambda\kappa;\mu\nu} - g_{\mu\kappa;\nu\lambda}\right) \tag{1.3.22}$$

All other terms being at least of first order in Γ. By permuting ν, κ and η cyclically. We obtain the Bianchi identities.

$$R_{\lambda\mu\nu\kappa;\eta} + R_{\lambda\mu\eta\nu;\kappa} + R_{\lambda\mu\kappa\eta;\nu} = 0 \tag{1.3.23}$$

Therefore, these equation are manifestly general covarient, so since they hold in general inertial system. We shall be particularly concerned with the contraction form of equation (1.3.23),recalling covarient derivatives of $g\lambda\nu$ also vanished, We find on contraction of λ with ν

$$g^{\lambda\nu}(R_{\lambda\mu\nu\kappa;\eta} + R_{\lambda\mu\eta\nu;\kappa} + R_{\lambda\mu\kappa\eta;\nu}) = 0$$

$$R_{\mu\kappa;\eta} - R_{\mu\eta;\kappa} + R^{\nu}_{\mu\kappa\eta;\nu} = 0$$

Contraction one more μ with κ or using $g^{\mu\kappa}$, gives as

$$R_{;\eta} - R^{\mu}_{\eta;\mu} - R^{\nu}_{\eta;\nu} = 0$$

it becomes,

$$\left(R^{\mu}_{\eta} - \frac{1}{2}\delta^{\mu}_{\eta}R\right); \mu = 0$$

an equivalent but more familiar form is,

$$\left(R^{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right); \mu = 0$$

1.3.5 Energy-Momentum

A tensor quantities in physics that describe the density, flux of energy and momentum in 4-dimensional space and is a conserved quantities. Sometimes generalized to stress-energy denoted by $T^{\mu\nu}$. The energy-momentum tensor is the source of the gravitational field in the Einstein field equations of general relativity, just as mass density is the source of such a field in Newtonian gravity [19]. A great many macroscopic physical systems, including perhaps the universe itself, may be approximately regarded as a perfect fluid. A perfect fluid is defined as having at each point a velocity V, such that an observer moving with this velocity sees the fluid around him as isotropic. This will be the cases if the mean free path between collision is small compared with the length used by the observe. Now we shall translate the above definition of a perfect fluid in to a statement about the energy-momentum tensor. First supposed that we are in a frame of reference (distinguished by tide) in which the fluid is at rest at some particular position and time. At this space-time point, the perfect fluid hypothesis tells us that energy-momentum tensor take the form characteristic of spherical symmetry:

$$\tilde{T}^{ij} = P\delta_{ij}$$

$$\tilde{T}^{i0} = \tilde{T}^{0i} = 0$$

$$\tilde{T}^{00} = \rho$$

The coefficient ρ and p are called proper energy density and pressure, respectively. Now go into a reference frame at rest in the laboratory, and suppose that the fluid in this frame appear to be moving with velocity v. The connection between the comoving coordinate $\tilde{x}^{\beta}$ and the lab coordinate x^{α} is than,

$$x^{\alpha} = \Lambda^{\alpha}_{\beta}(v)\tilde{x}^{\beta}$$

with $\Lambda^{\alpha}_{\beta}(v)$ "boost" defined [19]. But $T^{\alpha\beta}$ is tensor, so in the lab frame it is

$$T^{\alpha\beta} = \Lambda^{\alpha}_{\gamma}(v)\Lambda^{\beta}_{\delta}(v)\tilde{T}^{\gamma\delta}$$

or explicitly

$$T^{ij} = p\delta_{ij} + (p+\rho)\frac{v_i v_j}{1-v^2}$$

$$T^{i0} = (\rho + p)\frac{v_i}{1-v^2}$$

$$T^{00} = \frac{\rho + pv^2}{1-v^2}$$

Now combine the above T^{ij},T^{i0} and T^{00} in to single equation to check that $T^{\alpha\beta}$ is a tensor.so that,

$$T^{\alpha\beta} = p\eta_{\alpha\beta} + (\rho + p)U^{\alpha}U^{\beta}$$

In the case the absence of gravity the energy-momentum tensor for perfect fluid similar to the above equation.

$$T^{\alpha\beta} = p\eta_{\alpha\beta} + (\rho + p)U^{\alpha}U^{\beta} \tag{1.3.24}$$

Since U^{α} is a 4-velocity vectors, defined by

$$U^{\alpha} = \frac{dx^{\alpha}}{d\tau} \tag{1.3.25}$$

$$U^{\beta} = \frac{dx^{\beta}}{d\tau}$$

For line element

$$d\tau^2 = \eta_{\alpha\beta}dx^{\alpha}dx^{\beta} \tag{1.3.26}$$

Now, using equation (1.3.25) and equation (1.3.26) we have,

$$\eta_{\alpha\beta}U^{\alpha}U^{\beta} = 1$$

Now here using the above consequently equation, the energy-momentum of perfect fluid than take the following form in its rest frame.

$$T^{\alpha\beta} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \tag{1.3.27}$$

In general $T^{\alpha\beta}$ is a 4-dimensional symmetric tensor. It is strictly the description of energy - momentum have been useful in a practical applications to define the structure of stellar and the study of cosmology.

In the presence of gravity, energy - momentum tensor for perfect fluid given by from equation (1.3.24) becomes,

$$T^{\mu\nu} = Pg_{\mu\nu} + (P + \rho)\, U^{\mu}U^{\nu} \tag{1.3.28}$$

is also symmetric $T^{\mu\nu} = T^{\nu\mu}$, and from Normalization conditions we can arrive to,

$$g_{\mu\nu}U^{\nu}U^{\mu} = -1$$

Moreover, the conservation of energy - momentum tensor can be defined by,

$$T^{\mu\nu};_{\mu} = 0$$

1.4 Einstein Field Equation

The stage is now set for deriving and understanding Einstein field equation. In newtonian gravity, the rest mass(matter) generate gravitational effect. Moreover gravity can only exist where there exist a matter around space. However in GR, we recognized that the rest mass(matter) is just one form of energy, and that the mass and energy are equivalent. Therefore, we should expect that in GR all source of both energy and momentum contributions to generate space-time curvature. This mean that in GR, the energy-momentum tensor is the source for space-time curvature, in the same sense that the mass density ρ is the source for the gravitational potential (ϕ) [19]. This motivate Einstein to make a conclusion that gravity is not only create by the presence of matter, it is in fact the product of the presence of energy. therefore, GR must present appropriate analogues of the two parts of dynamical picture:-(1) how particles moves in response to gravity; and (2) how particles generate gravitational effect. The first part was answered when we derived the geodesic equation as the analogue of the Newton second Law.The second part required finding of the analogue of the poisson equation [11].

$$\nabla^2\phi(x) = 4\pi G\rho(x)$$

Which specifies how matter curves space - time.

In a weak static field produced by non-relativistic mass density ρ, the time - time component of the metric tensor is approximately given by,

$$g_{00} \simeq -(1+2\phi) \tag{1.4.1}$$

Now , here ϕ is the Newton potential; determined by poisson equation, that is

$$\nabla^2\phi = 4\pi G\rho \tag{1.4.2}$$

Further more, the energy density T_{00} for non-relativistic matter is just equal its mass density.

$$T_{00} = \rho \tag{1.4.3}$$

by taking both side ∇^2 in equation (1.4.1), it shows that

$$\nabla^2 g_{00} = -8\pi G\rho$$

using equation (1.4.3) the above equation becomes,

$$\nabla^2 g_{00} = -8\pi G T_{00} \tag{1.4.4}$$

This field equation is suppose to hold for which static field generated by non-relativistic matter, and is not even lorentz invariant as it stand. However, equation (1.4.4) lead us to guess that the weak - field equation for a general distribution of matter $T_{\alpha\beta}$ of energy and momentum take the form,

$$G_{\alpha\beta} = \nabla^2 g_{\alpha\beta}$$

$$G_{\alpha\beta} = 8\pi G T_{\alpha\beta} \tag{1.4.5}$$

Where $G_{\alpha\beta}$ is a linear combination of the metric and its first and second derivative. It follows than from the principle of equivalence that the equations which govern gravitational field of arbitrary strength take the form;

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{1.4.6}$$

Where $G_{\mu\nu}$ is a tensor which reduce to $G_{\alpha\beta}$ for weak field and formed from the metric tensor of it's derivatives. Now let review some property of left side of equation (1.4.6).

1. by definition, $G_{\mu\nu}$ is a tensor

2. By assumption, $G_{\mu\nu}$ consists only of terms with N=2 derivative of the metric ;that is $G_{\mu\nu}$ contain only terms that are either linear in the second derivative or quadratic in the first derivative of the metric.

3. Since $T_{\mu\nu}$ is symmetric, $G_{\mu\nu}$

4. Since $T_{\mu\nu}$ is conserved (in the sense of covariant differentiation) so in $G_{\mu\nu}$:

$$G^{\mu}_{\nu;\mu} = 0$$

5. For a weak stationary field produce by non-relativistic matter the 00 component equation (1.4.5) must reduce to (1.4.6), so in this limit;

$$G_{00} \simeq \nabla^2 g_{00} \tag{1.4.7}$$

The property are all we will need to find $G_{\mu\nu}$. The most general way of contracting a field satisfying (1) and (2) is by contracting of the curvature tensor. Hence (1) and (2) required $G_{\mu\nu}$ to take the form ;

$$G_{\mu\nu} = C_1 R_{\mu\nu} + C_2 g_{\mu\nu} R \tag{1.4.8}$$

Where C_1 and C_2 are constant. This is automatically symmetric so property (3) satisfy. Now using the above relations it follow that,

$$g^{\mu\tau} G_{\mu\nu} = g^{\mu\tau} C_1 R_{\mu\nu} + C_2 g^{\mu\tau} g_{\mu\nu} R$$

than,

$$G^{\tau}_{\nu} = C_1 R^{\tau}_{\nu} + C_2 \delta^{\tau}_{\nu} R \tag{1.4.9}$$

Now take the covariant divergence of equation (1.4.9)

$$G^{\tau}_{\nu;\tau} = C_1 R^{\tau}_{\nu;\tau} + C_2 \delta^{\tau}_{\nu} R_{;\tau} \tag{1.4.10}$$

Since $\delta^{\tau}_{\nu} \neq 0$, for $\tau = \nu$ and use $R^{\tau}_{\nu} = \frac{1}{2}\delta^{\tau}_{\nu} R_{;\tau}$

$$G^{\tau}_{\nu;\tau} = \left(\frac{1}{2}C_1 + C_2 \delta^{\tau}_{\nu}\right) R_{;\tau}$$

It becomes

$$G^{\tau}_{\nu;\tau} = \left(\frac{1}{2}C_1 + C_2\right) R_{;\tau} \tag{1.4.11}$$

Using property (4) for the conservation of $G_{\mu\nu}$, we have $G^{\tau}_{\nu;\tau} = 0$ and than the above equation becomes,

$$(\frac{1}{2}C_1 + C_2) R_{;\tau} = 0$$

The is true if and only if,

$$C_2 = -\frac{1}{2}C_1$$

So,equation (1.4.8) becomes,

$$G_{\mu\nu} = C_1 (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) \tag{1.4.12}$$

To fix the constant C_1 a non-relativistic system always has $|T_{ij}| \ll |T_{00}|$, so we are concerned here with a case where $|G_{ij}| \ll |G_{00}|$,

$$|G_{ij}| \cong 0$$

So that equation (1.4.12) becomes,

$$R_{ij} \cong \frac{1}{2} g_{ij} R$$

For weak field limit, $g_{\alpha\beta} \cong \eta_{\alpha\beta}$, moreover $g_{ij} \cong \eta_{ij}$, the above equation becomes ,

$$R_{ij} \cong \frac{1}{2} \eta_{ij} R$$

Now using the property of metric tensor i.e $\eta_{ij} = 1$, for i = j = 1,2,3 and it brings the above equation,

$$R_{ij} \cong \sum_{i=j}^{3} \frac{1}{2} \eta_{ij} R$$

$$R_{ij} \cong \frac{3}{2} R$$

In general

$$R_{kk} \cong \frac{3}{2} R$$

The curvature scalar therefore given by,

$$R \cong R_{kk} - R_{00} \cong \frac{3}{2} R - R_{00}$$

$$R \cong 2R_{00} \tag{1.4.13}$$

Now using the condition for weak field limit, we see that equation (1.4.12) becomes for 00 component,

$$G_{00} \cong C_1 (R_{00} - \frac{1}{2} g_{00} R)$$

$$G_{00} \cong C_1 (R_{00} - \frac{1}{2} g_{00} (2R_{00}))$$

The above equation becomes,

$$G_{00} = C_1 (2R_{00}) \tag{1.4.14}$$

To calculate R_{00} for weak field limit, we use the linear part of curvature tensor by ignoring $\Gamma\Gamma$ part. By recalling back equation (1.3.20),

$$R_{\lambda\mu\nu\kappa} = \frac{1}{2} \left(g_{\lambda\nu;\kappa\mu} - g_{\mu\nu;\kappa\lambda} - g_{\lambda\kappa\nu\mu} + g_{\mu\kappa;\nu\lambda} \right)$$

Since the field is static all time derivative vanished and the components becomes,

$$R_{0000} \cong 0$$

and

$$R_{i0j0} \cong \frac{1}{2} g_{00;ij} \cong \frac{1}{2} \nabla^2 g_{00}$$

where $g_{00;ij} = \nabla^2 g_{00}$
The above equation for the non vanished term of curvature tensor can be written as,

$$R_{\lambda 0 \nu 0} \cong \frac{1}{2} g_{00;\lambda\nu} \cong \nabla_2 g_{00} \qquad (1.4.15)$$

Now by contracting over two indices of the non-vanishing curvature in the above equation (1.4.15)

$$R_{00} \cong g^{\lambda\nu} R_{\lambda 0 \nu 0}$$

$$R_{00} \cong R_{i0j0} - R_{0000} \qquad (1.4.16)$$

Insert the equation developed from equation (1.4.15) and equation (1.4.16) in to equation (1.4.14) becomes,

$$G_{00} \cong 2C_1(R_{i0j0} - R_{0000}) \cong 2C_1(\frac{1}{2} \nabla^2 g_{00})$$

it becomes,

$$G_{00} = C_1 \nabla^2 g_{00} \qquad (1.4.17)$$

Finally, compare equation (1.4.18), with equation (1.4.7) the relations satisfied if and only if $C_1 = 1$, therefore we can arrived to ,

$$G_{00} = \nabla^2 g_{00}$$

So, from equation (1.4.6) and equation (1.4.12) for $G_{\mu\nu}$ can be written as,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} \qquad (1.4.18)$$

The Einstein field equations (EFE; also known as Einstein's equations) is the set of 10 equations in Albert Einstein's general theory of relativity that describes the fundamental interaction of gravitation as a result of spacetime being curved by mass and energy

1.5 Introducing Of Cosmology Constant into Einstein Field Equation

After completing his theory of GR, Einstein was interested to find a static solution of his field equations with the idea in cooperated with Mach's principle, [19]. As the consequence, Einstein himself introduced, "Cosmological Considerations in the General Theory of Relativity" in 1917, is rightly regarded as the first step in modern theoretical cosmology. Einstein included the cosmological constant as a term in his field equations for general relativity because he was dissatisfied that otherwise his equations did not allow, apparently, for a static universe: gravity would cause a universe that was initially at dynamic equilibrium to contract. To counteract this possibility, Einstein added the cosmological constant [7]. Perhaps the most striking novelty introduced by Einstein was the very idea of a cosmological model, an exact solution to his new gravitational field equations that gives a global description of the universe in its entirety. Moreover, Einstein also point out that the introduced a positive cosmological constant with the belief of constructing a static solution. The idea is that, the constant introduces is a repulsive force which can counterbalance the attractive force of gravity leading to the "static Einstein universe" [19]. Therefore the modified Einstein's field equations with the cosmological constant is,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{1.5.1}$$

where Λ is a new free parameter, the cosmological constant. Indeed, the left-hand side of equation (1.5.1) is the most general local, coordinate-invariant, divergence less, symmetric, two-index tensor we can construct solely from the metric and its first and second derivatives.where R and g describe the structure of spacetime, T pertains to matter and energy affecting that structure. When Λ is zero, this reduces to the original field equation of general relativity. When T is zero, the field equation describes empty space (the vacuum). In fact, adding the cosmological constant to Einstein's equations does not lead to a static universe at equilibrium because the equilibrium is unstable: if the universe expands slightly, then the expansion releases vacuum energy, which causes yet more expansion. Schwarzschild de-Sitter at the same year in 1917 presented exact solutions to "static Einstein universe", which had both static and dynamic character, that allows a redshift-distance relation [19]. The de-Sitter's prediction is considered as the first step towards the theoretical discovery of expanding universe. Recent observational data and results in modern cosmology revealed that the dark energy which is described in majority by the cosmological constant is of dominant importance in the dynamics of our Universe [7]. Measurements conducted by Wilkinson Microwave Anisotropic Probe (WMAP) indicate that almost three fourth of total mass-energy in the Universe is Dark Energy and the leading theory of dark energy is based on the cosmological constant characterized by repulsive pressure which was introduced by Einstein in 1917 to obtain a static cosmological model [27]. Later on Zeldovich interpreted this quantity physically as a vacuum energy of quantum fluctuation whose size is of the order of $\sim 3 * 10^{-56} cm^{-2}$.

Einstein's introducing cosmological constant inspired a small group of theorists to study cosmological models using his new gravitational theory, and the ideas developed during these early days have been a crucial part of cosmology ever since [19]. We can understated the physical properties of these models and their possible connections to astronomical observations was the central problem facing relativistic cosmology in the 20s [27]. By the early 30s, there was widespread consensus that a class of models describing the expanding universe was in at least rough agreement with astronomical observations. But this achievement was certainly not what Einstein had in mind in introducing the first cosmological model [19]. Einstein's was not simply a straightforward application of his new theory to an area where one would expect the greatest differences from Newtonian theory. Instead, Einstein's foray into cosmology was a final attempt to guarantee that a version of "Mach's principle" holds [29]. The Mach idea that inertia is due only to matter shaped Einstein's work on a new theory of gravity, but he soon realized that this might not hold in his "final" theory on November 1915. The 1917 paper should thus be read as part of Einstein's ongoing struggle to clarify the conceptual foundations of his new theory and the role of Mach's principle, rather than treating it only as the first step in relativistic cosmology. Einstein's work in cosmology illustrates the payoff of focusing on foundational questions such as the status of Mach's principle. In the course of an exchange with the Dutch astronomer Willem De Sitter, Einstein came to insist that on the largest scales the universe should not evolve over time-in other words, that it is static. Although he originally treated this as only a simplifying assumption, Einstein later brandished the requirement that any reasonable solution must be static to rule out an anti-Machian cosmological model discovered by De Sitter. Thus Einstein's concern with Mach's principle led him to introduce the first cosmological model, but he was also blind to the more dramatic result that his new gravitational theory naturally leads to dynamical models [2]. Even when expanding universe models had been described by Alexander Freidmann in 1922 constructed a matter dominated expanding universe without a cosmological constant. Then, the possibility that the universe is expanding led Einstein to abandoned,with Einstein calling it the "biggest blunder [he] ever made", the idea of a static universe including the cosmological constant, and Georges Lemaître constructed an expanding model which originated from such an asymptotically static state ("static Einstein universe") in the distant past [19]. However, Einstein rejected them as physically unreasonable. Einstein's work in cosmology was also not informed by a thorough understanding of contemporary empirical work. The shift in theoretical cosmology brought about by Einstein's work occurred at the same time as a shift in the observational astronomer's understanding of the nature of spiral nebulae and large scale structure of the cosmos. Others with greater knowledge of contemporary astrophysics, including Arthur Eddington, De Sitter, Lemaître, and Richard Tolman, made many of the more productive contributions to relativistic cosmology.

Chapter 2

General Relativity Magnetohydrodynamic In Schwarzschild-de Sitter Space-time

2.1 Introduction

In this thesis, we apply magnetohydrodynamics (MHD), which is an extension of fluid dynamics (i.e., additional terms are added into the equations of fluid dynamics), to study phenomena in the universe. Magnetohydrodynamics (MHD) studies the dynamics of an electrically conducting fluid under the influence of a magnetic field. If there is no magnetic field present, the problem reduces to traditional fluid dynamics. However, in most astrophysical settings, the fluids are highly conductive and observed to be magnetized [21]. Magnetohydrodynamic shock waves in the near relativistic regime have been obtained with the Columbia University Plasma Laboratory Electromagnetic High-Energy Shock Tube (Gross, 1971; Taussig, 1973). The theoretical analysis of these experiments is rather difficult and has been obtained by numerical simulation in a nonrelativistic framework (Liberman and Velikovich, 1985) [13]. A small increase in the attained speed would require a proper relativistic magneto-fluid dynamical calculation. The simplest model for a relativistic medium is that of a relativistic fluid. When the medium interacts electromagnetically and is electrically highly conducting the simplest description is in terms of relativistic magneto-fluid dynamics. From the mathematical viewpoint relativistic fluid dynamics (RFD) and magneto-fluid dynamics (RMFD) have mainly been treated in the framework of general relativity, that is, as describing possible sources of the gravitational field. This means that both the RFD and RMFD equations have been studied in conjunction with Einstein's equations. General Relativity (GR) is a beautiful scheme for describing the gravitational field. This theory is believed to apply to all forms of interactions, especially between large scale gravitational structures. It has been proven that black holes exist on the basis for to study the effects they exert on their surroundings. They greatly affect the surrounding plasma medium with their enormous gravitational fields. Since all compact objects have strong gravitational fields near their surfaces [1], it is important to study the general relativistic effects on physical processes, like electromagnetic processes taking place in their vicinity. The GRMHD equations help us to study stationary configurations and dynamic evolution of a conducting fluid in a magnetosphere. Moreover, a relativistic jets flow have been discovered in several different classes of objects including AGN (pearson and Zensus 1987; Bitetta, Sparks, and Macchetto 1999), micro-quarsars (mirabel and Rodriquez 1994; Tingay et al. 1995) [10], and gamma-ray bursts(Kulkarni et al. 1999). It is believed that a rapidly spinning black hole exist at the center of each of these objects and that the violent phenomena that occur near the hole is responsible for the jets formation. Dynamics of magnetized plasma around black hole is one of the most promising candidate of the process exist [8]. In order to understand the basic physics of dynamics of magnetize plasma fluid around SMBH hosted by AGN .Now here, we are going to develop a analytical method of a GRMHD equation demanded in the background of SdS geometry. The method is based on the GR formulation of the law of particle number conservation, energy-momentum conservation, max-well equation and ohm's law with zero electrical resistance (ideal MHD condition) on curved space-time. It is concerned with the dynamic of relativistic, electronically-conducting fluid (plasma) in the presence of a magnetic field. Here, we concentrate on purely ideal GRMHD, by neglecting the presence of viscosity and heat conduction in the limit of infinite conductivity,i.e., the fluid is assumed to be a perfect conductor.

2.2 Metrics

The metric ($g_{\mu\nu}$) with cosmology constant is a geometric tool that relates distances in space-time, a kind of generalized pythagorean theorem where the time coordinate is included as well. The underlying physics is more important than the relative coordinates, so all equations are written in the invariant language of tensors, or multi-indexed objects. The Einstein summation convention shortens the notation by assuming an implied sum over repeated indices.In fact, the Lorentzian form of the metric, or the (-+++) signature asymmetry of time with space, helps to explain the presence of a gravitational force in curved spacetime. With this in mind, the Schwarzschild-de sitter metric for a spherically symmetric vacuum spacetime (valid outside a star or black hole),in coordinates (t, r, θ, ϕ).The general metric with cosmology constant can be defined by,

$$g_{\mu\nu} = \begin{pmatrix} -e^{2\Phi} & 0 & 0 & 0 \\ 0 & (1-\frac{2Gm}{r}-\frac{r^2\Lambda}{3})^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 sin^2\theta \end{pmatrix} \tag{2.2.1}$$

Moreover, $g^{\mu\nu}g_{\mu\nu} = 1$, so that inverse line element becomes

$$g^{\mu\nu} = \begin{pmatrix} -e^{-2\Phi} & 0 & 0 & 0 \\ 0 & (1-\frac{2Gm}{r}-\frac{r^2\Lambda}{3}) & 0 & 0 \\ 0 & 0 & r^{-2} & 0 \\ 0 & 0 & 0 & r^{-2} sin^{-2}\theta \end{pmatrix}$$

2.3 Schwarzschild-de sitter metric

It describes the, spherically symmetric vacuum solution to the Einstein field equations with a positive cosmological constant, Since it is a static cosmological model mode of isotopic and homogenous by removing all matter from the universe. The resulting expanding universe had the density of matter will eventual becomes negligible and the expanding universe will approach to the schwarzschild-de sitter universe. For a vanishing cosmological constant the Schwarzschild solution follows, for vanishing matter the metric gives the de Sitter cosmology.

$$ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu}$$

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\varphi\varphi}d\phi^2$$

Using the components of the metric tensor in equ(2.2.1), the above equation become,

$$ds^2 = -(1-\frac{2GM}{r}-\frac{r^2\Lambda}{3})dt^2 + (1-\frac{2GM}{r}-\frac{r^2\Lambda}{3})^{-1}dr^2 + r^2d\theta^2 + r^2sin^2\theta d\phi^2 \tag{2.3.1}$$

we can re-write this in the form of some potential, Φ as,

$$ds^2 = -e^{2\Phi}dt^2 + \left(1 - \frac{2GM}{r} - \frac{r^2\Lambda}{3}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2$$

Due to spherical symmetry and no time dependence, we only have the r component of the metric and therefore the pressure P and the energy-density ϵ will only depend on this component, and thus a calculation of this component is required, let start calculating using equation (SdS) and equation (affine). Then the non-vanished christopher symbol Γ using equation (1.3.9)

$$\Gamma^r_{tt} = \left(1 - 2\frac{m}{r}r^2\frac{\lambda}{3}\right)\Phi' e^{2\Phi}, \qquad \Gamma^r_{rr} = \frac{\frac{\lambda r}{3} - \frac{m}{r^2} + r\frac{m'}{r^2}}{1 - 2\frac{m}{r} - \frac{r^2\lambda}{3}}$$

$$\Gamma^r_{\theta\theta} = -r(1 - 2\frac{m}{r} - \frac{r^2\lambda}{3})$$

$$\Gamma^r_{\phi\phi} = -r\left(1 - 2\frac{m}{r} - \frac{r^2\lambda}{3}\right)\sin^2\theta, \qquad \Gamma^t_{rt} = \Phi'$$

$$\Gamma^\phi_{\theta\phi} = \cot\theta,$$

$$\Gamma^\theta_{\phi\phi} = -\cos\theta\sin\theta, \qquad \Gamma^\phi_{r\phi} = \Gamma^\theta_{r\theta} = \frac{1}{r}$$

2.3.1 Ricci tensor and Ricci Scalar

Now using the mathematical formation designed in the previous chapter for Ricci tensor and Ricci scalar, it is possible to calculate the non-vanishing components for each of them are $(R_{tt}, R_{rr}, R_{\theta\theta}, R_{\phi\phi})$. Now by using,

$$R_{\mu\kappa} = \Gamma^\lambda_{\mu\lambda,\kappa} - \Gamma^\lambda_{\mu\kappa,\lambda} + \Gamma^\eta_{\mu\lambda}\Gamma^\lambda_{\kappa\eta} - \Gamma^\eta_{\mu\kappa}\Gamma^\lambda_{\lambda\eta}$$

$$\begin{aligned} R_{tt} &= \Gamma^\lambda_{tt,r} + \Gamma^\lambda_{t\lambda,t} + \Gamma^t_{tt}\Gamma^\lambda_{t\lambda} - \Gamma^t_{t\lambda}\Gamma^\lambda_{tt} \\ &+\Gamma^\lambda_{tt,r} + \Gamma^\lambda_{t\lambda,t} + \Gamma^r_{tt}\Gamma^\lambda_{r\lambda} - \Gamma^r_{t\lambda}\Gamma^\lambda_{rt} \\ &+\Gamma^\lambda_{tt,r} + \Gamma^\lambda_{t\lambda,t} + \Gamma^\theta_{tt}\Gamma^\lambda_{\theta\lambda} - \Gamma^\theta_{t\lambda}\Gamma^\lambda_{\theta t} \\ &+\Gamma^\lambda_{tt,r} + \Gamma^\lambda_{t\lambda,t} + \Gamma^\phi_{tt}\Gamma^\lambda_{\phi\lambda} - \Gamma^\phi_{t\lambda}\Gamma^\lambda_{\phi t} \end{aligned} \tag{2.3.2}$$

Now, after a certain mathematics manipulation for λ = t, r, θ, ϕ. Then

$$R_{tt} = \Gamma^r_{tt;r} + \Gamma^r_{tt}[\Gamma^t_{rt} + \Gamma^r_{rr} + \Gamma^\theta_{r\theta} + \Gamma^\phi_{r\phi}] - [\Gamma^t_{tr}\Gamma^r_{tt} + \Gamma^r_{tt}\Gamma^t_{rt}]$$

For simplicity $K = (1 - \frac{2}{m}r - \frac{\Lambda r^2}{3})$
and insert the non-vanishing chistoffel symbol then, R_{tt} becomes

$$R_{tt} = \frac{d}{dr}\left(K\Phi^{'}e^{2\Phi}\right) + K\Phi' e^{2\Phi}\left[\Phi' + (\frac{\frac{\Lambda r}{3} - \frac{m}{r^2} + \frac{m'r}{r^2}}{1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}}) + \frac{2}{r}\right] - \left[2K\Phi'^2 e^{2\Phi}\right]$$

$$= k'\Phi' e^{2\Phi} + K\Phi\text{"} e^{2\Phi} + K\Phi'(e^{2\Phi})' + K\Phi' e^{2\Phi}[\Phi' + (\frac{\frac{\Lambda r}{3} - \frac{m}{r^2} + \frac{m'r}{r^2}}{1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}}) + \frac{2}{r}] - [2K\Phi'^2 e^{2\Phi}]$$

Use, $\quad (e^{2\phi})' = 2\phi' e^{2\phi} \qquad k' = 2(\frac{m - r'm}{r^2} - \frac{\lambda r}{3})$, then

$$= 2\phi' e^{2\phi}\left(\frac{m - r'm}{r^2} - \frac{\lambda r}{3}\right) + K\Phi\text{"} e^{2\Phi} + 2\phi'^2 K e^{2\phi} + K\Phi' e^{2\Phi}\left[\Phi' + (\frac{\frac{\Lambda r}{3} - \frac{m}{r^2} + \frac{m'r}{r^2}}{1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}}) + \frac{2}{r}\right] - \left[2K\Phi'^2 e^{2\Phi}\right]$$

after rearrange,

$$= \left(\Phi\text{"} + \Phi'^2\right)Ke^{2\Phi} + 2\Phi' e^{2\Phi}(\frac{m - r'm}{r^2} - \frac{\lambda r}{3}) + K\Phi' e^{2\Phi}\left(\frac{2}{r} + (\frac{\frac{\Lambda r}{3} - \frac{m}{r^2} + \frac{m'r}{r^2}}{k})\right)$$

$$= \left(\phi\text{"} + \phi'^2\right)Ke^{2\Phi} + \Phi' e^{2\Phi}(2\frac{m - m'r}{r^2} - \frac{\lambda r}{3}) + K\Phi' e^{2\Phi}\left(\frac{2}{r} + (\frac{\frac{\Lambda r}{3} - \frac{m}{r^2} + \frac{m'r}{r^2}}{k})\right)$$

$$= \left(\Phi\text{"} + \Phi'^2\right)Ke^{2\Phi} + \Phi' e^{2\Phi}\left(\frac{m}{r} - \frac{m'r}{r^2} - \frac{\lambda r}{3} + \frac{2}{r}(1 - \frac{2m}{r} - \frac{\lambda r^2}{3})\right)$$

$$= (\Phi\text{"} + \Phi'^2)Ke^{2\Phi} + \Phi' e^{2\Phi}(\frac{-3m}{r^2} - \frac{m'r}{r^2} - \lambda r + \frac{2}{r})$$

$$R_{tt} = e^{2\Phi}\left[(\Phi\text{"} + \Phi^2)(1 - \frac{2}{m}r - \frac{\Lambda r^2}{3}) + \phi'(\frac{2r - 3m - m'r}{r^2} - \lambda r)\right] \tag{2.3.3}$$

Using the same technique we will investigate the rest components of Ricci tensor
For R_{rr}

$$R_{rr} = -\Gamma^t_{rt;r} - \Gamma^\theta_{r\theta;r} - \Gamma^\phi_{r\phi;r} + \Gamma^r_{rr}\left[\Gamma^t_{rt} + \Gamma^\theta_{r\theta} + \Gamma^\phi r\phi\right] - \left[\Gamma^t_{rt}\Gamma^t_{rt} + \Gamma^\theta_{r\theta}\Gamma^\theta_{r\theta} + \Gamma^\phi_{r\phi}\Gamma^\phi_{r\phi}\right]$$

$$= -\frac{d\Gamma^t_{rt}}{dr} - \frac{d\Gamma^\theta_{r\theta}}{dr} - \frac{d\Gamma^\phi_{r\phi}}{dr} + \Gamma^r_{rr}\left[\Gamma^t_{rt} + \Gamma^\theta_{r\theta} + \Gamma^\phi r\phi\right] - \left[(\Gamma^t_{rt})2 + (\Gamma^\theta_{r\theta})^2 + (\Gamma^\phi_{r\phi})^2\right]$$

$$= \frac{-d}{dr}\left(\Phi'\right) - \frac{d}{dr}\left(\frac{1}{r}\right) - \frac{d}{dr}\left(\frac{1}{r}\right) + \left(\frac{\frac{\Lambda r}{3} - \frac{m}{r^2} + \frac{m'r}{r^2}}{1 - \frac{2m}{r} - \frac{r^2\Lambda}{3}}\right)(\Phi' + \frac{1}{r} + \frac{1}{r}) - \left(\Phi'^2 + \frac{1}{r^2} + \frac{1}{r^2}\right)$$

$$= \Phi" + \frac{2}{r^2} + \left(1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}\right)^{-1}\left(\frac{\frac{\Lambda r3}{3} - m + m'r}{r^2}\right)(\frac{r\Phi' + 2}{r}) - \left(\Phi'2 + \frac{2}{r^2}\right)$$

$$R_{rr} = \left(1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}\right)^{-1}\left[\frac{(\frac{\Lambda r^3}{3} - m + m'r)(r\Phi' + 2)}{r^3}\right] - (\Phi" + \Phi'^2) \qquad (2.3.4)$$

For $R_{\theta\theta}$,

$$R_{\theta\theta} = \Gamma^r_{\theta\theta;r} - \Gamma^\phi_{\phi\theta;\theta} + \Gamma^r_{\theta\theta}[\Gamma^\theta_{r\theta} + \Gamma^t_{rt} + \Gamma^\phi_{r\phi} + \Gamma^r_{rr}] - [\Gamma^\theta_{r\theta}\Gamma^r_{\theta\theta} + \Gamma^r_{\theta\theta}\Gamma^r_{r\theta} + \Gamma^\phi_{\theta\phi}\Gamma^\phi_{\phi\vartheta}]$$

$$= \frac{d}{dr}\left[-r - 2m - \frac{\Lambda r^3}{3}\right] - \frac{d}{d\theta}[\cot\theta] + [-rK]\left[\frac{2}{r} + \Phi' + \left(\frac{\frac{\Lambda r}{3} - \frac{m}{r^2} + \frac{m'r}{r^2}}{1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}}\right)\right] - \left[-2K + \cot^2\theta\right]$$

$$= -1 + 2m' + \Lambda r^2 + \frac{1}{\sin^2\theta} - 2K - r\Phi' K - r\left[\frac{\Lambda r}{3} - \frac{m}{r^2} + \frac{m'r}{r^3}\right] + 2K - \cot^2\theta$$

$$= 2m' - m' + r^2\Lambda - \frac{r^2\Lambda}{3} + \frac{m}{r} + \Phi'\left[-r + 2m + \frac{r^3\Lambda}{3}\right]$$

$$R_{\theta\theta} = m' + \frac{m}{r} + \frac{2\Lambda r^2}{3} + \Phi'\left[-r + 2m + \frac{r^3\Lambda}{3}\right] \qquad (2.3.5)$$

For $R_{\phi\phi}$

$$R_{\phi\phi} = \Gamma^\theta_{\phi\phi;\theta} + \Gamma^r_{\phi\phi;r} + \Gamma^r_{\phi\phi}\left[\Gamma^t_{rt}\Gamma^r_{rr} + \Gamma^\theta_{r\theta} + \Gamma^\phi_{r\phi}\right] - 2\Gamma^r_{\phi\phi} - \Gamma^\theta_{\phi\phi}\Gamma^\phi_{\theta\phi}$$

after certain manipulation, we have

$$= \sin^2\theta - \cos^2\theta - K\sin^2\theta - rK\sin^2\theta - rK\Phi\sin^2\theta - r\left[\frac{r\Lambda}{3} - \frac{m}{r^2} + \frac{m'}{r}\right]\sin^2\theta - 2K\sin^2\theta + 2K\sin^2\theta + \cos^2\theta$$

Cancel similar terms,

$$R_{\phi\phi} = \sin^2\theta - K\sin^2\theta - rK\Phi\sin^2\theta - r\left[\frac{r\Lambda}{3} - \frac{m}{r^2} + \frac{m'}{r}\right]\sin^2\theta - rK\Phi\sin^2\theta$$

$$= \sin^2\theta[1 - \left[K + rK + r\left(\frac{r\Lambda}{3} - \frac{m}{r^2} + \frac{m'}{r}\right)\right] - rK\Phi\sin^2\theta$$

Use the value of K and expand

$$R_{\phi\phi} = \sin^2\theta\left[1 - (1 - \frac{2m}{r} - \frac{\Lambda r^2}{3} + r - 2m - \frac{\Lambda r^3}{3} + \frac{\Lambda r^2}{3} - \frac{m}{r} + m')\right]$$

$$-r\left[1-\frac{2m}{r}-\frac{\Lambda r^2}{3}\right]\Phi\sin^2\theta$$

again cancel terms,

$$R_{\phi\phi}=\sin^2\theta\left[m'+\frac{m}{r}+\frac{2\Lambda r^2}{3}\right]-\left[r-2m-\frac{\Lambda r^3}{3}\right]\Phi\sin^2\theta$$

$$=\sin^2\theta\left[m'+\frac{m}{r}+\frac{2\Lambda r^2}{3}+\Phi'(-r+2m+\frac{r^3\Lambda}{3})\right]$$

$$R_{\phi\phi}=\sin^2\theta\left[m'+\frac{m}{r}+\frac{2\Lambda r^2}{3}+\Phi'(-r+2m+\frac{r^3\Lambda}{3})\right]$$

$$R_{\phi\phi}=\sin^2\theta R_{\theta\theta} \tag{2.3.6}$$

Now the Ricci scalar can be obtain by using equ(2.3.3), equ(2.3.4), equ(2.3.5), equ(2.3.6), it's definition

$$R=g^{\mu\kappa}R_{\mu\kappa}$$

$$R=g^{tt}R_{tt}+g^{rr}R_{rr}+g^{\theta\theta}R_{\theta\theta}+g^{\phi\phi}R_{\phi\phi}$$

$$=-e^{-2\Phi}e^{2\Phi}\left[(\Phi"+\Phi^2)\left(1-\frac{2}{m}r-\frac{\Lambda r^2}{3}\right)+\phi'\left(\frac{2r-3m-m'r}{r^2}-\lambda r\right)\right]+$$

$$(1-\frac{2m}{r}-\frac{r^2\Lambda}{3})\left(1-\frac{2m}{r}-\frac{\Lambda r^2}{3}\right)^{-1}\left[\frac{(\frac{\Lambda r^3}{3}-mm'r)\,(r\Phi'+2)}{r^3}\right]-(\Phi"+\Phi'^2)+$$

$$r^{-2}\left[m'+\frac{m}{r}+\frac{2\Lambda r^2}{3}+\Phi'\left(-r+2m+\frac{r^3\Lambda}{3}\right)\right]+$$

$$r^{-2}\sin^{-2}\theta\sin^2\theta\left[m'+\frac{m}{r}+\frac{2\Lambda r^2}{3}+\Phi'(-r+2m+\frac{r^3\Lambda}{3})\right]$$

$$=-(\Phi"+\Phi'^2)\left(1-\frac{2m}{r}-\frac{\Lambda r^2}{3}\right)-\Phi'\left(\frac{2r-m'r-3m}{r^2}-r\Lambda\right)+\frac{(\frac{r^2\Lambda}{3}-m+m'r)(r\Phi'+2)}{r^3}$$

Collect similar terms, we have

$$=-2-\left(\Phi"+\Phi'^2\right)(1-\frac{2m}{r}-\frac{\Lambda r^2}{3})+\Phi'\left(2\Lambda r+\frac{2m'r}{r^2}+\frac{6m}{r^2}-\frac{4m'}{r^2}\right)+\frac{4m'}{r^2}+2\Lambda$$

$$R=2\left[\frac{2m'}{r^2}+\Lambda+\Phi'\left(\frac{m'r+3m-2r}{r^2}+\Lambda r\right)-(\Phi''+\Phi'^2)(1-\frac{2m}{r}-\frac{\Lambda r^2}{3})\right] \tag{2.3.7}$$

2.4 General Relativity Energy-Momentum Tensor with cosmology constant

2.4.1 Hydrodynamic

The stress energy-momentum tensor $T_{\mu\nu}$ of a perfect fluid is given in terms of pressure P and the energy-density ϵ of a given stellar object and is defined by;

$$T_{\mu\nu}^{fluid} = Pg_{\mu\nu} + (\epsilon + p)\, U^{\mu}U^{\nu} \tag{2.4.1}$$

where u^{μ} is the four-velocity vector of the fluid, ϵ is the energy density in the co-moving frame, and p is the thermal pressure of the fluid, also measured by the co-moving observer. A fluid described by the above equation is commonly know as perfect fluid, and is widely used to model the properties of isolated rotating relativistic stars, and to study the fluid dynamics around compact objects.

Note that:- A perfect fluid is defined by the property that, in the local rest frame, it allows no energy fluxes and no anisotropic stresses. Therefore, at a given space-time point, in the local rest frame,[11]$U^{\mu} = (1, 0, 0, 0)$.Moreover,

$$\epsilon = (\rho c^2 + \varepsilon)$$

From Normalization conditions we have,

$$g_{\mu\nu}U^{\nu}U^{\mu} = -1 \tag{2.4.2}$$

Using equ.(2.2.1),equ.(2.4.1) and equ.(2.4.2). The matrix element of $T_{\mu\nu}$, becomes

$$T_{\mu\nu}^{fluid} = \begin{pmatrix} -e^{2\Phi}\epsilon & 0 & 0 & 0 \\ 0 & p(1 - \frac{2M}{r} - \frac{r^2\Lambda}{3})^{-1} & 0 & 0 \\ 0 & 0 & pr^2 & 0 \\ 0 & 0 & 0 & pr^2\sin^2\theta \end{pmatrix} \tag{2.4.3}$$

Moreover, the inverse will be

$$T^{\mu\nu}_{fluid} = \begin{pmatrix} -e^{-2\Phi}\epsilon & 0 & 0 & 0 \\ 0 & p(1 - \frac{2M}{r} - \frac{r^2\Lambda}{3}) & 0 & 0 \\ 0 & 0 & pr^{-2} & 0 \\ 0 & 0 & 0 & pr^{-2}\sin^{-2}\theta \end{pmatrix}$$

2.4.2 Electromagnetic

The energy-momentum tensor of the electromagnetic field is given by the following expression,

$$T^{\mu\nu}_{em} = \frac{1}{\mu_0}(F^{\mu}_{\gamma}F^{\nu\gamma} - \frac{1}{4}F_{\gamma\delta}F^{\gamma\delta}g^{\mu\nu}) \tag{2.4.4}$$

where $F^{\mu\nu}$ electromagnetic field strength tensor. This tensor can be decomposed in terms of the electric field E^{μ}, and the magnetic field, B^{μ}, measured by the comoving observer, as

$$F^{\mu\nu} = E^{\mu}u^{\nu} - E^{\nu}u^{\mu} + \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}(u_{\alpha}B_{\beta} - u_{\beta}B_{\alpha})$$

On the other hand, according to the Ohm's law, $j^{\mu} = \sigma E^{\mu}$, where the conduction current, j^{μ}, is related to the electric field through the constitutive relation, $j^{\mu} = \sigma E^{\mu}$, where σ is the conductivity. Now, if we suppose that the material is a perfect conductor, then $\sigma \to \infty$, and the only way to have a finite conduction current is that $E^{\mu} = 0$. This approximation is the basis for the ideal magnetohydrodynamics, which is very useful to describe highly conducting astrophysical fluids where the effect of the magnetic field cannot be neglected, such as neutron stars, accretion fluids, magnetized winds, etc [27]. As a consequence of this approximation,

$$E^{\mu} = F^{\mu\nu}u_{\nu} = 0$$

(i.e. the electric field measured by the comoving observer is zero) in order to keep the current finite.; so it is possible to write the energy-momentum tensor in equ(2.4.4) as, [27, 28]

$$T^{\mu\nu}_{em} = \mid b \mid^2 u^{\mu}u^{\nu} + \frac{1}{2} \mid b \mid^2 g^{\mu\nu} - b^{\mu}b^{\nu} \tag{2.4.5}$$

$$\mid b \mid^2 = b^{\mu}b_{\mu}$$

and

$$b^{\mu} = \frac{B^{\mu}}{\sqrt{4\pi}}. \tag{2.4.6}$$

Moreover, the 4-vector b^{μ} is spacelike and satisfies the property $b^{\mu}u_{\mu} = 0$. The tensor $T^{\mu\nu}_{em}$ describes the energy and the momentum of the magnetic field in a well-conductor fluid.

2.5 General Relativity Magnetohydrodynamic Equations

2.5.1 Energy-momentum tensor of a magnetized perfect fluid

It can be written as the sum of energy-momentum tensor of a perfect fluid and the electromagnetic field tensor part. Now here we adopt the ideal magnetohydrodynamic limits and assume infinite conductivity (flux-freezing condition), where the electric field in the rest frame is vanished,

$$T^{\mu\nu} = T^{\mu\nu}_{fluid} + T^{\mu\nu}_{em}$$

The latter can be expressed solely in terms of the magnetic field b^μ measured by a comoving observer. In this case,

$$T^{\mu\nu} = Pg_{\mu\nu} + (\epsilon + p)\, U^\mu U^\nu + \mid b \mid^2\, u^\mu u^\nu + \frac{1}{2} \mid b \mid^2\, g^{\mu\nu} - b^\mu b^\nu \tag{2.5.1}$$

which is the total energy-momentum tensor for a perfect magneto-fluid. Now by using equ.(2.4.2),equ(2.4.6) and equ(2.5.1), we can find each component of $T^{\mu\nu}$ and its inverse, $T_{\mu\nu}$

$$T_{\mu\nu} = \begin{pmatrix} e^{2\Phi}(\epsilon + \frac{3B^2}{8\pi}) & 0 & 0 & 0 \\ 0 & (p - \frac{B^2}{8\pi})(1 - \frac{2M}{r} - \frac{r^2\Lambda}{3})^{-1} & 0 & 0 \\ 0 & 0 & (p - \frac{B^2}{8\pi})r^2 & 0 \\ 0 & 0 & 0 & (p - \frac{B^2}{8\pi})r^2 \sin^2\theta \end{pmatrix} \tag{2.5.2}$$

,and the inverse

$$T^{\mu\nu} = \begin{pmatrix} e^{-2\Phi}(\epsilon + \frac{3B^2}{8\pi}) & 0 & 0 & 0 \\ 0 & (p - \frac{B^2}{8\pi})(1 - \frac{2M}{r} - \frac{r^2\Lambda}{3}) & 0 & 0 \\ 0 & 0 & (p - \frac{B^2}{8\pi})r^{-2} & 0 \\ 0 & 0 & 0 & (p - \frac{B^2}{8\pi})\frac{r^{-2}}{\sin^2\theta} \end{pmatrix}$$

2.5.2 Tolmn-Oppenheimer-Volkoff Equations

In spirit of completion, we now present the TOV equation with the schwarzschild-de sitter geometry background, we began by using Einstein equation that is completely to determine the structure of a spherical symmetric body of isotopic material which is in static gravitational, so that it is possible to determine the upper bound limit of the stars mass and size. Now we need two parameters that useful to describe the structure of an object itself. Now here we used a perfect fluid as a model for the distribution of matter. A perfect fluid is completely characterized by its rest mass density (ρ) and isotopic pressure (p). Therefore the object can be described in the rest (local) frame by these two essential parameters. Real fluid are "sticky" and conduct heat. But a perfect fluid are idealized model in which these possibilities are neglected specifically [39]. So that, in perfect fluid have no share-stress, viscosity and heat conducting. Now by using Einstein field equation state above,

For tt-component

$$R_{tt} - \frac{1}{2}g_{tt}R + \Lambda g_{tt} = 8\pi T_{tt}$$

Now by using equ(2.2.1), equ(2.3.3), equ(2.3.7) and equ(2.5.2)

$$8\pi\left(\epsilon + \frac{3B^2}{8\pi}\right)e^{2\Phi} = e^{2\Phi}\left[(\Phi" + \Phi^2)\left(1 - \frac{2}{m}r - \frac{\Lambda r^2}{3}\right) + \phi'(\frac{2r - 3m - m'r}{r^2} - \lambda r)\right] +$$

$$\frac{1}{2}2\left[\frac{2m'}{r^2} + \Lambda + \Phi'(\frac{m'r + 3m - 2r}{r^2} + \Lambda r) - \left(\Phi'' + \Phi'^2\right)(1 - \frac{2m}{r} - \frac{\Lambda r^2}{3})\right]e^{2\Phi} + \Lambda e^{2\Phi}$$

$$8\pi\left(\epsilon + \frac{3B^2}{8\pi}\right) = \frac{2m'}{r^2}$$

$$4\pi(\epsilon + \frac{3B^2}{8\pi}) = \frac{dm}{r^2 dr}$$

$$\frac{dm}{dr} = 4\pi r^2\left(\epsilon + \frac{3B^2}{8\pi}\right) \tag{2.5.3}$$

From the above equation can determine the structural distribution matter.

For rr-component,

$$R_{rr} - \frac{1}{2}g_{rr}R + \Lambda g_{rr} = 8\pi T_{rr}$$

Now by using equ(2.2.1), equ(2.3.4), equ(2.3.7) and equ(2.5.2)

$$8\pi\left(p - \frac{B^2}{8\pi}\right)(1 - \frac{2m}{r} - \frac{r^2\Lambda}{3})^{-1} = (1 - \frac{2m}{r} - \frac{\Lambda r^2}{3})^{-1}\left[\frac{(\frac{\Lambda r^3}{3} - m + m'r)(r\Phi' + 2)}{r^3}\right] - (\Phi" + \Phi'^2) -$$

$$\frac{1}{2}2\left[\frac{2m'}{r^2} + \Lambda + \Phi'(\frac{m'r + 3m - 2r}{r^2} + \Lambda r) - \left(\Phi'' + \Phi'^2\right)(1 - \frac{2m}{r} - \frac{\Lambda r^2}{3})\right](1 - \frac{2m}{r} - \frac{r^2\Lambda}{3})^{-1}$$

$$+\Lambda(1-\frac{2m}{r}-\frac{r^2\Lambda}{3})^{-1}$$

$$8\pi(p-\frac{B^2}{8\pi})=\Phi'(\frac{\Lambda' r}{3}-\Lambda r\frac{4m}{r^2}\frac{2}{r})+\frac{2\Lambda}{3}-\frac{2m}{r^3}$$

$$2\Phi'(\frac{r-2m}{r^2}-\frac{\Lambda r}{3})=8\pi(p-\frac{B^2}{8\pi})+\frac{2m}{r^3}-\frac{2\Lambda}{3}$$

$$\Phi'=\frac{4\pi r^3(p-\frac{B^2}{8\pi})+m-\frac{\Lambda r^3}{3}}{r(r-2m-\frac{\Lambda r^3}{3})}$$

$$\frac{d\Phi}{dr}=\frac{4\pi r^3(p-\frac{B^2}{8\pi})+m-\frac{\Lambda r^3}{3}}{r(r-2m-\frac{\Lambda r^3}{3})} \tag{2.5.4}$$

From the above equation one can determine the gravitational potential energy by using appropriate boundary condition.

Another relevant issue arises from the conservation equation, $\nabla_\nu T^{\mu\nu}=0$, $\nabla_\nu(\rho U^\nu)=0$, which is useful for the evolution of a magnetized fluid is to be determined. Moreover it relates the pressure gradient in the radial direction to the fluid density and magnetic field. Choose $\mu=r$

$$\nabla_\nu T^{\mu\nu}=0$$

$$\nabla_\nu T^{\mu\nu}=\frac{\partial T^{\mu\nu}}{\partial x^\nu}+T^{\Lambda\nu}\Gamma^{\mu}_{\Lambda\nu}+T^{\Lambda\mu}\Gamma^{\nu}_{\Lambda\nu} \tag{2.5.5}$$

$$\frac{\partial T^{rr}}{\partial r}+T^{tt}\Gamma^r_{tt}+T^{rr}\Gamma^r_{rr}+T^{\theta\theta}\Gamma^r_{\theta\theta}+T^{\phi\phi}\Gamma^r_{\phi\phi}+T^{rr}[\Gamma^t_{tr}+\Gamma^r_{rr}+\Gamma^\theta_{\theta r}+\Gamma^\phi_{\phi r}]=0$$

$$\frac{d}{dr}\left((p-\frac{B^2}{8\pi})(1-\frac{2m}{r}-\frac{\Lambda r^2}{3})\right)+\Phi' e^{2\Phi}(1-\frac{2m}{r}-\frac{\Lambda r^2}{3})e^{-2\Phi}(\epsilon+\frac{3B^2}{8\pi})$$

$$+(p-\frac{B^2}{8\pi})(1-\frac{2m}{r}-\frac{\Lambda r^2}{3})(\frac{\frac{\Lambda r}{3}-\frac{m}{r^2}+\frac{m' r}{r^2}}{1-\frac{2m}{r}-\frac{\Lambda r^2}{3}})-(r)(1-\frac{2m}{r}-\frac{\Lambda r^2}{3})r^{-2}(p-\frac{B^2}{8\pi})$$

$$-(p-\frac{B^2}{8\pi})r^{-2}\frac{1}{\sin^2\theta}\left(r(1-\frac{2m}{r}-\frac{\Lambda r^2}{3})\right)\sin^2\theta+(p-\frac{B^2}{8\pi})(1-\frac{2m}{r}-\frac{\Lambda r^2}{3})\left(\Phi'+\frac{2}{r}+(\frac{\frac{\Lambda r}{3}-\frac{m}{r^2}+\frac{m' r}{r^2}}{1-\frac{2m}{r}-\frac{\Lambda r^2}{3}})\right)=0$$

$$(1-\frac{2m}{r}-\frac{\Lambda r^2}{3})\frac{d}{dr}(p-\frac{B^2}{8\pi})+\Phi'(\epsilon+\frac{3B^2}{8\pi}+p-\frac{B^2}{8\pi})(1-\frac{2m}{r}-\frac{\Lambda r^2}{3})=0$$

$$\frac{d}{dr}\left(p-\frac{B^2}{8\pi}\right)=-\left(\epsilon+\frac{B^2}{4\pi}+p\right)\Phi'$$

by using equ(2.5.4) the above equation become,

$$\frac{d}{dr}\left(p-\frac{B^2}{8\pi}\right)=-\left(\epsilon+\frac{B^2}{4\pi}+p\right)\frac{4\pi r^3(p-\frac{B^2}{8\pi})+m-\frac{\Lambda r^3}{3}}{r(r-2m-\frac{\Lambda r^3}{3})} \tag{2.5.6}$$

Then the TOV equation summarized using(2.5.3)(2.5.4)(2.5.6),

$$\frac{dm}{dr}=4\pi r^2\left(\epsilon+\frac{3B^2}{8\pi}\right)$$

$$\frac{d\Phi}{dr}=\frac{4\pi r^3(p-\frac{B^2}{8\pi})+m-\frac{\Lambda r^3}{3}}{r(r-2m-\frac{\Lambda r^3}{3})}$$

$$\frac{d}{dr}\left(p-\frac{B^2}{8\pi}\right)=-\left(\epsilon+\frac{B^2}{4\pi}+p\right)\frac{4\pi r^3(p-\frac{B^2}{8\pi})+m-\frac{\Lambda r^3}{3}}{r(r-2m-\frac{\Lambda r^3}{3})} \tag{2.5.7}$$

Recall that in the case of a relativistic star with $\rho=\bar{\rho}$, it is not necessary to use the unrealistic notion of an incompressible fluid. One can think of the fluids with pressure growing as radius decreases, having a composition that varies from one radius to another. Assuming, $\rho=\bar{\rho}$ we can integrate the structure equations analytically.

$$m=\frac{4\pi}{3}R^3\left(\rho+\frac{3B^2}{8\pi}\right) \tag{2.5.8}$$

we obtain from the mass formula (2.5.3), At the surface of the star (r = R), we get the total mass of a star,

$$M=\frac{4\pi}{3}R^3\left(\rho+\frac{3B^2}{8\pi}\right) \tag{2.5.9}$$

Now ,we can easily rearrange the radial component of the metric tensor using equ.(2.6.13)

$$(1-\frac{2m}{r}-\frac{\Lambda r^2}{3})=(1-\frac{r^2}{\beta^2}) \tag{2.5.10}$$

where we have introduced a new parameter, β by the relation

$$\frac{1}{\beta^2}=\frac{1}{3}(8\pi\rho+\Lambda) \tag{2.5.11}$$

At the surface of the star ,there

$$(1-\frac{2M}{R}-\frac{\Lambda R^2}{3})=(1-\frac{R^2}{\beta^2}) \tag{2.5.12}$$

and, we can see immediately that the radial metric coefficient of the interior spacetime is smoothly matched to the corresponding metric coefficient of the exterior Schwarzschild-de

Sitter spacetime of the mass parameter M = m(R). If $\rho = \bar{\rho}$, the modified TOV equation (2.5.7) became,

$$\frac{d(p-\frac{B^2}{8\pi})}{(\rho+P+\frac{B^2}{4\pi})(3P+\rho-\frac{3B^2}{8\pi})} = -\frac{4\pi}{3}\frac{rdr}{(1-\frac{r^2}{\beta^2})} \tag{2.5.13}$$

$$\frac{dp}{(\rho+P+\frac{B^2}{4\pi})(3P+\rho-\frac{3B^2}{8\pi})} = -\left[\frac{4\pi}{3}\frac{r}{(1-\frac{r^2}{\beta^2})}+\frac{B^2}{8\pi}\right]dr \tag{2.5.14}$$

Since B is the magnetic field part of MHD, assumed constant in plasma fluid. Now, which have to be integrated from the surface of the star (black hole) (R = r), where P(R) = 0, down to the center of the star at r = 0. For a non-zero cosmological constant we find the pressure at a radius r to be give by the relation,

$$P(r) = \frac{\rho(\rho-\frac{\Lambda}{4\pi}-\frac{3B}{8\pi})\left[(1-\frac{r^2}{\beta^2})^{\frac{1}{2}}-(1-\frac{R^2}{\beta^2})^{\frac{1}{2}}\right]}{3\rho(1-\frac{R^2}{\beta^2})^{\frac{1}{2}}-(\rho-\frac{\Lambda}{4\pi}-\frac{3B}{8\pi})(1-\frac{r^2}{\beta^2})^{\frac{1}{2}}}+\frac{B^2r}{8\pi}-\frac{B^2R}{8\pi} \tag{2.5.15}$$

The maximum pressure at the center of the star, where r = 0

$$P(r=0) = \frac{\rho(\rho-\frac{\Lambda}{4\pi}-\frac{3B}{8\pi})\left[1-(1-\frac{R^2}{\beta^2})^{\frac{1}{2}}\right]}{3\rho(1-\frac{R^2}{\beta^2})^{\frac{1}{2}}-(\rho-\frac{\Lambda}{4\pi}-\frac{3B^2}{8\pi})}-\frac{B^2R}{8\pi} \tag{2.5.16}$$

Finally, we can determine the gravitational potential energy that is necessary for jets formation around active galactic nuclei. using the relationship between potential energy and pressure in TOV's equations(2.5.7), given as

$$\frac{d\Phi}{dr} = \frac{4\pi r^3(p-\frac{B^2}{8\pi})+m-\frac{\Lambda r^3}{3}}{r(r-2m-\frac{\Lambda r^3}{3})} \tag{2.5.17}$$

$$\frac{d\Phi}{dr} = \frac{4\pi r^3\left(\frac{\rho(\rho-\frac{\Lambda}{4\pi}-\frac{3B}{8\pi})\left[(1-\frac{r^2}{\beta^2})^{\frac{1}{2}}-(1-\frac{R^2}{\beta^2})^{\frac{1}{2}}\right]}{3\rho(1-\frac{R^2}{\beta^2})^{\frac{1}{2}}-(\rho-\frac{\Lambda}{4\pi}-\frac{3B}{8\pi})(1-\frac{r^2}{\beta^2})^{\frac{1}{2}}}+\frac{B^2r}{8\pi}-\frac{B^2R}{8\pi}-\frac{B^2}{8\pi}\right)+m-\frac{\Lambda r^3}{3}}{r(r-2m-\frac{\Lambda r^3}{3})} \tag{2.5.18}$$

$$\Phi(r) = \int\frac{4\pi r\left(\frac{\rho(\rho-\frac{\Lambda}{4\pi}-\frac{3B}{8\pi})\left[(1-\frac{r^2}{\beta^2})^{\frac{1}{2}}-(1-\frac{R^2}{\beta^2})^{\frac{1}{2}}\right]}{3\rho(1-\frac{R^2}{\beta^2})^{\frac{1}{2}}-(\rho-\frac{\Lambda}{4\pi}-\frac{3B}{8\pi})(1-\frac{r^2}{\beta^2})^{\frac{1}{2}}}+\frac{B^2r}{8\pi}-\frac{B^2R}{8\pi}-\frac{B^2}{8\pi}\right)+\frac{4\pi r}{3}(\rho-\frac{\Lambda}{4\pi})}{(1-\frac{r^2}{\beta^2})}dr \tag{2.5.19}$$

To this end to develop angular momentum,

$$\rho u.\nabla u = -\nabla p - \rho\nabla\Phi_G + \mu_0 J\nabla.L$$

$$L = \frac{1}{\mu_0 J}\left[\rho u\frac{du}{dr} + \frac{dp}{dr} + \rho\frac{d\Phi}{dr}\right]dr$$

Where u is velocity, to determine the other dynamical parameter such as called angular momentum with a prefer boundary condition from,

$$L = \frac{1}{\mu_0 J}\left[\rho u\frac{du}{dr} + \frac{dp}{dr} + \frac{4\pi r^3\rho\left[\frac{\rho(\rho-\frac{\Lambda}{4\pi}-\frac{3B}{8\pi})\left[(1-\frac{r^2}{\beta^2})^{\frac{1}{2}}-(1-\frac{R^2}{\beta^2})^{\frac{1}{2}}\right]}{3\rho(1-\frac{R^2}{\beta^2})^{\frac{1}{2}}-(\rho-\frac{\Lambda}{4\pi}-\frac{3B}{8\pi})(1-\frac{r^2}{\beta^2})^{\frac{1}{2}}} + \frac{B^2 r}{8\pi} - \frac{B^2 R}{8\pi} - \frac{B^2}{8\pi}\right] + \frac{4\pi r}{3}(\rho - \frac{\Lambda}{4\pi})}{r(r - 2m - \frac{\Lambda r^3}{3})}\right]dr$$

2.6 Jet Formation Mechanism

An active galaxy is a galaxy that has a very small core of extremely high powered emissions emanating from the center of the galaxy. The core is very bright and may be highly variable compared to the rest of the galaxy. Some active galaxies have jets emanating from two sides of the center [5]. It is believed that almost all galaxies have super-massive black holes at their centers. "Active" galaxies have an Active Galactic Nucleus (AGN) and are often referred to as "AGN" galaxies. But, not all galaxies have an "active" super-massive black hole. For example, the black hole in our Milky Way Galaxy is not active. To be active, the galaxy must have a source of gas, dust, and/or other debris that the super-massive black hole can readily consume. As a galaxy ages, its black hole eventually runs out of local consumable materials and the black hole becomes "dormant" (but not dead). If a fresh supply of material is devored, a dormant super-massive black hole can begin to emit high powered jets again. This happens from time to time when two galaxies collide, or a star or nebula (cloud of gas) gets pulled into the gravity domain of a super-massive black hole. There is an accretion disk around the black hole which accelerates the materials to close to light speed and some Jet of the material is ejected out of the disk in the form of jets [5]. All AGN galaxies exhibit the same basic processes. Namely, a super-massive "active" black hole at the center and an accretion disk around the black hole. They all have a bright central core fed by local gas and/or a nearby star. They differ mainly in intensity of input. The more intense the galactic center, the higher the radiation energy it emits [30]. While

Figure 2.1: Active Galaxy Overview(To watch the movie, please go to the online version of this review article at http://www.livingreviews.org/lrr-2008-7.)

it is still not fully understood "exactly" how jets are formed, most astro-physicists believe that jet's power comes from the accretion disc. Accretion discs around some other stellar objects are still produce jets, but the jets around black holes are by far the most active and approach the speed of light. So, that the jet speed is about the same speed as the escape velocity from the accretion disk [31]. This makes the speed of a jet from a black hole disk near the speed of light, while jets from newly born stars are much slower. When matter is ejected at speeds approaching the speed of light, these jets are called "relativistic" jets. The most popular jet formation hypothesis is that the twisting of magnetic fields in the accretion disk collimate the outflow of plasma along the rotating axis of the black hole so that jets emerge from each face of the disk [5]. Some accretion discs produce jets of twin, highly collimated, and fast outflows that emerge in opposite directions from close to the disc [11]. The direction of the jet ejection is determined either by the angular momentum axis of the accretion disc or the spin axis of the black hole. The jet production mechanism and indeed the jet composition on small scales are able to understood at present due to the

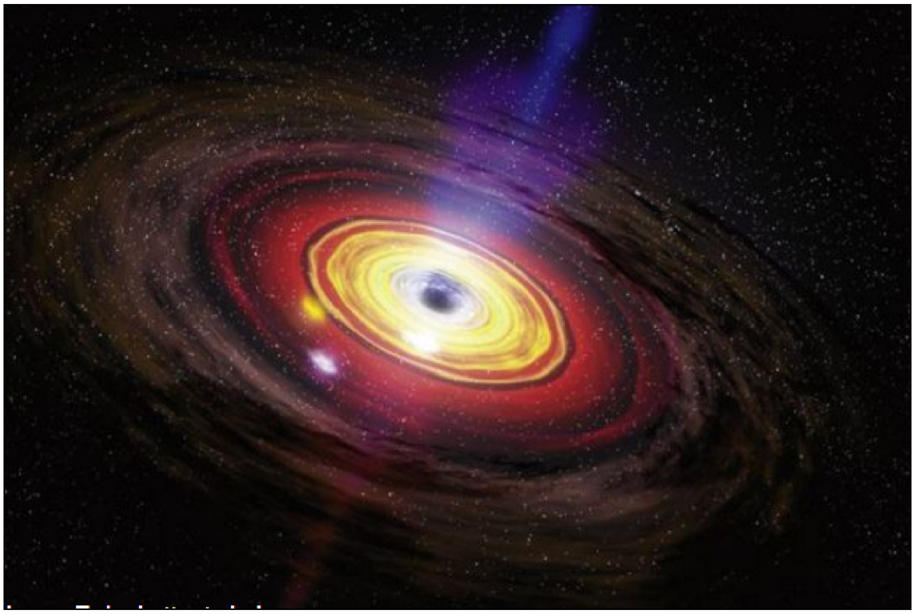

Figure 2.2: Active Galactic Nuclei(To watch the movie, please go to the online version of this review article at http://www.livingreviews.org/lrr-2008-7.)

resolution of astronomical instruments.

Chapter 3

Result and Discussion

The influence of the repulsive cosmological constant on the black-hole space-time structure can be properly represented by the dimensionless cosmological parameter $y = \frac{\Lambda R^2}{3}$. For SdS black holes admitting existence of stable circular geodesics,i.e., existence of accretion discs. For astrophysical relativistic SdS BH, the strong gravity near black hole horizon $r_{bh} = 3.1M$ weak with distance grow and at $r >> M$ was described by Newtonian theory, however, the Newtonian theory loose its validity near the so called static radius, where the repulsive effect of the cosmology constant start to be relevance up to the strong gravitating region near the cosmological horizon $r_c \sim y^{\frac{-1}{2}} R$. Therefore the cosmology constant has relevance influence on the geometry structure of gravitating compact objects around active galactic nuclei

. Now here by settings an appropriate boundary condition tried to get the potential energy released during the accretion. But theoretical strong gravitational energy most probably nearly at block hole horizon. The energy extracted from a rotating black hole if particles and field penetrate its ergo-sphere (region between event horizon and static limit for rotation). As can be seen $\frac{d\Phi}{dr}$ it tell us that, a more promising way to extract energy from rotating block hole via the existence of strong magnetic field.

Interpretation of the derived General Relativity Magnetohydrodynamic equation

We have derived the GRMHD equation from TOV equation for an accretion of massive objects like AGN usually possessing SMBHs from EFE. Accordingly, we did derived the analytical equation for pressure, distributed of particles and gravitational potential energy, under very limited boundary condition such as considering mean density, etc.

Numerical analysis:

Eq. (2.6.21) can be linearized to give

$$P(r) = P_t + P_B + P_\Lambda + P_{\Lambda B},$$

where P_t: is the pure thermal pressure, which is the dominant one due to high density.This pressure is purely due to gravitational collapse and build as thermal pressure. " On the other hand, P_B is the magnetic pressure. As we observe from the linearization it varies inversely with r i.e

$$P_B \sim \frac{1}{r}.$$

P_Λ is the pressure contributed from cosmological constant. It is given by

$$P_\Lambda \approx \frac{\rho r^2 \Lambda}{12}$$

While $P_{\Lambda B}$ is the pressure resulted from the field and cosmology constant coupling. It is of second order that we neglect here. To give an insight to what extent the magnetic term and the cosmological constant term pressures contribute in drifting or dragging jets (jets flow) AGNs here we present our numerical data generated by Mathematica (version 7) as in table 3.1 below and graphs 3.1 respectively. The assumed boundary condition is the SdS geometry, i.e., between the black hole horizon and cosmological horizon.

r(pc)	12	13	14	15	16	17	18	19	20
$p_B(\mu pascal)$	10^5	10^4	10^3	10^2	10	1	10^{-1}	10^{-2}	10^{-3}
$p_\Lambda(\mu pascal)$	10^{-9}	10^{-7}	10^{-5}	10^{-3}	10^{-1}	10	10^3	10^5	10^7

Figure 3.1: Table shows the Magnetic pressure and cosmological pressure with radius

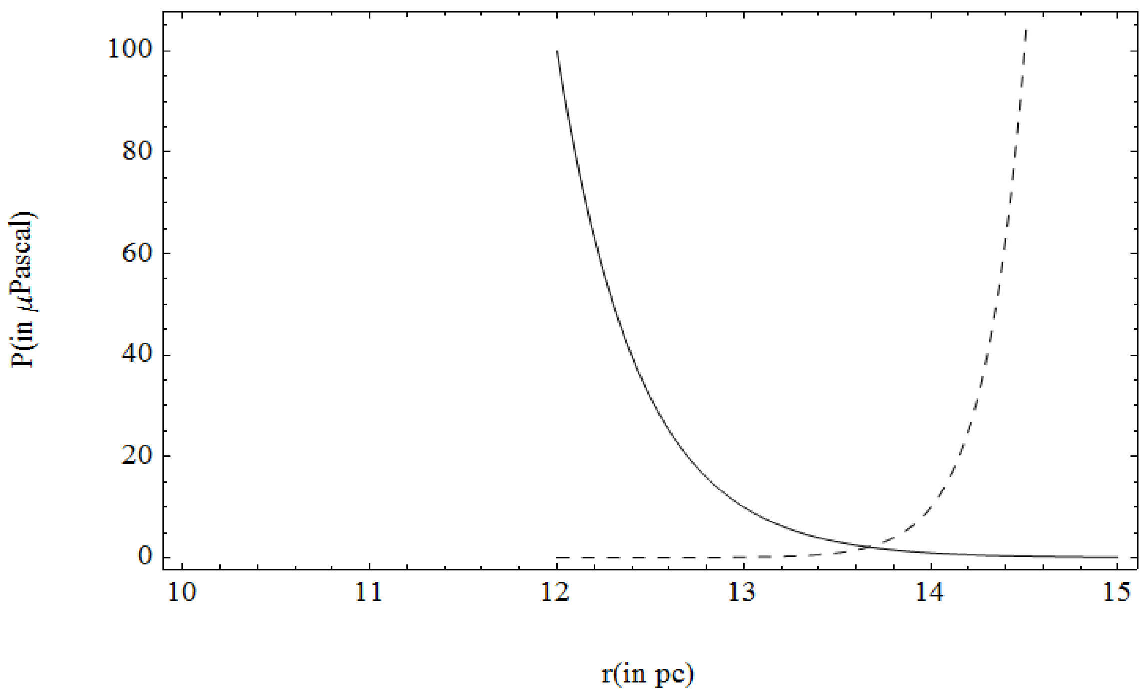

Figure 3.2: It shows the Magnetic pressure and cosmological pressure versus radius. Bold line represents Magnetic pressure (P_B) and Broken line represents Cosmological pressure (P_Λ)

Chapter 4

Conclusion

General theory of relativity is the theory of gravitation and geometry of spacetime. It generalizes the spacial theory of relativity and Newton law of universal gravitation. The matter and geometry of spacetime are related by the Einstein field equations ($R_{\mu\nu}+\frac{1}{2}g_{\mu\nu}R+g_{\mu\nu}\Lambda = 8\pi GT_{\mu\nu}$), where $G_{\mu\nu}$ is Einstein field tensor that described the geometry of spacetime where as $T_{\mu\nu}$ is energy-momentum tensor that is described the distribution of matter. The energy-momentum tensor for magnetized fluid was found from the sum of hydrodynamic and electromagnetic tensor, that is important to study the distribution of stellar structure, i.e. $T_{\mu\nu} = Pg_{\mu\nu} + (P + \rho)U^{\mu}U^{\nu}+ \mid b \mid^2 u^{\mu}u^{\nu} + \frac{1}{2} \mid b \mid^2 g^{\mu\nu} - b^{\mu}b^{\nu}$. Generally, the spacetime geometry and gravitation are described by tensors specially second rank (0; 2) tensors like Metric tensor,Riemann curvature tensor, Ricci tensor, Ricci scalar, Einstein field tensor and energy-momentum tensor in addition to Affine connections. Using schwarzschild-de sitter space-time metric. we derived the basic TOV's equations, by using Einstein field equation, by considering $\rho = \bar{\rho}_0 = \rho_{mean}$, the dynamically parameters such as pressure, gravitational potential energy are analytical derived and generated numerically value by using mathematica version 7. From the numerical result, We concluded that the source of energy for the formation relativistic jets ejected from black hole horizon comes due to the existence of strong magnetic pressure around SMBH host by AGN. In AGN, relativistic jets are thought to be formed as the result of accretion onto SMBH in the presence of accretion disk. Moreover, the magnetic pressure is the dominant pressure at the surface of black hole horizon but, far from black hole horizon the pressure due to cosmology constant becomes dominant. This pressure is termed as cosmological pressure. Therefore, I concluded that cosmology constant had observable effects on the dynamics of relativistic jets around AGN and the pressure in agreement with theoretical perception. Also the influence of cosmology constant on the geometrical structure of gravitating compact objects (SMBHs) around AGN was observed in the metric tensor ($g_{\mu\nu}$).

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