

# EARTHS ATMOSPHERIC EFFECT IN ASTRONOMICAL PHOTOMETRY/SPECTROSCOPY EXTINCTION 

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A THESIS SUBMITTED TO THE COLLAGE OF NATURAL SCIENCE DEPARTMENT OF PHYSICS IN FULFILLMENT OF THE

REQUIREMENTS FOR THE DEGREE OF
MSC. IN PHYSICS (ASTROPHYSICS)
JIMMA UNIVERSITY
JIMMA, ETHIOPIA
OCTOBER, 2017

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Date: October, 2017

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Title: EARTHS ATMOSPHERIC EFFECT IN ASTRONOMICAL

PHOTOMETRY/SPECTROSCOPY EXTINCTION
Department: A College of Natural Sciences
Physics Department
Degree: MSc.
Convocation: October
Year: 2017

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TO FAMILY

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## Acknowledgements

First of all, I would like to thank the Almighty God. I am deeply indebted to my Advisor Mr. Tolu Biressa for his guidance, constructive comments, consistent support and thanks for providing me an interesting topic. Besides my advisor, I extend my gratitude to my Co-Advisor Mr. Milkessa Gebeyehu for his professional support. Without their encouragement and guidance, this work would never have happened. Thank you all for your endless patience. Finally to my families and for my friends for their constructive ideas and support over the years.

## Abstract

Today the science astronomy for the study of origins and evolutions has at an advancing level both theoretically and observationally. However, there are a number of works for further developments. For example, the data extracted on earth is highly extinct due to stellar winds, background radiations and planetary atmospheres. In this work we did on the effect of atmospheric extinction on astronomical photometry and spectroscopy analytically. The method we applied is the Planck's radiation law where we did assume an exponential atmosphere that obeys the Beer-Bouguet-Lambert law of the optical depth equation. The results are in agreement with observational works and the standard theories. However, this work is limited to high approximations.

Keys: Astronomical-objects, Radiation, Magnitudes, Optical-depth, Atmospheric-extinction.

## General Introduction

## I. Background

Earths atmosphere is essential to life. This ocean of fluids and suspended particles surrounds Earth and protects it from the hazards of outer space. It insulates the inhabitants of Earth from the extreme temperatures of space and stops all but the larger meteoroids from reaching the surface. Further more, it filters out most radiation dangerous to life. Without the atmosphere, life would not be possible on Earth. The atmosphere contains the oxygen we breath.

Yet the same atmosphere that makes life possible hinders our understanding of Earth's place in the universe. It is the main noise sources of ground-based astronomical observations. Our only means for investigating distant stars, nebulae, and galaxies is to collect and analyze the electromagnetic radiation these objects emit into space. However, most of this radiation is absorbed or distorted by the atmosphere before it can reach a ground-based telescope. Only visible light and some radio waves, infrared, and ultraviolet light survive the passage from space to the ground. That limited amount of radiation has provided astronomers enough information to estimate the general shape and size of the universe and categorize its basic components, but there is much left to work[11].

It is essential to study the entire spectrum rather than just limited regions of it. The atmospheric extinction coefficients and night sky brightness are key parameters characterizing the quality of an astronomical observational site, in addition to the seeing and the number
of clear night. There is the brightness reduction of celestial objects as their light passes through the earth atmosphere.

Parrao and Schuster in 2003 points out [9]that precise atmospheric extinction determinations are needed not only for stellar photometry but also for any sort of photometry, spectroscopy, spectrophotometry and imaging whenever accurate, absolute and well-calibrated photometric measurements are required for the derivation of physical parameters in the studies of galaxies, nebulae, planets, and so forth. The night sky brightness is caused by the scattered starlight, the air glow, zodiacal light and the artificial light pollution from the nearby cities. The night sky brightness limits the detection depth of a telescope. The lower the night sky brightness, the fainter the stars that could be detected and the more astronomical information that could be collected.

The outlines of my work is organized as follows; Observational Parameters like, Apparent Magnitudes, Absolute Magnitudes, Visual Magnitudes, Photographic Magnitudes, Bolometric Magnitudes, Bolometric Correction, Luminosity, Color and Color indexes are introduced in the first chapter. Here the magnitude difference between the stars is defined and the equation for the flux ratio of the stars is derived from the fundamental equation(magnitude difference between the stars). Also other equations defining other parameters are presented in this chapter. Different figures carrying different information of the topics which taken from different sources are included in this chapter.

In chapter two, definitions for Black-Body and Black-Body Radiation are given; Plancks spectrum is discussed; Plancks distribution is derived; the equation for the total intensity is derived from Planck's law. And finally taking the ratio of the intensity and the StefanBoltzmann constant, the equations of flux and luminosity with respect to the temperature is derived. Wien's displacement law is also derived from the intensity.

In chapter three, atmospheric extinction, atmospheric emission, absorption \&scattering, atmospheric windows,optical depth, atmospheric seing and air mass are presented. The
equations that relates incident flux and out put flux with respect to the optical depth is derived and by rearranging the derived equation the equation of optical depth is also derived.

## II. Literature Review

The light coming from stars and other astronomical objects reaches us after traveling through the Earth's atmosphere. In this process it is scattered and absorbed. This leads to the attenuation of light which is called the Earth's atmospheric extinction. It is measured by extinction coefficient, which depends on various factors such as atmospheric conditions, altitude of the site and the wave length of the incoming light. The atmospheric extinction coefficients vary from night to night as meteorological conditions strongly affect them. [2] Up to the end of the Middle Ages, the most important means of observation in astronomy was the human eye. It was aided by various mechanical devices to measure the positions of celestial bodies in the sky. At the beginning of the 17th century telescope was invented in Holland and in 1609 Galileo Galilei made his first astronomical observations with this new instrument. Astronomical photography was introduced at the end of the 19th century, and during the last few decades many kinds of electronic detectors have been adopted for the study of electromagnetic radiation from space. The electromagnetic spectrum from the shortest gamma rays to long radio waves can now be used for astronomical observations[7]. Now a day electromagnetic spectrum from the shortest gamma rays to long radio waves can be used for astronomical observations. Outside the atmosphere, observations can be made with satellites and spacecraft. Yet, the great majority of astronomical observations are carried out from the surface of the Earth [7].

The atmosphere affects observations in many other ways as well. The air is never quite steady, and there are layers with different temperatures and densities. This causes convection and turbulence. When the light from a star passes through the unsteady air, rapid
changes in refraction in different directions result. Thus, the amount of light reaching a detector, e. g. the human eye, constantly varies. The star is said to scintillate In general atmospheric extinction is the reduction of the intensity of radiation as a result of absorption and scattering by the earth's atmosphere. It is the dimming of light in its passage through the earth's atmosphere. About one sixth of the amount of perpendicularly incident light is extinguished in the visible domain.

## III . Statement of the Problem

Ground-based astronomical observations are affected by the Earth's atmosphere. Light from astronomical objects is scattered and absorbed by air molecules and aerosols. This extinction effect can cause a significant loss of flux, depending on the wavelength and weather conditions. The signal of the targeted object is further deteriorated by background radiation, which is caused by light from other astronomical radiation sources scattered into the line of sight and emission originating from the atmosphere itself.

## IV . Research Questions

- How the earth's atmosphere affects radiations coming from astronomical sources?
- What is the effect of earth's atmosphere on astronomical photometry and spectroscopy?
- How big the earth's atmosphere change the magnitudes of brightness astronomical objects?


## V. Objectives

## I. General objective

To study the effect of earth's atmosphere on astronomical photometry and spectroscopy

## II. Specific Objectives

- To study spectral distribution of stars through Earth's Atmosphere.
- To study the extinction of astronomical data due to earth's atmospheric effect.
- To study the effect of the earth's atmosphere on the magnitudes of brightness astronomical objects


## VI. Methodology

Planck,s radiation law is being used to further derive relevant parameters like brightness magnitudes(such as apparent, absolute, bolometric magnitudes), and other observable parameters. In particular the atmospheric extinction is being considered by assuming an exponential atmosphere. Then the optical thickness related to extinction is an exponential one, where the Beer-Bouguer-Lambert law is considered. Finally the analytically derived equations are used to generate numerical data computationally using MATHEMATICA to analysis.

## Chapter 1

## Basic theory of Observational Astronomy

Observational astronomy is a division of the astronomical science that is concerned with getting data, in contrast with theoretical astrophysics which is mainly concerned with finding out the measurable implications of physical models. It is the practice of observing celestial objects by using telescopes and other astronomical apparatus.

As a science, astronomy is somewhat hindered in that direct experiments with the properties of the distant universe are not possible. However, this is partly compensated by the fact that astronomers have a vast number of visible examples of stellar phenomena that can be examined. This allows for observational data to be plotted on graphs, and general trends recorded.

Galileo Galilei was the first person known to have turned a telescope to the heavens and to record what he saw. Since that time, observational astronomy has made steady advances with each improvement in telescope technology.[1]

A traditional division of observational astronomy is given by the region of the electromagnetic spectrum observed:

- Optical astronomy is the part of astronomy that uses optical components (mirrors, lenses and solid-state detectors) to observe light from near infrared to near ultraviolet wavelengths.

Visible-light astronomy (using wavelengths that can be detected with the eyes, about 400 700 nm ) falls in the middle of this range.

- Infrared astronomy deals with the detection and analysis of infrared radiation (this typically refers to wavelengths longer than the detection limit of silicon solid-state detectors, about $1 \mu \mathrm{~m}$ wavelength). The most common tool is the reflecting telescope but with a detector sensitive to infrared wavelengths. Space telescopes are used at certain wavelengths where the atmosphere is opaque, or to eliminate noise (thermal radiation from the atmosphere).
- Radio astronomy detects radiation of millimetre to dekametre wavelength. The receivers are similar to those used in radio broadcast transmission but much more sensitive. See also Radio telescopes.
- High-energy astronomy includes X-ray astronomy, gamma-ray astronomy, and extreme UV astronomy, as well as studies of neutrinos and cosmic rays.

Optical and radio astronomy can be performed with ground-based observatories, because the atmosphere is relatively transparent at the wavelengths being detected. Observatories are usually located at high altitudes so as to minimise the absorption and distortion caused by the Earth's atmosphere. Some wavelengths of infrared light are heavily absorbed by water vapor, so many infrared observatories are located in dry places at high altitude, or in space. The atmosphere is opaque at the wavelengths used by X-ray astronomy, gamma-ray astronomy, UV astronomy and (except for a few wavelength "windows") far infrared astronomy, so observations must be carried out mostly from balloons or space observatories. Powerful gamma rays can, however be detected by the large air showers they produce, and the study of cosmic rays is a rapidly expanding branch of astronomy.

Stellar photometry came into use in 1861 as a means of measuring stellar colors. This
technique measured the magnitude of a star at specific frequency ranges, allowing a determination of the overall color, and therefore temperature of a star. [1] By 1951 an internationally standardized system of UBV-magnitudes (Ultraviolet-BlueVisual) was adopted [1].

Photoelectric photometry using the CCD is now frequently used to make observations through a telescope. These sensitive instruments can record the image nearly down to the level of individual photons, and can be designed to view in parts of the spectrum that are invisible to the eye. [1].

### 1.1 The Magnitude and Color System

All information we know about stars and galaxies comes from observing electromagnetic radiation, more precisely from two small windows where the Earth's atmosphere is nearly transparent. The first one is around the range of visible light plus some windows in the infra-red (IR), while the second one is the radio window in the wavelength range of, $1 \mathrm{~cm} \leq$ $\lambda \leq 30 \mathrm{~m}[8]$

### 1.1.1 Magnitudes

Optical astronomers uses astronomical magnitude system to talk about several different kinds of measurements, such as the observed brightnesses (energy fluxes), of stars and the luminosity (total power output in EMR) of stars. The historical roots of the magnitude system go way back to the first star catalog, compiled by a Greek named Hipparchus [10] some 2200 years ago. Hipparchus divided the stars into six brightness classes, and he called the stars that appeared brightest (to the naked eye), first magnitude stars, and the faintest visible stars the sixth magnitude stars. Much later, when astronomers were able to make
more exact measurements of the brightnesses of stars, they found that, the Hipparchus magnitude scale was roughly logarithmic. That is, each magnitude step corresponded to a fixed brightness ratio or factor. Based on the Hipparchus magnitude system, but using modern brightness measurements, astronomers have decided to define a magnitude system. With the use of photographic plates, the magnitude scale could be defined quantitatively. Pogson [8] found 1856 that the magnitude difference $\Delta \mathrm{m}=5$ corresponds to a ratio of energy fluxes of $\frac{f_{2}}{f_{1}}=100$, where the energy flux f is defined as the energy going through the area per time. Pogson defined the ratio of the brightnesses of classes $n$ and $n+1$ as $\sqrt[5]{100}=$ 2.521. Thus, each magnitude is exactly $100^{\frac{1}{5}}$ or about 2.512 times as bright as the next. An increase of 1 mag means a lower flux by about 2.5 . It is best to think about magnitudes as a short hand way of writing ratios of quantities. Say we have two stars, with flux $f_{1}$ and $f_{2}$, We can define the magnitude difference between the stars as,

$$
\begin{equation*}
m_{1}-m_{2}=-2.5 \log _{10}\left(\frac{f_{1}}{f_{2}}\right) \tag{1.1.1}
\end{equation*}
$$

[The log is the common logarithm (base 10) in astronomical usage. Thus a $1 \%$ change in flux is a change of 0.0108 in magnitude. [12]] Clearly, if the flux ratio is 100 , the magnitude difference is 5 . 1.1.1 is the fundamental equation needed to define and deal with magnitudes. We can rearrange (1.1.1 to give the flux ratio if the magnitude difference is known,

$$
\begin{equation*}
\frac{f_{1}}{f_{2}}=10^{-0.4\left(m_{1}-m_{2}\right)} \tag{1.1.2}
\end{equation*}
$$

The most common use for magnitudes is for expressing the apparent brightness of stars. To give a definite number for a magnitude of a star, instead of just the magnitude difference between pairs of stars, we must pick a starting place, or zero point, for the magnitude system. To oversimplify some what we pick the star Vega, and say it has magnitude of 0.00 . Then the magnitude of any other star is simply related to the flux ratio of that star and

Vega as follows:

$$
\begin{equation*}
m_{1}=-2.5 \log _{10}\left(\frac{f_{1}}{f_{v e g a}}\right) \tag{1.1.3}
\end{equation*}
$$

These magnitudes are called apparent magnitudes, because they are related to the flux of the star, or how bright the star appears to us.

### 1.1.2 Apparent Magnitude

The apparent magnitude $(\mathrm{m})$ of a star is, a measure of its apparent brightness as seen by an observer on Earth. The brighter the object appears, the lower the numerical value of its magnitude.

Apparent magnitude is an irradiance or illuminance, i.e incident flux per unit area, from all directions. Of course a star is a point light source, and the incident light is only from one direction. Apparent magnitude per square degree is a radiance, luminance, intensity, or "specific intensity". This is sometimes also called "surface brightness". Still another unit for intensity is magnitudes per square arcsec, which is the magnitude at which each square arcsec of an extended light source shines. Only visual magnitudes can be converted to photometric units. $\mathrm{U}, \mathrm{B}, \mathrm{R}$ or I magnitudes are not easily convertible to luxes, lumens etc, because of the different wavelengths intervals used. The conversion factors would be strongly dependent on e.g the temperature of the black-body radiation or, more generally, the spectral distribution of the radiation. The conversion factors between V magnitudes and photometric units are only slightly dependent on the spectral distribution of the radiation. 4]

### 1.1.3 Absolute magnitude

The absolute magnitude of a star is defined as the apparent magnitude we would measure if the star were placed a distance of $10 \mathrm{pc}(\sim 32$ light years) from an observer. 8 (1 pc $=$
$3 \times 10^{18} \mathrm{~cm}$ ) The apparent magnitude, m , is related to the absolute magnitude, M , by

$$
\begin{align*}
& m=M+5 \log \frac{d}{10 p c}  \tag{1.1.4}\\
& \Rightarrow m-M=5 \log \frac{d}{10 p c} \tag{1.1.5}
\end{align*}
$$

Absolute magnitude is related to the true brightness or luminosity of an object. To derive an objects absolute magnitude, one must measure the apparent magnitude, and also know the distance to the object and the amount of any obscuring dust between us and the object. Absolute magnitudes for stars generally range from -10 mag to +17 mag . [4] The absolute magnitude for galaxies can be much lower (brighter). For example, the giant elliptical galaxy M87 has an absolute magnitude of -22mag. Many stars visible to the naked eye have an absolute magnitude which is capable of casting shadows from a distance of 10pc; Rigel (-7.0mag), Deneb ( -7.2 mag ), Naos ( -6.0 mag ), and Betelgeuse ( -5.6 mag ). If the distance of a star is $\mathrm{r} p \mathrm{pc}$ [4, Its brightness diminishes as the square of the distance between star and observer. The apparent magnitude of the star will therefore be greater, by an additive term $\log _{2.5} \frac{r^{2}}{r_{0}^{2}}$ than its absolute magnitude. Distance modulus $\mu_{0}$ is given by,

$$
\begin{equation*}
\mu_{0} \equiv m-M=2.5 \log \frac{r^{2}}{r_{0}^{2}}=5 \log \frac{r}{r_{0}}=5 \log r-5 \tag{1.1.6}
\end{equation*}
$$

### 1.1.4 Visual magnitude

In daylight the human eye is most sensitive to radiation with a wavelength of about 550 nm , the sensitivity decreasing towards red (longer wavelengths) and violet (shorter wavelengths). The magnitude corresponding to the sensitivity of the eye is called the visual magnitude $m_{v}$.


Figure 1.1: Transmission Of The Earths Atmosphere(Attenuation of electromagnetic radiation by the atmosphere) [source, PH507 Astrophysics Dr Dirk Froebrich page, 10].

### 1.1.5 Photographic magnitude

Because the flux of star light varies with wavelength, the magnitude of a star depends upon the wavelength at which we observe. Originally, photographic plates were sensitive only to blue light, and the term photographic magnitude $m_{p g}$ still refers to magnitudes centered around $420 \mathrm{~nm}[4$ (in the blue region of the spectrum).

### 1.1.6 Bolometric Magnitudes

The flux of any object varies with wavelength. To measure all the EMR from a body, we would have to observe at all wavelengths of EMR, from gamma rays to the longest radio waves. Quantities integrated over all wavelengths are called bolometric quantities, e.g the


Figure 1.2: The Electromagnetic Spectrum.[Source, A Concise Introduction to Astrophysics Lecture Notes for FY2450 M. Kachelrie ß page 12]
bolometric luminosity of the Sun is the total power put out by the Sun in all wavelengths of EMR. Bolometric magnitudes are difficult to actually measure. The object must be observed with a number of different telescopes and detectors e.g ground based telescopes for the optical portion of the spectrum, satellite telescopes for the ultraviolet and X rays, which don't penetrate the atmosphere, ground or space telescopes for the infrared, space telescopes for the very short radio ( mm and sub mm range) and ground based radio telescopes for the longer radio waves. The wavelength of peak emission is of course related to the effective temperature of the star by Stefan-Boltzmann law [10. The wavelength of the peak flux, for most stars, is in or near the visible region of the spectrum. Fortunately, most
stars emit the vast majority of their total power within a reasonable interval in wavelength around the wavelength of their peak emission. This is less true for some other objects, for example quasars and other active galactic nuclei, which can emit significant energy over a very wide range of wavelengths. Bolometric quantities are important to the theorist, as they represent the total amount of energy output from an astronomical object. However, obervations must be limited to certain wavelengths regions, either by the atmosphere, or by the detectors used. The optical region is that region limited by the atmosphere on the short wavelength. Filters are optical components that only allow certain wavelengths to pass through them.

### 1.1.7 Bolometric Correction

Normally the bolometric brightness of a star can only be obtained by means of observations spanning the entire spectrum. The bolometric correction, BC, is defined as the difference between the bolometric and visual magnitudes of a star. The bolometric correction is always positive

$$
\begin{equation*}
B C=m_{v}-m_{b o l} \tag{1.1.7}
\end{equation*}
$$

## Apparent Magnitudes



Figure 1.3: Apparent Magnitudes[Source,PH507 Astrophysics Dr Dirk Froebrich, page,7]

### 1.1.8 Luminosity

Luminosity depends on the distance and extinction as well as relativistic effects. The radiated power L (Luminosity, [W]), ignoring extinction, is given by an inverse square law:

$$
\begin{gather*}
F=\frac{L}{4 \pi d^{2}} \Rightarrow F=\frac{E}{A t}=\frac{L}{A}  \tag{1.1.8}\\
\Rightarrow L=4 \pi d^{2} F \tag{1.1.9}
\end{gather*}
$$

where $A=4 \pi d^{2}$
we recover the inverse square law for the energy flux at the distance $r>R$ outside of the star. The validity of the inverse square law $F(r) \propto \frac{1}{r^{2}}$ relies on the assumptions that no radiation is absorbed and that relativistic effects can be neglected. The later condition requires in particular that the relative velocity of observer and source is small compared to the velocity of light.

The total luminosity L of a star is given by the product of its surface' $\mathrm{A}=4 \pi R^{2}$ and the radiation emitted per area $\sigma T^{4}$,

$$
\begin{equation*}
L=4 \pi R^{2} \sigma T^{4} \tag{1.1.10}
\end{equation*}
$$

### 1.1.9 Luminosity vs. Color of Stars

In 1911, Ejnar Hertzsprung investigated the relationship between luminosity and colors of stars in within clusters. In 1913, Henry Norris Russell did a similar study of nearby stars. Both found that the color (temperature, spectral type) was related to the luminosity. [4]

### 1.2 The color of a star

Rather than just have one apparent magnitude, measured across the entire visible spectrum we can use a filter to restrict the incoming light to a narrow waveband. If, for instance, we use a filter that only allows light in the blue part of the spectrum, we can measure a star's blue apparent magnitude, B. Similarly if we use a filter that approximates the eye's visual response which peaks in the yellow-green part of the spectrum we measure the magnitude V of a star. Color is defined as the difference between the magnitude of a star in one filter and the magnitude of the same star in a different filter.

### 1.2.1 Color

The observed brightness of a star depends on whether it is seen by eye, recorded on a photographic plate, or detected by means of a radio telescope. For different astronomical objects the spectral energy distribution, SED, the ratio of energy emitted, e.g, in the optical domain, the infrared, or radio regime, varies widely. The color or SED of an object can be roughly described by observing it through a variety of filters or with different detectors in several different spectral regions.

A quantitative way to measure the color of stars is to use filters which sensitivity is centered at different wavelengths $\lambda_{i}$ and compare their relative brightness is,

$$
\begin{equation*}
m_{\lambda_{1}}-m_{\lambda_{2}}=-2.5 \lg \frac{F_{\lambda_{1}}}{F_{\lambda_{2}}} \tag{1.2.1}
\end{equation*}
$$

Then $m_{\lambda_{1}}-m_{\lambda_{2}}$ is called the color or the color index of the object. e.g the difference in magnitudes measured with a filter for blue and visible light,

$$
\begin{equation*}
B-V \equiv m_{B}-m_{V}=-2.5 \lg \frac{F_{B}}{F_{v}} \tag{1.2.2}
\end{equation*}
$$

One widely used filter system in the optical region of the spectrum is called the UBV system. The letters correspond to different filters: U for ultraviolet, B for blue, and V
for visual. The central wavelengths of the filters are, $\lambda_{U} \approx 3600 \AA, \lambda_{B} \approx 4400 \AA, \lambda_{V} \approx$ $5500 \AA$. The passband, or wavelength range passed is roughy $1000 \AA$ for each filter in the broadband UBV system[10] e.g the B filter passes only light from about $3900 \AA$ to $4900 \AA$. We define magnitudes in each filter e.g $m_{V}$ or sometimes just V is the magnitude in the V filter, for instance. The color of an object related to the variation of flux with wavelength. Using broadband filters, like UBV we define the color index as the difference between the magnitudes in two colors, e.g $B-V=m_{B}-m_{V}$ defines the $\mathrm{B}-\mathrm{V}$ color index. we see that a magnitude difference corresponds to a flux ratio. The ratio is the flux at B, relative to the flux at V , of the same object, instead of different objects. (1.2.3)

$$
\begin{equation*}
B-V=m_{B}-m_{V}=-2.5 \log \left(\frac{f_{B}}{f_{V}}\right)+\text { constant } \tag{1.2.3}
\end{equation*}
$$

where $f_{B}$ is the flux averaged over the B filter and $f_{V}$ is the flux averaged over the V filter. The "constant" appears in 1.2.3 because of the way we define the zero point of the color system.

If,

$$
\begin{equation*}
B-V=0, \text { then, } f_{B}=f_{V} \tag{1.2.4}
\end{equation*}
$$

However, this is not how the color system is defined. Historically, astronomers picked a set of A stars (including Vega) and defined the average color of these stars to have all colors equal to 0.00 . For an A star, $f_{B}$ is not equal to $f_{V}$, so that a non-zero constant is needed in the $(1.2 .3$ ) to make the color come out to 0.00 . Thus, the $\mathrm{B}-\mathrm{V}$ color of Vega is 0.00 , pretty much "by definition". Fig (1.4) shows spectrophotometry of two stars to illustrate the relation between spectrum and colors. One star is a yellow star $(B-V=0.63)$, about the same color as the Sun. The other star is a very hot, blue star $(\mathrm{B}-\mathrm{V}=-0.32)$. The flux is expressed in magnitudes, here just a shorthand way to write $\log \left(f_{\nu}\right)$. Note that the flux of the stars as a function of wavelength behaves quite differently for the two stars, the yellow stars flux increasing with increasing wavelength, while the blue star's flux decreases
with increasing wavelength. Because magnitudes are essentially the logarithm of a flux, they are inconvenient for adding fluxes.


Figure 1.4: Spectrophotometry of a blue (top) and a yellow (bottom) star. [source; An Introduction to Astronomical Photometry Using CCDs W. Romanishin University of Oklahoma]

It is clear that the color of a star is connected to its temperature. The color magnitudes are normalized such that their differences $m_{\lambda_{1}}-m_{\lambda 2}$ are zero for a specific type of stars with surface temperature $\mathrm{T} \approx 10000 \mathrm{~K}$ [8]

## Chapter 2

## Black-Body Radiation

The word black indicates that the radiation is perfectly absorbed and reradiated by the object. The spectrum of light radiated by such an idealised black body is described by a universal spectrum called the Planck's spectrum. It depends on the absolute temperature T of the radiation

$$
\begin{equation*}
E_{s}=n_{1} E_{1}+n_{2} E_{2}+\ldots+E_{k} n_{k} \tag{2.0.1}
\end{equation*}
$$

where $n_{k}$ is the number of photons with energy $E_{k}$.

$$
\begin{gather*}
Z(T, \nu)=\sum_{n=0}^{\infty} e^{-\beta E_{s}} .  \tag{2.0.2}\\
=\sum_{n=0}^{\infty} e^{-\beta\left(n_{1} E_{1}+n_{2} E_{2}+\ldots+E_{k} n_{k}\right)} .  \tag{2.0.3}\\
=\sum_{n_{1}=0}^{\infty} e^{-\beta n_{1} E_{1}} \sum_{n_{2}=0}^{\infty} e^{-\beta n_{2} E_{2}} \ldots .  \tag{2.0.4}\\
=\prod_{k} \sum_{n_{k}=0}^{\infty} e^{-\beta n_{k} E_{k}} \cdot=\prod_{k}\left[\frac{1}{1-e^{-\beta E_{k}}} \cdot\right] \tag{2.0.5}
\end{gather*}
$$

$$
\begin{gather*}
\bar{n}_{k}=\frac{\sum_{s} n_{k} e^{-\beta E_{s}}}{\sum e^{-\beta E_{s}}} \\
=\frac{\sum n_{1} n_{2} \ldots n_{k} e^{-\beta\left(n_{1} E_{1}+n_{2} E_{2}+\ldots+E_{k} n_{k}\right)}}{z} \\
=\frac{1}{Z}\left[\frac{\partial}{\partial\left(-\beta E_{k}\right)} \sum e^{-\beta\left(n_{1} E_{1}+n_{2} E_{2}+\ldots+E_{k} n_{k}\right)}\right] \\
\bar{n}_{k}=\frac{\partial}{\partial\left(-\beta E_{k}\right)}\left[\sum-\ln \left(1-e^{-\beta E_{k}}\right)\right]=\frac{e^{-\beta E_{k}}}{1-e^{-\beta E_{k}}} \\
\bar{n}_{k}=\frac{1}{e^{\beta E_{k}}-1}\left(\text { Planck }^{\prime} \text { sdistribution }\right) \tag{2.0.6}
\end{gather*}
$$

where $\bar{n}_{k}$ is the mean number of photons in state k

### 2.1 Continuous Spectra

Continuous emission spectra can originate in recombinations and freefree transitions. In recombination, an atom captures a free electron whose energy is not quantized; in freefree transitions, both initial and final states are unquantized. Thus the emission line can have any frequency whatsoever. Similarly, ionizations and freefree transitions can give rise to a continuous absorption spectrum. Each spectrum contains a continuous component, or continuum, and spectral lines. Sometimes, however, the lines are so closely packed and so broad that they seem to form a nearly continuous spectrum.

A blackbody is defined as an object that does not reflect or scatter radiation shining upon it, but absorbs and reemits the radiation completely. A blackbody is a kind of an ideal radiator, which cannot exist in the real world. Yet many objects behave very much as if they were blackbodies. The radiation of a blackbody depends only on its temperature, being perfectly independent of its shape, material and internal constitution. The wavelength
distribution of the radiation follows Plancks law, which is a function of temperature only. The intensity at a frequency $\nu$ of a blackbody at temperature T is

$$
\begin{equation*}
B_{\nu}(T)=B(\nu ; T)=\frac{2 h \nu^{3}}{c^{2}} \frac{1}{e^{\frac{h \nu}{k T}}-1}(\text { Planckslaw }) \tag{2.1.1}
\end{equation*}
$$

where,
$\mathrm{h}=$ the Planck constant $=6.63 \times 10^{-34} \mathrm{~J} \mathrm{~s}$,
$\mathrm{c}=$ the speed of light $\approx 3 \times 10^{8} \mathrm{~ms}^{-1}$,
$\mathrm{k}=$ the Boltzmann constant $=1.38 \times 10^{-23} \mathrm{JK}^{-1}$
$\left[B_{\nu}\right]=W m^{-2} H z^{-1}$ sterad $^{-1}$
Blackbody radiation can be produced in a closed cavity whose walls absorb all radiation incident upon them (and coming from inside the cavity). The walls and the radiation in the cavity are in equilibrium; both are at the same temperature, and the walls emit all the energy they receive. Since radiation energy is constantly transformed into thermal energy of the atoms of the walls and back to radiation, the blackbody radiation is also called thermal radiation.

The spectrum of a blackbody given by Plancks law is continuous if the size of the radiator is very large compared with the dominant wavelengths [7]. In the case of the cavity, this can be understood by considering the radiation as standing waves trapped in the cavity. The number of different wavelengths is larger, the shorter the wavelengths are compared with the size of the cavity.We already mentioned that spectra of solid bodies are continuous; very often such spectra can be quite well approximated by Plancks law. We can also write Plancks law as a function of the wavelength. We require that,

$$
\begin{equation*}
B_{\nu} d \nu=-B_{\lambda} d \lambda \tag{2.1.2}
\end{equation*}
$$

$\Rightarrow$ The wavelength decreases with increasing frequency;

### 2.2 Planck's law

From $\mathrm{c}=\lambda \nu$, we have, $d c=d(\lambda \nu)=0$ becouse c is constant

$$
\begin{equation*}
\Rightarrow \frac{d \nu}{d \lambda}=\frac{-c}{\lambda^{2}} \tag{2.2.1}
\end{equation*}
$$

substituting 2.2 .1 in to 2.1.2 and rearranging,

$$
\begin{equation*}
B_{\lambda}=B \nu \frac{c}{\lambda^{2}} \tag{2.2.2}
\end{equation*}
$$

substituting 2.1.1 in to 2.2.2 and rearranging, we have,

$$
\begin{equation*}
B_{\lambda}(T)=\frac{2 h c^{2}}{\lambda^{5}} \frac{1}{e^{\frac{h c}{\lambda k T}}-1} \tag{2.2.3}
\end{equation*}
$$

$\left[B_{\lambda}\right]=W m^{-2} m^{-1}$ sterad $^{-1}$
The functions $B_{\nu}$ and $B_{\lambda}$ are defined in such a way that the total intensity can be obtained in the same way using either of them:

$$
\begin{equation*}
B(T)=\int_{0}^{\infty} B_{\nu} d \nu=\int_{0}^{\infty} B_{\lambda} d \lambda \tag{2.2.4}
\end{equation*}
$$

Using (2.1.1) in the first of 2.2 .4

$$
\begin{gather*}
B(T)=\int_{0}^{\infty} B_{\nu}(T) d \nu=\int_{0}^{\infty} \frac{2 h \nu^{3}}{c^{2}} \frac{1}{e^{\frac{h \nu}{k T}}-1} d \nu  \tag{2.2.5}\\
B(T)=\frac{2 h}{c^{2}} \int_{0}^{\infty} \frac{\nu^{3} d \nu}{e^{\frac{h \nu}{k T}}-1} \tag{2.2.6}
\end{gather*}
$$

If

$$
\begin{equation*}
x=\frac{h \nu}{k T}, \Rightarrow \nu=\frac{k T}{h} x, \Rightarrow \nu^{3}=\frac{k^{3} T^{3}}{h^{3}} x^{3}, \& d \nu=\frac{k T}{h} d x \tag{2.2.7}
\end{equation*}
$$

substituting 2.2.7 in 2.2.6 and rearranging,

$$
\begin{equation*}
B(T)=\frac{2 h}{c^{2}} \frac{k^{4} T^{4}}{h^{4}} \int_{0}^{\infty} \frac{x^{3} d x}{e^{x}-1} \tag{2.2.8}
\end{equation*}
$$

the integral,

$$
\begin{gather*}
\int_{0}^{\infty} \frac{x^{3} d x}{e^{x}-1}=\frac{\pi^{4}}{15}  \tag{2.2.9}\\
\Rightarrow B(T)=\frac{2 h}{c^{2}} \frac{k^{4} T^{4}}{h^{4}} \frac{\pi^{4}}{15} \tag{2.2.10}
\end{gather*}
$$

The Stefan-Boltzmann constant $\sigma$ is given by,

$$
\begin{equation*}
\sigma=\frac{2 \pi^{5} k^{4}}{15 h^{3} c^{2}} \tag{2.2.11}
\end{equation*}
$$

taking the ratios of 2.2 .10 and 2.2 .11 and rearranging

$$
\begin{gather*}
\Rightarrow B \pi=\sigma T^{4}  \tag{2.2.12}\\
\Rightarrow L=4 \pi R^{2} \sigma T^{4}  \tag{2.2.13}\\
F=B \pi=\frac{2 \pi h \nu^{3}}{c^{2}} \frac{1}{e^{\frac{h \nu}{k T}}-1}  \tag{2.2.14}\\
F=B_{\lambda} \pi=\frac{2 \pi h c^{2}}{\lambda^{5}} \frac{1}{e^{\frac{h c}{\lambda k T}}-1} \tag{2.2.15}
\end{gather*}
$$

If the star is assumed to radiate like a black-body, we have $F=\sigma T^{4}$, which gives $L=4 \pi \sigma R^{2} T^{4}$.

There for, (2.2.12) becomes,

$$
\begin{equation*}
F=B \pi=\sigma T^{4} \tag{2.2.16}
\end{equation*}
$$

Substituting 2.2.16 in to (1.1.1)

$$
\begin{equation*}
m_{1}-m_{2}=-2.5 \log \left[\frac{\pi B_{1}}{\pi B_{2}}\right] \tag{2.2.17}
\end{equation*}
$$

Using 2.2.3

$$
\begin{equation*}
\frac{B_{\lambda_{1}}}{B_{\lambda_{2}}}=\left(\frac{\lambda_{2}^{5}}{\lambda_{1}^{5}}\right)\left(\frac{e^{\frac{h c}{\lambda_{2} k T}}-1}{e^{\frac{c c}{\lambda_{1} k T}}-1}\right) \tag{2.2.18}
\end{equation*}
$$

The temperature T solved from equation 2.2 .18 is a colour temperature.
for $\frac{h c}{\lambda k T} \gg 1, e^{\frac{h c}{\lambda k T}}-1 \approx e^{\frac{h c}{\lambda k T}}$
Equation 1.1.1 becomes

$$
\begin{gather*}
m_{1}-m_{2}=-2.5 \log \left[\left(\frac{\lambda_{2}}{\lambda_{1}}\right)^{5}\left(\frac{e^{\frac{h c}{\lambda_{2} k T}}}{e^{\frac{h c}{\lambda_{1} k T}}}\right)\right]  \tag{2.2.19}\\
m_{1}-m_{2}=12.5 \log \left(\frac{\lambda_{1}}{\lambda_{2}}\right)+\frac{2.5 h c}{k T}\left(\frac{1}{\lambda_{1}}-\frac{1}{\lambda_{2}}\right) \log e  \tag{2.2.20}\\
m_{1}-m_{2}=-2.5 \log \left(\frac{\nu_{2}}{\nu_{1}}\right)^{3}+\frac{h}{k T}\left(\nu_{2}-\nu_{1}\right) \log e \tag{2.2.21}
\end{gather*}
$$

### 2.3 Approximate forms

From the equation, 2.1.1)

For $\frac{h \nu}{k T} \gg 1$
we have, $e^{\frac{h \nu}{k T}} \gg 1$ then,

$$
\begin{align*}
& B_{\nu}(T) \approx B(\nu ; T)=\frac{2 h \nu^{3}}{c^{2}} \frac{1}{e^{\frac{h \nu}{k T}}} \\
& \quad \Rightarrow B_{\nu}(T) \approx \frac{2 h \nu^{3}}{c^{2}} e^{-\frac{h \nu}{k T}} \tag{2.3.1}
\end{align*}
$$

and to have the corresponding wavelength-dependent version, using

$$
B_{\lambda}(T)=\frac{2 h c^{2}}{\lambda^{5}} \frac{1}{e \frac{h \nu}{\lambda k T}-1}
$$

when $\lambda \approx \lambda_{\max }$ or $\frac{h c}{\lambda k T} \gg 1$
we have, $e^{\frac{h c}{\lambda k T}} \gg 1$ then,

$$
\begin{align*}
& B_{\lambda}(T) \approx \frac{2 h c^{2}}{\lambda^{5}} \frac{1}{e^{\frac{h \nu}{\lambda k T}}} \\
& B_{\lambda}(T) \approx \frac{2 h c^{2}}{\lambda^{5}} e^{\frac{-h c}{\lambda k T}} \tag{2.3.2}
\end{align*}
$$

when

$$
\lambda \gg \lambda_{\max } \text { or } \frac{h c}{\lambda k T} \ll 1
$$

we have

$$
\begin{gather*}
e^{\frac{h c}{\lambda K T}} \approx 1+\frac{h c}{\lambda k T} \\
B_{\lambda}(T) \approx \frac{2 h c^{2}}{\lambda^{5}} \frac{1}{e^{\frac{h c}{\lambda k T}}-1} \\
B_{\lambda}(T) \approx \frac{2 h c^{2}}{\lambda^{5}} \frac{1}{1+\frac{h c}{\lambda k T}-1} \\
B_{\lambda}(T) \approx \frac{2 h c^{2}}{\lambda^{5}} \frac{1}{\frac{h c}{\lambda k T}} \\
B_{\lambda}(T) \approx \frac{2 h c^{2}}{\lambda^{5}} \frac{\lambda k T}{h c}=\frac{2 c k T}{\lambda^{4}} \tag{2.3.3}
\end{gather*}
$$

Using (2.3.3) in 2.2.16),

$$
\begin{equation*}
F=\frac{2 c k T \pi}{\lambda^{4}} \tag{2.3.4}
\end{equation*}
$$

And Using the definition of luminosity

$$
\begin{equation*}
L=4 \pi R^{2} \frac{2 c k T \pi}{\lambda^{4}} \tag{2.3.5}
\end{equation*}
$$

### 2.4 Effective Temperature $T_{e}$

It is the most important quantity describing the surface temperature of a star. It is defined as the temperature of a blackbody which radiates with the same total flux density as the star. The flux density on the surface is,

$$
F=\sigma T_{e}^{4}
$$

The total flux is $L=4 \pi R^{2} F$, where R is the radius of the star, and the flux density at a distance $r$ is

$$
\begin{gather*}
F(r)=\frac{L}{4 \pi r^{2}} \\
F(r)=\frac{L}{4 \pi r^{2}}=\frac{4 \pi R^{2} F}{4 \pi r^{2}}=\frac{R^{2}}{r^{2}} F \\
F(r)=\frac{R^{2}}{r^{2}} F  \tag{2.4.1}\\
F(r)=\frac{R^{2}}{r^{2}} \sigma T_{e}^{4}  \tag{2.4.2}\\
F(r)=\left(\frac{R}{r}\right)^{2} \sigma T_{e}^{4}  \tag{2.4.3}\\
F(r)=\left(\frac{\alpha}{2}\right)^{2} \sigma T_{e}^{4} \tag{2.4.4}
\end{gather*}
$$

where $\alpha=\frac{2 R}{r}$ is angular diameter of the star. The flux density $F_{\lambda}$ on the surface of the star is obtained from Plancks law, $F_{\lambda}=\pi B_{\lambda}$

$$
\begin{equation*}
F_{\lambda}(r)=\frac{R^{2}}{r^{2}} F_{\lambda} \tag{2.4.5}
\end{equation*}
$$

Also if $\alpha$ is known,

$$
\begin{equation*}
F_{\lambda}(r)=\left(\frac{\alpha}{2}\right)^{2} \pi B_{\lambda}\left(T_{b}\right) \tag{2.4.6}
\end{equation*}
$$

where $T_{b}$ is the brightness temperature
If the intensity at frequency $\nu$ is $I \nu$ the brightness temperature is obtained from

$$
I_{\nu}=B_{\nu}\left(T_{b}\right)=\frac{2 h \nu^{3}}{c^{2}} \frac{1}{e^{\frac{h \nu}{k T_{b}}}-1}
$$

For $h \nu \ll k T_{b}$ the above equation be comes,

$$
\begin{gather*}
I_{\nu}=B_{\nu}\left(T_{b}\right)=\frac{2 h \nu^{3}}{c^{2}} \frac{1}{1+\frac{h \nu}{k T_{b}}-1} \\
I_{\nu}=B_{\nu}\left(T_{b}\right)=\frac{2 h \nu^{3}}{c^{2}} \frac{k T_{b}}{h \nu} \\
B_{\nu}\left(T_{b}\right) \approx \frac{2 k \nu^{2}}{c^{2}} T_{b}  \tag{2.4.7}\\
\Rightarrow F=\frac{2 k \nu^{2}}{c^{2}} T_{b}  \tag{2.4.8}\\
\Rightarrow T_{b}=\frac{c^{2}}{2 k \nu^{2}} I_{\nu}=\frac{\lambda^{2}}{2 k} I_{\nu} \tag{2.4.9}
\end{gather*}
$$

This is the expression for the radio astronomical brightness temperature

## Chapter 3

## Optical Depth And Atmospheric Extinction

The light coming from stars and other astronomical objects reaches us after traveling through the earths atmosphere. In this process it is scattered and absorbed. This leads to the attenuation of light which is called the earths atmospheric extinction. In this process, light incident on the Earth's atmosphere from an extraterrestrial source is diminished by passage through the Earth's atmosphere [4]. Thus, sources will always appear less bright below the Earth's atmosphere than above it.

### 3.1 Optical Thickness

If the space between the radiation source and the observer is not completely empty, but contains some interstellar medium, 1.1.5 no longer holds, because part of the radiation is absorbed by the medium or scattered away from the line of sight. All these radiation losses are called the extinction. Since the medium absorbs and scatters radiation, the flux L will decrease with increasing distance r .

In the interval $[r, r+d r]$, the extinction $d L$ is proportional to the flux L and the distance travelled in the medium:

$$
\begin{equation*}
d L=-\alpha L d r \tag{3.1.1}
\end{equation*}
$$



Figure 3.1: The interstellar medium absorbs and scatters radiation; this usually reduces the energy flux L in the solid angle $\omega(\mathrm{dL} \leq 0)$ [Source,Fundamental Astronomy, Fifth Edition, page 89]

The factor $\alpha$ is called the opacity and it tells how effectively the medium can obscure radiation.
$[\alpha]=m^{-1}$
$\alpha=0 \Rightarrow$ for a perfect vacuum
$\alpha=\infty \Rightarrow$ for murky substance
The dimensionless quantity, the optical thickness $\tau$ is given by

$$
\begin{equation*}
d \tau=\alpha d r \tag{3.1.2}
\end{equation*}
$$

substituting this in the above we get

$$
\begin{align*}
& d L=-L d \tau  \tag{3.1.3}\\
& \Rightarrow \frac{d L}{L}=-d \tau
\end{align*}
$$

Integrating this from the source (where $\mathrm{L}=L_{0}$ and $\mathrm{r}=0$ ) to the observer:

$$
\begin{gathered}
\int_{L_{0}}^{L} \frac{d L}{L}=-\int_{0}^{d \tau} d \tau \\
\ln \frac{L}{L_{0}}=-\tau
\end{gathered}
$$

$$
\begin{align*}
& \Rightarrow \frac{L}{L_{0}}=e^{-\tau} \\
& \Rightarrow L=L_{0} e^{-\tau} \tag{3.1.4}
\end{align*}
$$

Where $\tau$ is the optical thickness of the material between the source and the observer and L is the observed flux.

The flux L falls off exponentially with increasing optical thickness. $\alpha=0 \Rightarrow$ Empty space (perfectly transparent)

If $F_{0}$ is the flux density on the surface of a star and $\mathrm{F}(\mathrm{r})$ is the flux density at a distance $r$, then the flux is expressed as

$$
\begin{aligned}
& F(r)=\frac{L}{4 \pi r^{2}} \\
\Rightarrow & L=4 \pi r^{2} F(r)
\end{aligned}
$$

and

$$
\begin{aligned}
F_{0} & =\frac{L_{0}}{4 \pi R^{2}} \\
\Rightarrow L_{0} & =4 \pi R^{2} F_{0}
\end{aligned}
$$

R is the radius of the star. substituting these values in the equation $\Rightarrow L=L_{0} e^{-\tau}$, we get,

$$
\begin{gather*}
4 \pi r^{2} F(r)=4 \pi R^{2} F_{0} e^{-\tau} \\
\Rightarrow F(r)=\frac{R^{2}}{r^{2}} F_{0} e^{-\tau}  \tag{3.1.5}\\
\frac{F(r)}{F_{0}}=\frac{R^{2}}{r^{2}} e^{-\tau} \tag{3.1.6}
\end{gather*}
$$

$$
\begin{gather*}
m-m_{0}=-2.5 \log \left(\frac{F(r)}{F_{0}}\right)=-2.5 \log \frac{R^{2}}{r^{2}} e^{-\tau}  \tag{3.1.7}\\
m-m_{0}=-2.5 \log \left(\frac{R}{r}\right)^{2}+1.086 \tau \tag{3.1.8}
\end{gather*}
$$

### 3.2 Atmospheric Extinction

EMR is attenuated by its passage through the atmosphere
Attenuation $=$ scattering + absorption
Scattering is the redirection of radiation by reflection and refraction
Attenuation is wavelength dependent

### 3.2.1 Absorption

On UV side primary absorption is Ozone $\left(O_{3}\right)$ and,
On IR side water vapor and $\mathrm{CO}_{2}$.

### 3.2.2 Scattering

Due to scattering, the direction and energy of a photon are changed Scattering of light by molecules is called Rayleigh scattering after the British physicist John William Strutt, the third Baron Rayleigh (1842-1919) who first described this phenomenon mathematically 3 (Bucholtz, 1995).

- Rayleigh Scattering (molecular scattering) is scattering by molecules and particles whose diameters are $\ll$ wavelength.

It is primarily due to oxygen and nitrogen molecules. Scattering intensity is proportional to $\lambda^{-4}$ responsible for blue sky.

Scattering of light by aerosol is called Mie scattering after the German physicist Gustav Mie (1868-1957) who first described this phenomenon mathematically. Aerosol particles have sizes in the same range as the wavelength of light ( 100-1000 nm), so they scatter light differently than molecules, which are much smaller [6] (Hodkinson, Greensleaves, 1963).

- Mie Scattering particles have a mean diameter 0.1 to 10 times the incident wavelength.

Examples: water vapor, smoke particles, fine dust etc. Scattering intensity is proportional to $\lambda^{-4}$ to $\lambda^{0}$ (depending on particle diameter)

Absorption and scattering of photons by collision with air molecules or particles During the absorption process a photon is destroyed and its energy transferred to the molecule, leading sometime to subsequent emission.

Clear atmosphere has both Rayleigh and Mie scattering. Their combined influence is $\lambda^{-0.7}$ to $\lambda^{-2}$

During sun rise and sun set sun light is attenuated due to Rayleigh scattering and Mie scattering from a particularly long passage through Earth's atmosphere, [5] and the Sun is sometimes faint enough to be viewed comfortably with the naked eye or safely with optics (provided there is no risk of bright sunlight suddenly appearing through a break between clouds).

### 3.2.3 Refraction.

Since light is refracted by the atmosphere, the direction of an object differs from the true direction by an amount depending on the atmospheric conditions along the line of sight. Since this refraction varies with atmospheric pressure and temperature, it is very difficult to predict it accurately. If the object is not too far from the zenith, the atmosphere between the object and the observer can be approximated by a stack of parallel planar layers, each of which has a certain index of refraction $n_{i}$. Outside the atmosphere, we have $\mathrm{n}=1$.


Figure 3.2: Refraction of a light ray travelling through the atmosphere

If the true zenith distance be z and the apparent one is $\zeta$, using fig (3.2) we obtain the following equations for the boundaries of the successive layers

$$
\begin{aligned}
& \sin z= n_{k} \sin z_{k} \\
& n_{k} \sin z_{k}= n_{(k-1)} \sin z_{(k-1)} \\
& \cdot \\
& \cdot \\
& \cdot \\
& \cdot \\
& n_{2} \sin z_{2}=n_{1} \sin z_{1} \\
& n_{1} \sin z_{1}=n_{0} \sin \zeta
\end{aligned}
$$

When the refraction angle $R=z-\zeta$ is small and is expressed in radians, we have,

$$
\begin{gather*}
n_{0} \sin \zeta=\sin z \\
n_{0} \sin \zeta=\sin (R+\zeta) \\
=\sin R \cos \zeta+\cos R \sin \zeta \\
n_{0} \sin \zeta=R \cos \zeta+\sin \zeta \\
n_{0} \sin \zeta-\sin \zeta=\cos \zeta \\
\left(n_{0}-1\right) \sin \zeta=R \cos \zeta \\
\Rightarrow R=\left(n_{0}-1\right) \tan \zeta \tag{3.2.1}
\end{gather*}
$$

The index of refraction depends on the density of the air, which further depends on the pressure and temperature. When the altitude is over $15^{0}$ we can use an approximate formula

$$
\begin{equation*}
R=\frac{P}{273+T} 0.00452^{0} \tan (90-a) \tag{3.2.2}
\end{equation*}
$$

where a is the altitude in degrees, T temperature in degrees Celsius, and P the atmospheric pressure in hectopascals (or, equivalently, in millibars).

At lower altitudes the curvature of the atmosphere must be taken into account. An approximate formula for the refraction is then

$$
\begin{equation*}
R=\frac{P}{273+T} \frac{0.1594+0.0196 a+0.00002 a^{2}}{1+0.505 a+0.0845 a^{2}} \tag{3.2.3}
\end{equation*}
$$

### 3.2.4 Atmospheric emission

Daytime scattering of sunlight prevents observations in visible and near infrared Also You can think of the atmosphere as an absorbing slab. Consider the beam of light from some star that will hit our telescope mirror. Just outside the atmosphere, the beam has flux $f_{\text {inc }}$ (for flux incident). At the telescope, the flux of this beam is less due to absorption and scattering of light out of the beam. We will call the observed flux at our telescope(see Figure 3.3


Figure 3.3: Beam of light from an object that will hit our telescope. (There are, of course, many more beams from the object that will NOT hit our telescope mirror!) $f_{\text {inc }}$ is the flux in the beam outside the atmosphere. fobs is the flux as it enters the telescope.

Obviously, if we look straight up (at the zenith) we have the minimum possible path
length through the atmosphere (for a given altitude of observatory). At an angle $\theta_{z}$, from the zenith (called the zenith angle) the amount of air we look through, relative to that at zenith, is simply given by $\operatorname{secant} \theta_{z}$ (see Figure 3.4 ):


Figure 3.4: Air mass equals secant of zenith angle, except close to the horizon where we have to take the curvature of the atmosphere into account.

When we are looking straight up we say we are looking through "1 air mass". At other zenith angles, we look through "secant $\theta$ air masses" (NOTE: The secant $\theta$ formula strictly applies only in an infinite at slab. Because the atmosphere is curved (due to the curvature of the Earth), air mass is not exactly secant $\theta$ but the difference between the real air mass and secant $\theta$ is significant only for lines of sight near the horizon (theta approaching 90 degrees). Also note that the fact that the atmosphere becomes less dense as we go up does not change the secant $\theta$ formula, as long as the density is the same at different places at the same altitude above sea level, which it is).

1 clear unit air mass transmits $82 \%$ in the visual, i.e. it dims by 0.215 mag . 4]
From the previous discussion, it is obvious that the relation of $f_{\text {obs }}$ (at zenith) and $f_{\text {inc }}$ can be specified by the optical depth of the atmosphere at zenith. Lets call that $\tau_{1}$. How can we determine $\tau_{1}$ ? One way would be to measure $f_{o b s}$ from a particular star with our telescope, then measure $f_{\text {inc }}$ above the atmosphere, either by boosting our telescope into
space (too expensive!) or somehow blowing away the atmosphere (which would greatly inconvenience people on the Earth!) What we can do is measure the change in extinction in the atmosphere at different air masses (different zenith angles) and extrapolate the observed flux to 0 air mass (above the atmosphere). One common way to do this is to measure the flux of a star, waiting until the star rises or sets some, so that the zenith angle changes, then measure the flux again. From these two fluxes, the optical depth at zenith (or as we see below, a closely related quantity called the extinction coefficient) can be derived. An example: say we observe a star at an air mass of $1.2\left(\tau=1.2 \tau_{1}\right)$ and we measure a flux which we will call $f_{1.2}$. Several hours later, when the star is lower in the sky at an air mass of 2.1 ( $\left.\tau=2.1 \tau_{1}\right)$, we measure a flux of 0.6 times that at 1.2 air mass $\left(f_{2.1}=0.6 f_{1.2}\right)$. One can easily calculate $\tau_{1}$ from the following two equation (Solve for $\tau_{1}$ by dividing one equation by the other, or by substitution. )
$f_{1.2}=f_{\text {inc }} e^{-1.2 \tau_{1}}$
$f_{2.1}=0.6 f_{1.2}=f_{\text {inc }} e^{-2.1 \tau_{1}}$
Instead of talking about $\tau_{1}$ directly, photometrists usually characterize the opacity of the atmosphere by a quantity called the absorption coefficient, usually designated by K. It has "units" of magnitudes per unit air mass. K is simply the ratio of $f_{\text {inc }}$ and $f_{\text {obs }}$ at the zenith, expressed in magnitudes

$$
\begin{equation*}
K=2.5 \log \left(\frac{f_{\text {inc }}}{f_{\text {obs }}(\theta=0)}\right) \tag{3.2.4}
\end{equation*}
$$

### 3.3 Atmospheric Windows

The general atmospheric transmittance across the whole spectrum of wavelengths is shown in Fig(3.5) The atmosphere selectively transmits energy of certain wavelengths. The spectral bands for which the atmosphere is relatively transparent are known as atmospheric windows. Atmospheric windows are present in the visible part and the infrared regions of
the EM spectrum. In the visible part transmission is mainly effected by ozone absorption and by molecular scattering. The atmosphere is transparent again beyond about $\lambda=1 \mathrm{~mm}$, the region used for microwave remote sensing. It is a regions in the EM spectrum, where


Figure 3.5: Atmospheric Windows[Source,Satellite Remote Sensing and GIS Applications in Agricultural Meteorology, page,35]
energy can be fully transmitted or those wavelengths that are relatively easily transmitted through the atmosphere

The windows: UV and visible: $0.30-0.75 \mu \mathrm{~m}$
Near infrared: $0.77-0.91 \mu m$
Mid infrared: $1.55-1.75 \mu m, 2.05-2.4 \mu m$
Far infrared:3.50-4.10 $\mu m, 8.00-9.20 \mu m, 10.2-12.4 \mu m$
Microwave: $7.50-11.5 \mathrm{~mm}, 20.0+\mathrm{mm}$
The atmospheric windows are important for RS sensor design

### 3.4 The Transfer of Radiation

When observing an astronomical source we may be looking through a cloud of matter which lies along the line of sight. This matter may absorb the radiation from the source, scatter
it or in fact emit further radiation. Each of these will change the source's apparent intensity.

### 3.4.1 Absorption



Consider light shining into a sparse cloud of perfectly absorbing spheres of cross section $\sigma_{\nu}$ and number density $n$. As the beam of area $d A$ propagates a distance $d s$ into the cloud it encounters a total absorbing cross section of $\sigma_{\nu} n d s, d A$, so we expect a fraction $\sigma_{\nu} n d s$ of the beam to be absorbed.

$$
\begin{equation*}
d I_{\nu}=-I_{\nu} n \sigma_{\nu} d s \equiv-\alpha_{\nu} I_{\nu} d s \tag{3.4.1}
\end{equation*}
$$

where $\alpha_{\nu}=n \sigma_{\nu}$ is defined as the fractional loss of intensity per unit length, with dimensions $m^{-1}$. It follows that the photon mean free path $l_{\nu}=\frac{1}{\alpha_{\nu}}$

### 3.4.2 Emission

An excited atom can return to its ground state through two distinct mechanisms:
(i) the atom emits energy spontaneously; (ii) it is stimulated into emission by the presence of electromagnetic radiation. The amount of stimulated emission is proportional to $I_{\nu}$, as was the amount of absorption. Therefore for simplicity it can be considered to be negative absorption and its effect included in $\alpha_{\nu}$.

We define a spontaneous emission coefficient $j_{\nu}$ which is the energy emitted per unit

time per unit volume per unit solid angle per unit frequency. This has unit $W m^{-3} s r^{-1} H z^{-1}$

$$
\begin{equation*}
d E=j_{\nu} d V d \Omega d \nu d t \tag{3.4.2}
\end{equation*}
$$

Thus on crossing a length $d s$ a beam's specific intensity is increased by spontaneous emission by

$$
\begin{equation*}
d I_{\nu}=j_{\nu} d s \tag{3.4.3}
\end{equation*}
$$

### 3.5 The optical depth

A medium is said to be opaque or optically thick if on average a photon cannot pass through the medium without absorption. Conversely, a transparent medium is said to be optically thin. Both these properties are functions of wavelength; for example, a pane of glass is optically thin in the optical, but optically thick in the infrared.

We define the optical depth $\tau_{\nu}$,

$$
\begin{equation*}
\tau_{\nu}=\int \alpha_{\nu} j_{\nu} \tag{3.5.1}
\end{equation*}
$$

### 3.6 The Radiative Transfer Equation

Combining emission and absorbtion gives the Radiative Transfer Equation

$$
\begin{equation*}
\frac{d I_{\nu}}{d s}=-\alpha_{\nu} I_{\nu}+j_{\nu} \tag{3.6.1}
\end{equation*}
$$

This can be rewritten as,

$$
\begin{equation*}
\frac{d I \nu}{d \tau}=-I_{\nu}+S_{\nu} \tag{3.6.2}
\end{equation*}
$$

where we have defined the source function $s_{\nu}=\frac{j_{\nu}}{\alpha_{\nu}}$ This is the value approached by $I_{\nu}$ given sufficient optical depth.

For the case of a homogeneous cloud with a constant source function and optical depth $\tau_{\nu}$ with initial incident intensity $I_{0}$, the emergent intensity is,

$$
\begin{equation*}
I_{\nu}=I_{0} e^{-\tau_{\nu}}+\left(1-e^{-\tau_{\nu}}\right) S_{\nu} \tag{3.6.3}
\end{equation*}
$$

This makes a lot of sense: the emergent radiation is the sum of the incident intensity attenuated by the total optical depth plus the sum of each section of cloud emission attenuated by the optical depth from that point to the receiver.

### 3.7 Air mass

Air mass is the path length that light from a celestial object takes through Earths atmosphere relative to the length at the zenith. Air mass refers to the thickness of the atmosphere for a given source position. The air mass is 1 at the zenith and roughly 2 at an altitude of $30^{\circ}$. For small zenith angles $(z)$ it is calculated as the secant of $z$.

$$
\begin{equation*}
A=\sec (z)=\frac{1}{\cos (z)} \tag{3.7.1}
\end{equation*}
$$



Figure 3.6: Observing a source through a Homogeneous Cloud

In a curved atmosphere the air mass is usually smaller than 40 , and (3.7.1) does not apply. A good approximation is ( z in degrees)

The path length of light through the atmosphere depends on the location of an object in the sky. The level of extinction of the stars light depends on its zenith angle. If a light beam of flux F passes through a thickness of material dX, with an opacity, $\tau$ (related to the absorption), the extinction loss the amount absorbed or scattered $d F$ is,

$$
\begin{equation*}
d F=-F \tau d X \Rightarrow \frac{d F}{F}=-\tau d X \tag{3.7.2}
\end{equation*}
$$

Integrating both sides,

$$
\begin{gather*}
\ln \frac{F}{F_{0}}=-\tau X  \tag{3.7.3}\\
\frac{F}{F_{0}}=e^{-\tau X}  \tag{3.7.4}\\
\log \frac{F}{F_{0}}=-\tau X \log e \tag{3.7.5}
\end{gather*}
$$

$$
\begin{equation*}
-2.5 \log F+2.5 \log F_{0}=0.4343 \tau x \log e \tag{3.7.6}
\end{equation*}
$$

$$
\begin{equation*}
m-m_{0}=K X \tag{3.7.7}
\end{equation*}
$$

where the K is called the extinction coefficient. The value of the extinction coefficient k depends on the observation site and time and also on the wavelength, since extinction increases strongly towards short wavelengths. And X is the total path length traversed by the light.

The intensity is given by

$$
\begin{gather*}
I=I_{0} e^{-\tau \sec \theta}  \tag{3.7.8}\\
\frac{I}{I_{0}}=e^{-\tau \sec \theta}  \tag{3.7.9}\\
m_{1}-m_{0}=-2.5 \log \left(\frac{I}{I_{0}}\right) \tag{3.7.10}
\end{gather*}
$$

Using (3.7.10)

$$
\begin{equation*}
m_{1}-m_{0}=2.5 \tau \sec \theta \log e \tag{3.7.11}
\end{equation*}
$$



Figure 3.7: If the zenith distance of a star is z , the light of the star travels a distance $\frac{H}{\cos z}$ in the atmosphere; H is the height of the atmosphere[Source,Fundamental Astronomy, Fifth Edition, page 90]

## Chapter 4

## Result and Discussion

The Set Of Summarized Relevant Equations are;

1. Planck's radiation law
a) Without extinction

$$
\begin{equation*}
B_{\lambda}=\frac{2 h c^{2}}{\lambda^{5}} \frac{1}{e^{\frac{h c}{\lambda k T}}-1} \tag{4.0.1}
\end{equation*}
$$

b) The intensity with extinction, where exponential atmosphere is assumed or where the Beer-Bouguet-Lambert law is considered for optical depth

$$
\begin{equation*}
I_{\lambda}=I_{0 \lambda} e^{-\tau} \tag{4.0.2}
\end{equation*}
$$

Or in terms of $B$

$$
\begin{equation*}
B_{\lambda_{e x}}=\frac{2 h c^{2}}{\lambda^{5}} \frac{1}{e^{\frac{h}{\lambda k T}}-1} e^{-\tau} \tag{4.0.3}
\end{equation*}
$$

2. Magnitudes
a) Using the relative apparent magnitude relation we derive the excess magnitude created in the atmospheric extinction

$$
\begin{equation*}
\Delta m=m_{\text {ext }}-m_{\text {noext }}=2.5 \tau \sec \theta \log [e] \tag{4.0.4}
\end{equation*}
$$

b) Also the excess magnitude in the bolometric magnitude as the result of the atmosphere is

$$
\begin{equation*}
\Delta M_{b o l}=2.5 \tau \sec \theta \log [e] \tag{4.0.5}
\end{equation*}
$$

With a particular temperature, $T=12000 K$ of typical stellar, the effect of atmosphere on the intensity and magnitudes received at the observation point on earth with air mass of $20 \& 70$ degrees both seeing with optical depth 0f $0.2 \& 0.8$ have been used to generate a numerical data computationally using MATIMATICA. The results are displayed as in 4.1 $\& 4.2$.


Figure 4.1: Effect of atmosphere on intensity. Intensity without extinction is being compared in the presence of extinction

AS we can see from the plots of 4.1 the intensity observed decreases in the present of atmosphere. The more optical depth of the atmosphere is the more it decreases the intensity. Moreover, the more air mass the seeing taking is the more the extinct it is.

The same phenomena also observed in the magnitudes as we see from 4.2.


Figure 4.2: Effect of atmosphere on magnitudes. Both apparent and bolometric magnitudes without extinction are being compared in the presence of extinction

## Chapter 5

## Summary and Conclusion

Astronomical object's observation is affected by different factors like atmospheric extinction such as Absorption Scattering, Atmospheric emission, clouds, dust particles in the air, air mass and motion of the air. That this extinction reduces the electromagnetic radiation that coming from the object to the earth.

It is important to make observation repeatedly rather than observing ones, in different time in a place with no dust in air, no cloud, and more or less no motion of air as well as at high altitude and small zenith angle to decrease air mass.

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# JIMMA UNIVERSITY <br> COLLEGE OF NATURAL SCIENCES PERFORMANCE CERTIFICATE FOR MASTER'S DEGREE 

Name of Student: Munawar Muzeyen Ali ID No. SMSC 00911/06
Graduate Program: Summer, MSc.

1. Course Work Performance

| Course <br> Code | Course Title | Cr. hr | Number <br> Grade | Rank ** | Remark |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Phys699 | MSc. Thesis | $\mathbf{6}$ | $\mathbf{7 7 . 4}$ | V. Good |  |

** Ecellent, Very Good, Good, Satisfactory, Fail.
Thesis Title

## EARTH'S ATMOSPHERIC EFFECT IN ASTRONOMICAL PHOTOMETRY/SPECTROSCOPY EXTINCTION

2. Board of Examiners decision Mark $\mathbf{X}$ in one of the boxes. Pass $\mathbf{X}$ Failed $\square$
If failed, give reasons and indicate plans for re-examination.
$\qquad$
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3. Approved by: Name and Signature of members of the examining Board, and Department Head

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# School of Graduate Studies Jimma University College of Natural Sciences MSc. Thesis Approval Sheet 

We the undersigned, number of the Board of Examiners of the final open defense by Munawar Muzeyen Ali have read and evaluated his/her thesis entitled
"Earth's Atmospheric Effect In Astronomical Photometry/Spectroscopy Extinction" and examined the candidate. This is therefore to certify that the thesis has been accepted in partial fulfilment of the requirements for the degree Master of Science in Physics(Astrophysics).

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