# EFFECTS OF INTERFACIAL LAYER ON REFRACTIVE INDEX AND PROPAGATION OF WAVES IN SMALL SPHERICAL METAL/DIELECTRIC COMPOSITES

By

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The undersigned hereby certify that they have read and recommend to the School of Graduate Studies for acceptance a thesis entitled "**Effects of Interfacial Layer on Refractive Index and Propagation of Waves in Small Spherical Metal/Dielectric Composites**" by **Dessalegn Kenate Tuge** as a requirement for the degree of **Masters of Science in Physics(Condensed Matter Physics)** .

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# Abstract

The increasing interest in optical properties of the composite media is dictated by the fact that small particles, which are randomly embedded in a linear dielectric host matrix, can be manufactured with the help of modern technology with desirable properties[1]. It is known that the particles with metal covering can considerably enhance the refractive index of the incident electromagnetic waves and reduce the group velocity if their frequency approaches to the frequency of the metal surface plasmon's[2]. In this work in comparison with no interfacial layer factor we have shown the effects of interfacial layer on the refractive index, and propagation of waves in metal/dielectric composite separated by interfacial layers which is randomly embedded in a linear dielectric host matrix. The corresponding theoretical, and numerical descriptions are in terms of the interfacial factor $(I)$ by making use of the Drude model. Finally, both the interfacial layer property and the percentage of the volume fraction of the metallic particles in the composite has a nonlinear optical response which strongly effect on the refractive index, and propagation of waves of the composite when the dielectric functions of the interfacial layer is more metal-like property than dielectric-like property.

**Key Words**: dielectric function, propagation of waves, optical properties, refractive index, interfacial layer.

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# Chapter 1

# Introduction

## 1.1 Back ground of the study

The nonlinear optical properties in disordered metal/dielectric composites have received much attention because of their potential application to optical correlator device and phase-conjugator[3]. A typical system is composed of nonlinear (or linear) metallic particles randomly embedded in a linear (or nonlinear) dielectric host. Detailed analysis of the dielectric properties of such composite reveals the existence of many phenomena such as nonlinear microscopy[4], ultrafast laser systems[5], optical routing and switching based on optically induced bistability both on networks[6], and on chips[7]. Many authors have studied the nonlinear optical response of such composite media[3]. But in this paper we are interested to study the theoretically the effects of interfacial layers on the refractive index, and propagation of waves in small spherical metal/dielectric composite separated by interfacial layer which is randomly embedded in a linear dielectric host.

As far as interfacial layer is concerned, the refractive index and propagation of waves of such composite are discussed analytically and numerically. The contribution of the interfacial layer on the linear and nonlinear response of the medium strongly effects on the propagation of electromagnetic wave, and refractive index in the optical material and can even result in the permanent modification of its physical properties. In turn, the linear and nonlinear optical features of composite materials with metal nanostructures are dominated by surface plasma oscillations. The fact that the surface plasmon (SP) strongly depends on size, shape, distribution of metal nanoparticles as well as on surrounding dielectric host offers an opportunity for manufacturing of new promising nonlinear materials, nanodevices, and optical elements. Metals have a fast and strong nonlinear response[8] and may be good candidates for nonlinear optical applications if they are combined with dielectrics[9]. Combining metals and dielectrics has two main purposes: One allowing light to enter more deeply into metals, and the other is achieving light localization which in turn leads to an enhanced nonlinear response.

The velocity of propagation of light in a material system can be controlled and modified to a large extent, ($v_g << c$), superluminal light ($V_g >> c$), and ($v_g$) negative have been reported[5]. The pioneering demonstrations of slow and fast light were all based on the exploitation of narrow spectral resonances, mainly created by electromagnetically-induced transparency[5] or coherent population oscillation[10]. Narrow spectral resonances induce an anomalous situation for the optical propagation, since any sharp spectral change in the medium transmission results in a steep linear variation of the effective refractive index along wavelength. This in turn results in a strong group velocity change at the exact center of the resonance[11].

## 1.2 Statement of the Problem :

The problem is to find the contribution of the interfacial layer effect on the refractive index, and propagation of wave of small spherical metal/dielectric composite separated by interfacial layer which is randomly embedded in a linear dielectric host matrix.

## 1.3 Objectives of the Study:

### 1.3.1 General Objectives:

The general objective of this study is: To analyze and describe the contribution of the interfacial layer effects on the propagation of wave and refractive index in small spherical metal/dielectric composite separated by interfacial layer which is randomly embedded in a linear dielectric host analytically and numerically.

### 1.3.2 Specific Objectives:

1/ To study the interfacial layer effect on the refractive index of small spherical particles of metal/dielectric composite separated by interfacial layer which is embedded in a linear dielectric host matrix, analytically and numerically.
2/ To study the interfacial layer effect on the propagation of waves of small spherical particle for metal/dielectric composite separated by interfacial layer which is randomly embedded in a linear dielectric host matrix, analytically and numerically.

## 1.4 Significance of the study:

The contribution of the linear and nonlinear response of the interfacial layer on the refractive index and propagation of waves in small sphericalmetal/dielectric composite materials needs to improve the optical properties of different materials by changing the properties of the interfacial layer factor, their size, and shape.

# Chapter 2

# Review Literature

## 2.1 The Maxwell Equations

The solution of the electromagnetism equations can be expressed by the Maxwell equation. Maxwell's hypothesis was confirmed in 1887 by Hertz who was able to produce and to detect electromagnetic waves[8].

The Maxwell equations relate the space and time derivatives of the electric and magnetic fields to each other throughout the continuous medium.If we adopt Gaussian units, next we present the Maxwell equations in a material medium [8].

$$\nabla \cdot \mathbf{D}(\mathbf{r},t) = 4\pi\rho(\mathbf{r},t), \tag{2.1.1}$$

$$\nabla \cdot \mathbf{B}(\mathbf{r},t) = 0. \tag{2.1.2}$$

$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\frac{1}{c}\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t}, \tag{2.1.3}$$

$$\nabla \times \mathbf{H}(\mathbf{r},t) = \frac{4\pi}{c}\mathbf{J}(\mathbf{r},t) + \frac{1}{c}\frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t}, \tag{2.1.4}$$

where $\mathbf{r}$ is the 3-dimensional coordinate vector, $t$ is time $\mathbf{D}(\mathbf{r},t)$ is electric displacement, $\rho(\mathbf{r},t)$ is the charge density, $\mathbf{B}(\mathbf{r},t)$ is the magnetic induction, $\mathbf{E}(\mathbf{r},t)$ is the electric field, $\mathbf{H}(\mathbf{r},t)$ is the magnetic field, and $\mathbf{J}(\mathbf{r},t)$ is the current density. The electric displacement and the magnetic induction are connected to the electric and magnetic field, respectively, as

$$\mathbf{D}(\mathbf{r},t) = \mathbf{E}(\mathbf{r},t) + 4\pi\mathbf{P}(\mathbf{r},t), \tag{2.1.5}$$

$$\mathbf{B}(\mathbf{r},t) = \mathbf{H}(\mathbf{r},t) + 4\pi\mathbf{M}(\mathbf{r},t) \tag{2.1.6}$$

where $\mathbf{P}(\mathbf{r},t)$, and $\mathbf{M}(\mathbf{r},t)$ are the polarization and magnetization of the medium respectively. These quantities describe the electromagnetic response of the medium. The current density is given by the following equation;

$$\vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}, \tag{2.1.7}$$

where $\sigma$ is the conductivity of the medium.

At optical frequencies the materials are usually non-magnetic, so $\mu = 1$. Therefore, the magnetization can be omitted. Based on this approximation, the optical response of a medium to an electromagnetic perturbation is completely described only by the relation between the polarization and the electric field inducing it.

## 2.2 Nonlinear Optics

In order to describe an optical nonlinearity, we consider how polarization $\mathbf{P}(t)$, of a material depends on an applied optical filed $\mathbf{E}(t)$. In the case of linear optics the induced polarization depends linearly on the electric filed strength described by the relationship,

$$\mathbf{P}(t) = \chi^{(1)}\mathrm{E}(t), \tag{2.2.1}$$

where, $\chi^{(1)}$ is the linear susceptibility. In nonlinear optics, the optical response can be described by generalizing equation (2.2.1) as a power series in the field strength $\mathbf{E}(t)$ as

$$\mathbf{P}(t) = \chi^{(1)}\mathbf{E}(t) + \chi^{(2)}\mathbf{E}^2(t) + \chi^{(3)}\mathbf{E}^3(t) + ... \tag{2.2.2}$$

The quantities $\chi^{(2)}$and $\chi^{(3)}$ are known as the second- and third-order nonlinear optical susceptibilities, respectively. The applied filed $\mathbf{E}$ is of the order of the characteristics of atomic electric filed as shown in ref[11]. $E_{at} = \frac{e}{4\pi\epsilon_o a_o^2}$, where, $a_o$ is Bohr radius of Hydrogen atom. Numerically, $E_{at} = 5.14 \times 10^{11} v/m$. Thus, nonlinear polarization induced in the media by propagating monochromatic electromagnetic wave is responsible for optical harmonic generation. By considering the Maxwell equations given from equations $(2.1.1 - 2.1.4)$ the electromagnetic wave equation can be derived as follow,

$$\nabla \times \nabla \times \mathbf{E} + \frac{4\pi}{c^2}\sigma\frac{\partial \mathbf{E}}{\partial t} + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} + 4\pi\frac{\partial^2 \mathbf{P}}{\partial t^2} = 0 \tag{2.2.3}$$

In principle, one now requires a full microscopic theory of the optical response of a particular material to relate the macroscopic electric field $\mathbf{E}$ to the polarization $\mathbf{P}$, By substituting equations (2.1.1) and (2.1.2) into equation (2.1.3) ,we have

$$\nabla \times \nabla \times \mathbf{E} + \frac{4\pi}{c^2}\sigma\frac{\partial \mathbf{E}}{\partial t} + \frac{1+\chi^{(1)}}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} + 4\pi\frac{\partial^2 \mathbf{P}^{NL}}{\partial t^2} = 0 \tag{2.2.4}$$

For weak incident optical field the nonlinear component of polarization can be omitted, and equation (2.2.4) becomes,

$$\nabla \times \nabla \times \mathbf{E} + \frac{4\pi}{c^2}\sigma\frac{\partial \mathbf{E}}{\partial t} + \frac{1+\chi^{(1)}}{c^2}\frac{\partial^{(2)} \mathbf{E}}{\partial t^2} = 0 \tag{2.2.5}$$

the well known ordinary wave equation[12].

## 2.3 Lorentz local Field

The concept of a local field was originally introduced by Lorentz[13]. The response of a medium to an external electric field cannot be explained exactly by means of the macroscopic electric fields[14]. The external field drives the bound charges of the medium apart and induces a collection of dipole moments[7]. In an optically dense medium, the interaction of the induced dipoles is taken into account by a local field factor, which relates the macroscopic fields to the local ones. For example, local field in a homogeneous medium can be related to the macroscopic average field according to

$$\mathbf{E}_{Loc} = L\mathbf{E} \tag{2.3.1}$$

Where $L$ is the Lorentz local-field correction factor, and $\mathbf{E}$ is the macroscopic average field. To find the local field acting on a typical dipole moment of the medium, one surrounds the dipole of interest with an imaginary spherical cavity of radius, $r << \lambda$. For material with linear optical response, the local field determines the microscopic polarization which is given as,

$$p = \alpha E_{loc} \tag{2.3.2}$$

The macroscopic polarization of the medium is obtained by averaging equation (2.3.2) over the integrated volume $V$ as follows[15].

$$\mathbf{p} = \frac{1}{v}\int_v p dv = N\alpha\mathbf{E}, \tag{2.3.3}$$

where $dv$ is the unit volume for integration and $N$ is the density of microscopic constituents.

The polarization in terms of the external electromagnetic field is,

$$\mathbf{P} = \chi^{(1)}\mathbf{E}. \tag{2.3.4}$$

In order to express linear susceptibility in terms of microscopic polarization, we express $\mathbf{E}_{loc}$ in terms of $\mathbf{E}_{ext}$ as

$$\mathbf{E}_{Loc} = \mathbf{E} + \frac{4\pi}{3}\mathbf{P} \tag{2.3.5}$$

The dipole moment induced in a single molecule (or atom) of the medium is,

$$p = \alpha \; E_{loc} \tag{2.3.6}$$

The macroscopic polarization of the material is,

$$P = Np \tag{2.3.7}$$

By substituting equation (2.3.4) into (2.3.5), and the result obtained into equation (2.3.6), and then the obtained expression into equation (2.3.7), we have

$$P = N\alpha[E + \frac{4\pi}{3}\chi^{(1)}E] \tag{2.3.8}$$

And again by equating equation (2.3.4) with equation (2.3.8) we have an expression for the linear susceptibility

$$\chi^{(1)} = \frac{N\alpha}{1 - \frac{4\pi}{3}N\alpha}. \tag{2.3.9}$$

To define optical susceptibility in terms of dielectric function, we know the electric field displacement $\mathbf{D}$ is $\epsilon\mathbf{D} = \mathbf{E} + 4\pi\mathbf{p}$ , where linear polarization $p = \chi^{(1)}E$ and by eliminating the field $\mathbf{E}$, we find that

$$\chi^{(1)} = (\varepsilon^{(1)} - 1)/4\pi. \tag{2.3.10}$$

where, $\varepsilon^1$ is the dielectric permittivity of the medium, by equating equations(2.3.9), and (2.3.10), we obtain the well-known Lorentz-Lorenz (or Clausius-Mossotti) relation.

$$\frac{\varepsilon^{(1)}-1}{\varepsilon^{(1)}+2}=\frac{4\pi}{3}N\alpha. \tag{2.3.11}$$

By rearranging equation (2.3.11) we can describe the linear susceptibility as,

$$\chi^{(1)}=(\frac{\varepsilon^{(1)}+2}{3})N\alpha. \tag{2.3.12}$$

The Lorentz local electric field can be obtained by substituting equation (2.3.12) into equation (2.3.4) and then relating the result obtained with equation ($P = N\alpha E_{Loc}$) we have,

$$E_{Loc}=(\frac{\varepsilon^{(1)}+2}{3})E. \tag{2.3.13}$$

By equating equation (2.3.1) with equation (2.3.13), the Lorentz local field correction factor ($L$) depends on the macroscopic electric field as wheel as the dielectric properties of the medium as,

$$L=\frac{\varepsilon^{(1)}+2}{3} \tag{2.3.14}$$

therefor, equation (2.3.13), is the Lorentz local-field correction factor [1]. Equation (2.3.14) for the local-field correction factor is valid in the case of homogeneous media[7].

## 2.4 Refractive Index

The development of nanostructure media with electronic and optical properties are vastly different from that usually found in natural material. In the case of metal-dielectric composite media, their applications can practically limited by absorption of incident electromagnetic radiation due to the presence of metal components. In different research it was proposed to use active (amplifying) host-matrix in order to compensate absorption at metallic inclusion[1][16].

In this paper we will discuss the refractive index of a small spherical metal/dielectric composite materials with Drude model (a classical model) together with Taylor expansion. For our calculation of index of refraction for both pure metal with interfacial layer, and metal/dielectric composite materials separated by interfacial layer which are randomly embedded in a linear dielectric host matrix. The refractive index $n$ is given by the square root of the dielectric constant $\varepsilon$ . It is planned to analyze the interfacial layer effect on the refractive index of small spherical particles for metal/dielectric composite which is embedded in a linear dielectric host matrix. The refractive index is given by[17]

$$n = \sqrt{\varepsilon} = \sqrt{1 + 4\pi\chi} \tag{2.4.1}$$

## 2.5 Phase Velocity and Group Velocity

The concept of group velocity was first introduced in[18]. The first recorded observation of the group velocity of a wave is presented in ref[18]. A continuous wave light beam propagating in a medium with refractive index $n$ has a phase velocity

$$V_p = \frac{c}{n} \tag{2.5.1}$$

where, $c$ is the speed of light in vacuum, and the refractive index is given by $n = \frac{kc}{\omega}$ for the phase velocity. Let us next consider the propagation of a pulse through a material system. A pulse is necessarily composed of a spread of optical frequencies, given as $\phi = kz - \omega t$ , where $k = \frac{n\omega}{c}$ . No change in $\phi$ to first order in $\omega$. That is, $\frac{d\phi}{d\omega} = 0$ or $\frac{dn}{d\omega}\frac{\omega z}{c} + \frac{nz}{c} - t = 0$, which can be written as $z = v_g t$ , where the group velocity is given by

$$\begin{aligned} v_g &= \frac{c}{n_g} = \frac{c}{n + \omega\frac{dn}{d\omega}} \\ n_g &= n + \omega\frac{dn}{d\omega} \end{aligned} \tag{2.5.2}$$

Here $n_g$ is group index and $n_p$ is phase index. We see that $n_g \neq n_p$ by a term that depends on the dispersion $\frac{dn}{d\omega}$ of the refractive index. Slow and fast light effects invariantly make use of the rapid vibration of refractive index that occurs in the vicinity of a material resonance.Slow light can be obtained by making $\frac{dn}{d\omega}$ larger and positive (larger normal dispersion), and fast light occurs when $\frac{dn}{d\omega}$ is larger and negative (larger anomalous dispersion). In theoretical treatment of pulse propagation[2], it is continent to expand the propagation constant $k(\omega)$ in a power series about the central frequency $\omega_o$ of the optical pulse as

$$k(\omega) = k_o + k_1(\omega - \omega_o) + \frac{1}{2}k_2(\omega - \omega_o)^2 + - - -- \tag{2.5.3}$$

where $k_o = k(\omega_o)$ is the mean wave vector magnitude of the optical pulse, $k_1 = \frac{dk}{d\omega}/\omega = \omega_o = \frac{1}{V_g} = \frac{n_g}{c}$ is the inverse of the group velocity, and $k_2 = \frac{d^2k}{d\omega^2}/\omega = \omega_o = \frac{d(\frac{1}{v_g})}{d\omega} = \frac{1}{c}\frac{dn_g}{d\omega}$ is a measure of the dispersion in the group velocity. Because the transit time through a material medium of length $z$ is given by $t = \frac{z}{V_g}$ but $k_1 = \frac{1}{v_g}$ which implies that $t = zk_1$ the spread in transit times is given approximately by $\Delta t \simeq zk_2\Delta\omega$, where $\Delta\omega$ is a measure of frequency bandwidth of the pulse. Group velocity keeps its physical meaning as long as the propagation distance of slow and fast light through the dispersive medium is much less than a narrow spectral bandwidth for the pulse. Present interest in negative group velocity based on anomalous dispersion in a composite medium, where the sign of the phase velocity is the same for incident and transmitted waves, and energy flows inside the composite medium in the opposite direction to the incident energy flow in vacuum. In this paper we analyze interfacial layer effects on the propagation of waves in small spherical metal/dielectric composite separated by interfacial layer.

# Chapter 3

# Materials and Methods

## 3.1 Materials

The thesis is aimed at theoretical study and numerical analysis of the interfacial layer effect on the refractive index and propagation of waves in small spherical particles of metal/dielectric composite separated by interfacial layer which is embedded in a linear dielectric host. The theory is supposed to be developed in the long wave approximation, which means that the wavelength of radiation is much greater than the typical size of inclusions. Because of the complexity of the equations of the electrodynamics of the composite media even with usage of different approaches such as Maxwell - Garnet formula, it would be necessary to employ different mathematical codes such as Mat-lab and Mathematica softwares. The apparatus that will be applied to carry out the theoretical part of the thesis are external hard disc, flash discs and softwares for simulating the dielectric functions of the composite materials.

## 3.2 Methodology

### 3.2.1 Analytical method

In our thesis one of the important methods is solving the problem analytically which is the most important input for the numerical computation.

### 3.2.2 Numerical method

For determining the most important parameter for the interfacial layer effect we follow to compute the analytical results with some computational tools in Mat-Lab codes.

# Chapter 4

# Data Analysis and Discussion

## 4.1 Effects of Interfacial Layer on the Refractive Index and Propagation of Waves

### 4.1.1 Introduction

It is shown that in different papers the presence of two enhancement in small spherical metal/dielectric composite particles with no interfacial layer. Also it is shown in several papers a one enhancement for pure metals in a host matrix[1],[6],[16]. In this work we consider the effect of the interfacial layer on the refractive index of small spherical pure metal particle with interfacial layer placed in a linear host and the effects of interfacial layers on the refractive index of small spherical composite of metal/dielectric particles separated by interfacial layer which are randomly embedded in a linear host matrix. The interfacial layer separating metal from dielectric core composites is described by the interfacial layer factor $I$ that may be positive or negative which represent the dielectric functions of the interfacial layer placed between the composite. Furthermore, we consider $f$ (volume fraction of metal particles in the linear host matrix) that may effect the refractive index of the composite.

### 4.1.2 Drude Model

The electric behavior of metals and semiconductors with high electron concentration as well as of plasmas is determined by the collective excitation of the free charges. The displacement of the free charges against the ionic trunks results in positively and negatively charged clouds which exert an attractive force on each other. For metals there is no spring to connect free electrons to ions. So that resonance frequency $\omega_o = 0$. Thus, the dielectric function of metal $\varepsilon_m$ is chosen in the Drude form.

$$\varepsilon_m = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}, \tag{4.1.1}$$

where $\omega_P$ is the plasma frequency expressed by $\omega_p = \sqrt{\frac{Ne^2}{m\epsilon_o}}$ , $\omega$ is the frequency of the incident wave, $e$ is the charge of electron, $\epsilon_o$ is the permittivity of free space,$m$ is the mass of electron, $N$ is the concentration of electron, and $\gamma$ is the damping constant. Then the real and imaginary part of equation (4.1.1) become,

$$\varepsilon_m' = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2 + \gamma^2} \tag{4.1.2}$$

and,

$$\varepsilon_m'' = \frac{\gamma\omega_p^2}{\omega^3 + \omega\gamma^2} \tag{4.1.3}$$

respectively. Equation (4.1.1)-(4.1.3), are called Drude model[19]. Let us introduce now dimensionless frequency, $z$, and $\upsilon$ as, $z = \frac{\omega}{\omega_p}$ , and $\upsilon = \frac{\gamma}{\omega_p}$. So, equations (4.1.2), and (4.1.3) become

$$\varepsilon_m' = \varepsilon_\infty - \frac{1}{z^2 + \gamma^2}, \tag{4.1.4}$$

and

$$\varepsilon_m'' = \frac{\gamma}{z(z^2 + \gamma^2)}, \tag{4.1.5}$$

respectively.

## 4.2 Analytical Descriptions of the Effect of Interfacial Layer for Refractive Index in Composite

We consider a composite in which a pure nonlinear metallic particles with interfacial layer embedded in a linear host matrix, and also a nonlinear spherical particles of metal/dielectric separated by interfacial layer which is randomly embedded in a linear dielectric host matrix respectively. In the electrostatic approximation, when the wavelength $\lambda$ of the incident electromagnetic radiation is much greater than a typical size of the inclusion, the distribution of the electric potential in the system is described by the following expressions[18]. Let us see first the analytical descriptions of the effects of the Interfacial layer on the refractive index of the composite for the case of pure metal with interfacial layer.

### 4.2.1 Analytical Descriptions of the Effect of Interfacial Layer for Refractive Index in Pure Metal/Interfacial Layer Composite.

For metal containing interfacial layer the electrostatic potential distribution in the composite is expressed as,

$$\begin{aligned} \Phi_m &= -E_{ap}Arcos\theta, r \leq r_1, \\ \Phi_i &= -E_{ap}(Br - Cr^{-2})cos\theta, r_1 \leq r_{1+t} \leq r, \\ \Phi_h &= -E_{ap}(r - Dr^{-2})cos\theta, r_{1+t} \leq r, \end{aligned} \tag{4.2.1}$$

where, $\phi_m$, $\phi_i$, and $\phi_h$ are potentials of the pure metal, interfacial layer, and linear host matrix respectively, $E_{ap}$ is the applied field, $r_1$, and $r_1 + t$ is the radius of pure metal, and the interfacial layer, respectively. Also $t$ is the thickness of the interfacial layer, and $A, B, C$, and $D$ are the unknown coefficients. By equating their electrostatic potential and electric displacement at the boundaries, we obtain expression for the induced dipole moment of the polarizability $D$ , and the local-field correction factor $A$ as

$$\begin{aligned} A &= \frac{3\varepsilon_h}{2\varepsilon_h + \varepsilon_m + \frac{2I}{r_1}}, \\ B &= \frac{2\varepsilon_h}{2\varepsilon_h + \varepsilon_m + \frac{2I}{r_1}}, \\ C &= \frac{-\varepsilon_h}{2\varepsilon_h + \varepsilon_m + \frac{2I}{r_1}} r_1^3, \\ D &= (\frac{\varepsilon_m + \frac{2I}{r_1} - \varepsilon_h}{2\varepsilon_h + \frac{2I}{r_1} + \varepsilon_m})(r_1 + t)^3. \end{aligned} \tag{4.2.2}$$

Here $I$ is the Interfacial layer factor defined as $I = \lim t\varepsilon_s$ as thickness $t$ tends to zero, and the dielectric functions of the interfacial layer goes to infinity ($\varepsilon_s \to \infty$ )[20]. Now, we consider the effect of interfacial layer through the limit $t \to 0$, namely the interfacial property is concentrated on a surface of approximate zero thickness ($t$); then only $t\varepsilon_s$ can be seen as a significant quantity. Here I is just called interfacial factor[21]. Generally speaking, for sharp and smooth interface is equal to zero ($i, eI = 0$), there is no jump in the normal component of the electric displacement across the interface; whereas, for imperfect interface it is denoted by $I \neq 0$, in this case the electric displacement jumps across the interface. We further remark that I can be taken as positive or negative values, which is resemble because the dielectric functions of metallic particles is made up of a large negative real and a small positive imaginary[22]. When I is taken as negative value, the interface exhibits metal-like behavior but for $I = +ve$ value, the interface exhibits dielectric-like behavior. Therefore, the polarization of an individual pure metal with interfacial layer embedded in a linear host matrix can be presented in the form of equation (4.2.2). Where the polarizability of the composite is

$$\alpha = \frac{\varepsilon_m + \frac{2I}{r_1} - \varepsilon_h}{2\varepsilon_h + \frac{2I}{r_1} + \varepsilon_m} \tag{4.2.3}$$

Here the real $\alpha'$ and imaginary $\alpha''$ parts of the polarizability of equation (4.2.3) is described as,

$$\alpha' = \frac{(\varepsilon'_m + \frac{2I}{r_1} - \varepsilon'_h)(2\varepsilon'_h + \frac{2I}{r_1} + \varepsilon'_m) + \varepsilon''^2_m}{(2\varepsilon'_h + \frac{2I}{r_1} + \varepsilon'_m)^2 + \varepsilon''^2_m}, \tag{4.2.4}$$

and

$$\alpha'' = \frac{3\varepsilon''_m\varepsilon'_h}{(2\varepsilon'_h + \frac{2I}{r_1} + \varepsilon'_m)^2 + \varepsilon''^2_m}, \tag{4.2.5}$$

separately . The refractive index is related with the dielectric functions by the following expressions

$$n = \sqrt{\varepsilon} = \sqrt{1 + 4\pi\alpha} \tag{4.2.6}$$

Consider the refractive index of the composites with real inclusions, when $\gamma$ the damping factor is not extremely small but a finite being of the order of $10^{-2}$. According to the Clausius-Mossotie formula the effective dielectric functions $\varepsilon$ of the composite can be written as

$$\frac{\varepsilon - \varepsilon_h}{\varepsilon + 2\varepsilon_h} = \frac{4\pi}{3} DN, \tag{4.2.7}$$

where, $D$ is given by equation (4.2.2), and $N$ is a density number of the inclusions. With the help of the volume fraction of spherical metallic particles in a linear dielectric host matrix $f = (4\pi/3)r_2^3 N$, the dielectric function of the composite is expressed by

$$\varepsilon = \varepsilon_h[1 + 3f\alpha] \tag{4.2.8}$$

In this expressions we neglect the higher order because of $f\alpha << 1$. The real and imaginary parts of equation (4.2.8) is described by the following expressions;

$$\varepsilon_1 = \varepsilon_h' + 3f\varepsilon_h'\alpha' \tag{4.2.9}$$

and

$$\varepsilon_2 = 3f\varepsilon_h'\alpha'' \tag{4.2.10}$$

respectively. By substituting equation (4.2.4) into equation (4.2.9) and equation (4.2.5) into (4.2.10) and then by equating the real and imaginary parts of the refractive index $n' + in''$ with the real and imaginary parts of the dielectric functions $\sqrt{\varepsilon_1 + i\varepsilon_2}$, we obtain a coupled equations for the real $n'$ and imaginary $n''$ parts of the refractive index of the composite[23].

$$\begin{aligned} n'^2 - n''^2 &= \varepsilon_1, \\ 2n'n'' &= \varepsilon_2, \end{aligned} \tag{4.2.11}$$

where $n'$ is the real refractive index of the medium and $n''$ is the imaginary part of the refractive index of the composite[23]. Comparison of the above equation yields the expression for the real $n'$ and imaginary $n''$ parts of the refractive index of metal containing interfacial layer composite,

$$n' = \sqrt{\frac{1}{2}(\varepsilon_1 + \sqrt{\varepsilon_1^2 + \varepsilon_2^2})},$$
$$n'' = \sqrt{\frac{1}{2}(-\varepsilon_1 + \sqrt{\varepsilon_1^2 + \varepsilon_2^2})}, \quad (4.2.12)$$

respectively. Similarly we can obtain the real $n'$, and imaginary $n''$ parts of the refractive index of the pure metal when the value of $I$ in equation (4.2.3) becomes zero.

### 4.2.2 Analytical Descriptions of the Effects of Interfacial Layer for Refractive Index in Metal/Dielectric Composites.

Here we are interested to see the effects of the interfacial layer on the refractive index of metal/dielectric composite separated by interfacial layer embedded in a linear dielectric host. In the electrostatic approximation, when the wavelength $\lambda$ of the incident electromagnetic radiation is much greater than a typical size of the inclusion, the distribution of the electric potential in the system is described by the following expressions[18].

$$\Phi_d = -E_{ap}Arcos\theta, r \leq r_1,$$
$$\Phi_i = -E_{ap}(Br - Cr^{-2})cos\theta, r_1 \leq r_{1+t},$$
$$\Phi_m = -E_{ap}(Dr - Er^{-2})cos\theta, r_{1+t} \leq r_2, \quad (4.2.13)$$
$$\Phi_h = -E_{ap}(r - Fr^{-2})cos\theta, r \geq r_2.$$

Where, $\phi_d, \phi_i, \phi_m$, and $\phi_h$ are potentials of the dielectric, interfacial layer, metal, and the linear dielectric host matrix respectively. $E_{ap}$ is the applied filed, $r$ and $\theta$ are the spherical coordinates of the observation point ($E_{ap}$ is chosen along the z-axis )

$r_1$, $r_{1+t}$, and $r_2$ are the radius of dielectrics, interfacial layer, and metal respectively. $A, B, C, D, E$ and $F$ are the unknown coefficients. Using the continuity conditions of the potential and displacement vector at the boundaries of dielectric-interfacial layer, interfacial layer-metal, and metal-host matrix, we obtain a system of linear algebraic equations for $A, B, C, D, E$ and $F$. The solutions of this system can be given as

$$\begin{aligned} A &= \frac{9\varepsilon_m\varepsilon_h}{2ph}. \\ F &= \{1 - \frac{3\varepsilon_h[(3/p-1)\varepsilon_m + b]}{2h}\}r_2^3. \end{aligned} \tag{4.2.14}$$

where,

$$h = \varepsilon_m^2 + k\varepsilon_m + b\varepsilon_h. \tag{4.2.15}$$

Here $k = (3/2p-1)b + (3/p-1)\varepsilon_h$, $p = 1-(r_1/r_2)^3$ is a metal fraction in the inclusion, $b = \varepsilon_d + \frac{2I}{r_1}$ where, $I$ is the interfacial layer factor, $\varepsilon_d, \varepsilon_m$, and $\varepsilon_h$ are the dielectric functions (DFs) of the dielectric, metal, and the linear host matrix, respectively. Therefore, the polarization of an individual small spherical metal inclusion separated by interfacial layer from a dielectric core which is embedded in a linear dielectric host matrix can be presented in the form of equation (4.2.14) can be,

$$\begin{aligned} F &= \alpha r_2^3, \\ \alpha &= 1 - \frac{3\varepsilon_h[(3/p-1)\varepsilon_m + b]}{2h} \end{aligned} \tag{4.2.16}$$

Here $F$ is the effective polarization of the composite. The expressions of equation (4.2.13) holds true for the Raman particles when the radius of inclusion $r_2$ is much less than the wavelength of incident radiation $\lambda$. Further, it is known that the frequency dependence of metals over the optical frequency range is described by the Drude model given in equation (4.1.4) and (4.1.5). In this paper we consider the case when the host matrix is real. Now we obtain expressions for the real $\alpha'$ and imaginary $\alpha''$

parts of the polarizability which is in the form of

$$\alpha' = 1 - \frac{3\varepsilon_h}{2}\frac{[(3/p-1)\varepsilon'_m + b]h' + \varepsilon_h[(3/p-1)\varepsilon''_m]h''}{h'^2 + h''^2}, \tag{4.2.17}$$

and,

$$\alpha'' = \frac{3\varepsilon_h}{2}\frac{[(3/p-1)\varepsilon'_m + b]h'' + \varepsilon_h[(3/p-1)\varepsilon''_m]h'}{h'^2 + h''^2}, \tag{4.2.18}$$

respectively. Also the real and the imaginary part of $h$ is given in the following expressions

$$\begin{aligned} h' &= \varepsilon'^2_m + k\varepsilon'_m + b\varepsilon_h - \varepsilon''^2_m, \\ h'' &= \varepsilon''_m(k + 2\varepsilon'_m). \end{aligned} \tag{4.2.19}$$

An analytic analysis of the obtained expressions can be done in the model of a very weak damping of plasma vibrations in the metal part of the inclusion when we consider $\gamma << 1$, which is negligible. In this case from the imaginary part of equation (4.1.5) can be consider $\varepsilon''_m \sim \gamma << 1$. The minimum of the denominators in (4.2.17) and (4.2.18) gives the maximum value of polarization at the condition of

$$\varepsilon'^2_m + k\varepsilon'_m + b\varepsilon_h = 0. \tag{4.2.20}$$

This equation (4.2.20) has two roots that in turn gives two resonant frequencies. The imaginary party of the polarizability is responsible for the absorption of electromagnetic waves in the composites. By substituting equation (4.2.17) into equation (4.2.9) and equation (4.2.18) into (4.2.10), and then by equating the real and imaginary parts of the obtained expressions of the dielectric functions with the real and imaginary parts of the refractive index, we obtain the expressions for the real $n'$, and imaginary parts of the refractive index of metal/dielectric composite separated by interfacial layer as shown in equation (4.2.12). Similarly we can obtain the real $n'$,

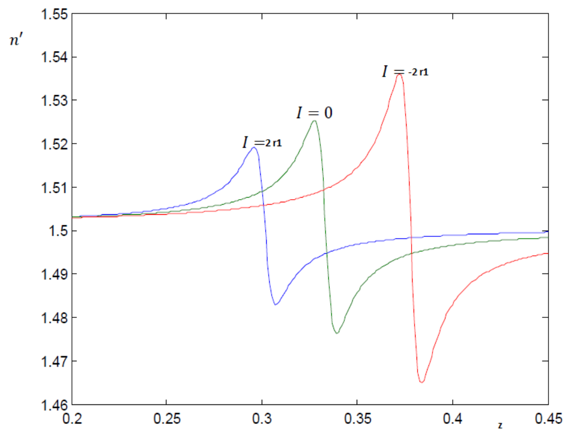


Figure 4.1: Refractive index $n'$ of pure metal versus $z$ for three different values of the interfacial layer factor, for the value of $f = 0.001$, and the numerical values of the composite parameters:

$$\varepsilon_h = 2.25,\ \varepsilon_h^{''} = 0, \varepsilon_\infty = 4.5, \omega_p = 1.6 \times 10^{16} rad/s, \text{ and} \gamma = 0.0115$$

and imaginary $n''$ parts of the refractive index of the metal/dielectric composite when the value of $I$ in the expression $b = \varepsilon_d + \frac{2I}{r_1}$ is equal to zero.

## 4.3 Numerical Analysis of the Effects of Interfacial Layer on the Refractive Index of Composite

By considering the case of the linear dielectric host matrix, we analyzed and describe the real part of the refractive index of the composite based on the following Figures. In Figure 4.1 for constant value of the volume fraction of metallic particles $f = 0.001$, we observe that for the three different values of interfacial layer factor $I = -2r_1, 0, 2r_1$, there is one maximum refractive index of the composite each for each of the three interfacial layer factors at different plasma resonance frequency, i.e, we observe one

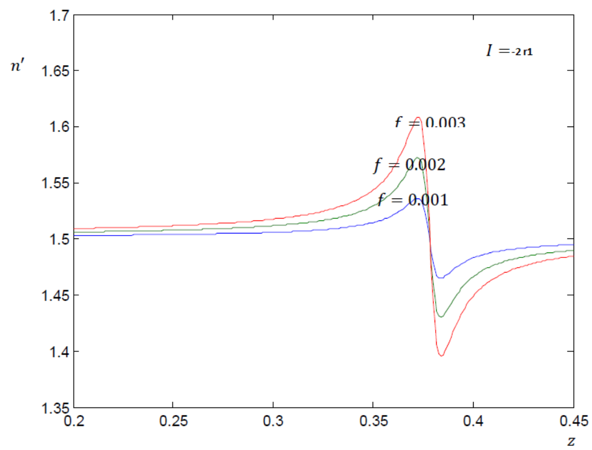


Figure 4.2: Refractive index $n'$ of pure metal composite versus $z$ for three different values of the percentage of metal $f$ in linear host matrix, when the rest of the parameters are the same as in Fig 4.1

region of anomalous dispersion for each of the three values of the interfacial layer factor $I$ at different resonance frequencies. For $I = 2r_1$ the maximum refractive index is $n' = 1.519$ at the frequency of $z = 0.296$, for $I = 0$ the maximum refractive is index $n' = 1.525$ is obtained at the frequency $z = 0.328$, and for $I = -2r_1$ the maximum refractive index is $n' = 1.536$ at the frequency of $z = 0.372$. Here we found that as the dielectric functions $\varepsilon_s$ of the interfacial layer property is changed from dielectric-like property ($I = 2r_1$) to metal-like property ($I = -2r_1$). The resonance frequency of the composite is increased while, the corresponding refractive index of the composite is enhanced.

In Fig 4.2 for three different values of the percentage of the volume fraction of the metal particles in the linear host matrix, we observe different maximum values of the refractive index for a given value of the interfacial layer factor $I = -2r_1$. i.e for $f = 0.001$ the refractive index $n'_{max} = 1.536$, for $f = 0.002$ the refractive index

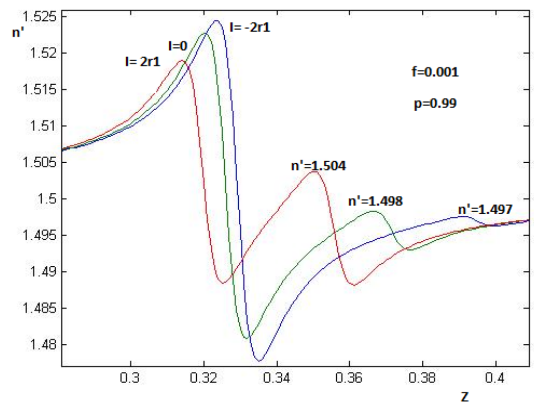


Figure 4.3: Refractive index $n'$ of metal/dielectric composite verses $z$ with interfacial layer factors $I = -2r_1, I = 0, 2r_1$ for $f = 0.001$, and $p = 0.99$, when $\varepsilon_d = 6$ and the rest of the parameters are the same as in Fig 4.1

$n'_{max} = 1.572$, and for $f = 0.003$ the refractive index $n'_{max} = 1.608$. The increasing in the volume fraction of the metallic particles $f$ in the linear host matrix increases the refractive index of the composite.

In Figure 4.3 the refractive index is plotted as a function of frequency for $f = 0.001$, $p = 0.99$, and $I = -2r_1, 0, 2r_1$. From the graphical representation we observe that for three different values of the interfacial layer factors different values for the first, and second maxima of the real parts of the refractive index of the composite of small spherical metal/dielectric particles which is embedded in a linear host matrix at different frequencies. For example, for $I = -2r_1$ the first maxima is $n'_1 = 1.525$, the second maxima is $n'_2 = 1.497$, for $I = 0$ the $1^{st}$, and $2^{nd}$ maxima is $n'_1 = 1.523$, and $n'_2 = 1.498$, respectively. And for $I = 2r_1$ the $1^{st}$ and $2^{nd}$ maxima is $n'_1 = 1.519$, and $n'_2 = 1.504$ respectively. From the above data we have seen three different maxima of refractive index of the composite at different plasma resonance frequency because of

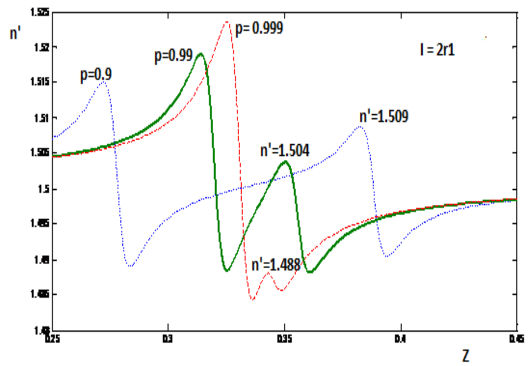


Figure 4.4: Refractive index $n'$ of metal/dielectric composite verses $z$ with interfacial layer factors $I = 2r_1$ for three different values of metal fraction $p = 0.9, 0.99, 0.999$, when$f = 0.001$, $\varepsilon_d = 6$ and the rest of the parameters are the same as in Fig 4.1

different value of the interfacial factor $I$. As the dielectric function of the interfacial layer $\varepsilon_s$ is changed from dielectric-like property to metal-like property the refractive index of the composite is enhanced, while the plasma resonance frequency is increased. But when the dielectric function of the interfacial layer $\varepsilon_s$ is changed from metal-like to dielectric-like property the refractive index of the composite is enhanced while, the plasma resonance frequency is decreased. If there is no interfacial layer effect on the refractive index of the composite, the peak of the refractive index of the composite is reduced. This shows the interfacial layer plays a role on the refractive index composite. And it is noticed that such role is more important for metal-like interfacial layer than dielectric-like interfacial layer.

In figure 4.4 by keeping $I = 2r_1$, and $f = 0.001$ we observe for the three different values of the metal fraction in the inclusion $p$, different values of the first, and second

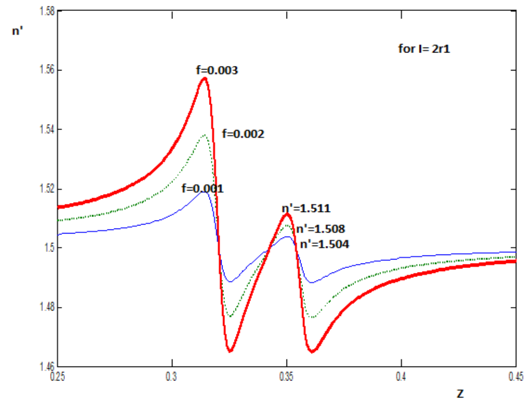


Figure 4.5: Refractive index $n'$ of metal/dielectric composite verses $z$ with interfacial layer factors $I = 2r_1$ for three different values of percentage of particles $f = 0.001, 0.002, 0.003$, when $\varepsilon_d = 6$.the rest of the parameters are the same as in Fig 4.1

maxima of the real parts of the refractive index of the composite at different frequencies. For example, for $p = 0.9$ the first maxima is $n'_1 = 1.515$ at frequency 0.272, the second maxima is $n'_2 = 1.509$ at frequency 0.383, for $p = 0.99$ the $1^{st}$,and $2^{nd}$ maxima is $n'_1 = 1.519$ at frequency 0.314,and $n'_2 = 1.504$ at frequency 0.35, respectively. And similarly for $p = 0.999$ the $1^{st}$ and $2^{nd}$ maxima is $n'_1 = 1.524$ at frequency 0.325,and $n'_2 = 1.488$ at frequency 0.343 respectively. Here the second maxima of the refractive index of the composite is decreasing and even for $I = -2r_1$ for the case of $p = 0.999$ second enhancement is very small.

In Figure 4.5 by keeping $I = 2r_1$, and $p = 0.99$ we observe for the three different values of the volume fraction $f$of the metallic particles in the linear host matrix for different values of the first, and second maxima of the real parts of the refractive index of the composite. For example for $f = 0.001$ the first maxima is $n'_1 = 1.519$ at frequency 0.314, the second maxima is $n_2' = 1.504$ at frequency 0.35, for $f = 0.002$ the $1^{st}$, and $2^{nd}$ maxima is $n_1' = 1.538$ at frequency 0.314, and $n_2' = 1.508$ at frequency 0.35 respectively. And similarly for $f = 0.003$ the $1^{st}$ and $2^{nd}$ maxima is $n_1' = 1.557$ at frequency 0.314,and $n_2' = 1.511$ at frequency 0.35,respectively. Here we found that increasing the value of $f$ increases the refractive index of the composite. So for different values of $f$ we observe three different values of the $1^{st}$ maxima of the refractive index of the composite almost at the same resonance frequency, and the same is true for the $2nd$ maxima. So the value of the parameter $f$ is not significantly frequency dependent.

## 4.4 Analytical Descriptions of Effect of Interfacial Layer for Propagation of Waves

The pioneering demonstrations of slow and fast light were all based on the exploitation of narrow spectral resonances, mainly created by electromagnetically-induced transparency[24] or coherent population oscillation[25]. In this chapter we peresent that the composites of spherical metal with interfacial layer embedded in a linear host matrix and the composite of small spherical metal/dielectric composite separated by interfacial layer which is embedded in a linear dielectric host matrix strongly absorb and refract light on one and two resonance frequencies, respectively. In our study we consider a linear dielectric host matrix which is represented as real $\varepsilon_h'$ and imaginary $\varepsilon_h''$ but in our study we neglect the imaginary part. For linear host matrix consideration the numerical values for the real and imaginary parts of the the polarizability of pure metal with interfacial layer and for metal/dielectric composite by interfacial layer are shown $\alpha_m,' \alpha''_m$ and $\alpha,' \alpha''$ in the Figures $4.1 - 4.4$ and 4.5. We set the the volume fraction $f = 0.001$. Equation (4.2.3), and equation (4.2.16) help us to obtain the analytical results to ignore the dipole-dipole interaction between the inclusions. The real parts of the refractive index of metal with interfacial layer are described in Figures $4.1 - 4.2$, and the real parts of the refractive index of metal/dielectric composite separated by interfacial layer are described in Figures $4.3 - 4.5$ for a linear dielectric host matrix. In this chapter we are interested to see the effects of interfacial layer on the propagation of electromagnetic waves in the composite of both pure metal with interfacial layer, and metal/dielectric composite separated by interfacial layer respectively. By substituting the analytical description of the real parts of the refractive index of the above composite expressed in equation (4.2.12) into equation (2.5.1) we obtain the analytical description of the group velocity for the composite separately.

The optical pulse propagating through highly dispersive[22],[25] show a negative value of the group velocity ($V_g$). In such consideration,the value of group velocity may be negative for two cases the $1^{st}$ prediction is when the peak of the transmitted pulse will exit the material before the peak of the incident pulse emerges the material, and in addition to that the $2^{nd}$ consideration is when the pulse will appear to propagate in the backward direction within the material[11]. We calculate the group velocity ($v_g$) with the help of the known formula[11],

$$\begin{aligned} v_g(z) &= \frac{c}{n_g}, \\ n_g(z) &= n'(z) + z\frac{dn'}{dz}, \end{aligned} \tag{4.4.1}$$

which is called the group velocity of a wave packet. Here $c$ is the speed of light in vacuum and $n_g$ is the group velocity index[11]. Let us consider the narrow wave packet centered at $k_o$ is given by the following equation $E(t) = \int_{-\infty}^{\infty} e^{-i(kx-w_o t)} e^{\frac{-(k-k_o)^2}{2\Delta^2}} dk$ This relation is obtained from $V_g = d\omega/dk$, where, $\omega$ is the frequency and $k$ is the wave vector using the definition of the real part of the refractive index $n' = kc/\omega(k)$, the group velocity $v_g$ appears in the second term of the Taylor series expansion

$$\omega_k = \omega_{k_0} + V_g(k_0)(k - k_0) + \frac{1}{2}\frac{dV_g}{dk}|_{k=k_0}(k - k_0)^2 + ... \tag{4.4.2}$$

around the center of the wave packet $k_0$. Therefore, (4.4.1) has a meaning of the group velocity in the case of a week dispersion of $\omega(k)$. For a strong dispersion, the higher terms in (4.4.2) must be taken into account and in this case $V_g$ losses its physical meaning[11].

## 4.5 Numerical Analysis of the Effects of Interfacial Layer on the Group Velocity of Composite

Based on the analytical descriptions of the group velocity the numerical descriptions of the group velocity of the composite of pure metal with interfacial layer and metal/dielectric composite separated by interfacial layer are described in the following Figure $4, 6 - 4.8$ and $4.9 - 4.11$ separately.

In Figure 4.6 we observe that for the value of interfacial layer factor $I = 0$, for constant $f = 0.01$. The supperluminal light or the negative group velocity $v_g/c < 0$ is observed between frequency ranges of $(0.33, 0.35)$ in this frequency range the negative group velocity or the propagation of light is rapidly increasing until it reaches the minimum of the group index $n_g$, which is the maximum point for the negative group velocity of the composite i.e $v_g = -0.15(c)$.

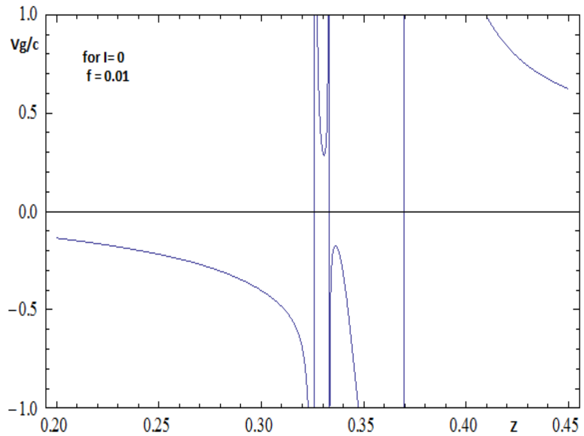


Figure 4.6: The graph of normalized group velocity by speed of light $v_g/c$ versus $z$ of the composite of pure metal, when $f = 0.01$, $\varepsilon'_h = 2.25$, and $\varepsilon_\infty = 4.5$

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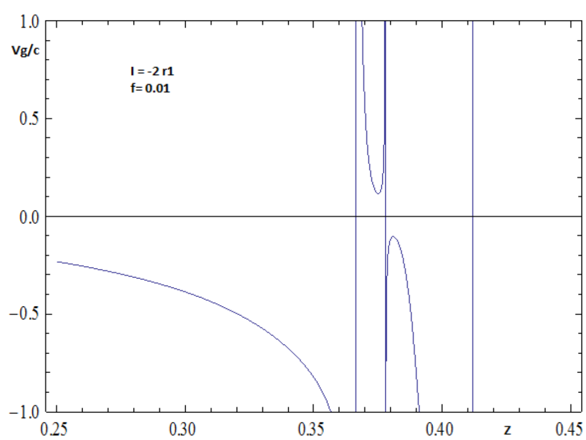


Figure 4.7: The graph of normalized group velocity by speed of light $v_g/c$ versus $z$ of the composite of pure metal with interfacial layer factor $I = -2r_1$, when $f = 0.01$,$\varepsilon'_h = 2.25$, and $\varepsilon_\infty = 4.5$

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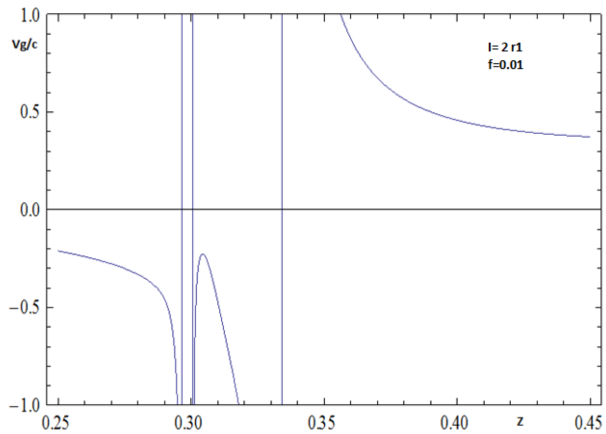


Figure 4.8: The graph of normalized group velocity by speed of light $v_g/c$ versus $z$ of the composite of pure metal with interfacial layer factor $I = 2r_1$, when $f = 0.01$, $\varepsilon'_h = 2.25$, and $\varepsilon_\infty = 4.5$

.

In Figure 4.7 we observe that for the value of interfacial layer factor $I = -2r_1$, i.e, when the dielectric function of the interfacial layer is metal-like property,for constant $f = 0.01$. The supperluminal light or the negative group velocity $v_g/c < 0$ is observed between frequency ranges $(0.378, 0.391)$ in this frequency range the negative group velocity or the propagation of light is rapidly increasing until it reaches the minimum of the group index $n_g$, which is the maximum point for the negative group velocity of the composite i.e $v_g = -0.13(c)$.

In Figure 4.8 we observe that for the value of interfacial layer factor $I = 2r_1$, i.e when the dielectric function of the interfacial layer is dielectric-like property, for a constant $f = 0.01$. The supperluminal light or the negative group velocity $v_g/c < 0$ is observed between frequency ranges of $(0.30, 0.319)$ in this frequency range the negative group velocity or the propagation of light is rapidly increasing until it reaches the minimum of the group index $n_g$, which is the maximum point for the negative group

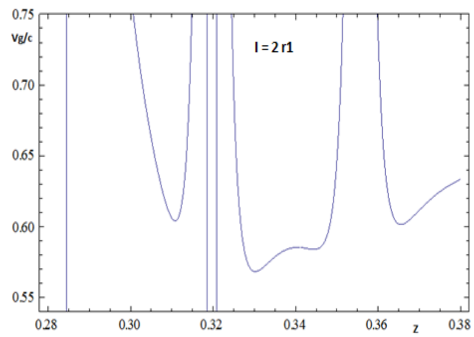


Figure 4.9: The graph of normalized group velocity by speed of light $v_g/c$ versus $z$ of the composite of metal/dielectric particles separated by interfacial layer with interfacial layer factor $I = 2r_1$, $f = 0.001$, and $p = 0.99$

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velocity of the composite i.e $v_g = -0.21(c)$.

In Figure 4.9 we observe that for the value of interfacial layer factor $I = 2r_1$, i.e when the dielectric function of the interfacial layer factor is dielectric-like property, for a constant $p = 0.99$, and $f = 0.01$. NO supperluminal light (or negative group velocity) $v_g/c < 0$ is observed between frequency ranges $(0.25, 0.40)$. In this frequency range the refractive index of the composite is minimum and also the value of absorption is less so that the group velocity is positive i.e slow propagation of wave $v_g/c < 1$.

In Figure 4.10 we observe that for the value of interfacial layer factor $I = 0$, $p = 0.99$ and $f = 0.01$. The supperluminal light or the negative group velocity $v_g/c < 0$ is observed between frequency ranges $0.322, 0.324$ in this frequency range the group velocity or the propagation of light is rapidly increasing until it reaches the minimum of the group index $n_g$ , which is the maximum point for the negative group

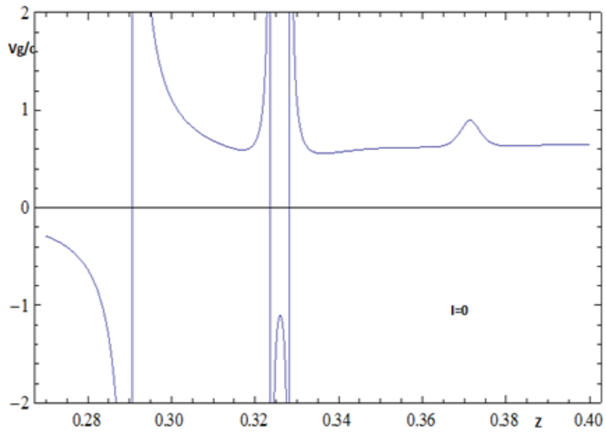


Figure 4.10: The graph of normalized group velocity by speed of light $v_g/c$ versus $z$ of the composite of metal/dielectric particles separated by interfacial layer with interfacial layer factor $I = 0$, $f = 0.001$, $p = 0.99$

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velocity of the composite i.e $v_g = -1.18(c)$.

In Figure 4.11 we observe that for the value of interfacial layer factor $I = -2r_1$,$p = 0.99$, and $f = 0.01$. The supperluminal light or the negative group velocity $v_g/c < 0$ is observed between frequency ranges$0.325, 0.335$ in this frequency range the negative group velocity or the propagation of light is rapidly increasing until it reaches the minimum of the group index, which is the maximum point for the negative group velocity of the composite i.e, $v_g = -0.75(c)$.

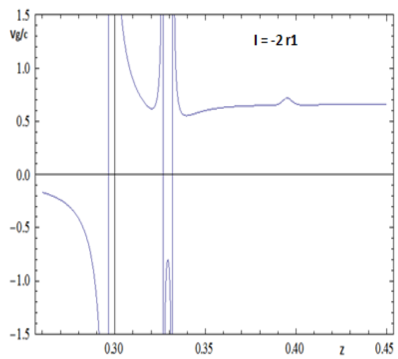


Figure 4.11: The graph of normalized group velocity by speed of light $v_g/c$ versus $z$ of the composite of metal/dielectric particles separated by interfacial layer with interfacial layer factor $I = -2r_1$, $f = 0.001$, $p = 0.99$

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# Chapter 5

# Conclusion and Recommendation

## 5.1 Conclusion

We have calculated the real and imaginary parts of the refractive index and group velocity for both composite of small spherical pure metal with interfacial layer, and small spherical particles of metal/dielectric separated by interfacial layer which is randomly embedded in a linear dielectric host. We have discussed the analytical descriptions, and the numerical analysis of refractive index, and the propagation of wave of such composite with Drude model together. As it is shown in the analytical descriptions and numerical analysis, we can conclude the following in this work.

Firstly:

In this work we show the effects of the interfacial layer on the refractive index of the composite.
When the dielectric function of the interfacial layer is changed from dielectric-like property to metal-like property the refractive index of the composite is more enhanced while, the frequency $\omega$ of the incident electromagnetic field is increased. But

when the dielectric functions of the interfacial layer is changed from metal-like property to dielectric-like property the refractive index of the composite is less enhanced while, the frequency $\omega$ of the incident electromagnetic field is decreased.

Maximum refractive index of the composite is obtained at less frequency of the incident electromagnetic filed when the dielectric functions of the interfacial layer is dielectric-like property but when the dielectric functions of the interfacial layer is metal-like property the maximum refractive index of the composite is obtained at high frequency of the incident electromagnetic field.

The refractive index of the composite is more enhanced when the dielectric function of the interfacial layer is metal-like property than dielectric-like property.

At a given frequency of the the incident electromagnetic field the refractive index of the composite is more enhanced when the percentage of the volume fraction of the metallic particles in the inclusion is increased .

Secondly:

We show the effects of the interfacial layer on the propagation of wave of the composite.

As it is shown in Fig 4.6 - 4.8 for the case of pure metal with interfacial layer which is randomly embedded in a linear dielectric host, for the value of $f = 0.01$ in comparison with no interfacial layer the maximum extreme value of the negative group velocity is observed when the dielectric function ($\varepsilon_s$) of the interfacial layer is metal-like property than dielectric-like property.

As it is shown in Fig 4.9 - 4.11 for the case of small spherical metal/dielectric composite separated by interfacial layer which is embedded in a linear dielectric host, by decreasing the percentage of the volume fraction of metallic particles in the inclusion from 0.01 to 0.001.

We found: No negative group velocity is is observed in the composite between frequency range (0.25 - 0.4) of incident electromagnetic field, when the dielectric function of the interfacial layer is dielectric-like property but negative group velocity is found in the composite between frequency (0.325 - 0.335)of the incident electromagnetic field when the dielectric function of the interfacial layer is metal-like property.

Finally, based on the above results we conclude the following:

The interfacial layer has an effect on the refractive index and propagation of waves in small spherical metal/dielectric composite separated by interfacial layer which is embedded in a linear dielectric host.
It is also possible to control the negative group velocity of the composite by using the dielectric function $\varepsilon_s$ of the interfacial layer dielectric-like property than metal-like property
It is also possible to observe a slow light propagation in a composite media when the dielectric property of the interfacial layer is metal-like property than dielectric-like property.

## 5.2 Recommendation

In our study we discussed the contribution of interfacial layer on the linear and nonlinear response of the composite media which strongly effects on the refractive index and propagation of waves of the optical materials. We expect that it will stimulate the development of experiments.

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## DECLARATION

I hereby declare that this thesis is my original work and has not been presented for a degree in any other University. All sources of material used for the thesis have been duly acknowledged.

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This thesis has been submitted for examination with my approval as University advisor.

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