GRAVITATIONAL LENSING IN THE STANDARD ΛCDM COSMOLOGY

By

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To My Family, especially to my Wife and my Mother

Table of Contents

List of Tables

List of Figures

Abstract

Gravitational lensing is one of the powerful methods for astrophysical investigations where General Relativity (GR) is being applied. But, the appropriate parameters incorporated in GR are still at debate among the scientific communities. Especially, the incorporation of cosmological constant Λ was controversial from its origin to its significance. However, recent progress indicates that cosmological models require a form of dark matter-energy sector that will behave in an old enough universe with energy content in the form of Λ. So, here we worked out the effect of Λ in gravitational lensing where an effective refractive index in vacuum is considered to accommodate it. Thus GR equations were being used to derive lensing equation in the presence of Λ with simplifying boundary conditions. The adopted method is so simple but agrees with the one derived from complex and more boundary value considerations. Moreover, its significant at cosmological scale goes up to 2%, that cannot be neglected.

Keywords:Gravitational Lensing, Cosmological Conctant Λ

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Chapter 1

General Introduction

1.1 Background of the Study

Gravitational lensing is one of the powerful methods for astrophysical investigations where General Relativity (GR) is being applied. The appropriate parameters incorporated in GR are still at debate among the scientific communities.

Within the last 20 years gravitational lensing has changed from being considered a geometric need to a helpful and in some ways unique tool of modern astrophysics [3].

The term gravitational lensing is used as one of the test of General relativity in the deflection of light by massive bodies and the associated phenomenon. Although these deflection of light at the solar limb was very successfully established as the first experiment to confirm a prediction of Einstein's theory of General Relativity in 1919, it took more than half a century to establish this phenomenon observationally in some other environment[18]. By now almost many different realizations of lensing are known and observed.

Gravitational lensing is the bending of light by matter - displays a number of attractive features as an academic discipline. Its principles are very easy to understand and to explain due to its being a geometrical effect. Its ability to produce optical illusions is fascinating to scientists and Particular people a like. And - most importantly of course - its usefulness for a number of astrophysical problems makes it an attractive tool in many branches of astronomy.

The great interest in gravitational lensing comes from the fact that this phenomenon can be used as an astrophysical and cosmological tool.

Even before more examples becomes known the gravitational lensing effect inspired theorists as to the potential it would have for astrophysics and specially Cosmology. Then the Cosmology can be done with the Cosmological Model or standard Model. Out of these Model the ΛCDM(Lambda cold dark matter) model is very know model in cosmology.

The ΛCDM or Lambda-CDM model is a parameterization of the Big Bang cosmological model in which cosmological issues like age of astronomical objects, origins, size, and age of the universe we live in is estimated. The universe contains a cosmological constant, which can be taken as fluid denoted by Lambda (Greek Λ), associated with dark energy, and cold dark matter (abbreviated CDM)

In the general theory of relativity, light rays follow null geodesics, i.e., the minimum paths in a curved space-time, when a light ray from a far source interacts with the gravitational field due to a massive body.

Consistent with Einstein's Theory of General Relativity, gravitational lensing involves studying how the gravitational field of a massive object will bend light. Mean while, redshift attempts to gauge the speed at which other galaxies are moving away from ours by measuring the extent to which their light is shifted towards the red end of the spectrum (i.e. its wavelength becomes longer the faster the source is moving away) [2].

Gravitational lensing is especially useful when it comes to determining how the Universe came to be. Our current cosmological model, known as the Lambda Cold Dark Matter

(Lambda CDM) model, states that Dark Energy is responsible for the late-time acceleration in the expansion of the Universe, and that Dark Matter is made up of massive particles that are responsible for cosmological structure formation.

Based on this standard (ΛCDM) model, my study aim to identify the effect of cosmological constant in lensing system by Vacuum fluid Approach Within point mass model and several issues like whether Λ is geometrical constant or a pure scalar function.

1.2 Statement of the Problem

Since the discovery of Einstein's GTR, its field equations have already initiated viable researches including cosmological issues like age of astronomical objects, origins, size, and age of the universe we live in. Today, we can relatively tell how old our unverse is and so on using gravitational lensing. However, the models of this lensing itself is not yet complete. Within the framework of the standard ΛCDM model there are several issues like whether Λ is geometrical constant or a pure scalar function and so on need still investigation.

Research questions

- How does gravity affect light geodesy?
- What is the appropriate lensing equation in the presence of cosmological constant?
- What are the relevant parameters entering in the lensing equations that determine the age, size, origin of our universe?

1.3 Objectives

1.3.1 General Objectives

To study gravitational lensing in the standard ΛCDM cosmology.

1.3.2 Specific Objectives

- To determine the effect of gravity on light geodesy and its implications.
- To derive lensing equation in the presence of cosmological constant.
- To extract and describe relevant parameters entering in the lensing equations that determine the age, size, origin of our universe.

1.4 Research Methodology

General theory of relativity is considered in the presence of positive cosmological constant to derive gravitational lensing equation with the simple point source boundary condition. The background medium is considered as a continues smeared fluid of varying refractive index as a function of radial distance and cosmological constant. All the angular distances also consider the cosmological constant and the expanding universe scenario by way of the transformation between the static and co-moving coordinates. The lensing equation assumes the deflectors (the lens) and the sources positions angular distances through the observed redshifts by Hubble and Lemaitre law. Then, the analytically derived Lens equations are being used to calculate some numerical values to compare with observation, For the computation Mathematica 11 is used.

1.5 Acronyms

In this thesis there is abbreviated words that one can not know the meaning of them but the researcher and other person who related with the field knows and the researcher can use them, so we give the meaning of them as follow

- GTR or GR = General theory of relativity.

- GL = Gravitational lensing
- ΛCDM= Lambda cold dark matter.
- EFE = Einstein's field equation.
- CMB=Cosmic microwave background.
- FGRS=Field galaxy redshift survey.
- WMAP= Wilkinson microwave anisotropy probe

and other such like abbreviated words can be used in these thesis.

Chapter 2

Introduction to General Theory of Relativity

General relativity is the geometric theory of gravitation. One of Einsteins great insights was to make general relativity a geometric theory of gravitation.

In special relativity, spacetime is the arena for physics. Spacetime consists of events, which require four numbers for their complete specification: three numbers to give the spatial location with respect to some chosen coordinate grid, and one number to give the time. Geometrically, spacetime is represented by a four-dimensional manifold (surface), each point in the manifold corresponding to an event in spacetime.

The general theory of relativity is a classical field theory of gravitation in which all variables are assumed to be continuous and are uniquely specified.[20, 22]

The basic philosophy of general relativity is to relate the geometry of space time, which determines the motion of matter, to the density of matter-energy, known as the stress energy tensor. This relation is accomplished through the Einstein field equations. The geometry of space-time is dictated by the metric tensor which defines the properties of that geometry and basically describes how travel in one coordinate involves another coordinate, so that

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{2.0.1}$$

The elements of the metric tensor are dimensionless; for ordinary Euclidean space they are all unity if $\mu = \nu$ and zero otherwise.

General relativity is defined on a four dimensional Riemannian manifold. [20] Coordinates in this non-Euclidian space are denoted by $x_\mu = (x^0, x^1, x^2, x^3)$.

Now the field equations relate second derivatives of the metric tensor to the properties of the local matter-energy density expressed in terms of the stress-energy tensor. Specifically the Einstein field equations are

$$G_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu} \tag{2.0.2}$$

Here

- $G_{\mu\nu}$is known as the Einstein tensor and
- $T_{\mu\nu}$is the stress energy tensor in physical units (say grams per cubic centimeter).
- The quantity G/c^2 is a very small number in any common system of units, which shows that the departure from Euclidean space is small unless the stress-energy is exceptionally large.

In GR it is assumed that all matter moves in an effective pseudo-Riemannian metric space-time with a universal coupling, governed by the Einstein Equivalence Principle, consisting of two parts;[14]

- **The Weak Equivalence Principle**: Given the same initial positions and velocities, subject only to gravity particles will follow the same trajectories, or geodesics. In other words, particles all fall with the same acceleration regardless of composition and consequently gravity is universal.

- **The Strong Equivalence Principle**: The laws of physics take the same form in a freely-falling reference frame as in SR. Effectively, gravity can always be eliminated at a point.

The first three tests of General Relativity were proposed by Einstein, the gravitational redshift, the deflection of light by massive bodies and the perihelion shift of Mercury.

2.1 Einstein Field Equation

The Einstein field equations (EFE; also known as Einsteins equations) comprise the set of equations in Albert Einsteins general theory of relativity that describe the fundamental interaction of gravitation as a result of spacetime being curved by mass and energy. Similar to the way that electromagnetic fields are determined using charges and currents via Maxwells equations.[14]

The EFE are used to determine the spacetime geometry resulting from the presence of mass energy and linear momentum, i.e; they determine the metric tensor of spacetime for a given arrangement of stress energy in the spacetime.

Einstein's equation tells us how the presence of matter curves space-time, and so we need to describe the matter under consideration. The Einstein field equations (EFE) may be written in the form

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{2.1.1}$$

where

- $R_{\mu\nu}$- is the Ricci curvature tensor,
- R -is the scalar curvature,
- $g_{\mu\nu}$ -is the metric tensor,

- Λ -is the cosmological constant,
- G -is Newtons gravitational constant,
- c -is the speed of light in vacuum, and
- $T_{\mu\nu}$ -is the stress energy tensor.

The EFE is a tensor equation relating a set of symmetric 4 x 4 tensors. Each tensor has 10 independent components. The four Bianchi identities reduce the number of independent equations from 10 to 6, leaving the metric with four gauge fixing degrees of freedom, which correspond to the freedom to choose a coordinate system.

In fact, when fully written out, the EFE are a system of ten coupled, nonlinear, hyperbolic-elliptic partial differential equations.

One can write the EFE in a more compact form by defining the Einstein tensor.

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \tag{2.1.2}$$

Which is a symmetric second-rank tensor that is a function of the metric. The EFE can then be written as

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{2.1.3}$$

In standard units, each term on the left has units of $1/length^2$. With this choice of Einstein constant as $8\pi G/c^4$, then the stress-energy tensor on the right side of the equation must be written with each component in units of energy-density (i.e., energy per volume = pressure).

Using geometrized units where G = c = 1, this can be rewritten as

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{2.1.4}$$

The expression on the left represents the curvature of spacetime as determined by the metric; the expression on the right represents the matter/energy content of spacetime. The

EFE can then be interpreted as a set of equations dictating how matter/energy determines the curvature of spacetime.

2.2 Cosmological constant

Einstein modified his original field equations to include a cosmological constant term Λ proportional to the metric

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi \mathrm{G}}{c^4} T_{\mu\nu}$$

Since Λ is constant, the energy conservation law is unaffected.

The cosmological constant term was originally introduced by Einstein.

Despite Einstein's motivation for introducing the cosmological constant term, [5] there is nothing inconsistent with the presence of such a term in the equations. For many years the cosmological constant was almost universally considered to be 0. However, recent improved astronomical techniques have found that a positive value of Λ is needed to explain the accelerating universe. However, the cosmological constant is negligible at the scale of a galaxy or smaller.

Einstein thought of the cosmological constant as an independent parameter, but its term in the field equation can also be moved algebraically to the other side, written as part of the stressenergy tensor

$$T_{\mu\nu}^{(vac)} = -\frac{\Lambda c^4}{8\pi \mathrm{G}} g_{\mu\nu}$$

The resulting vacuum energy density is constant and given by

$$\rho^{(vac)} = \frac{\Lambda c^2}{8\pi \mathrm{G}}$$

The existence of a cosmological constant is thus equivalent to the existence of a non-zero vacuum energy. Thus, the terms "cosmological constant" and "vacuum energy" are now used interchangeably in general relativity.

A positive vacuum energy density resulting from a cosmological constant implies a negative pressure, and vice versa. If the energy density is positive, the associated negative pressure will drive an accelerated expansion of the universe, as observed.

2.2.1 Positive Value of Cosmological Constant

Observations announced in 1998 of distanceredshift relation for Type Ia supernovae(Supernova Cosmology Project (Perlmutter et al. (1999)) indicated that the expansion of the universe is accelerating. When combined with measurements of the cosmic microwave background radiation these implied a value of $\Omega_\Lambda \sim 0.7$(Baker et al. (1999)) [15] a result which has been supported and refined by more recent measurements. There are other possible causes of an accelerating universe, such as quintessence, but the cosmological constant is in most respects the simplest solution. Thus, the current standard model of cosmology, the Lambda-CDM model, includes the cosmological constant, which is measured to be on the order of $10^{-52}m^{-2}$, in metric units. It is often expressed as $10^{-35}s^{-2}$ or 10^{-122}(Barrow and Shaw (2011)) in other unit systems. The value is based on recent measurements of vacuum energy density, $\rho_{vacuum} = 5.96 \times 10^{-27}$ kg/m^3 or $10^{-47}GeV^4$,in other unit systems. As

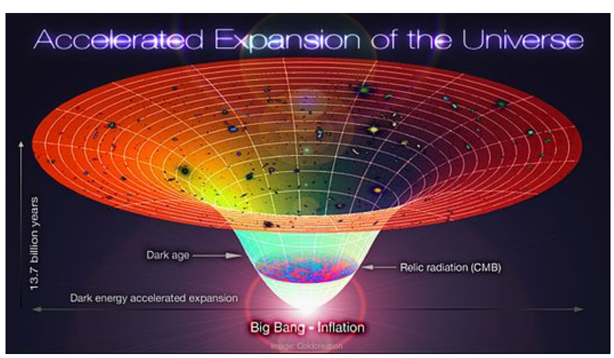

Figure 2.1: Lambda-CDM, accelerated expansion of the universe. The time-line in this schematic diagram extends from the Big Bang/inflation era 13.7 Byr ago to the present cosmological time.

was only recently seen, by works of 't Hooft, Susskind and others, a positive cosmological constant has surprising consequences, such as a finite maximum entropy of the observable universe.[2]

In addition to the above when formulating general relativity, Einstein believed that the Universe was static, but found that his theory of general relativity did not permit it. This is simply because all matter attracts gravitationally; none of the solutions we have found correspond to a static Universe with constant a. In order to arrange a static Universe, he proposed a change to the equations, something he would later famously call his "greatest blunder". That was the introduction of a cosmological constant.

The introduction of such a tenn is permitted by general relativity, and although Einstein's original motivation has long since faded, it is currently seen as one of the most important and enigmatic objects in cosmology. The cosmological constant Λ appears in the Friedmann equation as an extra term, giving

$$H^2 = \frac{8\pi \mathrm{G}}{3}\rho - \frac{\kappa}{a^2} + \frac{\Lambda}{3} \tag{2.2.1}$$

In principle, Λ can be positive or negative, though the positive case is much more commonly considered. Einstein's original idea was to balance curvature, Λ and ρ to get H(t) $= 0$ and hence a static Universe. In fact, this idea was rather misguided, since such a balance proves to be unstable to small perturbations, and hence presumably couldn't arise in practice. Nowadays, the cosmological constant is most often discussed in the context of Universes with the flat Euclidean geometry, $\kappa = 0$

The effect of Λ can be seen more directly from the acceleration equation. By using the Friedmann equation as given above, gives

$$\frac{\ddot{\mathrm{a}}}{\mathrm{a}} = -\frac{4\pi \mathrm{G}}{3}(\rho + \frac{3p}{c^2}) + \frac{\Lambda}{3} \tag{2.2.2}$$

A positive cosmological constant gives a positive contribution to $\ddot{a}$, and so acts effectively as a repulsive force. In particular, if the cosmological constant is sufficiently large, it can overcome the gravitational attraction represented by the first term and lead to an accelerating Universe [6].

Chapter 3

Gravitational Lensing

3.1 Introduction

As we know in classical optics lensing is the process by which transparent apparatus like glasses, human eye refract light. As the lens is curved, the image gets focused to a smaller area Depending on the lens we use, it can make objects bigger or clearer.

By GR principle gravity is deflect all objects including light. The deflection of light by gravity is know as gravitational lensing.

A massive object has far reaching gravitational field and it causes the light passing near or through it to bent. So, the light will be refocused elsewhere. The bending will be more if the object is massive as the gravitational field will be huge.

A gravitational lens is a distribution of matter (such as a cluster of galaxies) between a distant light source and an observer, that is capable of bending the light from the source as the light travels towards the observer. This effect is known as gravitational lensing, and the amount of bending is one of the predictions of Albert Einstein's general theory of relativity.[9]

General relativity predicts that the path of light will follow the curvature of spacetime as it passes near a star. This effect was initially confirmed by observing the light of stars or distant quasars being deflected as it passes the Sun.

Gravitational lensing is the astrophysical phenomenon where by the propagation of light is

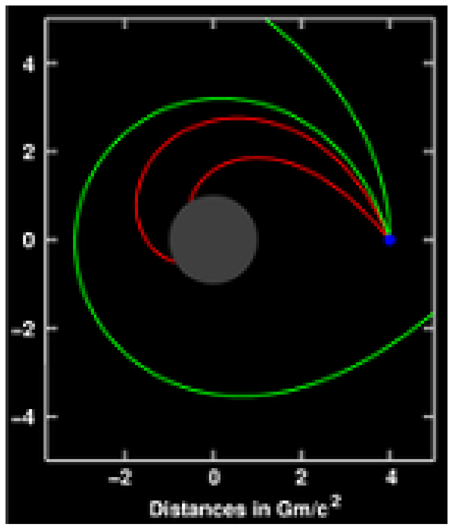

Figure 3.1: Deflection of light (sent out from the location shown in blue) near a compact body (shown in gray)

affected by the distribution of mass in the universe. As photons travel across the universe, their trajectories are perturbed by the gravitational effects of mass concentrations with respect to those they would have followed in a perfectly homogeneous universe.

Also Reported by [15] "A consequence of the relativistic phenomenon of light rays bending around gravitating masses is that masses can serve as gravitational lenses if the distances are right and the gravitational potential is sufficient".

Further more "When a light-ray passes through the gravitational field of a large mass it is bent in the same sense as a converging lens. The gravitational field of a spherical mass is a somewhat odd lens in that the amount of bending decreases away from the axis. This means that an extended object on the axis is focused into a ring (an Einstein ring) by an intervening spherical mass."

3.2 Historical Development of Gravitational Lensing

3.2.1 The Early Years, Before General Relativity

As referred in [11] The Newtonian theory of gravitation predicts that the gravitational force F on a particle of mass m is proportional to m, so that the gravitational acceleration $\mathbf{a} = \mathbf{F/m}$ is independent of m. Therefore, the trajectory of a test particle in a gravitational field is independent of its mass but depends, for a given initial position and direction, only on the velocity of the test particle. About 200 years ago, several physicists and astronomers speculated that, if light could be treated like a particle, light rays may be influenced in a gravitational field as well. John Mitchell in 1784, in a letter to Henry Cavendish, and later Johann von Soldner in 1804, mentioned the possibility that light propagating in the field of a spherical mass M (like a star) would be deflected by an angle $\hat{\alpha} = 2GM/(c^2\xi)$, where G and $\rfloor$ are Newton constant of gravity and the velocity of light, respectively, and ξ is the impact parameter of the incoming light ray. At roughly the same time, Pierre-Simon Laplace in 1795 noted that the gravitational force of a heavenly body could be so large, that light could not flow out of it (Laplace 1975), i.e., that the escape velocity $v_e = \sqrt{2GM/R}$ from the surface of a spherical mass M of radius R becomes the velocity of light, which happens if $R = Rs = 2GM/c^2$, nowadays called the Schwarzschild radius of a mass M.

3.2.2 Gravitational Light Deflection in GR

All these results were derived under the assumption that light some how can be considered like a massive test particle; this was of course well before the concept of photons was introduced [11]. After the formulation of general relativity by Albert Einstein in 1915 the behavior of light in a gravitational field be studied on a firm physical ground. Before the final formulation of GR, Einstein published a paper in 1911 where he recalculated the results of Mitchell and Soldner (of whose work he was unaware) for the deflection angle. Only after the completion of GR did it become clear that the Newtonian value of the deflection angle was too small by a factor of 2. In the general theory of relativity, the deflection is

$$\hat{\alpha} = \frac{4GM}{c^2\xi} = 1''.75(\frac{M}{M_\odot})(\frac{\xi}{R_\odot})^{-1} \tag{3.2.1}$$

The deflection of light by the Sun can be measured during a total solar eclipse when it is possible to observe stars projected near the Solar surface.

The first observation of light [14] deflection was performed by noting the change in position of stars as they passed near the Sun on the celestial sphere. The observations were performed in 1919 by Arthur Eddington, Frank Watson Dyson, and their collaborators during the total solar eclipse on May 29.The solar eclipse allowed the stars near the Sun to be observed. light deflection then slightly changes their positions. A measurement of the deflection in 1919, with a sufficient accuracy to distinguish between the Newtonian and the GR value, provided a tremendous success for Einsteins new theory of gravity.

Soon thereafter,[11] Lodge (1919) used the term lens in the context of gravitational light deflection, but noted that it has no focal length. Chwolson (1924) considered a source perfectly coaligned with a foreground mass, concluding that the source should be imaged as a ring around the lens. Einstein, in 1936, after being approached by the Czech engineer Rudi Mandl, wrote a paper where he considered this lensing effect by a star, including both the image positions, their separation, and their magnifications. He concluded that the angular separation between the two images would be far too small (of order milli-arcseconds) to be resolvable, so that there is no great chance of observing this phenomenon (Einstein 1936).

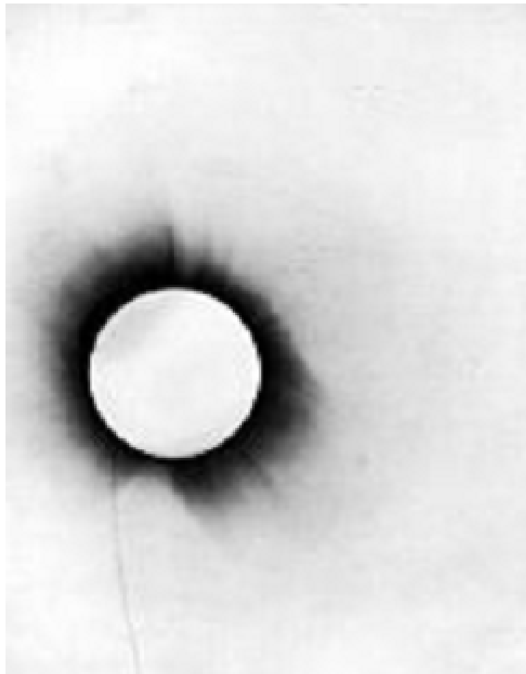

Figure 3.2: One of Eddington's photographs of the 1919 solar eclipse experiment, presented in his 1920 paper announcing its success

3.2.3 The Revival of Lensing

Until the beginning of the 1960s the subject rested, but in 1963/4, three authors independently reopened the field [11]: Klimov (1963), Liebes (1964) and Refsdal (1964a,b). Klimov considered lensing of galaxies by galaxies, whereas Liebes and Refsdal mainly studied lensing by point-mass lenses. Their papers have been milestones in lensing research; for example, Liebes considered the possibility that stars in the Milky Way can act as lenses for stars in M31 we shall see in ML, this is a truly modern idea. Refsdal calculated the difference of the light travel times between the two images of a source since light propagates along different paths from the source to the observer, there will in general be a time delay which can be observed provided the source is variable, such like a supernova. Refsdal pointed out that the time delay depends on the mass of the lens and the distances to the lens and the source, and concluded that, if the image separation and the time delay could be measured, the lens mass and the Hubble constant could be determined [11].

In 1963, the first quasars were detected: luminous, compact (quasi-stellar) and very distant sources hence, a source population had been discovered which lies behind Zwickys nebulae, and finding lens systems amongst them should be a certainty. In 1963, a Dutch astronomer

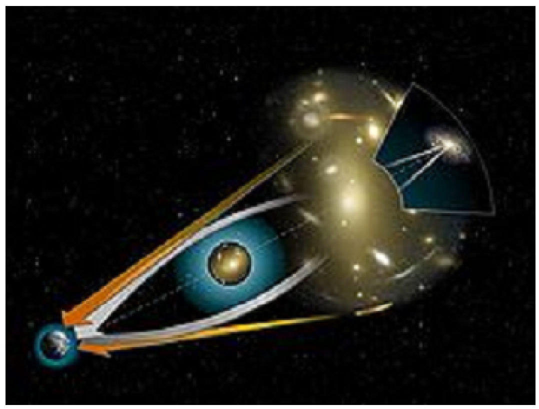

Figure 3.3: Bending light around a massive object from a distant source. The orange arrows show the apparent position of the background source. The white arrows show the path of the light from the true position of the source.

named Maarten Schmidt identified the first quasar (Schmidt 1963) using optical and radio telescopes. Quasars are distant galaxies that harbour an active nucleus, consisting of a supermassive black hole accreting matter in the shape of a disk.

Around the same time, [11] three astrophysicists independently revived the interest for gravitational lensing. In 1963, the USSR scientist Yu Klimov provided a mathematical description of lensing by galaxies (Klimov 1963a,c,b). In a 1964 paper, Sidney Liebes studied the probability of a stellar lens detection (Liebes 1964). The same year, Sjur Refsdal published two papers on the subject: the first one proposes a geometrical optics description of a point-mass lens. The second one highlights the possibility of using gravitational lensing observations to measure the Hubble parameter, giving for the first time a cosmological application to this phenomenon [4].

Finally, in 1979, the first observation of gravitational lensing of a quasar by a galaxy is confirmed. Dennis Walsh, Bob Carwell and Ray Weymann observed a pair of twin quasars, with the same spectrum at the same redshift, separated by a short distance (Walsh et al. 1979). They suggested immediately upon discovery that this twin object was in fact two images of the same background quasar formed through gravitational lensing. Shortly after, the elliptical galaxy responsible for the lensing had been identified by Stockton (1980). The second lensing candidate was discovered that same year (Weymann et al. 1980).

3.3 Form of Gravitational Lensing

Gravitational lensing is a consequence of one of the most famous predictions of Einsteins General Relativitythe idea that light is bent in a gravitational field.

There are three main forms of gravitational lensing:

3.3.1 Strong Gravitational Lensing

The first strong gravitational lens, discovered in 1979, was indeed linked to a quasar (QSO 0957+561 [19]), and although the phenomenon was expected on theoretical grounds, it left the astronomers surprise. The existence of two objects separated by about 6"(6 arcsec) and characterized by an identical spectrum led to the conclusion that they were the doubled image of the same quasar, clearly showing that Zwicky was perfectly right and that galaxies may act as gravitational lenses.

Afterwards, also the lens galaxy was identified, and it was established that its dynamical mass, responsible for the light deflection, was at least ten-times larger than the visible mass.

This double quasar was also the first object for which the time delay (about 420 days) between the two images [19], due to the different paths of the photons forming the two images, has been measured. This has also allowed obtaining an independent estimate of the lens galaxy dynamical mass. Observations can also show four images of the same quasar, as in the case of the so-called Einstein Cross, or when the lens and the source are closely aligned, one can observe the Einstein ring.

The macroscopic effect of multiple images formation is generally called strong lensing, which also consists of the formation of arcs, as those clearly visible in the deep sky field images by the Sloan Digital Sky Survey (SDSS). The sources of strong lensing events are often quasars, galaxies, galaxy clusters and supernovae, whereas the lenses are usually galaxies or galaxy clusters. The image separation is generally larger than a few tenths of an arcsec, often up to a few arcsecs.

Strong gravitational lensing is nowadays a powerful tool for investigation in astrophysics [9]. Strong lensing gives a unique opportunity to measure the dynamical mass of the lens object using, for example, the mass estimator $M(< R_E) = \pi \Sigma_{cr} \theta_E^2$.

Light rays leaving a source in different directions are focused on the same point by the intervening galaxy or cluster of galaxies. These are called strong lenses.

The first strong lensing observation was of the doubly imaged quasar Q0957+561 by Walsh, Carswell, and Weymann (1979). An optical image of QSO 0957+561 taken by HSTs WFPCII camera is shown in Figure 3.4. The magnification produced by strong lensing affects the observable properties of active galaxies, quasars, and any other lensed sources. Strong lensing also may provide information for cosmology. For example, the time delay among the multiple images of a quasar can be used to measure the Hubble constant. The first large

Figure 3.4: HST image of QSO 0957+561.(Soucail et al.1987a)

luminous arc produced by strong lensing (Figure 3.5) was found in the massive nearby cluster, Abell 370, in 1986 by Lynds and Petrosian (1986) at Kitt Peak National Observatory (KPNO) and by Soucail et al. (1987a) at the Canada France Hawaii Telescope (CFHT).

Giant arcs are due to the lensing effect of rich clusters of galaxies on background galaxies, with huge magnifications that can distort the galaxy shapes into long arcs around the clusters cores. The cluster Abell 2218 contains the most famous example of gravitationally lensed arcs (Figure 3.6). Until recently, the most massive galaxies and galaxy clusters have been the object of gravitational lensing studies. Galaxy groups are comprised of a lower density of galaxies than clusters, making them more difficult to detect.

After some controversy regarding whether ΛCDM (cold dark matter plus Cosmological Constant) simulations predict enough dark matter substructures to account for the observations, some indication is found of an excess of massive galaxy satellites), more recent analysis, taking also into account the uncertainty in the lens system ellipticity, finds results consistent with those predicted by the standard cosmological model. The strong lensing systems is main point we study under this thesis by Using ΛCDM Model. Three properties make strong gravitational lensing a most useful tool to measure and understand the

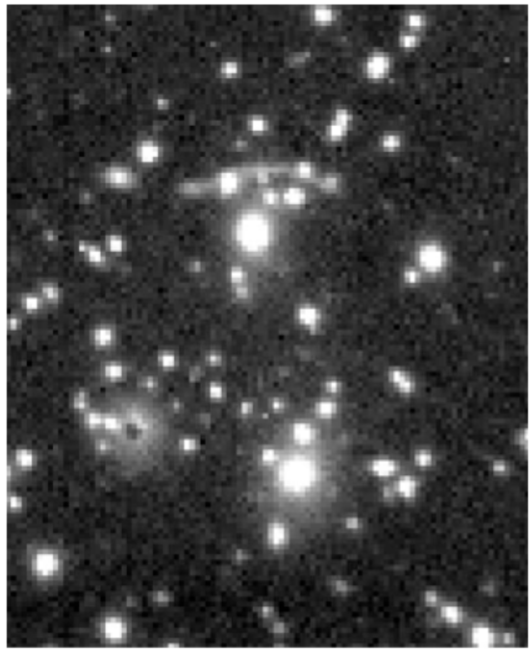

Figure 3.5: First observed giant gravitational arc (Soucail et al.1987a).

universe.

- **Firstly**,strong lensing observable - such as relative positions, flux ratios, and time delays between multiple images - depend on the gravitational potential of the foreground galaxy (lens or deflector) and its derivatives.

- **Secondly**, the lensing observable also depend on the overall geometry of the universe via angular diameter distances between observer, deflector, and source.

- **Thirdly**, the background source often appears magnified to the observer, sometimes by more than an order of magnitude.

As a result, gravitational lensing can be used to address three major astrophysical issues:

- Understanding the spatial distribution of mass at kpc and sub-kpc scale where baryons and DM interact to shape galaxies as we see them;

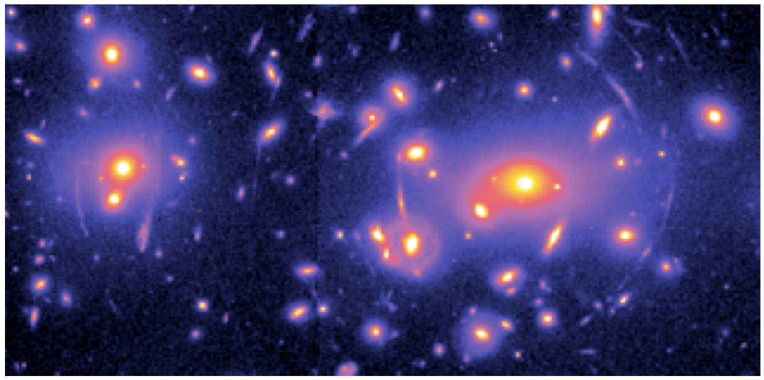

Figure 3.6: Foreground galaxies in the cluster Abell 2218 distort the images of background galaxies. Giant elliptical arcs surround the central region of the cluster at right.

- Determining the overall geometry, content, and kinematics of the universe;
- Studying galaxies, black holes, and active nuclei that are too small or too faint to be resolved or detected with current instrumentation.

3.3.2 Weak Lensing

In the deep field surveys of the sky, also arclets (i.e., single distorted images with an elliptical shape) and weakly distorted images of galaxies, with an almost invisible individual elongation, have been detected. This effect is known as *weak lensing* and is playing an increasingly important role in cosmology.

The weak lensings main feature is the shape deformation of background galaxies, whose light crosses a mass distribution (e.g., a galaxy or a galaxy cluster) that acts as a gravitational lens. Actually, gravitational lensing gives rise to two distinct effects on a source image: convergence, which is isotropic, and shear, which is anisotropic. In the weak lensing regime, the observer makes use of the shear, that is the image deformation (sometimes related to the galaxy orientation), while the convergence effect is not used, since the intrinsic luminosity and the size of the lensed objects are unknown.

The first weak lensing event was detected in 1990 as statistical tangential alignment of galaxies behind massive clusters [12], but only in 2000, coherent galaxy distortions were measured in blind fields, showing the existence of the cosmic shear [12].The weak lensing cannot be measured by a single galaxy, but its observation relies on the statistical analysis of the shape and alignment of a large number of galaxies in a certain direction.

There are at least two major issues in weak lensing studies, one mainly relying on the theory, the other one on observations: the former concerns finding the best way to reconstruct the intervening mass distribution from the shear field $\gamma = (\gamma_1, \gamma_2)$, the latter with looking for the best way to determine the true ellipticity of a faint galaxy, which is smeared out by the instrumental point spread function PSF). To solve these issues, several approaches have been proposed, which can be distinguished into two broad families: direct and inverse methods. On the theoretical side, the direct approaches are: the integral method, which consists of expressing the projected mass density distribution as the convolution of γ by a kernel, and the local inversion method, which instead starts from the gradient of ϕ (e.g., under [9] and the references therein). The inverse approaches work on the lensing potential, and they include the use of the maximum likelihood or the maximum entropy methods to determine the most likely projected mass distribution that reproduces the shear field. The inverse methods are particularly useful since they make it possible to quantify the errors in the resultant lensing mass estimates, as, for instance, errors deriving from the assumption of a spherical mass model when fitting a non-spherical system.

The inverse methods allow one also to derive constraints from external observations, such as X-ray data on galaxy clusters strong lensing or CMB lensing. In particular, one can compare mass measurements from weak lensing and X-ray observations for large samples of galaxy clusters.

weak lensing observations showed that the mass was largely concentrated around the galaxies themselves, and this enabled a clear, independent measurement of the amount of dark matter.

3.3.3 Microlensing

The Microlensing lensing is the phenomenon that occurs when θ_E is smaller than the typical telescope angular resolution, as in the case of stars lensing the light from background stars. If the source and the lens are aligned (first panel on the left), the circular symmetry of the problem leads to the formation of a luminous annulus having radius θ_E around the lens position. Otherwise, increasing the θ_S value, the secondary image gets closer to the lens position, while the primary image drifts apart from it, and in the limit of $\theta_s \gg \theta_E$, the microlensing phenomenon tends to disappear. However, observing multiple images during a microlensing event is practically impossible with the present technology. [19] For instance, in the case in which the phenomenon is maximized, corresponding to the perfect alignment.

3.4 Applications Gravitational Lensing

3.4.1 Measure Mass and Mass Distributions

Gravitational light deflection is determined by the gravitational field through which light propagates. This in turn is related to the mass distribution via the Poisson equation (or its GR generalization). It is essential to realize that this simple fact implies that gravitational light deflection is independent of the nature of the matter and of its state lensing is equally sensitive to dark and luminous matter, and to matter in equilibrium or far out of it. On the negative side, this implies that lensing alone cannot distinguish between these forms of matter, but on the positive side, it also cannot miss one of these matter forms. Hence, lensing is an ideal tool for measuring the total mass of astronomical bodies, dark and luminous.

Weak lensing studies of clusters estimate the mass distribution to much larger radii than the strong lensing regime [21], and like strong lensing effects, probe for asymmetries and substructures in the cluster mass. In fact, substructure in the mass distribution of lens galaxies has been detected, thereby confirming one of the robust predictions of the Cold Dark Matter model for our Universe. In addition, the mass distribution of galaxies at large radii, where one runs out of local dynamical tracers, can be studied statistically using an effect called galaxygalaxy lensing.

3.4.2 Constraining the Number Density of Mass Concentrations

The probability for a lensing event to occur (e.g., the fraction of high-redshift sources that are multiply imaged, or the fraction of stars undergoing microlensing) depends on the projected number density of potential lenses [21]. Hence, by investigating statistically well-defined samples of sources and their lensed fraction, we can infer the number density of lenses. Examples of such studies are estimates of the number density of compact objects in the dark halo of our Galaxy, the redshift evolution of the number density of galaxies acting as strong lenses, and the number density of clusters producing strong and weak lensing signals. Upper limits on the number of lensing events can also be translated into upper bounds on the number density of putative lenses: e.g., the fact that nearly all multiply-imaged sources have a visible lens galaxy puts strong upper bounds on the number density of dark lenses (they can at most provide a few percent of the galaxy-mass objects), and the non-detection of lens systems with image separations of tens of milli-arcseconds provides bounds on the number density of compact galaxies with masses $\sim 10^9 M_\odot$. In fact, by now lensing has put stringent constraints on the population of compact massive objects in the Universe over an extremely broad range of mass scales, from $\sim 10^{-3} M_\odot$ (from upper limits on the variability of distant quasars) to $\sim 10^{16} M_{\odot}$ (from the absence of very wide pairs of quasars), with only a few mass gaps within this range. Even lower-mass objects ($\sim 10^{-6} M_{\odot}$) can be ruled out as significant contributors to the dark matter in our Milky Way.

3.4.3 Cosmological Parameters

Following Refsdals idea, the Hubble constant can be obtained from the time delay in multiple image systems. This method has the advantage of being independent of the usual distance ladder used in determinations of H_0, and it also measures the Hubble constant on a truly cosmic scale, in contrast to the quite local measurements based on Cepheid distances. Despite the determination of time delays in a number of systems, values for H_0 by lensing are burdened with the uncertainties of the lens models; however, there is a trend toward slightly lower values of the Hubble constant than obtained from Cepheids. Other cosmological parameters can also be obtained from lensing [21]. For example, the fraction of lensed high-redshift quasars when combined with the distribution of image separations can be used to estimate the cosmological model. Weak lensing by the large-scale structure is sensitive to the matter density parameter and the normalization of the density fluctuations, and significant constraints on these parameters have been obtained. In particular in combination with results from the anisotropy of the cosmic microwave background, future cosmic shear studies will provide an invaluable probe of the equation of state of the dark energy. Weak lensing has also successfully been used to determine the bias parameter, which describes the relation between the statistical distribution of galaxies and the underlying dark matter, and for which only few alternative methods are available.

3.4.4 Lenses as Natural Telescopes

Since a lens can magnify background sources, these appear brighter than they would without a lens. This makes it easier to investigate these sources in detail, e.g. through spectroscopic observations. In some cases, this magnification is even essential to detect the sources in the first place, provided their lensed brightness just exceeds the detection threshold of a survey or of the current instrumental sensitivity. This magnification effect has in fact yielded spectacular results, such as very detailed spectra of very distant galaxies, the detection of some of the highest redshift galaxies behind cluster lenses, and the detection of very faint sub-millimeter sources in cluster fields.

With the lenses as magnifiers, larger effective angular resolution of the sources is obtained. Galaxies acting as sources for giant arcs can therefore be resolved in unprecedented detail, at least in one dimension. The host galaxy of quasars, which is difficult to study in unlensed objects owing to the large brightness contrast between the active nucleus and the surrounding host, can be studied much more easily when lensing allows the spatial resolution of the host in many cases, the host galaxy is in fact mapped into an Einstein ring.

3.4.5 Searches for Planets

As refereed in [12], the light curves of Galactic microlensing events are affected by companions of the main lens. For example, light curves of binary stars are readily identified as such, provided their separation falls into a favorable range determined by the geometry of the lens system. Because of that, even planets will leave an observable trace in the microlensing light curves if they are situated at the right radius from the star and at the right orbital phase. Although these traces can be quite subtle, and last for a short time only, current observing campaigns aimed at the search for planets have the sensitivity for their detection, and several candidate events for the detection of planetary signals in microlensing light curves have been reported. Indeed, microlensing is considered to be the simplest (and cheapest) possibility to detect the presence of low-mass planets around distant stars (ML). These few examples should suffice to illustrate the broad range of applications of gravitational lensing; the ever increased publication rate of articles investigating and applying gravitational lensing underlines the timeliness of the subject.

3.5 General Lens System

The description of a mass distribution as a point mass (Schwarzschild lens) is only sufficient for GL considerations [9]. Even if the deflector is a star, its gravitational field is distorted in most realistic cases, either because the star is part of a galaxy which provides a tidal gravitational field, or a disturbance is due to galaxies lying near the line-of-sight to the source therefore, introduces an additional distortion.Hence, the Schwarzschild lens is an idealization; however, it is extremely useful, not only because of its simple properties, but also because such a simple model provides relations, e.g., between lens mass, the distances to lens and source, and angular separation between the images of a lensed source, which are of the same order of-magnitude as those for more realistic lenses.whereas a Schwarzschild lens (and other matter distributions with spherical symmetry) can produce ring-shaped images of arbitrarily small sources, a general lens does not have this property.

3.5.1 Description of a general lensing situation

Fig. 3.8 is a typical lensing situation.[9] Consider the source sphere S_s, i.e., a sphere with radius D_s , centered on the observer 0, and, correspondingly, the deflector sphere S_d with radius D_d , i.e., the distance to the center of the lens L [12]. Here we assume that the lens has a velocity relative to a comoving observer which is much smaller than the velocity of light. We call the straight line through 0 and L the *optical axis*, which serves as a reference

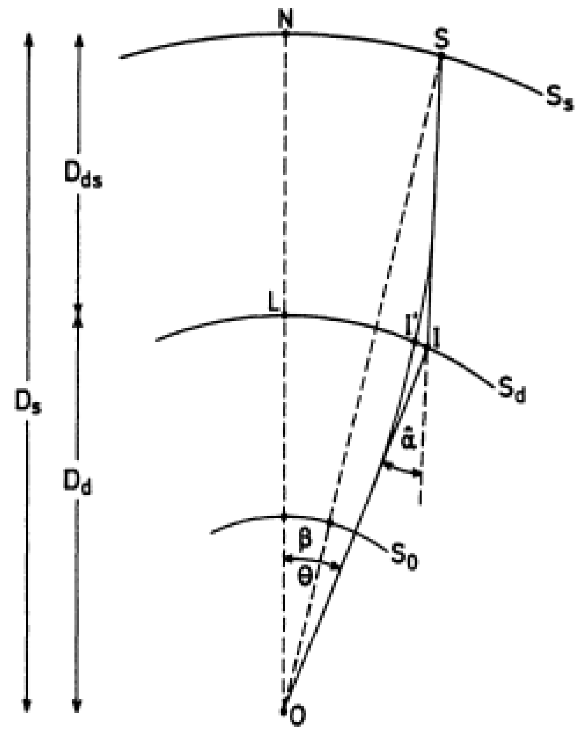

Figure 3.7: A general GL system; the 'center' of the lens is at L, and the line through L and the observer is the 'optical axis'. Relative to that, the source S has an undisturbed angular position β. A light ray SI'O from the source is deflected by an angle $\hat{\alpha}$, so that an image of the source is observed at position θ. Due to the smallness of all angles present, we can replace the real light ray by its approximation SIO, and the source and lens spheres by their tangent planes.(Source [9])

line; it intersects the source sphere at N. In addition, consider the observer sphere S_o which is the apparent "sky" of the observer. On S_o, the source would have angular position β if the light rays from the source S were not influenced by the gravitational field of the deflector. However, since the lens does bend light rays, the straight line SO is no longer a physical ray path. Rather, there are light rays which connect source and observer but which are curved near S_d. One such ray Sl'O is drawn, together with its approximation SID, consisting of the two asymptotes of the real ray. The angle $\hat{\alpha}$ between the two asymptotes SI and IO is the deflection angle caused by the matter distribution L. The observer will thus see the source at the position θ on his sphere S_o.

3.5.2 The deflection angle

General relativity predicts that a dense object bends the space-time continuum in its vicinity.In the general relativistic case, however, gravity affects both the spatial and time component of the photons path, so that the actual bending is twice this value. Thus, we define the angle of deflection, otherwise known as the Einstein angle, as[1].

A simple sketch of the deflection is shown on Figure 3.8. The deflection angle $\hat{\alpha}$ on a light ray passing by a mass M at a distance b is:

$$\hat{\alpha} = \frac{4GM}{bc^2}. \tag{3.5.1}$$

where G is the gravitational constant and c the speed of light.

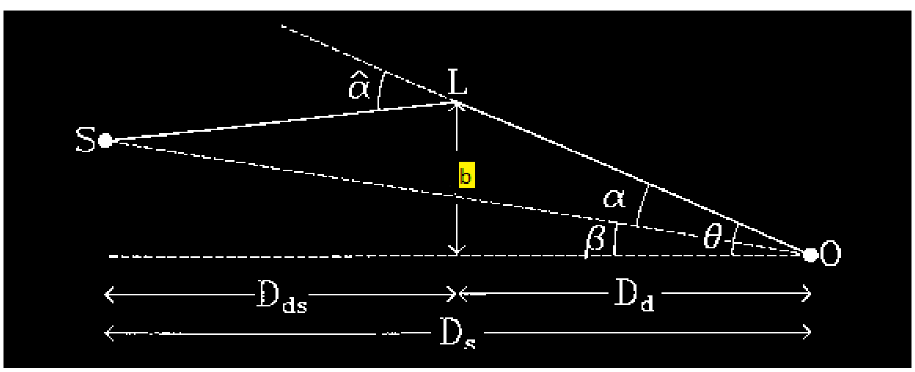

Figure 3.8: The deflection of a light by a mass M, O is the observer, b is the impact parameter

3.5.3 The lens equation

Let us consider first a point-like mass M. The aim is to express the deflection angle as a function of observables quantities. Figure 3.9 represents the situation of an observer O observing the image I of a source S lensed by L. D_L, D_S and D_{LS} are the respective

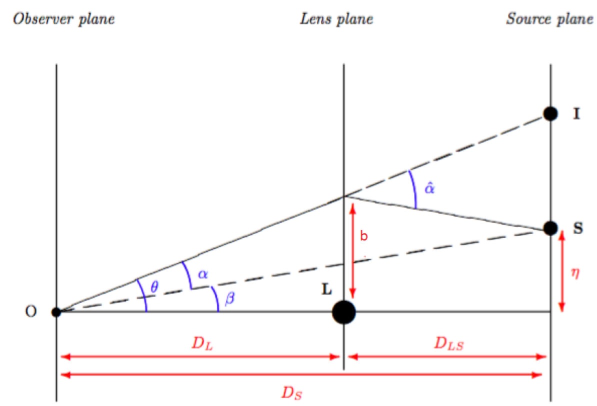

Figure 3.9: A diagram of gravitational lensing by a lens L of a light ray emitted by a source S into an image I as seen by the observer O Credit: Chantry (2009)[1]

distances between the observer, the lens, and the source. At cosmological scales, it can be assumed that these distances are much larger than the typical size of a galaxy. This yields two reasonable working hypotheses: first, the thin lens approximation, which means that all the mass of the lens is concentrated in a plane at a distance D_L. The same approximation is tacitly assumed for the background source. Second, the small-angle approximation makes it possible to approach a few trigonometrical functions of the angles by the size of the angles themselves. α is the apparent angle between the source and its lensed image, as measured by the observer, whereas $\hat{\alpha}$ is that same angle, as measured from the lens plane

They are linked by the following reduced deflection angle equation

$$\alpha = \frac{D_{LS}}{D_S}\hat{\alpha} \tag{3.5.2}$$

β is the angular position of the source and θ is the angular position of the lensed image, both as seen by the observer. θ is usually the only observable angle. b is the distance from the lens to the intersection between the light ray and the lens plane.

Under the small-angle hypothesis, using trigonometry, we have

$$\theta D_S = \beta D_S + \hat{\alpha} D_{LS} \tag{3.5.3}$$

By using Equation 3.5.2 equation 3.5.3 can be rewritten as:

$$\beta = \theta - \alpha(\theta) \tag{3.5.4}$$

then by substituting for $\alpha(\theta)$

$$\beta = \theta - \hat{\alpha}(\theta)\frac{D_{LS}}{D_S} \tag{3.5.5}$$

The above equation 3.5.5 is the lens equation. These relations are valid under the assumption that the mass M is point-like, which is unphysical.

"Einstein rings"

For a point lens of mass M the deflection angle is given by equation (3.5.1). Plugging into equation (3.5.5) and using the relation $b = D_L\theta$ (from Figure 3.9) one obtains:

$$\beta(\theta) = \theta - \frac{4GM}{c^2\theta}\frac{D_{LS}}{D_L D_S} \tag{3.5.6}$$

For the special case in which the source lies exactly behind the lens ($\beta = 0$), due to the symmetry a ring-like image occurs whose angular radius is called **Einstein radius** or Einstein ring θ_E:

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_L D_S}} \tag{3.5.7}$$

The Einstein radius defines the angular scale for a lens situation.

Ring-shaped images of point sources can only occur in lensing situations with axially symmetric matter distributions; they arise solely from symmetry. On the other hand, extended sources can have ring-shaped images even if the lens is not perfectly symmetric,Such images are frequently called "Einstein rings".

3.6 Angular Distance in Gravitational Lensing

There are many ways to specify the distance between two points, because in the expanding Universe, the distances between comoving objects are constantly changing, and Earth-bound observers look back in time as they look out in distance [8]. The unifying aspect is that all distance measures somehow measure the separation between events on radial null trajectories, i.e, trajectories of photons which terminate at the observer.

3.6.1 Cosmographic parameters

The Hubble constant H_0

is the constant of proportionality between recession speed v and distance d in the expanding Universe [8]

$$v = H_0 d$$

The subscripted "0" refers to the present epoch because in general H changes with time. The dimensions of H_0 are inverse time, but it is usually written as

$$H_0 = 100hkms^{-1}Mpc^{-1}$$

where h is a dimensionless number parameterizing our ignorance. The inverse of the Hubble constant is the Hubble time t_H

$$t_H = \frac{1}{H_0}$$

and the speed of light c times the Hubble time is the Hubble distance D_H

$$D_H = \frac{c}{H_0} = \frac{c}{H_0(1+Z)} \tag{3.6.1}$$

Where Z-is the redshift, for Gravitational Lensing at source Z_S, at Lens Z_L

The mass density ρ of the Universe and the value of the cosmological constant Λ

They are dynamical properties of the Universe, affecting the time evolution of the metric [8]. They can be made into dimensionless density parameters Ω_M and Ω_Λ by

$$\Omega_M = \frac{8\pi G\rho_0}{3H_0^2}$$

$$\Omega_\Lambda = \frac{\Lambda}{3H_0^2}$$

A third density parameter Ω_K measures the "curvature of space" and can be defined by the relation

$$\Omega_M + \Omega_K + \Omega_\Lambda = 1 \tag{3.6.2}$$

Assuming the observed Flatness $\Omega_K = 0, \Omega_M = 0.27, \Omega_\Lambda = 0.73$

3.6.2 Comoving distance (line-of-sight)

A small comoving distance δD_C between two nearby objects in the Universe is the distance between them which remains constant with epoch if the two objects are moving with the Hubble flow [8]. In other words, it is the distance between them which would be measured with rulers at the time they are being observed (the proper distance) divided by the ratio of the scale factor of the Universe then to now. In other words the proper distance multiplied by (1+z). The total line-of-sight comoving distance D_C from us to a distant object is computed by integrating the infinitesimal δD_C contributions between nearby events along the radial ray from z = 0 to the object.

As adopted by [8] comoving distance

$$E(Z) = \sqrt{\Omega_M(1+Z)^3\Omega_K(1+Z)^2 + \Omega_\Lambda} \tag{3.6.3}$$

Since the speed of light is constant, this is a proper distance divided by the scale factor, which is the definition of a comoving distance. The total line-of-sight comoving distance is then given by integrating these contributions, or

$$D_C = \frac{c}{H_0(1+Z)} \int_0^z \frac{dz'}{E(z')} \tag{3.6.4}$$

where D_H is the Hubble distance defined above.

In some sense the line-of-sight comoving distance is the fundamental distance measure in cosmography since, as will be seen below, all others are quite simply derived in terms of it.

3.6.3 The angular diameter distance

The angular diameter distance D_A between source and Observer at redshifts Z_S and the observer and Lens at Z_L, frequently used in gravitational lensing [8]. It is not found by subtracting the two individual angular diameter distances.

by using eqn.3.6.4 above is

- The angular Distance observer to Lens D_L is

$$D_L = \frac{c}{H_0(1+Z_L)} \int_0^{z_L} \frac{dz'}{E(z')} \tag{3.6.5}$$

- The angular Distance observer to Source D_S is

$$D_S = \frac{c}{H_0(1+Z_S)} \int_0^{z_S} \frac{dz'}{E(z')} \tag{3.6.6}$$

- The angular Distance Lens to Source D_{LS} is

$$D_{LS} = \frac{c}{H_0(1+Z_S)} \int_{Z_L}^{Z_S} \frac{dz'}{E(z')} \tag{3.6.7}$$

where $D_H = \frac{c}{H_0(1+Z)}$ is the Hubble Distance, c-is speed of Light H_0-is Hubble Constant

Chapter 4

Gravitational Lensing in the standard ΛCDM Cosmology

4.1 Introduction

During the 1980s, most research focused on cold dark matter with critical density in matter, around 95% CDM and 5% baryons: these showed success at forming galaxies and clusters of galaxies, but problems remained; notably, the model required a Hubble constant lower than preferred by observations, and observations around 1988-1990 showed more large-scale galaxy clustering than predicted. These difficulties sharpened with the discovery of CMB anisotropy by COBE in 1992, and several modified CDM models, including ΛCDM and mixed cold and hot dark [10] matter, came under active consideration through the mid-1990s.

The ΛCDM model then became the leading model following the observations of accelerating expansion in 1998, and was quickly supported by other observations: in 2000, the BOOMER and microwave background experiment measured the total (matter-energy)density to be close to 100% of critical,whereas in 2001 the 2nd FGRS galaxy redshift survey measured the matter density to be near 25%; the large difference between these values supports a positive Λ or dark energy. Much more precise spacecraft measurements of the microwave background from WMAP in 2003 - 2010 and Planck in 2013 - 2015 have continued to support the model and pin down the parameter values, most of which are now constrained below 1% uncertainty.[7, 16, 17].

In the twenty-first century, gravitational lensing is a highly active field of astrophysical research. Since the first conference exclusively devoted to gravitational lensing was held in Lige, France, in 1983, there have been similar international conferences every year.

The reason for the field's growth is that, today, gravitational lenses are much more than just an interesting general relativistic phenomenon. Now that a significant number of lens systems has been identified, lensing is used more and more as an observation tool, allowing us to answer astrophysical as well as cosmological questions, from estimates of the amount of dark matter contained in the lens mass to the determination of fundamental parameters of the big bang models[19].

The thesis was mainly adopted by Considering GR in the presence of positive cosmological constant to derive gravitational lensing equation and effect of cosmological constant through **vacuum fluid approach** with the simple point Mass source model. The background medium is considered as a continues smeared fluid of varying **refractive index** as a function of radial distance and cosmological constant. All the angular distances also consider the cosmological constant and the expanding universe scenario by way of the transformation between the static and co-moving coordinates. The lensing equation assumes the deflectors (the lens) and the sources positions angular distances through the observed redshifts by Hubble law. Then, the analytically derived Lens equations are being used to generate some numerical values to compare with observation, For the computation Mathematica 11 is used.

4.1.1 Index of Refractive

Starting from the field equations of general relativity, light deflection can be calculated by studying geodesic curves. It turns out that light deflection can equivalently be described by Fermats principle, as in geometrical optics.

Fermat's Principle: A light ray from a source S (spacetime event) to an observer 0 (timelike curve) follows a trajectory that is a stationary value of the arrival times t, measured relative to the observer's proper time, of all paths from S to 0 [13].

We first need an index of refraction n because Fermats principle says that light will follow a path along which the travel time,will be extremal

$$\int \frac{n}{c} dl \tag{4.1.1}$$

As in geometrical optics, we thus search for a path, $\vec{x}(l)$, for which the variation

$$\delta \int_{A}^{B} n(\vec{x}(l)) dl = 0 \tag{4.1.2}$$

In order to find the index of refraction, we make a first approximation: we assume that the lens is weak, and that it is small compared to the overall dimensions of the optical system composed of source, lens and observer. There are multiple ways at gravitational lensing which agree in the limit which is commonly applied. In by far the most astrophysical applications, the Newtonian gravitational potential ϕ is small, $|\phi|/c^2 \ll 1$, and the lensing mass distribution moves slowly with respect to the cosmological rest frame.

Under such conditions, gravitational lensing can be described by a small perturbation of the weak lens perturbs this metric such that

$$\eta_{\mu\nu} \rightarrow g_{\mu\nu} = \begin{pmatrix} 1+\frac{2\phi}{c^2} & 0 & 0 & 0 \\ 0 & -(1-\frac{2\phi}{c^2}) & 0 & 0 \\ 0 & 0 & -(1-\frac{2\phi}{c^2}) & 0 \\ 0 & 0 & 0 & -(1-\frac{2\phi}{c^2}) \\ \end{pmatrix}$$

for which the line element becomes

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = (1+\frac{2\phi}{c^2})c^2dt^2 - (1-\frac{2\phi}{c^2})(d\vec{x})^2 \tag{4.1.3}$$

Also for the Schwarzschild de-sitter Metrics

$$g_{\mu\nu} = \begin{pmatrix} 1+\frac{2\phi}{rc^2}-\frac{\Lambda r^2}{3} & 0 & 0 & 0 \\ 0 & -(1-\frac{2\phi}{rc^2}-\frac{\Lambda r^2}{3}) & 0 & 0 \\ 0 & 0 & -(1-\frac{2\phi}{rc^2}-\frac{\Lambda r^2}{3}) & 0 \\ 0 & 0 & 0 & -(1-\frac{2\phi}{rc^2}-\frac{\Lambda r^2}{3}) \\ \end{pmatrix}$$

for which the line element becomes

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -(1+\frac{2\phi}{rc^2}-\frac{\Lambda r^2}{3})dt^2 + (1-\frac{2\phi}{rc^2}-\frac{\Lambda r^2}{3})^{-1}(d\vec{x})^2 \tag{4.1.4}$$

Now light propagates or projected at null geodesic (ds = 0), from which we gain

$$(1+\frac{2\phi}{rc^2}-\frac{\Lambda r^2}{3})dt^2 = (1-\frac{2\phi}{rc^2}-\frac{\Lambda r^2}{3})^{-1}(d\vec{x})^2 \tag{4.1.5}$$

The light speed in the schwarzschild de-sitter metric is thus

$$c' = \frac{|d\vec{x}|}{dt} = c\sqrt{\frac{1+\frac{2\phi}{rc^2}-\frac{\Lambda r^2}{3}}{1-\frac{2\phi}{rc^2}-\frac{\Lambda r^2}{3}}} \approx c(1+\frac{2\phi}{rc^2}-\frac{\Lambda r^2}{3}), \tag{4.1.6}$$

where we have used that $\phi/c^2 \ll 1$ by assumption. The index of refraction is thus

$$n = \frac{c}{c'} = \frac{1}{1+\frac{2\phi}{rc^2}-\frac{\Lambda r^2}{3}} \approx 1-\frac{2\phi}{c^2}-\frac{\Lambda r^2}{3} \tag{4.1.7}$$

With $\phi \leq 0$, $n \geq 1$, and the light speed c' is lower than in vacuum.

n will typically depend on the spatial coordinate $\vec{x}$ and perhaps also on proper time τ. Let $\vec{x}(l)$ be a light path. Then the light travel time is proportional to

$$\int_A^B n[\vec{x}(l)]dl, \tag{4.1.8}$$

and the light path follows from

$$\delta \int_A^B n[\vec{x}(l)]dl = 0 \tag{4.1.9}$$

This is a standard variational problem, which leads to the well known Euler equations. In our case we write

$$dl = |\frac{d\vec{x}}{d\tau}|d\tau \tag{4.1.10}$$

with a curve parameter τ which is yet arbitrary, and find

$$\delta \int_A^B d\tau n[\vec{x}(l)]|\frac{d\vec{x}}{d\tau}| = 0 \tag{4.1.11}$$

The expression

$$n[\vec{x}(l)]|\frac{d\vec{x}}{d\tau}| = L(\dot{\vec{x}}, \vec{x}, \tau) \tag{4.1.12}$$

takes the role of the Lagrangian in analytic mechanics, with

$$\dot{\vec{x}} = \frac{d\vec{x}}{d\tau} \tag{4.1.13}$$

Finally, we have

$$|\frac{d\vec{x}}{d\tau}| = |\dot{\vec{x}}| = (\dot{\vec{x}}^2)^{1/2} \tag{4.1.14}$$

Using these expressions, we find the Euler equations

$$\frac{d}{d\tau}\frac{\partial L}{\partial \dot{\vec{x}}^2} - \frac{\partial L}{\partial \vec{x}} = 0. \tag{4.1.15}$$

Now

$$\frac{\partial L}{\partial \vec{x}} = |\dot{\vec{x}}| \frac{\partial n}{\partial \vec{x}} = (\vec{\nabla} n)|\dot{\vec{x}}|, \frac{\partial L}{\partial \dot{\vec{x}}} = n \frac{\dot{\vec{x}}}{|\dot{\vec{x}}|} \dot{\vec{x}} \tag{4.1.16}$$

$\dot{\vec{x}}$ is a tangent vector to the projected light path, which we can assume to be normalized by a suitable choice for the curve parameter τ. We thus assume $|\dot{\vec{x}}| = 1$ and write $\vec{e} = \dot{\vec{x}}$ for the unit tangent vector to the projected light path. Then, we have

$$\frac{d}{d\tau}(n\vec{e}) - \vec{\nabla} n = 0 \tag{4.1.17}$$

or

$$n\dot{\vec{e}} + \vec{e}.[(\vec{\nabla} n)\dot{\vec{x}}] = \vec{\nabla} n \tag{4.1.18}$$

$$\Rightarrow n\dot{\vec{e}} = \vec{\nabla} n - \vec{e}(\vec{\nabla} n.\vec{e}) \tag{4.1.19}$$

The second term on the right hand side of eqn 4.1.19 is the derivative along the projected light path, thus the whole right hand side is the gradient of n perpendicular to the light path. Thus

$$\dot{\vec{e}} = \frac{1}{n} \vec{\nabla} \perp n = \vec{\nabla} \perp \ln n \tag{4.1.20}$$

As $n = 1 - 2\phi/rc^2 - \frac{\Lambda r^2}{3}$ and $\phi/c^2 \ll 1$, $\ln n \approx -2\phi/rc^2 - \frac{\Lambda r^2}{3}$, and

$$\dot{\vec{e}} = -\frac{2}{c^2} \vec{\nabla} \perp \phi - \frac{\Lambda r^2}{3} \tag{4.1.21}$$

4.2 The effect of cosmological constant in gravitational lensing through vacuum fluid approach

4.2.1 Deflection angle from Refractive index

The total deflection angle of the light path is the integral over the refractive index along the light path,

$$\hat{\vec{\alpha}} = \int_A^B \vec{\nabla}_\perp (\phi - \frac{\Lambda r^2}{3}) d\tau \tag{4.2.1}$$

Where the first term of RHS of the above equation is gravitational potential in GR ($\phi = 2GM/rc^2$) and the 2nd term is the Cosmological potential term.

The deflection is thus the integral over the pull of the gravitational potential and cosmological potential perpendicular to the projected light path. Note that $\vec{\nabla}(\phi - \frac{\Lambda r^2}{3})$ points away from the lens center, so $\hat{\vec{\alpha}}$ points towards it [20].

Which is the gradient of the dimension-less Newtonian potential and Cosmological potential perpendicular to the deflected light ray, integrated along the light ray and multiplied by two. This factor of two comes from the fact that the perturbed schwarzschild de-sitter metric has equal perturbations in both its temporal and spatial components.

Now From refractive index gradient integral scenario of eqn 4.1.21 the total deflection angle

$$\begin{aligned} \frac{\partial n}{\partial r} &= \alpha dr \\ \alpha &= -\int \frac{1}{n}\frac{\partial n}{\partial r} dr \end{aligned} \tag{4.2.2}$$

Deflection angle in terms of matter and Cosmological constant

Now from the above total deflection angle, the deflection angle interms of matter and Cosmological constant Λ for refractive index ($n = 1 + \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{3}$) is

$$\begin{aligned} \alpha &= -\int_{-D_{LS}}^{D_L} \vec{\nabla}\left(\tfrac{2GM}{c^2 r} - \tfrac{\Lambda r^2}{3} \right) \\ &= -\int_{-D_{LS}}^{D_L} \frac{1}{r}\left(\tfrac{2GM}{c^2 r^2} - \tfrac{2\Lambda r}{3} \right) \\ \alpha &= -\int_{-D_{LS}}^{D_L} \frac{1}{r}\left(\tfrac{2GM}{c^2 r^3} - \tfrac{2\Lambda}{3} \right) \end{aligned} \tag{4.2.3}$$

$$\alpha(\Lambda, m) = -\int_{-D_{LS}}^{D_L} \left(\tfrac{2GM}{c^2 r^3} - \tfrac{2}{3}\Lambda \right) ydx \tag{4.2.4}$$

The integration limit, for the origin of the plane of the lens located at the center of the lens, is all right. But the integration limit in the final result for the correction term is switched only from the source to the plane of the lens.This effect neglects the lensing from the plane of the lens to the observer, and so contradicts the starting assumption of the lensing system being considered with varying refractive index from its center outwards or the reverse.

So with this comment we will have the following improved approximation on the effect of the cosmological constant on lensing.

a. deflection angle contribution due to matter The first integral term of eqn.4.2.4 is represented by $\alpha(m)$. Of course it is the deflection angle contribution due to matter.

$$\alpha(m) = -\int_{-D_{LS}}^{D_L} \left(\tfrac{2GM}{c^2 r^3} \right) y dx \tag{4.2.5}$$

As D_L and D_{LS} get very large it is possible to replace the limit of integration from $-\infty$ to $+\infty$. So

$$\alpha(m) = -\int_{-\infty}^{+\infty} \left(\tfrac{2GM}{c^2 r^3} \right) y dx \tag{4.2.6}$$

$$\alpha(m) = \frac{2GM}{c^2 y} \int_{-\infty}^{+\infty} \left(\tfrac{1}{r^3} \right) dr \tag{4.2.7}$$

For our Spacetime is spherical symmetry r has the r,θ, and ϕ components

$$r = \begin{bmatrix} r\sin\theta\cos\phi \\ r\sin\theta\sin\phi \\ r\cos\theta \end{bmatrix}$$

Thus

$$\frac{\partial r}{\partial r} = \begin{vmatrix} r\sin\theta\cos\phi \\ r\sin\theta\sin\phi \\ r\cos\theta \end{vmatrix} = \left| \tfrac{\partial r}{\partial r} \right| = 1$$

$$\frac{\partial r}{\partial \theta} = \begin{bmatrix} r\cos\theta\cos\phi \\ r\cos\theta\sin\phi \\ -r\sin\theta \end{bmatrix} = \left| \tfrac{\partial r}{\partial \theta} \right| = r$$

$$\frac{\partial r}{\partial \phi} = \begin{bmatrix} -r\sin\theta\sin\phi \\ r\sin\theta\cos\phi \\ 0 \end{bmatrix} = \left| \frac{\partial r}{\partial \phi} \right| = r\sin\theta$$

The surface element spaning from θ to $\theta + d\theta$ and ϕ to $\phi + d\phi$ at constant spherical surface r

$$\left|| \frac{\partial r}{\partial \theta}\hat{r} \times \frac{\partial r}{\partial \phi}\hat{r} \right| |d\theta d\phi = r^2 \sin\theta d\theta d\phi$$

Hence, by integrating by part

$$\int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi} \sin\theta d\theta \int_{r=0}^{\infty} \frac{1}{r^3} dr = 4$$

Then now

$$\begin{aligned} \alpha(m) &= \frac{2GM}{c^2 y} \int_{-\infty}^{+\infty} \left(\tfrac{1}{r^3} \right) dr \\ \alpha(m) &= \frac{2GM}{c^2 y} \left(\int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi} \sin\theta d\theta \int_{r=0}^{\infty} \frac{1}{r^3} dr \right) \end{aligned}$$

This is easily integrated to give us

$$\alpha(m) = \frac{4GM}{c^2 y} \tag{4.2.8}$$

It seems that the integral depends on y, but the matter contribution is just within its strong field. Hence for effective matter contribution of Einstein photon deflection

$$\alpha(m) = \frac{4GM}{c^2 b} \tag{4.2.9}$$

where b is the closest distance by the photon to the lensing.

Of course it is possible to have some additional terms from second to other higher order terms in GM/c^2, which revives the Robertson - Walker metric expansion form.

b. deflection angle due to cosmological Constant contribution The integral of the cosmological effect part as in equation 4.2.4 is trivially integrated to give us

$$\alpha(\Lambda) = \frac{2}{3}\Lambda y \int_{-D_{LS}}^{D_L} dx = \frac{2}{3}\Lambda y D_S \tag{4.2.10}$$

Here we note that the effect is vacuum dominance and therefore one cannot treat the effect in similar manner as that of the matter. With this understanding the

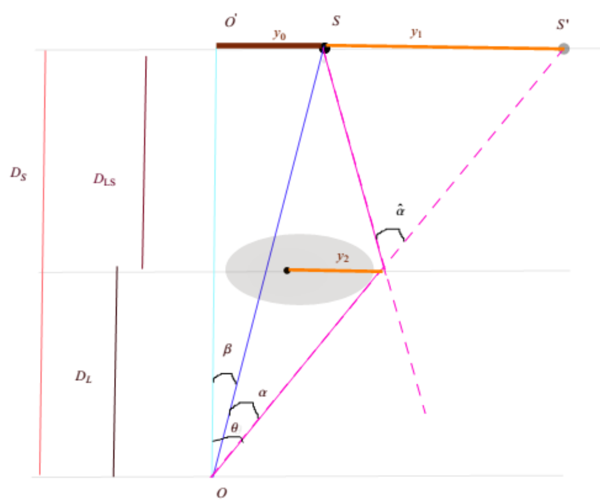

Figure 4.1: Sketch of a gravitational-lens system of point mass(self sketched):

deflection angle arising due to cosmology varies with y over the whole space extended along the path of the photon. So in equation 4.2.10 the value of y averaged over all the path length of the photon must be used (fig.4.1). Though, it still needs further analysis(future work), we can reasonably approximate the average value of y as in the following manner;

Let y_1 is the average value of y along the path of photon from the source to the plane of the lens. That means it varies from βD_S to the closest distance b or θD_L. Then for order of cosmological distances it is quite reasonable to average it as,

$$\begin{aligned} y_1 &= \frac{1}{2}(y_0 + y_I) \\ \tan\beta &= \frac{y_0}{D_s} \\ y_0 &= \beta D_s \\ \tan\theta &= \frac{y_I}{D_L} \\ y_I &= \theta D_L \end{aligned}$$

Since D_{LS} and D_L are nearly the same order of magnitudes we can use D_L as D_{LS} and for very small angle $\tan\beta \approx \beta, \tan\theta \approx \theta$. Then for y_1 is the average of y_0 and y_I

$$y_1 = \frac{1}{2}(\theta D_L + \beta D_s) \tag{4.2.11}$$

In a similar way we define y_2 as the average of y over the path of the photon travel from the plane of the lens to the observer given by

$$y_2 = \frac{1}{2}\theta D_L \tag{4.2.12}$$

Since D_L and D_{LS} are nearly the same order of magnitudes, we can once again reasonably average y over y_1 and y_2 to obtain

$$y_{av} = \frac{1}{2}(\theta D_L + \frac{1}{2}\beta D_S) \tag{4.2.13}$$

So, the contribution of cosmological constant to the deflection of light in the vicinity of eqs. 4.2.10, 4.2.13 is given by

$$\begin{aligned}
\alpha(\Lambda) &= \int_{-D_{LS}}^{D_L} -\frac{2}{3}\Lambda y dx \\
&= -\frac{2}{3}\Lambda y \int_{-D_{LS}}^{D_L} dx \\
&= -\frac{2}{3}\Lambda y x|_{-D_{LS}}^{D_L} \\
&= -\frac{2}{3}\Lambda y (D_L - (-D_{LS})) \\
\alpha(\Lambda) &= -\frac{2}{3}\Lambda y D_S
\end{aligned}$$

From the curved space-time background of vacuum fluid source the angular distance is not additive i.e;

$$D_S \neq D_L + D_{LS} \tag{4.2.14}$$

Instead we use the distance of redshift is used

$$D_S(1 + z_s) = D_L(1 + z_L) + D_{LS}(1 + z_s) \tag{4.2.15}$$

$$\begin{aligned}
\alpha(\Lambda) &= -\frac{2}{3}\Lambda(\frac{1}{2}(\theta D_L + \frac{1}{2}\beta D_S))D_S \\
&= -\frac{2}{3}\Lambda \times \frac{1}{2}(\theta D_L + \frac{1}{2}\beta D_S)D_S
\end{aligned}$$

$$\alpha(\Lambda) = -\frac{1}{3}\Lambda(\theta D_L + \frac{1}{2}\beta D_S)D_S \tag{4.2.16}$$

Now by eqs. 4.2.11 ,4.2.16 the angle of deflection for lensing through *vacuum fluid approach* is given by

$$\alpha(\Lambda, m) = \frac{4GM}{c^2 b} - \frac{1}{3}\Lambda(\theta D_L + \frac{1}{2}\beta D_S)D_S \tag{4.2.17}$$

Or

$$\alpha(\Lambda, m) = \frac{4GM}{c^2 D_L \theta} - \frac{1}{3}\Lambda(\theta D_L + \frac{1}{2}\beta D_S)D_S \tag{4.2.18}$$

Where we replaced b by θDL

4.2.2 The Lens Equation

Assuming spherical spacetime, From fig.4.2 Let

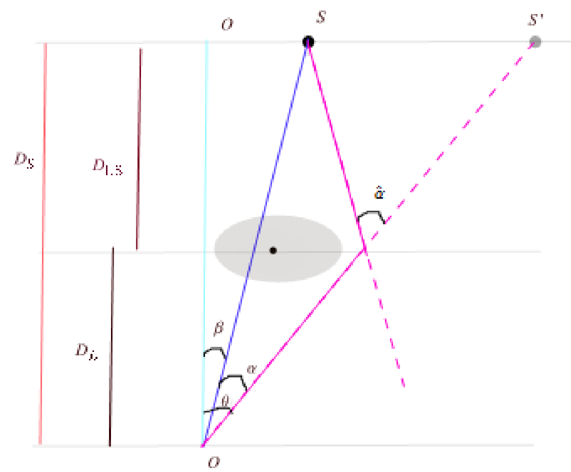

Figure 4.2: Sketch of a gravitational-lens system of point mass(self sketched): The optical axis runs from the observer O through the centre of the lens to O'. The angle between the source S and the optical axis O' is β, the angle between the image S' and the optical axis O' is θ. The light ray towards the image is bent by the deflection angle $\hat{\alpha}$, measured at the lens. The reduced deflection angle α is measured at the observer

$$\begin{aligned} \widehat{O'S} &= \beta D_s \\ \widehat{O'S'} &= \theta D_s \\ \widehat{SS'} &= \alpha D_s \\ \widehat{SS'} &= \hat{\alpha} D_{LS} \end{aligned}$$

But we have

$$\begin{aligned} \alpha D_s &= \hat{\alpha} D_{LS} \\ \hat{\alpha} &= \frac{\alpha D_s}{D_{LS}} \end{aligned}$$

Then now the lens equation measured at observer is derived as follows

$$\begin{aligned} \widehat{O'S'} &= \widehat{O'S} + \widehat{SS'} \qquad (4.2.19) \\ &= \beta D_s + \hat{\alpha} D_{LS} \\ \widehat{O'S'} &= \theta D_s = \beta D_s + (\frac{\alpha D_s}{D_{LS}}) D_{LS} \\ \vec{\theta} &= \vec{\beta} + \vec{\alpha} \end{aligned}$$

If we rewrite eqn.interms of θ

$$\vec{\alpha}(\theta) = \vec{\theta} - \vec{\beta}(\theta) \qquad (4.2.20)$$

Again

$$\begin{aligned} \widehat{O'S} &= \widehat{O'S'} - \widehat{SS'} \qquad (4.2.21) \\ \beta D_s &= \theta D_S - \hat{\alpha} D_{LS} \end{aligned}$$

By dividing both side D_S finally we get lens equation

$$\beta = \theta - \hat{\alpha} \frac{D_{LS}}{D_S} \qquad (4.2.22)$$

By noting that, for small angles and with the angle expressed in radians, the point of nearest approach y at an angle α for the lens L on a distance D_L is given by $y = \theta D_L$,

For a source right behind the lens, $\theta D_L = 0$, and the lens equation for a point mass gives a characteristic value for θ that is called the Einstein radius, denoted θ_E. Putting $\beta D_S = 0$ and solving for θ gives the Einstein radius for a point mass provides a convenient linear scale to make dimensionless lensing variables. The Einstein radius most prominent for a lens typically halfway between the source and the observer.

Substituting eq. 4.2.18 in the lens equation 4.2.22 we get **lens equation with the effect of Cosmological constant**;

$$\beta = \theta - \frac{D_{LS}}{D_S} \left(\frac{4GM}{c^2 D_L \theta} - \frac{1}{3}\Lambda(\theta D_L + \frac{1}{2}\beta D_S) D_S \right) \qquad (4.2.23)$$

This is the Fundamental lensing equation we derive in the presence of cosmological constant.

Alignment

When the source is exactly behind the lens, the angular position of the source(S) and the Optical sight(O) becomes Align;i.e $\beta = 0$ then

$$\beta = \theta - \frac{D_{LS}}{D_S}\left(\frac{4GM}{c^2\theta D_L} - \frac{1}{3}\Lambda(\theta D_L + \frac{1}{2}\beta D_S)D_S\right)$$

$$o = \theta - \frac{D_{LS}}{D_S}\left(\frac{4GMD_S}{c^2\theta D_L} - \frac{1}{3}\Lambda b D_S - 0\right)$$

$$\theta \quad = \quad \left(\frac{4GMD_{LS}}{D_S c^2\theta D_L} - \frac{1}{3}\Lambda\theta D_{LS}D_L\right)$$

From these

$$\theta^2 = \theta_E^2 - \frac{1}{3}\Lambda D_L D_{LS} \tag{4.2.24}$$

Now by representing the Einsteins ring radius $\theta_{E\Lambda}^2$ with cosmological correction in Schwarzschild - de Sitter metric in terms of the purely Schwarzschild metric θ_E as

$$\theta_{E\Lambda}^2 = \frac{\theta_E^2}{1 + F_\Lambda} \tag{4.2.25}$$

Where θ_E is given by

$$\theta_E^2 = \frac{4GM}{c^2}\frac{D_{LS}}{D_L D_S} \tag{4.2.26}$$

And F_Λ is the correction factor to Einstein ring radius due to cosmological constant given by

$$F_\Lambda = \frac{1}{3}\Lambda D_L D_{LS} \tag{4.2.27}$$

In conclusion we observe that the Einstein radius is affected by the factor

$$\frac{1}{1 + F_\Lambda} \tag{4.2.28}$$

And therefore the deflection of light due to the presence of cosmological constant decreases.

Chapter 5

Result and discussion

Observational data Source

Cultech astrophysical lensing data **https://www.cfa.harvard.edu/castles.**

To study the effect of Λ in lensing we implement eqn. 4.2.23 with point mass assumption.

Thus, from the data source Einstein ring system is being used, where the selection is determined by the number of images.

Moreover, the particular case $\beta = 0$, i.e; when the source is exactly behind the lens is considered for in view of the smallness of Λ for better imagination.

So, eqn. 4.2.23 becomes

$$\theta^2 = \theta_E^2 - \frac{1}{3}\Lambda D_L D_{LS} \tag{5.0.1}$$

Where

$$\theta_E^2 = \frac{4GMD_{LS}}{c^2 D_L D_S} \tag{5.0.2}$$

Is Einstein Ring.

And thus, the cosmological effect part is

$$F_\Lambda = \frac{1}{3}\Lambda D_L D_{LS} \tag{5.0.3}$$

As can be seen from eqn.5.0.1 for $\Lambda = 0$ just Einstein's ring.

5.1 Data analysis for the four(4)Einstein's ring extracted from observation

In our fundamental equation 5.0.1 and 5.0.2 we use the angular distances given in eqn.3.6.5 -3.6.7 summarized as

$$D_L = \frac{c}{H_0(1+Z_L)} \int_0^{z_L} \frac{dz'}{E(z')}$$

$$D_S = \frac{c}{H_0(1+Z_S)} \int_0^{z_S} \frac{dz'}{E(z')}$$

$$D_{LS} = \frac{c}{H_0(1+Z_S)} \int_{Z_L}^{Z_S} \frac{dz'}{E(z')}$$

And the correction part

$$F_\Lambda = \frac{\Omega_\Lambda}{3} \frac{1}{(1+Z_S)(1+Z_L)} \int_0^{Z_L} \frac{dz'}{E(z')} \int_{Z_L}^{Z_S} \frac{dz'}{E(z')}$$

Furthermore, the Einstein ring is determined by the velocity dispersion given by: [9].

$$\theta_E = \frac{4\pi\sigma^2}{c^2} \frac{D_{LS}}{D_S} \tag{5.1.1}$$

Table 5.1: Data of selected ring of lens system.

$N\underline{0}$	Lens System	z_S	z_L	$\sigma(km/s)$	F_Λ	$\theta_2(\Lambda = 0)$	$\theta_1(\Lambda \neq 0)$
1	Q0047-2808	3.60	0.48	229	0.01887	1.0962	1.048
2	CFRS03.1077	2.941	0.938	256	0.01831	1.0748	1.02826
3	HST15433+5352	2.092	0.497	108	0.01936	1.0450	1.01304
4	MG1549+3047	1.17	0.11	227	0.00840	1.0918	1.04177

The worked out data is displayed in table 5.1.

For this table we considered a flat cosmological model defined by the parameters $\Omega_\Lambda = 0.73$,$\Omega_M = 0.27$ and $H_0 = 71 km/s/Mpc$.

From the above table we see that the significance of Cosmological constant is 2%.

Chapter 6

Summary and Conclusion

By using simple point mass models for the Lens, we have derived the lensing equation in the presence of cosmological constant,furthermore from eqn.4.2.22 we summarize,that

- $\frac{4GM}{c^2 b}$-is the matter contribution, and its just within the strong field. hence for this its also known as Einstein photon deflection, around 43%,
  - b is the closest distance by the photon to the lensing
- $\frac{1}{3}\Lambda D_{LS} D_L$-is the cosmological constant contribution to the deflection of lights and also its known as the correction factor, its Contributes about 2% in my result

The contribution of Λ is completely involved in the form of the angular diameter distance D_A. no modifications due to Λ appear even if the second-order terms in G are included.also some authors have shown that the gravitational lensing effects are strongly dependent on the value of the cosmological constant and hence they provide with useful means to test the cosmological constant.

Finally

1. The cosmological constant Λ does appear in the Lens equation of light,as geometrical optics

2. Nevertheless the bending angle of light α can be affected by Λ by a very small significance correction,or the deflection of light due to the presence of cosmological constant decreases, and

3. Its significance is around 2%.

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