PROPAGATION OF LIGHT IN PHOTONIC CRYSTALS

By

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To My Family.

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Abstract

The study starts from reviewing historical background and fundamental properties of photonic crystals to analytical analysis of electromagnetism in mixed dielectric media. Light propagation in one-, two- and three-dimensional photonic crystals and how light propagation is inhibited through photonic band gap, along with photonic band gap formation are theoretically studied. The proper effect of dielectric function, group velocity, group velocity dispersion and group index are studied analytically. Maxwell's equation and Bloch's theorem are introduced to show how the light wave (EMW) propagates in photonic crystals analytically. We show theoretically and analytically slow light and delay bandwidth product with a remarkably low group velocity is a promising solution for buffering and time-domain processing of optical signals. Photonic-crystal devices are especially attractive for generating slow light, as they are compatible with on-chip integration and room temperature operation, and can offer wide-bandwidth and dispersion-free propagation.
Key words: Group index, Periodic dielectric function, Photonic crystals, Silicon element.

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Chapter 1

Introduction

The search for new materials is one of the defining characteristics of modern science and technology. Novel optical devices are often the result of the fabrication of new materials. Recent advances in optical science and technology, such as development of laser, detectors and photonic devices have relied heavily on the advance in the areas research [1].
The physics of light propagation in photonic crystals has recently gained increasing interest because, in order to achieve a better system performance and integration, instead of electrons, researchers are turning to light as information carrier and working on constructing photon circuits [2].

Photonic crystals can be used as Omni-directional mirrors with high reflectivity due to the photonic band gap (PBG). One of the suggested potential applications is for a wave guided with strong optical confinement, in which photonic crystals are used as claddings. Low loss propagation through wave guides with sharp bends, branches, etc has been theoretically estimated [4] and some experimental demonstrations have been presented involving the use of large crystal for millimeter wave [5], however, experiments are much more difficult for light waves. Light propagation in straight wave guides has been recently investigated. Still the effect of photonic crystals cladding was not made clear, since the wave guide was as short as several micrometers.

Photonic crystals with forbidden photonic bands, which are known as photonic band gaps (PBG) materials have received considerable attention for the study of their fundamental physical properties as well as for potential applications in photonic devices since the work of Yablonovitch and John [1-3].

Photonic band gap (PBG) materials may be designed in one, two, and three dimensions. One dimensional periodic structure is the simplest photonic crystal usually used as reference model to understand the formation of the band diagram [4], showing stop band regions. Two-dimensional photonic crystals are more difficult to fabricate than one dimensional photonic crystal, but their increased complexity, even if, still quite less than three dimensional photonic crystals, is largely compensated by the wide field of applications in integrated photonic circuits. Three dimensional photonic crystals have attracted much attention as ideal optical materials, in which the complete photonic band gap (PBG) can control light perfectly without any losses.

1.1 Statement of the problem

Photonic crystals technologies have been playing a major role in the rapid development of today's information and communications technologies. Photonics has enabled high speed and wide-band width data handling beyond the limitation of electronics technologies. However it is also true that photonics is not versatile, and many of the data processing processes still require electronics. This limitation is deeply related to the fundamental properties of light in media. In comparison with electrons in media, it is difficult to confine or store light in a small volume, to control or change the speed of light, etc. All of these characteristics limit the way of manipulating light and make photonic data processing more difficult.

Photonic crystals are expected to solve these problems by letting us manipulate the behavior of light in media beyond the conventional limitations. The purpose of the study is, therefore, to investigate the important features of photonic crystals and light propagation in photonic crystals by addressing the following major issues

how light propagation is influenced by photonic crystals,

what fundamental properties of photonic crystals control the speed of light and change the direction of light and

how photonic crystals differ from electronic crystals.

For the above questions; to understand the properties of photonic crystals and propagation of light in photonic crystals, to determine how light is propagated in one-, two-, and three-dimensional photonic crystals and to determine how photonic crystals affect the behavior of visible light are designed as objectives of the study.

1.2 Significance of the study

This study will have its own contribution for the advancement of utilization of light in modern world. Thus, the engineering of photonic crystals will be highly developed and it will provide good opportunity on knowledge and application for the researcher and other scholars who will be interested to do more investigation in the field.

This study will address the properties of photonic crystals, photonic band gaps and the characteristics of light as it propagates in one dimensional, two dimensional, and three dimensional photonic crystals.

1.3 Limitation of the study

The limitation of the study were:

Time constraint and proximity with advisor during winter season

The problem of getting actual photonic crystals to check the propagation of light in photonic crystals experimentally to make the study more feasible

Chapter 2

Theoretical Background

Photonic crystals have been studied in one form or another since 1887, but no one used the term photonic crystal until over 100 years later - after Eli-Yablonovitch and Sajeev John published two milestone papers on photonic crystals in 1987 [8,9]. Before 1987, one-dimensional photonic crystals in the form of periodic multi-layer dielectric stacks (such as the Bragg mirror) were studied extensively. Lord Rayleigh started their study in 1887[10], by showing that such systems have a one-dimensional photonic band-gap, a spectral range of large reflectivity, known as a stop-band. Today, such structures are used in a diverse range of applications; from reflective coatings to enhancing light emitting diode (LED) efficiency to highly reflective mirrors in certain laser cavities. A detailed theoretical study of one-dimensional optical structures was done by Vladimir P. Bykov [11], who was the first to investigate the effect of a photonic band-gap on the spontaneous emission from atoms and molecules embedded within the photonic structure. Bykov also speculated as to what could happen if two- or three-dimensional periodic optical structures were used [12]. The concept of three-dimensional photonic crystals was then discussed by Ohtaka in 1979 [1], who also developed a formalism for the calculation of the photonic band structure. However, these ideas did not take off until after the publication of two milestone papers in 1987 by Yablonovitch and John. Both these papers concerned high-dimensional periodic optical structures, i.e., photonic crystals. Yablonovitch's main goal was to engineer photonic density of states to control the spontaneous emission of materials embedded

in the photonic crystal. John's idea was to use photonic crystals to affect localization and control of light.

After 1987, a number of research papers concerning photonic crystals began to grow exponentially. However, due to the difficulty of fabricating these structures at optical scales, early studies were either theoretical or in the microwave regime, where photonic crystals can be built on the more accessible centimeter scale. This fact is due to a property of the electromagnetic fields known as scale invariance. In essence, electromagnetic fields, as the solutions to Maxwell's equations, have no natural length scale; so solutions for centimeter scale structure at microwave frequencies are the same as for nanometer scale structures at optical frequencies.

In 1991, Yablonovitch had demonstrated the first three-dimensional photonic band-gap in the microwave regime. The structure that Yablonvitch was able to produce involved drilling an array of holes in a transparent material, where the holes of each layer form an inverse diamond structure; today it is known as Yablonovite [14].

In 1996, Thomas Krauss demonstrated a two-dimensional photonic crystal at optical wavelengths. This opened the way to fabricate photonic crystals in semiconductor materials by borrowing methods from the semiconductor industry. Today, such techniques use photonic crystal slabs, which are two dimensional photonic crystals "etched" into slabs of semiconductor. Total internal reflection confines light to the slab, and allows photonic crystal effects, such as engineering photonic dispersion in the slab. Researchers around the world are looking for ways to use photonic crystal slabs in integrated computer chips, to improve optical processing of communications; both on-chip and between chips [15]. Such techniques have yet to mature into commercial applications, but two-dimensional photonic crystals are commercial used in photonic crystal fibers [16]. Photonic crystal fibers were first developed by Philip Russell in 1998, and can be designed to possess enhanced properties over normal optical fibers.

Study has proceeded more slowly in three-dimensional than in two-dimensional photonic crystals. This is because of more difficulty in fabrication [16]. Three-dimensional photonic crystal fabrication had no inheritable semiconductor industry techniques to draw on. Another strand of research has tried to construct three-dimensional photonic structures from self-assembly; essentially letting a mixture of dielectric nano-spheres settle from solution into three-dimensionally periodic structures that have photonic band-gaps. The ever expanding field of biomimetics; the study of natural structures for better understanding and use them in design; is also helping researchers in photonic crystals [17,18].

2.1 Fundamentals of Photonic Crystals

Photonic crystals (PCs) are periodic dielectric structures that control the propagation of light. The periodicity of the dielectric constant removes the degeneracies of the free photon states at Bragg planes and produces a range of forbidden energies of the photons. This leads to the concept of photonic band gaps (PBGs) which underscores the analogy between electrons in semiconductors and photons in a photonic crystal. Any material which exhibits special periodicity in the refractive index is a photonic crystal. The periodicity defines the dimensionality of a photonic crystal. Usually one dimensional, photonic crystals pose a photonic band gap for only one particular direction of the incident light, two dimensional photonic crystals are able to stop the light beam in one particular plane, while some dielectric structures with a three dimensional periodicity, have no propagation modes in any direction at all for a range of frequencies, giving rise to a complete photonic band gap [19]. Photonic crystals can introduce photonic band gaps in a radiation spectra, i.e. they can serve as light conductors or insulators [4], they can modify the dispersion of the light, i.e. they can significantly reduce its group velocity [26] and they can also modify the diffraction of the light in that the diffraction can vanish in propagation through particularly prepared photonic crystals.

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Photonic crystals are composed of periodic dielectric or metallo dielectric nanostructures that affect the propagation of electromagnetic waves in the same way as the periodic potential in a semiconductor crystal affects the electron motion by defining allowed and forbidden electronic energy bands [21]. Essentially, photonic crystals contain regularly repeating internal regions of high and low dielectric constant. Photons propagate through this structure -or not- depending on their wavelength. Wavelengths of light that are allowed to travel are known as modes, and groups of allowed modes form bonds. Forbidden bands of wavelengths are called photonic band gaps. This gives rise to distinct phenomena such as inhibition of spontaneous emission, high-reflecting Omni-directional mirrors and low-loss- wave guiding, amongst others.

Since the basic phenomenon is based on diffraction, the periodicity of the photonic crystal structure has to be of the same length-scale as half of the wavelength of the electromagnetic waves i.e. 200nm (blue) to 350nm (red) for photonic crystals operating in the visible part of the spectrum-the operating regions of high and low dielectric constant have to be of this dimension. This makes the fabrication of optical photonic crystals cumbersome and complex.

The photonic crystals are artificially created materials that can do to photons what an ordinary semiconductor does to electrons: they can exhibit a band gap in which photons with certain energies cannot propagate inside the crystal, regardless of polarization and propagation direction.

Photonic crystals are characterized by three parameters: the lattice topology, the spatial period and the dielectric constants of the constituent materials. By suitable selection of these parameters, a gap in the electromagnetic dispersion relation can be created, within which the linear propagation of electromagnetic waves is forbidden. This forbidden frequency range is called the photonic band gap. It is said that a photonic band gap is complete, if a forbidden gap exists for all polarizations and all propagation directions. It is common to distinguish one-, two- and three-dimensional photonic crystals by the number of dimensions within which the periodicity has been introduced into the structure. Examples of one-, two- and three-dimensional photonic crystals are given in Figure (2.1). Necessary but not sufficient conditions to obtain a complete photonic band gap are a periodicity in the three spatial directions and a large difference in the dielectric constants of the constituent materials [22].

In 1987 E.Yablonovitch proposed to use a three-dimensional periodic medium, which he called a photonic crystal, to inhibit the spontaneous emission and to realize localized defect modes and consequently to enhance the spontaneous emission. In the same years, S. John [23] proposed the use of a disordered three-dimensional periodic medium to localize electromagnetic waves. Many interesting quantum optical phenomena such as the bound state of photons and non-exponential decay of the spontaneous emission were predicted. These ideas actively stimulated research area [24-27], which lead both to various unexpected results in the fundamental understanding of light-matter interaction and to various new optoelectronics and photonics applications

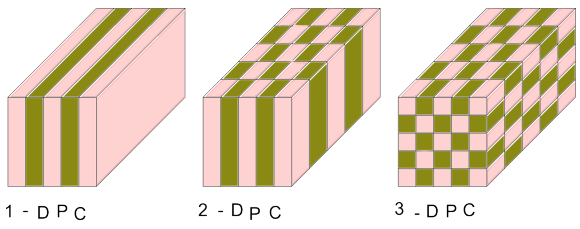

Figure 2.1: Examples of one, two and three-dimensional photonic crystals

2.2 The Origin of the Photonic Band gap

Photonic crystals are materials patterned with a periodicity in dielectric constant, which can create a range of forbidden frequencies called photonic band gap. Photons with energies lying in the band gap con not propagate through the medium. This provides the opportunity to shape and mould the flow of light for photonic information technology. Photonic band gap is a frequency band in which electromagnetic waves are forbidden irrespective of propagation direction in space. Inside a photonic band gap optical modes, spontaneous emission, and zero point fluctuation are all absent. Photonic band gap devices were introduced in optical applications, which have a property of preventing light from propagating in a certain frequency band [5]. Their structures are periodic in nature where the propagation of waves is not allowed for some frequencies for some frequency bands or directions, according to the Bragg phenomenon.

Photonic band gap formation can be regarded as the synergetic interplay between two distinct resonance scattering mechanisms. The first is the "macroscopic" Bragg resonance from a periodic array of scatters. This leads to electromagnetic stop gaps when the wave propagates in the direction of periodic modulation when an integer number, m=1,2,3,,of half wavelength coincides with the lattice spacing, L, $L = \frac{m\lambda}{2}$, of the dielectric microstructure. The second is a "microscopic" scattering resonance from a single unit cell of the material. In the illustration, this (maximum back scattering) occurs when precisely one quarter of the wavelength coincides with the diameter, 2r, of a single dielectric well of refractive index n. Photonic band gap formation is enhanced by choosing the materials parameters r, L, and n such that both the microscopic and microscopic resonance occur at the same frequency.

Photonic band gap formation is facilitated if the geometrical parameters of the photonic crystal are chosen so that both the microscopic and macroscopic resonance occurs at precisely the same wavelength. Both of these scattering mechanisms must individually be

quite strong. In practice, this means that the underlying solid material must have a very high refractive index contrast (typically about 3.0 or higher and it is to precisely achieve this contrast, holes are drilled in to the medium). These conditions on the geometry, scattering strength, and the purity of the dielectric material severely restrict the set of engineered dielectrics that exhibit a photonic band gap.

Here one can take advantage of the simplicity of the one-dimensional systems to explain the physical origin of the PBG. A one-dimensional photonic crystal is made of layers with alternating dielectric constant, as shown in Figure 2.2. This system repeats in the z-direction with period a, which will be of the order wavelengths of the light. A plane wave traveling in the z-direction, along the line of periodicity, will be scattered at the interface between two media. This gives rise to forward and backward propagating waves within the structures. These waves will interfere to form standing waves and just they are taken to be stopped.

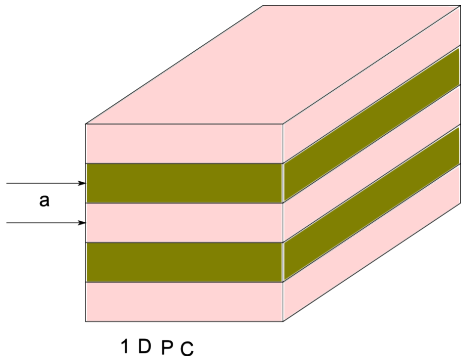

Figure 2.2: One dimensional photonic crystal with period 'a'. It consists of alternating layers with different dielectric constants

2.3 Properties of Photonic Crystals

2.3.1 Polarization

In the two-dimensional case, all derivatives of magnetic field and electric field with respect to z coordinate vanish and the study is restricted to the propagation along the cross-section plane of the crystal, i.e. the xy plane. This crucial property opens the possibility of separately studying the problems corresponding to the fundamental cases of polarization. These polarization modes are referred to as transverse magnetic (TM) and transverse electric (TE) [28] polarizations Figure(2.3.). In the case of the TM polarization the components of magnetic field are parallel to the xy plane and $\mathbf{E_z} \neq 0$. For the TE case $\mathbf{H_z} \neq \mathbf{0}$, and electric field components lie in the xy plane.

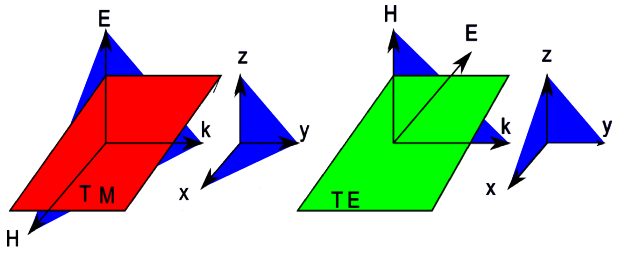

Figure 2.3: TM polarization; the component of magnetic field are parallel to the xy plane and $E_z \neq 0$. For the TE case the $H_z \neq 0$ and electric field components lie in the xy plane.

2.3.2 Density of States

The photonic density of states (DOS) plays an important role in understanding the optical properties of a photonic crystal because it describes the integral availability of allowed states in a certain frequency range regardless of band-index n or wave vector k. The calculation of the DOS provides a cross-check for the existence of a photonic band gap.

The total DOS is defined as :

$$\rho(\omega) = \sum_n \int_{1BZ} d^3k \cdot \delta(\omega - \omega_n(k)) \tag{2.3.1}$$

where the k-space integration covers the whole first Brillion zone (1BZ) and $\omega_n(k)$ is the eigen value for band-index n and wave vector k,δ is the Dirac delta function. In the homogeneous media $\rho(\omega)$ is proportional to ω^2. A vanishing density of state is the commensurate condition for a complete photonic band gap

.Density of states per unit volume per unit energy $\rho(E)$ with different dimension can be calculated.

2.3.3 Calculation of density of states for electronic crystals

To calculate density of three dimensional, two dimensional and one dimensional photonic crystals the property of delta function i.e.

$\int \delta(x-a)f(x)dx = f(a)$ and $\delta(\alpha x) = \frac{1}{\alpha}\delta(x)$ should be taken in to consideration.

For any dimensional state we use the following formula:

$g(E) = 2(\frac{L}{2\pi})^\alpha \int \delta(E - E_k)d^\alpha k$ For three dimensional states

$$g(E) = \sum_s \sum_k \delta(E - E_k) \tag{2.3.2}$$

where $E_k = \frac{\hbar^2 k^2}{2m} \Rightarrow dE_k = \frac{\hbar^2 k dk}{2m}$

$$g(E) = 2(\frac{L}{2\pi})^3 \int \delta(E - E_k)k^2 \sin\theta dk d\theta d\phi \tag{2.3.3}$$

but $\int_0^\pi \int_0^{2\pi} \sin\theta d\phi = 4\pi$

$$g(E) = 2(\frac{L}{2\pi})^3 \int \delta(E - \frac{\hbar^2 k^2}{2m})4\pi = 2\frac{V}{8\pi^3} \int 4\pi\delta(\frac{\hbar^2}{2m}\frac{2mE}{\hbar^2} - k^2)k^2 dk \tag{2.3.4}$$

$$g(E) = 2\frac{V}{8\pi^3}4\pi \int \frac{2m}{\hbar^2}\delta(\frac{2mE}{\hbar^2} - k^2)k^2 dk \tag{2.3.5}$$

using $\delta(\alpha x) = \frac{1}{\alpha}\delta(x)$ delta function property

$$g(E) = \frac{V}{\pi^2}\frac{2m}{\hbar^2}\int \delta(\frac{2mE}{\hbar^2} - k^2)k^2 dk \tag{2.3.6}$$

Let $y = k^2, dy = 2kdk \Rightarrow dk = \frac{dy}{2k} = \frac{dy}{2\sqrt{y}}$

$$g(E) = \frac{V}{\pi^2}\frac{2m}{\hbar^2}\int \delta(\frac{2mE}{\hbar^2} - y)y\frac{dy}{2\sqrt{y}} \tag{2.3.7}$$

$$g(E) = \frac{V}{\pi^2}\frac{2m}{\hbar^2}\frac{1}{2}\int \delta(\frac{2mE}{\hbar^2} - y)\sqrt{y}dy \tag{2.3.8}$$

By using delta function property, finally we get the desired equation

$$g(E) = \frac{V}{2\pi^2}(\frac{2m}{\hbar^2})\sqrt{\frac{2mE}{\hbar^2}} = \frac{V}{2\pi^2}(\frac{2m}{\hbar^2})^{\frac{3}{2}}E^{\frac{1}{2}} \tag{2.3.9}$$

By using the same technique we can drive equation for density of states for two dimensional bodies.

$$g(E) = \sum_s \sum_k \delta(E - E_K) = 2(\frac{L}{2\pi})^\alpha \int \delta(E - E_k)d^\alpha k \tag{2.3.10}$$

$$g(E) = 2(\frac{L}{2\pi})^2 \int \delta(E - E_k)kdkd\phi \tag{2.3.11}$$

but $\int_0^{2\pi} d\phi = 2\pi$ and by using this relation we get the following

$$g(E) = 2\frac{A}{4\pi^2}2\pi \int \delta\epsilon(E - \frac{\hbar^2k^2}{2m})kdk \tag{2.3.12}$$

let $y = k^2, dy = 2kdk \Rightarrow dk = \frac{dy}{2k} = \frac{dy}{2\sqrt{y}}$ we get the following

$$g(E) = \frac{A}{\pi}\int \delta(\frac{\hbar^2}{2m}(\frac{2mE}{\hbar^2} - k^2))\sqrt{y}\frac{dy}{2\sqrt{y}} \tag{2.3.13}$$

$$g(E) = \frac{A}{2\pi}\frac{2m}{\hbar^2}\int \delta(\frac{2mE}{\hbar^2} - y)dy = \frac{A}{2\pi}\cdot\frac{2m}{\hbar^2} \tag{2.3.14}$$

Finally density of states for one dimension can be calculated by using the techniques

$$g(E) = \frac{1}{2\pi}\int \delta(E - E_k)dk \tag{2.3.15}$$

Like the previous techniques by using properties of delta function we derive the following

$$g(E) = \frac{1}{2\pi}\int \delta(\frac{\hbar^2}{2m}(\frac{2mE}{\hbar^2} - k^2))dk \tag{2.3.16}$$

Let $y = k^2, dy = 2kdk \Rightarrow dk = \frac{dy}{2k} = \frac{dy}{2\sqrt{y}}$ and using delta property

$$g(E) = \frac{1}{2\pi}\frac{2m}{\hbar^2}\frac{1}{2}\int \delta(\frac{2mE}{\hbar^2} - y^2)\frac{dy}{\sqrt{y}} = \frac{1}{2\pi}\frac{m}{\hbar^2}\int \delta(\frac{2mE}{\hbar^2} - y^2)(\sqrt{y})^{-1}dy \tag{2.3.17}$$

Using the dirac delta function i.e. $\int \delta(y_0 - x)f(x)dx = f(x)$, in our case $f(x) = f(\sqrt{x})^{-1} \Rightarrow \int \delta(\frac{2mE}{\hbar^2} - y^2)(\sqrt{y})^{-1}dy = \frac{1}{\sqrt{\frac{2mE}{\hbar^2}}}$

$$g(E) = \frac{1}{2\pi}\frac{m}{\hbar^2}(\frac{\hbar^2}{2m})^{\frac{1}{2}}E^{\frac{-1}{2}} \tag{2.3.18}$$

2.3.4 Calculation of density of states for photonic crystals

For photonic crystals, density of states per unit volume per unit angular frequency (ω) can be calculated for one-, two-, and three-dimensional photonic crystals.

In this part we use the same techniques as the electronic crystals.

For one dimensional crystals

$$g(\omega) = \sum \delta(\omega - \omega_k) \tag{2.3.19}$$

$$g(\omega) = \frac{L}{2\pi}\int \delta(\omega - \omega_k) \tag{2.3.20}$$

where $\omega_k = ck$ is dispersion relation of photons and $d\omega_k = cdk \Rightarrow dk = \frac{d\omega_k}{c}$

$$g(\omega) = \frac{L}{2\pi}\int \delta(\omega - \omega_k)\frac{d\omega_k}{c} = \frac{L}{2\pi C}\int \delta(\omega - \omega_k)d\omega_k \tag{2.3.21}$$

using dirac delta property $\int \delta(\omega - \omega_k)d\omega = 1$

$$g(\omega) = \frac{L}{2\pi c} \tag{2.3.22}$$

For two-dimensional photonic crystals

$$g(\omega) = \sum \delta(\omega - \omega_k) \tag{2.3.23}$$

$$g(\omega) = (\frac{L}{2\pi})^2\int \delta(\omega - \omega_k)kdkd\phi \tag{2.3.24}$$

but $\int_0^{2\pi} d\phi = 2\pi and dk = \frac{\omega k}{c}$

$$g(\omega) = (\frac{L}{2\pi})^2\int \delta(\omega - \omega_k)2\pi\frac{\omega_k}{c}\frac{d\omega_k}{c} \tag{2.3.25}$$

$$g(\omega) = (\frac{A}{4\pi^2})\frac{2\pi}{c^2}\int \delta(\omega - \omega_k)\omega_k d\omega_k \tag{2.3.26}$$

From the property of dirac delta function, we have

$\int(\omega - \omega_k)\omega_k d\omega_k = f(\omega_k) \Rightarrow f(\omega_k) = \omega_k = \omega$

$$g(\omega) = \frac{A\omega}{2\pi c^2} \tag{2.3.27}$$

Finally for three-dimensional photonic crystals:

$$g(\omega) = \sum \delta(\omega - \omega_k) \tag{2.3.28}$$

$$g(\omega) = (\frac{L}{2\pi})^3 \int \delta(\omega - \omega_k)k^2 \sin\theta dk d\theta d\phi \tag{2.3.29}$$

but $\int_0^\pi \int_0^{2\pi} \sin\theta d\theta d\phi = 4\pi and dk = \frac{\omega_k}{c}$

$$g(\omega) = \frac{V}{8(\pi)^3}\int \delta(\omega - \omega_k)4\pi k^2 dk \tag{2.3.30}$$

$$g(\omega) = \frac{V}{8(\pi)^3}4\pi \int \delta(\omega - \omega_k)\frac{(\omega_k)^2}{c^2}\frac{d\omega_k}{c} \tag{2.3.31}$$

$$g(\omega) = (\frac{V}{2(\pi^2)}) \cdot \frac{1}{c^3}\int \delta(\omega - \omega_k)\omega_k^2 d\omega_k \tag{2.3.32}$$

using the property of delta function i.e.

$\int \delta(\omega - \omega_k)\omega_k^2 d\omega_k = f(\omega_k)^2 = \omega^2$

$$g(\omega) = \frac{V}{2\pi^2} \cdot \frac{\omega^2}{c^3} \tag{2.3.33}$$

2.3.5 Group Velocity

The group velocity is the speed at which the wave envelope, the overall shape of an amplitude varying wave, propagates. This is frequently considered to be the velocity of the information transported by light signals, which is correct for most cases. Nevertheless, as it will be discussed below, there are some exceptions to this statement.

The group velocity of a wave in a certain medium corresponds to the velocity of light in vacuum, c, divided by the group index of the medium, n_g, and it is given by the equation

$$v_g = \frac{c}{n_g} = \frac{d\omega}{dk} \tag{2.3.34}$$

where ω is the angular frequency and k is the angular wave number. The variation ω with k is called the dispersion relation, and hence, the group velocity is given by the slope of the dispersion relation. The phase velocity is the speed at which the phase of every particular frequency component of the wave travels and it is given by

$$v_p = \frac{\omega}{k} \tag{2.3.35}$$

It can be seen that for media in which ω varies linearly with k, the phase and the group velocity coincide. However, in general, for dispersive media, phase and group velocities differ and both vary with frequency. Coming back to the point of slow light, it has been defined as the phenomenon of light propagation at reduced group velocity. Bearing in mind equation (2.3.34), it is straightforward to see that slow light relies on increasing the group index of the medium, which is given by

$$n_g = n + \omega \frac{dn}{d\omega} \tag{2.3.36}$$

The refractive index of the material n varies with frequency ω and can be slightly altered by making use of a plurality of phenomena, such as electro-optic or thermo-optic effects. When aiming at a significant change in the group index, it is the second term in equation (2.3.36), $\frac{dn}{d\omega}$, the one that dominates, i.e. the dispersive behavior of the media. This is the reason why sharp spectral resonances are always behind slow light techniques.

The group velocity of the radiation modes has very important role in the light propagation and optical response in the photonic crystal. The group velocity of the eigen modes is defined as the gradient of the dispersion curves that is the derivative of the angular frequency ω with respect to the wave vector k. The dispersion for light in an isotropic dielectric material is given by the equation:

$$\omega(k) = \frac{ck}{\sqrt{\varepsilon}} \tag{2.3.37}$$

where c is the speed of light, k is a wave vector and ε is a dielectric constant, given by the properties of the dielectric medium. This equation shows that the energy of light varies linearly with momentum, with zero momentum corresponding to zero energy.

2.4 Electromagnetism in Mixed Dielectric Media

In order to study the propagation of light in photonic crystals, we should turn to the Maxwell equations. After specializing to the case of a mixed dielectric medium, we cast the Maxwell equations as a linear Hermitian eigen value problem. From this formulation, in close analogy with the Schrödinger equation of quantum mechanics comes a variety of useful properties, including the orthogonality of modes and the electromagnetic variational theorem. Finally, we show electromagnetic problems with different overall length and dielectric scales can be related.

All of macroscopic electromagnetism, including the propagation of light in photonic crystal, is governed by four macroscopic Maxwell equations. They are:

$$\nabla \cdot \mathbf{B} = 0 \tag{2.4.1}$$

$$\nabla \cdot \mathbf{D} = 4\pi\rho \tag{2.4.2}$$

$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0 \tag{2.4.3}$$

$$\nabla \times \mathbf{H} - \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} = \frac{4\pi}{c}\mathbf{J} \tag{2.4.4}$$

where (respectively)$\mathbf{E}$ and $\mathbf{H}$ are the macroscopic electric and magnetic fields,$\mathbf{D}$and $\mathbf{B}$ are the displacement and magnetic induction fields, and ρ and J are charge density and current density.

We will restrict ourselves to propagation within a mixed dielectric medium, a composite of regions of homogeneous dielectric material with no free charges or currents. This composite need not be periodic It is good approximation to employ the following standard assumptions; First, We assume the field strengths are small enough so that we are in the linear regime. Second, we assume the material is macroscopic and isotropic, so that $\mathbf{E}(\mathbf{r},\omega)$ and $\mathbf{D}(\mathbf{r},\omega)$ are related by a scalar dielectric constant $\varepsilon(\mathbf{r},\omega)$. Third, we ignore any explicit frequency dependence of the dielectric constant. Instead we simply choose the value of the dielectric constant appropriate to the frequency range of the physical system we are considering. Fourth, we focus only on low-loss dielectrics; which means we can treat $\varepsilon(\mathbf{r})$ as purely real. When all is taken in to consideration, we have $\mathbf{D}(\mathbf{r}) = \varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r})$. For most dielectric materials of interest, the magnetic permeability is very close to unity and we may set B=H With all of these assumptions in place, the Maxwell equation become

$$\nabla \cdot \mathbf{H}(\mathbf{r},t) = 0 \tag{2.4.5}$$

$$\nabla \cdot \varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r},t) = 0 \tag{2.4.6}$$

$$\nabla \times \mathbf{E}(\mathbf{r},t) + \frac{1}{c}\frac{\partial \mathbf{H}(\mathbf{r},t)}{\partial t} = 0 \tag{2.4.7}$$

$$\nabla \times \mathbf{H}(\mathbf{r},t) - \varepsilon(\mathbf{r})\frac{\partial \mathbf{E}(\mathbf{r},t)}{\partial t} = 0 \tag{2.4.8}$$

We have restricted ourselves to linear lossless materials. In general both E and H are complicated functions of time and space. But since Maxwell equations are linear, we can separate out the time dependence by expanding the fields in to a set of harmonic models. In this and the following sections we will concern ourselves with the restriction that the Maxwell equation impose on a field pattern that happens to very harmonically with time. This is no great limitation, since we know by Fourier analysis that we can build any solution with an appropriate combination of these harmonic modes. Often we will

refer to them simply as modes. We employ the familiar trick of using a complex-valued field for mathematical convenience; remember to take the real part to obtain the physical fields. This allows us to write a harmonic mode as a certain field pattern times a complex exponential

$$\mathbf{H}(\mathbf{r},t) = \mathbf{H}(\mathbf{r})e^{i\omega t} \tag{2.4.9}$$

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})e^{i\omega t} \tag{2.4.10}$$

To find the equations for the model profile of a given frequency, we insert the above equations in to equation (2.4.5) and(2.4.6.) The two divergence equations give the simple conditions

$$\nabla \cdot \mathbf{H}(\mathbf{r}) = 0 \tag{2.4.11}$$

$$\nabla \cdot \mathbf{D}(\mathbf{r}) = 0 \tag{2.4.12}$$

$$\nabla \cdot \varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r}) = 0 \tag{2.4.13}$$

These equations have a simplest physical interpretation. There are no point sources or sinks of displacement and magnetic fields in the medium. Alternatively, the field configurations are built up of electromagnetic waves that are transverse. That is if we have a plane wave $H(r) = a^{ik\cdot r}$, equation (2.4.12) requires that $a \cdot \mathbf{k} = 0$. We can focus our attention on the other two of the Maxwell equations as long as we are always careful to enforce this transversality requirement. The two curl equations relate E(r) to H(r):

$$\nabla \cdot \mathbf{E}(\mathbf{r}) + \frac{i\omega}{c}\mathbf{H}(\mathbf{r}) = 0 \tag{2.4.14}$$

$$\nabla \cdot \mathbf{H}(\mathbf{r}) - \frac{i\omega}{c}\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r}) = 0 \tag{2.4.15}$$

We can decouple these equations in the following way. Divide equation of (2.4.15) by $\varepsilon(\mathbf{r})$, and then take the curl. Then use the first equation to eliminate $\mathbf{E}(\mathbf{r})$. The result is an equation entirely in $\mathbf{H}(\mathbf{r})$:

$$\nabla \times (\frac{1}{\varepsilon(\mathbf{r})}\nabla \times \mathbf{H}(\mathbf{r})) = (\frac{\omega}{c})^2\mathbf{H}(\mathbf{r}) \tag{2.4.16}$$

This is the master equation. In addition to the divergence equation (2.4.11), it completely determines H(r). Our strategy will be the following: for a given photonic crystal $\varepsilon(\mathbf{r})$, solve the master equation to find the modes H(r) for a given frequency subject to transversality requirement.

Then we use equation (2.4.12) to recover E(r):

$$\mathbf{E}(\mathbf{r}) = (\frac{-ic}{\omega\varepsilon(\mathbf{r})})\nabla \times \mathbf{H}(\mathbf{r}) \tag{2.4.17}$$

2.5 Bloch modes, dispersion and the plane wave method

The plane wave method is a direct adaptation of electronic band structure methods, and allows optimal account of the crystal lattice symmetry in determining the electromagnetic properties of photonic crystal. By using Maxwell's equations for an inhomogeneous dielectric medium without charges or currents together with the constitutive relations for a non magnetic dielectric composite i.e ($\mathbf{D} = \varepsilon_0\varepsilon(\mathbf{r})\mathbf{E}$) and ($\mathbf{B} = \mu_0\mathbf{H}$) we can drive the next equation

$$\nabla \times (\nabla \times \mathbf{E}(\mathbf{r})) + [1 - \varepsilon(\mathbf{r})]\frac{\omega^2}{c^2}\mathbf{E}(r) = \frac{\omega^2}{c^2}\mathbf{E}(\mathbf{r}) \tag{2.5.1}$$

Assuming , harmonic time dependence with frequency ω, and the speed of light c in vacuum. For photonic crystals the dielectric constant $\varepsilon(\mathbf{r})$ is by definition a periodic function, which is often piece wise constant for fabricated composites. The borrowing of the electronic band structure methods to solve (2.5.1) is inspired by the similarity of this wave equation with the time independent Schrödinger equation for an electron in a periodic potential[31].Indeed the physics appear simpler for photons as they do not interact with each other. However several differences are readily apparent.Since we assume the dielectric constituents to be non magnetic it is more advantageous to solve the magnetic field wave equation

$$\nabla \times [\frac{1}{\varepsilon(\mathbf{r})}\nabla \times \mathbf{H}(\mathbf{r})] = \frac{\omega^2}{c^2}\mathbf{H}(\mathbf{r}) \tag{2.5.2}$$

The benefit of using the magnetic field wave equation solely rests on the fact that the operator $\nabla \times \frac{1}{\varepsilon(\mathbf{r})}\nabla\times$ is hermmitian. As a result it is clear that its eigen values are real. Furthermore it follows that nondegenerate magnetic field eigenmodes are orthogonal and can be classified according to symmetry properties. In addition the hermitian nature of the magnetic field operator facilitates variational and perturbational calculations. In contrast the electric field eigenvalue equation (2.5.1) is not hermitian eigenvalue problem, causing inferior convergence of electric field plane-wave methods. This symmetry between magnetic field and electric field methods disappears for more general problems where the magnetic permeability is also spatially dependent. A special dependence of the magnetic permeability is taken in to account by generalizing $\mathbf{D} = \varepsilon(\mathbf{r})\mathbf{E}(r)$ and $\mathbf{B} = \mu_0 H$ to

$$\mathbf{B} = \mu_0 \mu \mathbf{H}(\mathbf{r}) \tag{2.5.3}$$

Due to the periodicity of the dielectric constant $\varepsilon(\mathbf{r})$, Bloch's theorem is applicable to the eigenmodes of the eigen value problem Eq.(2.5.2) and asserts that eigenmodes can be decomposed as

$$\mathbf{H}_{n,k(\mathbf{r})} = e^{ik\cdot\mathbf{r}} u_{n,k(\mathbf{r})} \tag{2.5.4}$$

Where $u_{n,k(\mathbf{r})}$ has the periodicity of crystal lattice (photonic crystals). Such Bloch mode is periodic up a phase factor $e^{ik\cdot(\mathbf{r})}$.At the edge of the Brillion zone the dispersion relation$\omega_n(k)$ folds back, and thus organize in to bands, labeled by n. All the modes can be uniquely labeled with a Bloch wave vector k with in the first Brillion zone, and the integer index n.

2.6 Slow light in photonic crystals

2.6.1 Slow light theory and the delay-bandwidth product

The group velocity of light is given by the inverse of the first-order dispersion $(\frac{dk}{d\omega})^{-1}$, where k and ω are the wave number and angular frequency, respectively[30]. The group index $n_g \equiv \frac{c}{v_g} = c\frac{dk}{d\omega}$ is regarded as a slow-down factor from the velocity, c.

Note that, in any material, the material index itself is neither very large nor easy to modify; it can be changed by no more than several percent by any form of external stimulation. Still, n_g is greatly enhanced in materials or structures with large first-order dispersion.

Slow light has been observed in various media, but to be useful it needs to be achieved over a wide bandwidth. Detailed analysis of the delay band width product (DBP) has been reported in numerous papers; a simple treatment is shown here. Let us denote the material index or the modal equivalent index as n. Using the relation $k = \omega(\frac{c}{n})$,

$$n_g = c\frac{dk}{d\omega} = \frac{d(n\omega)}{d\omega} = n + \omega\frac{dn}{d\omega} \tag{2.6.1}$$

When n_g is much greater than n,the DBP and its normalized form are given by

$$\Delta t \Delta f \cong \frac{L\Delta n}{\lambda} \tag{2.6.2}$$

$$n_g\frac{\Delta f}{f} \cong \Delta n \tag{2.6.3}$$

respectively, where Δt is the delay of light at a wavelength λ over a propagation length of L,Δf is the frequency bandwidth centered at a frequency of $f = \frac{\omega}{2\pi}$, and Δn is the change of n in the bandwidth. The time duration of one optical bit is approximately given by $(\Delta f)^{-1}$, although an accurate value depends on the modulation format. Therefore, the DBP $\Delta t \Delta f$ is a good indication of the highest buffering capacity that the slow-light device potentially provides. On the other hand, the normalized form can be more useful when devices that have different lengths and different operating frequencies are compared. The shortest spatial length of one bit ΔL is approximated by

$$\Delta L \approx \frac{\lambda}{\Delta n} \tag{2.6.4}$$

Equations (2.6.2, 2.6.3) and (2.6.4) indicate that Δn is the dominant factor for achieving a large buffering capacity. In materials and structures that have low dispersion, Δn changes linearly with Δf when Δf is much greater than f. On the other hand, Δn can be

maximized in highly dispersive media independently of Δf by optimizing photo excitation and structural design, for example, and, according to equations (2.6.2) and(2.6.3), a large n_g and long Δt are obtained by narrowing Δf.

2.6.2 Slow light in highly dispersive structures

The device with which slow light was first observed in 2001 was a silicon photonic-wire waveguide (PWW) [30], which is widely used in silicon photonic devices [31,32]. It is a simple rectangular channel waveguide with a high index contrast between the silicon core and air or SiO_2 cladding. The propagation loss of this waveguide is sometimes measured from the finesse of the internal Fabry-Perot resonance. In the first observation, the group index n_g was evaluated from the relation $n_g = \frac{\lambda_2}{2L\Delta\lambda_r}$,($\Delta\lambda_r$ is the peak spacing of the resonances) as around four to five. This was not caused by the resonance but by the large dispersion arising from the high index contrast, which largely changes the propagation constant (k in the propagation direction) with respect to ω, particularly near the cut-off of the waveguide mode [33]. This result suggests that the dispersion term in equation (2.6.1) can be comparable to or larger than n itself even in a simple waveguide.

A photonic crystal waveguide (PCW) is usually fabricated on a silicon-on-insulator (SOI) or III-V compound semiconductor substrate by using standard semiconductor processes, including high-resolution lithography, selective dry etching and wet etching. A typical structure consists of air holes with a diameter of 240 nm and a lattice constant (lattice spacing) of 450 nm for a target wavelength of $\lambda = 1.55\mu m$.

Present technology means that such structures, etched to a depth of around 200 nm, can be achieved with a disorder within several nanometers. The major component of the higher-order dispersion is the group-velocity dispersion (GVD), given by $\frac{d(\nu_{\mathbf{g}})^{-1}}{d\omega} = \frac{d^2k}{d\omega^2}$. It usually becomes extremely large near the band edge; a typical GVD constant is of the order of 100 ps $nm^{-1}mm^{-1}$, which is 10^6 times larger than that of single-mode silica fibers. Because of this, dispersion-compensated and zero-dispersion slow light are very important. Even though a high buffering capacity is potentially expected from a large DBP in a PCW device specifically designed for wide band slow light, the net capacity is finally determined by how the GVD is suppressed.

Chapter 3

Methods and Materials

3.1 Formulation of models

Theoretical calculations of photonic crystals are, in principle, exact, because Maxwell's equations are derived from first principles. Therefore, the power of computations is comparable to that of experiments in characterization of photonic crystals. The aim of this chapter is to present an overview of most common computational methods and modeling tools applicable to photonic crystals analysis as a function of the phase velocity, group velocity, group index, group velocity dispersion and dispersion relation.

Let we consider Taylor expansion of the wave vector evaluated at $\omega = \omega_0$

$$k(\omega) = k(\omega_0) + (\frac{\partial k}{\partial \omega})|_{\omega=\omega_0}(\omega - \omega_0) + \frac{1}{2}(\frac{\partial^2 k}{\partial \omega^2})|_{\omega=\omega_0}(\omega - \omega_0)^2 + ... \tag{3.1.1}$$

where $k(\omega_0) = k_0$ is mean angular vector: The phase velocity (the speed at which the phase of every particular frequency component of the wave travels)can be derived as

$$\frac{1}{\upsilon_{ph}} = n\frac{\omega_0}{c} \tag{3.1.2}$$

$$\frac{k_0}{\omega_0} = \frac{1}{\upsilon_{ph}} \Rightarrow \upsilon_{ph} = \frac{\omega_0}{k_0} \tag{3.1.3}$$

The group velocity (the speed at which the wave envelope, the overall shape of an amplitude varying, propagates), can be calculated by the following derived formula

$$\frac{\partial k}{\partial \omega|_{\omega=\omega_0}} = \frac{1}{\upsilon_g} \tag{3.1.4}$$

The group velocity dispersion (GVD); major component component of higher-order dispersion as a function of frequency

$$(\frac{\partial^2 k}{\partial \omega^2})|_{\omega=\omega_0} = D_\omega \tag{3.1.5}$$

The group index of photonic crystals can be calculated by the derived formula

$$\frac{1}{\upsilon_g} = \frac{\partial k}{\partial \omega}|_{\omega=\omega_0} = \frac{1}{c}[n(\omega_0) + \omega_0 \frac{\partial n}{\partial \omega|_{\omega=\omega_0}}] \tag{3.1.6}$$

$$\upsilon_g = \frac{c}{[n(\omega_0) + \omega_0 \frac{\partial n}{\partial \omega}|_{\omega_0}]} = \frac{c}{n_g(\omega_0)} \Rightarrow n_g(\omega_0) = n(\omega_0) + \omega_0 \frac{\partial n}{\partial \omega}|_{\omega_0} \tag{3.1.7}$$

Again from group velocity dispersion we have the following

$$D = D_\omega = \frac{\partial^2 k}{\partial \omega^2}|_{\omega_0} \tag{3.1.8}$$

$$D_\lambda = \frac{\partial}{\partial \lambda}(\frac{1}{\nu_g})|_{\lambda=\lambda_0} \tag{3.1.9}$$

We have the following relations $\frac{\partial}{\partial \lambda} = \frac{\partial \omega}{\partial \lambda} \cdot \frac{\partial}{\partial \omega}$, and $\omega = \frac{2\pi c}{\lambda}$, $\frac{\partial \omega}{\partial \lambda} = \frac{-2\pi c}{\lambda^2}$;

‘ from these relations we will have

$$D_\lambda = \frac{-2\pi c}{\lambda^2} D_\omega \tag{3.1.10}$$

The two type dispersion can be determined from equation (3.1.10) as when the value of the dispersion as a function of wavelength is greater than zero ($D > 0$), it is said to be normal dispersion, where as if it is less than zero ($D < 0$), it is said to be anomalous dispersion.

Now let we turn to Helmholtz equation

$$\Delta u(r, \omega) + \frac{\omega^2}{c^2} \varepsilon(\omega) u(r, \omega) = 0 \tag{3.1.11}$$

Lastly the dispersion relation as a function of dielectric function is calculated

$$k^2(\omega) = \frac{\omega^2}{c^2} \varepsilon(\omega) \tag{3.1.12}$$

3.2 Methods and softwares used

The study deals on determining the properties of photonic crystals, how light is propagated in one-, two-, and three-dimensional photonic crystals. Moreover how photonic band gaps formed and work. The study is supported by different materials such as published articles, thesis, and dissertations which are the main sources of theory. We employed analytical method to study the properties of light propagation in photonic crystals considering dielectric function, group index, group velocity and group velocity dispersion as the main parameters.

In the study we use Taylor expansion to derive equation that describes waves (EMW) as a function of periodic dielectric function, slow light in photonic crystals as a function of group index, and the group velocity of light in photonic crystals as a function of index of refraction and frequency. Inkscape is used for drawing one-, two-, and three-dimensional photonic crystals in addition TE and TM polarization graph.

Chapter 4

Results and Discussion

4.1 Properties of Model Dielectric Crystal

Let us consider (SiO_2) model photonic crystal as a crystal material. The reason why we choose this dielectric photonic crystal is due to:

The wide availability, good thermal and mechanical property, and relative high dielectric constant ($\varepsilon = 13$). The remarkable properties of Si and SiO_2 make possible the integration of an increasing number of devices on ever larger Si wafers. This material is well understood, that it has a good mechanical and chemical stability and sufficiently large nonlinear optical coefficient.

4.2 Analytical Description of Main Parameters

Based on our model equation, we derived the following equations

The derived formula for phase velocity as function of frequency

$$\upsilon_{ph} = \frac{c}{n\omega_0} \tag{4.2.1}$$

The result equation for group velocity as function of frequency

$$\frac{\partial k}{\partial \omega}|_{\omega=\omega_0} = \frac{1}{\upsilon_g} \tag{4.2.2}$$

The group velocity dispersion calculated as

$$D_\omega = \frac{\partial^2 k}{\partial \omega^2}|_{\omega=\omega_0} \tag{4.2.3}$$

The group index as a function of frequency and index of refraction

$$n_g(\omega_0) = n(\omega_0) + \omega_0 \frac{\partial n}{\partial \omega}|_{\omega_0} \tag{4.2.4}$$

The final equation for dispersion relation as a function of dielectric function and frequency is

$$DR = (k^2(\omega)) = \frac{\omega^2}{c^2}\varepsilon(\omega) \tag{4.2.5}$$

Light whose group υ_g velocity is markedly slowed down in comparison with c, (the velocity of light in vacuum) is called slow light. It is believed that slow light may find application in compact optical delay lines and optical buffers. It is also expected to enhance non linear effects because the optical signals are specially compressed and their internal intensity is enhanced the low υ_g. Thus far, large material dispersion and structural dispersion have been studied to generate slow light. Photonic crystal waveguides consisting of line defect in a two dimensional photonic crystal slab shows a low υ_g converging to zero and a correspondingly large group index $n_g = \frac{c}{\upsilon_g}$ diverging to infinity owing to the large first-order structural dispersion $\frac{dk}{d\omega}$ at the photonic band edge. However, the band-width of slow light becomes narrower as υ_g approaches to zero. Moreover, the wave form of high-speed optical signals is severely distorted by the large second-order dispersion (group velocity dispersion $d(\frac{d\omega}{dk})$ of slow light.

Chapter 5

Summary and Conclusion

Photonic band gap materials are a powerful tool for tailoring light propagation properties. This Chapter has put the emphasis on light propagation in photonic crystals as a function of group velocity, group index, group velocity dispersion, and dielectric function. Dielectric function, group index and group velocity were the major parameters. By taking these main parameters in to account, photonic band gaps are designed. In addition the geometrical parameters of photonic crystals should be chosen so that both the macroscopic and microscopic resonance occur at precisely the same wavelength.

Slow light with a group velocity several tens to several hundreds of times lower than c is attainable with present PCW-based technology. All slow light techniques reviewed rely on slowing down the information or the energy transported by light signals, more than on slowing the photons themselves. Among the plurality of foreseen slow-light applications we have mainly high lighted two, due to its high technological and societal expected impact. The development of optical buffer memories is of utmost importance for the deployment of all-optical networks.

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