THE RATE OF STAR FORMATION IN INTERACTING MOLECULAR CLOUDS WITH STELLAR ACTIVITIES

By

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TO MY FAMILY

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Abstract

According to the current understanding stars are formed from dust molecular clouds (MCs) mostly from hydrogen gas interstellar medium (ISM). It is also believed that external agents like gravity shock-waves probably responsible to trigger to formation and evolution. Today, lot of attention has given to study the evolutionary scenario of stellar systems. This is partly in due as the result of both observational development and new computational techniques. But, still the analytical work is awaiting much to do to develop basic principles. To this end here we have analytically worked out on the evolution of mass transfers and their rates of evolution by assuming the standard three component interacting system: molecular clouds, atomic gases and stellar population and evolution. The results well agree with the earlier numerical works of the standard three-component system.

Key:molecular-clouds, star-formation-rate ,stellar-mass, interstellar-medium, atomic-gas

Chapter 1

Introduction

I. Background

Stars are formed in molecular clouds in the interstellar medium, which consist mostly of molecular hydrogen (primordial elements made a few minutes after the beginning of the universe and dust). The dust originates from the cool surfaces of super giants, massive stars in a late stage of stellar evolution. The clouds can range in size from less than a light year to several hundred light years across and range in mass from 10 to 10 million on solar masses. Stars are born from the gravitational collapse of dense cores within giant molecular clouds.

A molecular cloud breaks in to smaller and smaller piece in a hierarchial manner until the fragment reach stellar mass.In each of these fragments the collapsing gas radiates away the energy gained by the release of gravitational potential energy. As the density increases the fragment become opaque and are thus less efficient radiating away their energy. This raise the temperature of the cloud and inhibit further fragmentation. The fragment now condense in to rotating spheres of gas that serve as stellar embryos [16]. Complicating this picture of a collapsing cloud are the effect of turbulence, macroscopic flows, rotation, magnetic fields and the cloud geometry.

Both rotational and magnetic fields can hinder the collapse of a cloud.Turbulence is instrumental in causing fragmentation of the cloud and, the smallest scale it promotes collapse[9].

During the collapse, the density of the cloud increases towards the center and thus the middle region became optically opaque first. This occur when the density is about$10^{-13}\frac{g}{cm^3}$. A core region now we call first hydrostatic core, forms where the collapse is essentially halted. It continue to increase in temperature as determined by the virial theorem. The gas falling toward this opaque region collides with it and creates shock waves that heat the core.

When the core temperature reaches about 2000K, the thermal energy dissociates the H_2 molecule. This is followed by the ionization of the hydrogen and helium atom.

After the density of in falling material has reached about $10^{-8}\frac{g}{cm^3}$, that material is sufficiently transparent to allow energy radiated by the protostar to escape. The combination of convection with in the protostar and radiation from its exterior allow the star to contract further. This continues until the gas is hot enough for the internal pressure to support the protostar against further gravitational collapse. when this accretion rate is nearly completed the resulting object is protostar[11].

There are actually two main types of star formation

(i) Large scale or simply the formation of many stars at ones.

(ii) small scale, the formation of only a few stars. What is needed for large scale star formation? We need to have a lot of the basic material that goes into stars, hydrogen and helium.

Stars are believed to form from the collapse of **giant molecular clouds(GMCs)** the gas loss pressure, flows towards the center of halo potential well while its density increase. Once the gas density exceeds the density of the dark matter, the gas continuous to collapse under its own gravitational potential. In the presence of efficient cooling, collapse continues until matter becomes dense enough to enable the formation of stars .

The giant molecular clouds (GMCs) or the large cloud of gas and dust,they have very distinctive characteristics:

- Masses of these clouds are typically on the order of millions of solar masses, and on some occasions up to billions of solar masses (that's the giant part).
- They are cool, around 10K.
- Gasses are generally found in molecular form, with such molecules as H_2, CO_2, H_2O, SiO, CO etc...

II. Literature Review

In this review we focus on molecular clouds as a fundamental formation sites ,rather than on the large scale-process that form the clouds and sets their properties .Molecular clouds are shaped in to a complex filamentary structure by supersonic turbulence ,with only a small fraction of the clouds mass channeled in to collapsing protostar over a free-fall of the system .In recent years the physics of supersonic turbulence has been widely explored with computer simulations ,leading to statistical models of this fragmentation process and to the prediction of the star formation rate ,as a function of fundamental physical parameters molecular clouds ,such as the virial parameters,the compressive fraction of the turbulent driver ,and the ratio of gas to magnetic pressure .Infra red space telescopes ,as well as ground based observations have provided un precedented probes of the filamentary structure of molecular clouds and the location of star with in them[15]

The recent recognition of massive near by clouds with little star formation rates in even near by clouds of similar mass can vary considerably as much as an order of magnitud[8].Therefore ,systematic and comparative observational studies of the physical properties of local molecular clouds and this relation of these properties to the varying levels of star formation activity with in them could lead to new insights concerning the underlying physics controlling the star formation rates in molecular gas.

III. Statement of the Problem

Present-day stars and planets formation are the most problem facing astro physics research .Stars are formed from dust molecular clouds(MCs)mostly from hydrogen gas interstellar medium (ISM). But the rate at which the stars are formed in giant molecular clouds (GMC) is the problem ranging from observational limitations to theoretical developments remains unresolved.

Research Question

- How stars form in interacting molecular clouds?
- At what rate the stars form in interacting molecular clouds of stellar activity?

IV . Objectives

I. General objective

To study the rate of star formation in interacting molecular clouds with stellar activities.

II. Specific Objectives

- To study and drive the dynamical parameters involving star formation rate in interacting molecular clouds.
- to drive the rate of star formation in interacting Molecular clouds with stellar activity.

V. Methodology

Our approach was theoretical and analytical analysis:

- We have provided preliminary boundary conditions and set the relevant Boltzmann transport equation.
- The result of the relevant parameters derived from the the first step were studied and examined.
- Some numerical data were generated from the formalism using mathematica 7 and comparison is made with the observational results.
- Generate theoretical data from the theoretical formula using computation. For the numerical computation MATHEMATICA 7 is used.

- At the end, we made summery and conclusion.

The general design of this work: In chapter one we give the theories of stars and molecular cloud formation and the governing stellar evolutionary equations. In chapter two we give the implications of Boltzmann transport equations (BTE) in relation to conservation laws, and we present our work of transport phenomena in star-forming molecular clouds as outlined in the methodology.In chapter three we present the star formation rate of molecular clouds .In chapter four we discuss the results of our work and finally in chapter five we give our summary and conclusion.

Chapter 2

Basic theory of star formation and hydrodynamic equations

2.1 Basic definitions and concepts of stars formation

A star is an object that (1) radiates energy from an internal source and (2) is bound by its own gravity. The first criterion excludes objects like planets, comets and brown dwarfs where both are not hot enough for nuclear fusion. The second criterion excludes trivial objects that radiate (e.g. glowing coals). A star is born out of an interstellar (molecular) gas cloud, lives for a certain amount of time on its internal energy supply, and eventually dies when this supply is exhausted. The definition imposes that stars can have only a limited range of masses, between $\sim 0.1 M_{\odot}$ and$\sim$ $1000 M_{\odot}$. Stars are considered to be isolated in space, so that their structure and evolution depend only on intrinsic properties (mass and composition). For most single stars in the Galaxy this condition is satisfied to a high degree (compare for instance the radius of the Sun with the distance to its nearest neighbor Proxima Centauri. However, for stars in dense clusters, or in binary systems, the evolution can be influenced by interaction with neighboring stars. Also, stars are formed with a homogeneous composition, a reasonable assumption since the molecular clouds out of which they form are well-mixed. We will often assume a so-called quasi-solar composition (X = 0.70, Y = 0.28 and Z = 0.02), even though recent determinations of solar abundances have revised the solar metallicity down to Z = 0.014. where x,y and z in units of m_H. In practice there is relatively little variation in composition from star to star, so that the initial mass is the most important parameter that determines the evolution of a star. Moreover, spherical symmetry, which is promoted by self-gravity a good approximation for most stars. Deviations from spherical symmetry can arise if non-central forces become important relative to gravity, in particular rotation and magnetic fields. Although many stars are observed to have magnetic fields, the field strength (even in highly magnetized neutron stars) is always negligible compared to gravity. Finally, understanding the structure and evolution of stars, and their observational properties, requires laws of physics involving different areas (e.g. thermodynamics, nuclear physics, electrodynamics, plasma physics)[7] Stars are the fundamental unit of luminous matter in the universe, and they are responsible directly or indirectly, for most of what we see when we observe it.

Star formation occur as a result of the action of gravity on a wide range of scales. On galactic scale the tendency of interstellar matter to condense under gravity in to star forming clouds is countered by galactic tidal forces, and star formation can occur only where the gas become dense enough for its self gravity to over come these tidal force,for example in spiral arms. on the intermediate scales of star forming giant molecular clouds (GMCs),turbulence and magnetic fields may be the most important effects counteracting gravity, and star formation may involve the dissipation of turbulence and magnetic fields[3].

2.2 Sites of star formation

Star formation occur near the center of some galaxies, including our own milky way galaxy, but this nuclear star formation is often obscured by interstellar dust and its existence is inferred only from the infra red radiation emitted by dust heated by the embedded young star. The gas from which stars form,whether in spiral arms or in galactic nuclei, is concentrated in massive and dense "molecular clouds"whose hydrogen is nearly all in molecular form. Some near by molecular clouds are seen as dark clouds against the bright back ground of the Milky way because their interstellar dust absorbs the starlight from the more distant stars.

In some near by dark clouds many faint young stars are seen,most distinctive among which are the T Tauri star, whose variability, close association with the dark cloud, and relatively high luminosity for their temperature indicate that they are extremely young and have gas of typically only about 1Myr.These T Tauri stars are the youngest known visible stars, and they are "pre- main -sequence" stars that have not yet become hot enough at their centers to burn hydrogen and begin the main sequence phase of evolution .Some of these young stars are embedded in particularly dense small dark clouds, which are thus the most clearly identified sites of star formation[13].

2.3 Cloud formation

Since molecular clouds are transient features, it follows that they are constantly being formed and destroyed. It is necessary to understand the process by which they are continually being reassembled from more dispersed gas. The rate at which interstellar gas is presently being collected in to star forming molecular clouds in our galaxy is related to the star formation rate and it can be estimated empirically from the observed

star formation rate and the efficiency of star formation in molecular clouds.The total rate of star formation in our galaxy is of the order of $3M\odot$ of per year. Since only about few percent or less of the mass of a typical molecular cloud converted in to stars; it implies that at least $150M_{\odot}$ of gas per year is being turned in to star forming molecular clouds. The total amount of gas in our galaxy is about $5 \times 10^9 M\odot$.
Two possible formation mechanism for molecular clouds that have been considered are (1)cloud growth by random collisions and coalescence and (2) gravitational instability or swing amplification.

2.4 Cloud Collapse and Fragmentation

A giant molecular cloud must begin forming stars soon after the cloud it self has formed, since relatively few of the largest molecular clouds are not forming star. Even if as many as half of all molecular clouds are not forming stars, the time delay between the formation of a molecular cloud and the on set of star formation in it can not exceed the subsequent duration of the star formation activity which is of the order of 10Myr and comparable to the internal dynamical time scale. Since it takes some what longer than this to build large molecular clouds it is likely that star formation begin already in molecular clouds while they are still being assembled;more over, star formation must begins with in a time not much longer than the dynamical or free-fall time of such a cloud. Collapse and star formation can occur in the densest part of a cloud even if the cloud as a whole is not collapsing, and this must in fact be what usually happens because there is no evidence that most star forming clouds are under going any rapid over all collapse, and there is even evidence that many of them are being dispersed.

Star formation involves the collapse of a cloud or part of a cloud under gravity and the association fragmentation of the cloud in to smaller and smaller bound clumps this is expected to occur because molecular clouds typically contain many times the "Jeans mass", which is the minimum mass for gravitational bound fragments [12].

The near-constant low temperature across molecular clouds is an important feature of the star formation process because of its influence on the Jeans mass, and it is what makes possible the collapse of pre stellar cloud cores with masses as small as one solar mass. The Jeans mass is the critical mass at which a cloud becomes unstable and starts to collapse, as it possesses insufficient pressure support to balance the force of gravity.In the absence of pressure or other support, gravitational collapse of such a cloud will occur in a free fall time:

$$\tau_{ff} = \sqrt{\frac{3\pi}{32G\rho_o}}$$

where ρ_o is the mean density of the cloud .The star formation process in molecular clouds appears to be fast. Once the collapse of a cloud region sets in, it rapidly forms an entire cluster of stars with in 10^6 years or less. The resulting stellar population is widely dispersed throughout the cloud and, since collapsing clumps are frequently destroyed by shock interaction, the overall star formation rate is low[1]

The observation show that the gas (which contain mainly molecular hydrogen, H_2)is highly clumpy, and virtually all molecular gas distribute in GMCs.

The molecular cloud always rotate due to differential in the disk in which they are formed. If a collapse conserves angular momentum, this would imply a rotation period of well bellow 1sec of the emerging star. This means that angular momentum

has to be transferred during the collapse.
Further more potential energy of the clouds $E_p \alpha - \frac{GM^2}{r}$ is released during the collapse. This energy therefore must be radiated or transported away, despite the high opacity of the surrounding medium.
present-day star formation in our galaxy is observed to take place in cold molecular clouds which appear to be in a state of highly compressible magnetohydrodynamic (MHD) turbulent on large scales from hundred to thousands of parsec
An interstellar cloud of gas will remain in hydrostatic equilibrium as long as the kinetic energy of the gas pressure is in balance potential energy of the internal gravitational force. Mathematically this is expressed using the virial theorem which state that to maintain equilibrium the gravitational potential energy must equal twice the internal thermal energy [10].

$$2K + U = 0 \text{ Virial theorem}$$

If $2K <\mid U \mid$ the cloud will collapse under the force of gravity.The gravitational potential energy can be written as

$$U \simeq \frac{3}{5}(\frac{GM_c^2}{R_c})$$

Where$M_c and R_c$ are respectively the mass and the radius of the cloud
The average kinetic energy per particle is $K = \frac{3}{2}kT$ where k is Boltsman constant. Thus, the total internal kinetic energy of the cloud is just
$K = \frac{3}{2}NkT$, where N is the total number of particles. We can write N in terms of the mass and the mean molecular weight.

$$N = \frac{M_c}{\mu MH}$$

We can therefore write the condition for gravitational collapse $(2K <\mid U \mid$

Where K-is internal kinetic energy and U-is gravitational potential energy.

$$\frac{3M_cKT}{\mu MH} < \frac{3GM_c^2}{5R_c}$$

$$\text{but,} R_c = (\frac{3M_c}{4\prod \rho_o})^{\frac{1}{3}}$$

Where ρ_o is the initial density of the cloud prior to collapse with the assumption that the cloud is a sphere of constant density.

by substituting we obtain the important concept of the Jeans mass.

$$M_J \simeq (\frac{5KT}{G\mu MH})^{\frac{3}{2}}(\frac{3}{4\prod \rho_o})^{\frac{1}{2}}$$

If a cloud is massive enough that the gas pressure is insufficient to support it, the cloud will under go gravitational collapse. The mass above which a cloud will under go such collapse is the Jeans mass. The Jeans mass depends on the temperature and density of the cloud, but is typically thousands to tens of thousands of solar mass.

$$M_J \backsimeq 3x10^4\sqrt{\frac{T^3}{M}}M_{\odot}$$

Where T is cloud temperature, M is cloud mass and $M_{\odot}$ is solar mass.

In triggered star formation one of several events might occur to compress a molecular cloud and initiate its gravitational collapse.Molecular cloud may collide with each other or a near by supernova explosion can be a trigger sending shocked matter in to the cloud at very high speed

.

2.5 Stages of star formation process:

According to the current understanding there is six stages in star formation process.

1. The initial free-fall collapse of the parent interstellar cloud.
2. Cloud fragmentation, leading to a range of stellar mass .
3. Formation of protostellar core. The star appears on the H-R diagram.
4. Accretion of the surrounding gas, generally through an accretion disk.
5. Dissociation of molecules and ionization of H and He.
6. Pre- main sequence phase.

   - star formation is considered to completed once the star appears on the "Zero Age Main sequence" (ZAMS).

2.6 Basic stellar evolutionary equations

The basic theory of stellar structure assumes spherical symmetry, so that all variables depend on only one thing, the distance (r) from the center of the star.On spherical shells of radius r, all physical variables (temperature, density, pressure chemical composition) are assumes to be uniform. The principle variables of stellar structure are pressure (P), temperature(T), density ρ, luminosity) through a shell at r L(r)and mass interior to r M(r). For an isolated static, spherically symmetric star four basic laws/equations needed to describe structure.

All physical quantities depend on the distance from the center of the star alone.

The fundamental hydrodynamic equations are being derived from the Boltzman transport equations. For ordinary stellar evolutions and formations the classical Maxwell-Boltzmann distribution is considered.

2.6.1 Homogeneous Boltzmann Transport Equation

In stellar astrophysical, modeling gas flows around stars or in interstellar space, the ideal gas assumption is very much accurate. Therefore, in our analysis of the stellar evolution including magnetic field dynamism we apply the classical Boltzmann statistical distributions and derive the dynamic equations from Boltzmann transport equations.

The Boltzmann transport equation in six dimensional position-velocity phase space basically expresses the change in the phase density within a differential volume, in terms of the flow through these faces, and the creation or destruction of particles within that volume. In the canonical position-momentum coordinate system, the Boltzmann transport equation (BTE) is given by

$$\sum_{i=1}^{3}[\dot{x}_i \frac{\partial f}{\partial x_i} + \dot{p}_i \frac{\partial f}{\partial p_i}] + \frac{\partial f}{\partial t} = s \Rightarrow BTE \tag{2.6.1}$$

Where $f \equiv f(x, \dot{x}, t)$ is the number density distribution functions, s is the rate of particle creation /destraction ,$\dot{x}_i = \frac{\partial x_i}{\partial t}$ and $\dot{p}_i = \frac{\partial p_i}{\partial t}$

This equation can be recast in vector notation as

$$\frac{\partial f}{\partial t} + \vec{v}.\vec{\nabla} f + \vec{F}.\vec{\nabla}_p f = s \tag{2.6.2}$$

Where $\vec{F}$ is force and $\vec{\nabla}_i$ is the momentum gradient.

In conservative field system since $\vec{F} = -\dot{\nabla}\Phi$ whereΦ is a scalar potential (eg gravitational scalar potential), then BTE will be given as :

$$\frac{\partial f}{\partial t} + \vec{v}.\nabla f - \frac{1}{m}\nabla\Phi.\nabla_v f = s \tag{2.6.3}$$

The potential gradient $\nabla\Phi$ has replaced the momentum time derivative while$\nabla_v$is a gradient with respect to velocity. The quantity m is the mass of a typical particle. It is also not unusual to find the BTE written in terms of the total stokes time derivative

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v}.\nabla \tag{2.6.4}$$

Where $\vec{v}$ is the flow velocity and $\frac{\partial}{\partial t}$ is the Eulerian time derivative. If we take$\nabla$to be a six-dimensional 'velocity' and r to be a six-dimensional gradient the BTE becomes

$$\frac{Df}{Dt} = s \tag{2.6.5}$$

If the creation/destruction rate of particles is zero (s = 0), we will obtain the homogeneous Boltzmann Transport Equation (BTE) given as

$$\frac{\partial f}{\partial t} + \vec{v}.\nabla f - \frac{1}{m}\nabla\Phi.\nabla_v f = 0 \tag{2.6.6}$$

This is Liouville's theorem The physical interpretation of Liouville Equation is the 6N-dimensional analogue of the equation of continuity of an incompressible fluid. It implies that the phase points of the ensemble are neither created nor destroyed. In Astrophysics it is called the Vlasov equation, or sometimes the Collision less Boltzmann Equation. It is used to describe the evolution of a large number of collision less particles moving in a gravitational potential. In the case of classical statistical mechanics, the number of particles N is very large, (of the order of Avogadro's number, for a laboratory-scale system). Setting $\frac{\partial \rho}{\partial t} = 0$gives an equation for the stationary states of the system and can be used to find the density of microstates accessible in a given statistical ensemble. For eg. in an equilibrium of the Maxwell-Boltzmann statistical distribution$\rho$is given as

$$\rho \alpha e^{\frac{H}{(k_B T)}}$$

Where H is the Hamiltonian, T is the temperature and k_B is the Boltzmann constant. The right-hand side of equation (2.6.1) is a measure of the rate at which particles are created or destructed in the phase space volume. Note that creation or destruction in phase space includes a good deal more than the conventional spatial creation or destruction of particles. To be sure, that type of change is included, but in addition processes which change a particle's position in momentum space may move a particle in or out of such a volume. From BTE the right-hand side is zero or the creation or destruction rate of particles is zero, this is known as Homogeneous Boltzmann Transport Equation Liouville's theorem of statistical mechanics[5]

2.6.2 Moments of the Boltzmann Transport Equation and Conservation Laws

By the moment of a function we mean the integral of some property of interest, weighted by its distribution function, over the space for which the distribution function is defined. The mean of a distribution function is simply the first moment of the distribution function, and the variance can be simply related to the second moment. In general, if the distribution function is analytic, all the information contained in the function is also contained in the moments of that function. The complete solution to the BTE is, in general, extremely difficult and usually would contain much more information about the system than we wish to know. The process of integrating the function over its defined space to obtain a specific moment removes or averages out much of the information about the function. This is a standard "trick" of mathematical physics and one which is employed over and over throughout this. Almost every instance of this type carries with it the name of some distinguished scientist or is identified with some fundamental conservation laws, but the process of its formulation and its origin are basically the same.

i) The Zero moment of Boltzmann Transport Equation and Conservation of Matter.

To derive conservation laws and energy balance we start from n_{th} functions of BTE, we have

$$M_n[f(x)] = \int x^n f(x) dx$$

The local spatial density is given as

$$\rho = m\int_{-\infty}^{+\infty} f(x,\vec{v})d\vec{v} \tag{2.6.7}$$

The related BTE is

$$\int_{-\infty}^{+\infty}(\frac{\partial f}{\partial t} + \sum_{i=1}^{3} v_i\frac{\partial f}{\partial x_i} + \sum_{i=1}^{3}\dot{v}_i\frac{\partial f}{\partial v_i})d\vec{v_i} = \int_{-\infty}^{+\infty} sd\vec{v}$$

The integral of the creation rate S over all velocity space becomes simply the creation rate for particles in physical space, which we call$\Im$. The first term from equation (2.6.7) is given as $\frac{1}{m}\frac{\partial\rho}{\partial t}$

$$\frac{\partial}{\partial t}(\int_{-\infty}^{+\infty} fd\vec{v}) + \int^{+\infty -\infty}(\vec{v}.\nabla f d\vec{v}) + \int_{-\infty}^{+\infty}(\dot{\vec{v}}.\nabla_v f)d\vec{v} = \Im \tag{2.6.8}$$

From equation (2.6.7) the second term can be define by the vector identity

$$\vec{v}.\nabla f = \nabla.(f\vec{v}) - f\nabla.\vec{v}$$

where$\vec{v}.\vec{\nabla}f = \vec{\nabla}.(f\vec{v})$

The third term from equation(2.6.7)is

$$\dot{\vec{v}}.\vec{\nabla}_v f = -\frac{\nabla\vec{\Phi}}{m}.\vec{\nabla}f \tag{2.6.9}$$

Now using the equation in equation(2.6.8)

$$\frac{1}{m}\frac{\partial\rho}{\partial t} + \int_{-\infty}^{+\infty}[\vec{v}.f\vec{v}d\vec{v}] - \int_{-\infty}^{+\infty}[\frac{\nabla\Phi}{m}.\vec{\nabla}_v f]d\vec{v} = \vec{\Im}$$

$$\frac{\partial\rho}{\partial t} + m\vec{\nabla}.[\int_{-\infty}^{+\infty}\vec{v}fd\vec{v}] - \vec{\nabla}\Phi.\int_{-\infty}^{+\infty}\vec{\nabla}_v fd\vec{v} = \vec{\Im}$$

From the above equation no particle with infinite velocity,then the last integral will be vanish.

$$\frac{\partial\rho}{\partial t} + m\nabla.[\int_{-\infty}^{+\infty}\vec{v}fd\vec{v}] = \vec{\Im} \tag{2.6.10}$$

$$M_1[f(v)] = \int f(v)dv$$

The mean flow velocity is U is a measure of the mean flow rate of the material,for a normalization scale

$$\vec{U} = \frac{\int_{-\infty}^{+\infty}\vec{v}f(\vec{v})d\vec{v}}{\int_{-\infty}^{+\infty}f(\vec{v})d\vec{v}} \tag{2.6.11}$$

From equation(2.6.7) $\frac{\rho}{m} = \int_{-\infty}^{+\infty}f(x,\vec{v})d\vec{v}$ using equation (2.6.7)and (2.6.10) in equation (2.6.11) we get the continuity equation

$$\frac{\partial\rho}{\partial t} + \nabla.(\rho\vec{U}) = \vec{\Im_m}$$

ii) The First Moment of Boltzmann Transport Equation - Euler-Lagrange Equations and Conservation of Momentum.

To produce an expression of the conservation of momentum let as multiply the Boltzman transport equation by the local particle velocity $\vec{v}$

$$\vec{v}[\int_{-\infty}^{+\infty} \frac{\partial f}{\partial t} d\vec{v} + \int_{-\infty}^{+\infty} \vec{v}.\nabla f d\vec{v} + \int_{-\infty}^{+\infty} (\dot{\vec{v}} \vec{\nabla_v} f) d\vec{v}] = \int_{-\infty}^{+\infty} \vec{v} s d\vec{v} \qquad (2.6.12)$$

From equation(2.13.1) the first term

$$[\int_{-\infty}^{+\infty} \vec{v} \frac{\partial f}{\partial t} d\vec{v} = \frac{\partial}{\partial t} \int_{-\infty}^{+\infty} \vec{v} f d\vec{v} = \frac{\partial}{\partial t}[\int_{-\infty}^{+\infty} f(\vec{v}) d\vec{v}] \qquad (2.6.13)$$

$$\frac{\int_{-\infty}^{+\infty} \vec{v} f(\vec{v}) d\vec{v}}{\int_{-\infty}^{+\infty} f d\vec{v}} = \frac{\partial}{\partial t}[\frac{\rho \vec{U}}{m}] = \frac{\partial}{\partial t}(n\vec{U})$$

The second term from equation (2.6.12)is

$$\int_{-\infty}^{+\infty} \vec{v}.\nabla f d\vec{v} = \int_{-\infty}^{+\infty} \vec{v}(\vec{v}.\vec{\nabla} f) d\vec{v} \qquad (2.6.14)$$

Where$\dot{\vec{v}} = -\frac{\nabla \Phi}{m}$

$$\int_{-\infty}^{+\infty} \vec{v}(\vec{v}.\vec{\nabla}_v f) d\vec{v} = \int_{-\infty}^{+\infty} \vec{v}(\frac{-\nabla \Phi}{m}.\vec{\nabla}_v f) dv$$

$$\int_{-\infty}^{+\infty} \vec{v}(\vec{v}.\vec{\nabla}_v f) d\vec{v} = -\frac{\vec{\nabla} \Phi}{m} \int_{-\infty}^{+\infty} \vec{v} \nabla f d\vec{v}$$

$$(\vec{\nabla}_v f)\vec{v} = \vec{\nabla}_v (f\vec{v}) - f(\vec{\nabla}_v \vec{v}) = \vec{\nabla}_v (f\vec{V}) - fI$$

Where I is the identity matrix.

$$\int_{-\infty}^{+\infty} \vec{v}(\vec{v}.\vec{\nabla}_v f)d\vec{v} = -\frac{\nabla\vec{\Phi}}{m}.\int_{-\infty}^{+\infty} \vec{\nabla}_v(fv)d\vec{v}$$

$$+\frac{\nabla\vec{\Phi}}{m}.\int_{-\infty}^{+\infty} fd\vec{v}$$

$$\int_{-\infty}^{+\infty} \vec{v}(\vec{v}.\vec{\nabla}_v f)d\vec{v} = -\frac{\nabla\vec{\Phi}}{m}.\int_{-\infty}^{+\infty} fd\vec{v}$$

$$= n\frac{\nabla\Phi}{m}$$

$$\frac{\partial}{\partial t}(n\vec{u}) + \int_{-\infty}^{+\infty} \vec{v}(\vec{\nabla}.(\vec{vf}))d\vec{v} + n\frac{\vec{\nabla\Phi}}{m} = \int_{-\infty}^{+\infty} s\vec{v}d\vec{v} \tag{2.6.15}$$

$$\frac{\partial}{\partial t}(n\vec{u}) = u\frac{\vec{\partial} n}{\partial t} + n\frac{\partial\vec{u}}{\partial t} \tag{2.6.16}$$

$$\frac{\partial n}{\partial t} = -\nabla.(n\vec{u}) + \int_{-\infty}^{+\infty} sd\vec{v} = -(\vec{u}.\vec{\nabla}_n + \vec{\nabla}_n.\vec{u})\int_{-\infty}^{+\infty} sd\vec{v}$$

From the continuity equation

$$\frac{\partial n}{\partial t} + \nabla.(n\vec{u}) = \int_{-\infty}^{+\infty} sd\vec{v} \tag{2.6.17}$$

Then equation (2.6.15)and (2.6.16) we get

$$\frac{\partial}{\partial t}(n\vec{u}) = U\frac{\partial n}{\partial t} + n\frac{\partial\vec{u}}{\partial t} - (\vec{u}.\nabla n + n\nabla.\vec{u})\vec{u} + \int_{-\infty}^{+\infty} \vec{u}sd\vec{v} \tag{2.6.18}$$

Using equation (2.6.15)and(2.6.18)

$$\frac{\partial}{\partial t}(n\vec{u}) = n\frac{\partial \vec{u}}{\partial t} - (\vec{u}.\vec{\nabla}_n + \vec{\nabla}_n.\vec{u})\vec{u} + \int_{-\infty}^{+\infty} \vec{u}sd\vec{v} \tag{2.6.19}$$

$$n\frac{\partial \vec{u}}{\partial t} - (\vec{u}.\vec{\nabla}_n + \vec{\nabla}_n.\vec{u})\vec{u} + \int_{-\infty}^{+\infty} \vec{v}(\vec{\nabla}.(\vec{v}f)d\vec{v} + n\frac{\vec{\nabla}\Phi}{m} \tag{2.6.20}$$

$$\textstyle\int_{-\infty}^{+\infty} s(\vec{v} - \vec{u})d\vec{v}$$

The velocity tensor$\vec{u}$ is given as

$$\vec{U} = \frac{\int_{-\infty}^{+\infty} \vec{v}f(\vec{v})d\vec{v}}{\int_{-\infty}^{+\infty} f(\vec{v})d\vec{v}} \tag{2.6.21}$$

$$\rho\frac{\partial u}{\partial t} + \rho(\vec{u}.\vec{\nabla})\vec{u} + \vec{\nabla}.(\rho(\vec{u} - \vec{u}\vec{u})) + n\vec{\nabla}\Phi = \int_{-\infty}^{+\infty} ms(\vec{v} - \vec{u})d\vec{v} \tag{2.6.22}$$

The quantity$\rho(\vec{u} - \vec{u}\vec{u})$is the pressure tensor.The pressure tensor the second moment of $f(v)$is $\vec{\rho}$ equal to

$$\frac{\int_{-\infty}^{+\infty} f(v)(\vec{v}-\vec{u})(\vec{v}-\vec{u})d\vec{v}}{\int_{-\infty}^{+\infty} f(v)} d\vec{v} \tag{2.6.23}$$

It describes the difference between the local flow $\vec{v}$ and the mean flow $\vec{u}$. The first velocity moment of the BTE becomes

$$\frac{\partial \vec{u}}{\partial t} + (\vec{U}.\nabla)\vec{u} = -\nabla\Phi - \frac{1}{\rho}\nabla P + \frac{1}{\rho}\int_{-\infty}^{\infty} ms(\vec{v}-\vec{u})d\vec{v} \tag{2.6.24}$$

This set of vector equations is the **Euler-Lagrange equations of hydrodynamic flow**. This assumption of local velocity leads to the simpler and more familiar expression for hydrodynamic flow

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} = -\nabla\Phi - \frac{\nabla p}{\rho} \tag{2.6.25}$$

Under the assumption of a nearly isotropic velocity field, P will be $P(\rho)$ and an expression known as an equation of state. From equation (2.6.24)the left-hand side is zero. The Euler-Lagrange equations of hydrodynamic flow is

$$\nabla P = -\rho\nabla\Phi \tag{2.6.26}$$

Which is known as the equation of hydrostatic equilibrium: This equation is usually an expression of the conservation of linear momentum. The zeroth moment of the BTE results in the conservation of matter, where as the first velocity moment equations which represent the conservation of linear momentum. The second velocity moment represent an expression for the conservation of energy.

iii) The Second Moment of Boltzmann Transport Equation - Ergodic equations and Energy Conservation

The Euler-Lagrange equations of hydrodynamic flow, which represent the first velocity moment of the transport equation. These are vector equations we obtain a scalar result by taking the scalar product of a position vector with the flow equations and integrating over all space with the system. The origin of the position vector is important only in the interpretation of some of the terms which will arise in the expression. The left-hand side of equation (2.6.24) is the total time derivative of the flow velocity $\bar{U}$ then, the first spatial moment equation becomes

$$\int_v \rho\vec{r}\frac{d\vec{u}}{dt}dv + \int_v \rho\vec{\vec{r}}.\nabla\Phi dv + \int_r \vec{r}.\nabla p dv = 0 \tag{2.6.27}$$

$$\int_v \rho\frac{dQ}{dt}dv = \frac{d}{dt}\int \rho Q dv \tag{2.6.28}$$

Since$\vec{u}$ is the time rate of change of position, since

$$\vec{u} = \frac{d\vec{r}}{dt} \tag{2.6.29}$$

$$r.\tfrac{d\vec{u}}{dt} = \tfrac{d}{dt}(\vec{r}.\vec{u}) - \tfrac{d\vec{r}}{dt}.\vec{u}$$

$$= \tfrac{d}{dt}(\vec{r}.\vec{U}) - \vec{U}\bar{U}$$

$$= \tfrac{d}{dt}(\vec{r}.\vec{U}) - U^2$$

$$= \tfrac{d}{dt} r \tfrac{d\vec{r}}{dt} - U^2$$

$$= \tfrac{1}{2}\tfrac{d}{dt}(\tfrac{d}{dt}(\vec{r}.\vec{r})$$

$$r\frac{d\vec{u}}{dt} = \frac{1}{2}\frac{d^2 r}{dt^2} - u^2 \tag{2.6.30}$$

The first integral of equation(2.6.24) by using equation (2.6.28)

$$\textstyle\int_v \rho\vec{r}\frac{d\vec{u}}{dt} = \frac{1}{2}\int_v \rho\frac{d^2\vec{r}}{dt^2}dv - \int_v \rho u^2 dv$$

$$\textstyle= \frac{1}{2}\frac{d^2}{dt^2}\int r^2\rho dv - U^2\int_v \rho dv$$

Where I is the moment of inertia is given by:

$$I = \int_v r^2 \rho dv$$

And also the kinetic energy in bulk motion

$$T = \frac{1}{2} \int_v \rho u^2 dv$$

This implies that the mass is given as

$$m = \int_v \rho dv$$

$$\frac{1}{2} \frac{d^2}{dt^2} I - mu^2$$

But$mu^2 = 2T$

$$\int_v \rho \vec{r}. \frac{d\vec{u}}{dt} = \frac{1}{2} \frac{d^2 I}{dt^2} - 2T \tag{2.6.31}$$

The third integral of equation(2.6.27)

$$\int_v \vec{r}.\vec{\nabla} p dv = \int_v \vec{r}.\nabla(\vec{r}p) dv - \int_v p(\vec{\nabla}.\vec{r}) dv = \oint_s p_s \vec{r}\hat{n} dA - 3\int_s p dv = \oint_s p_s \vec{r}\hat{n} dA - 3U$$

Where$\vec{\nabla}.\vec{r} = (i\frac{\partial}{\partial x} + j\frac{\partial}{\nabla y} + \frac{\partial}{\partial z}).(ix + jy + kz) = 1 + 1 + 1 = 3$ The internal kinetic energy density of an ideal gas is

$$\epsilon = \frac{3}{2} \frac{\rho k T}{\mu m H}$$

We can replace the pressure **P**

$p = \frac{2}{3}\epsilon$

The integral then yields twice the total internal kinetic energy of the system ,and the moment of equation becomes

$$\frac{1}{2}\frac{d^2I}{dT^2} = 2(T+U) - \int_v \rho\vec{v}.\nabla\Phi dv \qquad (2.6.32)$$

The last term on the right-hand side of the equation (2.6.27) is called the total potential energy .Also the expression is called Lagrange's identity and is also called the non-averaged form of the virial theorem.

$$\frac{1}{2}\frac{d^2I}{dt^2} - 2T - 2U + \int_v \rho\vec{r}.\vec{\nabla}\Phi dv \qquad (2.6.33)$$

A system in equilibrium, so that the time average of equation(2.6.32)remove the accelerative changes of the moment of inertia($< \frac{d^2I}{dt^2} >= 0$

$$2 < T > +2 < U > + < \Omega >= 0 \qquad (2.6.34)$$

T is kinetic energy in bulk motion and U is the internal energy and Ω is the total potential energy of the system.

The theorem which permits is the Ergodic theorem .

2.7 The summarized Boltzmann Transport and Hydrodynamic equation

From zero moment of Boltzmann transport equation conservation of matter we can have continuity equation.

$\frac{\partial\rho}{\partial t} + \nabla.(\rho\vec{U}) = \vec{\Im}_m$

From first moment of Boltzmann transport equation conservation of linear momentum we have hydrodynamic equation

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} = -\nabla\Phi - \frac{\nabla p}{\rho}$$

second moment of Boltzmann transport equation conservation of energy

$$\frac{1}{2}\frac{d^2 I}{dt^2} - 2T - 2U + \int_v \rho\vec{r}.\vec{\nabla}\Phi dv = 0 \qquad (2.7.1)$$

A system in equilibrium, so that the time average of equation(2.6.33)remove the accelerative changes of the moment of inertia($< \frac{d^2 I}{dt^2} >= 0$

$$2 < T > +2 < U > + < \Omega >= 0$$

In astronomy this theorem is called Ergodic theorem [6]. By now we have the basic mathematical tools to apply in stellar evolution. All evolutionary equations can be derived by applying appropriate boundary conditions to the BTEs.

2.8 Equation of state of an ideal gas

In thermodynamics, an equation of state provides the mathematical relation among variables such as temperature, pressure, density, and internal energy. Equations of state (EOS) are useful in describing the properties of fluids, mixtures of fluids, solids, and even the interiors of stars. For stars, the state usually describe the relation among pressure(P), temperature(T),density (n: number of density of particles or (ρ):mass density). Formulation of the Boltzmann Transport Equation (BTE) also provides an ideal setting for the formulation of the equation of state for a gas under wide-ranging conditions. The relationship between the pressure as given by the pressure tensor and the state variables (p, T, ρ) of the distribution function. The pressure tensor is $p(u - \vec{u}\vec{u})$. If $f(\vec{v})$ is symmetric in $\vec{v}, then \vec{u}$ must be zero (or there exist an inertial coordinate system in which $\vec{u}$ is zero, and the divergence of the pressure can be replaced by the gradient of a scalar which we call the gas pressure and will be given by

$$\vec{p} = \rho \frac{\int_{-\infty}^{+\infty} v^2 f(v) d\vec{v}}{\int_{-\infty}^{+\infty} f(v) d\vec{v}} \tag{2.8.1}$$

From the Maxwell-Boltzmann statics the distribution function of particles in terms of their velocity is given by

$$f(v) = constant.exp^{(\frac{-mv^2}{2Kt})} \tag{2.8.2}$$

The mean pressure is

$$\vec{p} = c\rho \frac{\int_{-\infty}^{+\infty} v^2 exp^{\frac{-mv^2}{2kT}}}{c\int_{-\infty}^{+\infty} exp^{\frac{-mv^2}{2kT}}}$$

$$\vec{p} = \rho \frac{\int_{-\infty}^{+\infty} v^2 exp^{-\alpha} v^2 dv}{\int_{-\infty}^{+\infty} exp^{-\alpha} v^2 dv}$$

Where$\alpha = \frac{m}{2kT}$.The integral of the function is

$$\int_{-\infty}^{+\infty} v^2 exp(-\alpha V^2) dv = \frac{1}{4}\sqrt{\pi}\alpha^{\frac{-3}{2}}$$

$$\int_{-\infty}^{+\infty} exp(-\alpha V^2) dv = \sqrt{\frac{\pi}{\alpha}}$$

then,

$$\bar{p} = \frac{\rho\frac{1}{4}\sqrt{\Pi}\alpha^{\frac{-3}{2}}}{\sqrt{\frac{\Pi}{\alpha}}}$$

$$\bar{p} = \rho\frac{\frac{\sqrt{\Pi}\alpha^{\frac{-3}{2}}}{4}}{2\sqrt{\Pi}\alpha^{\frac{-1}{2}}}$$

$$= \rho\frac{\alpha^{-1}}{2} = \frac{\rho}{2\alpha}$$

but,$\alpha = \frac{m}{2kT}$ then the mean pressure is

$$\bar{p} = \frac{\rho}{\frac{2m}{2KT}} = \frac{\rho KT}{m}$$

$$\bar{p} = nkT$$

2.9 Fundamental Equations of Stellar Structure

(1) **Conservation of mass**:

For a spherical shell of thickness dr is

$$\frac{dM_r}{dr} = 4\pi r^2 \rho(r) \tag{2.9.1}$$

written in terms of integral,this is

$$M_{(r)} = \int_0^r 4\pi r^2 \rho_r dr \tag{2.9.2}$$

However, over its lifetime a star's radius will change by many orders of magnitude while its mass will remain relatively constant. Moreover, the amount of nuclear reactions occurring inside a star depends on the density and temperature not where it is in the star. A better and more natural way to treat stellar structure is therefore to use mass as the independent parameter, rather than r. Thus

$$\frac{dr}{dM} = \frac{1}{4\pi r^2 \rho} \tag{2.9.3}$$

This is the Lagrangian form of the equation (rather than the Eulerian form). All the equations of stellar structure will be expressed in the Lagrangian form, and most of the parameters will be expressed in per unit mass, rather than per unit size or volume.

(2)Conservation of Energy (at each radius, the change in the energy flux equals the local rate of energy of release). Consider the net energy per second passing outward through a shell at radius r. If no energy is created in the shell, then the amount of energy in equals the amount of energy out, and $\frac{dL}{dr} = 0$ However, if additional energy is released or absorbed with in the shell, then $\frac{dL}{dr}$ will be non-zero. Let's define ϵ as the energy released per second by a unit mass of matter. Then:

$$\frac{dL}{dr} = 4\pi r^2 \rho \epsilon \tag{2.9.4}$$

or,in the Lagrangian

$$\frac{dL}{dr} = \epsilon_n + \epsilon_\nu + \epsilon_g \tag{2.9.5}$$

Note thatϵ has three components .

1.ϵ_n, the total energy created by nuclear reactions.

2.ϵ_ν, the energy in to neutrinos, and

3.ϵ_g, the energy produced or lost by gravitational expansion or contraction .Thus

$$\frac{dL}{dr} = \epsilon_n + \epsilon_\nu + \epsilon_g \quad (2.9.6)$$

In general, the contribution from reactions will always be positive, while the energy in neutrinos will always be lost from the system.

(3) Equation of Energy Transport (relation between the energy flux and the local gradient of temperature). Assume that the star is in thermal equilibrium at each radius the gas is neither heating up nor cooling down with time. The transport equation also describes how energy is transported through the layers of the star, i.e. how the gas affects the radiation as it travels through. Depends on local density, opacity and temperature gradient. Let the rate of energy generation per unit mass be ϵ. Then:

$$dL = 4\pi r^2 \rho dr \times q\frac{dL}{dr} = 4\pi r^2 \rho \epsilon \quad (2.9.7)$$

(4) Equation of Hydrostatic Equilibrium.

The force of gravity pulls the stellar material towards the center. It is resisted by the pressure force due to the thermal motions of the gas molecules. The first equilibrium condition is that these forces in equilibrium. Radial forces acting on the element:

$$Gravityinward : F_g = -\frac{Gm\Delta m}{r^2} \quad (2.9.8)$$

Pressure (net force due to difference in pressure between upper and lower face):

$F_p = p(r)ds - p(r+dr)ds - [p(r) + \frac{dp}{dr}]ds = -\frac{dp}{dr}drds$

Mass of element :$\Delta m = \rho drds$

applying Newton's second law ($F = ma$)

$$\Delta m\ddot{r} = F_g + F_p = -\frac{Gm\Delta m}{r^2} - \frac{dp}{dr}drds \qquad (2.9.9)$$

Acceleration =0 every where if star static.

setting acceleration to zero,and substituting for Δm:

$$0 = -\frac{Gm\rho drds}{r^2} - \frac{dp}{dr}drds \qquad (2.9.10)$$

$$\frac{dp}{dr} = -\frac{Gm}{r^2}\rho \qquad (2.9.11)$$

Basic equations are supplemented by Equations of State (pressure of a gas as a function of density and temperature) opacity (how transparent it is to radiation) Nuclear Energy Generation Rate as $f(\rho; T)$. **Equation of State in Stars:** Interior of a star contains a mixture of ions, electrons and radiation (photons). For most stars (exception very low mass stars and stellar remnants) the ions and electrons can be treated as an ideal gas and quantum effects can be neglected.

Total pressure $p = p_i + p_e + p_r = p_{gas} + p_r$

where

p_i is the pressure of the ions

p_e is the pressure of the electron

p_r is the radiation pressure **gas pressure.** The equation of state for the state for the ideal gas is :

$p_{gas} = nkT$

Where n is the number of particles per unit volume;$n = n_i + n_e$, wheren_i andn_e are the number of densities of ions and electrons. In terms of the mass density ρ:

$$p_{gas} = \frac{\rho}{\mu m_H} KT$$

Where m_H is the mass of hydrogen and μ is the average mass of particle in units of m_H.

The ideal gas constant is :

$$R = \frac{k}{m_H} \Rightarrow p_{gas} = \frac{R}{\mu} \rho T$$

Radiation pressure:For black body radiation

$p_{gas} = \frac{1}{3} aT^4$

where a is radiation constant:

$$a = \frac{8\Pi^5 k^4}{15c^3 h^3} = \frac{4\sigma}{c}$$

Gas pressure is most important in **low-mass stars.**

Radiation pressure is most important in **high mass stars**.

2.10 Time scale of stellar evolution

There are three important time scales in the life of star.

2.10.1 The Nuclear(evolutionary)Time Scale

The time in which a star radiates away all the energy that can be released by nuclear reactions. An estimate of this time can be obtained if one calculates the time in which all available hydrogen is turned into helium. On the basis of theoretical considerations and evolutionary computations it is known that only just over 10 percent of the total mass of hydrogen in the star can be consumed before other, more rapid evolutionary mechanisms set in. Since 0.7 percent of the rest mass is turned into energy in hydrogen burning, the nuclear time scale will be.

$$\tau_n \sim \frac{K_n M C^2}{L}$$

WhereK_n is just the fraction of the rest mass available to a particular nuclear process, M is rest mass, L is stellar luminosity and c is speed of light.

$$\Rightarrow \tau_n = \frac{E_{nuclear}}{L}$$

$$\tau_n \approx \frac{0.007 \times 0.01 M C^2}{L}$$

For the Sun one obtains the nuclear time scale 10^{10}years, and thus

$$\tau_n \approx \frac{\frac{M}{M_{sun}}}{\frac{L}{L_{sun}}} \times 10^{10} a$$

2.10.2 Dynamical time scale

Measure of the time scale on which a star would expand or contract if the balance between pressure gradient and gravity was suddenly disrupted (some as free -fall time scale)

$$\tau_{dyn} = \frac{charactersticradius}{charactersticvelocity} = \frac{R}{\nu_{esc}}$$

Escape velocity from the surface of the star:

$$\nu_{esc} = \sqrt{\frac{2GM}{R}}$$

$$\tau_{dyn} = \sqrt{\frac{R^3}{2GM}} \tag{2.10.1}$$

In terms of mean density:

$$\tau_{dyn} = \frac{1}{\sqrt{G\bar{\rho}}} \tag{2.10.2}$$

Where $\bar{\rho}$ is the mean density of the star (molecular cloud).

2.10.3 Kelvin-Helmholtz Time scale

Kelvin-Helmholtz Time scale is the time in which a star would radiate away all its thermal energy if the nuclear energy production where suddenly turned off.

$$\tau_{kH} = \frac{U}{L}$$

Virial theorem: the thermal energy U is roughly equal to the gravitational potential energy.

$$\tau_{KH} = \frac{GM^2}{RL}$$

Important timescale: determines how quickly a star contracts before nuclear fusion starts-i.e. sets roughly the pre-main-sequence lifetime. Most stars, most of the time in hydrostatic and thermal equilibrium, with slow changes in structure and composition occurring on the (long) time scale τ_{nuc}as fusion occurs.

Dynamical time scale: timescale of collapsing star, supernova

Thermal/kelvin-Helmholtz Timescale of star before nuclear fusion starts, pre main-sequence lifetime.

Chapter 3

The rate of star formation in interacting molecular clouds with stellar activities

3.1 Star formation in molecular clouds and accretion rate

Once a molecular cloud is being fragmented and starts to form a progenitor star (the "baby-star"),2, it growth up by accreting matter from its environment. The rate at which it accretes is given by [2]

$$\frac{dM}{M^{3/2}} = \gamma dt \tag{3.1.1}$$

where γ is a proportionality constant related to radial oscillation of the spherically collapsing cloud.

3.2 A three-component model for star formation

Following the work of [4] here we assume a three-component star forming interacting systems.

The model contains three active components: (1) cool atomic clouds, (2) cool molecular clouds, and (3)active young stars. Each of these components may interact with the other components or with the external activating systems like galaxies, etc. To this end, the model is an open system connected with two mass reservoirs outside the system.

i) Cool atomic cloud: Its main component is neutral hydrogen, the most abundant chemical element in the Galaxy .The density varies over a large range, but direct star formation in these clouds seems not to occur. The cooling capacity of these clouds is not large enough to allow a sufficient condensation. This component is connected to an unlimited reservoir of new atomic gas outside the system.

ii) Cool molecular cloud: mainly consist of molecular hydrogen HII. The densities mast enormously and are generally much higher than in neutral clouds The temperature in such cloud decreases as its density increases as a consequence of large cooling capacity of the CO molecules. They have smaller dimensions than neutral clouds.

iii) Young, active stars: mostly accompanied by hot ionized HII gas.These stars strongly affect the surrounding gas clouds and are responsible for shock waves in these clouds. In this way, new condensation regions may be formed in the molecular clouds of the system.The presence of young stars therefore has a positive effect on the stellar birth rate.The capacity of influencing the other components ends when these young stars evolve to neutron stars. Although these remnants are still physically present in the system, their masses have stopped playing an active role in the star formation process. We therefore say that this mass has left the active star formation system. The second reservoir is hence a waste reservoir containing the stellar remnants.

3.3 Interaction of the system

The three mass components S for the total mass of active stars, M for the total mass of molecular clouds, and A for the total mass of atomic clouds. It is assumed that the total mass of the system remains constant; thus, we assume that the amount of mass lost by stellar evolution is exactly replaced by fresh atomic clouds entering the star formation region from the external sources. Now calling the total mass of the system T, we write

$$T = A + M + S \tag{3.3.1}$$

There are three kinds of interaction for the atomic cloud component:

a) First, there is a constant replenishment by new atomic clouds in an amount equal to the amount of mass leaving the active system by stellar evolution.The amount of new gas may therefore be considered as proportional to the amount of stellar mass S.We will call the proportional constant of this process K_1.

b) Secondly, the atomic component is increased as young, active stars lose mass by stellar wind. This process is also proportional to the number of stars and therefore to the amount of stellar mass. The proportional constant for this process is K_2.

c) The third interaction is the transformation of atomic into molecular clouds. This process is clearly proportional to the amount of atomic gas A, but since the transformation becomes more and more effective with the cooling capacity of the cloud, and since this capacity increases with the square of the density of molecular content, we assume that the transformation of atomic into molecular gas is proportional to the square of the molecular mass. This third processes loss of atomic gas and therefore is written with a minus sign in the differential equation and a proportional constant K_3

$$\frac{dA}{dt} = K_1S + K_2S - K_3AM^2 \tag{3.3.2}$$

Or

$$\frac{dA}{dt} = K_{12}S - K_3AM^2 \tag{3.3.3}$$

where K_{12} is the coupled constants K_1 & K_2. In fact a further analysis shows that this is a typical oscillation frequency of the couple.

The rate of the star formation may be considered as being proportional to the n^{th} power of the density of molecular cloud.Values of n can be selected between 0.5 and 3.5 or between 1 and 2. It is assumed that the presence of other young stars is a necessary condition for star formation since they will perturb the molecular cloud and provoke condensations. In this way we may also state that the star formation rate is proportional to the number of active stars already present. Let us call the proportional constant K_4. This process increases the mass of stellar component. Two other process decrease it: stellar evolution, for which we may use K_1, and mass loss by stellar wind, for which we again use K_2 Both processes are proportional to the amount of stellar mass. Thus, the equation describing the variation of stellar mass in the system is

$$\frac{dS}{dt} = K_4 S M^n - K_1 S - K_2 S \tag{3.3.4}$$

Or

$$\frac{dS}{dt} = K_4 S M^n - K_{12} S \tag{3.3.5}$$

Transformation of atomic into molecular gas, which increases for the variable M, and stellar formation, which decreases the amount of molecular mass. The equation for M will be

$$\frac{dM}{dt} = K_3 A M^2 - K_4 S M^n \tag{3.3.6}$$

Note that, basically eqn. 3.1.1 & eqn. 3.3.6 should represent the same physics. Consequently, it helps us to impose an additional condition in integrating the coupled system of differential equations.

The constant parameters $K_1,\ K_2,\ K_3$ and K_4 can be further transformed by introducing the dimensionless parameter constants k_1 & K_2 given by

$$k_1 = \frac{K_3 T^2}{K_{12}} \tag{3.3.7}$$

$$k_2 = \frac{K_4 T^n}{K_{12}} \tag{3.3.8}$$

The solution of $M(t), S(t),$ and $A(t)$

$$M(t) = \frac{M_0}{(1 + \frac{1}{2}\sqrt{M_0}\gamma t)^2} \tag{3.3.9}$$

$$S(t) = \frac{S_0 Exp\left(\frac{2k_2 K_{12}}{(2n-1)\gamma} T^{-n} M_0^{(2n-1)/2}\right)}{Exp\left(\frac{2k_2 K_{12}}{(2n-1)\gamma} \frac{T^{-n} M_0^{(2n-1)/2}}{(1+\frac{1}{2}\sqrt{M_0}\gamma t)^{2n-1}} + K_{12} t\right)} \tag{3.3.10}$$

$$A(t) = \frac{k_1}{k_2} M^{n-2} + \frac{\gamma M^{-\frac{1}{2}}}{k_2 K_{12}} \tag{3.3.11}$$

On the other hand $A(t)$ can also be determined from eqn. 3.3.1. Then, between this and eqn. 3.3.11 we can express the constant parameter γ in terms of the characteristic oscillation frequency K_{12} worked out at $t = 0$.

$$\gamma = \frac{K_{12}}{\sqrt{M_0}}[k_1 a_0 - k_2 s_0 m_0^{n-1}] \tag{3.3.12}$$

. Following [14], we introduce the dimensionless mass ratio parameters $m(t), s(t),$ and $a(t)$ corresponding respectively to $M(t), S(t),$ and $A(t)$ as

$$a = \frac{A}{T}, \quad m = \frac{M}{T}, \quad s = \frac{S}{T} \tag{3.3.13}$$

where then,

$$a(t) + m(t) + s(t) = 1 \tag{3.3.14}$$

Now γ in terms of the dimensionless mass ratios being evaluated at the initial condition, the total mass, the characteristic frequency and the two dimensionless k-constants is given as

$$\gamma = \frac{\sigma K_{12}}{\sqrt{m_0 T}} \tag{3.3.15}$$

where

$$\sigma = k_1 a_0 - k_2 s_0 m_0^{n-1} \tag{3.3.16}$$

Now, the complete solutions of the ratio masses of the system evolving in time are given by

$$m(t) = \frac{m_0}{(1 + \frac{1}{2}\sigma K_{12} t)^2} \tag{3.3.17}$$

$$s(t) = s_0 \frac{Exp\left[\frac{2k_2 m_0^n}{(2n-1)\sigma}\right]}{Exp\left[\frac{2k_2 m_0^n}{(2n-1)\sigma}\left(1 + \frac{1}{2}\sigma K_{12} t\right)^{1-2n} + K_{12} t\right]} \tag{3.3.18}$$

$$a(t) = 1 - [m + s](t) \tag{3.3.19}$$

Chapter 4

Result and discussion

Following the 3-component interacting system in star forming active region, such as dense spiral arms we have derive a complete analytical solutions of the mass transfers of the system and as well as the rate at which the masses being transferred chap3. 3.

Then we have generated numerical data computationally using MATHEMATICA for the evolution of the masses and the rate at which the masses being flow out or into each component in time. The plots of the results are as shown in Fig.4.1 and Fig. 4.2.

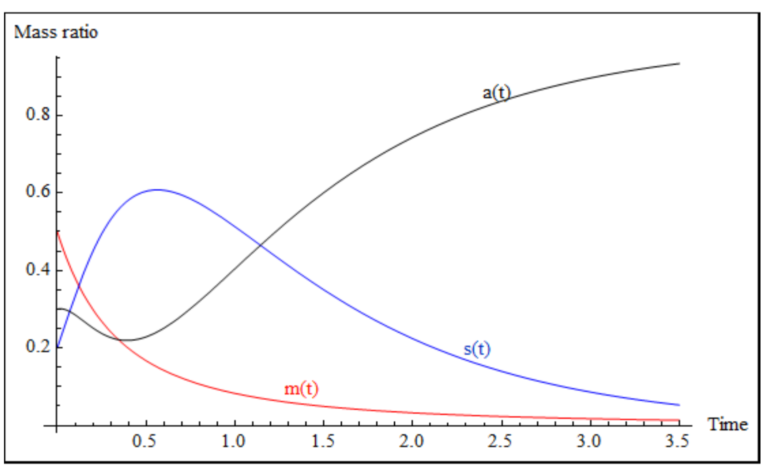

Figure 4.1: The evolution of the three-component mass system: $m(t)$-molecular mass, red;$s(t)$-stellar mass - blue; $a(t)$ - atomic mass - black

In plotting the graphs we have used the k-parameters as: $k_1 = 20$ & $k_2 = 25$; the ratio masses as $m_0 = 0.7$ & $s_0 = 0.2$ & $a_0 = 0.3$; $n = 1.7$ and the characteristic couple oscillation frequency is between 0 & 1 cycles per the order of the time of evolution during the formation. where t is in unit of second.

As we observe from the plots at the beginning both the stellar mass and the atomic mass decrease. While the stellar mass increases. But after a sufficient time the stellar mass stops for a moment and begins to decrease while the molecular gas continues decrease. On the other hand, the atomic gas turns to increase at the expense of the decrease in the other two. This, is true as one expects from m the standard theory of formation.

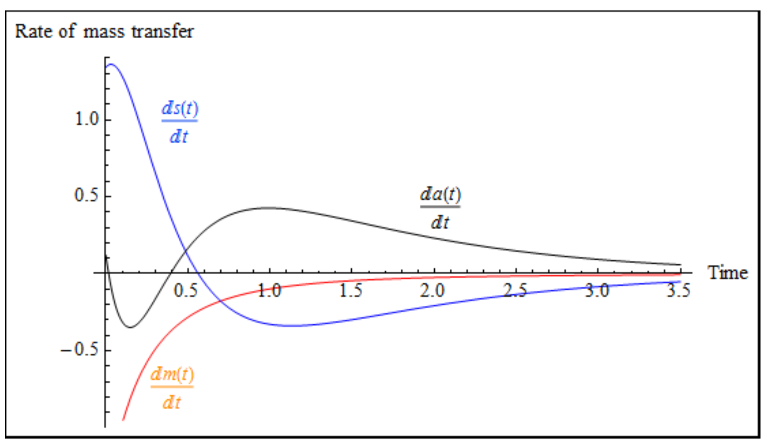

Figure 4.2: The evolution of mass transfer rates of the three-component system: $\frac{dm(t)}{dt}$-molecular mass rate, red; $\frac{ds(t)}{dt}$-stellar mass rate - blue; $\frac{da(t)}{dt}$ - atomic mass rate - black

The rate of mass transfers of the three components of the systems are all in different characteristics. The molecular gas ever continues to increase asymptotically until it exacts. The stellar rate of transfer at the beginning radically decreases, stops for a while and gradually increases till it comes to stops. On the other hand, the atomic gas behaves in three ways. First for a short period of time it decreases radically, then increases for a longer period of time relatively and then gradually continues to decrease until it comes to halt.

Chapter 5

summary and conclusion

Star formation occur as a result of the action of gravity on a wide range of scales. On galactic scale the tendency of interstellar matter to condense under gravity in to star forming clouds is countered by galactic tidal forces, and star formation can occur only where the gas become dense enough for its self gravity to over come these tidal force,for example in spiral arms. On the intermediate scales of star forming giant molecular clouds (GMCs),turbulence and magnetic fields may be the most important effects counteracting gravity, and star formation may involve the dissipation of turbulence and magnetic fields. Oem tn top of these, there is external agents that will trigger the system to begin the formation such as shock waves external to the system.

Due to the complex system and complicated theories needed to work out in stellar formation and evolution, we here we have worked out the three-component interaction system of formation. As we have worked out in chap3 3 and discussed the results in chap4 **??** we have successfully derived analytical solutions to the dynamical evolution of the masses of the interacting system including their rate of evolution. However, we believe that this work needs further development and inputs to give more meanings to the parameters therein introduced.

Bibliography

[1] Ascensso, *Astron.astrophys*, et.al 2012.

[2] Thesgaye Eebba& Tolu Biressa, *Magnetic field evolution of pre-main sequence stars*, Master's thesis, Jimma University College of Natural Sciences Graduate Studies, Phys, Dept., October 2014.

[3] J.H Bodenheimer, *principle of star formation*, 2011.

[4] Paolo Farinella Bruno Bertotti and David Vokrouhlicky, *Dynamics and evolution, space physics, and spacetime structure*, Kluwer Academic publisher **293** (2003).

[5] II George W.Collins, *The fundamentals of stellar astrophysics*, 2003.

[6] II George W.collins, *The fundamentals of stellar astrophysics*, 2003.

[7] Tadele Guta, *Stellar evolution: The case of accreting main sequence stars, masters thesis*, Master's thesis, october 2016.

[8] Charles J.Lada, *The star formation rate of molecular clouds*, et.al 2003.

[9] H. Karttune, *Fundamental astronomy*, 2007.

[10] Sun Kwok, *Physics and chemistry of the interstellar medium*, 2009.

[11] Richard Larson, *Numerical calculation of the dynamics of collapsing protostar*, 1969.

[12] Richard B Larson, *The evolution of molecular clouds*, Sol Phys (1966).

[13] Richard B. Larson, *The physics of star formation*, 2003.

[14] R. Ghoneim M.A sharaf, *A mathematical model of star formation in galaxy*, 2012.

[15] Paodan.p and Nordland .A, *The star formation rate of molecular clouds*, 2009.

[16] Prilink, *Introduction to the theory of stellar structure and evolution*, Cambridge university Dina, 2000.