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A Refined Analysis of Squeezing Amplification in the One-Mode Subharmonic Generation

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I. Introduction

One-mode subharmonic generation is one of the most interesting and widely studied quantum optical processes. In this process a pump photon of frequency 2ω is down converted into a pair of signal photons each of frequency ω. A theoretical analysis of the statistical and squeezing properties of the signal mode produced by one-mode subharmonic generation has been made by a number of authors [1-7]. Among other things, it has been predicted that the signal mode has a maximum squeezing of 50% below the vacuum-state level [4-7].

It is to be recalled that the Hamiltonian describing the process of subharmonic generation consists of the operators $\hat{a}^2$ and $\hat{a}^{\dagger 2}$. And the quantum analysis of the signal mode is usually carried out employing the operators $\hat{a}$ and $\hat{a}^\dagger$ with the commutation relation $[\hat{a}.\hat{a}^\dagger] = 1$. However, such analysis leads, among others, to one-half of the mean photon number of the signal mode [1-7]. This is surely the mean number of one set of the signal photons, consisting of one photon from each pair [6-7]. Since the other set of the signal photons is not included in such analysis, we seek to resolve this problem by applying the commutation relation $[\hat{a}.\hat{a}^\dagger] = 2$.

We therefore seek to analyze the statistical and squeezing properties of the signal mode applying the solution of c-number Langevin equations. We use this solution to calculate, in particular, the mean photon number and quadrature squeezing of the signal mode.

II. The Q function

We first obtain c-number Langevin equations, associated with the normal ordering, for the signal mode produced by one-mode subharmonic generation. The process of one-mode subharmonic generation is described by the Hamiltonian

$$\hat{H} = i\mu(\hat{b}^\dagger - \hat{b}) + \frac{i\lambda}{2}(\hat{b}^\dagger\hat{a}^2 - \hat{b}\hat{a}^{\dagger 2}), \tag{1}$$

where $\hat{a}(\hat{b})$ is the annihilation operator for the signal mode (pump mode), λ is the coupling constant, and μ is proportional to the amplitude of the coherent light driving the pump mode. Applying (1) and taking into account the interaction of the pump and signal modes with two independent vacuum reservoirs, the master equation for the cavity modes can be written as

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$$\frac{d\hat{\rho}}{dt} = \mu(\hat{b}^\dagger\hat{\rho} - \hat{\rho}\hat{b}^\dagger + \hat{\rho}\hat{b} - \hat{b}\hat{\rho}) + \frac{\lambda}{2}(\hat{\rho}\hat{b}\hat{a}^{\dagger 2} - \hat{b}\hat{a}^{\dagger 2}\hat{\rho} + \hat{b}^\dagger\hat{a}^2\hat{\rho} - \hat{\rho}\hat{b}^\dagger\hat{a}^2)$$
$$+\frac{\kappa}{2}\Big(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}\Big) + \frac{\kappa}{2}\Big(2\hat{b}\hat{\rho}\hat{b}^\dagger - \hat{b}^\dagger\hat{b}\hat{\rho} - \hat{\rho}\hat{b}^\dagger\hat{b}\Big), \tag{2}$$

in which κ is the cavity damping constant for the signal mode as well as the pump mode. Now employing the relations

$$\frac{d}{dt}\langle\hat{A}\rangle = Tr(\frac{d\hat{\rho}}{dt}\hat{A}) \tag{3}$$

and

$$[\hat{a}, \hat{a}^\dagger] = 2, \tag{4}$$

along with (2), one readily obtains

$$\frac{d}{dt}\langle\hat{a}(t)\rangle = -\kappa\langle\hat{a}(t)\rangle - 2\lambda\langle\hat{a}^\dagger(t)\hat{b}(t)\rangle, \tag{5}$$

$$\frac{d}{dt}\langle\hat{a}(t)\hat{a}(t)\rangle = -2\kappa\langle\hat{a}^2(t)\rangle - 4\lambda\langle\hat{a}^\dagger(t)\hat{a}(t)\hat{b}(t)\rangle - 2\lambda\langle\hat{b}(t)\rangle, \tag{6}$$

$$\frac{d}{dt}\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle = -2\kappa\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle - 2\lambda\Big(\langle\hat{a}^2(t)\hat{b}^\dagger(t)\rangle + \langle\hat{a}^{\dagger 2}(t)\hat{b}(t)\rangle\Big). \tag{7}$$

We note that the c-number equations corresponding to Eqs. (5), (6), and (7) are

$$\frac{d}{dt}\langle\alpha(t)\rangle = -\kappa\langle\alpha(t)\rangle - 2\lambda\langle\alpha^*(t)\beta(t)\rangle, \tag{8}$$

$$\frac{d}{dt}\langle\alpha(t)\alpha(t)\rangle = -2\kappa\langle\alpha^2(t)\rangle - 4\lambda\langle\alpha^*(t)\alpha(t)\beta(t)\rangle - 2\lambda\langle\beta(t)\rangle, \tag{9}$$

$$\frac{d}{dt}\langle\alpha^*(t)\alpha(t)\rangle = -2\kappa\langle\alpha^*(t)\alpha(t)\rangle - 2\lambda\Big(\langle\alpha^2(t)\beta^*(t)\rangle + \langle\alpha^{*2}(t)\beta(t)\rangle\Big). \tag{10}$$

On the basis of Eq. (5), one can write

$$\frac{d}{dt}\alpha(t) = -\kappa\alpha(t) - 2\lambda\alpha^*(t)\beta(t) + f_\alpha(t), \tag{11}$$

where $f_\alpha(t)$ is a noise force whose properties remain to be determined. We note that Eq. (8) and the expectation value of Eq. (11) will have identical forms if

$$\langle f_\alpha(t)\rangle = 0. \tag{12}$$

Moreover, it can be readily verified using (11) that

$$\frac{d}{dt}\langle\alpha(t)\alpha(t)\rangle = -2\kappa\langle\alpha^2(t)\rangle - 4\lambda\langle\alpha^*(t)\alpha(t)\beta(t)\rangle + 2\langle\alpha(t)f_\alpha(t)\rangle. \tag{13}$$

Comparison of Eqs. (9) and (13) shows that

$$\langle\alpha(t)f_\alpha(t)\rangle = -\lambda\langle\beta(t)\rangle. \tag{14}$$

A formal solution of Eq. (11) can be written as

$$\alpha(t) = \alpha(0)e^{-\kappa t} + \int^t e^{-\kappa(t-t')}[f_\alpha(t') - 2\lambda\beta(t')\alpha^*(t')]dt'. \tag{15}$$

We then notice that

$$\langle\alpha(t)f(t)\rangle = \langle\alpha(0)f_\alpha(t)\rangle e^{-\kappa t} + \int_0^t e^{-\kappa(t-t')}[\langle f_\alpha(t)f_\alpha(t')\rangle - 2\lambda\langle\beta(t')\alpha^*(t')f_\alpha(t)\rangle]dt'. \tag{16}$$

On account of the assertion that a noise force at time t should not affect a cavity mode variable at earlier time, we have

$$\langle\alpha(t)f_\alpha(t)\rangle = \int_0^t e^{-\kappa(t-t')}\langle f_\alpha(t)f_\alpha(t')\rangle dt', \tag{17}$$

so that in view of (14), there follows

$$\int_0^t e^{-\kappa(t-t')}\langle f(t)f(t')\rangle dt' = -\lambda\langle\beta(t)\rangle. \tag{18}$$

Now on the basis of the relation

$$\int_0^t e^{-a(t-t')}\langle f(t)g(t')\rangle dt' = D, \tag{19}$$

we assert that

$$\langle f(t)g(t')\rangle = 2D\delta(t-t'), \tag{20}$$

where a is a constant and D is a constant or some function of time t. We then see that

$$\langle f_\alpha(t)f_\alpha(t')\rangle = -2\lambda\langle\beta(t)\rangle\delta(t-t'). \tag{21}$$

Furthermore, it can be verified applying (11) and its complex conjugate that

$$\frac{d}{dt}\langle\alpha^*(t)\alpha(t)\rangle = -2\kappa\langle\alpha^*(t)\alpha(t)\rangle - 2\lambda\langle\beta(t)\alpha^{*2}(t)\rangle - 2\lambda\langle\beta^*(t)\alpha^2(t)\rangle + \langle\alpha^*(t)f_\alpha(t)\rangle + \langle\alpha(t)f^*_\alpha(t)\rangle. \tag{22}$$

On comparing this with (10), we observe that

$$\langle\alpha^*(t)f_\alpha(t)\rangle + \langle\alpha(t)f^*_\alpha(t)\rangle = 0. \tag{23}$$

In addition, using (15) and its complex conjugate, we easily get

$$\langle\alpha(t)f^*_\alpha(t)\rangle = \int_0^t e^{-\kappa(t-t')}\langle f^*_\alpha(t)f_\alpha(t')\rangle dt' \tag{24}$$

and

$$\langle\alpha^*(t)f_\alpha(t)\rangle = \int_0^t e^{-\kappa(t-t')}\langle f_\alpha(t)f^*_\alpha(t')\rangle dt'. \tag{25}$$

Now taking into account (23), (24), (25), and assuming that

$$\langle f^*_\alpha(t)f_\alpha(t')\rangle = \langle f_\alpha(t)f^*_\alpha(t')\rangle, \tag{26}$$

we arrive at

$$\int_0^t e^{-\kappa(t-t')}\langle f_\alpha^*(t) f_\alpha(t')\rangle dt' \;=\; \int_0^t e^{-\kappa(t-t')}\langle f_\alpha(t) f_\alpha^*(t')\rangle dt' = 0. \tag{27}$$

Therefore, on account of (19) and (20), we see that

$$\langle f_\alpha^*(t) f_\alpha(t')\rangle = \langle f_\alpha(t) f_\alpha^*(t')\rangle \;=\; 0. \tag{28}$$

It is worth mentioning that (12), (21), and (28) describe the correlation properties of the noise force $f_\alpha(t)$ which is associated with the normal ordering.

On the other hand, we wish to determine the correlation properties of the noise force associated with $\beta(t)$. To this end, employing the relation described by (3) and the commutation relation

$$[\hat{b}, \hat{b}^\dagger] = 1, \tag{29}$$

along with (2), one readily obtains

$$\frac{d}{dt}\langle \hat{b}(t)\rangle = -\frac{\kappa}{2}\langle \hat{b}(t)\rangle + \frac{\lambda}{2}\langle \hat{a}^2(t)\rangle + \mu, \tag{30}$$

$$\frac{d}{dt}\langle \hat{b}(t)\hat{b}(t)\rangle = -\kappa\langle \hat{b}^2(t)\rangle + 2\mu\langle \hat{b}(t)\rangle + \lambda\langle \hat{a}^2(t)\hat{b}(t)\rangle, \tag{31}$$

$$\begin{aligned}\frac{d}{dt}\langle \hat{b}^\dagger(t)\hat{b}(t)\rangle = {} & -\kappa\langle \hat{b}^\dagger(t)\hat{b}(t)\rangle + \mu\Big(\langle \hat{b}^\dagger(t)\rangle + \langle \hat{b}(t)\rangle\Big) \\ & + \frac{\lambda}{2}\Big(\langle \hat{a}^2(t)\hat{b}^\dagger(t)\rangle + \langle \hat{a}^{\dagger 2}(t)\hat{b}(t)\rangle\Big).\end{aligned} \tag{32}$$

We see that the c-number equations corresponding to Eqs. (30), (31), and (32) are

$$\frac{d}{dt}\langle \beta(t)\rangle = -\frac{\kappa}{2}\langle \beta(t)\rangle + \frac{\lambda}{2}\langle \alpha^2(t)\rangle + \mu. \tag{33}$$

$$\frac{d}{dt}\langle \beta(t)\beta(t)\rangle = -\kappa\langle \beta^2(t)\rangle + 2\mu\langle \beta(t)\rangle + \lambda\langle \alpha^2(t)\beta(t)\rangle, \tag{34}$$

$$\begin{aligned}\frac{d}{dt}\langle \beta^*(t)\beta(t)\rangle = {} & -\kappa\langle \beta^*(t)\beta(t)\rangle + \mu\Big(\langle \beta^*(t)\rangle + \langle \beta(t)\rangle\Big) \\ & + \frac{\lambda}{2}\Big(\langle \alpha^2(t)\beta^*(t)\rangle + \langle \alpha^{*2}(t)\beta(t)\rangle\Big).\end{aligned} \tag{35}$$

On the basis of Eq. (33), we can write

$$\frac{d}{dt}\beta(t) = -\frac{\kappa}{2}\beta(t) + \frac{\lambda}{2}\alpha^2(t) + \mu + f_\beta(t), \tag{36}$$

where $f_\beta(t)$ is a noise force the properties of which remain to be determined. We note that Eq. (33) and the expectation value of Eq. (36) will have identical forms if

$$\langle f_\beta(t)\rangle = 0. \tag{37}$$

Moreover, it can be readily verified using (36) that

$$\frac{d}{dt}\langle\beta(t)\beta(t)\rangle = -\kappa\langle\beta^2(t)\rangle + 2\mu\langle\beta(t)\rangle + \lambda\langle\alpha^2(t)\beta(t)\rangle + 2\langle\beta(t)f_\beta(t)\rangle. \tag{38}$$

Comparison of Eqs. (34) and (38) shows that

$$\langle\beta(t)f_\beta(t)\rangle = 0. \tag{39}$$

A formal solution of (36) can be written as

$$\beta(t) = \beta(0)e^{-\kappa t/2} + \int_0^t e^{-\kappa(t-t')/2}\Big(f_\beta(t') + \frac{\lambda}{2}\alpha^2(t') + \mu\Big)dt'. \tag{40}$$

We note that

$$\langle\beta(t)f_\beta(t)\rangle = \int_0^t e^{-\kappa(t-t')/2}\langle f_\beta(t')f_\beta(t)\rangle dt', \tag{41}$$

so that in view of (39) and (41) together with (19) and 20), there follows

$$\langle f_\beta(t')f_\beta(t)\rangle = 0. \tag{42}$$

Furthermore, employing Eqs. (36), we readily obtain

$$\begin{aligned}\frac{d}{dt}\langle\beta^*(t)\beta(t)\rangle &= -\kappa\langle\beta^*(t)\beta(t)\rangle + \frac{\lambda}{2}\Big[\langle\alpha^{*2}(t)\beta(t)\rangle + \langle\alpha^2(t)\beta^*(t)\rangle\Big]\\ &\quad +\mu\Big(\langle\beta(t)\rangle + \langle\beta^*(t)\rangle\Big) + \langle\beta^*(t)f_\beta(t)\rangle + \langle\beta(t)f^*_\beta(t)\rangle.\end{aligned} \tag{43}$$

On comparing this with Eq. (35), we have

$$\langle\beta^*(t)f_\beta(t)\rangle + \langle\beta(t)f^*_\beta(t)\rangle = 0. \tag{44}$$

In addition, with the aid of (40) and its complex conjugate, we easily get

$$\langle\beta(t)f^*_\beta(t)\rangle = \int_0^t e^{-\frac{\kappa(t-t')}{2}}\langle f^*_\beta(t)f_\beta(t')\rangle dt' \tag{45}$$

and

$$\langle\beta^*(t)f_\beta(t)\rangle = \int_0^t e^{-\frac{\kappa(t-t')}{2}}\langle f_\beta(t)f^*_\beta(t')\rangle dt'. \tag{46}$$

On account of Eq. (44), we have

$$\int_0^t e^{-\frac{\kappa(t-t')}{2}}\langle f^*_\beta(t)f_\beta(t')\rangle dt' + \int_0^t e^{-\frac{\kappa(t-t')}{2}}\langle f_\beta(t)f^*_\beta(t')\rangle dt' = 0. \tag{47}$$

And assuming that

$$\langle f^*_\beta(t)f_\beta(t')\rangle = \langle f_\beta(t)f^*_\beta(t')\rangle, \tag{48}$$

we arrive at

$$\int_0^t e^{-\frac{\kappa(t-t')}{2}}\langle f^*_\beta(t)f_\beta(t')\rangle dt' = \int_0^t e^{-\frac{\kappa(t-t')}{2}}\langle f_\beta(t)f^*_\beta(t')\rangle dt' = 0. \tag{49}$$

Thus with the aid of (19) and (20), we see that

$$\langle f^*_\beta(t')f_\beta(t)\rangle = \langle f_\beta(t')f^*_\beta(t)\rangle = 0. \tag{50}$$

Now on account of Eqs. (37), (42), and (50), we can drop the noise force in Eq. (36) and write

$$\frac{d}{dt}\beta(t) = -\frac{\kappa}{2}\beta(t) + \frac{\lambda}{2}\alpha^2(t) + \mu. \tag{51}$$

Applying the large time approximation scheme to this equation, we have

$$\beta(t) = \frac{2\mu}{\kappa} + \frac{\lambda}{\kappa}\alpha^2(t). \tag{52}$$

Then on substituting Eq. (52) into (11) and (21) and dropping terms second order in λ, we obtain

$$\frac{d}{dt}\alpha(t) = -\kappa\alpha(t) - 2\varepsilon\alpha^*(t) + f_\alpha(t) \tag{53}$$

and

$$\langle f_\alpha(t)f_\alpha(t')\rangle = \langle f_\alpha(t')f_\alpha(t)\rangle = -2\varepsilon\delta(t-t'), \tag{54}$$

where

$$\varepsilon = \frac{2\mu\lambda}{\kappa}. \tag{55}$$

In order to obtain the solution of Eq. (53), we introduce a new variable defined by

$$\alpha_\pm(t) = \frac{1}{2}\Big[\alpha^*(t) \pm \alpha(t)\Big]. \tag{56}$$

It can then be shown using (53) and its complex conjugate that

$$\frac{d\alpha_\pm(t)}{dt} = -\frac{1}{2}\xi_\pm\alpha_\pm(t) + \frac{1}{2}(f^*(t) \pm f(t)), \tag{57}$$

in which

$$\xi_\pm = \kappa \pm 2\varepsilon. \tag{58}$$

According to Eq. (57) together with (58), the equation of evolution of α_- does not have a well behaved solution for $\kappa < 2\varepsilon$. We then identify $\kappa = 2\varepsilon$ as the threshold condition. For $2\varepsilon < \kappa$, the solution of Eq. (57) can be written as

$$\alpha_\pm(t) = \alpha_\pm(0)e^{-\frac{1}{2}\xi_\pm t} + \frac{1}{2}\int_0^t e^{-\frac{1}{2}\xi_\pm(t-t')}[f^*(t') \pm f(t')]dt'. \tag{59}$$

Now with the aid of (56) and (59), one readily gets

$$\alpha(t) = F_+(t)\alpha(0) + F_-(t)\alpha^*(0) + E_+(t) - E_-(t), \tag{60}$$

in which

$$F_\pm(t) = \frac{1}{2}\Big[e^{-\frac{1}{2}\xi_+ t} \pm e^{-\frac{1}{2}\xi_- t}\Big] \tag{61}$$

and

$$E_{\pm}(t) = \frac{1}{2}\int_0^t e^{-\frac{1}{2}\xi_{\pm}(t-t')}\Big[f^*(t') \pm f(t')\Big]dt'. \tag{62}$$

We now proceed to calculate the Q function for the signal mode assumed to be initially in a coherent state. The Q function is expressible in terms of the antinormally ordered characteristic function as

$$Q(\alpha,\alpha^*,t) = \frac{1}{\pi^2}\int d^2\eta\phi_a(\eta^*,\eta,t)e^{\eta^*\alpha-\eta\alpha^*}. \tag{63}$$

Employing the identity

$$e^{\hat{A}}e^{\hat{B}} = e^{\hat{B}}e^{\hat{A}}e^{[\hat{A},\hat{B}]}, \tag{64}$$

the characteristic function $\phi_a(\eta^*,\eta,t)$ takes the form

$$\begin{aligned}\phi_a(\eta,\eta^*,t) &= exp\Big[-a(t)\eta^*\eta - \frac{b(t)}{2}(\eta^2+\eta^{*2})\\ &+(F_+(t)\eta - F_-(t)\eta^*)\alpha_0^* - (F_+(t)\eta^* - F_-(t)\eta)\alpha_0\Big],\end{aligned} \tag{65}$$

in which

$$a(t) = 1 - \frac{\varepsilon}{(\kappa+2\varepsilon)}[1-e^{-(\kappa+2\varepsilon)t}] + \frac{\varepsilon}{(\kappa-2\varepsilon)}[1-e^{-(\kappa-2\varepsilon)t}] \tag{66}$$

and

$$b(t) = \frac{\varepsilon}{(\kappa+2\varepsilon)}[1-e^{-(\kappa+2\varepsilon)t}] + \frac{\varepsilon}{(\kappa-2\varepsilon)}[1-e^{-(\kappa-2\varepsilon)t}]. \tag{67}$$

Finally, substituting (65) into Eq. (63) and then carrying out the integration, the Q function for the signal mode is found to be

$$Q(\alpha,\alpha^*,t) = \frac{q(t)}{\pi}exp\Big[-u\alpha^*\alpha - \frac{v}{2}(\alpha^2+\alpha^{*2}) + p(t)\alpha + p^*(t)\alpha^*\Big], \tag{68}$$

in which

$$\begin{aligned}q(t) &= [u^2-v^2]^{\frac{1}{2}}exp\Big[-\Big(u(F_+^2(t)+F_-^2(t)) + 2vF_+(t)F_-(t)\Big)\alpha_0^*\alpha_0\\ &-uF_+(t)F_-(t)\Big(\alpha_0^2+\alpha_0^{*2}\Big) - \frac{v}{2}(F_+^2(t)+F_-^2(t))\Big(\alpha_0^2+\alpha_0^{*2}\Big)\Big],\end{aligned} \tag{69}$$

$$p(t) = u\Big(F_+(t)\alpha_0^* + F_-(t)\alpha_0\Big) + v\Big(F_+(t)\alpha_0 + F_-(t)\alpha_0^*\Big), \tag{70}$$

with

$$u(t) = \frac{a(t)}{a^2(t)-b^2(t)} \tag{71}$$

and

$$v(t) = \frac{b(t)}{a^2(t)-b^2(t)}. \tag{72}$$

We find the Q function for the signal mode initially in a vacuum state upon setting $\alpha_0^* = \alpha_0 = 0$ to be

$$Q(\alpha,\alpha^*,t) \;=\; \frac{[u^2-v^2]^{\frac{1}{2}}}{\pi} exp\Big[-u\alpha^*\alpha-\frac{v}{2}(\alpha^2+\alpha^{*2})\Big]. \tag{73}$$

One can easily check that the Q functions described by Eqs. (68) and (73) are normalized to unity.

III. The Density Operator

Here we seek to determine the density operator for the signal mode. Suppose $\hat{\rho}'(\hat{a}^\dagger,\hat{a})$ is density operator for a certain light beam. Then upon expanding this density operator in normal order

$$\hat{\rho}'(t) \;=\; \sum_{kl} C_{kl}\hat{a}^{\dagger k}\hat{a}^l \tag{74}$$

and employing the completeness relation for coherent state

$$\int \frac{d^2\alpha}{\pi}|\alpha\rangle\langle\alpha| = \hat{I}, \tag{75}$$

one easily finds

$$\hat{\rho}'(t) \;=\; \frac{1}{\pi}\int d^2\alpha \sum_{kl} C_{kl}\alpha^{*k}|\alpha\rangle\langle\alpha|\hat{a}^l. \tag{76}$$

Thus in view of the identity

$$|\alpha\rangle\langle\alpha|\hat{a}^l = \left(\alpha+\frac{\partial}{\partial\alpha^*}\right)^l|\alpha\rangle\langle\alpha|, \tag{77}$$

there follows

$$\hat{\rho}'(t) \;=\; \int d^2\alpha Q\left(\alpha^*,\alpha+\frac{\partial}{\partial\alpha^*},t\right)|\alpha\rangle\langle\alpha|, \tag{78}$$

where

$$Q\left(\alpha^*,\alpha+\frac{\partial}{\partial\alpha^*},t\right) \;=\; \frac{1}{\pi}\sum_{kl} C_{kl}\alpha^{*k}\left(\alpha+\frac{\partial}{\partial\alpha^*}\right)^l. \tag{79}$$

IV. The Global Mean Photon Number

Here we wish to calculate the mean photon number for the one-mode subharmonic light. The mean photon number, for the cavity light, is defined by

$$\bar{n} = \langle\hat{a}^\dagger(t)\hat{a}(t)\rangle. \tag{80}$$

Employing Eq. (73), the mean photon number of the signal mode is given by

$$\bar{n} \;=\; -\frac{\varepsilon}{(\kappa+2\varepsilon)}[1-e^{-(\kappa+2\varepsilon)t}] + \frac{\varepsilon}{(\kappa-2\varepsilon)}[1-e^{-(\kappa-2\varepsilon)t}]. \tag{81}$$

Thus at steady state we see that

$$\bar{n}_{ss} = \frac{4\varepsilon^2}{\kappa^2 - 4\varepsilon^2}. \tag{82}$$

We observe that the mean photon number given by (82) is twice that of a twin signal light beam.

V. The Local Mean Photon Number

We calculate the local mean photon number in a given frequency interval employing the power spectrum for the signal mode. The power spectrum for a cavity light with central frequency ω_0 is expressible as

$$P(\omega) = \frac{1}{\pi} Re \int_0^\infty \langle \hat{a}^\dagger(t)\hat{a}(t+\tau)\rangle_{ss} e^{i(\omega-\omega_0)\tau} d\tau. \tag{83}$$

The two-time correlation function for the cavity light can be written as

$$\langle \hat{a}^\dagger(t)\hat{a}(t+\tau)\rangle = Tr[\hat{\rho}'(t)\hat{a}^\dagger(0)\hat{a}(\tau)]. \tag{84}$$

Now introducing (78) into (84), we have

$$\langle \hat{a}^\dagger(t)\hat{a}(t+\tau)\rangle = \int d^2\alpha Q\left(\alpha^*, \alpha + \frac{\partial}{\partial \alpha^*}, t\right)\alpha^* Tr\left[\hat{\rho}'(0)\hat{a}(\tau)\right], \tag{85}$$

in which

$$\hat{\rho}'(0) = |\alpha\rangle\langle\alpha|. \tag{86}$$

We note that

$$Tr[\hat{\rho}'(0)\hat{a}(\tau)] = Tr[\hat{\rho}'(\tau)\hat{a}(0)]. \tag{87}$$

Furthermore, replacing (α, α^*, t) by $(\lambda, \lambda^*, \tau)$ in Eq. (78), the density operator $\hat{\rho}'(\tau)$ can be written as

$$\hat{\rho}'(\tau) = \int d^2\lambda Q(\lambda^*, \lambda + \frac{\partial}{\partial \lambda^*}, \tau)|\lambda\rangle\langle\lambda|. \tag{88}$$

Thus applying (88) in (87), we get

$$Tr[\rho'(\tau)\hat{a}(0)] = \int d^2\lambda Q\left(\lambda^*, \lambda + \frac{\partial}{\partial \lambda^*}, \tau\right)\lambda. \tag{89}$$

Moreover, replacing (α, α^*, t) by $(\lambda, \lambda^*, \tau)$ and $(\alpha_0, \alpha_0^*, t)$ by (α, α^*, τ) in Eq. (68), the $Q(\lambda, \lambda^*, \tau)$ function for the signal mode can be put in the form

$$Q(\lambda, \lambda^*, \tau) = \frac{q(\tau)}{\pi} exp\left[-u\lambda^*\lambda - \frac{v}{2}(\lambda^2 + \lambda^{*2}) + p(\tau)\lambda + p^*(\tau)\lambda^*\right]. \tag{90}$$

Thus combination of (85) and (90) leads to

$$\langle \hat{a}^\dagger(t)\hat{a}(t+\tau)\rangle = \frac{\varepsilon}{(\kappa - 2\varepsilon)} e^{-\frac{1}{2}(\kappa-2\varepsilon)\tau} - \frac{\varepsilon}{(\kappa + 2\varepsilon)} e^{-\frac{1}{2}(\kappa+2\varepsilon)\tau}. \tag{91}$$

Finally, substituting (91) into Eq. (83) and then carrying out the integration over τ, the power spectrum of the signal mode turns out to be

$$P(\omega) = \overline{n}\left\{\frac{(\kappa^2-4\varepsilon^2)}{8\pi\varepsilon}\left[\left(\frac{1}{\Omega^2+(\frac{\kappa-2\varepsilon}{2})^2}\right)-\left(\frac{1}{\Omega^2+(\frac{\kappa+2\varepsilon}{2})^2}\right)\right]\right\}, \tag{92}$$

where $\Omega = \omega - \omega_0$.

We thus realize that the steady-state local mean photon number in the interval between $\omega^{'} = -\lambda$ and $\omega^{'} = \lambda$ can be written as

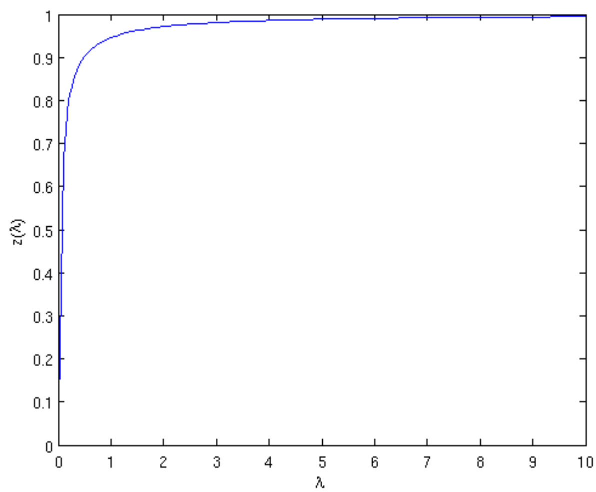

Figure 1: A plot of $z(\lambda)$ [Eq. (96)] versus λ for κ=0.8 and $\varepsilon = 0.35$.

$$\overline{n}_{\pm\lambda} = \int_{-\lambda}^{\lambda} P(\omega')d\omega', \tag{93}$$

where $\omega' = \omega - \omega_0$. Therefore, using (92) and (93) and the fact that

$$\int_{-\lambda}^{+\lambda}\frac{d\omega'}{\omega'^2+a^2} = \frac{2}{a}tan^{-1}\left(\frac{\lambda}{a}\right), \tag{94}$$

we readily obtain

$$\overline{n}_{\pm\lambda} = \overline{n}z(\lambda), \tag{95}$$

where

$$z(\lambda) = \frac{1}{2\pi\varepsilon}\left[(\kappa+2\varepsilon)tan^{-1}\left(\frac{2\lambda}{\kappa-2\varepsilon}\right)-(\kappa-2\varepsilon)tan^{-1}\left(\frac{2\lambda}{\kappa+2\varepsilon}\right)\right]. \tag{96}$$

One can easily get from Fig. 1 that $z(0.5)$ = 0.9019, $z(1)$ = 0.9496, $z(2)$ = 0.9713, and $z(3)$ = 0.9815. Then combination of this results with Eq. (95) yields $\overline{n}_{\pm 0.5}$ = 0.9019 $\overline{n}$, $\overline{n}_{\pm 1}$ = 0.9496 $\overline{n}$, $\overline{n}_{\pm 2}$ = 0.9713 $\overline{n}$, and $\overline{n}_{\pm 3}$ = 0.9815 $\overline{n}$. We immediately see that a large part of the total mean photon number is confined in a relatively small frequency interval.

VI. Global Quadrature Squeezing

We now proceed to calculate the global quadrature squeezing of the one-mode subharmonic light. We can define the quadrature variance of the cavity signal mode by

$$(\Delta a\pm)^2 = 2 + \langle : \hat{a}_\pm(t), \hat{a}_\pm(t) : \rangle, \tag{97}$$

where

$$\hat{a}_+(t) = \hat{a}^\dagger(t) + \hat{a}(t) \tag{98}$$

and

$$\hat{a}_-(t) = i(\hat{a}^\dagger(t) - \hat{a}(t)), \tag{99}$$

are the plus and minus quadrature operators for the cavity light. The first term on the right hand-side of Eq. (97) represents the quadrature variance of the cavity vacuum-state. It is also the commutator of the annihilation and creation operators representing the signal mode. Then Eq. (97) can be put in the form

$$(\Delta a_\pm)^2 \;=\; 2 + 2\langle \hat{a}^\dagger \hat{a}\rangle \pm \langle \hat{a}^{\dagger 2}\rangle \pm \langle \hat{a}^2\rangle. \tag{100}$$

Thus combination of (81) and (100) yields

$$(\Delta a_\pm)^2 = 2 \mp \frac{4\varepsilon}{(\kappa \pm 2\varepsilon)}\left[1 - e^{-(\kappa \pm 2\varepsilon)t}\right]. \tag{101}$$

We observe that the signal mode is in a squeezed state and the squeezing occurs in the plus quadrature.

To this end, we calculate the quadrature squeezing of the cavity signal mode relative to the quadrature variance of the cavity vacuum-state. We then define the quadrature squeezing of the cavity signal mode by

$$S_+ = \frac{2 - (\Delta a_\pm)^2}{2}, \tag{102}$$

so that on account of (101), there follows

$$S_+ = \frac{2\varepsilon}{(\kappa + 2\varepsilon)}\left[1 - e^{-(\kappa + 2\varepsilon)t}\right]. \tag{103}$$

Moreover, on taking into account (103), we see that at steady-state and threshold

$$S_+ = \frac{1}{2}. \tag{104}$$

We then note that at steady state and at threshold there is a 50% global squeezing of the cavity signal mode below the cavity vacuum-state level.

VII. Local Quadrature Squeezing

Here we obtain the local quadrature squeezing of the signal mode employing the spectrum of quadrature fluctuations. We first define the spectrum of quadrature fluctuations for a given cavity light with central frequency ω_0 by

$$S_{\pm}(\omega) = \frac{1}{\pi} Re \int_0^{\infty} \langle \hat{a}_{\pm}(t), \hat{a}_{\pm}(t+\tau)\rangle_{ss} e^{i(\omega-\omega_0)\tau} d\tau, \tag{105}$$

in which

$$\hat{a}_{+}(t+\tau) = (\hat{a}^{\dagger}(t+\tau) + \hat{a}(t+\tau)) \tag{106}$$

and

$$\hat{a}_{-}(t+\tau) = i(\hat{a}^{\dagger}(t+\tau) - \hat{a}(t+\tau)). \tag{107}$$

Then in view of Eqs. (98), (99), (106), and (107) along with (105), we have

$$\langle \hat{a}_{\pm}(t), \hat{a}_{\pm}(t+\tau)\rangle = \Big[\pm \langle \hat{a}(t)\hat{a}(t+\tau)\rangle + \langle \hat{a}(t)\hat{a}^{\dagger}(t+\tau)\rangle + \langle \hat{a}^{\dagger}(t)\hat{a}(t+\tau)\rangle \pm \langle \hat{a}^{\dagger}(t)\hat{a}^{\dagger}(t+\tau)\rangle \Big]. \tag{108}$$

Following the same procedure employed as in section five, one can readily establish that

$$\langle \hat{a}(t)\hat{a}^{\dagger}(t+\tau)\rangle_{ss} = \frac{\varepsilon}{(\kappa-2\varepsilon)} e^{-\frac{1}{2}(\kappa-2\varepsilon)\tau} - \frac{\varepsilon}{(\kappa+2\varepsilon)} e^{-\frac{1}{2}(\kappa+2\varepsilon)\tau}, \tag{109}$$

$$\langle \hat{a}^{\dagger}(t)\hat{a}^{\dagger}(t+\tau)\rangle_{ss} = -\frac{\varepsilon}{(\kappa-2\varepsilon)} e^{-\frac{1}{2}(\kappa-2\varepsilon)\tau} - \frac{\varepsilon}{(\kappa+2\varepsilon)} e^{-\frac{1}{2}(\kappa+2\varepsilon)\tau}, \tag{110}$$

and

$$\langle \hat{a}(t)\hat{a}(t+\tau)\rangle_{ss} = -\frac{\varepsilon}{(\kappa-2\varepsilon)} e^{-\frac{1}{2}(\kappa-2\varepsilon)\tau} - \frac{\varepsilon}{(\kappa+2\varepsilon)} e^{-\frac{1}{2}(\kappa+2\varepsilon)\tau}. \tag{111}$$

Now on account of (91), (109), (110), and (111) together with (108), we find

$$\langle \hat{a}_{\pm}(t), \hat{a}_{\pm}(t+\tau)\rangle_{ss} = (\Delta a_{\pm})^2_{ss} e^{-\frac{1}{2}(\kappa\pm 2\varepsilon)\tau}. \tag{112}$$

Finally, introducing (112) into (105) and then carrying out the integration over τ, the spectrum of the quadrature fluctuations for the signal mode is found to be

$$S_{\pm}(\omega) = (\Delta a_{\pm})^2_{ss} \left(\frac{\frac{(\kappa\pm2\varepsilon)}{2\pi}}{\Omega^2 + [\frac{\kappa\pm2\varepsilon}{2}]^2} \right), \tag{113}$$

where $\Omega = \omega - \omega_0$.

The local quadrature variance in the interval $\omega' = -\lambda$ and $\omega' = \lambda$ can then be written as

$$(\Delta a_{\pm\lambda})^2 = \int_{-\lambda}^{\lambda} (S_{\pm}(\omega'))^2 d\omega', \tag{114}$$

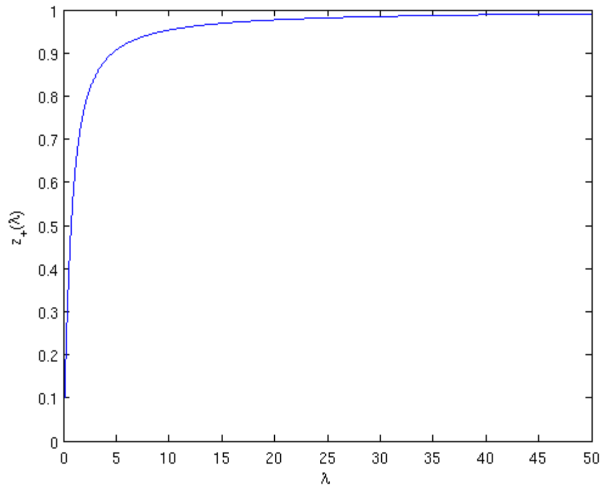

Figure 2: A plot of $z_+(\lambda)$ [Eq. 116] versus λ for κ=0.8 and $\varepsilon = 0.35$.

in which $\omega' = \omega - \omega_0$.
Then upon integrating Eq. (113) in the interval between $\omega' = -\lambda$ and $\omega' = \lambda$, using the relation described by (94), we readily get

$$(\Delta a_{\pm\lambda})^2 = \left(1 \mp \frac{2\varepsilon}{\kappa \pm 2\varepsilon}\right)\left[\frac{2}{\pi} tan^{-1}\left(\frac{2\lambda}{\kappa \pm 2\varepsilon}\right)\right], \tag{115}$$

in which

$$z_{\pm}(\lambda) = \frac{2}{\pi} tan^{-1}\left(\frac{2\lambda}{\kappa \pm 2\varepsilon}\right). \tag{116}$$

We easily obtain from Fig. 2 that z(+5)=0.906, z(+15)=0.968, z(+25)=0.981, and z(+50)=0.990. Then combination of this results with Eq. (115) yields $(\Delta a_{\pm 5})^2$=0.906 $(\Delta a_+)^2$, $(\Delta a_{\pm 15})^2$=0.968 $(\Delta a_+)^2$, $(\Delta a_{\pm 25})^2$=0.981 $(\Delta a_+)^2$, and $(\Delta a_{\pm 50})^2$=0.990 $(\Delta a_+)^2$. We immediately see that a large part of the quadrature variance of the signal mode is confined in a relatively small frequency interval.

We note that the quadrature variance of the vacuum state in the interval between $\omega' = -\lambda$ and $\omega' = \lambda$ can be obtained by setting $\varepsilon = 0$ in Eq. (115). We then get

$$(\Delta a_{\pm\lambda})^2_v = (\Delta a_{\pm})^2_v z_v(\lambda), \tag{117}$$

where

$$z_v(\lambda) = \frac{2}{\pi} tan^{-1}\left(\frac{2\lambda}{\kappa}\right). \tag{118}$$

We next calculate the local quadrature squeezing of the signal mode relative to the local quadrature variance of vacuum state. We define the local quadrature squeezing of the cavity light in the interval between $\omega' = -\lambda$ and $\omega' = \lambda$ by

$$S_{\pm\lambda} = \frac{(\Delta a_{\pm\lambda})^2_v - (\Delta a_{\pm\lambda})^2}{(\Delta a_{\pm\lambda})^2_v}. \tag{119}$$

Then combination of Eqs. (115), (117), and (119) leads to

$$S_{\pm\lambda} = 1 - \left(\frac{\kappa}{\kappa + 2\varepsilon}\right)\frac{tan^{-1}(\frac{2\lambda}{\kappa+2\varepsilon})}{tan^{-1}(\frac{2\lambda}{\kappa})}. \tag{120}$$

We immediately see that the quadrature squeezing of the cavity light in a given frequency in-

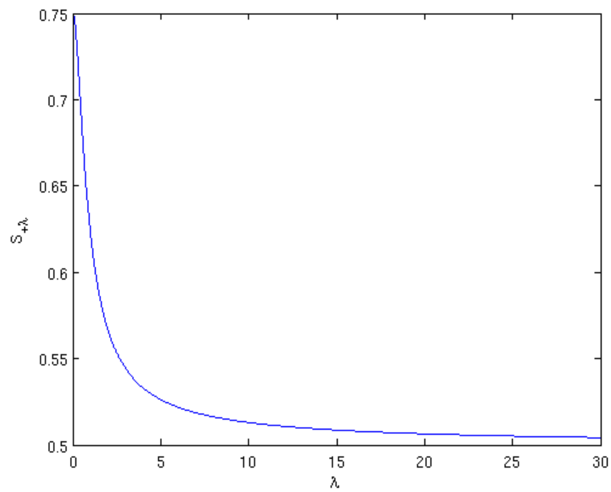

Figure 3: A plot of $S_{\pm\lambda}$ [Eq. 120] versus λ for κ=0.8 and $\varepsilon = 0.4$.

terval is not equal to that of the cavity light in the entire frequency interval. We see from the plot in Fig.3 that the maximum local quadrature squeezing is 75% and occurs in the ± 0.01 frequency interval. In addition, we note that the local quadrature squeezing approaches to the global quadrature squeezing as λ increases.

VIII. Conclusion

It has been established that the mean photon number of the signal mode, obtained following the usual procedure, is just half of the actual mean photon number. The mean photon number calculated, employing the usual procedure, is certainly the mean photon number of a twin signal light beam. In view of this we have asserted that the usual procedure of analysis is valid for a light mode represented in the pertinent Hamiltonian by first order annihilation and creation operators whose commutation relation $[\hat{a}, \hat{a}^{\dagger}]$=1.
Therefore, we have analyzed the photon statistics and quadrature squeezing of the signal mode applying the commutation relation $[\hat{a}, \hat{a}^{\dagger}]$=2. We have found that the mean photon number to be twofold of that of a twin signal light beam. And a large part of the mean photon number is confined in a relatively small frequency interval. In addition, we have shown that the local quadrature squeezing of the signal mode is in general greater than the global quadrature squeezing and approaches to the global quadrature squeezing as λ increases . Moreover, the one-mode subharmonic light beams have a maximum squeezing of 75% below the vacuum state level and occurs in ± 0.01 frequency interval.

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