TRANSPORT PHENOMENA IN STAR FORMING MOLECULAR CLOUDS

By

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To the wounder full of my advisor

To my teachers, who taught me, what I know at Jimma University, College of Natural Sciences, Department of Physics

To my wife Mulat GH.

To my daughter Rahewa Teame

To G/Michael G/Hiwot

Table of Contents

Abstract

The present generation scientific status in broad, generally or otherwise with speculation historically linked to the open sky observations and the physical interpretations of the phenomena, "Astronomy of the ancient people." The interpretations were more philosophical while the observational tests were much constrained. But, today we are contended with diverse but interdisciplinary sciences, as it has to be. Irrespective of this fact, Astronomy is still the field of natural philosophy as well as the science of discovery including, the origin, evolution and age of the universe at the other extreme development of the science as the measure of the knowledge of human beings. The transport phenomena by energy is a crucial physical process in star-forming molecular clouds. As a molecular cloud collapses, gravitational potential energy is converted(transformed) into thermal energy and radiated away, if this thermal energy is not lost, the resulting pressure would halt the collapse.

Magnetic and gravitational forces can both play important roles in transforming angular momentum in star-forming molecular cloud formation. During star-forming molecular clouds, the amount of energy inflow is increasing but the amount of energy outflow is decreasing , the radius of the cloud reduces, and temperature increases(heats up).

As a cloud of interstellar gas collapses to form a star, approximately half of the potential energy would be transformed to thermal energy and the other half would be radiated in the form of electromagnetic radiation. There is an overall progress in astronomy and astrophysics, several problems ranging from observational limitations to theoretical developments have remained unresolved. For example, the origin, evolution and structure of stars, galaxies and interstellar media **ISM** are not yet fully developed. However, according to the current astrophysical understanding, most of the substances that make up our world are formed in stars. Meanwhile, the process of star formation is inextricably tied up with the formation and early evolution of planetary systems. It is generally believed that stars are formed from dust molecular clouds made up of mostly from hydrogen gas. The questions : How these molecular clouds **MCs** form into stars? What dynamical quantities responsible for star-formation? How these dynamical systems affect or responsible for star-formation? What is the coupling dynamics between these quantities and how do they evolve? Are some fundamental questions to be answered in star-formation and stellar evolution. The current picture is that seeded magnetic field, turbulence and gravity play role in the formation and evolution of interstellar clouds and in star formation. The existing models about how these parameters play role are not yet concisely and concretely established. The main objective of this project is to study the role of transport phenomena (particle-energy) and the dynamical controlling parameters responsible for stellar evolutionary scenario.

Key words :stellar-formation-dynamism:transport phenomena:molecular clouds

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Introduction

Understanding the formation, structure, and evolution of stars and stellar systems remains one of the most basic pursuits of astronomical science, and is a prerequisite to obtaining an understanding of the universe as a whole. So currently astrophysical attention has increasingly focusing on the details and refinements that make the current models of stars so quantitatively accurate. On the other hand, recent developments in cosmology may actually have elevated that discipline to diversified research areas. The theoretical foundations of galactic structure seem to be in a close relation to that of stellar structure as scientific studies reveal.

The observational evidences for star origin, formation, structure and properties are all intrinsic. Observational astronomers gather these important information from the spectrums that come out of them. So based on the kind of spectrum that comes out we have diverse field of observation like: Gama-ray astronomy, x-ray astronomy, radio astronomy, etc.

What theoretical physicists(astrophysicist) do is to model the opbservational facts based on physical principles(natural laws). To this end they model the principles in terms of microscopic and macroscopic parametrs like temperature, pressure, density, magnetic field. These parameters are all directly or indirectly either modeled based on hypothesis or derived from the observational spectrum. For detailed information one can refer [1], [9] & [10]. So understanding the physics underlying the structure formation and evolution of molecular clouds (MCs) is an important cornerstone to a predictive statistical theory of star formation (McKee & Ostriker2007 and the references therein [11]).

Despite the fact that, there is an overall progress in the field, several problems ranging from observational limitations to theoretical developments have remained unresolved. For eg. the evolution of structure and transport process and the dynamical controlling parameters within, from, and between stars such as variable magnetic fields, accretion, convection, shocks, pulsations, and winds responsible for (eg. Carpenter, Kenneth G.et. al 2009 [4]) are all not well established. The energy transport near the surface of the Sun and other Sun-like stars dominated by convection excites sound waves traveling throughout the stars enable to study their interiors. However, as reviews report the interaction of these phenomena is poorly understood.

Motivated by this scientific background and rationale, we have worked on a specific research topic: **Transport Phenomena in Star Forming Molecular Clouds**.

Statement of the problem

To be precise one can raiuse the following questions:Where stars are being formed? How stars will evolve out of dusts? How molecular stars are formed? How the molecular clouds form into stars? What dynamical quantities responsible for star-formation? How stellar interiors evolve? How these dynamical systems affect or responsible for star-formation? What is the coupling dynamics between these quantities and how do they evolve? Are some fundamental questions to be answered in star - formation and evolution.

The focus of our research is to answer partially on the mechanims of transportation in star forming molecular clouds. In particular during star formation where we discuss on the dynamical parameters involved in transporting matter and energy-momentum.

Objectives

1. **General Objective** Study transport phenomena in stellar evolution.

2. **Specific Objectives**

   - To study the dynamic parameters in transport phenomena.
   - To study the controlling mechanisms of the dynamic parameters in stellar formation and structure.
   - To analyse and derived concluding remarks on the current theoretical picture of star formation in MCs.

Method

Our approach was a pure theoretical analysis, where the relevant parameters are derived from the Boltzmann transport equations (BTE). The main steps we have followed were:

1. Detailed mathematical derivation and theoretical analysis that involve magnetohydrodynamics equations coupled with gravity where the BTE is considered.

2. Implementing boundary conditions derived the relevant dynamic parameters and analysed

3. The relevant particle-energy transport system is then analyzed in relation to star-forming molecular clouds.

4. Generate theoretical data from the theoretical formula using computation. For the numerical computation MATHEMATICA 10 is used.

5. The results is compared and analyzed with the recent literatures and observational data. Observational data is used from free science source National Aeronautics Space and Administration (NASA), and European Space Agency (ESA)

6. Summary and conclusion is given.

The general design of this work: In chapter one we give the basic theory of stellar formation. and molecular cloud formation and the the governing stellar evolutionary equations. In chapter two we give basic statistical distributions and derive basic princioples from BTE. In chapter three we implement BTE in to analyze the relevant particle-energy transport system in relation to conservation laws. In chapter four we discuss the results of our work and finally in chapter five we give our summary and concluding remarks.

Chapter 1

Basic Theory of Stellar Formation and Structure

1.1 Introduction

Stars are the fundamental units of luminous matter in the universe and they are responsible, directly or indirectly, for most of what we see when we observe it. They also serve as our primary tracers of the structure and evolution of the universe and its contents. Consequently, it is of central importance in astrophysics to understand how stars form and what determines their properties. The generally accepted view that stars form by the gravitational condensation of diffuse matter in space is very old, indeed almost as old as the concept of universal gravitational attraction itself, having been suggested by Newton in 1692. Star formation occurs as a result of the action of gravity on a wide range of scales, and different mechanisms may be important on different scales, depending on the forces opposing gravity. On galactic scales, the tendency of interstellar matter to condense under gravity into star-forming clouds is counteracted by galactic tidal forces, and star formation can occur only where the gas becomes dense enough for its self gravity to overcome these tidal forces, for example in spiral arms. On the intermediate scales of star-forming 'giant molecular clouds', turbulence and magnetic fields may be the most important effects counteracting gravity, and star formation may involve the dissipation of turbulence and magnetic fields. On the small scales of individual pre stellar cloud cores, thermal pressure becomes the most important force resisting gravity, and it sets a minimum mass that a cloud core must have to collapse under gravity to form stars. After such a cloud core has begun to collapse, the centrifugal force associated with its angular momentum eventually becomes important and may halt its contraction, leading to the formation of a binary or multiple system of stars. When a very small central region attains stellar density, its collapse is permanently halted by the increase of thermal pressure and an embryonic star or 'protostar' forms and continues to grow in mass by accretion. Magnetic fields may play a role in this final stage of star formation, both in mediating gas accretion and in launching the bipolar jets that typically announce the birth of a new star.

Understanding the structure and evolution of stars, and their observational properties, requires laws of physics involving different areas e.g. thermodynamics, nuclear physics, electrodynamics, plasma physics.

1.2 Interstellar Dust and Sites of Star Formation

The interstellar medium(ISM) is a mixture of $\sim$ 90 percent gas and $\sim$one percent dust by mass which permeates the space between stars. Interstellar hydrogen amounts $\sim$ 89 percent of the gas content in the ISM and it is found in a variety of chemical forms, temperatures, densities which characterize different phases coexisting in the ism. The ISM is powered by energy emitted by stars[13]. Interstellar space is filled with various gas and dust. In certain concentration of these materials gives rise to nebulae (a cloud where star is born), and their locations are entirely random. Most of the star formation in galaxies occurs in spiral arms, which are marked primarily by their concentration of luminous young stars. Star formation occurs also near the centers of some galaxies, including our own Milky Way galaxy. But this nuclear star formation is often obscured by interstellar dust and its existence is inferred only from the infrared radiation emitted by dust heated by the embedded young stars. The gas from which stars form, whether in spiral arms or in galactic nuclei, is concentrated in massive and dense molecular clouds whose hydrogen is nearly all in molecular form. Some nearby molecular clouds are seen as dark clouds against the bright background of the Milky Way because their interstellar dust absorbs the starlight from the more distant stars. In some nearby dark clouds many faint young stars are seen, most distinctive among which are the T Tauri stars, whose variability, close association with the dark clouds, and relatively high luminosities for their temperatures indicate that they are extremely young and have ages of typically only about 1 million years (Herbig 1962; CohenandKuhi 1979). These T Tauri stars are the youngest known visible stars, and they are pre-main-sequence stars that have not yet become hot enough at their centers to burn hydrogen and begin the main-sequence phase of evolution. Some of these young stars are embedded in particularly dense small dark clouds, which are thus the most clearly identified sites of star formation[7].

1.3 Cloud formation

Since molecular clouds are transient features, it follows that they are constantly being formed and destroyed. The rate at which interstellar gas is presently being collected into star -forming molecular clouds in our galaxy is related to the star formation rate, and it can be estimated empirically from the observed star formation rate and the efficiency of star formation in molecular clouds. The total rate of star formation in our galaxy is of the order of $3M_{sun}$ per year, and since only about few percent or less of the mass of a typical molecular cloud converted into stars; it implies that at least $150M_{sun}$ of gas per year is being turned into star - forming molecular clouds.since the total amount of gas in our galaxy is about $5 \times 10^9 M_{sun}$, the average time required to collect gas into giant molecular cloud must then be about $30M_{sun}$. A similar estimate for the solar neighborhood, where the time scales are somewhat longer than the Galactic average, yields a formation time scale for molecular clouds of about 50Myear. This estimated formation time is not much longer than the cloud life time of 20Myear, thus the formation of molecular clouds must itself be a rather rapid process, and cannot take many dynamical timescales. Since the timescales for the formation, internal evolution, and destruction of molecular clouds are all of the same order. These processes probably cannot be clearly separated in time, and they may all go on simultaneously in different parts of a star-forming complex. Two possible formation mechanisms for molecular clouds:

i. Cloud growth by random collisions and coalescence and

ii. Gravitational instability The first possibility, i.e., the building of large clouds from smaller ones by random collisions and coalescence, predicts formation times of at least 100Myear for giant molecular clouds, and therefore probably cannot be the primary formation mechanism because this is longer than the cloud formation timescale estimated empirically, in any case, most collisions between smaller clouds are probably disruptive and so are not likely to result in coalescence. The second possible mechanisms, i.e. large-scale gravitational instability and amplification effects in the Galactic gas layer, is almost certainly more important because it can collect gas into large complexes on a time scale that is only about 40Myear in the solar neighborhood, in good agreement with the estimated cloud formation time. Evidences show that gravitational instability effects are indeed primarily responsible for both molecular cloud formation and star formation in galaxies is provided by the fact that star formation is observed to occur only where the surface density of gas in galactic disks exceeds a threshold which is close to the critical value predicted for the onset of gravitational instability. The formation of massive molecular clouds by gravitational instability or swing amplification effects in a turbulent interstellar medium (ISM) must also involve complex smaller-scale processes, including collisions between the small clouds that were present in the initial medium and the building up of large clouds by accretion processes. Collisions almost certainly play a role in the building up of large molecular clouds, it nevertheless seems clear that purely random collisions cannot build them fast enough and that more ordered large-scale motions are therefore required, such as those that are involved in the formation of large cloud complexes or spiral arm segments in galaxies. This conclusion is of course, consistent with the fact that most of the star formation in galaxies is observed to occur in large complexes or spiral arm segments [8].

1.4 Cloud Collapse and Fragmentation

A giant molecular cloud must begin forming stars after the cloud itself has formed, since relatively few of the largest molecular clouds are not forming stars. Even if as half of all molecular clouds are not forming stars. The time delay between the formation of a molecular cloud and the onset of star formation in it cannot exceed the subsequent duration of the star formation activity, which is of the order of 10Myear. Since it takes somewhat longer than this to build large molecular clouds, it is likely that star formation begins in molecular cloud, star formation must begin within a time not much longer than the dynamical or free-fall time of such a cloud. Collapse and star formation can occur in the densest part of a cloud even if the cloud as a whole is not collapsing, and because there is no evidence that most star -forming clouds are undergoing any rapid overall collapse. Star formation involves the collapse of a cloud or part of a cloud under gravity and associated fragmentation of the cloud into smaller and smaller bound clumps; because molecular clouds typically contain many times the 'Jeans mass', which is the minimum mass for gravitational bound fragments. It is possible, however, that much of the small-scale structure that eventually develops into stars and groups of stars is present from the beginning because star -forming clouds are assembled from gas that has much small-scale structure. Fragmentation does not occur in star -forming clouds, at least in the sense that small-scale density fluctuations are strongly amplified with time, is provided by the fact that one of the few large molecular clouds that is not presently forming stars contains relatively few small clumps compared with clouds that are forming stars. If this cloud is at an early stage of evolution and will later evolve to a star-forming stage, this suggests that many more small clumps will be formed in this process. For a variety of reasons, including the effects of initial asymmetries, magnetic fields, and turbulence, the collapse of the densest parts of molecular clouds will almost certainly not be spherical but will tend to produce flattened or filamentary structures. Two basic types of processes could be involved:

(1) the observed dense clumps and cloud cores might originate from small density

fluctuations in molecular clouds that are amplified by their self-gravity; such a gravitational fragmentation process might, in principle, generate a hierarchy of progressively smaller and denser clumps[5]. (2) Alternatively, the observed clumpy structure might be generated by supersonic turbulent motions that compress the gas in shocks; a hierarchy of compressed regions or clumps might be produced by a hierarchy of turbulent motions. Almost certainly, both gravity and turbulence play important roles in fragmenting molecular clouds into the observed dense star-forming clumps. GMCs are highly clumped, so that a typical molecule is in region with a density significantly greater than average [2].

1.5 Basic Stellar Evolutionary Equations

The basic theory of stellar structure assumes spherical symmetry, so that all variables depend on only one thing, the distance (r) from the center of the star. On spherical shells of radius r, all physical variables (temperature, density, pressure chemical composition) are assumes to be uniform. The principle variables of stellar structure are pressure (P), temperature(T), density (ρ), luminosity through a shell at r L(r) and mass interior to r (M_r).

For an isolated, static, spherically symmetric star, four basic laws/equations are needed to describe structure.

All physical quantities depend on the distance from the center of the star alone.

1.6 The Basic Equations of Stellar Structure

(1) **Conservation of mass**:

For a spherically symmetric star, the mass interior to some radius R is

$$M(r) = \int_0^R 4\pi r^2 \rho(r) dr \tag{1.6.1}$$

Where ρ is density mass, Written in terms of the differential, this is

$$\frac{dM}{dr} = 4\pi r^2 \rho(r) dr \tag{1.6.2}$$

However, over its lifetime, a star's radius will change by many orders of magnitude, while its mass will remain relatively constant. Moreover, the amount of nuclear reactions occurring inside a star depends on the density and temperature not where it is in the star. A better and more natural way to treat stellar structure is therefore to use **mass** as the independent parameter, rather than r. Thus

$$\frac{dr}{dM} = \frac{1}{4\pi r^2 \rho} \tag{1.6.3}$$

This is the Lagrangian form of the equation (rather than the Eulerian form). All the equations of stellar structure will be expressed in the Lagrangian form, and **most of the parameters** will be expressed in **per unit mass**, rather than per unit size or volume.

(2) **Conservation of Energy** (at each radius, the change in the energy flux equals the local rate of energy of release). Consider the net energy per second passing outward through a shell at radius r. If no energy is created in the shell, then the amount of energy **in** equals the amount of energy **out**, and $\frac{dL}{dr} = 0$. However, if additional energy is released or absorbed within the shell, then $\frac{dL}{dr}$ will be non-zero. Let's define

ϵ as the energy released per second by a unit mass of matter. Then

$$\frac{dL}{dr} = 4\pi r^2 \rho \epsilon \tag{1.6.4}$$

Where L is the luminosity. In the Lagrangian form

$$\frac{dL}{dr} = \epsilon \tag{1.6.5}$$

Note that ϵ has three components.

1. ϵ_n, the total energy created by nuclear reactions.

2. ϵ_ν, the energy input into neutrinos, and

3. ϵ_g, the energy produced or lost by gravitational expansion or contraction.

Thus

$$\frac{dL}{dr} = \epsilon_n + \epsilon_\nu + \epsilon_g \tag{1.6.6}$$

In general, the contribution from reactions will always be positive, while the energy in neutrinos will always be lost from the system.

(3) **Equation of Energy Transport**: (relation between the energy flux and the local gradient of temperature).

Assume that the star is in thermal equilibrium at each radius the gas is neither heating up nor cooling down with time. The transport equation also describes how energy is transported through the layers of the star, i.e. how the gas affects the radiation as it travels through. Depends on local density, opacity and temperature gradient. Let the **rate of energy generation per unit mass** be ϵ. Then:

$$dL = 4\pi r^2 \rho dr \times q \frac{dL}{dr} = 4\pi r^2 \rho \epsilon \tag{1.6.7}$$

(4) **Equation of Hydrostatic Equilibrium**

The force of gravity pulls the stellar material towards the center. It is resisted by the pressure force due to the thermal motions of the gas molecules. The first equilibrium condition is that these forces are in equilibrium.

Radial forces acting on the element:

$$\text{Gravity inward}: F_g = -\frac{Gm\Delta m}{r^2}$$

Pressure (net force due to difference in pressure between upper and lower faces):

$$F_p = p(r)dS - p(r+dr)dS = p(r)dS - \left[p(r) + \frac{dp}{dr}xdr\right]dS = -\frac{dp}{dr}drdS$$

Mass of element: $\triangle m = \rho dr dS$

Applying Newton's second law ($'F = ma'$)

$$\triangle m\ddot{r} = F_g + F_p = -\frac{Gm\triangle m}{r^2} - \frac{dp}{dr}drdS$$

Acceleration $= 0$ everywhere if star static.

Setting acceleration to zero, and substituting for Δm:

$$0 = -\frac{Gm\rho dr dS}{r^2} - \frac{dp}{dr}drdS \tag{1.6.8}$$

$$\frac{dp}{dr} = -\frac{Gm}{r^2}\rho \tag{1.6.9}$$

The basic equations are supplemented by

∗ Equations Of State (pressure of a gas as a function of density and temperature)

∗ Opacity (how transparent it is to radiation)

∗ Nuclear Energy Generation Rate as $f(\rho, T)$

Equation Of State In Stars: Interior of a star contains a mixture of ions, electrons, and radiation (photons). For most stars (exception very low mass stars and stellar remnants) the ions and electrons can be treated as an ideal gas and quantum effects can be neglected.

$Total pressure P = P_i + P_e + P_r = P_{gas} + P_r$

Where

P_i is the pressure of the ions

P_e is the pressure of the electrons

P_r is the radiation pressure.

Gas Pressure: The equation of state for the ideal gas is :

$$P_{gas} = nkT$$

Where n is the number of particles per unit volume, T is temperature and k is the Boltzmann constant; $n = n_i + n_e$, where n_i and n_e are the number of densities of ions and electrons. In terms of the mass density ρ:

$$P_{gas} = \frac{\rho}{\mu m_H} kT$$

Where m_H is the mass of hydrogen and μ is the average mass of particles in units of m_H.

The **ideal gas constant** is:

$$R = \frac{k}{m_H} \Rightarrow P_{gas} = \frac{R}{\mu} \rho T$$

Radiation Pressure: For black body radiation

$$P_{gas} = \frac{1}{3} aT^4$$

Where a is the radiation constant:

$$a = \frac{8\pi^5 k^4}{15c^3h^3} = \frac{4\sigma}{c}$$

Gas Pressure is most important in **low-mass stars**.

Radiation pressure is most important in **high mass stars**.

1.7 Time scales of Stellar Evolution

There are three important timescales in the life of stars.

1.7.1 Dynamical time scale

Dynamical time scale is the measure of the time scale on which a star would expand or contract if the balance between pressure gradients and gravity was suddenly disrupted (same as free-fall time scale):

$$\tau_{dyn} = \frac{characristicradius}{characteristicvelocity} = \frac{R}{v_{esc}}$$

Escape velocity from the surface of the star is given by

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

G is gravitational constant.

$$\tau_{dyn} = \sqrt{\frac{R^3}{2GM}} \tag{1.7.1}$$

In terms of mean density, then becomes

$$\tau_{dyn} = \frac{1}{\sqrt{G\bar{\rho}}} \tag{1.7.2}$$

Where $\bar{\rho}$ is the mean density of the star (molecular cloud).

1.7.2 Kelvin-Helmholtz Time scale

Thermal timescale is the time required for sun to radiate all its reservoirs of thermal energy.

$$\tau_{KH} = \frac{U}{L}$$

Virial theorem: the thermal energy U is roughly equal to the gravitational potential energy

$$\tau_{KH} = \frac{GM^2}{RL}$$

Important timescale: determines how quickly a star contracts before nuclear fusion starts-i.e. sets roughly the pre-main-sequence lifetime. Most stars, most of the time in hydrostatic and thermal equilibrium, with slow changes in structure and composition occurring on the (long) time scale τ_{nuc} as fusion occurs.

•**Dynamical time scale**: timescale of collapsing star, supernova

•**Thermal/kelvin-Helmholtz** Timescale of star before nuclear fusion starts, pre-main-sequence lifetime.

Chapter 2

Statistical Distributions and Boltzmann Transport Equations

2.1 Fundamental Principles and Statistical Distributions

The phase space (higher-dimension space) includes the momentum -distribution of particles which make up the star as well as their location. The three cartesian coordinates represent the spatial volume and the three cartesian coordinates represent the components of the velocity of the particles. The volume of the space is

$$dV = dx_1 dx_2 dx_3 dv_1 dv_2 dv_3 \tag{2.1.1}$$

If the number of particles in small volume dV is N, then the phase density is given by

$$f(x_1, x_2, x_3, v_1, v_2, v_3)dV = N \tag{2.1.2}$$

A macro state of a system is said to be specified when the number of particles in each phase space volume dV is specified. It the phase density is specified everywhere, the the macro state of the system is specified. In addition to the number of particles in each volume, it makes a difference which particles are in which volumes. If the specification of individual particles can be accomplished, then it can be said that a micro state is specified. In a system which is continually rearranging itself by collisions, the most probable macro state becomes the most likely state in which to find the system. A system which is in its most probable macro state is said to be in statistical equilibrium. If the total number of particles N to be arranged sequentially among m volumes, then the total number of sequences is simply N!. However, within each volume (the ith volume), N_i particles N_i! indistinguishable sequences which must be removed when the allowed number of particles is counted. Thus, the total number of allowed micro states in a given macro state is

$$W = \frac{N!}{\prod_{i=1} N_i!} \tag{2.1.3}$$

The particle distribution of the most probable macro state is unique and is the equilibrium macro state. A system which is reached its equilibrium macro state is in strict thermodynamic equilibrium. The statistical distribution of micro state verses macro state given by equation (2.1.3) is the Maxwell-Boltzmann statistics. The phase space volumes are indeed differential and arbitrarily small. There is a limit to how well the position and momentum (velocity, if the mass is known) of any particle can be determined. Within that phase space volumes, particles are indistinguishable and the limit is the Heisenberg uncertainty principle and it is stated as

$$\Delta p \Delta x \geq \frac{h}{2\pi} = \hbar, \tag{2.1.4}$$

the differential cell volumes into compartments of size h^3 so that the total number of compartments

$$n = \frac{dx_1 dx_2 dx_3 dp_1 dp_2 dp_3}{h^3} \tag{2.1.5}$$

the total number of micro state per macro state is

$$W = \prod_i W_i \tag{2.1.6}$$

Where W_i is the number of micro states per cell of phase space which can be expressed in terms of the number of particles N_i in cell.

2.2 Statistical Equilibrium For A Gas

To find the macro state of a steady equilibrium for a gas, we follow basically the same procedures regardless of the statistics of the gas. In general, we wish to find that macro state for which the number of microstates is a maximum. So by varying the number of particles in a cell volume we will search for: $dW = 0$. Or equivalently $d \ln W = 0$, since $\ln W$ is a monotonic function of W [14]. The use of logarithms is that it makes easier to deal with the factorials through the use of Stirring's formula for the logarithm of a factorial of a large number.

In general we have three statistical distributions where one is classical and the other two are quantum: Maxwell-Boltzmann (MB) for classical gases, Bose-Enistein (BE) for quantum gases of family Bosons and for quantum gases of family Fermions.

The maximum macro state W of the distributions are:

$$\ln W_{MB} = \ln N! - \sum_i \ln(N_i!) \qquad \textit{Maxwell-Boltzmann} \tag{2.2.1}$$

$$\ln W_{BE} = \sum_i \ln(n + N_i - 1)! - \ln N_i! - \ln(n-1)! \quad \textit{Bose-Enistein} \tag{2.2.2}$$

$$\ln W_{FD} = \sum_i \ln(2n)! - (2n - N_i)! - \ln N_i 1 \qquad \textit{Fermi-Dirac} \tag{2.2.3}$$

Imposing the condition for the most probable macro state we will find the additional constraints, arising from conservation laws (particle number and energy), on the system which have not been directly incorporated into these equations. These can be achieved by taking the variations of the total particle density and total energy of the system given as:

$$\delta\left[\sum_i N_i\right] = \delta N = 0 \quad (2.2.4)$$

$$\delta\left[\sum_i W_i N_i\right] = \sum_i W_i \delta N_i = 0 \quad (2.2.5)$$

Where $\mathbf{w}_i$ is the energy of an individual particle. These additional constraints represent new information about the system. One of the standard methods of solving these equations in order to extract the new information is the method of Lagrange multipliers. Since equations (2.2.4 & 2.2.5)represent quantities which are zero we can multiply them by arbitrary constants (say β_i) and add constants to them (say α_i) equations 2.2.1 - 2.2.3 to get

$$MB : \sum_i \ln N_i - \ln \alpha 1 + \ln \beta 1 W_i \delta N_i = 0 \quad (2.2.6)$$

$$BE : \sum_i \{\ln\left[\frac{(n+N_i)}{N_i}\right] - \alpha 2 - \beta 2 W_i\}\delta N_i = 0 \quad (2.2.7)$$

$$FD : \sum_i \{\ln\left[\frac{2n-N_i}{N_i}\right] - \ln \alpha 3 - \beta 3 W_i\}\delta N_i = 0 \quad (2.2.8)$$

The solutions of these equations are respectively given as:

$$MB : N_i/\alpha_1 = 2n/N_i \quad (2.2.9)$$

$$BE : n/N_i = \alpha_2 \exp(W_i \beta_2) - 1 \quad (2.2.10)$$

$$FD : 2n/N_i = \alpha_3 \exp(W_i \beta_3) + 1 \quad (2.2.11)$$

All that remains is to develop a physical interpretation of the undetermined Parameters α_j andβ_j . For example let us look at Maxwell-Boltzmann statistics how this can be done.

Then,

$$N_i = \alpha_1 e^{-W_i/(kT)} \tag{2.2.12}$$

If the cell volumes of phase space are not all the same size, it may be necessary to weigh the number of particles to adjust for the different cell volumes. We call these weight functions g_i. Then,

$$N = \sum_i g_i N_i = \alpha_i \sum_i g_i e^{-W_i/(kT)} \equiv \alpha_1 U(T) \tag{2.2.13}$$

The parameter $U(T)$ is called the partition function and it depends on the composition of the gas and the parameter T alone. Now if the total energy of the gas is E , then,

$$\begin{aligned} E &= \sum_i g_i W_i N_i \\ &= \sum_i g_i W_i \alpha_1 e^{-w_i/(kT)} \\ &= \frac{\left[\sum_i W_i g_i 1 N_i e^{-W_i/(KT)}\right]}{U(T)} \\ &= NKT \left(\frac{\dot{l}nU}{\dot{l}nT}\right) \end{aligned} \tag{2.2.14}$$

For a free particle like that found in a monatomic gas, the partition function U is:

$$U(T) = \frac{(2\pi mkT)^{2/3} v}{h^3} \tag{2.2.15}$$

Where, V is the specific volume of the gas, m is the mass of the particle, and T is the kinetic temperature.

Now using Eqs. 2.2.14 & 2.2.15 we obtain the familiar classical energy of gases given as:

$$E = \frac{3}{2} NkT \tag{2.2.16}$$

This is only correct if, T is the kinetic temperature. Thus we arrive at a self-consistent solution if the parameter T is to be identified with the kinetic temperature. The situation for a photon gas in the presence of material particles is not simple, because the matter acts as a source and sinks for photons. Now we can no longer apply the constraint $dN = 0$. This is equivalent to adding $\ln \alpha_2 = 0$ $(\alpha_2 = 1)$ to the equations of condition. It is also possible to show in a similar fashion that $\beta_2 = 1/(KT)$ in Bose-Einstein statics so that the appropriate solution to eq. 2.2.10 is

$$\frac{N_i}{n} = \frac{1}{e^{(hv/KT)} - 1} \tag{2.2.17}$$

Where the photon energy w_i has been replaced by hv in a volume h^3.

The distinguishability condition of this statistics looks for the number of phase space given by

$$n = \frac{2}{h^3} dx_1 dx_2 dx_3 dp_1 dp_2 dp_3 \tag{2.2.18}$$

We can replace the rectangular form of the momentum volume,$dp_1 dp_2 dp_3$, by It's spherical counterpart $4\pi p^2 dp$ and remembering that the momentum of a photon is, hv/c , we get:

$$\frac{dN}{v} = \frac{8\pi v^3}{c^3 \frac{1}{e^{(\frac{hv}{c})}} - 1} \tag{2.2.19}$$

Here we have replace N_i with dN. This assumes that the number of particles in any phase space volume is small compared to the total number of particles. Since the energy per unit volume dE_v is just $h_v dN/V$, we get the relation known as Planck's law or sometimes as the black body law:

$$dE_v = \frac{8\pi v^2}{c^3} \frac{1}{e^{\left(\frac{hv}{KT}\right)} - 1} dv = \frac{4\pi}{c} B_v(T) \tag{2.2.20}$$

The parameter $B_v(T)$ is known as the Planck function. This, then, is the law for photons which are in strict thermodynamic equilibrium. If we were to consider the

Bose-Einstein result for particles and let the number of Heisenberg compartments be much larger than the number of particles in any volume, we would recover the result for Maxwell-Boltzmann statistics. This is further justification for using the Maxwell-Boltzmann result for ordinary gases.

2.2.1 Thermodynamic Equilibrium - Strict and Local

In stars, as throughout the universe, photons outnumber material particles by a large margin and continually undergo interactions with matter. Indeed, it is the interplay between the photon gas and the matter. If both components of the gas(photons and particles) are in statistical equilibrium, then we should expect the distribution of the photons to be given by Planck's law and the distribution of particle energies to be given by the Maxwell-Boltzmann statistics. In some cases, when the density of matter becomes very high and the various cells of phase space become filled, it may be necessary to use Fermi-Dirac statistics to describe some aspects of the matter. When both the photon and the material matter components of the gas are in statistical equilibrium with each other, we say that the gas is in **strict thermodynamic equilibrium**. If the photons depart from their statistical equilibrium (i.e., from Planck's law), but the material matter continues to follow Maxwell-Boltzmann Statistics (i.e., to behave as if it were still in thermodynamic equilibrium), we say that the gas is in **local thermodynamic equilibrium** (LTE)[14]and[10].

2.3 Boltzmann Transport Equation (BTE)

In stellar astrophysical, modeling gas flows around stars or in interstellar space, the ideal gas assumption is very much accurate. Therefore, in our analysis of the stellar evolution including magnetic field dynamism we apply the classical Boltzmann statistical distributions and derive the dynamic equations from Boltzmann transport equations.

The Boltzmann transport equation in six dimensional position-velocity phase space basically expresses the change in the phase density within a differential volume, in terms of the flow through these faces, and the creation or destruction of particles within that volume. In the canonical position-momentum coordinate system, the Boltzmann transport equation (BTE) is given by

$$\boxed{\sum_{i=1}^{3}(\dot{x}_i\frac{\partial f}{\partial x_i}+\dot{p}_i\frac{\partial f}{\partial p_i})+\frac{\partial f}{\partial t}=S} \quad \text{-} \; BTE \tag{2.3.1}$$

where $f \equiv f(x, \dot{x}; t)$ is the number densty distribution function, S is the rate of particle creation/destruction, $\dot{x}_i = \frac{\partial x_i}{\partial t}$ and $\dot{p}_i = \frac{\partial p_i}{\partial t}$

This equation can be recast in vector notation as

$$\frac{\partial f}{\partial t}+\vec{v}\cdot\vec{\nabla}f+\vec{F}\cdot\vec{\nabla}_p f=S \tag{2.3.2}$$

where $\vec{F}$ is force and $\vec{\nabla}_p$ is the momentum gradient.

In conservative field system since $\vec{F} = -\vec{\nabla}\Phi$ where Φ is a scalar potential (eg. gravitational scalar potential), then, BTE will be given as

$$\frac{\partial f}{\partial t}+\vec{v}.\nabla f-\frac{1}{m}\nabla\Phi.\nabla_v f=S \tag{2.3.3}$$

The potential gradient $\nabla\Phi$ has replaced the momentum time derivative while ∇_v is a gradient with respect to velocity. The quantity m is the mass of a typical particle. It is also not unusual to find the BTE written in terms of the total stokes time derivative

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \tag{2.3.4}$$

where $\vec{v}$ is the flow velocity and $\frac{\partial}{\partial t}$ is the Eulerian time derivative.

If we take $\vec{v}$ to be a six-dimensional 'velocity' and ∇ to be a six-dimensional gradient the BTE becomes

$$\frac{Df}{Dt} = S \tag{2.3.5}$$

2.4 Boltzmann Transport Equation and Liouville's theorem

If the creation/destruction rate of particles is zero ($S = 0$), we will obtain the homogeneous Boltzmann Transport Equation (BTE) given as

$$\boxed{\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f - \frac{1}{m} \nabla \Phi . \nabla_v f = 0} \qquad \textit{The Liouville's theorem} \tag{2.4.1}$$

The physical interpretation of Liouville Equation is the 6N-dimensional analogue of the equation of continuity of an incompressible fluid. It implies that the phase points of the ensemble are neither created nor destroyed.

In Astrophysics it is called the Vlasov equation, or sometimes the Collision less Boltzmann Equation. It is used to describe the evolution of a large number of collision less particles moving in a gravitational potential.

In the case of classical statistical mechanics, the number of particles N is very large, (of the order of Avogadro's number, for a laboratory-scale system). Setting $\frac{\partial \rho}{\partial t} = 0$ gives an equation for the stationary states of the system and can be used to find the density of microstates accessible in a given statistical ensemble. For eg. in an equilibrium of the Maxwell-Boltzmann statistical distribution ρ is given as

$$\rho \propto e^{H/(k_B T)}$$

where H is the Hamiltonian, T is the temperature and k_B is the Boltzmann constant.

2.5 The Moments of BTE and Conservation Laws

The equations of fluid dynamics can b e derived by calculating moments of the Boltzmann equation for quantities that are conserved in collisions of the particles.

The n^{th} moment of a function f with primary variable x is

$$M_n\left[f(x)\right] = \int x^n f(x)dx \tag{2.5.1}$$

2.5.1 The zeroth moment of BTE and the Continuity Equation

When $n = 0$ as in eq. 2.5.1 we derive the local spatial density given as

$$\boxed{\rho = m\int_{-\infty}^{+\infty} f(x,\vec{v})d\vec{v}} \tag{2.5.2}$$

The related BTE is

$$\int_{-\infty}^{+\infty}\left(\frac{\partial f}{\partial t} + \sum_{i=1}^{3} v_i\frac{\partial f}{\partial x_i} + \sum_{i=1}^{3}\dot{v}_i\frac{\partial f}{\partial v_i}\right)d\vec{v_i} = \int_{-\infty}^{+\infty} Sd\vec{v} \tag{2.5.3}$$

The integral of the creation rate S over all velocity space becomes the creation rate for particles in physical space, which we call $\Im$.

In the conservative field system (eq.2.3.3), the zeroth moment of BTE is given as

$$\frac{\partial}{\partial t}\int_{-\infty}^{+\infty} fd\vec{v} + \int_{-\infty}^{+\infty}\vec{v}\cdot\nabla fd\vec{v} - \int_{-\infty}^{+\infty}\frac{\vec{\nabla}\Phi}{m}\cdot\vec{\nabla}_v fd\vec{v}\nabla = \Im \tag{2.5.4}$$

In view of eq. 2.5.2, the first integral of the left hand side of eq.2.5.4 is given by

$$\frac{\partial}{\partial t}\int_{-\infty}^{+\infty} f d\vec{v} = \frac{1}{m}\frac{\partial \rho}{\partial t} \tag{2.5.5}$$

The second integral of the left hand side of eq. 2.5.4 is simplified to yield (See appendix A.1)

$$\int_{-\infty}^{+\infty} \vec{v}\cdot\vec{\nabla} f d\vec{v} = \vec{\nabla}\cdot\int_{-\infty}^{+\infty} \vec{v} f d\vec{v} \tag{2.5.6}$$

Now using eqs. 2.5.3 - 2.5.6

$$\frac{\partial \rho}{\partial t} + m\vec{\nabla}\cdot\int_{-\infty}^{+\infty} \vec{v} f d\vec{v} - \vec{\nabla}\Phi\cdot\int_{-\infty}^{+\infty} \vec{\nabla}_v f d\vec{v} = \Im m \tag{2.5.7}$$

For realistic physical system with finite velocity the second integral of the left hand side equation has to vanish. Then,

$$\frac{\partial \rho}{\partial t} + m\vec{\nabla}\cdot\int_{-\infty}^{+\infty} \vec{v} f d\vec{v} = \Im m \tag{2.5.8}$$

Using the normalized mean flow velocity $\vec{u}$, a measure of the mean flow rate of the material defined as

$$\vec{u} = \frac{\int_{-\infty}^{+\infty} \vec{v} f(\vec{v}) d\vec{v}}{\int_{-\infty}^{+\infty} f(\vec{v}) d\vec{v}} \tag{2.5.9}$$

the zeroth moment of BTE yields the continuity equation

$$\boxed{\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{u}) = \Im m} \tag{2.5.10}$$

In the absence of creation field, the continuity equation gives the familar **local matter conservation**.

2.5.2 The First Moment of BTE and the Euler-Lagrange Equations of Hydrodynamic Flow

Multiplying the BTE by the local particle velocity $\vec{v}$ and integrating over all velocity space will produce momentum like moments given as

$$\int_{-\infty}^{+\infty} \vec{v}\frac{\partial f}{\partial t} d\vec{v} + \int_{-\infty}^{+\infty} \vec{v}\cdot\vec{\nabla} f d\vec{v} + \int_{-\infty}^{+\infty} (\dot{\vec{v}}\cdot\vec{\nabla}_v f) d\vec{v} = \int_{-\infty}^{+\infty} \vec{v} S d\vec{v} \tag{2.5.11}$$

The integral of this equation are not a simple scalars or vectors, but are the vector outer products called tensors.

Using the expressions of local spatial density ρ and the mean velocity $\vec{u}$, the first integral of the left hand side of this equation is given by (See appendix A.2.1.)

$$\int_{-\infty}^{+\infty} \vec{v}\frac{\partial f}{\partial t} d\vec{v} = n\frac{\partial \vec{u}}{\partial t} - (\vec{u}.\vec{\nabla} n + \vec{\nabla} n.\vec{u})\vec{u} + \int_{-\infty}^{+\infty} \vec{u} S d\vec{v} \tag{2.5.12}$$

where $n = \rho/m$, the number density.

The second integral of the left hand side of this moment like equation as discussed earlier is

$$\int_{-\infty}^{+\infty} \vec{v}\cdot\vec{\nabla} f d\vec{v} = \int_{-\infty}^{+\infty} \vec{v}(\vec{v}\cdot\vec{\nabla} f) d\vec{v} \tag{2.5.13}$$

The third integral of the left hand side of eq. 2.5.11 is as worked out in appendix A.2.2 given by

$$\int_{-\infty}^{+\infty} \vec{v}(\vec{v}\cdot\vec{\nabla}_v f) d\vec{v} = n\frac{\vec{\nabla}\Phi}{m} \tag{2.5.14}$$

Using eqs. 2.5.12 - 2.5.14 in eq. 2.5.11 we obtain

$$n\frac{\partial \vec{u}}{\partial t} - (\vec{u}.\vec{\nabla} n + \vec{\nabla} n.\vec{u})\vec{u} + \int_{-\infty}^{+\infty} \vec{v}(\vec{\nabla}.(\vec{v} f) d\vec{v} + n\frac{\vec{\nabla}\Phi}{m} = \int_{-\infty}^{+\infty} S(\vec{v} - \vec{u}) d\vec{v} \tag{2.5.15}$$

Defining the velocity tensor $\overleftrightarrow{u}$ as

$$\overleftrightarrow{u} = \frac{\int_{-\infty}^{+\infty} \vec{v}\vec{v} f(\vec{v}) d\vec{v}}{\int_{-\infty}^{+\infty} f(\vec{v}) d\vec{v}} \tag{2.5.16}$$

Now usung eq. 2.5.15 & eq. 2.5.16 we find

$$\rho\frac{\partial u}{\partial t}+\rho(\vec{u}.\vec{\nabla})\vec{u}+\vec{\nabla}.(\rho(\vec{\vec{u}}-\vec{u}\vec{u}))+n\vec{\nabla}\Phi=\int_{-\infty}^{+\infty}mS(\vec{v}-\vec{u})d\vec{v} \tag{2.5.17}$$

The quantity $\rho(\vec{u}-\vec{u}\vec{u})$ is called the pressure tensor. Then we define the mean pressure tensor of f(v) as $\overleftrightarrow{p}$ equal to

$$\overleftrightarrow{p}=\frac{\int_{-\infty}^{+\infty}f(v)(\vec{v}-\vec{u})(\vec{v}-\vec{u})d\vec{v}}{\int_{-\infty}^{+\infty}f(v)d\vec{v}} \tag{2.5.18}$$

It describes the different between the local flow $\vec{v}$ and the mean flow $\vec{u}$.

Finally the first velocity moment of the BTE is given by

$$\boxed{\frac{\partial\vec{u}}{\partial t}+(\vec{u}.\nabla)\vec{u}=-\nabla\Phi-\frac{1}{\rho}\nabla P+\frac{1}{\rho}\int_{-\infty}^{+\infty}mS(\vec{v}-\vec{u})d\vec{v}} \tag{2.5.19}$$

This set of vector equations are called **Euler-Lagrange equations of hydrodynamic flow**.

On the other hand the assumption of excessive collisions where $\vec{v}$ is considered to be random and the assumption S to be symmetrical implies the integral over all velocity space vanishes. Then Euler-Lagrange equations of hydrodynamic flow of BTE is given by

$$\boxed{\frac{\partial\vec{u}}{\partial t}+(\vec{u}.\nabla)\vec{u}=-\nabla\Phi-\frac{\nabla P}{\rho}} \tag{2.5.20}$$

Under the assumption of a nearly isotropic velocity field, $\mathbf{P}$ will be $\mathbf{P}(\rho)$ and an expression known as an equation of state. From equation (3.5.14)the left-hand side is zero. The Euler-Lagrange equations of hydrodynamic flow is

$$\nabla P=-\rho\nabla\Phi \tag{2.5.21}$$

Which is known as the **equation of hydrostatic equilibrium**. This equation is usually an expression of the c**onservation of linear momentum**. The zeroth moment of the BTE results in the conservation of matter, where as the first velocity moment equations which represent the conservation of linear momentum. The second velocity moment represent an expression for the conservation of energy.

2.6 Boltzmann Transport Equation and the Virial Theorem

The Euler-Lagrange equations of hydrodynamic flow are vector equations and represent vectors. We can obtain a scalar result by taking the scalar product of a position vector with the flow equations and integrating over all space with the system. The origin of the position vector is important only in the interpretation of some of the terms which will arise in the expression.

So now the spatial first moment of the Euler-Lagrange equations of hydrodynamic flow eq. 2.5.20.

$$\int_V \vec{r} \cdot \left(\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} + \nabla\Phi + \frac{\nabla P}{\rho} \right) dV = 0 \tag{2.6.1}$$

Working out this equation (See appendix A.3) gives us

$$\frac{1}{2}\frac{d^2 I}{dt^2} - 2T - 2U + \Omega = 0 \tag{2.6.2}$$

Where I is the moment of inertia, T is kinetic energy in bulk motion and U is the internal energy and Ω is the total potential energy of the system.

This equation 2.6.2 is known as the ***Non-averaged form of the virial theorem***. For a system in equilibrium, the time average of eq.2.6.2 removes the accelerative changes of the moment of inertia(($< \frac{d^2I}{dt^2} >= 0$)) so that

$$2 < T > +2 < U > + < \Omega >= 0 \quad (2.6.3)$$

The theorem which permits is the Ergodic theorem.

2.7 Stellar Evolution, Hydrodynamics and Magnetohydrodynamics

2.7.1 Hydrodynamics

Astrophysical fluids are complex, with a number of different components: neutral atoms and molecules, ions, dust grains (often charged), and cosmic rays. The magnetic fields generally ties all these fluids together except where gradients are very steep, as in shocks. It is the study of the motion of the fluids (liquids and gas). Although fluids are made of particles, it is sufficient to treat a fluid as a continuous substance in many situations. Moreover, in the fluid approximation, we treat the ensemble of particles as a single fluid. To describe an ensemble of particles precisely we need to know the position and velocity of each particle [2]. It is the study of fluid flow (gas and liquids) in the motion. To describe an ensemble of particles precisely we need to the position and velocity of each particle. Although fluids are made of particles. If the number of particles is large enough to perform statistics then it makes sense to describe the ensemble with a distribution function n.

$$\delta N = N(r, u, t)\delta r^3 \delta u^3 \quad (2.7.1)$$

Where δN is the number of particles in a small volume in position/velocity space at time t, r is the space coordinate (a vector with as many components as the space has dimensions) and **u** is the velocity. A small volume in physical space ($i.e., \delta r^3 = \delta x \delta y \delta z$) can contain particles with completely different velocities.

2.8 The Hydrodynamic Equation

There are three equations of hydrodynamics which come from the conservation of momentum, mass, and energy laws. They are partial differential equations (PDEs) containing the time derivatives of the velocity and the two thermodynamics variables. There are three equations of hydrodynamics equations; which come from conservation of (mass, momentum, and energy)laws[14].

2.8.1 The Continuity Equation

The first equation of hydrodynamics is that of continuity, or the conservation of mass stated as, the rate at which mass accumulates in an element of volume is equal to the net rate at which it flows in through that elements boundaries. The rate of mass flow through any unit element of area $\rho\vec{u}.\hat{n}$, where ρ, is the density, $\vec{u}$ is the velocity and $\hat{n}$ is the unit normal to the surface. Using vector calculus, the net mass flow from an infinitesimal element of unit volume is given by $\nabla.(\rho\vec{u})$, and the contained mass is ρ, so that the equation of continuity is

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho\vec{u}) = 0 \tag{2.8.1}$$

$$\frac{\partial}{\partial t}\int \rho dV = -\int_S \rho\vec{u}.\vec{n}dS = -\int_V \nabla.(\rho\vec{u})dV \tag{2.8.2}$$

This gives us the equation of continuity which describes the conservation of mass in a fluid volume:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho\vec{u}) = 0 \tag{2.8.3}$$

If ρ is constant the fluid is called incompressible, and $\nabla.\vec{u} = 0$

2.8.2 Conservation of Momentum

The second equation of hydrodynamics is equivalent to $\mathbf{F} = ma$, or the conservation of momentum. The momentum density of the gas is $\rho\bar{u}$. As from the continuity equation above, the change of the total momentum in volume V is determined by how much momentum is flowing in or out and by any external force acting on the volume of gas, which can be expressed as pressure force-$P\mathbf{I}$ acting on each surface of V

$$\frac{\partial}{\partial t}\int_S \rho\vec{u}dV = -\int_S \rho\vec{u}\vec{u}.\vec{n}dS - \int_S P\mathbf{I}.\vec{n}dS \tag{2.8.4}$$

Where we introduce the unit tensor $\mathbf{I}$, P is the pressure. Using Gauss's theorem, we obtain:

$$\frac{\partial}{\partial t}\int \rho\vec{u}dV = -\int_V \nabla.(\rho\vec{u}\vec{u} + P\mathbf{I})dV \tag{2.8.5}$$

which leads to:

$$\frac{\partial(\rho\vec{u})}{\partial t} + \nabla.(\rho\vec{u}\vec{u}) + \nabla P = 0 \tag{2.8.6}$$

equation (3.2.5) can be rewritten as:

$$\vec{u}\frac{\partial\rho}{\partial t} + \rho\frac{\partial\vec{u}}{\partial t} + \vec{u}\nabla.(\rho\vec{u}) + \rho\vec{u}.\nabla\vec{u} + \nabla P = 0 \tag{2.8.7}$$

using equation(3.2.1) one obtains:

$$\rho\frac{\partial\vec{u}}{\partial t} + \rho\vec{u}.\nabla\vec{u} + \nabla P = 0 \tag{2.8.8}$$

In this form it is called Euler's equation. Viscous stresses and gravity and other body forces may be readily added, if pressure -they are "sources terms " for momentum, in analogy with the sources of mass one might add to the continuity equation (3.2.8) may be rewritten, after some manipulation and the use of equation (3.2.1), in the form

$$\frac{\partial}{\partial t}(\rho\vec{u}) + \nabla.(\rho\vec{u}\vec{u} + P) = 0 \tag{2.8.9}$$

Where $\vec{u}\vec{u}$ is a dyad and P is the stress tensor. For in viscid fluids P is the scalar pressure p multiplied by a unit tensor, equation (3.2.5) is written in a form analogous to equation (3.2.1), so that it is obviously a conservation law for the momentum density $\rho\vec{u}$; $(\rho\vec{u}\vec{u} + P)$ is the momentum flux density tensor. There generally is no independent hydrodynamic equation derivable form the conservation of angular momentum. Taking the cross product of a radius vector with the momentum equation would give an equation for the conservation of angular momentum [8].

2.8.3 Conservation of Energy

an element of gas has two forms of energy : an amount $\frac{1}{2}\rho v^2$ of kinetic energy per unit volume and internal (thermal)energy $\rho\varepsilon$ per unit volume, where ε, the internal energy per unit mass, depends on the temperature T of the gas. According to the equipartition theorem of elementary kinetic theory, each degree of freedom of each gas particle is assigned a mean energy $\frac{1}{2}kT$. For a monatomic gas the only degrees of freedom are the three orthogonal directions of translational motion and

$$\varepsilon = \frac{3kT}{2\mu m_H}$$

The energy conservation equation becomes :

$$\frac{\partial}{\partial t}\int_v \rho(\varepsilon + \frac{1}{2}v^2)dv = -\int_s \nabla(\rho(\varepsilon + \frac{1}{2}v^2 + p)dv) = 0 \tag{2.8.10}$$

$$\frac{\partial}{\partial t}\int_v \rho(\varepsilon + \frac{1}{2}v^2)dv + \int_s \nabla.(\rho(\varepsilon + \frac{1}{2} + p)dv) = 0$$

The conservation laws of mass, momentum and energy then become

$$\nabla.(\rho v) = 0 \tag{2.8.11}$$

$$\rho(v.\nabla)v + \nabla P = f \tag{2.8.12}$$

$$\nabla.[(\frac{1}{2\rho}v^2 + \rho\varepsilon + P)v] = f.v \tag{2.8.13}$$

Substituting the first of these equations in the third implies

$$\mathbf{f.v} = \rho v.\nabla(\frac{1}{2}v^2 + \varepsilon + \frac{P}{\rho})$$

$$f = \rho v(v.\nabla)v + \nabla P =$$

$$\rho v.\nabla(\frac{1}{2}v^2 + \rho\varepsilon\frac{P}{\rho}) = f.v \tag{2.8.14}$$

While equation (2.2.8), the Euler equation, shows that

$$f.v = \rho v(v.\nabla)v + v.\nabla P = \rho v.\nabla(\frac{1}{2}v^2) + v.\nabla P \tag{2.8.15}$$

$$\nabla.[(\frac{1}{2}\rho v^2 + \rho\varepsilon + P)v] = (\rho(v.\nabla)v + \nabla P).v = \rho v(v.\nabla)v + v.\nabla P$$

$$\nabla.[(\frac{1}{2}\rho v^2 + \rho\varepsilon + P)v] = \rho v(v.\nabla)v + v.\nabla P\nabla.[(\frac{1}{2}v^2 + \varepsilon + \frac{P}{\rho})v] = \rho v(v.\nabla)v + v.\nabla P$$

Where $(v.\nabla)v = \frac{1}{2}v^2 - v \times \nabla \times v$

$$\rho v.\nabla(\frac{1}{2}v^2) + v.\nabla P = \rho v.\nabla(\frac{v^2}{2} + \varepsilon + \frac{P}{\rho})$$

$$\rho v.\nabla(\frac{v^2}{2}) + v.\nabla P = \rho v.\nabla(\frac{v^2}{2}) + \rho v.\nabla(\varepsilon + \frac{P}{\rho})$$

$$v.\nabla P = \rho v.\nabla(\varepsilon + \frac{P}{\rho})$$

hence, eliminating **f.v** from equation (2.2.10) we get

$$v.\nabla P = \rho v.\nabla(\varepsilon + \frac{P}{\rho})$$

Expanding $\nabla(\frac{P}{\rho})$ and rearranging

$$v.[\nabla\varepsilon + P\nabla(\frac{1}{\rho}) = 0$$

By the definition of the operator, this means that,if we travel a small distance along a streamline of the gas,i.e.,if we follow the velocity **v**, the increments $d\varepsilon$ and $d(\frac{1}{\rho})$ in ε and $\frac{1}{\rho}$ must be related by

$$d\varepsilon + pd(\frac{1}{\rho}) = 0$$

But from the expression for the internal energy

$$\varepsilon = \frac{3}{2}\frac{kT}{\mu m_H}$$

And the perfect gas law is given by

$$P = \frac{\rho kT}{\mu m_H}$$

This requires that

$$d(\frac{3kT}{2\mu m_H}) + \frac{\rho kT}{\mu m_H}d(\frac{1}{\rho}) = 0$$

$$\frac{3kT}{2\mu m_H}dT + \frac{\rho kT}{\mu m_H} = 0$$

Where $\frac{3}{2}d(\frac{1}{\rho}) = -\frac{1}{\rho^2}d\rho$ Which is equivalent to

$$\frac{3}{2}dT - \frac{\rho T}{\rho^2}d\rho = 0$$

$$\frac{3}{2}dT - \frac{T}{\rho}d\rho = 0$$

$$\frac{3}{2}dT = \frac{T}{\rho}d\rho$$

$$\frac{3}{2}\frac{1}{T}dT - \frac{1}{\rho}d\rho = 0$$

$$\int \frac{3}{2}\frac{dT}{T} - \int \frac{d\rho}{\rho} = 0$$

$$\frac{3}{2}\ln(T) - \ln(\rho) = 0$$

$$\ln(T^{\frac{3}{2}} - \rho) = 0$$

$$\ln(\frac{T^{\frac{3}{2}}}{\rho}) = 0$$

$$\frac{T^{\frac{3}{2}}}{\rho} = constant$$

$$\rho^{-1}T^{\frac{3}{2}} = constant$$

Finally, we get

$$P\rho^{\frac{-5}{3}} \tag{2.8.16}$$

2.9 Magnetohydrodynamics

The MHDEs are essentially an extension of the HDEs with one extra variable: the magnetic field **B**. There is one extra term in the momentum equation and a new partial differential equations (PDEs) called the induction equation.

2.9.1 Magnetohydrodynamics

Magnetohydrodynamics (MHD) (magneto fluid dynamics or hydromagnetics) is the study of the magnetic properties of electrically conducting fluids. Examples of such magneto-fluids include plasmas, liquid metals, and salt water or electrolytes. The field of MHD was initiated by Hannes $Alfvèn$. For which he received the Nobel Prize in Physics in 1970. The set of equations that describe MHD are a combination of the Naiver-Stokes equations of fluid dynamics and Maxwell's equations. MHD applied to astrophysics and cosmology since of baryonic matter content of the universe is made up of plasma, including stars, interstellar medium and so on. Many astrophysical systems are not in local thermal equilibrium, and therefore require an additional kinematic treatment to describe all the phenomena within the system where the dynamic cause is contained in Lorentz force that involves both electric and magnetic fields. So MHD equations (MHDEs) are essentially an extension of the hydrodynamic equations with one extra variable, the magnetic field **B** as primary variable.

2.10 Magnetohydrodynamic Equations and Conservation Laws

2.10.1 Magnetohydrodynamic Equations

The set of equations that describe MHD are a combination of the Naiver-Stokes equations of fluid dynamics and Maxwell's equations.

MHD treats the large-scale dynamics of an ionized gas, i.e. an electrically conducting fluid, with a magnetic field $\vec{B}$ throughout. So it is an extension of HD to include the magnetic stresses. We need, then, to write down the appropriate induction equation for the time variation of $\vec{B}$. The essential feature of an electrically conducting fluid is its inability to sustain a significant electric field in its own moving frame of reference, resulting in the magnetic field being carried along bodily with the moving fluid. Magnetic fields both simplify and complicate astrophysical fluid flows: they simplify them by tying the different components of the fluids together, and they complicate them by introducing an array of new phenomena. A full description of the behavior of a plasma requires determining the distribution function of each component under the influence of Maxwell's equations. MHD simplifies this problem enormously by assuming that the time variations are slow. As a result, the displacement current $(1/c)\partial\mathbf{E}/\partial t$ can be neglected.

$$F_{Lor} = q(E + v \times B) \tag{2.10.1}$$

So the well known Lorentz force per univt volume [] fromelectrodynamics becomes

$$\mathbf{f} = \frac{1}{\mu_0}\left((\mathbf{B}.\nabla)\mathbf{B} - \nabla\mathbf{B}\ ^2\right) \tag{2.10.2}$$

individual particle. So, the total force per unit volume (also called Lorentz force) is,

$$\begin{aligned} F_{Lor} = &\quad F_i + F_e && (2.10.3) \\ = &\quad (n_i q_i + n_e q_e)E + (n_i q_i \frac{v_i}{c} + n_e q_e \frac{v_e}{c})B \end{aligned}$$

Where F_i is the total force on the ions, n_i, q_i and v_i are the number density, charge and mean velocity of ions and the quantities with subscript e refer to electrons. Here we have assumed that all ions have the same charge and there are no neutral particles. Now let$\epsilon\frac{(n_i q_i + n_e q_e)}{n_e q_e}$is the fractional charge imbalance ratio as well as the drift velocity as the mean velocity of the electrons relative to the fluid, i.e. $v_{drift} \equiv (v_e - u)$. Since the ions carry almost all of the momentum, fluid velocity $u \simeq v_i$. Hence, equation (2.6.14) becomes [1]

$$F_{Lor} = n_e q_e \left[\epsilon E + (\epsilon\frac{u}{c} + \frac{v_{drift}}{c}) \times B\right] \tag{2.10.4}$$

Note: that in the Earth we need $\epsilon < 10^{-36}$. So electric field does not over Come gravity and cause it to explode and that in almost all astrophysical content ϵ is negligible.

Hence, we drop terms with ϵ(despite the fact that normally $v_{drift} \ll u$). Now let us introduce electric current $J = n_e q_e v_{drift}$ to simplify the Lorentz force to[12]

$$\begin{aligned} F_{Lor} &= n_e q_e \left[\epsilon E + (\epsilon \frac{u}{c} + \frac{v_{drift}}{c}) \right] \times B \\ &= n_e q_e \epsilon E + \frac{n_e q_e \epsilon u}{c} + \frac{n_e q_e v_{drift}}{c} \times B \\ &= (n_e q_e \epsilon E + \frac{n_e q_e \epsilon u}{c} + \frac{J}{c}) \times B \\ &= (0 + 0 + \frac{J}{c}) \times B \end{aligned} \tag{2.10.5}$$

Because ϵ is negligible.

$$F_{Lor} = \frac{J}{c} \times B \tag{2.10.6}$$

However, the origin of this relative velocity of electrons to the ions comes essentially from the difference in force on the two species. So the electrons experience a force relative to the fluid given by[6]: $F_{Lor} = F_i + F_e$

$$F_i - F_{Lor} = n_e q_e (E + \frac{u}{c} \times B) \tag{2.10.7}$$

This force will accelerate the electrons relative to the ions. And in fluids with normal conductivity properties the drift velocity (and therefore current) established will be proportional to this acceleration. This gives Ohm's law.

$$J = \sigma (E + \frac{u}{c} \times B) \tag{2.10.8}$$

Where, σ is conductivity of the fluid which depends on mean free path, temperature, etc. Now remember Maxwell's equations,

$$\nabla \cdot E = 4\pi \rho_e \tag{2.10.9}$$

$$\nabla \cdot B = 0 \tag{2.10.10}$$

$$\nabla \times E = \frac{1}{c}\frac{\partial}{\partial t}B \tag{2.10.11}$$

Where,ρ_e is the net charge density. But if $\nabla \cdot B = 0$ is satisfied at some point in time, $\nabla \times E = \frac{1}{c}\frac{\partial}{\partial t}B$ ensures that it is satisfied at all other times, since the divergence of curl of any vector fluid is zero. In standard MHD we make an approximation that the charge density ρ_e is small, also the displacement current can be neglected, since $4\pi J \gg \frac{\partial E}{\partial t}$ so from Ohm's law we get

$$\begin{aligned}
\nabla \times B = &\quad \frac{4\pi}{c}J + \frac{1}{c}\frac{\partial}{\partial t}E \qquad (2.10.12)\\
J = &\quad \sigma(E + \frac{u}{c} \times B)\\
\frac{J}{\sigma} = &\quad E + \frac{u}{c} \times B\\
\Rightarrow E = &\quad \frac{J}{\sigma} - (\frac{u}{c}) \times B\\
\nabla \times E = &\quad -\frac{1}{c}\frac{\partial}{\partial t}B\\
\nabla \times \frac{J}{\sigma} - (\frac{u}{c}) \times B = &\quad -\frac{1}{c}\frac{\partial}{\partial t}B\\
\Rightarrow \frac{\partial}{\partial t}B = &\quad -c(\nabla \times \frac{J}{\sigma} - (\frac{u}{c}) \times B)
\end{aligned}$$

From equation bellow we get the relation between J and B

$$\frac{\partial}{\partial t}B = \quad -\nabla \times (\nabla \times B - \frac{cJ}{\sigma}) \tag{2.10.13}$$

$$\nabla \times B = \tfrac{4\pi}{c}J + \tfrac{1}{c}\tfrac{\partial}{\partial t}E$$

$$\nabla \times B = \quad \frac{4\pi}{c}J$$

$$c(\nabla \times B) = \quad 4\pi J$$

$$J = \quad \frac{c}{4\pi}\nabla \times B$$

$$\frac{\partial}{\partial t}B = \quad -\nabla \times (\nabla \times B - \frac{cJ}{\sigma}$$

$$\frac{\partial}{\partial t}B = \quad -\nabla \times (u \times B - \frac{c}{\sigma}(\frac{c}{4\pi}\nabla \times B))$$

$$\frac{\partial}{\partial t}B = \quad -\nabla \times (u \times B - \frac{c^2}{4\pi\sigma}\nabla \times B)$$

$$\frac{\partial}{\partial t}B = \quad \nabla \times (u \times B - \eta\nabla \times B)$$

$$\frac{\partial}{\partial t}B = \quad -\nabla \times (u \times B - (\nabla(\nabla \times B))$$

$$\frac{\partial}{\partial t}B = \quad \nabla \times (\nabla \times B - \eta\nabla \times B)$$

$$\frac{\partial}{\partial t}B = \nabla \times (u \times B - \eta(\nabla \times (\nabla \times B)) \tag{2.10.14}$$

Where $\eta = \frac{c^2}{4\pi\sigma}$ is magnetic diffusivity with units cm/s. Also substituting for J in to equation (2.8.5), Lorentz force per unit volume becomes,

$$F_{Lor} = \frac{J}{c} \times B \tag{2.10.15}$$

$$F_{Lor} = \frac{c}{4\pi}\frac{1}{c}\nabla \times B \times B \tag{2.10.16}$$

$$F_{Lor} = \frac{c}{4\pi}\nabla \cdot \frac{1}{c} \times B \times B \tag{2.10.17}$$

$$F_{Lor} = \frac{1}{4\pi}(\nabla \times B) \times B \tag{2.10.18}$$

Thus E and J have been eliminated. Comparing this with the original hydrodynamic equations, here we have one additional variable B, one additional induction equation and one additional term equation(2.8.17) in the momentum equation. But, if we assume the electrical conductivity σ is uniform, and using the constraint $\nabla \cdot B = 0$ and the vector identity, $\nabla \times \nabla \times A = A(\nabla \cdot A) - \nabla^2 A$ becomes,

$$\frac{\partial}{\partial t} B = \nabla \times (u \times B - \eta(\nabla \times (\nabla \times B)) \tag{2.10.19}$$

$$\frac{\partial}{\partial t} B = \nabla \times (u \times B) - \eta \left[B(\nabla \cdot B) - \nabla^2 B \right] \tag{2.10.20}$$

$$\frac{\partial}{\partial t} B = \nabla(u \times B) + \eta \nabla^2 B \tag{2.10.21}$$

$$\frac{\partial}{\partial t} B = \nabla(u \times B) + \frac{1}{4\pi\sigma} \nabla^2 B \tag{2.10.22}$$

Where $\sigma = \frac{1}{4\pi\sigma}$. Equation (2.4.21) is called **magnetic field evolution equation** or **induction equation**. This equation, together with the fluid mass, momentum and energy equations form a close set of equations for the MHD state variables (r,V, p,B). Which tells us:

• How the magnetic field will evolve in time. As B changes, the Lorentz force provides a back reaction on the plasma producing - a force that modifies the velocity[14].

• It also suggests that the motion of a conducting liquid in an applied magnetic field will induce a magnetic field in the medium.

Chapter 3

Transport Phenomena In Star-Forming Molecular Clouds

3.1 Sources of Energy In Star-Forming Molecular Clouds

Gravitational Energy

The total gravitational potential energy is

$$\Omega \leq -\frac{3}{5}\frac{GM^2}{R} \tag{3.1.1}$$

The right-hand side of the inequality is the gravitational potential energy for a star uniform density sphere, which provides upper limit for the energy. For the gravitational potential energy of a polytrope:

$$\Omega = -\frac{3}{5-n}\frac{GM^2}{R} \tag{3.1.2}$$

For a star in convective equilibrium (that is $n = \frac{3}{2}$) the factor multiplying $\frac{GM^2}{R}$ becomes $\frac{6}{7}$. For a polytrope of index 5, $\Omega \to -\infty$ implying an infinite central concentration of material. The **Virial theorem** is given as

$$2U + \Omega = 0 \tag{3.1.3}$$

Where **U** is the total internal kinetic energy of the gas which includes all motions of the particles making up the gas. The internal kinetic energy density of a differential gas element of the gas is

$$d\mathbf{U} = \frac{3}{2}RTdm = \frac{3}{2}(C_p - C_v)Tdm \tag{3.1.4}$$

Where R is the gas constant,C_p is the heat capacity at constant pressure,C_v the heat capacity at constant volume and $\frac{C_p}{C_v} = \gamma$.The internal heat energy of a differential mass element is

$$du = C_v T dm \tag{3.1.5}$$

From equation (2.9.4) and (2.9.5) Tdm is eliminate and integrating the energy densities of the entire star,

$$\mathbf{U} = \frac{3}{2} < \gamma - 1 > U \tag{3.1.6}$$

Where **U** is the total heat energy or the the total internal energy. The quantity $< \gamma - 1 >$ is the value of $\gamma - 1$ averaged over the star. If γ is assume that constant throughout the star, then the Virial theorem becomes

$$3(\gamma - 1)U + (\Omega) = 0 \tag{3.1.7}$$

The total energy **E** is the sum of the internal energy and the gravitational energy, then we can express the Virial theorem in the following ways:

$$u = \frac{-\Omega}{3(\gamma - 1)} \tag{3.1.8}$$

$$\mathbf{E} = -(3\gamma - 4)u = \frac{3\gamma - 4}{3(\gamma - 1)}(\Omega) \tag{3.1.9}$$

Rotational Energy

The magnitude of the rotational energy that we can expect by noting that

(1) the moment of inertia of the star will always be less than that of a sphere of uniform density and

(2) there is a limit to the angular velocity ω_c at which the star can rotate Thus, for a centrally condensed star

$$\omega^2 \leq \frac{8GM}{R_p^3} \tag{3.1.10}$$

$I_Z < \frac{2}{5}MR^2$

Which implies that the rotational energy must be bounded by

$$E_{rot} = \frac{1}{2}I_Z\omega^2 < \frac{8}{135}\frac{GM^2}{R} \tag{3.1.11}$$

Nuclear Energy

Most of the energy to be gained from nuclear fusion occurs by the conversion of hydrogen to helium and less than one-half of that energy can be obtained by all other fusion processes that carry helium to iron.

3.2 Energy Transport Equation

3.2.1 Energy Transport by Radiation

Stars are hotter at the center, hence the energy must flow from the center to the surface.There are three modes of energy transport: conduction, convection and radiation. Most stars are in a long-lived state of thermal equilibrium, in which energy generation in the stellar center exactly balances the radiative loos from the surface. Photons that carry the energy are continually scattered, absorbed and re-emitted in random directions. Because stellar matter is very opaque to radiation, the photon mean free path ℓ_{ph} is very small (typically $\ell_{ph} \ll R$).

3.2.2 Heat Diffusion by Random Motions

Fick's law of diffusion states that, when there is a gradient ∇n in the density of particles of a certain type, the diffusive flux **J**-i.e. the net flux of such particles per unit area per second-is given by

$$\mathbf{J} = -D\nabla n \tag{3.2.1}$$

Here D is the *diffusion coefficient*, which depends on the average particle velocity $\bar{v}$ and their mean free path ℓ. Therefore a gradient in the energy flux ∇U gives rise to a net energy flux

$$\mathbf{F} = -D\nabla U \tag{3.2.2}$$

Since a gradient in energy density is associated with a temperature gradient, $\nabla U = (\frac{\partial U}{\partial T})_V \nabla T$, we can write this as an equation for heat conduction,

$$\mathbf{F} = -K\nabla T \tag{3.2.3}$$

with

$$K = \frac{1}{3}\bar{v}\ell C_v \tag{3.2.4}$$

Where K is the conductivity.

3.2.3 Radiative Diffusion of Energy

In stars the photon mean free path in much smaller than the stellar dimension($\ell_{ph} \ll R$) so the transport of energy by photons can be considered as a diffusive process. For photons, we cab take $\bar{u}$ = c and $U = aT^4$. Hence the specific heat (per unit volume) is $C_v = \frac{dU}{dT} = 4aT^3$. The photon mean free path is given by

$$\ell_{ph} = \frac{1}{\kappa\rho} \tag{3.2.5}$$

The quantity κ is the opacity and the radiative conductivity is

$$K_{rad} = \frac{4}{3}\frac{acT^3}{\kappa\rho} \tag{3.2.6}$$

The radiative energy flux per unit time and area is

$$F_{rad} = -K_{rad}\nabla T = -\frac{4}{3}\frac{acT^3}{\kappa\rho}\nabla T \tag{3.2.7}$$

In spherical symmetric star the flux is related to the luminosity, $F_{rad} = \frac{l}{4\pi r^2}$. The temperature gradient is

$$\frac{\partial T}{\partial r} = -\frac{\kappa\rho}{16\pi acT^3}\frac{l}{r^2} \tag{3.2.8}$$

From mass conservation dr is given as $dr = \frac{dm}{4\pi r^2\rho}$. Then

$$\frac{\partial T}{\partial m} = -\frac{3}{64\pi^2 ac}\frac{\kappa l}{r^4 T^3} \tag{3.2.9}$$

This the temperature gradient required to carry the entire luminosity l by radiation. It gives the fourth stellar equation. In hydrostatic equilibrium,

$$\frac{dT}{dm} = \frac{dP}{dm}.\frac{dT}{dP} = -\frac{Gm}{4\pi r^4}\frac{T}{P}\frac{d\log T}{d\log P} \tag{3.2.10}$$

Where $\frac{dP}{dm}$ is given by

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4} \tag{3.2.11}$$

and from equation (2.10.9) $\frac{dT}{dm}$ is

$$\frac{dT}{dm} = -\frac{3\kappa}{64\pi^2 acr^4}\frac{l}{T^3} \tag{3.2.12}$$

$$-\frac{3\kappa}{64\pi^2 acr^4}\frac{l}{T^3} = -\frac{Gm}{4\pi r^4}\frac{d\log T}{d\log P} \tag{3.2.13}$$

Where the dimensionless radiative temperature gradient is given by

$$\nabla_{rad} = (\frac{d\log T}{d\log P})_{rad} \tag{3.2.14}$$

Then,

$$\nabla_{rad} = \frac{3}{16\pi acG}\frac{\kappa lP}{mT^4} \tag{3.2.15}$$

This describes the logarithmic variation of T with depth (where depth is now expressed by the pressure) for a star in HE if energy is transported only by radiation. The radiative diffusion equations derived above are independent of frequency ν, since the flux F is integrated over all frequencies. However, in general the opacity coefficient κ_ν depends on frequency. If $\mathbf{F}_\nu d\nu$ represents the radiative flux in the frequency interval $[\nu, \nu + d\nu]$, then equation(2.10.2) must be replaced by

$$\mathbf{F}_\nu = -D_\nu \nabla U_\nu = -D_\nu \frac{\partial U_\nu}{\partial T}\nabla T \tag{3.2.16}$$

Where

$$D_\nu = \frac{1}{3}c\ell_\nu = \frac{c}{\kappa_\nu \rho} \tag{3.2.17}$$

The energy density U_ν in the same frequency interval and $U_\nu = h\nu n(\nu)$,

$$U_\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{\exp(\frac{h\nu}{kT}) - 1} \tag{3.2.18}$$

Which is proportional to the Planck function for the intensity of black-body radiation. The total flux is obtained by integrating equation (2.10.16) over all frequencies,

$$\mathbf{F} = -\left[\frac{1}{3\rho}\int_0^\infty \frac{1}{\kappa_\nu}\frac{\partial U_\nu}{\partial T} d\nu\right] \nabla T \tag{3.2.19}$$

This is equation (2.10.2) but with conductivity

$$K_{rad} = \frac{c}{3\rho}\int_0^\infty \frac{1}{\kappa_\nu}\frac{\partial U_\nu}{\partial T} d\nu \tag{3.2.20}$$

Comparing with equation (2.10.6) shows that the proper average of frequency as it appears in equation (2.10.8) or equation (2.10.9) is

$$\frac{1}{\kappa} = \frac{1}{4aT^3}\int_0^\infty \frac{1}{\kappa_\nu}\frac{\partial U_\nu}{\partial T}d\nu \tag{3.2.21}$$

This is the so-called *Rosseland mean opacity.* The factor $4aT^4$ appearing (2.10.21) is equal to

$$\int_0^\infty \left[\frac{\partial U\nu}{\partial T}\right] d\nu$$

3.2.4 Conductive Transport of Energy

Collisions between the gas particles (ions and electrons) can also transport heat. Under normal (ideal gas) conditions, however, the collisions conductivity is very much smaller than the radiative conductivity. The average particle velocity $\bar{v} = \sqrt{\frac{3kT}{m}} \ll c$. So normally we can neglect heat conduction compared to radiative diffusion of energy. At very high densities, when $\ell_e \gg \ell_{ph}$, electron conduction becomes a much more efficient way of transporting energy than radiative diffusion. The energy flux due to heat conduction can be written as

$$\mathbf{F}_{cd} = -K_{cd}\nabla T \tag{3.2.22}$$

such that the sum of radiative and conductive fluxes is

$$\mathbf{F} = \mathbf{F}_{rad} + \mathbf{F}_{cd} = -(K_{rad} + K_{cd})\nabla T \tag{3.2.23}$$

The conductive opacity is K_{cd} given by

$$K_{cd} = \frac{4acT^3}{3\ell_{cd}\rho} \tag{3.2.24}$$

The we can write the combine flux due to radiation and conduction in the same form as as the radiative flux, equation (2.10.7),

$$\mathbf{F} = -\frac{4acT^3}{3\kappa\rho}\nabla T \tag{3.2.25}$$

with

$$\frac{1}{\kappa} = \frac{1}{\kappa_{rad}} + \frac{1}{\kappa_{cd}}$$

This result simply means that the transport mechanisms with the largest flux will dominate, that is, the mechanism for which the stellar matter has the highest transparency.

3.3 Energy Inflow In Star-Forming Molecular Clouds

A star is forming when molecular cloud is collapse and there is more inflow energy due to the gravitational potential energy (stored energy), but the outflow energy is less. A cloud with radius R, mass M, and temperature T will collapse to form a star, if the total energy of the cloud < 0, i.e., the gravitational potential energy exceeds the thermal energy of the cloud:

$$PE_{grav} + KE_{cloud} = 0$$

Where PE_{grav} is given as

$$E_{grav} = \int_0^R dE = -G\int_0^R \frac{M(r)}{r}dM$$

Where $M(r) = 4\pi r^3\rho$ and $dM = 4\pi r^2\rho dr$

$$PE_{grav} = -G\int_0^R \left[4\pi r^3\rho\right]\left[\frac{4\pi r^2\rho}{r}\right]dr$$

$$= -G\int_0^R \left[\frac{4\pi r^3\rho}{3}\frac{4\pi r^2\rho}{r}\right]dr$$

$$= -G \int_0^R \left[\frac{4\pi r^3 \rho}{3} 4\pi r \rho \right] dr$$

$$= -G \int_0^R \left[\frac{4\pi r^3 \rho}{3} .4\pi r \rho \right] dr$$

$$= -G \int_0^R \left[\frac{4\pi r^4 \rho}{3} .4\pi \rho \right] dr$$

$$= -\frac{G}{5} \left[\frac{4\pi R^3 \rho}{3} .4\pi R^2 \rho \right]_0^R$$

$$= -\frac{G}{5} \left[\frac{4\pi R^5 \rho}{3} .4\pi \rho \right]_0^R$$

Where $\frac{3M}{R} = 4\pi R^2 \rho$, and $M = 4\pi R^2 \rho$, then

$$= -\frac{G}{5} \left[\frac{3M}{R} .M \right] = -\frac{3GM^2}{5R} \tag{3.3.1}$$

Then, the gravitational potential energy for uniform sphere is

$$= -\frac{3GM^2}{5R} \tag{3.3.2}$$

The thermal energy of the cloud is

$$KE_{cloud} = \frac{3}{2} NkT$$

Where N is the total number of particles on cloud. Assuming an **isothermal** (constant temperature) and constant density, r cloud, we can solve the critical radius (Jean's radius) at which the cloud will collapse.

$$\frac{3GM^2}{5R} = \frac{3NkT}{2}$$

But the number of gas particles can be written as

$$N = \frac{M}{m_H}$$

where m_H average mass per particles in the cloud, assumed to be hydrogen.

$$\frac{3GM^2}{5R} = \frac{3MkT}{2m_H}$$

$$\frac{3GM}{5R} = \frac{3kT}{m_H}$$

Where M is $M = \frac{4\pi R^3}{3}\rho$

$$\frac{3G}{5R}\frac{4\pi R^3}{3}\rho = \frac{3kT}{2m_H}$$

Solving for R gives and $\rho = n_H.m_H$

$$R_J = \sqrt{\frac{15kT}{8.\pi G\rho m_H}} \tag{3.3.3}$$

$$= \sqrt{\frac{15kT}{8.\pi G m_H^2.n_H}} \tag{3.3.4}$$

As the cloud collapse, it loses total energy, which is radiated away. **Energy of gas cloud**:

$$E = \frac{M}{m_H}\frac{3kT}{2} - \frac{3}{5}\frac{GM^2}{R}$$

If **E** < 0 then gas cloud collapse.

If **E** > 0 then gas can support itself. If the mass is given by

$$M = \frac{4}{3}\pi R^3 n.m_H$$

$$E = \frac{3kTM}{2m_H} - \frac{3G}{5}\left[\frac{4\pi n m_H}{3}\right]^{\frac{1}{3}} M^{\frac{5}{3}} \tag{3.3.5}$$

by increasing the mass, we can always cause the gravity to dominate that the gas cloud collapse. **Critical size** and mass are called the Jean's length and mass

$$R_J = \sqrt{\frac{15kT}{8\pi G m_H^2.n}}$$

A cloud is:

Bound and collapse if the total energy $E_{tot} < 0$.

Unbound and expands if the total energy $E_{tot} > 0$.

$$E_{tot} = E_{GR} + E_{KE}$$

For a cloud of classical gas particles

$$E_{KE} = \frac{3}{2}NkT$$

Where $E_{KE} \propto T$, E_{tot} is minimized if T is minimized. Therefore clouds collapse (possibly into stars)if they are cold.

Re-express the Jean's criterion for collapse:

$E_{tot} < 0$ or $E_{GR} > E_{KE}$

The Virial theorem states that for an object that is bound by gravity (as opposed to mechanical forces, as in rock) and is stable because of counteracting pressure forces, that kinetic energy W and potential energy U satisfy

$$2W + U = 0$$

The total energy is

$$E = W + U = \frac{U}{2} < 0$$

It can be shown that the Virial Theorem (VT) holds for a planet of mass m in a circular orbit of radius r around a star of mass M:

$$W = \frac{mv^2}{2}$$

And $U = -\frac{GMm}{r}$ For a circular orbit $\frac{mv^2}{2} = \frac{GMm}{r^2}$ from which the kinetic energy W can calculated in terms of U.

Application of Stars: for a star, the VT is also satisfied statistically. Consider a star of mass having uniform density $\rho = \frac{3M}{4\pi R^3}$. The mass interior to a radius r is $M(r) = M \times \frac{(}{r}R)^3$. Using this, the potential energy of the entire star is

$$U = -\int_0^R dr \frac{GM(r)4\pi r^2 \rho}{r} = -\int_0^R dr \frac{GM4\pi r^2}{r}.\frac{3M}{4\pi R^3}.\frac{r^3}{R^3}$$
$$= -\frac{3}{5}\frac{GM^2}{R} \tag{3.3.6}$$

For the kinetic energy we consider only thermal energy of mono-particles. We have

$$W =< N\frac{1}{2}mv^2 >= \frac{3}{2}NkT = \frac{3}{2}\frac{MkT}{\mu m_H} \tag{3.3.7}$$

Where k = Boltzmanns' constant = $1.38x10^{-16}ergK^{-1}$, m_H = mass of the hydrogen atom (i.e. the proton) = $1.67x10^{-24}g$, and μ is the mean atomic weight, slightly greater than 1, that takes into account that the stars are predominantly hydrogen combined with helium. The total energy is

$$E = W + U = -\frac{U}{2} = -\frac{3}{5}\frac{GM^2}{R} \tag{3.3.8}$$

Mean temperature of a star: using the VT we can solve for the temperature T using $W = -\frac{U}{2}$,

$$T = \frac{GM\mu m_H}{5kR} \tag{3.3.9}$$

Critical Condition: for collapse of a cloud with radius R to occur, need either:

$$M_{cloud} > M_J = \frac{3kTR}{2G\mu m_H} \Rightarrow Jean'smass$$

$$\rho_{cloud} > \rho_J = \frac{3}{4\pi M^2}\left[\frac{3kT}{2G\mu m_H}\right]^3 \Rightarrow Jean'sdensity$$

Gravitational potential energy is

$$\Omega = -\int \frac{Gm(r)}{r}dm = -\frac{(4\pi\rho)^2}{3}\int_0^R r^4 dr = -\frac{3GM^2}{5R}$$

Where $m(r) = \frac{4\pi r^3}{3}\rho \ dm = 4\pi r^2 \rho dr \ \rho = \frac{3M}{4\pi R^3}$

Kinetic energy is

$$U = \frac{3}{2}kT \times \frac{M}{\mu m_H}$$

The condition of the collapse

$$\frac{3GM^2}{5R} > \frac{3kTM}{\mu m_H}$$

Initially, large scale structure is collapsed. But, as the collapsing continue, the density inside the cloud increases. $M_J \propto \rho_0^{\frac{-1}{2}}$ with $M_J \propto \rho_0^{\frac{-1}{2}} T^{\frac{-3}{2}}$

As a cloud of interstellar gas collapses to form a star, approximately half of the potential energy would be transformed to thermal energy and the other half would be radiated in the form of electromagnetic radiation.The energy outflow in star forming molecular cloud formation in radiation energy from the black body is given by

$$E_{outflow} = \frac{4\pi}{c} \int B_\nu(T) d\nu$$

Where B_ν is

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\frac{h\nu}{kT} - 1} d\nu$$

$$E_{outflow} = \frac{4\pi}{c} \int \frac{2h\nu^3}{c^2} \frac{1}{\exp\frac{h\nu}{kT} - 1} d\nu$$

$$E_{outflow} = \frac{4\pi}{c} \int \frac{2h\nu^3}{c^2} \frac{1}{\exp\frac{h\nu}{kT} - 1} d\nu$$

To integrate the function let $x = \frac{h\nu}{kT}$ and $dx = \frac{h}{kT} d\nu$, $d\nu = \frac{kT}{h} dx$ Therefore the outflow energy is

$$E_{outflow} = \frac{8\pi h}{c^3} \int \left[(\frac{kT}{h})^3 \frac{kT}{h} \right] \frac{x^3}{\exp x - 1} dx$$

$$E_{outflow} = \frac{8\pi h}{c^3} (\frac{kT}{h})^4 \int x^3 (\exp x - 1)^{-1} dx$$

The integral is

$$\int x^3(\exp x - 1)^{-1}dx = \frac{\pi^4}{15}$$

$$E_{outflow} = \frac{8\pi}{(ch)^3}(kT)^4.\frac{\pi^4}{15} = \frac{8\pi^5k^4T^4}{15c^3h^3}$$

The Stefan-Boltzmann constant is

$$\sigma = \frac{2\pi^5k^4}{15h^3c^2}$$

and $a = \frac{4\sigma}{c}$ The outflow energy due to radiation is

$$E_{outflow} = aT^4 \tag{3.3.10}$$

The thermal energy is given as $\frac{3}{2}NkT$.

The Virial Theorem For an Ideal Gas: The pressure of a gas is related to its internal energy. The pressure of an ideal gas is given by

$$P = nkT = \frac{\rho kT}{\mu m_u}$$

Where $n = \frac{N}{V}$ is the number of particles per unit volume, and μ is the mass of a gas particle in atomic mass units. The kinetic energy per particle is $\epsilon_k = \frac{3}{2}kT$, and the internal energy of an ideal monatomic gas is equal to the kinetic energy of its particles. The internal energy per unit mass is then

$$u = \frac{3}{2}\frac{kT}{\mu m_u} = \frac{3}{2}\frac{P}{\rho} \tag{3.3.11}$$

$$\int (\frac{P}{\rho})dm = \frac{2}{3}udm = \frac{2}{3}E_{int} \tag{3.3.12}$$

Where E_{int} is the total internal energy of the star. The Virial theorem for an ideal gas is therefore

$$E_{int} = -\frac{1}{2}E_{gr} \tag{3.3.13}$$

Also for **equations of state** other than an ideal gas a relation between pressure and internal energy exists, which we can write generally as

$$u = \frac{3}{2}\frac{P}{\rho} \tag{3.3.14}$$

3.4 Application of the Virial Theorem

Virial Theorem (VT) be used to estimate conditions for cloud collapse:

1. If $2K > U \Rightarrow$ gas pressure (energy) will exceed gravitational potential energy and expand.

2. If $2K < U \Rightarrow$ gravitational potential energy will exceed gas pressure and collapse.

The boundary between these two cases describes the critical condition for stability.

We know that

$$\rho = \frac{M_c}{V_c} = \frac{3M_c}{4\pi R_c}$$

Where M_c is the mass of the cloud and R_c radius of the cloud.

The radius of the cloud is

$$R_c = \left[\frac{3M_c}{4\pi\rho}\right]^{\frac{1}{3}}$$

The number of particles is $N = \frac{M_c}{\mu m_H}$ The total energy of the cloud is

$$\frac{3M_c kT}{\mu m_H} - \frac{3GM_c^2}{5}\left[\frac{3M_c}{4\pi\rho}\right]^{\frac{-1}{3}} = 0$$

The Jeans mass is written as

$$M_J = \left[\frac{5kT}{G\mu m_H}\right]^{\frac{3}{2}}\left[\frac{3}{4\pi\rho}\right]^{\frac{1}{2}}$$

If $M_c > M_J \Rightarrow$ the cloud will collapse.

$$M_J = \left[\frac{5k}{G\mu m_H}\right]^{\frac{3}{2}}\left[\frac{3}{4\pi}\right]^{\frac{1}{2}}\left[\frac{T^3}{\rho}\right]^{\frac{1}{2}}$$

$$= \left[\frac{375k^3}{4\pi\mu^3G^3m_H^3}\right]^{\frac{1}{2}}\left[\frac{T^3}{\rho}\right]^{\frac{1}{2}}$$

The radius of the cloud (Jeans radius) in terms of the mass density (ρ) and mass of the cloud is

$$R_J = \left[\frac{375}{4\pi}\right]^{\frac{1}{2}}\left[\frac{k}{G\mu m_H}\right]^{\frac{3}{2}}\left[\frac{T}{\rho}\right]^{\frac{3}{2}}\left[\frac{3}{4\pi}\right]$$

$$= \left[\frac{15}{4\pi}\right]^{\frac{3}{2}}\left[\frac{k}{G\mu m_H}\right]^{\frac{3}{2}}\left[\frac{T}{\rho}\right]^{\frac{3}{2}} = \left[\frac{15k}{4\pi\mu G m_H}\right]^{\frac{1}{2}}\left[\frac{T}{\rho}\right]^{\frac{1}{2}}$$

If $R_c > R_J$: stable.

If $R_c < R_J$: unstable and collapse.

Chapter 4

Result and Discussion

A star is forming when the inflow energy due to gravity (gravitational potential energy) is greater than the outflow of energy (thermal energy and radiate away). In the outflow also there is a particle (matter) and energy. The gravitational potential energy is $E_{GRA} > E_{KE}$. Also more inflow of number of particles and less outflow of number of particles in star forming. By the Virial theorem expression states that equilibrium occurs when, 2KE+PE = 0, then if $PE > 2KE$ the cloud will collapse or star is forming and if $PE < 2KE$ the cloud will expand star is not forming. The outflow of energy is in the form of thermal energy and radiation energy.
The total energy of a star is the sum of its gravitational potential energy, its internal energy and its kinetic energy E_{kin} (due to bulk motions of gas inside the star, not the thermal motions of the gas particles):

$$E_{tot} = E_{gr} + E_{int} + E_{kin} \tag{4.0.1}$$

The star is bound as long as its total energy is negative. For a star in hydrostatic equilibrium we can set $E_{kin} = 0$. Furthermore for a star in hydrostatic equilibrium the Virial theorem holds, so the E_{tot}

$$E_{tot} = E_{int} + E_{gr}$$

The bulk velocity of a gas flow makes a transition between supersonic and subsonic values. The sound waves in plasmas provided by a steady adiabatic, spherical. The gas velocity will, in general be close to the free-fall velocity v_{ff} and therefore highly supersonic near to the surface of the accreting star. The gas must make a transition to a subsonic 'settling' flow, in which the gas very small near the stellar surface. The gas velocity v, the surface of the star is pushing into the gas at a supersonic speed $\cong v_{ff}$. From the energy equation for the gas is the perturbations adiabatic, or isothermal. For small perturbations,

$$P = P' = K(\rho + \rho') \tag{4.0.2}$$

where K = constant, with $\gamma = \frac{5}{3}$ (adiabatic) or $\gamma = 1$ (isothermal). Linearizing the continuity equation and the Euler equation and using the fact that $\nabla P_0 = \mathbf{f}$, we get

$$\frac{\partial \rho'}{\partial t} + \rho_0 \nabla.\mathbf{v'} = 0 \tag{4.0.3}$$

$$\frac{\partial \mathbf{v'}}{\partial t} + \frac{1}{\rho_0}\nabla P = 0 \tag{4.0.4}$$

From equation (4.1.1) P is purely a function of ρ, so $\nabla P' = (\frac{dP}{d\rho})_0 \nabla \rho'$ to first order, where the subscript zero implies that the derivative is to evaluated for the equilibrium solution, i.e. $(\frac{dP}{d\rho})_0 = \frac{dP_0}{d\rho_0}$. Thus, equation (4.1.3) becomes, from equations (4.0.4) and (4.0.2) by operating with $\nabla.$ and $\frac{\partial}{\partial t}$ respectively and then subtracting, gives

$$\frac{\partial}{\partial t\hbar}\left[\frac{\partial \rho'}{\partial t} + \rho_0 \nabla.\mathbf{v'}\right] = \nabla.\left[\frac{\partial \mathbf{v'}}{\partial t} + \frac{1}{\rho_0}c_s^2 \nabla \rho'\right] \tag{4.0.5}$$

$$\frac{\partial^2 \rho'}{\partial t^2} + c_s^2 \nabla^2 \rho' \tag{4.0.6}$$

$$c^{2_s} = \left[\frac{dP}{d\rho}\right]_0 \tag{4.0.7}$$

Equation (4.0.5) will be recognized as the wave equation, with the wave speed c_s. The small perturbations about HE propagate through the gas as sound waves with speed c_s. From equations (4.0.1), (4.0.7) the sound speed c_s can have two values

$$\text{adiabatic}: c_s^{ad} = \left[\frac{5P}{3\rho}\right]^{\frac{1}{2}} = \left[\frac{5kT}{3\mu m_H}\right]^{\frac{1}{2}} \propto \rho^{\frac{1}{3}} \tag{4.0.8}$$

$$\text{isothermal}: c_s^{iso} = \left[\frac{P}{\rho}\right]^{\frac{1}{2}} = \left[\frac{kT}{\mu m_H}\right]^{\frac{1}{2}} \tag{4.0.9}$$

In star-forming molecular cloud formation, there is more inflow energy due to gravity (gravitational potential energy/stored energy) and there is less outflow of energy due to electromagnetic radiation by photons and neutrinos. The cool gas by the energy convective transport is sinking down/inflow and the hot gas by the radiative energy transport rising up/outflow for star-forming molecular cloud formation. In star-forming molecular cloud formation gravity is the dominant parameter to contract the cloud and the outward pressure is expanding, but in star-forming molecular cloud formation is neglect. The other parameters are the temperature, density, and pressure decreasing in star-forming molecular cloud formation.

Chapter 5

Summary and Conclusion

A star is an object that radiates energy from an internal source and bound by its own gravity in which most of the substances that make up our world are formed in it. Its formation is a complex process that involves a number of diverse physical phenomena. Although the central process in star formation is the gravitational collapse of a dense, cold molecular clouds cores in interstellar medium (ISM), turbulence and magnetic field are also the most important physical processes that determine a star formations in which turbulence plays a dual role, both creating over densities to initiate gravitational contraction or collapse, and countering the effects of gravity in these over dense regions (e.g.,McKee1 and Ostriker2007), while magnetic fields play a role in the final stage of star formation, both in mediating gas accretion and in launching the bipolar jets that typically announce the birth of a new star. Broadly the problem of star formation can be divided in to two categories. Microphysics deals with how individual stars (or binaries) form, the dynamical evolution of them (their properties which is determined by the properties of the medium out of which they form) etc. Macrophysics deals with the formation of systems of stars, ranging from clusters to galaxies, and the processes that determine the distributions of physical conditions within star forming region (ISM) etc. The basic theory of stellar structure is spherical symmetry, so that all variables depend on only one variable, the distance (r) from the center of the star. On spherical shells of radius r, all physical variables (temperature, density, pressure, and chemical composition)are uniform. The principle variables of stellar structure are pressure(P), temperature(T), $density(\rho)$, luminosity through a shell at r L(r) and mass interior to r (M_r). Energy is transported by photons carry energy away from the star's center (radiation), cells of hot gas move up and cool gas sink down (convection), and collisions between electrons can move energy outwards (conduction).If the gravitational and pressure forces are seriously out of balance, the star contracts or expands significantly in a time t_d.

Chapter 6

Significance of the Study

On one hand, it serves to get MSc. degree in physics where then, in turn, serve as educator and researcher at educational and research institutes. On the other hand, it means that, the project contributes science professionals who are going to educate, involve in science research and produce researchers where the current attention given by Ethiopian national educational science policy has envisaged. Globally, the result of the project will contribute scientific work to researchers and educators as references for the development of the science.

Appendices

Appendix A

Further ellaboration on BTEs

A.1 The zeroth moment of BTE

Using the vector identity

$$\vec{v} \cdot \vec{\nabla} f = \vec{\nabla} \cdot (f\vec{v}) - f\vec{\nabla} \cdot \vec{v} \tag{A.1.1}$$

In the momentum space, the position coordinates and the momentum (velocity) coordinates are independent so that $\vec{\nabla} \cdot \vec{v} = 0$. Then,

$$\begin{aligned}
\int_{-\infty}^{+\infty} \vec{v} \cdot \vec{\nabla} f d\vec{v} &= \int_{-\infty}^{+\infty} \vec{\nabla} \cdot (f\vec{v}) d\vec{v} - \int_{-\infty}^{+\infty} f(\vec{\nabla} \cdot \vec{v}) d\vec{v} \\
&= \int_{-\infty}^{+\infty} \vec{\nabla} \cdot (f\vec{v}) d\vec{v}
\end{aligned} \tag{A.1.2}$$

Since in the momentum space, the position coordinates and the momentum (velocity) coordinates are independent then, eq. A.1.2 is given by

$$\int_{-\infty}^{+\infty} \vec{v} \cdot \vec{\nabla} f d\vec{v} = \vec{\nabla} \cdot \int_{-\infty}^{+\infty} \vec{v} f d\vec{v} \quad \text{QED} \tag{A.1.3}$$

A.2 The first moment of BTE

A.2.1 $\int_{-\infty}^{+\infty} \vec{v}\frac{\partial f}{\partial t} d\vec{v}$ as in eq. 2.5.12

$$\int_{-\infty}^{+\infty} \vec{v} f d\vec{v} = \int_{-\infty}^{+\infty} f d\vec{v} \left(\frac{\int_{-\infty}^{+\infty} \vec{v} f d\vec{v}}{\int_{-\infty}^{+\infty} f d\vec{v}} \right) \tag{A.2.1}$$

The first factor of the right hand side of eq. A.2.1 is obviously the number density n as defined earlier. While the second factor (in the bracket) is the mean velocity $\vec{u}$. Then,

$$\begin{aligned} \int_{-\infty}^{+\infty} \vec{v}\frac{\partial f}{\partial t} d\vec{v} &= \frac{\partial (n\vec{u})}{\partial t} \\ \text{Or} \quad \int_{-\infty}^{+\infty} \vec{v}\frac{\partial f}{\partial t} d\vec{v} &= n\frac{\partial \vec{u}}{\partial t} + \vec{u}\frac{\partial n}{\partial t} \end{aligned} \tag{A.2.2}$$

By the continuity equation eq. 2.5.10

$$\begin{aligned} \frac{\partial n}{\partial t} &= -\nabla.(n\vec{u}) + \int_{-\infty}^{+\infty} S d\vec{v} \\ &= -(\vec{u}.\vec{\nabla} n + \vec{\nabla} n.\vec{u}) + \int_{-\infty}^{+\infty} S d\vec{v} \end{aligned} \tag{A.2.3}$$

Now using eqs. A.2.2 & A.2.3 we get the relation given by (as required)

$$\int_{-\infty}^{+\infty} \vec{v}\frac{\partial f}{\partial t} d\vec{v} = n\frac{\partial \vec{u}}{\partial t} - (\vec{u}.\vec{\nabla} n + \vec{\nabla} n.\vec{u})\vec{u} + \int_{-\infty}^{+\infty} \vec{u} S d\vec{v} \tag{A.2.4}$$

From the continuity equation

$$\frac{\partial n}{\partial t} + \nabla.(n\vec{u}) = \int_{-\infty}^{+\infty} S d\vec{v} \tag{A.2.5}$$

Then equations (2.3.18)and (2.3.19) we get

$$\frac{\partial}{\partial t}(n\vec{u}) = \vec{u}\frac{\partial n}{\partial t} + n\frac{\partial \vec{u}}{\partial t} - (\vec{u}.\nabla n + n\nabla.\vec{u})\vec{u} + \int_{-\infty}^{+\infty} \vec{u} S d\vec{v} \tag{A.2.6}$$

Using equations (2.3.17) and (2.3.20)

$$\frac{\partial}{\partial t}(n\vec{u}) = n\frac{\partial \vec{u}}{\partial t} - (\vec{u}.\vec{\nabla} n + \vec{\nabla} n.\vec{u})\vec{u} + \int_{-\infty}^{+\infty} \vec{u} S d\vec{v} \tag{A.2.7}$$

$$n\frac{\partial \vec{u}}{\partial t} - (\vec{u}.\vec{\nabla}_n + \vec{\nabla}_n.\vec{u})\vec{u} + \int_{-\infty}^{+\infty} \vec{v}(\vec{\nabla}.(\vec{v}f)d\vec{v} + n\frac{\vec{\nabla}\Phi}{m} = \int_{-\infty}^{+\infty} S(\vec{v}-\vec{u})d\vec{v} \quad \text{(A.2.8)}$$

A.2.2 $\int_{-\infty}^{+\infty}(\dot{\vec{v}} \cdot \vec{\nabla}_v f)d\vec{v}$ as in eq. 2.5.12

Using the vector algebra

$$\begin{aligned} (\vec{\nabla}_v f)\vec{v} &= \vec{\nabla}_v(f\vec{v}) - f(\vec{\nabla}_v \vec{v}) \\ &= \vec{\nabla}_v(f\vec{v}) - f\mathrm{I} \end{aligned} \quad \text{(A.2.9)}$$

where I is the identity matrix, the third integral of the left hand side of eq. 2.5.11 is

$$\int_{-\infty}^{+\infty} (\dot{\vec{v}} \cdot \vec{\nabla}_v f)d\vec{v} = \int_{-\infty}^{+\infty} \dot{\vec{v}} \cdot \vec{\nabla}_v(f\vec{v})d\vec{v} - \int_{-\infty}^{+\infty} \dot{\vec{v}} \cdot f d\vec{v} \quad \text{(A.2.10)}$$

Note that $\dot{\vec{v}} = -\frac{\vec{\nabla}\Phi}{m}$. And morever $\vec{v}$ and the postion coordinates are independent orthogonal phase space coordinates so that eq. A.2.10 can be recast as

$$\int_{-\infty}^{+\infty} (\dot{\vec{v}} \cdot \vec{\nabla}_v f)d\vec{v} = -\frac{\vec{\nabla}\Phi}{m} \cdot \int_{-\infty}^{+\infty} \vec{\nabla}_v(f\vec{v})d\vec{v} + \frac{\vec{\nabla}\Phi}{m} \cdot \int_{-\infty}^{+\infty} f d\vec{v} \quad \text{(A.2.11)}$$

For a bound system, the first integral of the right hand side of eq. A.2.11 must vanish. Then using the relation

$$n = \int_{-\infty}^{+\infty} f d\vec{v}$$

in eq, A.2.11 we obtain the desired equation given as

$$\int_{-\infty}^{+\infty} (\dot{\vec{v}} \cdot \vec{\nabla}_v f)d\vec{v} = n\frac{\vec{\nabla}\Phi}{m} \quad \text{QED} \quad \text{(A.2.12)}$$

A.3 BTE and the Virial Theorem

Recall the BTE given as in eq. 2.5.20

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} = -\nabla\Phi - \frac{1}{\rho}\nabla P \quad \text{(A.3.1)}$$

The left hand side of this equation is just the total average acceleration of the system

$$\begin{aligned} \frac{d\vec{u}}{dt} &= \frac{\partial \vec{u}}{\partial t} + \sum_i \frac{\partial \vec{u}}{\partial x_i}\dot{\vec{x_i}} \\ &= \frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} \end{aligned} \tag{A.3.2}$$

The spatial first moment of(taking the dot product with $\vec{r}$) eq.A.3.1 after some rearrangements is

$$\int_v \rho\vec{r}\cdot\frac{d\vec{u}}{dt}dV + \int_v \rho\vec{r}\cdot\nabla\Phi dV + \int_v \vec{r}.\nabla p dV = 0 \tag{A.3.3}$$

By vectors product property we have

$$\begin{aligned} \frac{d}{dt}(\vec{r}.\vec{u}) &= \vec{r}\cdot\frac{d\vec{u}}{dt} + \frac{d\vec{r}}{dt}\cdot\vec{u} \\ &= \vec{r}\cdot\frac{d\vec{u}}{dt} + u^2 \end{aligned} \tag{A.3.4}$$

The left hand side of equation A.3.4 is

$$\frac{d}{dt}(\vec{r}\cdot\vec{u}) = \frac{1}{2}\frac{d^2r^2}{dt^2} \tag{A.3.5}$$

Using eqs. A.3.4 & A.3.5 the first integral of the left hand side of equation A.3.3 is

$$\int_v \rho\vec{r}\cdot\frac{d\vec{u}}{dt} = \frac{1}{2}\frac{d^2}{dt^2}\int_v r^2\rho dV - u^2\int_v \rho dV \tag{A.3.6}$$

By now we have two well defined quantities, the moment of inertia I and the kinetic energy T respectively given by

$$I = \int_v r^2\rho dV \tag{A.3.7}$$

$$T = \frac{1}{2}\int_v \rho u^2 dV \tag{A.3.8}$$

Then plugging eqs. A.3.7 A.3.8, eq. A.3.6 can be recast as

$$\int_v \rho\vec{r}\cdot\frac{d\vec{u}}{dt} = \frac{1}{2}\frac{d^2I}{dt^2} - 2T \tag{A.3.9}$$

Using vector product property the third integral of the left hand side of equation A.3.3 is expanded to give

$$\int_v (\vec{r}\cdot\vec{\nabla})pdV = \int_v \vec{\nabla}\cdot(rp)dV - \int_v p(\nabla\vec{\cdot}\vec{r})dV \tag{A.3.10}$$

Noting $\nabla\cdot\vec{r} = 3$ and using Gauss divergence theorem, eq. A.3.10 becomes

$$\int_v (\vec{r}\cdot\vec{\nabla})pdV = \oint_{\text{over all surface}} p_s\vec{r}\cdot\hat{n}dA - 3\int_v pdV \tag{A.3.11}$$

where $\hat{n}$ is unit normal to the surface while p_s is the pressure at the surface.

In the MB distribution the pressure is $p = nkT$. So using eq. 2.2.6 the internal energy density of the Maxwellian gas is

$$\varepsilon = \frac{3}{2}p \tag{A.3.12}$$

On the other hand the surface integral of eq. A.3.11 over all space vanishes, otherwise it diverges. Then eq. A.3.10 will be recast as

$$\int_v (\vec{r}\cdot\vec{\nabla})pdV = -2\int_V \varepsilon dV = -2U \tag{A.3.13}$$

where U is the total internal energy of the system. The second left hand term of eq. A.3.3 due to the work of Clausis, Lagrange and Jacob is considered as the negative of the total potential energy Ω of the system.

$$\int \rho\vec{r}\cdot\nabla\Phi dV = -\Omega \tag{A.3.14}$$

Finally the non-averged spatial first moment of BTE by eqs. A.3.3, A.3.9, A.3.13 & A.3.14 is

$$2T + 2U + \Omega = \frac{1}{2}\frac{d^2I}{dt^2} \tag{A.3.15}$$

While the averaged spatial first moment of BTE reduces to

$$2 < T > +2 < U > +\Omega >= 0 \quad \text{QED} \tag{A.3.16}$$

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