TRANSPORT PHENOMENA IN STELLAR INTERIORS

By

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To my father who was faithful and praying for my success.

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Abstract

As stellar astrophysics has developed, attention has increasingly become focused on the details and refinements that make the current models of stars so quantitatively accurate. Despite the fact that, there is an overall progress in astronomy and astrophysics, several problems ranging from observational limitations to theoretical developments have remained unresolved. For example, the origin, evolution and structure of stars, galaxies and interstellar media are not yet fully developed. However, according to the current astrophysical understanding, most of the substances that make up our world are formed in stars. Meanwhile, the process of star formation is inextricably tied up with the formation and early evolution of planetary systems. It is generally believed that stars are formed from dust molecular clouds made up of mostly from hydrogen gas. While stars burning phase is believed to be in plasma state at large involving nulear fusion at the core.This thesis work focuses on transport phenomena during the burning stage of a star within its interior which is one of the active research area. Our approach is a pure theoretical analysis where the relevant parameters are derived from the classical Boltzman transport equations. The result indicates that the current model seems to handle medium size stars like our sun but the complicated reletivistic stars such as super gaint single stars, binary stars and accreting stars need further developments.

Key words: Stellar evolution-Transport Phenomena-Nuclear reaction.

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Introduction

As current understanding shows, generally most of the substances that make up our world are formed in stars. However, the process of star formation is inextricably tied up with the formation and early evolution of planetary systems formed from dust molecular clouds made up of mostly from hydrogen gas. Consequently, the study of the origin, formation, evolution and structure of stars and stellar systems presently is enormous. (for eg. Carpenter, Kenneth G.et. al 2009, Rees 2005) are all not well established. The energy transport near the surface of the sun and other sun-like stars dominated by convection excites sound waves traveling throughout the stars enable to study their interiors[3].

But, several problems ranging from observational limitations to theoretical developments have remained unresolved. Observationally traceable parameters that the evolution of structure and transport process inherited within, from, and between stars such as variable magnetic fields, accretion, convection, shocks, pulsations, and winds responsible for (eg. Carpenter, Kenneth G.et. al 2009, Rees 2005) are all not well established[5].

In general, though, there is an overall progress in stellar astrophysics, several questions remain to be solved. How the molecular clouds form into stars? What dynamical quantities responsible for star formation? How stellar interiors evolve? What dynamical systems and parameters affect or responsible for stellar formation and as well as evolution? How the controlling parameters evolve themselves? Are some fundamental questions to be answered in stellar formation and evolution[11].

Motivated by this introductory scientific background, we have interested to work on the particular topic transport phenomena (particle-energy) within the stellar interiors in order to draw some useful evolutionary scenario of the relevant parameters as of the existing classical model about stellar evolution[9].

Objectives

General Objective

To study transport phenomena (particle-energy) within the stellar interiors

Specific Objectives

1. To study the mechanisms of transport phenomena within the interior of stellar system.

2. To study and derive the dynamical parameters involving in transport phenomena within the stellar interiors.

3. To derive rate of matter-energy conversion during star burning.

Method

Our approach is a pure theoretical analysis where the relevant parameters are derived from the classical Boltzman transport equations. The main steps are:

1. Detailed mathematical derivation and theoretical analysis that involve magneto hydrodynamics equations coupled with gravity.

2. i) First we give the preliminary boundary conditions

   ii) Derive the relevant equations from step 1.

   iii) The relevant particle - energy transport system is then analysed in relation to stellar evolu: tion.

3. Generate theoretical data from the theoretical formula using computation. For the numerical computation MATHEMATICA 10 is used.

4. Simulate the theoretical data with observational data and compare. Observational data is used from free science source National Aeronautics and Space Administration (NASA) and European Space Agency (ESA)

5. The results is compared and analyzed with the recent literatures and observational data.

6. Summary and conclusion is given

The general design of this work is: In chapter 1 we give the theories of stars and the governing statistical assuptions used in the work. In chapter 2 we give the implications of Boltzmann transport equations in relation to conservation laws. In chapter 3 we present our work of transport phenomena within the interiors of stars as outlined in the methdology. In chapter 4 we discuss the results of our work and finally in chapter 5 we give our summary and concluding remarks.

Chapter 1

Basic theory of stars, statistical assumptions and Boltzman transport equation

1.1 Stars

A simple definition for a star is an object that (1) radiates energy from an internal source and (2) is bound by its own gravity. The first criterion excludes objects like planets, comets and brown dwarfs where both are not hot enough for nuclear fusion. The second criterion excludes trivial objects that radiate (e.g. glowing coals)[8].

A star is born out of an interstellar (molecular) gas cloud, lives for a certain amount of time on its internal energy supply, and eventually dies when this supply is exhausted. The definition imposes that stars can have only a limited range of masses, between ~ 0.1 and $\sim 1000 M_s$. The thesis subject area is a burning (radiating) star limited to transport phenomena withen the interiors[8].

Understanding the structure and evolution of stars, and their observational properties, requires laws of physics involving different areas (e.g. thermodynamics, nuclear physics, electrodynamics, plasma physics).

1.2 Statistical Equilibrium For A Gas

To find the macrostate of a steady equilibrium for a gas, we follow basically the same procedures regardless of the statistics of the gas. In general, we wish to find that macrostate for which the number of microstates is a maximum. So by varying the number of particles in a cell volume we will search for: $dW = 0$. Or equivalently $d \ln W = 0$, since $\ln W$ is a monotonic function of W [6]. The use of logarithms is that it makes easier to deal with the factorials through the use of Stirring's formula for the logarithm of a factorial of a large number[6].

In general we have three statistical distributions where one is classical and the other two are quantum: Maxwell-Boltzmann (MB) for classical gases, Bose-Enistein (BE) for quantum gases of family Bosons and for quantum gases of family Fermions.

The maximum macro state W of the distributions are:

$$\ln W_{MB} = \ln N! - \sum_i \ln(N_i!) \qquad \textit{Maxwell-Boltzmann} \tag{1.2.1}$$

$$\ln W_{BE} = \sum_i \ln(n + N_i - 1)! - \ln N_i! - \ln(n-1)! \quad \textit{Bose-Enistein} \tag{1.2.2}$$

$$\ln W_{FD} = \sum_i \ln(2n)! - (2n - N_i)! - \ln N_i 1 \qquad \textit{Fermi-Dirac} \tag{1.2.3}$$

Imposing the condition for the most probable macrostate we will find the additional constraints, arising from conservation laws (particle number and energy), on the system which have not been directly incorporated into these equations. These can be achieved by taking the variations of the total particle density and total energy of the system given as:

$$\delta \left[\sum_i N_i \right] = \delta N = 0 \tag{1.2.4}$$

$$\delta \left[\sum_i W_i N_i \right] = \sum_i W_i \delta N_i = 0 \tag{1.2.5}$$

Where $\mathbf{w}_i$ is the energy of an individual particle. These additional constraints represent new information about the system. One of the standard methods of solving these equations in order to extract the new information is the method of Lagrange multipliers. Since equations (1.2.4 & 1.2.5)represent quantities which are zero we can multiply them by arbitrary constants (say β_i) and add constants to them (say α_i) equations 1.2.1 - 1.2.3 to get

$$MB : \sum_i [\ln N_i - \ln \alpha 1 + \ln \beta 1 W_i] \, \delta N_i = 0 \quad (1.2.6)$$

$$BE : \sum_i \{\ln \left[\frac{(n + N_i)}{N_i} \right] - \alpha 2 - \beta 2 W_i\} \delta N_i = 0 \quad (1.2.7)$$

$$FD : \sum_i \{\ln \left[\frac{(2n - N_i)}{N_i} \right] - \alpha 3 - \beta 3 W_i\} \delta N_i = 0 \quad (1.2.8)$$

The solutions of these equations are respectively given as:

$$MB : N_i/\alpha_1 = 2n/N_i \quad (1.2.9)$$

$$BE : n/N_i = \alpha_2 \exp(W_i \beta_2) - 1 \quad (1.2.10)$$

$$FD : 2n/N_i = \alpha_3 \exp(W_i \beta_3) + 1 \quad (1.2.11)$$

All that remains is to develop a physical interpretation of the undetermined Parameters α_j andβ_j . For example let us look at Maxwell-Boltzmann statistics how this can be done.

Then,

$$N_i = \alpha_1 e^{-W_i/(kT)} \quad (1.2.12)$$

If the cell volumes of phase space are not all the same size, it may be necessary to weigh the number of particles to adjust for the different cell volumes. We call these weight functions g_i. Then,

$$N = \sum_i g_i N_i = \alpha_i \sum_i g_i e^{-W_i/(kT)} \equiv \alpha_1 U(T) \quad (1.2.13)$$

The parameter $U(T)$ is called the partition function and it depends on the composition of the gas and the parameter T alone. Now if the total energy of the gas is E , then,

$$\begin{aligned} E &= \sum_i g_i W_i N_i \\ &= \sum_i g_i W_i \alpha_1 e^{-w_i/(kT)} \\ &= \frac{\left[\sum_i W_i g_i 1 N_i e^{-W_i/(KT)}\right]}{U(T)} \\ &= NKT \left(\frac{\dot{\l}nU}{\dot{\l}nT} \right) \end{aligned} \tag{1.2.14}$$

For a free particle like that found in a monatomic gas, the partition function U is:

$$U(T) = \frac{(2\pi mkT)^{2/3} v}{h^3} \tag{1.2.15}$$

Where, V is the specific volume of the gas, m is the mass of the particle, and T is the kinetic temperature.

Now using Eqs. 1.2.14 & 1.2.15 we obtain the familiar classical energy of gases given as:

$$E = \frac{3}{2} NkT \tag{1.2.16}$$

This is only correct if, T is the kinetic temperature. Thus we arrive at a self-consistent solution if the parameter T is to be identified with the kinetic temperature. The situation for a photon gas in the presence of material particles is not simple, because the matter acts as a source and sinks for photons. Now we can no longer apply the constraint $dN = 0$. This is equivalent to adding $\ln \alpha_2 = 0$ $(\alpha_2 = 1)$ to the equations of condition. It is also possible to show in a similar fashion that $\beta_2 = 1/(KT)$ in Bose-Einstein statics so that the appropriate solution to eq. 1.2.10 is

$$\frac{N_i}{n} = \frac{1}{e^{(hv/KT)} - 1} \tag{1.2.17}$$

Where the photon energy w_i has been replaced by hv in a volume h^3.

The distinguishability condition of this statistics looks for the number of phase space given by

$$n = \frac{2}{h^3} dx_1 dx_2 dx_3 dp_1 dp_2 dp_3 \tag{1.2.18}$$

We can replace the rectangular form of the momentum volume,$dp_1 dp_2 dp_3$, by It's spherical counterpart $4\pi p^2 dp$ and remembering that the momentum of a photon is, hv/c , we get:

$$\frac{dN}{v} = \frac{8\pi v^3}{c^3 \frac{1}{e^{(\frac{hv}{c})}} - 1} \tag{1.2.19}$$

Here we have replace N_i with dN. This assumes that the number of particles in any phase space volume is small compared to the total number of particles. Since the energy per unit volume dE_v is just $h_v dN/V$, we get the relation known as Planck's law or sometimes as the black body law:

$$dE_v = \frac{8\pi v^2}{c^3} \frac{1}{e^{\left(\frac{hv}{KT}\right)} - 1} dv = \frac{4\pi}{c} B_v(T) \tag{1.2.20}$$

The parameter $B_v(T)$ is known as the Planck function. This, then, is the law for photons which are in strict thermodynamic equilibrium. If we were to consider the Bose-Einstein result for particles and let the number of Heisenberg compartments be much larger than the number of particles in any volume, we would recover the result for Maxwell-Boltzmann statistics. This is further justification for using the Maxwell-Boltzmann result for ordinary gases.

1.2.1 Thermodynamic Equilibrium - Strict and Local

In stars, as throughout the universe, photons outnumber material particles by a large margin and continually undergo interactions with matter. Indeed, it is the interplay between the photon gas and the matter. If both components of the gas(photons and particles) are in statistical equilibrium, then we should expect the distribution of the photons to be given by Planck's law and the distribution of particle energies to be given by the Maxwell-Boltzmann statistics. In some cases, when the density of matter becomes very high and the various cells of phase space become filled, it may be necessary to use Fermi-Dirac statistics to describe some aspects of the matter. When both the photon and the material matter components of the gas are in statistical equilibrium with each other, we say that the gas is in **strict thermodynamic equilibrium**. If the photons depart from their statistical equilibrium (i.e., from Planck's law), but the material matter continues to follow Maxwell-Boltzmann Statistics (i.e., to behave as if it were still in thermodynamic equilibrium), we say that the gas is in **local thermodynamic equilibrium** (LTE)[6]and[10].

1.3 Boltzmann Transport Equation (BTE)

In stellar astrophysical, modeling gas flows around stars or in interstellar space, the ideal gas assumption is very much accurate. Therefore, in our analysis of the stellar evolution including magnetic field dynamism we apply the classical Boltzmann statistical distributions and derive the dynamic equations from Boltzmann transport equations.

The Boltzmann transport equation in six dimenstional postion-velocity phase space basically expresses the change in the phase density within a differential volume, in terms of the flow through these faces, and the creation or destruction of particles within that volume. In the canonical postion-momentum coordinate system, the Boltzmann transport equation (BTE) is given by

$$\boxed{\sum_{i=1}^{3}(\dot{x}_i\frac{\partial f}{\partial x_i}+\dot{p}_i\frac{\partial f}{\partial p_i})+\frac{\partial f}{\partial t}=S} \quad \text{- } BTE \tag{1.3.1}$$

where $f \equiv f(x, \dot{x}; t)$ is the number densty distribution function, S is the rate of particle creation/destruction, $\dot{x}_i = \frac{\partial x_i}{\partial t}$ and $\dot{p}_i = \frac{\partial p_i}{\partial t}$

This equation can be recast in vector notation as

$$\frac{\partial f}{\partial t}+\vec{v}\cdot\vec{\nabla}f+\vec{F}\cdot\vec{\nabla}_p f=S \tag{1.3.2}$$

where $\vec{F}$ is force and $\vec{\nabla}_p$ is the momentum gradient.

In conservative field system since $\vec{F}=-\vec{\nabla}\Phi$ where Φ is a scalar potential (eg. gravitational scalar potential), then, BTE will be given as

$$\frac{\partial f}{\partial t} + \vec{v}.\nabla f - \frac{1}{m}\nabla\Phi.\nabla_v f = S \tag{1.3.3}$$

The potential gradient $\nabla\Phi$ has replaced the momentum time derivative while ∇_v is a gradient with respect to velocity. The quantity m is the mass of a typical particle. It is also not unusual to find the BTE written in terms of the total stokes time derivative

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \tag{1.3.4}$$

where $\vec{v}$ is the flow velocity and $\frac{\partial}{\partial t}$ is the Eulerian time derivative.

If we take $\vec{v}$ to be a six-dimensional 'velocity' and ∇ to be a six-dimensional gradient the BTE becomes

$$\frac{Df}{Dt} = S \tag{1.3.5}$$

1.4 Boltzmann Transport Equation and Liouville's theorem

If the creation/destruction rate of particles is zero ($S = 0$), we will obtain the homogeneous Boltzmann Transport Equation (BTE) given as

$$\boxed{\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f - \frac{1}{m}\nabla\Phi.\nabla_v f = 0} \qquad \textit{The Liouville's theorem} \tag{1.4.1}$$

The physical interpretation of Liouville Equation is the 6N-dimensional analogue of the equation of continuity of an incompressible fluid. It implies that the phase points of the ensemble are neither created nor destroyed.

In Astrophysics it is called the Vlasov equation, or sometimes the Collisionless Boltzmann Equation. It is used to describe the evolution of a large number of collisionless particles moving in a gravitational potential.

In the case of classical statistical mechanics, the number of particles N is very large, (of the order of Avogrados number, for a laboratory-scale system). Setting $\frac{\partial \rho}{\partial t} = 0$ gives an equation for the stationary states of the system and can be used to find the density of microstates accessible in a given statistical ensemble. For eg. in an equilibrium of the Maxwell-Boltzmann statistical distribution ρ is given as

$$\rho \propto e^{H/(k_B T)}$$

where H is the Hamiltonian, T is the temperature and k_B is the Boltzmann constant.

1.5 The Moments of BTE and Conservation Laws

The equations of fluid dynamics can b e derived by calculating moments of the Boltzmann equation for quantities that are conserved in collisions of the particles.

The n^{th} moment of a function f with primary variable x is

$$M_n\left[f(x)\right] = \int x^n f(x)dx \tag{1.5.1}$$

1.5.1 The zeroth moment of BTE and the Continuity Equation

When $n = 0$ as in eq. 1.5.1 we derive the local spatial density given as

$$\boxed{\rho = m\int_{-\infty}^{+\infty} f(x,\vec{v})d\vec{v}} \tag{1.5.2}$$

The related BTE is

$$\int_{-\infty}^{+\infty}\left(\frac{\partial f}{\partial t} + \sum_{i=1}^{3} v_i \frac{\partial f}{\partial x_i} + \sum_{i=1}^{3} \dot{v}_i \frac{\partial f}{\partial v_i}\right) d\vec{v_i} = \int_{-\infty}^{+\infty} S d\vec{v} \tag{1.5.3}$$

The integral of the creation rate S over all velocity space becomes the creation rate for particles in physical space, which we call $\Im$.

In the conservative field system (eq.1.3.3), the zeroth moment of BTE is given as

$$\frac{\partial}{\partial t}\int_{-\infty}^{+\infty} f d\vec{v} + \int_{-\infty}^{+\infty} \vec{v}\cdot\nabla f d\vec{v} - \int_{-\infty}^{+\infty} \frac{\vec{\nabla}\Phi}{m}\cdot\vec{\nabla}_v f d\vec{v}\nabla = \Im \tag{1.5.4}$$

In view of eq. 1.5.2, the first integral of the left hand side of eq.1.5.4 is given by

$$\frac{\partial}{\partial t}\int_{-\infty}^{+\infty} f d\vec{v} = \frac{1}{m}\frac{\partial \rho}{\partial t} \tag{1.5.5}$$

The second integral of the left hand side of eq. 1.5.4 is simplified to yield (See appendix A.1)

$$\int_{-\infty}^{+\infty} \vec{v}\cdot\vec{\nabla} f d\vec{v} = \vec{\nabla}\cdot\int_{-\infty}^{+\infty} \vec{v} f d\vec{v} \tag{1.5.6}$$

Now using eqs. 1.5.3 - 1.5.6

$$\frac{\partial\rho}{\partial t} + m\vec{\nabla}\cdot\int_{-\infty}^{+\infty} \vec{v} f d\vec{v} - \vec{\nabla}\Phi\cdot\int_{-\infty}^{+\infty} \vec{\nabla}_v f d\vec{v} = \Im m \tag{1.5.7}$$

For realistic physical system with finite velocity the second integral of the left hand side equation has to vanish. Then,

$$\frac{\partial\rho}{\partial t} + m\vec{\nabla}\cdot\int_{-\infty}^{+\infty} \vec{v} f d\vec{v} = \Im m \tag{1.5.8}$$

Using the normalized mean flow velocity $\vec{u}$, a measure of the mean flow rate of the material defined as

$$\vec{u} = \frac{\int_{-\infty}^{+\infty} \vec{v} f(\vec{v}) d\vec{v}}{\int_{-\infty}^{+\infty} f(\vec{v}) d\vec{v}} \tag{1.5.9}$$

the zeroth moment of BTE yields the continuity equation

$$\boxed{\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{u}) = \Im m} \tag{1.5.10}$$

In the absence of creation field, the continuity equation gives the familar **local matter conservation**.

1.5.2 The First Moment of BTE and the Euler-Lagrange Equations of Hydrodynamic Flow

Multiplying the BTE by the local particle velocity $\vec{v}$ and integrating over all velocity space will produce momentum like moments given as

$$\int_{-\infty}^{+\infty} \vec{v}\frac{\partial f}{\partial t} d\vec{v} + \int_{-\infty}^{+\infty} \vec{v}\cdot\vec{\nabla} f d\vec{v} + \int_{-\infty}^{+\infty} (\dot{\vec{v}}\cdot\vec{\nabla}_v f) d\vec{v} = \int_{-\infty}^{+\infty} \vec{v} S d\vec{v} \tag{1.5.11}$$

The integral of this equation are not a simple scalars or vectors, but are the vector outer products called tensors.

Using the expressions of local spatial density ρ and the mean velocity $\vec{u}$, the first integral of the left hand side of this equation is given by (See appendix A.2.1.)

$$\int_{-\infty}^{+\infty} \vec{v}\frac{\partial f}{\partial t} d\vec{v} = n\frac{\partial \vec{u}}{\partial t} - (\vec{u}.\vec{\nabla} n + \vec{\nabla} n.\vec{u})\vec{u} + \int_{-\infty}^{+\infty} \vec{u} S d\vec{v} \tag{1.5.12}$$

where $n = \rho/m$, the number density.

The second integral of the left hand side of this moment like equation as discussed earlier is

$$\int_{-\infty}^{+\infty} \vec{v} \cdot \vec{\nabla} f d\vec{v} = \int_{-\infty}^{+\infty} \vec{v}(\vec{v} \cdot \vec{\nabla} f) d\vec{v} \tag{1.5.13}$$

The third integral of the left hand side of eq. 1.5.11 is as worked out in appendix A.2.2 given by

$$\int_{-\infty}^{+\infty} \vec{v}(\vec{v} \cdot \vec{\nabla}_v f) d\vec{v} = n\frac{\vec{\nabla}\Phi}{m} \tag{1.5.14}$$

Using eqs. 1.5.12 - 1.5.14 in eq. 1.5.11 we obtain

$$n\frac{\partial \vec{u}}{\partial t} - (\vec{u}.\vec{\nabla} n + \vec{\nabla} n.\vec{u})\vec{u} + \int_{-\infty}^{+\infty} \vec{v}(\vec{\nabla}.(\vec{v} f) d\vec{v} + n\frac{\vec{\nabla}\Phi}{m} = \int_{-\infty}^{+\infty} S(\vec{v} - \vec{u}) d\vec{v} \tag{1.5.15}$$

Defining the velocity tensor $\overleftrightarrow{u}$ as

$$\overleftrightarrow{u} = \frac{\int_{-\infty}^{+\infty} \vec{v}\vec{v} f(\vec{v}) d\vec{v}}{\int_{-\infty}^{+\infty} f(\vec{v}) d\vec{v}} \tag{1.5.16}$$

Now usung eq. 1.5.15 & eq. 1.5.16 we find

$$\rho\frac{\partial u}{\partial t} + \rho(\vec{u}.\vec{\nabla})\vec{u} + \vec{\nabla}.(\rho(\vec{\vec{u}} - \vec{u}\vec{u})) + n\vec{\nabla}\Phi = \int_{-\infty}^{+\infty} mS(\vec{v} - \vec{u}) d\vec{v} \tag{1.5.17}$$

The quantity $\rho(\vec{u} - \vec{u}\vec{u})$ is called the pressure tensor. Then we define the mean pressure tensor of f(v) as $\vec{\vec{\rho}}$ equal to

$$\overleftrightarrow{p} = \frac{\int_{-\infty}^{+\infty} f(v)(\vec{v} - \vec{u})(\vec{v} - \vec{u}) d\vec{v}}{\int_{-\infty}^{+\infty} f(v) d\vec{v}} \tag{1.5.18}$$

It describes the different between the local flow $\vec{v}$ and the mean flow $\vec{u}$.

Finally the first velocity moment of the BTE is given by

$$\boxed{\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} = -\nabla\Phi - \frac{1}{\rho}\nabla P + \frac{1}{\rho}\int_{-\infty}^{+\infty} mS(\vec{v} - \vec{u}) d\vec{v}} \tag{1.5.19}$$

This set of vector equations are called **Euler-Lagrange equations of hydrodynamic flow**.

On the other hand the assumption of excessive collisions where $\vec{v}$ is considered to be random and the assumption S to be symmetrical implies the integral over all velocity space vanishes. Then Euler-Lagrange equations of hydrodynamic flow of BTE is given by

$$\boxed{\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} = -\nabla\Phi - \frac{\nabla P}{\rho}} \tag{1.5.20}$$

Under the assumption of a nearly isotropic velocity field, $\mathbf{P}$ will be $\mathbf{P}(\rho)$ and an expression known as an equation of state. From equation (3.5.14)the left-hand side is zero. The Euler-Lagrange equations of hydrodynamic flow is

$$\nabla P = -\rho\nabla\Phi \tag{1.5.21}$$

Which is known as the **equation of hydrostatic equilibrium**. This equation is usually an expression of the c**onservation of linear momentum**. The zeroth moment of the BTE results in the conservation of matter, where as the first velocity moment equations which represent the conservation of linear momentum. The second velocity moment represent an expression for the conservation of energy.

1.6 Boltzmann Transport Equation and the Virial Theorem

The Euler-Lagrange equations of hydrodynamic flow are vector equations and represent vectors. We can obtain a scalar result by taking the scalar product of a position vector with the flow equations and integrating over all space with the system. The origin of the position vector is important only in the interpretation of some of the terms which will arise in the expression.

So now the spatial first moment of the Euler-Lagrange equations of hydrodynamic flow eq.

1.5.21.

$$\int_V \vec{r} \cdot \left(\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} + \nabla\Phi + \frac{\nabla P}{\rho} \right) dV = 0 \tag{1.6.1}$$

Working out this equation (See appendix A.3) gives us

$$\frac{1}{2}\frac{d^2 I}{dt^2} - 2T - 2U + \Omega = 0 \tag{1.6.2}$$

Where I is the moment of inertia, T is kinetic energy in bulk motion and U is the internal energy and Ω is the total potential energy of the system.

This equation 1.6.2 is known as the ***Non-averaged form of the virial theorem***. For a system in equilibrium, the time average of eq.**??** removes the accelerative changes of the moment of inertia($(< \frac{d^2 I}{dt^2} >= 0)$) so that

$$2 < T > +2 < U > + < \Omega >= 0 \tag{1.6.3}$$

The theorem which permits is the Ergodic theorem.

1.7 Hydrodynamics

Astrophysical fluids are complex, with a number of different components: neutral atoms and molecules, ions, dust grains (often charged), and cosmic rays. The magnetic fields generally ties all these fluids together except where gradients are very steep, as in shocks. It is the study of the motion of the fluids (liquids and gas). Although fluids are made of particles, it is sufficient to treat a fluid as a continuous substance in many situations. Moreover, in the fluid approximation, we treat the ensemble of particles as a single fluid. To describe an ensemble of particles precisely we need to know the position and velocity of each particle [4] .

1.8 Equation of State for an Ideal Gas

In thermodynamics, an equation of state provides the mathematical relation among variables such as temperature, pressure, density, and internal energy. Equations of state (EOS) are useful in describing the properties of fluids, mixtures of fluids, solids, and even the interiors of stars. For stars, the of state usually describe the relation among pressure(P), temperature(T),density (n: number of density of particles or (ρ) :mass density). Formulation of the Boltzmann Transport Equation (BTE) also provides an ideal setting for the formulation of the equation of state for a gas under wide-ranging conditions. The relationship between the pressure as given by the pressure tensor and the state variables (p,T,ρ) of the distribution function. The pressure tensor is $\mathbf{p}(\vec{u} - \vec{u}\vec{u})$. If f($\vec{v}$) is symmetric in $\vec{v}$, then $\vec{u}$ must be zero (or there exist an inertial coordinate system in which $\vec{u}$ is zero), and the divergence of the pressure can be replaced by the gradient of a scalar, which call the gas pressure, and will be given by

$$\vec{p} = \rho \frac{\int_{-\infty}^{+\infty} v^2 f(v) d\vec{v}}{\int_{-\infty}^{+\infty} f(v) d\vec{v}} \tag{1.8.1}$$

From the Maxwell-Boltzmann statistics, the distribution function of particles, in terms of their velocity, is given by

$$f(v) = constant. \exp(\frac{-mv^2}{2kT}) \tag{1.8.2}$$

The mean pressure is

$$\begin{aligned} \vec{p} &= c\rho \frac{\int_{-\infty}^{+\infty} v^2 \exp(\frac{-mv^2}{2kT})}{c\int_{-\infty}^{+\infty} \exp(\frac{-mv^2}{2kT})} \\ &= \rho \frac{\int_{-\infty}^{+\infty} v^2 \exp(-\alpha v^2) dv}{\int_{-\infty}^{+\infty} \exp(-\alpha v^2) dv} \end{aligned}$$

Where $\alpha = \frac{m}{2kT}$ The integral of the function is

$$\int_{-\infty}^{+\infty} v^2 \exp(-\alpha v^2) dv = \frac{1}{4}\sqrt{\pi}\alpha^{\frac{-3}{2}}$$

and the integral of the denominator is given as

$$\int_{-\infty}^{+\infty} \exp(-\alpha v^2) dv = \sqrt{\frac{\pi}{\alpha}}$$

Then,

$$\bar{p} = \frac{\rho \frac{1}{4}\sqrt{\pi}\alpha^{\frac{-3}{2}}}{\sqrt{\frac{\pi}{\alpha}}}$$

$$\bar{p} = \frac{\rho \frac{\sqrt{\pi}\alpha^{\frac{-3}{2}}}{4}}{2\sqrt{\pi}\alpha^{\frac{-1}{2}}}$$

$$= \rho\frac{\alpha^{-1}}{2} = \frac{\rho}{2\alpha}$$

But, $\alpha = \frac{m}{2kT}$ then, the mean pressure is

$$\bar{p} = \frac{\rho}{\frac{2m}{2kT}} = \frac{\rho kT}{m} \tag{1.8.3}$$

1.8.1 Conservation of mass

Let the mass density of our gaseous fluid be given by ρ, which is a function of position vector r and time t. Consider an arbitrary volume in the fluid V . The total mass in V is

$$M = \int_V \rho dV \tag{1.8.4}$$

The principle of mass conservation is that the mass can't change except by a net flux of material flowing through the boundary S of the volume:

$$\frac{dM}{dt} \equiv \int_V \frac{\partial \rho}{\partial t} dV = -\int_S \rho v \cdot dS \tag{1.8.5}$$

The divergence theorem gives

$$\int_S \rho v \cdot dS = \int_V \nabla \cdot (\rho v) dV \tag{1.8.6}$$

Since V is arbitrary, the integrands of the volume integrals must be the same, And

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \tag{1.8.7}$$

differential form of the statement of mass conservation. The advection derivative $v \cdot \nabla\rho$ is a directional derivative that measures the changes of ρ in the direction of v

If$\nabla \cdot v < 0$, converging flow (**compresion**).

If$\nabla \cdot v > 0$, diverging flow (**dilation**).

If$\nabla \cdot$, the plasma is (**incompressible**).

1.8.2 Conservation of Momentum

The momentum density of the gas $\rho\overrightarrow{u}$ (which is equal to the mass flux). The total momentum in a volume V is therefore the volume integral over $\rho\overrightarrow{u}$.In principle we can do the same trick as above, with a volume integral over $\rho\overrightarrow{u}$ and a surface integral over $\rho\overrightarrow{u}\,\overrightarrow{u}\cdot\overrightarrow{n}$ But here we must also take into account the forces that act on the surface by the gas surrounding the volume. At any position on the surface, the force acting by the gas outside the volume onto the gas inside the volume is -P$\overrightarrow{n}$. We can therefore write:

$$\frac{\partial}{\partial t}\int \rho\overrightarrow{u}dV = -\int_{\partial V}\rho\overrightarrow{u}\,\overrightarrow{u}ds - \int_{\partial V}P\overrightarrow{n}ds \tag{1.8.8}$$

We can write $P\overrightarrow{n}$ using identity and Gauss's theorem, the above equation becomes,

$$\frac{\partial}{\partial t}\int \rho\overrightarrow{u}dV = -\int_{V}\nabla\cdot(\rho\overrightarrow{u}\,\overrightarrow{u} + IP)dV \tag{1.8.9}$$

Where I, is a unit vector identity. Thus it arise at the PDE

$$_{\partial t}(\rho\overrightarrow{u}) + \nabla\cdot(\rho\overrightarrow{u}\,\overrightarrow{u} + IP) = 0 \tag{1.8.10}$$

Where$(\rho\overrightarrow{u}\,\overrightarrow{u} + IP)$is stress tensor of the fluid.in index notation (tensor)this becomes.

$$\partial t(\rho\overrightarrow{u}) + \partial_k(\rho u_i u_k + \delta_{ik}P) = 0 \tag{1.8.11}$$

Here if we include the force of gravity which is the divergence of the gravitational potential Φ, we obtain

$$\partial_t(\rho u_i) + \partial_k(\rho u_i u_k + \delta_{ik}P) = -\rho\partial_i\Phi \tag{1.8.12}$$

1.8.3 Conservation of energy

Energy exists in many forms. Here we concentrate on the two most basic ones: the thermal (internal) specific energy e and the kinetic specific energy $e_{kin} = \frac{u^2}{2}$ So the total energy is the volume integral of $\rho(e + \frac{u^2}{2})$ and the advection of energy the control volume surface is the surface integral of $(\rho(e+\frac{u^2}{2})\overrightarrow{u}\cdot\overrightarrow{n}$ So the energy conservation equation becomes,

$$\frac{\partial}{\partial t}\int \rho(e+\frac{u^2}{2})dv = -\int_{\partial V}\rho(e+\frac{u^2}{2})\overrightarrow{u}\cdot\overrightarrow{n}ds - \int_{\partial V}P\overrightarrow{u}\cdot\overrightarrow{n}ds \tag{1.8.13}$$

Then with the same analogy and using Gauss's theorem, we have

$$\frac{\partial}{\partial t}\int \rho(e+\frac{u^2}{2})dV + \int \nabla\cdot[(\rho e+\frac{1}{2}\rho u^2+P)\overrightarrow{u}] = 0 \tag{1.8.14}$$

Since this must be valid for all control volume V in PDE it becomes,

$$\partial t(\rho e_{tot}) + \nabla\cdot[(\rho e_{tot}+P)\overrightarrow{u} = 0 \tag{1.8.15}$$

This is called energy conservation equation. where,$(e_{tot} = e+\frac{u^2}{2})$ is the total specific energy of the fluid.

Chapter 2

Energy And Momentum Transport in Stellar Interiors

2.1 Introduction

So far, we have been equipping with the basic and fundamental principles of stellar process. These principles are now at hand to use. In fact the plan behind is to use them in our topic focus, Transport phenomena within a medium sized burning star that includes both momentum-energy and particle transport phenomena. It is important to describe the sources of energy, particle so on and the mechanisms involved in the dynamics in time.

2.2 Sources of Energy in Stellar interiors

We have come to that place in the study of stellar structure where we must be mindful of the flow of energy through the star. After all, stars do shine. So far, we have been able to learn much about the equilibrium structure of a star without considering that it is really a structure in a steady state, rather than one in perfect strict equilibrium. The basic reason that we have been able to ignore the flow of energy through the star is that, during a dynamical time, a very small fraction of the stored energy in the star escapes from the star.

Although a star is not, strictly speaking, an equilibrium structure, it comes closer to being one than most any other object in the universe. However, before delving into the actual movement of energy within the star, we must first identify the sources of that energy as well as the processes which impede its flow. This will also give us the chance to discuss the stores of energy within the star since these certainly represent a potential supply of flowing energy with which to generate the stellar luminosity. One of the great mysteries of the late nineteenth and early twentieth century's was the source of the energy required to sustain the luminosity of the sun. By then, the defining solar parameters of mass, radius, and luminosity were known with sufficient precision to attempt to relate them. For instance, it was clear that if the sun derived its energy from chemical processes typically yielding less that 1012 erg/g, it could shine no longer than about 10,000 years at its current luminosity. It is said that Lord Kelvin, in noting that the liberation of gravitational energy could only keep the sun shining for about 10 million years, found it necessary to reject Charles Darwin's theory of evolution because there would have been insufficient time for natural selection to provide the observed diversity of species.[6]

2.2.1 Gravitational Energy

It is generally conceded that the sun has shone at roughly its present luminosity for at least the past 2 billion years and has been in existence for nearly 5 billion years. With this in mind, let us begin our study of the sources of stellar energy with an inventory of the stores of energy available to the sun. Perhaps the most obvious source of energy is that suggested by Lord Kelvin, namely gravitation. From the integral theorems, it is known that the maximum potential gravitational energy is Given by [6]:

$$\Omega = \frac{3}{5}\frac{GM^2}{R} \tag{2.2.1}$$

Remember that the gravitational energy is considered negative by convention; a rather larger magnitude of energy may be available for a star that is more centrally concentrated than a uniform- density sphere. We may acquire a better estimate of the gravitational potential energy by using the results for a polytrope. Chandrasekhar obtains for the gravitational potential energy of a polytrope [6] :

$$\Omega = \frac{3}{5-n}\frac{GM^2}{n} \tag{2.2.2}$$

For a star in convective equilibrium (that is, n = 3/2) the factor multiplying GM^2/R becomes 6/7 or nearly unity. Note that for a polytrope of index $5, \Omega \rightarrow -\infty$ implying an infinite central concentration of material. This is also one of the polytropes for which there exists an analytic solution and $\xi 1 = \infty$. Thus, one has the picture of a mass point surrounded by a mass less envelope of infinite extent. It is not at all obvious that the total gravitational energy would be available to permit the star to shine. Some energy must be provided in the form of heat, to provide the pressure which supports the star. We may use the Virial theorem to estimate how much of the gravitational energy can be utilized by the luminosity. Consider a star with no mass motions, so that the macroscopic kinetic energy T in equation (1.6.3) is zero. Let us also assume that the equilibrium state is good enough that we can replace the time averages by the instantaneous values. Then the Virial theorem becomes

$$2U + \Omega = 0 \tag{2.2.3}$$

Remember that$U$ is the total internal kinetic energy of the gas which includes all motions of the particles making up the gas. Now we know from thermodynamics that not all the internal kinetic energy is available to do work, and it is therefore not counted in the internal energy of the gas. The internal kinetic energy density of a differential mass element of the gas is

$$dU = \left(\frac{3}{2}\right) RTdm = \left(\frac{3}{2}\right)(c_p - c_v)Tdm \tag{2.2.4}$$

Where: the relationship of the gas constant R to the specific heats was given in Chapter 1 [equation (1.2.15)]. However, from the definition of specific heats [equation (2.2.4)], the internal heat energy of a differential mass element is

$$dU = c_v T dm \tag{2.2.5}$$

Eliminating Tdm from equations (**??** and (**??** and integrating the energy densities of the entire star, we get

$$U = \left(\frac{3}{2}\right) \langle \gamma - 1 \rangle U \tag{2.2.6}$$

Where: U is the total internal heat energy or just the total internal energy. The quantity $< \gamma - 1 >$ is the value of$\gamma - 1$ averaged over the star. For simplicity, let us assume that γ is constant through out the star. Then the Virial theorem becomes

$$3(\gamma - 1)U + \Omega = 0 \tag{2.2.7}$$

Remembering that the total energy E is the sum of the internal energy and the gravitational energy, we can express the Virial theorem in the following ways:

$$\begin{aligned} U &= \frac{-\Omega}{3(\gamma - 1)} \\ E &= -(3\gamma - 4)U \\ E &= \frac{3\gamma - 4}{3(\gamma - 1)}\Omega \end{aligned} \tag{2.2.8}$$

It is clear that for $\gamma > 4/3(that is, n < 3)$, the total energy of the star will be negative. This simply says that the star is gravitationally bound and can be in equilibrium. So we can look for the physically reasonable poly tropes to have indices less than or equal to3. The case of $n = 3$ is an interesting one that we shall return to later, for it represents radiation dominated gas. In the limit of complete radiation dominance, the total energy of the configuration will be zero.

2.2.2 Rotational Energy

While utilizing the Virial theorem to estimate the gravitational energy, we set the mass motions of the star to zero so that the macroscopic kinetic energy T was zero. However, stars do rotate, and we should not forget to count the rotational energy in the inventory of energies. We may place a reasonable upper limit on the magnitude of the rotational energy that we can expect by noting that

1) The moment of inertia of the star will always be less than that of a sphere of uniform density and

2) There is a limit to the angular velocity ωc at which the star can rotate. Thus, for a centrally condensed star

$$E_{rot} \leq \frac{1}{2} I_{max} \omega_{max}^2 \tag{2.2.9}$$

2.2.3 Nuclear Energy

Of course, the ultimate upper limit for stored energy is the energy associated with the rest mass it self. It is also the common way of estimating the energy available from nuclear sources. Indeed, that fraction of the rest mass which becomes energy when four hydrogen atoms are converted to one helium atom provides the energy to sustain the solar luminosity. Clearly most of the energy to be gained from nuclear fusion occurs by the conversion of hydrogen to helium and less than one-half of that energy can be obtained by all other fusion processes that carry helium to iron. All these entries are generous upper limits. For example, the sun rotates at less than .5 percent of its critical velocity, it was never composed of 100 percent hydrogen and will begin to change significantly when a fraction of the core hydrogen is consumed, and not all the gravitational energy could ever be converted to energy for release. In any event, only nuclear processes hold the promise of providing the solar luminosity for the time required to bring about agreement with the age of the solar system as derived from rocks and meteorites. However, the time scales of Table 3.2 are interesting because they provide an estimate of how long the various energy sources could be expected to maintain some sort of equilibrium configuration [6].

2.2.4 Nuclear Reaction And Elemental Abundance In Stellar Interiors

Synthesis Of Elements In The Star

There are eight possess in the chemical reaction in the core of the star during it's evolution.

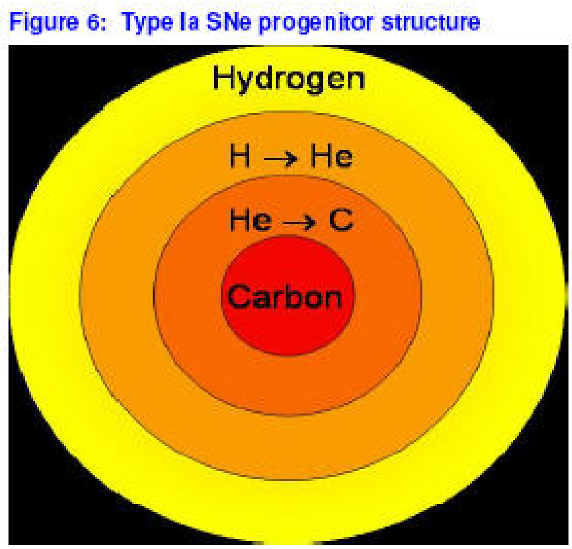

Figure 2.1: The figure shows that the onion skin model of the stellar structure which shows the chemical reaction in the core produces the heavier element plus energy in the form of radiation

1. Hydrogen burning

Following the fundamental papers by Bethe and Critchfield (1938) and Bethe (1939), B^2FH described the laboratory experiments and the derived reaction rates of the various proton captures of the *pp* chain and the CNO cycle. The details of the rates of the individual reactions in the *pp* chain determine the energy spectrum of the resulting neutrinos, which is crucial in attempts to understand the solar neutrino problem. In addition they discussed the p capture by the neon isotopes to produce Na(also mentioned in Bethe's 1939 paper). In Secs. IV and VII, Parker and Champagne bring us up to date on the laboratory rates of hydrogen burning reactions from the pp reaction through the CNO cycle, the NeNa cycle and the MgAl cycle. In addition the hot CNO cycle plays an important role in nova explosions, while the rp process discussed by Boyd in Sec. XIII may be responsible for the production of certain p-rich isotopes[12]and[2].

2. Helium burning

By the time of B^2FH helium burning to produce^{12}C pik (1951) and its rate estimated by Salpeter (1952). In Sec. VI. G. Hale describes the present state of the experiments that located and had been suggested by O measured the width of the vital 7.65 MeV level in ^{12}C(predicted by Hoyle) and the complicated combination of experiment and theory that is necessary to estimate the rate of the $^{12}C(\alpha,\gamma)^{16}O$ reaction. The rate of the $^{12}C(\alpha,\gamma)^{16}O$ reaction relative to the $3-\alpha$ process deter- mines the carbon/oxygen ratio in massive stars, and this is crucial for the later evolution of such a star and its resulting nucleosynthesis. Unfortunately, 40 years after B^2FH, the rate of the $^{12}C(\alpha,\gamma)^{16}O$reaction is still not well determined. Fortunately the material that is returned to the interstellar medium by stars that are less massive than $11M\bigodot$ and evolve into whitedrawfs has been enriched by matter that has experienced only partial helium burning, so the uncertainty in the ratio of reaction rates plays a minor role[12][2].

3. The α process

B^2FH suggested that further α captures could extend nucleosynthesis beyond ^{16}O to $^{20}Ne,^{24}Mg$, etc., up to the very stable, doubly magic nucleus, 40 Ca. However,after experiments showed that the $^{16}O(\alpha,\gamma)^{20}Ne$ rate is very slow in stellar interiors, it became evident that carbon and oxygen burning are responsible for the origin of species from Ne

to$S$, with the nuclei consisting of integral numbers of α particles dominating the abundance curve in this region[12][2].

4. The e process

At very high temperatures, about 4 or $5 \times 10^9 K$, so many reactions take place that the nuclei settle down to statistical equilibrium dominated by the most tightly bound nuclei around ^{56}Fe. Such conditions are reached only in supernovae. The observation of γ rays from SN1987A due to the deexcitation of ^{56}Fe, resulting from the β decay of ^{56}Ni, has demonstrated the importance of the production of iron-peak species in supernova explosions. Modern calculations of the iron-peak abundances are discussed by Meyer in Sec. XV[12][2].

5. The s process

Beyond the iron-peak, utilizing neutrons produced by reactions such as $^{13}C(\alpha, n)^{16}O$ and $^{22}Ne(\alpha, n)^{25}Mg$, nuclei can be produced along or adjacent to the valley of stability via a process in which sequential neutron captures take place on a time scale that is slow compared to the beta-decay lifetime of these nuclei. This process can continue all the way up to lead and bismuth; beyond bismuth the resulting nuclei alpha decay back to Pb and Tl isotopes. In Secs. X and XI Ka ?ppeler and Smith bring us up to date on laboratory measurements, stellar models (also discussed in Sec. III by Iben), and abundance studies of the s-process elements [12][2].

6. The r process

B^2FH showed that, in addition to the s process, there must be another neutron capture process in which the sequential neutron captures take place on a time scale which is much more rapid than the beta decay of the resulting nuclei. This process produces the much more neutron-rich progenitors that are required to account for the second set of abundance peaks that are observed about 10 mass units above the s-process abundance peaks corresponding to the neutron magic numbers, $N = 50$ and 82. Historically, the r process has been associated with SN explosions, and in the past decade interest has focused more specifically on the neutrino-heated atmosphere surrounding the newly formed neutron star as the r-process site. In Sec. XII Hoffman and Timmes review both the physics and astrophysical scenario of rapid neutron capture during the explosion of massive supernovae[12][2].

7. The p process

There are some relatively rare proton-rich nuclei such as 92 Mo that are impossible to produce by n capture alone. They may be produced by p capture at high enough temperatures to overcome the huge coulomb barrier or by (γ, n) reactions during supernova explosions. Recent work on (p, γ) and (γ, n) reactions including the rp process are reviewed by Boyd in Sec. XIV [12][2].

8. The x process

None of the above processes can produce D, Li, Be, or B, all of which are burned by p capture at low temperatures in stars but hardly ever (except for 7 Li) produced in stars. B^2FH did not know how they were produced so they ascribed their synthesis to the x process. Modern cosmological models of big bang nucleosynthesis are tuned to produce D,^3He,^4He , and some ^7Li to fit observations of these species in very metal-poor stars and other astrophysical sources. The observations and theories of production of ^7Li, Be, and B in stars are reviewed by Boesgaard in Sec. V. For a brief review of the production of D, ^3He, ^4He, and ^7Li in the early universe with further references see Olive and Schramm (1996)[12][2].

2.3 Time Scale

One of the most useful notions in stellar astrophysics for establishing an intuitive feel for the significance of various physical processes is the time required for those processes to make a significant change in the structure of the star. To enable us to estimate the relative importance of these processes, we shall estimate the time scales for several of them. In Chapter 2 we used the free-fall time of the sun to establish the fact that the sun can be considered to be in hydrostatic equilibrium. The statement was made that this time scale was essentially the same as the dynamical time scale. So let us now turn to estimating the time required for dynamical forces to change a star[6].

2.3.1 Dynamical Time Scale

The Virial theorem provides us with a ready way of estimating the dynamical time scale, for in the form given, it must hold for all $1/r^2$ forces. Consider a star which is not in equilibrium because the internal energy is too low. As it enters the non-equilibrium condition, the star's kinetic energy will also be small. Thus, the Virial theorem would require

$$\begin{aligned} \frac{d^2I}{dt^2} &\approx \Omega \\ \frac{d^2I}{dt^2} &\approx \frac{1}{\tau_d^2} \\ \tau_d^2 &= -\frac{1}{\Omega} = \frac{\frac{2}{5}MR^2}{\frac{3}{5}\frac{GM^2}{R}} \\ \tau_d &= \left(\frac{\frac{2}{3}R^3}{GM}\right)^{\frac{1}{2}} \end{aligned} \tag{2.3.1}$$

Where: $\tau_d = dynamicaltime$ Considering a rapid collapse and taking the average value of accelerative change in moment of inertia the dynamical time is given by equation [equation (2.3.1)] above[6].

2.3.2 Kelvin -Helmholtz Time Scale

Now we turn to some of considerations that led Lord Kelvin to reject the Darwinian theory of evolution. These involve the gravitational heating of the sun. If you imagine the early phases of a star's existence, when the internal temperature is insufficient to ignite nuclear fusion, then you will have the physical picture of a cloud of gas which is slowly contracting and is thereby being heated. Ultimately some of the energy generated by this contraction will be released from the stellar surface in the form of photons. As long as the process is slow compared to the dynamical time scale for the object, the Virial theorem equation hold and $< T > \approx 0$. Thus

$$\frac{1}{2}\langle\Omega\rangle = -\langle U\rangle \tag{2.3.2}$$

which implies that one-half of the change in the gravitational energy will go into raising the internal kinetic energy of the gas. The other half is available to be radiated away. This was the mechanism that Lord Kelvin proposed was responsible for providing the solar luminosity and he suggested a lifetime for such a mechanism to be simply the time required for the luminosity to result in a loss of energy equal to the present gravitational energy. If we estimate the latter by assuming that the star of interest is of uniform density, then

$$\tau_{KH} = \frac{-\Omega}{L} = \frac{\frac{5}{3}GM^2}{RL} \tag{2.3.3}$$

This equation is known as Kelvin-Helmholtz gravitational contraction time and it is same as the life time obtained from the gravitational energy. Since the star is simply cooling off and having it's internal energy re-supplied by gravitational contraction, this ties scale can be referred as thermal time scale τ_{th} as the time required for luminosity to result in energy loss is equal to internal heat energy, and we can relate that to Kelvin-Helmholtz time by means of Virial theorem as:

$$\tau_{th} = \frac{\langle\Omega\rangle}{L} = \frac{\langle\Omega\rangle}{\langle 3(\gamma-1)\rangle L} \approx \frac{\tau_{KH}}{\langle 3(\gamma-1)\rangle} \tag{2.3.4}$$

Where τ_{KH} is Kelvin Helmoltz time scale[6].

2.3.3 Nuclear Time Scale

In the beginning of this section we estimated the lifetime of the sun which could result from the dissipation of various sources of stored energy. By far the most successful at providing a long life was nuclear energy. The conversion of hydrogen to iron provided for a lifetime of some 140 billion years. However, in practice, when about 10 percent of the hydrogen is converted to helium in stars like the sun, major structural changes will begin to occur and the star will begin to evolve. We can define a time scale for these events in a manner analogous to our other time scales as time scale on which the star will exhaust its supply of nuclear fuel if it keeps burning it at the current rate: Energy release from fusing one gram of hydrogen to helium is $6x10^{18}erg$, so:

$$\tau_n = \frac{k_n M c^2}{L} \tag{2.3.5}$$

Where:k, Mc^2 and L are opacity, total energy and luminosity respectively[6]

Hydrogen Burning Time Sc-ale

Hydrogen burning is responsible for all of the energy production of stars on the main sequence. After stars have evolved off the main sequence following the development of a chemical inhomogeneity, hydrogen burning in a shell still remains an important energy source. Thus the synthesis of elements in hydrogen burning is going on continuously, and the range in timescales for particular stars is dependent only on their initial masses after condensation, and the point in their evolution at which they eject material which has been synthesized. Thus these time-scales may range from $\sim 10^6$ years for massive O and B stars to times which are of the order of though less than the age of the galaxy (since we have no evidence for the existence today of primeval stars of pure hydrogen).

Helium Burning -Time Scale

It is believed that helium burning takes place in stars which have evolved onto the giant branch of the HR diagram. In this region central temperatures of $\sim 10^8$ degrees and. densities of $\sim 10^5$ g/cc are reached in the helium core, according to the theoretical calculations of Hoyle and Schwarzschild (Ho55), and under these conditions synthesis of C^{12} becomes possible (Sa52, Co57, Sa57). The further hehum-burning reactions leading to production of 0^{16}, Ne^{20}, andMg^{24} also can take place under suitable conditions though the amounts of these isotopes synthesized will decrease as the Coulomb barriers get larger (cf. Sec. III). The timescale for helium burning is therefore governed by the lifetime of stars subsequent to their becoming red giants. Since their mode of evolution off the giant branch is not yet understood, it is difficult to make accurate estin1ates of this time, but calculations of Hoyle and Schwarzschild and Hoyle and Haselgrove (Ho56a) suggest that a time-scale of $10'108$ years for stars to evolve in this region is reasonable.

2.4 Means of energy transport in stellar interiors

2.4.1 Means of energy transport in Non Accreting stellar interiors

The energy that a star radiates from its surface is generally replenished from sources or reservoirs located in its hot central region. This represents an outward energy flux at every layer in the star, and it requires an effective means of transporting energy through the stellar material. This transfer of energy is possible owing to a non-zero temperature gradient in the star. While radiation is often the most important means of energy transport, and its is always present, it is not the only means. In stellar interiors, where matter and radiation are always in local thermodynamic equilibrium (Chapter 2) and the mean free paths of both photons and gas particles are extremely small, energy (heat) can be transported from hot to cool regions in two basic ways:

1). Random thermal motions of the particles - either photons or gas particles - by a process that can be called heat diffusion. In the case of photons, the process is known as radiative diffusion. In the case of gas particles (atoms, ions, electrons) it is usually called heat conduction.

2). Collective (bulk) motions of the gas particles, which is known as convection. This is an important process in stellar interiors, not only because it can transport energy very efficiently, it also results in rapid mixing. Unfortunately, convection is one of the least understood ingredients of stellar physics. The transport of energy in stars is the subject of this chapter, which will lead us to two additional differential equations for the stellar structure[6].

Chapter 3

Transport Phenomena In Stellar Interiors

3.1 Introduction

The theory of stellar structure and evolution has been a cornerstone of astrophysics since its inception; stars even gave their name to this branch of physics. In order to put the methods developed and simulations performed in this thesis into context, we give a brief overview of the basic theory of stellar evolution and the numerical methods used for research in this field. This section loosely follows the detailed introduction given in Kippenhahn et al. (2013)[3].

Energy Generation In The Stellar Interior And Transport Phenomena

Stars in various stages of their life are basically the sole producer of all elements heavier than lithium in the Universe. Through nuclear reactions enabled by the high densities and temperatures in their interior, they convert the primordial elements H, He, and Li to heavier elements. Even though this conversion happens inside stars, it is the driving mechanism of the evolution of all baryonic matter in the Universe. This is because stars feed a part of the elements they produce back to the interstellar medium, from which new generations of stars and planetary systems are formed. Stars go through different so-called burning phases in which the conditions for fusion of a fuel element are met. For example, during the H-burning phase they turn hydrogen into helium, in the He-burning phase they produce carbon from helium. Nuclear burning occurs at the core or in radial shells around the core. Different phases of shell and core burning can overlap. The last of these phases is Si-burning, which ultimately forms iron. At this point the core of the star will collapse due to effects discussed below. Stars of low and intermediate mass ($\leq 8M_{\odot}$) never reach the last burning stages. In a very simplified picture this can be understood by the following argument. When a star has depleted the fuel for a burning stage at its core, nuclear energy generation stops and the core starts to contract. If the critical density and temperature for the ignition of the next burning phase can be reached, it will commence releasing energy through this burning process. If it cannot be reached, the core will stay inert, while the previous burning stages still occur in shells around the core. The final outcome of this system is a white dwarf star, which is the remaining degenerate core, surrounded by a planetary nebula formed by the outer layers of the star that were gravitationally unbound.Even for supernova explosions the structure of the progenitor star is of importance. In this introduction we give a brief overview of models of stellar evolution and discuss how multidimensional hydrodynamics could provide an improvement to some of the deficiencies of these models.

3.2 Transport Phenomena And Evolution Of Star

During central hydrogen burning on the main sequence, we have seen that stars are In thermal equilibrium ($\tau_{nuc} \approx \tau_{KH}$) with the surface luminosity balanced by the nuclear power generated in the center. After the main sequence a hydrogen-exhausted core is formed inside which nuclear energy production has ceased. This inert helium core is surrounded by a hydrogen-burning shell and a H-rich envelope. For such an inert core to be in thermal equilibrium requires $l_m = R_m Q_{nuc} dm = 0$ and hence $dT/dr \alpha l = 0$, which implies that the core must be isothermal to remain in TE. Such a stable situation is possible only under certain circumstances. If ideal-gas conditions hold in the core, then an upper limit exists for the mass of the core Mc relative to the total mass of the star. If $qc = Mc/M$ exceeds this limit, then the pressure within an isothermal core cannot sustain the weight of the overlying envelope. This was first pointed out by Schonberg and Chandrasekhar in 1942, who computed the limiting core mass fraction to be Astrophysicists and theoretical physicists have done lots of work on this question. We won't discuss any of the details but it's worth summarizing results very sloppily! (with apologies to astronomy classes) The basic idea is that all elements are produced by reactions in stars. The material in our sun (and solar system) has been cycled through at least several stars. Reasonable-since age of universe is substantially greater than that of our solar system[1]and [4].

3.2.1 Transport Phenomena In Non Accreting Young Star And It's Nuclear Evolution

The hydrogen burning phase

In this section we discuss in some detail the evolution of stars during hydrogen-core burning, until the onset of helium burning. Based on the above section, qualitative differences are to be expected between low-mass stars ($M \sim< 2M_{\odot}$) on the one hand and intermediate- and high-mass stars ($M \sim> 2M_{\odot}$) on the other hand. Therefore we discuss these two cases separately, starting with the evolution of higher-mass stars because it is relatively simple compared to low-mass stars. We use two detailed stellar evolution sequences, for stars of $5M_{\odot}$and $1M_{\odot}$ respectively, as examples for the general evolutionary behavior of stars in these two mass ranges[8].

According to Bethe (1939)Hydrogen burning will release energy as the following reaction.

The Proton Proton Chain

There are three PP chain. This are (PPI, PPII and PPIII)

PPI chain:

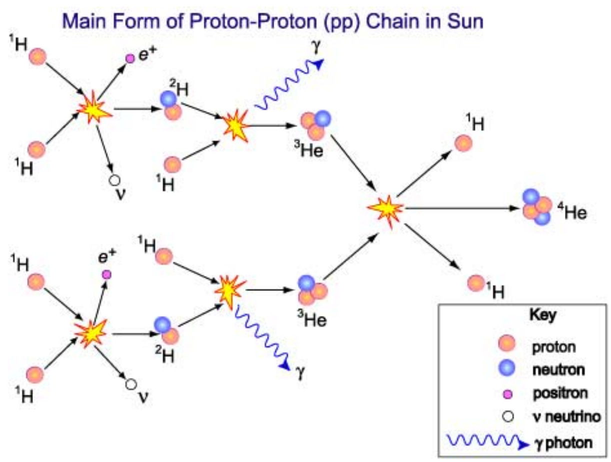

Figure 3.1: This figure shows the PP chain reaction by which H molecules fuse to He

$$1)\ {}^{1}H + {}^{1}H \rightarrow {}^{2}D + e^{+} + \nu + 1.44MeV$$

$$2)\ {}^{2}D + {}^{1}H \rightarrow {}^{3}He + \gamma + 5.49MeV$$

$$3)^{3}He + {}^{3}He \rightarrow {}^{4}He + {}^{1}H + {}^{1}H + 12.85MeV$$

The total energy released by one hydrogen molecule is 19.51Mev. But, for each conversion of ${}^{4}H \rightarrow {}^{4}He$, The reactions (1) and (2)have to occur twice and reaction (3) once. Then the neutrino in (1) carries away 0.26 MeV leaving 26.7 MeV to contribute to the luminosity. Reaction (1) is a weak interaction $\rightarrow$ bottleneck of the reaction chain .Therefore the energy released as radiation is 26.7Mev for each fusion of hydrogen molecule to helium. Typical reaction times for Temperature $T = 3 \times 10^{7}K$ and initial mass $M_0 = M_{\odot}$ are:

$$1)\ 1 \times 10^{10} yr$$

$$2)\ 6s$$

$$3)\ 10^{6} yr$$

PPII chain:

$$1)\ {}^{3}He +^{4} He \rightarrow {}^{7}Be + \gamma + 1.59Mev$$

$$2)\ {}^{7}Be + e^{-} \rightarrow {}^{7}Li + \nu + 0.86MeV$$

$$3)^{7}Li + {}^{1}H \rightarrow {}^{4}He +^{4} He + 17.35MeV$$

The total energy released by this process is 19.8Mev

PPIII chain:

$$1)\ {}^{7}Be + {}^{1}H \rightarrow {}^{8}B + \gamma + 0.14MeV$$

$$2)\ {}^{8}Be^{-} \rightarrow {}^{8}Be + e^{+} + \nu$$

$$3)\ {}^{8}Be \rightarrow {}^{4}He + {}^{4}He + 18.07MeV$$

The total energy released by this process is 18.21Mev. In both PPII and PPIII, a ${}^{4}He$ atom acts as a catalyst to the conversion of

${}^{3}He +^{1} H \rightarrow^{4} He + v$.

E_{total} is the same in each case but the energy carried away by the neutrino is different.

All three PP chains operate simultaneously in a H burning star containing significant ${}^{4}He$: details of the cycle depend on density, temperature and composition.

THE CNO CYCLE

For the core temperature ($T < 10^{8}K$) Carbon, nitrogen and oxygen serve as catalysts for the conversion of H to He.

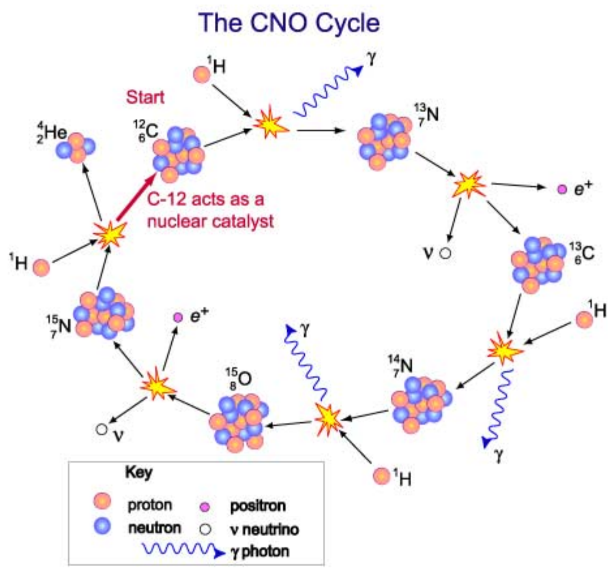

Figure 3.2: This figure shows that fore stars with mass greater than $2.5M_{\odot}$
The CNO cycle act as catalyst in the reaction H fuse to He

$$^{12}C + ^{1}H \rightarrow ^{13}N + \gamma$$

$$^{13}N \rightarrow ^{1}3C + e + +\nu$$

$$^{13}C + ^{1}H \rightarrow ^{14}N + \gamma$$

$$^{14}N + ^{1}H \rightarrow ^{15}O + \gamma$$

$$^{15}O \rightarrow ^{15}N + e + +\nu$$

$$^{15}N + ^{1}H \rightarrow ^{12}C + ^{4}He$$

Let us consider the young and burning star with the following Boundary condition

The initial mass of the star is (M_o)

Its elemental abundance is hydrogen (H)

The initial energy of the star is E_o. Where $E_o = M_o c^2$

The initial time is t_o. Where $t_o = 0$

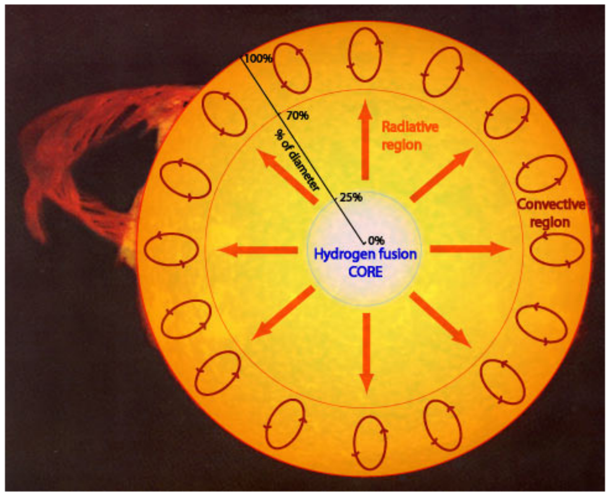

Figure 3.3: This figure shows that the hydrogen react in the core and emitting nuclear energy to support the star against gravity

The central density is ρ_c

The gravitational energy is dominant at this stage and the star collapse to the center and this increases the thermal energy of the core. So that,when the temperature of the core exceed the effective temperature T_{eff}, for hydrogen $T_{eff} =\sim> 10^6 k$ the Hydrogen molecules react to give Helium and some energy released as radiation.The energy will be transported to the surface from the core, and the particle or gas transport from the surface in ward dew to gravitational energy. The burning of hydrogen at the core is the source of radiation which exerts pressure to support the star from further collapse.At the end of Hydrogen burning, the evolution of the star can be given as:

The timescale τ_{MS} that a star spends on the main sequence is essentially the nuclear timescale for hydrogen burning, Another way of deriving essentially the same result is by realizing that, in the case of hydrogen burning, the rate of change of the hydrogen abundance X is related to the energy generation rate ϵ_{nuc}

$$-\epsilon_{nuc} = \frac{dX}{dt} qH \tag{3.2.1}$$

Were ϵ_{nuc}, X and qH are nuclear energy,the rate of change of Hydrogen abundance and the effective energy per unit mass of Hydrogen chain $({}^1_4H \longrightarrow^4 He + 2e^+ + 2v)$ $(qH = QH/4m_u)$. The change in mass during Hydrogen burning can be calculated as:

$$\frac{dM_H}{dt} = -\frac{4m_u}{QH}\langle L \rangle \tag{3.2.2}$$

where $\langle L \rangle$ is the time average luminosity

Here: M_H is the total mass of the star at the end of hydrogen burning. Note that while eq. (3.2.1) only strictly applies to regions where there is no mixing, eq (3.2.2) is also valid if the star has a convective core, because convective mixing only redistributes the hydrogen supply. If we now integrate over the main sequence lifetime we obtain for the total mass of hydrogen consumed. There fore

$$\bigtriangledown M_H = \frac{4m_u}{QH}\int_0^{\tau_{Ms}} Ldt = \frac{4m_u}{QH}\langle L_{\tau_{Ms}} \rangle \tag{3.2.3}$$

Now the time evolution of the main sequence is given by

$$\tau_{Ms} = X_o qc \frac{QH}{4m_u}\frac{M}{\langle L \rangle} \tag{3.2.4}$$

Where qc is the effective core mass fraction and X_o is the initial Hydrogen mass fraction. At the end of hydrogen burning the change in energy can be given by $E_H = E_o - \epsilon_{nuc}$. This energy is lost by the star during hydrogen burning. This energy will be transported to the surface from the core by radiation and convection.

The main sequence time τ_{Ms} of the star is given by:

$$\tau_{Ms} = 10^{10} yr \left(\frac{M_0}{M_\odot}\right)^{-3} \tag{3.2.5}$$

These indicate the life time of the star is depend on initial mass. For the star with initial mass M_0 is equal to $25M_\odot$ the main sequence time is about 10^6 years. And for the star with initial mass M_0 is equal to mass of the sun $M_0 = M_\odot$, $\tau_{Ms} = 1 \times 10^{10}$years. This indicate that the evolution time depends on the original mass of the star.

The mass versus $\tau_{Ms} graph$

The mass and the main sequence life time is related by the equation

$$\tau_{Ms} = 10^{10} yrs (M/M_{\odot})^{-3} \quad (3.2.6)$$

By generating the data we had drawon the following graph of the relation and compare the graph with the theoretical values.

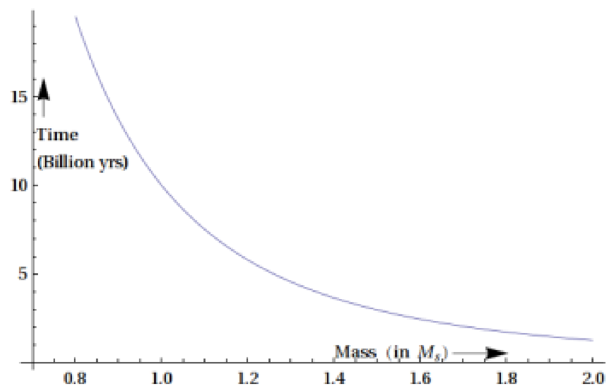

Figure 3.4: This Graph shows the mass and the main sequence life time relation of the burning star

3.2.2 The luminosity mass relation

The relation between mass and luminosity is given by the following equation.

$$L/L_{\odot} = (M/M_{\odot})^{3.5} \quad (3.2.7)$$

This equation shows that luminosity is directly proportional to mass. So if the star has large initial mass the rate of energy released from the surface of the star is also large.

3.2.3 Table showing The Relation Between Different Parameters

Note: This table had been taken from the reference [on line Astronomy e Text :Stellar evolution]. Hence our graph of mass main life time compared and fate with this table

$M/M_\odot$	$L/L_\odot$	$R/R_\odot$	τ_{ms}	$T_{eff}k$
0.01	0.003	0.16	2×10^{12}	2900
0.5	0.03	0.6	2×10^{11}	3800
0.75	0.3	0.8	3×10^{10}	5000
1	1	1	1×10^{10}	6000
1.5	5	1.4	2×10^{9}	7000
3	60	2.5	2×10^{8}	11000
5	600	3.8	7×10^{7}	17000
10	10000	5.6	2×10^{7}	22000
15	170000	6.8	1×10^{7}	28000
25	80000	8.7	7×10^{6}	35000
60	790000	15	3.4×10^{6}	44000

From the mass energy relation we can get the amount of energy lost by any star in the main sequence.

The mass loos By The Sun And Estimation Of Its Main Sequence Life Time

In the sun 1kg of hydrogen is react to produce 0.9963kg of helium. hence 0.007kg of hydrogen is lost or converted to energy. This is about 0.7 percent of total mass of hydrogen. Hence: Any star loos 0.7 percent of it's initial mass. This lost mass converted in to the γ ray at the core of the star which is then transported out ward to the surface and then emitted from the surface as EM wave of visible spectrum. The γ ray from the core absorbed due to interaction with hydrogen column as it transported out ward and its energy drop by exerting radiation pressure which prevent the star from further collapse by gravitational force[7]. In a fusion reaction in the Sun 0.7 percent of the mass involved is converted to energy, and if 10 percent of the mass of the Sun is available for fusion, then the total mass

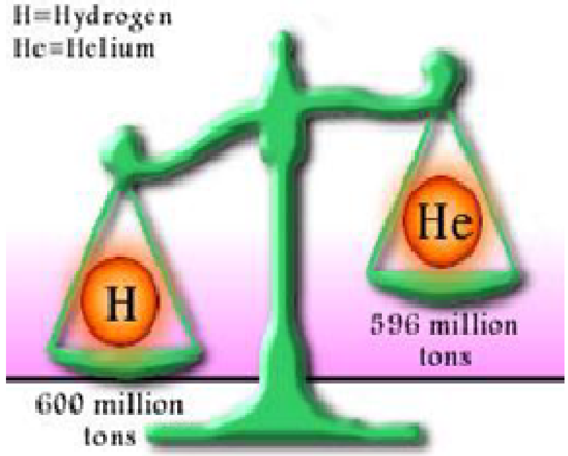

Figure 3.5: The Sun converts 600 million tons of hydrogen into 596 million tons of helium every second. The difference in mass is 4million tons is converted into energy in every second. The Sun will continue burning hydrogen during 5 billions years. Energy released by H-burning: $6.45 \times 10^{18} ergg^{-1}$ = 6.7 MeV/nuc Solar Luminosity: $3.85 \times 10^{33} ergs^{-1}$

that can be converted to energy is:

$$M_{loos} = 0.1 \times 0.007 \times 3.8 \times 10^{26} kg = 2.66 \times 10^{23} kg \tag{3.2.8}$$

In order to use this in the equation E=mc^2 we need to know the speed of light, which is $c = 3 \times 10^8 m/s$

$$E = (2.66 \times 10^{23} kg) \times (3 \times 10^8 m/s)^2 = 2.394 \times 10^{40} J \tag{3.2.9}$$

In other way also we can calculate the energy released by formation of a single PP chain reaction or CNO cycle reaction is 26.7Mev this is the same as E = ?m c2 so substituting in values gives

$$\begin{aligned} E &= \triangle Mc^2 \text{substituting this we get} \\ &= 0.0286(1.66 \times 10^{-27})(3 \times 10^8)^2 \qquad (3.2.10) \\ &= 4.3 \times 10^{-12} J \end{aligned}$$

Then, to figure out how long this lifetime is, you need to know the rate at which the Sun is losing energy. We know this from the Sun's luminosity, which is 3.78×10^{26}Joules/second. We can put this all together to figure out how long the Sun can shine (i.e., lose energy) at its present rate. The Sun loses energy at rate of 3.78×10^{26} Joules/second, and if it has 1.26×10^{44} Joules of available energy, then we can divide the two to determine how long the Sun can shine:

$$\tau_{Ms} = (3.78 \times 10^{26} J/s)/(1.26 \times 10^{44} J) = 3.33 \times 10^{17} s \tag{3.2.11}$$

We should convert this to more appropriate units of years,

$$\tau_{Ms} = \frac{3.33 \times 10^{17} s}{(60s/min) \times (60min/hr) \times (24hr/day) \times (365day/year)]} = 1.05 \times 10^{10} yr \tag{3.2.12}$$

Or 10.5 billion years.

The Helium burning phase

F0r stars with an initial mass greater than $8M_\odot$, the process of chemical reaction continues forward at the end of hydrogen burning the core collapse soon because the radiation pressure will be dominated by gravity and the temperature of the core rise. At this stage the core is Helium abundant and it get sufficient thermal energy [8]. At the temperature $T \sim 2 \times 10^8 k$, the Burning of Helium becomes possible.

If ^4He is sufficiently abundant, two further chains can occur. These are: When the helium ran out, the core collapse once again and the temperature increases so that the carbon, oxygen and nitrogen burning continues and the heavy elements like iron synthesized. This is the end of life of star. At the end of helium burning,our star has mass M_{He} and Energy E_{He}

$$\bigtriangledown M_{He} = \frac{4m_u}{QHe} \int_0^{\tau_{He}} L dt = \frac{4m_u}{QHe} \langle L_{\tau_{He}} \rangle \tag{3.2.13}$$

The energy lost by the star at the age of burning helium is greater than those at the age of hydrogen burning. Hence the luminosity increase.

The Carbon, Oxygen And Nitrogen Burning Phase

The seed nuclei are believed to be predominantly ^{12}C and ^{16}O: these are the main products of He burning, a later stage of nucleosynthesis. Once the silicon burning phase has produced an iron core the fate of the star is sealed. Since iron will not fuse to produce more energy, energy is lost by the productions of neutrinos through a variety of nuclear reactions. Neutrinos, which interact very weakly with matter, immediately leave the core taking energy with them. The core contracts and the star titers on the edge of oblivion. As the core shrinks, it increases in density. Electrons are forced to combine with protons to make neutrons and more neutrinos, called neutronization. The core cools more, and becomes an extremely rigid form of matter. This entire process only takes 1/4 of a second.

3.2.4 The Role Of Transport Phenomena In the Evolution Of Star

Comparing the mass of the final helium-4 atom with the masses of the four protons reveals that 0.007 or 0.7 percent of the mass of the original protons has been lost. This mass has been converted into energy, in the form of gamma rays and neutrinos released during each of the individual reactions. The total energy yield of one whole chain is 26.73MeV. Energy released as gamma rays will interact with electrons and protons and heat the interior of the Sun. Also kinetic energy of fusion products(e.g. of the two protons and the 2^4He from pp-I reaction) increases the temperature of plasma in the Sun. This heating supports the Sun and prevents it from collapsing under its own weight. Neutrinos do not interact significantly with matter and there fore do not help support the Sun against gravitational collapse. Their energy is lost: the neutrinos in the ppI, ppII and ppIII chains carry away (2.0,4.0, and 28.3)percent of the energy int hose reactions,respectively.

The transport phenomena plays a role in balancing some parameters which are important to keep the star in equilibrium. And also the transport phenomena is important to fulfill the low of conservation of energy, conservation of mass and conservation of momentum.

One of the balance parameter is energy. According to the Virial theorem,equation (1.6.3).

$$2\langle T\rangle + 2\langle U\rangle + \langle\Omega\rangle = 0 \tag{3.2.14}$$

The mass lost in all phase of nuclear reaction will be converted in to energy and transported out ward balancing the mass being transported in ward. By Einstein equation of mass energy relation,

$$E = Mc^2 \tag{3.2.15}$$

The radiation pressure and the gravitational pressure also balances.

Chapter 4

Result And Discussion

4.1 Introduction

We had worked out the transport phenomena in the stellar interiors specifically in burning star. The energy source of this star is the nuclear reaction in it's core. The nuclear reaction starts with hydrogen nuclei fusion to produce helium and the reaction continues by forming the heavier elements.

We had tried to address the transport phenomena In two different categories of of Young star in the main sequences whose core is hydrogen abundant. Now we may have the accreting and non accreting star.

4.1.1 For Non Accreting Star

1,The young star in main sequence, whose elemental abundance is hydrogen and it is gravitationally collapsed. At the core, this gravitational energy converted into thermal energy. For the star to be in equilibrium the following parameters must be balance. These are:
The in ward pressure P_g due to gravitational force have to be balanced by out ward pressure P_{rad} due to radiation from nuclear reaction at the core eg. Hydrogen burning.Equations(3.2.11)and (3.2.12) are equivalent.

2, The role of transport phenomena in this case is the particle flow in to the core in the interior of the star cause the increase in density at the center and this cause the temperature of the core to increase to effective temperature at which the hydrogen burning occur. The other is the energy transport outward from the core as radiation which exerts the outward radiation pressure due to photon particle interaction because of opacity.

3, The other role of transport phenomena is that the nuclear energy balances the mass lost by the nuclear reaction. By Einstein equation of mass energy relation (3.2.15). The mass converted to energy is responsible to support the star against gravity and the size of the star remain unchanged eventhough it loos the enormous of mass during its main sequence time. The sun loose 4million tons of it's mass in a second as we can see figure 3.2 It generate $4.65 \times 10^{18} ergg^{-1}$

4,The mass life time relation graph shows that as the mass of the star increases the amount of energy transported from the core to the surface is increases. That is why the life time of its nuclear reaction decrease.

5,The luminosity and mass relation graph shows that, as the initial mass increases of the star increases the luminosity also increases. this reveals that, the rate of energy released by the star increases. So that the tow graphs gives us the the same meaning.

6, For the burning star most of the energy is transported by radiation. Because about 70percent of the interior region of the star is radiative region. 7, The sun release $2.394 \times 10^{40} J$ of energy in it's entire of about 10billions of it's life of hydrogen burning.

8, The sun loose 4million tons of it's mass in every second during hydrogen burning stage.

9, The energy from nuclear fuel is transported by radiation, electron and neutrino diffusion through the column of the star.

10, The energy wasted in the column of the star due to the opacity of the star exerts the pressure force against gravitational force.

Chapter 5

Summary And Conclusion

5.1 Summary

The transport phenomena has an important implication in understanding the interior of the star. Because we study the star at large distance. And our observational data are steel depend on some parameters which comes to by transport phenomena. Fore example radiation.

From our result we realized that transport phenomena plays an important role in the process of evolution of star. It is also important to balance the energies and forces to keep the star in equilibrium during its main sequence. Because, at this time the star is constant in size and have constant luminosity. The rate of nuclear reaction is constant because of the governing parameters in the stellar interiors are feed buck to each other by the stellar equilibrium equations and the Boltzmann transport equation. The evolution of the star is determined by the amount of particle transported during the formation of the star.

5.2 Conclusion

The nuclear reaction after helium burning remain complex depending on the initial mass of the star. The end of stars life is determined by the type of nuclear reaction in it's core at different stage of it's life. We have to do a lot in this area for future.

In this work we didn't touch the case of accreting star and the energy released during the nuclear reaction of heavier elements due to the time and financial constraints. So we recommend this to the interesting area to be worked on for future.

Appendices

Appendix A

Further ellaboration on BTEs

A.1 The zeroth moment of BTE

Using the vector identity

$$\vec{v} \cdot \vec{\nabla} f = \vec{\nabla} \cdot (f\vec{v}) - f\vec{\nabla} \cdot \vec{v} \tag{A.1.1}$$

In the momentum space, the position coordinates and the momentum (velocity) coordinates are independent so that $\vec{\nabla} \cdot \vec{v} = 0$. Then,

$$\begin{aligned} \int_{-\infty}^{+\infty} \vec{v} \cdot \vec{\nabla} f d\vec{v} &= \int_{-\infty}^{+\infty} \vec{\nabla} \cdot (f\vec{v}) d\vec{v} - \int_{-\infty}^{+\infty} f(\vec{\nabla} \cdot \vec{v}) d\vec{v} \\ &= \int_{-\infty}^{+\infty} \vec{\nabla} \cdot (f\vec{v}) d\vec{v} \end{aligned} \tag{A.1.2}$$

Since in the momentum space, the position coordinates and the momentum (velocity) coordinates are independent then, eq. A.1.2 is given by

$$\int_{-\infty}^{+\infty} \vec{v} \cdot \vec{\nabla} f d\vec{v} = \vec{\nabla} \cdot \int_{-\infty}^{+\infty} \vec{v} f d\vec{v} \quad \text{QED} \tag{A.1.3}$$

A.2 The first moment of BTE

A.2.1 $\int_{-\infty}^{+\infty} \vec{v}\frac{\partial f}{\partial t} d\vec{v}$ as in eq. 1.5.12

$$\int_{-\infty}^{+\infty} \vec{v} f d\vec{v} = \int_{-\infty}^{+\infty} f d\vec{v} \left(\frac{\int_{-\infty}^{+\infty} \vec{v} f d\vec{v}}{\int_{-\infty}^{+\infty} f d\vec{v}} \right) \tag{A.2.1}$$

The first factor of the right hand side of eq. A.2.1 is obviously the number density n as defined earlier. While the second factor (in the bracket) is the mean velocity $\vec{u}$.

Then,

$$\begin{aligned} \int_{-\infty}^{+\infty} \vec{v}\frac{\partial f}{\partial t} d\vec{v} &= \frac{\partial (n\vec{u})}{\partial t} \\ \text{Or} \quad \int_{-\infty}^{+\infty} \vec{v}\frac{\partial f}{\partial t} d\vec{v} &= n\frac{\partial \vec{u}}{\partial t} + \vec{u}\frac{\partial n}{\partial t} \end{aligned} \tag{A.2.2}$$

By the continuity equation eq. 1.5.10

$$\begin{aligned} \frac{\partial n}{\partial t} &= -\nabla.(n\vec{u}) + \int_{-\infty}^{+\infty} S d\vec{v} \\ &= -(\vec{u}.\vec{\nabla}_n + \vec{\nabla}_n.\vec{u}) + \int_{-\infty}^{+\infty} S d\vec{v} \end{aligned} \tag{A.2.3}$$

Now using eqs. A.2.2 & A.2.3 we get the relation given by (as required)

$$\int_{-\infty}^{+\infty} \vec{v}\frac{\partial f}{\partial t} d\vec{v} = n\frac{\partial \vec{u}}{\partial t} - (\vec{u}.\vec{\nabla}n + \vec{\nabla}n.\vec{u})\vec{u} + \int_{-\infty}^{+\infty} \vec{u} S d\vec{v} \tag{A.2.4}$$

From the continuity equation

$$\frac{\partial n}{\partial t} + \nabla.(n\vec{u}) = \int_{-\infty}^{+\infty} S d\vec{v} \tag{A.2.5}$$

Then equations (2.3.18)and (2.3.19) we get

$$\frac{\partial}{\partial t}(n\vec{u}) = \vec{u}\frac{\partial n}{\partial t} + n\frac{\partial \vec{u}}{\partial t} - (\vec{u}.\nabla n + n\nabla.\vec{u})\vec{u} + \int_{-\infty}^{+\infty} \vec{u} S d\vec{v} \tag{A.2.6}$$

Using equations (2.3.17) and (2.3.20)

$$\frac{\partial}{\partial t}(n\vec{u}) = n\frac{\partial \vec{u}}{\partial t} - (\vec{u}.\vec{\nabla}n + \vec{\nabla}n.\vec{u})\vec{u} + \int_{-\infty}^{+\infty} \vec{u} S d\vec{v} \tag{A.2.7}$$

$$n\frac{\partial \vec{u}}{\partial t} - (\vec{u}.\vec{\nabla}_n + \vec{\nabla}_n.\vec{u})\vec{u} + \int_{-\infty}^{+\infty} \vec{v}(\vec{\nabla}.(\vec{v}f)d\vec{v} + n\frac{\vec{\nabla}\Phi}{m} = \int_{-\infty}^{+\infty} S(\vec{v}-\vec{u})d\vec{v} \quad \text{(A.2.8)}$$

A.2.2 $\int_{-\infty}^{+\infty}(\dot{\vec{v}} \cdot \vec{\nabla}_v f)d\vec{v}$ as in eq. 1.5.12

Using the vector algebra

$$\begin{aligned}(\vec{\nabla}_v f)\vec{v} &= \vec{\nabla}_v(f\vec{v}) - f(\vec{\nabla}_v \vec{v}) \\ &= \vec{\nabla}_v(f\vec{v}) - f\mathrm{I}\end{aligned} \quad \text{(A.2.9)}$$

where I is the identity matrix, the third integral of the left hand side of eq. 1.5.11 is

$$\int_{-\infty}^{+\infty}(\dot{\vec{v}} \cdot \vec{\nabla}_v f)d\vec{v} = \int_{-\infty}^{+\infty} \dot{\vec{v}} \cdot \vec{\nabla}_v(f\vec{v})d\vec{v} - \int_{-\infty}^{+\infty} \dot{\vec{v}} \cdot f d\vec{v} \quad \text{(A.2.10)}$$

Note that $\dot{\vec{v}} = -\frac{\vec{\nabla}\Phi}{m}$. And morever $\vec{v}$ and the postion coordinates are independent orthogonal phase space coordinates so that eq. A.2.10 can be recast as

$$\int_{-\infty}^{+\infty}(\dot{\vec{v}} \cdot \vec{\nabla}_v f)d\vec{v} = -\frac{\vec{\nabla}\Phi}{m} \cdot \int_{-\infty}^{+\infty} \vec{\nabla}_v(f\vec{v})d\vec{v} + \frac{\vec{\nabla}\Phi}{m} \cdot \int_{-\infty}^{+\infty} f d\vec{v} \quad \text{(A.2.11)}$$

For a bound system, the first integral of the right hand side of eq. A.2.11 must vanish. Then using the relation

$$n = \int_{-\infty}^{+\infty} f d\vec{v}$$

in eq, A.2.11 we obtain the desired equation given as

$$\int_{-\infty}^{+\infty}(\dot{\vec{v}} \cdot \vec{\nabla}_v f)d\vec{v} = n\frac{\vec{\nabla}\Phi}{m} \quad \text{QED} \quad \text{(A.2.12)}$$

A.3 BTE and the Virial Theorem

Recall the BTE given as in eq. **??**

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} = -\nabla\Phi - \frac{1}{\rho}\nabla P \quad \text{(A.3.1)}$$

The left hand side of this equation is just the total average acceleration of the system

$$\begin{aligned} \frac{d\vec{u}}{dt} &= \frac{\partial \vec{u}}{\partial t} + \sum_i \frac{\partial \vec{u}}{\partial x_i}\dot{\vec{x_i}} \\ &= \frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} \end{aligned} \tag{A.3.2}$$

The spatial first moment of(taking the dot product with $\vec{r}$) eq.A.3.1 after some rearrangements is

$$\int_v \rho\vec{r}\cdot\frac{d\vec{u}}{dt}dV + \int_v \rho\vec{r}\cdot\nabla\Phi dV + \int_v \vec{r}.\nabla p dV = 0 \tag{A.3.3}$$

By vectors product property we have

$$\begin{aligned} \frac{d}{dt}(\vec{r}.\vec{u}) &= \vec{r}\cdot\frac{d\vec{u}}{dt} + \frac{d\vec{r}}{dt}\cdot\vec{u} \\ &= \vec{r}\cdot\frac{d\vec{u}}{dt} + u^2 \end{aligned} \tag{A.3.4}$$

The left hand side of equation A.3.4 is

$$\frac{d}{dt}(\vec{r}\cdot\vec{u}) = \frac{1}{2}\frac{d^2r^2}{dt^2} \tag{A.3.5}$$

Using eqs. A.3.4 & A.3.5 the first integral of the left hand side of equation A.3.3 is

$$\int_v \rho\vec{r}\cdot\frac{d\vec{u}}{dt} = \frac{1}{2}\frac{d^2}{dt^2}\int_v r^2\rho dV - u^2\int_v \rho dV \tag{A.3.6}$$

By now we have two well defined quantities, the moment of inertia I and the kinetic energy T respectively given by

$$I = \int_v r^2\rho dV \tag{A.3.7}$$

$$T = \frac{1}{2}\int_v \rho u^2 dV \tag{A.3.8}$$

Then plugging eqs. A.3.7 A.3.8, eq. A.3.6 can be recast as

$$\int_v \rho\vec{r}\cdot\frac{d\vec{u}}{dt} = \frac{1}{2}\frac{d^2I}{dt^2} - 2T \tag{A.3.9}$$

Using vector product property the third integral of the left hand side of equation A.3.3 is expanded to give

$$\int_v (\vec{r} \cdot \vec{\nabla}) p dV = \int_v \vec{\nabla} \cdot (rp) dV - \int_v p(\nabla \vec{\cdot} \vec{r}) dV \tag{A.3.10}$$

Noting $\nabla \cdot \vec{r} = 3$ and using Gauss divergence theorem, eq. A.3.10 becomes

$$\int_v (\vec{r} \cdot \vec{\nabla}) p dV = \oint_{\text{over all surface}} p_s \vec{r} \cdot \hat{n} dA - 3 \int_v p dV \tag{A.3.11}$$

where $\hat{n}$ is unit normal to the surface while p_s is the pressure at the surface.

In the MB distribution the pressure is $p = nkT$. So using eq. 1.2.6 the internal energy density of the Maxwellian gas is

$$\varepsilon = \frac{3}{2} p \tag{A.3.12}$$

On the other hand the surface integral of eq. A.3.11 over all space vanishes, otherwise it diverges. Then eq. A.3.10 will be recast as

$$\int_v (\vec{r} \cdot \vec{\nabla}) p dV = -2 \int_V \varepsilon dV = -2U \tag{A.3.13}$$

where U is the total internal energy of the system. The second left hand term of eq. A.3.3 due to the work of Clausis, Lagrange and Jacob is considered as the negative of the total potential energy Ω of the system.

$$\int \rho \vec{r} \cdot \nabla \Phi dV = -\Omega \tag{A.3.14}$$

Finally the non-averged spatial first moment of BTE by eqs. A.3.3, A.3.9, A.3.13 & A.3.14 is

$$2T + 2U + \Omega = \frac{1}{2} \frac{d^2 I}{dt^2} \tag{A.3.15}$$

While the averaged spatial first moment of BTE reduces to

$$2 < T > +2 < U > +\Omega >= 0 \quad \text{QED} \tag{A.3.16}$$

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