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STATISTICAL MODELING ON NUMBER OF CHILDREN EVER BORN FROM WOMEN OF 15-49 YEARS IN ETHIOPIA

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STATISTICAL MODELING ON NUMBER OF CHILDREN EVER BORN FROM WOMEN OF 15-49 YEARS IN ETHIOPIA

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SCHOOL OF GRADUATE STUDIES, DEPARTMENT OF STATISTICS

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Tolesa Teshome

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LIST OF ACRONYM

WFS	World Fertility Survey
MOH	Ministry of Health
CSA	Central Statistical Agency
DHS	Demographic and Health Survey
EDHS	Ethiopian Demographic and Health Survey
UNPF	United Nation Population Fund
UNICEF	United Nation Children's Fund
WHo	World Health Organization
USAID	United States Agency for International Development
TGE	Transitional Government of Ethiopia
UNDP	United Nation Development Program
IMF	International Monetary Fund
MoFED	Ministry of Finance and Economic Development
MWUD	Ministry of Work and Urban Development
MDG	Millennium Development Goal
SNNPR	Southern Nation Nationalities and Peoples Region
WHR	World Health Report
TFR	Total Fertility Rate
MMR	Maternal Mortality Ratio
CEB	Total children Ever Born
WFS	World Fertility Survey
AIC	Akaike Information Criterion
GLM	Generalized Linear Model
GPRM	Generalized Poisson Regression Model
NBRM	Negative Binomial Regression Model
PRB	Population Reference Bureau
GLMM	Generalized Linear Mixed Model
UN	United Nation

DEDICATION

This work is dedicated to my lovely family, specially my wife Megertu Misgana and all their dedicated partnership for the Success of my life.

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ABSTARCT

Background: *High level of fertility and rapid population growth has an impact on the overall socio-economic development of the country in general and maternal and child health in particular; which leads to increased obstetric and medical risks of mothers. According to UN, 2009 reports more developed regions have fertility levels below replacement; whereas, least developing regions have five or above Five children per women. The core objective of this study was to identify key factors that influence high fertility of Women of Child bearing age and to assess between and within regional heterogeneity of determinants of fertility in Ethiopia.*

Method: *Data from the 2011 Ethiopia Demographic and Health Survey which is a nationally representative survey of mothers in the 15-49 years age groups were used to identify determinant factor of fertility for woman of child bearing age (n=4976) in Ethiopia. In this paper, the descriptive statistics of the total children ever born data exhibit the presence of over-dispersion in the data set; we have used Negative Binomial Regression Model and Generalized Poisson Regression Model. These two models have statistical advantages over standard Poisson regression model and are suitable for analysis of count data that exhibit either over-dispersion or under-dispersion and also generalized linear mixed models (GLMM) were used to assess between and within regional heterogeneity determinant of fertility in Ethiopia using 2011 EDHS data set.*

Results: *The results obtained from Generalized Poisson model, Negative Binomial model and GLMM showed that Age of mother, Age at first birth, Age at first marriage, status of education of parents, place of residence, Religion, contraceptive use and status of breast feeding were significantly affect number children ever born in house hold and only Age of mother between 20-39 years, Religion was positive effect for children ever born. It found that Generalized Poisson regression model has statistical advantages over standard Poisson regression model and Negative Binomial Regression model because it was suitable for analysis of count data that exhibit either over-dispersion or under-dispersion. For GLMM it was also found that model with two random intercepts was the best description of the data to address the between and within-regional heterogeneity of fertility.*

Conclusion: *We can conclude that delaying early marriage, extend Age at first birth, Using contraceptive method and Breast feeding were the most determinant factors in reducing number of children ever born in house hold.*

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1. INTRODUCTION

1.1. BACKGROUND

The national censuses of many countries include a question that asks women about the number of children they have ever born to them; these are referred to as children ever born (CEB) data. Demographers often use such data in statistical models of fertility and in epidemiology also such type of data called parity (number of live born children a woman has delivered). CEB data may be referred to as event count or count data. “An event count refers to the number of times an event occurs and is the realization of a non-negative integer-valued random variable” (Cameron & Trivedi, 1998).

Worldwide, about three quarters of all pregnancies are deemed either unwanted or unplanned, but account for almost three hundred thousand pregnancies that occur daily (partners in health, 2009).This figures are alarming, and pose a huge threat to the provision of quality social service such as education and health for most governments especially in developing countries, where resource are scarce and highly constrained.

The global population was about 5.4 billion in 2007 and 6.8 billion in 2009 with 5.6 billion (82% of the world total) living in the less developed regions (UN, 2009). The population of the more developed regions remained largely unchanged at 1.2 billion inhabitants. Three least developed countries including Bangladesh, Ethiopia and the Democratic Republic of the Congo were among the ten most populous countries in the world.

Whereas the population of more developed regions was rising at an annual rate of 0.34 per cent, that of the less developed regions was increasing four times as fast, 1.37 per cent annually, and the least developed countries as a group were experiencing even more rapid population growth, at 2.3 per cent per year (UN, 2009).

In recent decades, the world has witnessed a rapid pace of fertility decline in the majority of the developing countries. Overall, the total fertility rate (TFR) of the developing world dropped from 6.0 births per woman in the late 1960s to 2.9 birth per woman in 2000-2005 (UN, 2007; Bongaarts, 2008). Declines were most rapid in Asia, North Africa, and Latin America regions, where social and economic development has also been relatively rapid.

Sub-Saharan Africa also experienced significant declines despite its lagging development (*Bongaarts*, *2003)*. The emphasis on social development gained acceptance as a growing body of empirical research substantiated the view that public action in fertility decline had much to contribute both to better living conditions and to reducing population growth (*Dreze and Murthi*, *2001*).

The average total fertility rate for sub-Saharan Africa as a whole is more than five children per Women which is almost twice the world average of 2.5 (*Haub*,C., *et al*). More developed regions have fertility levels below replacement; whereas, least developing regions have five or above five children per women (*UN, 2009*).

Total fertility rate as a measure of fertility in Nigeria has achieved a marked reduction over the years, from 6.6 in 1965 to 5.7 in 2008. However, the pace of reduction is slow as population continues to increase rapidly from about 80 million in 1990 to approximately 170 million in 2012. The pattern of fertility varies widely across different regions and by socio-demographic characteristics in Nigeria. For instance, fertility peaks in age group 25-29 with 265 births per 1,000 women and declines thereafter. The general fertility rate is 194, which means that there were 194 births for every 1,000 women during the three-year period preceding the survey. The crude birth rate was 40.6 per 1,000 populations for the same period. Conscious of the relative consequence of fertility on population health and development indicators, the patterns of fertility in Nigeria has attracted attention of researchers for some time now both locally and internationally.

Ethiopia, like most countries in sub-Saharan Africa, is experiencing rapid population growth. With a 2014 population of approximately 96.5 million (increased from 2013's estimate of 95 million) (http://world population review.com/continents Africa –population/.)

According to the United Nations Population Fund (*UNPF,2007*), the country is ranked among African countries that have high fertility rate and is not considered to be among the countries at or near the start of the transition to low level of fertility (*Sibanda et al, 2003*). The country's high population growth rate is also mainly sustained by this high fertility rate. Cognizant of this, the government of Ethiopia has been implementing a National Population Policy starting from 1993 the specific objectives of which includes reducing the total fertility rate (TRF) to approximately four by the year 2015. Since the introduction of the national population policy, the fertility rate of the country has not shown a significant decline and the target of achieving a lower level of TFR still looks out of reach. During the 1990 National Family and Fertility Study the total fertility rate was reported to be 6.4 births per woman, showing a drop of only 0.5 children on average by 2005 (CSA, 2005).

However, as can be fairly expected, there is a stark difference in TFR of the urban and rural areas standing at 2.4 and 6 respectively (ibid). The need for the study of fertility can be overstressed because of its great impact on both population growth rate and on other social, economical and cultural parameters.The demographic pattern of developing countries is characterized by the co-existence of high fertility and high infant, and child mortality (Yohannes *et al.*, 2004).

In many cultures early marriage and child bearing are the norm. Early child bearing and high parity increase the woman’s chance of complications in child bearing. Pregnancies are most dangerous for women who are too young (less than 18 years), have too many births (more than four) and do not want another pregnancy and may resort to unsafe abortion (John Hopkins University, 1988). The fertility rate is the highest in sub-Saharan Africa than many parts of the world, mainly due to strong kinship networks and high economic and social values attached to children (Hinde and Mturi, 2000).

High fertility and rapid population growth have an impact on the overall socio-economic development of the country in general and maternal and child health in particular. Maternal and child mortality are two of the major health problems challenging healthcare organizations, especially in developing countries. The majority of maternal deaths are the direct result of complications encountered during pregnancy and arising from unsafe terminations (Gaym 2000; Merric 2002; Population Reports 1999). The World Health Report (WHR 2005) noted that unwanted, mistimed and unintended pregnancy is the most common cause of maternal mortality in developing countries.

This study aiming at modeling individual woman’s fertility level and predicts average number of children women will have bearing in mind of differential in their socio-demographic characteristics. The total number of children ever born per woman is uses as a measure of fertility since it is often uses in various demographic studies as a proxy for fertility estimation.

Naturally, total Number of Children ever Born (CEB) is a count outcome. We used Poisson regression model, Generalized Poisson Regression model and Negative Binomial Model which belongs to the family of generalized linear models (GLM) and Generalized Linear Mixed Model (GLMM) in order to analysis the data . Besides contributing to the body of knowledge on fertility issues in Ethiopia and beyond, this study will help individuals and policy makers in encouraging and promoting characteristics that are favorably disposed to lowering fertility in the country.

1.2. STATEMENT OF THE PROBLEM

As high fertility is associated with increased obstetric and medical risks of mothers, in order to reduce fertility and control population growth of the country, the Determinant/ factors that influence fertility should be clearly identified (Zhang, 2007). Experience of fertility transition countries also emphasizes the role of its determinant in fertility change (Angeles, 2008). Human fertility is a function of a variety of factors. The factor varies from place to place, depending on the specific conditions of the given area (Lindstrom and Kiros, 2001; Yohannes et *al.*, 2004). Adolescent fertility also known as teenage fertility refers to women could have given more than two live births before the age of 20 years and who did not breastfeed, as well as did not use family planning. The earliest age reported at first birth was 12 Years of age (Tewodros et al., 2010) .

Moreover, there are no enough previous studies on this area regarding of Statistical modeling of number children ever born from women of 15-49 years in Ethiopia using Generalized Linear Models. Most of the studies previously done are using only a case control studies and Logistic Regression models.

This study, has tries to fill the gaps in understanding un controlled Fertility by identifying determinant factors of number of children ever born from women of 15-49 years age in Ethiopia by Using Poisson Regression Model, Generalized Poisson Regression model and Negative Binomial Regression Model, Also cluster specific model. A proper understanding of these factors are of paramount importance in tackling the problem of uncontrolled fertility, which paves the way for the improvement of the prevailing socioeconomic problems of the country. Particularly, it would have a substantial contribution in the improvement of the health status of women and children.

The study Used clustered data from EDHS of 2011 by addressing the following research questions:

- ✓ Which covariates are the most determinant factors for modeling of children ever born from women of 15-49 years age in Ethiopia?
- ✓ Is there between and within regional heterogeneity of Fertility?

1.3. OBJECTIVE OF THE STUDY

1.3.1. GENERAL OBJECTIVE

General objective of the study is modeling Number of children ever born from women of 15-49 years in Ethiopia.

1.3.2. SPECIFIC OBJECTIVES

The specific objectives of the study are:

- To fit an appropriate statistical model and interpretable estimates of important Covariate for children ever born in Ethiopia.
- To identify key factors that influence high level of fertility of Women of Child bearing age in Ethiopia.
- To assess between and within regional heterogeneity of determinants of fertility

1.4. SIGNIFICANCE OF THE STUDY

The fertility level of Ethiopia especially in the rural area is unacceptably high. The higher the Fertility of women, the more the risk associated with each birth. In developing country like Ethiopia, pregnancy and child birth is 18 times more likely to end in the woman's death than in developed countries (John Hopkins University, 1999).

The results of this study will be very useful to understan the factors responsible for the fertility level would help in designing strategies to effectively implement any program to tackle uncontrolled fertility and in raising the status of women.

Generally, this research is expected to give Information Specially:

- It is expected that this study might be increase the understanding of uncontrolled fertility in Ethiopia especially at rural area.
- The results of the study might be appraising understanding of policymakers by clarifying the main determinant factors that affecting the Fertility of Women Child bearing age in Ethiopia.
- The study is also very important for further studies using count data models which are employed to reduce the problems of under and over dispersions.

2. LITERATURE REVIEW

2.1. OVERVIEW OF FERTILITY SITUATION

The micro economic theory of fertility developed by Becker (1960) and Becker and Lewis (1973) has attracted much attention in different empirical studies. Different studies that have attempted to test the quantity-quality tradeoff have found a negative correlation between family size and child quality that supports the theory (Rosenzweig and Wolpin, 1980; Li *.et al*, 2005). However, the relationship between household income and fertility decisions remains to be elusive in many empirical studies. In cases where the husband's income increases the positive relationship is likely to be prevailing as it results in an increased ability to support more children (Freedman and Thornton, 1982). On the other hand, an increase in the wife's earning from her participation in the labor force is shown to have a negative substitution effect by making childbearing a costly activity for the woman (McNown, 2003; Engelhardt et al, 2004).

A wide range of empirical studies also show the existence a consistent relationship between women's education and low fertility (Jain, 1981; Chaundhury, 1984; Axinn, 1993; Bledsoe and Cohen; 1993). According to Jain (1981) education actually affects fertility through two mechanisms. One explanation for the negative effect of female education on fertility is through increasing the potential for educated women to participate in labor force of the modern sectors of the economy. This is expected to increase the opportunity cost of women to rear children and hence, reduce fertility. Second, education of women can also affect fertility through two important intermediate variables – breastfeeding and use of contraceptives by increasing the awareness of women on the benefits of breastfeeding and family planning in general. However, there could also be other indirect ways that education can affect fertility.

Another determinant of fertility that is commonly employed in empirical studies is age-at-marriage. When women get married at a younger age the probability that they are likely to have more children is going to be high since the exposure to the risk of childbearing in their reproductive years is higher. Early marriage also makes it difficult for women to attain higher level of education. Field and Ambrus (2006), using a data from rural Bangladesh show that women attain less schooling as a result of marrying young. Hence, age-at-marriage may also affect fertility through the intermediate variable- education of women. This gives an indication that age-at-marriage may also affect fertility through education as an intermediate variable.

Although literature has reported a decline in the number of births world-wide since 1960, the birth rate is still high in sub-Sahara Africa, especially in Nigeria. Compared with the reported general decreasing fertility outcomes across the globe, sub-Saharan African countries continue to top the worlds' fertility charts. While the 2012 world's total fertility rate (TFR) is 2.4, it is 1.7 for more developed countries, 2.7 for less developed countries, 5.2 for sub-Saharan Africa, 5.5 for West Africa and 5.7 for Nigeria. Elsewhere in West Africa sub-region, Nigeria ranked 5th in high TFR behind Guinea-Bissau (5.8), Liberia (5.9), Burkina Faso (6.0) and Mali (6.6).

The federal government of Ethiopia clearly recognizes the importance of reducing fertility rates. A National Population Policy was initiated in 1993 when the current government took power, with the general objective of harmonizing the relationship between population dynamics and other factors that affect the country's development. The specific objectives of the policy include raising the contraceptive prevalence rate among married women from 4 percent in 1990 to 44 percent by 2015, raising the age of marriage from 15 to 18 years, and reducing the total fertility rate from 7.1 children in 1990 to 4 children in 2015. However, the most recent Demographic and Health Surveys (DHS) data show achieving these targets is at best a remote possibility. For example, in 2005 only 15 percent of married women used either a traditional or a modern method of contraception. And the decline in fertility rates has only been modest, declining to 5.4 children in 2005 (CSA 2005).

The Ethiopian Demographic and Health Surveys (EDHS) report reveals that the total fertility A rate (TFRs) of 2000 and 2005 was 5.5 and 5.4 respectively. Overall, utilization of health Services remains low for a number of reasons, including limitations in the services and delivery Capacities available, as well as the affordability and quality of the services (WHO, 2009). As Maternal deaths related to child-bearing is unacceptably very high in our country, knowing the Factors affecting the fertility levels of women at the individual and community levels in the rural Context of Ethiopia where the majority of women reside would help greatly in averting deaths Related to high fertility and thereby raising the status of women at large.

2.2. DETERMINANT FACTORS OF FERTILITY

Generally it is believed that high infant and under-five mortality causes high fertility through the insurance and replacement effect. The "insurance effect" assumes that the couples adjust their fertility because they expect some of their children to die. "Child replacement effect" involves a deliberate decision of couples to make up for the lost children and is based on the fat their previous child bearing (Gyimah, 2001). Analysis using data from rural Ethiopia supports child/infant mortality had a significant positive effect on the number of children ever born.

An increase in the number of children who have died raises the probability of attaining higher fertility (Yohannes et al., 2004). Similar results in South Africa were also found in the study of Dust (2005), in which he illustrated that under-five mortality had a significant positive effect on fertility status. That is, an increase in the under-five mortality rate increases fertility Significantly. As the number of children who died increased, women were exposed to a higher risk of uncontrolled fertility (Ramesh A., 2010).

The relationship between education attainment of parents and level of fertility generally noted in surveys of sub-Saharan African countries and other parts of the world has been an inverse one. Groups with high educational attainments (either husband or wife) have lower fertility than low educational groups (Dejene, 2000; Vilaysook, 2009). Education can affect birth rate through a number of channels including changes in the level of contraceptive knowledge, desire for children and economic productivity. Educated women are more likely to postpone marriage, have smaller families and use contraception more than uneducated women. The educational level of the parents (wife or husbands) influences access to modern knowledge and new ways of life. In addition, education tends to break down barriers to communication about family planning between spouses (Derebssa, 2002). Similarly it has important implications in raising family planning discussion like the use of contraception, which ultimately reduces the fertility level and helps to reach the replacement level of fertility with their husbands. Woman's education, directly and indirectly influences contraceptive use (Azhar and Pasha, 2008).

The husband's desire for more children, a preference for the sex of the next child, and the women's poor education attainment remain the main barriers to contraceptive use in Pakistan (Saleem and Pasha, 2008).

Previous empirical studies have found that the Fertility is related to factors such as age at first marriage, current marital status (polygyny (having more than one wife at the same time, use of contraceptives, individual wealth index, household wealth index, place of residence, income, education level, religion, ethnicity, Abortion**,** breastfeeding, age of mothers, age at first birth, Infant mortality, reproductive health, family size and others. These factors and model families also discussed as such:

Age at First marriage: The age at first marriage has a major effect on child bearing because women who marry early have on average a longer period of exposure to pregnancy and a greater number of life time births (CSA, 2006). In a study on differentials of fertility in Awassa, the age at first marriage was significantly associated with the level of fertility, the age at first sexual intercourse and the age at first birth (Samson and Mulugeta, 2009). Marriage is a leading social and demographic indicator of the exposure of women to the risk of pregnancy, especially in the case of low levels of contraceptive use, and, therefore, is important for an understanding of fertility. Women who marry early, for example at age fifteen, have roughly twice as many years of productivity as those marrying at age 30. But their productivity is more than twice that of those marrying age 30. This is because even though those marrying at 30 expose themselves to pregnancy half as many years as those marrying at age 15, their reproductive years are not as productive as the 15 years between age 15 and 30 due to reduced fecundity (biological potential to reproduce). In Ethiopia, the median age at marriage among women aged 25 – 49 was 16.1 years, and 79 percent of them were already married by age 20 and 49% were married at age 18 (CSA, 2006; Henry, 2006). Woman who live in urban areas and completed lower secondary school tended to have a higher age at first marriage than those who lived in the rural area and had lower education (Boupha *et al.,* 2005).

Woman's age: woman's age is a significant factor involved with the probability for her to get pregnant. Increasing infertility with age is a well-documented and very apparent problem in modern society. The longer women wait to have children, the higher the chance is for them to have fertility problems due to the quality of the eggs and other related issues (Vilaysook, 2009).

From the World Fertility Survey (WFS) findings, Weinberger (1987) reported that the singulate mean age at marriage increases steadily with education, with the largest difference averaging 2, 6 years occurring between those with 4-6 and those with seven or more years of education. Jejeebhoy and Cleland (1995) argued that the enhanced decision-making autonomy of educated women allows them to resist pressures for early marriage. Samara and Susheela (1996) noted that in Sub-Saharan Africa, the most educated women marry at least four years later than uneducated women. According to the analysis from a DHS conducted in 25 developing countries, Edwards(1996) noted that one of the main ways in which education affects fertility is by delaying marriage.

UNICEF, (2001) report revealed age at marriage is an important factor in child bearing. Global estimates suggested that girls aged 15-19 are twice as likely to die from childbirth compared with women in their twenties, while girls younger than age 15 face a risk that is five times as great. Indeed, more adolescent's girls at early age die from pregnancy – related causes than from any other cause (PRB, 2000).

Income: Income affects fertility through its effect on child survival which in turn affects maternal mortality, environmental contamination, nutritional status, personal illness, and controlling the use of medical services .The 2005 EDHS showed that Ethiopian women in the lowest wealth quintile have twice as many children as those in the highest wealth quintile. The fact that "….84 % of women in the lowest quintile have no education compared with 38 % in the highest quintile" shows the obvious fact that wealth and education go hand-in-hand and, together, make the biggest fertility impact. It is no wonder, then that the wealthy countries of the world have low fertility while most African countries plagued by poverty and illiteracy have, as a group, the highest fertility in the world (CSA, 2006). The lower the income levels the higher the child mortality. Higher child mortality is followed by a higher fertility in individuals (Dust K., 2005).

Religion: Religion continues to be associated with variations in the intermediate variables contraceptive because large differences by religion remain in contraceptive choice (CSA, 2006). Traditionally one of the indisputable generalizations in demography has been that Orthodox Christians have higher fertility rates than the Muslim (Yohannes *et al.,* 2003).

Place of residence: Women who lived in the urban area were more likely to use contraceptives than those who lived in rural areas. The fertility levels in urban and rural areas tend to be different (Boupha *et al.,* 2005). A longitudinal study of Nepal's fertility trend based on the Demographic Health Survey in 1996 and 2001 illustrated that the estimates of TFR and fertility level of women in the urban area were lower than women who lived in the rural area, because of differences in contraceptive use (Retherford and Thapa, 2003).

Contraceptive use: Contraceptive use is another substantial proximate factor affecting fertility among countries. At the same time, culture and socio-economic condition have significant roles in the use of contraceptive method. By and large, it is found that an increase in contraceptive prevalence rates is consistent with an increase in the proportion of woman who needs to avoid pregnancy, which then leads to a decrease in fertility (Feyisetan, 2000). The prevalence of use of contraceptive methods increases with the increase in the number of living children as well as education level of the respondent (Sajid et al., 2005; Azhar and Pasha, 2008). Similarly this association was also found in rural Tanzania where the number of living children and education were the main factors in use of contraception (Marchant *et al*., 2004). This was also found in Nepal where the sex preference was an important barrier to the increase of contraceptive use and the decline of fertility in the country (Tiziana *et al*., 2003).

Sub-Saharan Africa countries are characterized by low contraceptive prevalence. Low total fertility rate (TFR) can be associated with a high contraceptive prevalence rate. Countries like Kenya with low mean ages at first intercourse, marriage and birth have a lower total fertility rate (TFR) (less than 6) because its contraceptive prevalence rate is higher than 30 percent. It seems that countries with a prevalence rate of more than 40 percent have a total fertility rate (TFR) of less than 5.

Breast-feeding: Another proximate determinant which can affect fertility is breastfeeding. Jain et al (1981) stated that it is important to understand the relative contribution of breastfeeding and contraception in suppressing marital fertility of women with no education. Whether or not the average marital fertility of women in a country would rise with advancements in female education would depend upon the relative shifts in levels and effectiveness of these two intermediate factors. These shifts would depend, among other things, upon the accessibility to contraception, changes in infant feeding practices, and the extent to which breastfeeding is used deliberately for spacing or limiting purposes. In an analysis of breastfeeding patterns and its influence on fertility, it has been shown that in Indonesia, the average duration of breastfeeding decreases from about 20 months among women with no education to about 11 months among those with at least seven years of schooling.

Akmam(2002) argued that prolonged breastfeeding is one of the traditional practices that serves as a means of contraception. With increases in the levels of education of women, the period of breastfeeding tends to decrease. Breastfeeding practices are affected by education through knowledge, decision-making and emotional autonomy.

It is well known that breast-feeding is the major factor influencing the duration of postpartum infertility. The inhibitory mechanism by which breast-feeding acts to delay ovulation was not fully understood, but there is evidence that both the frequency and the duration of suckling play an important role (Hadia *et al*., 2009). Similar studies conclude that the fertility-inhibiting effect of postpartum infecundity resulting from prolonged breast-feeding is by far the most important proximate determinant of fertility. The duration of breast-feeding showed a significant difference between the two fertility profiles .Those mothers with prolonged breast-feeding showed a lower fertility status (Yohannes *et al*., 2004).

Education level: Numerous studies have been carried out using household-level data that confirm the findings from studies using aggregate data. To cite one example, an examination of the determinants of fertility in fourteen countries of sub-Saharan Africa by Ainsworth, Beegle, and Nyamete (1996) using household survey data shows an inverse correlation between female schooling and fertility in virtually all of the countries, though the relationship is non-linear: female primary schooling has an inverse relation with fertility in about half of the countries only but female secondary schooling is universally associated with lower fertility, and the strength of the correlation increases with increasing years of schooling. Among ever-married women, husband's schooling has no significant relation with fertility in about one-third of the countries. Moreover, in cases where both women's and men are schooling matter, women's schooling exerts a much larger negative effect on fertility than men's schooling.

Hoem *et al.* (2006) find that ultimate fertility decreases somewhat with an increasing educational level, but its dependence on the field of education is much more impressive. In particular, women educated for the teaching or health-care professions have less childlessness and a higher ultimate fertility than others. Conversely, women with an education for esthetic or (non-teacher) humanist occupations have unusually high fractions childless and low ultimate fertility. Women with religious educations stand out by having very high fractions childless but quite ordinary mean ultimate fertility nevertheless; such women have very little childbearing outside of marriage. Women with research degrees have remarkably ordinary childbearing behavior; they do not forego motherhood to the extent that some theories would predict.

However, a few recent studies indicate a diminishing negative effect of female education on fertility levels; in Norway (Kravdal & Rindfuss, 2008) and in United States (Shang & Weinberg, 2012). Moreover, some studies report a nil or positive relationship between education and fertility levels (Diamond, Newby & Varle, 1999).

Age at first birth: Childbearing at the young ages consequences a greater risk to maternal mortality and child mortality to the mother and the child respectively .It also inclines to restrict the educational and economic opportunities for all. The earliest age reported at first birth was 12 Years of age (Tewodros *et al.*, 2010)

2.3. OVERVIEW OF MODEL FAMILIES

Count data regression models are used when the dependent variables takes on non-negative integer values for each of the n observations. These values represent the number of times an event occurs in a fixed domain. Cameron and Trivedi (1996) and Long (1997) provide good overviews of standard count regression models.

Count Data: As the name implies, count data is data that arises from counting. They are the "realization of a nonnegative integer-valued random variable" (Cameron & Travedi, 1998). As such, the response values take the form of discrete integers (Zorn, 1996). Although the lower boundary can feasibly be any integer, it is usually the case that its value is zero. Strictly speaking, there can be no nonnegative numbers. Hence, the data are constrained by this lower bound of zero and no upper bound.

The well analysis of data is required in modeling the association between the response variable And the given set of covariates. Generalized Poisson regression, Standard Poisson regression Model and Negative Binomial Regression and cluster specific Model were used for the analysis of number of children ever born in a household of Ethiopia

To understand the nature of socio-economic and demographic factors related to rapid population growth, a generalized linear modeling has been used in this current study. But there may be more variability around the model's fitted values than is consistent with a Poisson formulation, that is, Over dispersion. To correct this problem, we have used Negative Binomial Regression and

Generalized Poisson Regression which are also the special case of Generalized Linear Model. But the main concern of this study is to explore the best Model that fit the data by explaining over dispersion. That means this study wants to explain that which model is better among Negative Binomial Regression and Generalized Poisson Regression for describing over dispersion of count data without considering cluster nature of the data. Femoye, Wulu and Singh (2001) noted that the Poisson regression model is not appropriate when a data set exhibit over-dispersion, a condition where the variance is more than the mean.

2.3.1. GENERALIZED LINEAR MODELS (GLM)

The basic count data regression models can be represented and understood using the GLM Frame work that emerged in the statistical literature in the early 1970s (Nelder and Wedderburn 1972).

Bryk and Raudenbush (1996) state, "There are important cases . . . for which the assumption of linearity and normality are not realistic, and no transformation can make them so" (p.291). Count data is likely to be one such case. Instead of deleting cases or transforming the data, it is more reasonable to specify a different distribution. As explained by Hox (2002), although it is nice to be able to transform data, "modeling inherently nonlinear functions directly is sometimes preferable, because it may reflect some 'true' developmental process" (pp. 93-94). In order for a model to be 'inherently nonlinear' (Hox, 2002), there must be no transformation that makes it linear. These nonlinear models belong to the class of generalized linear models (GLM).The following explanation of generalized linear models based on the seminal work of McCullagh and Nelder (1989) with additional clarification by Lawal (2003) and Agresti (1996). Lawal (2003) explains that generalized linear models are a subset of the traditional linear models that permit other possibilities than modeling the mean as a linear function of the covariates.

The Three components that specify a generalized linear model (GLM) are a random component, a systematic component, and a link function. Random component: identifies the response variable and its probability distribution. Systematic component (Linear predictor); specifies Explanatory variables used in a linear predictor function. Link function; specifies the function of expected value of the response variable that the model equates to the systematic component.

2.3.2. POISSON REGRESSION MODEL

The Poisson distribution is named after the French mathematician and physicist Siméon- Denis Poisson (1790–1840). Mathematicians at the time were concerned with developing the foundations of the field of probability, including various probability formulas, such as the binomial formula. Poisson methods are often more statistically powerful than traditional methods with count response variables when the population distribution is skewed and the distribution approximates the Poisson distribution.

The simplest distribution used for modeling count data is the Poisson distribution and thus Poisson regression is a special case of the GLM framework. The canonical link is $g(\mu) = \log(\mu)$ resulting in a log-linear relationship between mean and linear predictor. The variance in the Poisson model is identical to the mean, thus the dispersion is fixed at $\emptyset = 1$ and the variance function is $V(\mu) = \mu$ which implies that Poisson distribution is determined by one parameter μ, which is both the mean and variance of the distribution. When μ varies from subject to subject, this single parameter model can no longer be used to address the variation in μ.

The Poisson log linear regression is an extension of Poisson distribution to account for such heterogeneity. The name log linear stems from the fact that it is the logarithm of μ rather than μ itself that being modeled as a linear function of explanatory variables.

The generalized linear model with Poisson link function is particularly useful for response variable that are counts or frequencies and for which it is reasonable to assume an underlying Poisson distribution. For exploring the relationship between the mean of a Poisson vitiate and some explanatory variables of interest, the link function in a generalized linear model (GLM) is generally taken to be the logarithm, generating positive fitted values.

2.3.3. GENERALIZED POISSION REGRESSION MODEL (GPRM)

When the response \ dependent \variable is a count generated by processes in which the number of incidences is due to a rare or chance event, and that rare or chance event follows the principle of randomness. In such cases, Poisson regression model is applied to fit this type of data. In theory, data of the Poisson distribution should have its mean equal to its variance. But in practice, data arising from groups or individuals may be statistically dependent, so the observed variance of the data may be larger or smaller than the corresponding mean.

The generalized Poisson regression model has statistical advantages over standard Poisson regression model and is suitable for analysis of count data that exhibit either over-dispersion or under-dispersion. McCullagh and Nelder (1989), Lawal (2003), and Rice (1995) are the key references for the technical underpinnings for this model and distribution. The generalized linear "Poisson" model is considered to be the benchmark model for count data (Cameron & Triverdi, 1998). This is primarily attributed to the fact that the Poisson distribution has a nonnegative mean (Agresti, 1996).

For exploring the relationship between the mean of a Poisson covariate and some explanatory variables of interest, the link function in a generalized linear model (GLM) is generally taken to be the logarithm, generating positive fitted values. Since the response variables here is a count, Poisson regression is used to investigate the relationship of number of children ever born from women 15-49 years with the explanatory variables; explicitly, the model to be fit to the mean number of children.

2.3.4. NEGATIVE BINOMIAL REGRESSION MODELS

In practice, many real-life counting outcomes exhibit more variability than the nominal variance under the Poisson distribution, a condition called over dispersion a negative binomial is a generalized Poisson distribution that includes a dispersion parameter to accommodate the unobserved heterogeneity in the count data.

The way of modeling over-dispersed count data is to assume a negative binomial (NB) Distribution for $y_i|x_i$ which can arise as a gamma mixture of Poisson distributions. If ø is not known but to be estimated from the data, the negative binomial model is not a special case of the general GLM—however, an ML fit can easily be computed re-using GLM Methodology by iterating estimation of β given θ and vice versa.

Count data often vary more than we would expect if the response distribution truly were Poisson. The phenomenon of the data having greater variability than expected for a GLM is called over dispersion. Unlike the Poisson, the negative binomial distribution has an additional parameter so that the variance can exceed the mean.

Negative binomial GLMs for counts express μ in terms of explanatory variables, and it is common to use the log link too. The dispersion parameter α is often assumed to be constant at all predictor values.

2.3.5. CLUSTER SPECIFIC MODELS

Proper analysis of data is required in modeling the association between the response variable and the given set of covariates. Molenberghs & Verbeke broadly classified models in to two main model families (Molenberghs & Verbeke, 2005).

Cluster-specific models: the responses are assumed independent, given a collection of cluster-specific parameters. Generalized linear mixed model is one of subject specific family (Molenberghs & Verbeke, 2005). Based on the nature of sampling design and nature of data, some of the model families would be appropriate for this study is discussed as follow.

2.3.5.1. GENERALIZED LINEAR MIXED MODEL (GLMM)

Agresti explained that, generalized linear model (GLM) extend ordinary regression by allowing non-normal responses and a link function of the mean. The generalized linear mixed model is a further extension that permits random effects as well as fixed effects in the linear predictor (Agresti, 2007). Antonio & Beirlant defined GLMM as extend of GLM by allowing for random or cluster-specific effects in the linear predictor. These models are useful when the interest of the analyst lies in the individual response profiles rather than the marginal mean. The inclusion of random effects in the linear predictor reflects the idea that there is natural heterogeneity across subjects or clusters in some of their regression coefficients (Antonio & Beirlant, 2006). According to McCulloch clarification, GLMM is very versatile in that they can handle non-normal data, nonlinear models, and a random effects covariance structure. This can be used to incorporate correlations in models, model the correlation structure, identify sensitive subjects and can be used to handle heterogeneous variances. The modeling process is relatively straightforward, requiring the following decisions: what is the distribution of the data, what is to be modeled, what are the factors, and are the factors fixed or random? This all makes GLMM attractive for use in modeling. Unfortunately, computing methods for much of the class of GLMM is an area of active research. No general-purpose software exists and, tests and confidence intervals are asymptotic and approximate (McCulloch, 1997).

Generalized the above explanation, GLMM is an extension to generalized linear model (GLM) that includes random effects in the linear predictor, giving an explicit probability model that explains the origin of the correlations. The resulting cluster-specific parameter estimates are suitable when the focus is on estimating the effect of changing one or more components of the predictor on a given individual.

The key problem in GLMM is maximization of the marginal likelihood, obtained by integrating out the random effects. In general, no analytic expressions are available for the integrals and numerical approximations are needed. There are large statistical literatures on various methods like approximation of the data, approximation of the Integral (Molenberghs & Verbeke, 2005).

3. DATA AND METHODOLOGY

3.1. SOURCE OF DATA.

The source of data for this study was the 2011 Ethiopia Demographic and Health Survey (EDHS), which is obtained from Central Statistical Agency (CSA). It was the third survey Conducted in Ethiopia as part of the worldwide Demographic and Health Surveys project. The 2011 Ethiopian Demographic and Health Survey, was designed to provide estimates for the health and demographic variables of interest for the following domains. Ethiopia as a whole; urban and rural areas (each as a separate domain); and 11 geographic administrative regions (9 regions and 2 city administrations), namely: Tigray, Affar, Amhara, Oromiya, Somali, Benishangul-Gumuz, Southern Nations, Nationalities and Peoples (SNNP), Gambela and Harari regional states and two city administrations, that is, Addis Ababa and Dire Dawa. The principal objective of the 2011 EDHS is to provide current and reliable data on fertility and family planning behavior, child mortality, adult and maternal mortality, children's nutritional status, use of maternal and child health services, knowledge of HIV/AIDS, and prevalence of HIV/AIDS and anemia.

3.1.1. SAMPLING DESIGN OF EDHS 2011

The 2007 Population and Housing Census, conducted by the CSA, provided the sampling frame from which the 2011 EDHS sample was drawn. Administratively, regions in Ethiopia are divided into zones, and zones, into administrative units called weredas. Each wereda is further subdivided into the lowest administrative unit, called kebele. During the 2007 Census, each kebele was subdivided into census enumeration areas (EAs) or clusters, which were convenient for the implementation of the census. The 2011 EDHS sample was selected using a stratified, two-stage cluster sampling design.

Clusters were the sampling units for the first stage. The sample included 624 clusters, 187 in urban areas and 437 in rural areas. Households comprised the second stage of sampling. In the second stage, a fixed number of 30 households were selected for each cluster. A complete Listing of households was carried out in each of the selected clusters from September 2010 Through January 2011.

The 2011 EDHS used three questionnaires: the Household Questionnaire, the Woman's Questionnaire and the Man's Questionnaire. These questionnaires were adapted from model Survey instruments developed for the measure DHS project to reflect the population and Health issues relevant to Ethiopia. In addition to English, the questionnaires were translated Into three major local languages-Amharigna, Oromiffa, and Tigrigna.

Births' data set has one record for every child ever born women aged 15-49 of eligible women. Essentially, it is the full birth history of all women interviewed including its information on pregnancy and postnatal care as well as immunization and health for children born in the last 5 years. Data for the mother of each of these children was also included. This file can be used to calculate health indicators as well as fertility and mortality rates. The unit of analysis (case) in this file is the children ever born of eligible women.

All women aged 15-49 and all men aged 15-49 years were eligible for interview. In the interviewed Households 17,385 eligible women were identified for individual interview; complete Interviews were conducted for 16,515, yielding a response rate of 95%. A total of 11,654 women's were Interviewed regarding of number of children ever born woman has delivered from women of 15-49 years during survey. At the end, 4976 (42.6%) out of 11,654(70.5%) women's during the survey interview was included for the analysis.

3.1.2. VARIABLES IN THE STUDY

A household fertility decision may depend on different factors. The following is the list of dependent and independent variables used in this study.

The response and predictor (Explanatory) variables that served for the estimation of parameters were defined as follow.

Response (Dependent) variable:

- Number of children ever born a woman has delivered (CEB) in a family from women of 15-49 years.

Predictor (explanatory) variables:

The explanatory variables that would be included were explained as following. The choice of These variables are guided by different literatures as the determinant factors of Fertility among Women of Child Bearing Age 15-49 years. These Predictor variables can be divided in to two according to different literatures. First Socioeconomic and Demographic variables including: Income/Wealth Index, Educational level (for both Husband and wife), place of residence, Religion, Region, women's Occupation. Secondly Proximate determinants of Fertility (Reproductive variables) including: Age at first marriage, Age at first birth, Contraceptive Use, Breast feeding etc. These categories of the Predictor (independent) variables were coded starting from zero to make it appropriate for further analysis using different statistical models.

Table 3.1. Coding and description of explanatory (Independent variables) variables:

Variables	Explanation
Age of Respondent (**Agemo**)	This variable refers to age of mother's at the time of the survey and has three categories ranging from 15-49 years as: 1 for 15-19, 2 for 20-39,3 for 40-49
Age at First Birth **(Ageatfb)**	This variable indicates that the age at which mothers gets child at the time of survey and coded as: 0 for 15-19, 1 for 20-39
Educational of mother (**mothedu**)	Education of mother refers to whether the mother educated or not and it is categorized in to two groups; 0 for no education, 1 For educated
Mass Media (**Massm**)	Refers that whether the mothers heard about family planning by listening to radio, watching TV & reading Newspaper **etc** and Coded as 0 for No, 1 for Yes
Educational Level of Partner (**husedu**)	Similar to educational levels of Mothers and this is also categorized in to two categories: 0 for no education, 1 for Educated
Place of Residence (**Residence**)	This factor is Dichotomous explanatory variable (Urban and Rural) according to where the Women was living at the time of survey And coded as: 0 for Rural, 1 for Urban.

Religion (**Religion**)	Classification of this variable was developed according to previous Literature. These categories are orthodox, Protestant, Muslim and other religion as 1for Orthodox, 2 for Protestant, 3for Muslim & 4 for others.
Wealth status (Wealth)	Measured by a composite score of several indicators of household Possession .This was based on the questions about whether the household has items and facilities as piped water, toilet, type of floor used, electricity, radio, television and etc. Then according to the answer, each asset was given weight. Each household then was assigned a score according to each asset and the scores were summed for each household. It is coded as 1 for Poor, 2 for Middle & 3 for Rich
Contraceptive Uses (**Contrace**)	this factor indicates whether the mother uses contraceptive Effectively during the question of the survey which is Dichotomous variable and coded as: 0 for not use, 1 for Uses
Breast Feeding (**BF**)	This factor indicates that the current status that the Mother breast feed the children ever born during the time of Survey and coded as: 0 for no breast feed,1 for breast feed
Age at First Marriage (**Ageatfm**)	This covariates refers to the age at which mother's married at the time of survey and has two categories ranging from 15-49 years as: 0 for 15-19, 1 for 20-39

3.2. METHOD OF DATA ANALYSIS

A range of techniques has been developed for analyzing count response variables data. To understand the nature of socio-economic and demographic factors related to rapid population growth, a generalized linear modeling approach was used in this current study. But there was more variability around the model's fitted values than is consistent with a Poisson formulation, that is, Over dispersion. To correct this problem, we use Negative Binomial Regression, Generalized Poisson Regression models without considering clustering nature of the data, which is also the special case of Generalized Linear Model and cluster specific model was used.

3.2.1. GENERALIZED LINEAR MODELS (GLM)

A class of models that has gained increasing importance in the past several decades is the class of generalized linear models. The theory of generalized linear models originated with Nelder and Wedderburn (1972) and Wedderburn (1974), and was subsequently made popular in the monograph by McCullagh and Nelder (1989). This class of models extends the theory and methods of linear models to data with nonnormal responses.

The explanation of generalized linear models based on the seminal work of McCullagh and Nelder (1989) with additional clarification by Lawal (2003) and Agresti (1996). Lawal (2003) explains that generalized linear models are a subset of the traditional linear models that permit other possibilities than modeling the mean as a linear function of the covariates. Also generalized linear models (GLMs) extend ordinary regression models to encompass non normal Response distributions and modeling functions of the mean (Agresti, 2002).

The Three components that specify a generalized linear model (GLM) are a random component, a systematic component, and a link function. Random component, identifies the response variable Y and its probability distribution; a systematic component (Linear predictor) specifies Explanatory variables used in a linear predictor function; and a link function specifies the function of expected value of the response variable that the model equates to the systematic component. I.e. the Link function relates the mean of response variable to linear predictors. Therefore GLM is a linear model for a transformed mean of a response variable that has distribution in the natural exponential family.

The Exponential Family:

A random variable Y follows a distribution that belongs to the exponential family, if the Density function is of the form:

$$f(y;\theta,\emptyset)=\exp\{\emptyset^{-1}[y\theta-\varphi(\theta)]+c(y,\emptyset)\}\ldots\ldots\ldots\ldots 3.1$$

, for a specific set of unknown parameters θ and ϕ, and for known functions $\varphi\ (\cdot)$ and C (·, ·).

Where the parameter θ is called the canonical parameter and represents the location while, ϕ is Called the dispersion parameter and represents the scale parameter. Specially for the Poisson and Binomial distribution it is fixed to be one (Faraway, 2006). Thus, an important property of the GLM is the functional relation between mean and variance.

3.2.2. POISSON REGRESSION MODEL (PRM)

Count response models are a subset of discrete response regression models which are nonnegative integer responses with right skewed of the distribution. Poisson regression with log link is the standard or base count response regression model where other count models deal with data that violate the assumptions carried by the Poisson model. (J. M. Hilbe, 2011). Count data are very often analyzed under the assumption of a Poisson model (A. Agresti, 2002).

Poisson regression is among the most statistical models widely used to model count data. Hence, the simplest distribution used for modeling count data is the Poisson distribution and thus Poisson regression is a special case of the GLM framework. The canonical link is g(μ) = log(μ) resulting in a log-linear relationship between mean and linear predictor. The variance in the Poisson model is identical to the mean (equi-dispersion), thus the dispersion is fixed at $\phi = 1$ and the variance function is V (μ) = μ. The generalized linear model with Poisson link function is particularly useful for response variable that are counts or frequencies and for which it is reasonable to assume an underlying Poisson distribution.

For exploring the relationship between the mean of Poisson covariates and some explanatory variables of interest, the link function in a generalized linear model (GLM) is generally taken to be the logarithm, generating positive fitted values. Since the response variables here is a count, Poisson regression is used to investigate the relationship of number of children ever born with the explanatory variables; explicitly, the model to be fit to the mean number of children ever born.

In many empirical studies of fertility, the number of children ever born in a household in Ethiopia is modeled as a function of socio-economic variables. The commonly used model is the standard Poisson. This model is considered because the number of children ever born in a family is non-negative. However, this model has some restrictions in some situations. In standard Poisson regression model, the conditional mean and variance of the dependent variable is constrained to be equal (equi-dispersion) for each observation. In practice, this assumption is often violated since the variance can either be larger or smaller than the mean. More variation in the data may be present than is expected by the distributional assumption. This is called over-dispersion (also known as heterogeneity) which typically occurs when the observations are correlated or are collected from "clusters". That is, both over dispersion and under-dispersion can exist in the count data. If the equi-dispersion assumption is violated, the estimates in Poisson regression model are still consistent but inefficient. As a result, inference based on the estimated standard errors is no longer valid. As noted in Winkelmann and Zimmermann the number of children ever born in a household often does not follow equal-dispersion assumption. Therefore, the standard Poisson regression model which assumes equal-dispersion is not appropriate to model data about household fertility decision.

For Poisson regression models, the covariance matrix (and hence the standard errors of the parameter estimates) is estimated under the assumption that the chosen model is appropriate.

More precisely, consider a sample of n objects, and let Y_i become a count response and $x_i = (xi_{1,\dots} xi_p)^{\mathbf{T}}$ a vector of independent variables from ith subject ($1 \leq i \leq n$)

A random variable Y is said to have a Poisson distribution with parameter $\boldsymbol{\mu}$ if it takes integer values y= 0, 1, 2…. with probability:

$$\boldsymbol{P(Y_i = y_i) =} \quad \frac{\mathbf{exp}\,(-\boldsymbol{\mu_i})\boldsymbol{\mu_i^{y_i}}}{\boldsymbol{y_i!}} \text{ Where } \mu_i = \exp\,(\beta x_i),\ y_{i=0,1,2} \text{ ....3.2}$$

The parameter μ_i on the Poisson regression model may be written as a log-linear model: $log\mu_i = x\beta_i$

And also the log-likelihood function is written as:

$\ln(\mu_i) = \sum_{i=1}^{n} y_i \ln(\mu_i) - \mu_i - \ln(y_i!) = x_i^T \beta$ (Agresti; (1997))

Where, $\ln(\mu_i)$ is the log link function, **X_i= n_i * (p + 1)** dimensional vector of known covariates and β is a vector of estimable parameters.

The Poisson Regression is to be fitted to the mean number of children, μ_i can be expressed as the following independent variables (covariates).

Log $(\boldsymbol{\mu_i}) = \boldsymbol{\beta_0} + \boldsymbol{\beta_1 x_1} + \boldsymbol{\beta_2 x_2} + \cdots + \boldsymbol{\beta_k x_k}$ … … … … … … … … … 3.3

This also implies Equation 3.3 written as:

$$\boldsymbol{\mu_i = exp(\beta_0 + \beta_1 x1_i + \beta_2 x2_i + \cdots + \beta_k xk_i)}$$

Where μ_i is the expected number of children ever born for ith women, $x1_i$, $x2_i$.......xk_i are covariates which are determinant of fertility. $\beta_0, \beta_1 \ldots \ldots \beta_k$ Are poison regression coefficients. The parameters β can be estimated by the maximum likelihood estimation method. Under the Poisson, the mean μ_i is assumed to be constant or homogeneous within the classes. However, by defining a specific distribution for $\mu_i$heterogeneity within the classes is now allowed. For example, by assuming μ_i to be a gamma with mea $E(\mu_i) = \theta_i$ and $V(\mu_i) = \theta_i^2 v_i^{-1}$ i.e. One way to model over-dispersed count data is to use mixture models, for example, the gamma-Poisson mixture and y_{i/μ_i} to be a Poisson with conditional mean $E(y_i/\mu_i) = \mu_i$ it can be shown that the Marginal distribution of y_i follows a Negative Binomial distribution with Pdf:

$$\boldsymbol{P[Y_i = y_i] = \int \mathbf{Pr}(Y_i = y_i/\theta_i)\, f(\theta_i) d\theta_i = \frac{\gamma(y_i+v_i)}{\gamma(y_i+1)\gamma(v_i)} \left(\frac{v_i}{v_i+\mu_i}\right)^{v_i} \left(\frac{\mu_i}{v_i+\mu_i}\right)^{y_i}} \text{.......}\mathbf{3.4}$$

Where the mean is E $(y_i)=\mu_i$ and the variance $Var(y_i) = \mu_i + \mu_i^2 v_i^{-1}$. The Likelihood for Negative Binomial Regression Model may be written as:

$$l(\beta,\alpha) = \sum_i \left\{ \sum_{r=1}^{y_{i-1}} \log(1+\alpha r) \right\} - y_i \log(\alpha) - \log(y_i!) + y_i \log(\alpha\mu_i) -$$

$$(y_i + \alpha^{-1}) \log(1 + \alpha\mu_i)$$

Therefore the maximum likelihood estimates $(\widehat{\beta, \hat{\alpha}})$ can be obtained by Maximizing $\ell(\beta, \alpha)$

3.2.3. GENERALIZED POISSION REGRESSION MODEL (GPRM)

Suppose a count response variable follows a generalized Poisson distribution. To model number of children ever born in the house hold, we define as the number of children ever born per household. **Following Singh and Famoye** the probability of mass function is given by:

$$f(y_i;\mu_i,\alpha)=\left[\frac{\mu_i}{1+\alpha\mu_i}\right]^{y_i}\left(\frac{1+\alpha y_i}{y_i!}\right)^{y_i-1}\exp\left[-\frac{\mu_i(1+\alpha y_i)}{1+\alpha\mu_i}\right] \ldots\ldots\ldots\ldots 3.5$$

where $y_i = 0,1,2,\ldots\ldots$, $\mu_i=\mu_i(x_i)=\exp(x_i\beta)$

Where X_i ~ (***k*** **-1**) dimensional vector of explanatory variables including personal characteristics of both husband and wife in a family as well as some demographic attributes of the family, and β is a ***k*** dimensional vector of regression parameters. The mean and variance of y_i are given by:

$E\left(\frac{y_i}{x_i}\right)=\mu_i$ and $V\left(\frac{y_i}{x_i}\right)=\mu_i(1+\alpha\mu_i)^2$ respectively

The Generalized Poisson regression model in equation (3.5) above is a generalization of the Poisson regression (PR) model. When α =0 the probability mass function in (3.5) reduces to the PR model and then

$E\left(\frac{y_i}{x_i}\right)=V\left(\frac{y_i}{x_i}\right)$ This means equi-dispersion

In practical applications, this assumption is often not true since the variance can either be larger or smaller than the mean. If the variance is not equal to the mean, the estimates in PR model are still consistent but not efficient, which lead to the invalidation of inference based on the estimated standard errors.

- For α>0, $V\left(\frac{y_i}{x_i}\right)>E\left(\frac{y_i}{x_i}\right)$ and the GPRM in equation (3.5) represents over dispersed count data.
- For α<0, $V\left(\frac{y_i}{x_i}\right)<E\left(\frac{y_i}{x_i}\right)$ and the GPRM in equation (3.5) represents under dispersed count data.

In (3.5), α is called the dispersion parameter and can be estimated simultaneously with the coefficients in the GPRM model (3.5).

To identify possible over-dispersion in the data for a given model, divide the deviance by its degrees of freedom; this is called the dispersion parameter. If the deviance is reasonably "close" to the degrees of freedom (i.e., the scale parameter=1) then evidence of over-dispersion is lacking .i.e. Dispersion parameter (or scaled deviance) = Deviance/DF

- A scale parameter that is greater than 1 does not necessarily imply over-dispersion is present. This can also indicate other problems, such as an incorrectly specified model (omitted variables, interactions, or non-linear terms), an incorrectly specified functional form (an additive rather than a multiplicative model may be appropriate), as well as influential or outlying observations.

Also Equation (3.5) is an alternative model which can capture both over- and under-dispersion which is also called restricted generalized Poisson regression (RGPR) model by Famoye (1993).where $log(\mu_i) = \beta^T x_i$ it is called a" restricted " model because the dispersion parameter α is restricted to $1+\alpha\mu_i > 0$ and $1+\alpha y_i > 0$ [cui,kim and zhu(2006)]

3.2.3.1. METHOD OF PARAMETER ESTIMATION FOR GPRM

Maximum likelihood estimators often perform better than other types of estimation procedures in terms of being the most efficient use of data. Hence, maximum likelihood estimation is a very popular method of estimation in statistical practice. From the theory of maximum likelihood, it follows that in most standard situations, the maximum likelihood estimators have approximate normal distributions provided the sample size is relatively large. Maximum likelihood estimators also tend to be nearly unbiased and they also tend to have smaller variances than other unbiased estimators which make maximum likelihood estimation a very popular statistical estimation process.

To estimate (β, α) in the GPRM model (3.5), we need the log-likelihood function of the GPRM model, that is,

$$l(\alpha,\beta) = \sum_{i=1}^{n} y_i\, log\left(\frac{\mu_i}{1+\alpha\mu_i}\right) + (y_i - 1)\log(1+\alpha y_i) - \frac{\mu_i(1+\alpha y_i)}{1+\alpha\mu_i} - \log\,(y_i!)$$

The maximum likelihood equations for estimating α and β are obtained by taking the partial derivatives and equating to zero. Thus we get

$$\frac{\partial l(\alpha,\beta)}{\partial \alpha} = \sum_{i=1}^{n} \left\{ \frac{y_i \mu_i}{1+\alpha+\mu_i} + \frac{y_i(y_i-1)}{1+\alpha y_i} - \frac{\mu_i(y_i-\mu_i)}{(1+\alpha\mu_i)^2} \right\} = 0 \ldots\ldots\ldots..(*)$$

And

$$\frac{\partial l(\alpha,\beta)}{\partial \beta_r} = \sum_{i=1}^{n} \frac{y_i-\mu_i}{\mu_i(1+\alpha\mu_i)^2} \frac{\partial \mu_i}{\partial \beta_r} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (**)$$

Where r =1, 2, 3... K (where k is the number of estimated parameters)

Substituting $\mu_i = \exp(x_i \beta)$ equation (*) becomes

$$\frac{\partial l(\alpha,\beta)}{\partial \beta_1} = \sum_{i=1}^{n} \frac{y_i-\mu_i}{(1+\alpha\mu_i)^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (***)$$

$$\frac{\partial l(\alpha,\beta)}{\partial \beta_r} = \sum_{i=1}^{n} \frac{(y_i-\mu_i)x_i}{(1+\alpha\mu_i)^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (****)$$

here, r=2 ,3 , ...K

By using an iterative algorithm equations (**), (***) and (****) are solved simultaneously. The final estimate of β from fitting a Poisson regression model to the data is used as initial estimate of β for the iteration process. The initial estimate of α can be taken as zero or it may be obtained by equating the chi-square statistic to its degrees of freedom. This is given by:

$$\frac{\sum_{i=1}^{n}(y_i-\mu_i)^2}{V\left[\frac{y_i}{x_i}\right]} = n-k$$

When α<0 (the case of under dispersion), the value of α is such that $1+\alpha\mu_i > 0$ and $1+\alpha y_i > 0$ i.e. $\alpha > \min\left\{\frac{-1}{\max(\mu_i)}, \frac{-1}{\max(y_i)}\right\}$ as required in equation in (*).

An R-program is used to solve Equation (**), (***), and (****) simultaneously.

Now,

$$l(\hat{\alpha}\ \ \widehat{\beta}) = \sum_{i=1}^{n} \left\{ y_i \log\left[\frac{\hat{\mu}_i}{(1+\hat{\alpha}\ \hat{\mu}_i)}\right] + (y_i-1)\log(1+\hat{\alpha}\ y_i) - \frac{\hat{\mu}_i(1+\hat{\alpha}\, y_i)}{1+\hat{\alpha}\,\hat{\mu}_i} - \log(y_i!) \right\}$$

$$l(\hat{\alpha},\, \bar{y}) = \sum_{i=1}^{n} \left\{ y_i log\left(\frac{\bar{y}}{1+\widehat{\alpha\,\bar{y}}}\right) + (y_i - 1)\log(1+\hat{\alpha}\ y_i) - \frac{\bar{y}(1+\hat{\alpha}\,y_i)}{1+\hat{\alpha}\,\bar{y}} - \log\,(y_i!) \right\}$$

3.2.4. NEGATIVE BINOMIAL REGRESSION MODEL (NB)

Negative binomial regression models do not assume an equal mean and variance and particularly correct for over dispersion in the data, which is when the variance is greater than the conditional mean (Osgood, 2000; Paternoster & Brame, 1997).

The PMF is given by:

$$\boldsymbol{P(y_i) = \frac{\gamma\left(y_i + \frac{1}{k}\right)}{\gamma\left(\frac{1}{k}\right)\gamma(y_i+1)} P\left(\frac{1}{k}\right)(1-p)^{y_i}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{3.6}$$

The Negative Binomial is derived by rewriting the Poisson parameter for each observation. μ_i=Exp (βX_i+ ε_i) Where, Exp (ε_i)) is a gamma-distributed error term with mean 1

and variance k. Now the rewritten pmf of Negative Binomial (NB) distribution using the gamma function is given by:

$$\mathbf{P(Y_i) = \frac{\gamma(y_i + r)}{\gamma(r)\gamma(y_{i+1})}\ \ P^r\ \ [1-p]^{y_i}} \ldots\ldots\ldots\ldots \mathbf{3.7}$$

Where, Γ (.) is the gamma function, $\boldsymbol{P = \frac{r}{\mu_{i+r}}}$ $\mu_{i=E(y_i)}$ and r $= \frac{\mathbf{1}}{\boldsymbol{\alpha}}$ which is the inverse of dispersion parameter α

We have the mean and variance of the negative binomial:

$$\mathrm{E(Y_i) = \mu_i = \frac{r(1-p)}{p}} \qquad \text{and} \qquad \mathrm{V(Y_i) = \frac{1-p}{p^2}}$$

Now, Solving for P, $\boldsymbol{P = \frac{1}{(1+\alpha\mu)}}$ then after re-parameterization Pmf model above (3.7) becomes:

$$\mathbf{P(y_i) = \frac{\gamma\left(y_i + \frac{1}{\alpha}\right)}{\gamma\left(\frac{1}{\alpha}\right)\gamma(y_i+1)}\left(\frac{1}{1+\alpha\mu}\right)^{\frac{1}{\alpha}}\left(\frac{\alpha\mu}{1+\alpha\mu}\right)^{y_i}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.8$$

Hence the log-likelihood function of Negative Binomial model becomes:

$$\ln(\mu_i) = \sum_{i=1}^{n} y_i ln\left(\frac{\alpha\mu_i}{\alpha\mu_i + 1}\right) - \frac{1}{\alpha} ln(\alpha\mu_i + 1) + ln\gamma\left(y_i + \frac{1}{\alpha}\right) - ln\gamma(y_i + 1) - ln\gamma\left(\frac{1}{\alpha}\right)$$

$$\Longrightarrow \quad ln(\mu_i) = \beta_0 + \sum_{i=1}^{n} \beta_i x_i$$

Where k is the number of parameter in the fitted Model. And ln (μ_i) is the link function .Here the dispersion parameter alpha, $\alpha \geq 0$, if α=0 the distribution becomes reduces to a Standard Poisson distribution.

3.3. MODEL COMPARISION AND MODEL SELECTION

In this section, several Model comparison will be briefly discuss, Akaike Information Criteria (AIC) and Bayesian Schwartz Criteria (BIC). Since these measures are already familiar to those who used the Generalized Linear Model with Poisson error structure for claim count or frequency analysis, the same measures may also be implemented to the regression models of Negative Binomial model (NB) and Generalized Poisson Regression model (GPM) as well.

It can be useful to do model selection in generalized linear models (GLM). As we are basing our estimation on the log likelihood function, choosing our model based on large log likelihood (or on a small deviance) might seem to be a reasonable approach. When parameters get added to a model, the log likelihood must go up (or the deviance must go down). So we need to adjust are model selection criteria to take account of the number of predictor variables in a prospective model and the amount of information each predictor variable adds**.** One approach is a penalized likelihood approach, similar to Mallow's *Cp* for linear regression models. The idea is to pick the Model that minimizes:

$$\text{Deviance} + \text{Penalty}(p) \quad ....3.9$$

Where Penalty (p) is a penalty term which depends on p, the number of parameters in the model of interest, including the intercept**.**

The primary objective of model comparison is to choose the simplest model that provides the best fit to the data. When several maximum likelihood models are available, one can compare the performance of alternative model based on several likelihood measures .Two of most regularly used measures are Akaike information criterion AIC (1973) and Schwarz's Bayesian information criteria (BIC).

Akaike information criterion AIC (1973) derived a criterion from information theories, known as the Akaike information criterion (AIC). It is a tool for model selection of an estimated statistical model. It is not a test on the model in the sense of hypothesis testing. The AIC penalizes the likelihood by the number of covariance parameters in the model. In AIC data will base approximation for discrepancy between a candidate model and the true model. This is defined as:

$$\boldsymbol{AIC = -2\log(likelihood) + 2p} \text{......................3.10}$$

2log (likelihood) is Measures the goodness of fit of the model, which is penalized by model complexity in the second part 2P where P is the number of parameter in the model. Therefore The smaller AIC results in the better candidate model.

The other Model selection is Schwarz's Bayesian information criteria (BIC) will be use which is defined as:

$$\mathbf{BIC = Deviance + p * log\ (n)} \text{ ....3.11}$$

$\Rightarrow$ BIC= -2log (likelihood) + P* log(n) ,where P is the number of Parameter in the model and n is the number of sample size given in the data set and then the smaller BIC results in the better candidate model.

What tends to happen is that *AIC* will tend to pick models where the predictors choose should have at least moderate influence, whereas *BIC* tends to include variables with strong influence.

Also two of the most frequently used measures for goodness-of-fit in the Generalized Linear models are the scaled deviance and Pearson chi-squares.

The scaled deviance (G^2) is defined as:

$$G^2 = 2 \text{ Log } (L_{max} - L_{red}) \text{ ....3.12}$$

Deviance provides an alternative to likelihood. I.e. several further measures of model performance are based on likelihood function. In other words the Deviance given by:

$$D = 2(l(y;y) - l(\mu;y)$$

Where $l(y; y)$ and $l(\mu, y)$ are the models Log likelihood evaluated respectively under μ and y. For an adequate model D also has an asymptotic chi-square distribution with n-p degree freedom where P is the number parameter in the model and n is the number of sample size in the data given.

The Pearson chi-squares statistic is equivalent:

$$\sum_{i=1}^{n} \left(\frac{Z_i - \mu_i}{Var(Z_i)}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.13$$

For an adequate model, the statistic has an asymptotic chi-squares distribution with n - p degrees of freedom, where n denotes the number of rating classes and p the number of parameters.

3.4. TESTS FOR DISPERSION

The generalized Poisson regression model reduces to the Poisson regression model when the dispersion parameter α equals to zero. To assess justification of using Generalized Poisson Regression model (GPRM) over the PR model, we test the hypothesis:

$H_0: \alpha = 0$ vs $H_a: \alpha \neq 0$................3.14

The test of H_0 in (3.14) is for the significance of the dispersion parameter. Whenever H_0 is rejected, it is recommended to use the Generalized Poisson regression model or Negative Binomial Regression model in place of the PR model.

To carry out the test in (3.14), one can use the asymptotically normal Wald type " t " statistic defined as the ratio of the estimate of a to its standard error. Another way to test the null hypothesis of a equals to zero is to use the likelihood ratio statistic, which is approximately chi-square distribution with one degree of freedom when the null hypothesis is true. Both the likelihood ratio test and the Wald type "t" test are asymptotically equivalent.

3.5. CLUSTER SPECIFIC (SUBJECT SPECIFIC) MODELS

There are two general classes of models for analyzing clustered data: cluster specific (**CS**) and marginal or population averaged (PA). The most common **CS** approaches are Generalized Linear Mixed Models (GLMMS), which extend the class of generalized linear models by including random effects in the linear predictor (Breslow and Clayton 1993; Schall 1991; McGilchrist 1994; Goldstein 1991 and Longford 1994).

The responses are assumed to be independent, given a collection of cluster-specific parameters. Generalized linear mixed model is one of subject specific family (Molenberghs & Verbeke, 2005).

In most biomedical and biological data problems, interest often lies in understanding the response of individual patient characteristics and how this response is influenced by a given set of possible covariates (Myers et al.,2010). Cluster specific models are useful in such cases. Cluster specific models differ from the marginal models by inclusion of parameters that are specific to clusters or subjects within a population. Consequently, random effects will directly used in modeling the random variation in the dependent variable at different levels of the data.

3.5.1. GENERALIZED LINEAR MIXED MODEL (GLMM)

Generalized Linear Mixed Models (GLMM) has attracted considerable attention over the last years. The word "Generalized" refers to non-normal distributions for the response variable, and the word "Mixed" refers to random effects in addition to the usual fixed effects of regression analysis.

Generalized linear models (GLM) is one parts of subject specific models which extends ordinary regression by allowing non-normal responses and a link function of the mean. The generalized linear mixed model is a further extension that permits random effects as well as fixed effects in the linear predictor (Agresti, 2002).

Let $\boldsymbol{y_{ij}}$ denote the response of $\boldsymbol{i^{th}}$ individual children ever born from $\boldsymbol{j^{th}}$mother where $\boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}, \ldots \ldots\ \boldsymbol{n_j}$ is the value of the count outcome, the number of events (y_{ij} = 0, 1, 2 ,......)

Let $\boldsymbol{f}(\boldsymbol{b_j}/\boldsymbol{D})$ be the density of the $\boldsymbol{N}(\boldsymbol{0}, \boldsymbol{D})$ distribution for the random effect $\boldsymbol{b_j}$. Assumed conditionally on q-dimensional random effects $\boldsymbol{b_j}$ to be drawn independently from $\boldsymbol{N}(\boldsymbol{0}, \boldsymbol{D})$, the outcomes $\boldsymbol{y_{ij}}$ of $\boldsymbol{Y_j}$ are independent with the density of the form.

$$f_j(y_{ij}/b_j,\beta,\phi) = exp\{\phi^{-1}[y_{ij}\theta_{ij} - \psi(\theta_{ij})] + c(y_{ij},\phi)\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.15)$$

The mixed-effect Poisson regression model indicates the Expected number of counts.

Then the generalized linear mixed model (Molenberghs and Verbeke, 2005); with log link is defined as.

$$E(y_{ij}) = \mu_{ij} = exp(X'_{ij}\beta + Z'_{ij}b_j),$$

$$j = 1,2,\ldots \quad \ldots \quad \ldots m \qquad (3.16)$$

$$log\mu_{ij} = X'_{ij}\beta + Z'_{ij}b_j$$

Where $E(Y_{ij}/b_j) = \mu_{ij}$, is the mean response vector conditional on the random effects b_j , for mother in cluster j and, x_{ij} are known covariates, β are the Regression coefficient for the covariates, z_{ij} are random effect variable(s)usually just an intercept for clustered data and b_j are random effects ~N(0,$\sum_b$) how cluster j influences the observation within cluster.

3.5.1.1. Parameter Estimation for GLMM

Random-effects models can be fitted by maximization of the marginal likelihood, obtained by integrating out the random effects. Such likelihood may involve high-dimensional integrals that cannot be evaluated analytically. The likelihood of the data expressed as a function of unknown parameters is:

$$L(\beta, D, \phi) = \prod_{j=1}^{m} f_j(Y_j/\beta, D, \phi)$$

$$= \prod_{j=1}^{m} \int \prod_{j=1}^{n_j} f_{ij}(Y_{ij}/b_j, D, \phi) f(b_j/D) db_j \qquad 3.17$$

It is the integral over the unobserved random effects of the joint distribution of the data and random effects. The problem in maximizing (3.17) is the presence of m integrals over the q-dimensional random effects b_j With Gaussian data, the integral has a closed form solution and relatively simple methods exist for maximizing the likelihood or restricted likelihood. With non-linear models, numerical techniques are needed. The Laplace method (Molenberghs & Verbeke, 2005) will be designed to approximate integrals of the form:

$$\mathbf{I} = \int \mathbf{e}^{\mathbf{Q(b)}}\,\mathbf{db} \ldots\ldots\ldots\ldots\ldots\ldots.. \qquad \mathbf{3.18}$$

Where $Q(b)$ is a known, unimodal, and bounded function of a q-dimensional variable ***b.*** Let $\hat{\mathbf{b}}$ be the value of $\mathbf{b}$ for which Q is maximized. Then the second order Taylor expansion of $Q(b)$ is the form

$$\boldsymbol{Q(b) \approx Q(\hat{b}) + \frac{1}{2}(b-\hat{b})' Q''(\hat{b})(b-\hat{b})} \qquad \mathbf{3.19}$$

Where,$\boldsymbol{Q}''(\hat{\boldsymbol{b}})$ is the matrix of second-order derivative of Q, evaluated at $\hat{\mathbf{b}}$.Replacing $Q(b)$ in (3.11) by its approximation in (3.19) we obtained:

$$\mathbf{I} \approx (\mathbf{2\pi})^{\mathbf{q/2}} \left|-\mathbf{Q}(\hat{\mathbf{b}})\right|^{-\mathbf{1/2}} \mathbf{e}^{\mathbf{Q}(\hat{\mathbf{b}})}$$

Clearly, each integral (3.17) is proportional to an integral of the form (3.11) for functions $\boldsymbol{Q(b)}$ given by

$$\boldsymbol{Q(b) = \phi^{-1} \sum_{i=1}^{n_j} \left[y_{ij}\left(x'_{ij}\beta + Z'_{ij}b\right) - \psi(x'_{ij}\beta + Z'_{ij}b)\right] - \frac{1}{2} b' D^{-1}}$$

This is called the Laplace’s method or approximation of integrands. Note that the mode $\widehat{\boldsymbol{b}}$ of **Q** depends on the unknown parameters **β, ϕ,** and **D**, such that in each iteration of the numerical maximization of the likelihood will be recalculated conditionally on the current values for the estimates for these parameter.

3.5.1.2. VARIABLE SELECTION TECHNIQUE IN GLMM

A different approach to account for clustering is by using random components such as random intercepts. The model will also include the random effects, in this case, random intercepts to address the between and within-regional heterogeneity. These are introduced in the generalized linear mixed model due to the fact that, the probability of the number of children ever-born possibly varies for individuals within the same regions as well as individuals in different regions.

To select significant variables, firstly under the GLMM, model building strategy started by Fitting a model containing all possible covariates in the data. This was done by considering one (within regional variation) and two (between and within regional variations) random intercept respectively. In order to select the important factors related to the response variable, the backward selection procedure was used. The strategy is called backward because we are working backward from our largest starting model to a smaller final model. In this case, the procedure is used to remove covariates with non-significant p-values. This means that variables that did not contribute to the model based on the highest p-value was eliminated sequentially and each time a new model with the remaining covariates was refitted, until we remained with covariates necessary for answering our research question. Finally, the two models model with one random intercept (within regional variation) and two random intercept (between and within regional variations) were compared using model comparison techniques.

3.5.1.3. MODEL COMPARISON IN GLMM

This study used Likelihood ratio test and Information criteria to select the best model based on the values of asymptotic estimations.

Likelihood Ratio Test: In order to decide on the best of the two random effects models, two models fitted, one with the two random intercepts (between and within regional variations) and another with one random intercept (within regional variation). One can use the approximate restricted maximum likelihood ratio test (LRT) to compare these two models (Myers et al., 2010).

Let $LR_{full} = \mathbf{-2}\log likelihood$ value for full model and

$LR_{redu} = -2\, log\ likelihood$ Value for reduced model. Then, the likelihood ratio test statistic is given by:

$$\lambda = LR_{full} - LR_{redu}$$

The asymptotic null distribution of the likelihood ratio test statistic $\boldsymbol{\lambda}$ is a chi-square distribution with degrees of freedom equal to the difference between the numbers of parameters in the two models.

Akaike's information criterion (AIC): AIC is a measure model selection of an estimated statistical model. It is not a test on the model in the sense of hypothesis testing; rather it is a tool for model selection. The AIC penalizes the likelihood by the number of covariance parameters in the model, therefore

$$AIC = -2\log(L) + 2P$$

Where, L is the maximized value likelihood function for the estimated model and p is the number of parameters in the model. The model with the lowest AIC value is preferable.

3.5.1.4. MODEL CHECKING TECHNIQUE

In GLMM, it is assumed that the random effects are normally distributed and uncorrelated with the error term. Normality of the random effects is assessed using normal plot of each random effect. Normal Q-Q plot of estimated random effects is an important method for checking the normality (Myers et al., 2010).

4. ANALYSIS AND DISCUSSION

4.1. DESCRIPTIVE STATISTICS AND EXPLORATORY DATA ANALYSIS

Before any advance statistical analysis, it is better to examine the overview of the data. Among the sampled interviewed women from Ethiopia, 4976 (42.6%) out of 11,654 women's during the survey interview was included about the number of children they have.

From Appendix, Table 4.1 presents basic descriptive information that summarizes the determinant factors and children ever born in house hold. The total of 4976 mothers from nine regional states and two city administrations in Ethiopia were eligible for this study. Among these eligible mothers, only 136(76.4%) and 42(23.6%) mothers of Age between 15-19 years delivered one and two child respectively. For Age of mothers between 20-39years 1021(22.7%) mothers delivered two children where as 23(0.5%) mothers delivered 11 children .For Age of mothers between 40-49 years 5(1.7%) mothers deliver three children where as 66(22.4%) mothers delivered 9 children.

The Age at which the mothers delivered the first birth, 486(18.4%) mothers their Age between15-19years delivered two child where as 5(.2%) mothers delivered 12 children .Also for Age of mother at first birth between 20-39 years 527(22.6%) mothers delivered only one child and 5(0.2%) mothers delivered 12 children. This shows that 53.09% of mothers get first birth between 15-19 years than 46.90% of mothers get first birth between 20-39 years.

Educational status of mothers has decreasing proportion to the number of children ever born. There were 58.6% Uneducated mothers and 41.4% educated mothers. Around 68.2% mothers did not heard about family planning on mass media and only 31.8% mothers heard about family planning on mass media. Also 76.9% mothers' residence is Rural and only 23.1% mothers' lives in Urban.

In case of Religion of mothers 36.9% mothers follows orthodox among this 454(24.7%) of mothers delivered 2 children and only 5(.3%) of mothers delivered 12 children. 34.9% mothers follows Muslim and 358(20.6%) of mothers were delivered 2 children where as only 4(0.2%) mothers were delivered 12 children and 25.1% mother's follows protestant and 234(18.8%) mothers who were follows protestant delivered 2 children where as only 1(0.1%) mothers delivered 12 children. Only 3.1% mothers were follows other religion.

Based on whether the mother used/not contraceptive method, 65.9% mothers didn't use contraceptive method where as only 34.1 mothers used contraceptive method .Around 34.8% of mothers didn't breast feed their child and 69.3% of mother's breast fed their child.
Regarding of Age of mother at which they get married 73.8% of mothers married between age 15-19 years and only 26.1% of mothers married between 20-39 years.

4.1.1. TEST OF OVER DISPERSION

We have discussed the over view of test of dispersion in the methodology. To assess justification of using Poisson model over the NB model, we test the hypothesis:

$$\boldsymbol{H_0: \alpha = 0 \quad vs \quad H_1: \alpha \neq 0} \qquad \text{Where } \alpha \text{ is dispersion parameter.}$$

The test of H_o in above is for the significance of the dispersion parameter. Whenever H_o is rejected, it is recommended there is over dispersion in the data and Criteria for assessing Goodness of Fit can be:

Table.4.2 Test of over dispersion

Criteria	Models	DF	Value	Value/DF	p-value
Deviance	Poisson	4966	4701.0934	0.9467	<.0001
	NegBin	4966	4711.5923	0.9488	0.0003
Scaled Deviance	Poisson	4966	4701.0934	0.9467	<.0001
	NegBin	4966	4711.5923	0.9488	0.0003
Pearson Chi-Square	Poisson	4966	4944.3424	0.9956	<.0001
	NegBin	4966	4954.7904	0.9977	0.0014
Scaled Pearson X^2	Poisson	4966	4944.3424	0.9956	<.0001
	NegBin	4966	4954.7904	0.9956	0.0014

From table 4.5 in NB regression analyses, the deviance GOF statistics indicating over-dispersion was obtained. Because the Deviance divided by the df is higher than zero and the observed value of 0.9488 is significantly different from zero, with P-value 0.0003. When we have the "correct" model, outliers are not a problem, and the scaled deviance is large, over dispersion is said to occur when the fitted variance is larger than the mean.

4.1.2. MODELING NUMBER OF CHILDREN EVER BORN

Here we are used some models to modeling number of children ever born in house hold.

4.1.3. ANALYSIS OF NEGATIVE BINOMIAL REGRESSION MODEL

The model building strategy (variable selection) under Generalized Poisson regression model, and Negative binomial regression model, is started by fitting a model containing all possible covariates first for Poisson regression model. In order to select the important factors related to number of children ever born, the backward selection procedure was used. After fitting the full model for Poisson, covariates with the largest p-value of Wald test is removed and refitted the model with the rest of the covariates sequentially. Hence, Wealth index of household was the first covariates removed from the model with (p=0.7454)and Mass media either mother of child heard about family planning on media are the second covariates excluded from the model(p=0.6179) i.e. Wald test of P-value> 0.05. Finally the reduced model from Poisson Regression model is used as Reference for parameter estimation of Negative Binomial Regression model and Generalized Poisson Regression model.

In this study the mean of Number of children ever born was 3.75, which is much smaller than the variance 5. 69. This indicates that there is an over dispersion and tests of over dispersion given below on table 4.5, So that the standard Poisson regression model may not be an appropriate model to fit the data. Here Negative Binomial regression and generalized Poisson regression model allows the variance to be larger than the Mean; it is often an alternative in such situations.

For NB, Model left out with Age of mother, Age at first birth, Age at first marriage, mothers Educational status , husbands' Educational status, types of place of residence, Religion, Contraceptive use and status of breast feeding of mother.

The final model for Negative Binomial (NB) regression model after excluding insignificant parameter is as follows:

$$\begin{aligned}\log(\mu_i) = \beta_0 &+ \beta_1 Agemo1 + \beta_2 Agemoth2 + \beta_3 Agefb + \beta_4 Agefm + \beta_5 mothedu \\ &+ \beta_6 husedu + \beta_7 ResidenceU + \beta_8 Religionpr + \beta_9 Religionmus \\ &+ \beta_{10} ReligionOth + \beta_{11} contracepY + \beta_{12} B\end{aligned}$$

The subscripts in each covariates are defined as, *1= Age of mother 20-39, Age of mother 2 = 40-49, U=Urban, Pr = protestant, Mus = Muslim, Oth= Other, Y=yes, BF=breast feed, fb=age atfirst birth, fm=age at first marriage*

Table 4.3 Parameter estimates of NB and their corresponding standard errors alongside the p-values.

Effects	Level	*Parameter*	Estimates (s.e)	conf.int	*P-value*
Intercept		β_0	0.6685 (0.03186)	(0.5965, 0.7178)	<.0001
Ageofmoth.	15-19(Ref)	-	-	-	-
	20-39	β_1	0.02322 (0.01042)	(0.0017, 0.0426)	0.0334
	40-49	β_2	-0.7699 (0.02014)	(-0.8094,-0.7304)	<.0001
Agefb	15-19(Ref)	-	-	-	-
	20-39	β_3	-0.2111 (0.01823)	(-0.2468, -0.1753)	<.0001
Agefm	15-19(Ref)	-	-	-	-
	20-39	β_4	-0.07715 (0.02172)	(-0.1197, -0.0345)	0.0004
Mothedu	not educated(Ref)	-	-	-	-
	Educated	β_5	-0.2435 (0.01790)	(-0.2786, -0.2084)	<.0001
Husedu	not educated(Ref)	-	-	-	-
	Educated	β_6	-0.02385(0.01602)	(-0.0552, -0.0156)	0.008
Residence	Rural(Ref)	-	-	-	-
	Urban	β_7	-0.2167 (0.02220)	(-0.2602, -0.1732)	<.0001
Religion	Orthodox(Ref)		-	-	-
	Protestant	β_8	0.02007 (0.005676)	(0.00894, 0.03120)	<.0001
	Muslim	β_9	0.04129 (0.005594)	(0.03032,0.05225,)	<.0001
	Others	β_{10}	0.06312 (0.002700)	(0.05783, 0.06842)	<.0001
Contracep	No(Ref)	-	-	-	-
	Yes	β_{11}	-0.09512 (0.01704)	(-0.1285, -0.06172)	<.0001
BF	No(Ref)	-	-	-	-
	Yes	β_{12}	-0.06352 (0.01645)	(-0.09577,-0.03127)	0.0001

4.1.4. ANALYSIS OF GENERALIZED POISSON REGRESSION MODEL

Here generalized Poisson Regression model (GPRM) is different from Standard Poisson regression model it takes account in to dispersion parameter that we have discussed in methodology section and also used for both over and under dispersion data. Also variable selection for GPRM is the same with that NB.

The final model for GPRM regression model after excluding insignificant parameter is as follows:

$$\log(\mu_i) = \beta_0 + \beta_1 Agemo1 + \beta_2 Agemoth2 + \beta_3 Agefb + \beta_4 Agefm + \beta_5 mothedu \\ + \beta_6 husedu + \beta_7 ResidenceU + \beta_8 Religionpr + \beta_9 Religionmus \\ + \beta_{10} ReligionOth + \beta_{11} contracepY + \beta_{12} BF$$

The subscripts in each covariates are defined as, *1= Age of mother 20-39, Age of mother 2 = 40-49, U=Urban, Pr = protestant, Mus = Muslim, Oth= Other, Y=yes, BF=breast feed, fb=age atfirst birth, fm=age at first marriage*

Table .4.4 Parameter estimates of GPRM and their corresponding standard errors alongside the p-value.

Effects	Level	Parameter	Estimates (s.e)	conf.int	P-value
Intercept		β_0	0.6688(0.03185)	(0.6063, 0.7312)	<.0001
	15-19(Ref)	-	-	-	-
Ageofmoth	20-39	β_1	0.02306 (0.01042)	(0.00263, 0.04349)	0.0208
	40-49	β_2	-0.7697 (0.01042)	(-0.7911,-0.7503)	<.0001
	15-19(Ref)	-	-	-	-
Ageatfb	20-39	β_3	-0.2105(0.01821)	(-0.2462,-0.1748)	<.0001
	15-19(Ref)	-	-	-	-
Ageatfm	20-39	β_4	-0.07805(0.02170)	(-0.1206,-0.03552)	0.0003
	not educated (Ref)	-	-	-	-
mothedu	educated	β_5	-0.2511(0.01714)	(-0.2847,-0.2174)	<.0001
	not educated (Ref)	-	-	-	-
husedu	educated	β_6	-0.02377 (0.01600)	(-0.0551,-0.0076)	0.0001
	Rural(Ref)				
Residence	Urban	β_7	-0.2212(0.02196)	(-0.2642,-0.1781)	<.0001
	Orthodox(Ref)	-	-	-	-
	Protestant	β_8	0.02012(0.002682)	(0.01485,0.02539)	<.0001
Religion	Muslim	β_9	0.04124(0.002687)	(0.03597,0.04651)	<.0001
	Others	β_{10}	0.06317(0.002683)	(0.05790,0.06844)	<.0001
	No(Ref)	-	-	-	-
Contrace	Yes	β_{11}	-0.09625(0.01701)	(-0.1296,-0.06291)	<.0001
BF	No (Ref)	-	-	-	-
	Yes	β_{12}	-0.06348(0.01643)	(-0.09535,-0.03093)	0.0001

4.2. MODEL SELECTION CRITERIA

when more than one regression models are available for a given data set, one can compare performance of the given models based on some measures of criterion method ,One commonly used measure is the Akaike information criterion AIC. Here are model selection criteria.

Table. 4.5 Model selection criteria for PR, GPRM and NB regression models.

Model			
Criteria	Poisson	NB	GPRM
-2 Log Likelihood	20982	20839	19670
AIC (smaller is better)	21018	20877	19696
AICC (smaller is better)	21018	20877	19696
BIC (smaller is better)	21135	21000	19781

As we seen from this table the AIC of NB and GPRM are 20877 and 19696 respectively which are less than 21018 the AIC of Poisson, which implies that the Generalized Poisson regression model fit well the data than standard Poisson and Negative Binomial Regression model .From this we can expect that Generalized Poisson Regression model fits the data well.

4.3. PARAMETER INTERPRETATION OF GENERALIZED POISSON REGRESSION MODEL

The parameter estimates are almost similar for both Generalized Poisson regression model and Negative Binomial models. This is expected since estimates from both models are consistent. The results of standard errors of estimates from standard Poisson model are under estimated because the PR model does not consider the over-dispersion exhibited by the data. In this case the standard errors of the estimates from Generalized Poisson Regression model (GPRM) are more accurate since it considers the over dispersion showed by the data.

From the results the coefficient for Age of mother between 20-39 years was positive and statistically significant and here the log of the mean number of children ever born for Age of women 20-39 years is increases by 2.3 % higher than Age of women 15-19 years i.e. Between 20-39 years women in each category of predictor variable have exp(0.02306)=1.02343 times as many children as they did at 15-19 years where all other variable are fixed. And the log mean of number of children ever born for Age between 40-49 years a woman have 2.15 times less than the Age of mother between 15-19 years.

Age of mothers at first birth has negative coefficient and statistically significant effect and the log mean of number of children ever born for Age of mother at first birth between 20-39 years have 23.4 % times fewer than Age of mother between 15-19 years, which means that the number of CEB gets multiplied by exp (-0.2105) = 0.8102 at any fixed predictor variable. Here as Age of mother at first birth increases the fertility become decreases. Also, the coefficients of Age of mother at first marriage is negative and statistically significant and the log of the average of number of children ever born for Age of mother at first marriage between 20-39 years have 8% less than Age of mother between 15-19 years. That is the number of CEB gets multiplied by exp (-0.07805) =0.9249 where all other variable are fixed.

In case of education status at any fixed predictor variable educated woman have approximately 28.5% fewer children than uneducated woman. I.e. exp (-0.2511) = 0.7779 times as many children as UN educated woman. We seen that educated mother is associated with smaller family sizes than that of uneducated. The coefficient of education status of husband was negative and statistically significant and the log of the mean of number of children ever born for educated husband is 2.4% fewer than uneducated husband .That is exp (-0.02377) =0.9765 times as many children as UN educated husband where other explanatory variable hold constant.

Here the log of mean of number of children for urban woman have 24.8% fewer than rural woman. Which means that the number of CEB gets multiplied by exp (-0.2212) =0.8016 where all other variable are fixed. Therefore, we have seen that the effects of residence show that urban women have the lowest fertility. Protestant religion women have 2.03% more fertility than among orthodox religion. I.e. exp (0.02007) =1.0203 times as many children as orthodox religion women. Also Muslim religion women have 4.2% more fertility than among orthodox women religion, which means that exp (0.0412) = 1.0421 times as many children as orthodox religion women where all other variable are fixed.

The coefficient whether the mother used contraceptive or not was negative and statistically significant. The log mean of number of children ever born for mother who used contraceptive is 10% fewer than who don't used contraceptive. I.e. the number of CEB gets multiplied by exp(-0.09512)=0.902 where all other variable are fixed. The log of the mean of number of children for Breast feed mother is 6.5% fewer than who didn't breast feed mother. This shows that breast feed mother in each category of predictor variable have exp (-0.06348) = 0.9385 times As many children as who didn't breast feed.

4.4. ANALYSIS OF GENERALIZED LINEAR MIXED MODEL (GLMMs)

4.4.1. MODEL BUILDING IN GLMM

Here the Model included the random effects in this case, random intercepts to address between and within-regional variations. First, all main effect covariates and the two random intercepts model were fitted and as usual, insignificant covariates were removed sequentially starting from variables with highest p-value for fixed effect covariates. Then the saturated models for GLMM were fitted as follows where $\boldsymbol{b_{j\ and}\, u_{ij}}$, two random intercepts.

$$\begin{aligned}\log(\mu_i) = \beta_0 + \beta_1 Agemo1 + \beta_2 Agemoth2 + \beta_3 Agefb + \beta_4 Agefm + \beta_5 mothedu \\ + \beta_6 husedu + \beta_7 massmY + \beta_8 ResidenceU + \beta_9 Religiopr \\ + \beta_{10} Religionmus + \beta_{11} ReligionOth + \beta_{12} wealthM + \beta_{13} wealthR \\ + \beta_{14} contracepY + \beta_{15} BF + b_{j+} u_{ij}\end{aligned}$$

In order to decide which is better model of the two random effects models, two models were fitted, one the saturated model above with two random intercepts to estimate between and within regional variations And the other with one random intercept model to estimate within regional variation. AIC and Likelihood ratio tests (LRT) were used to compare the two models to select appropriate models.

Table 4.6 Information criteria for comparison of one and two random intercept models are given as:

Models	AIC	BIC	-2LogLik	Deviance	$\boldsymbol{\sigma_W}$	$\boldsymbol{\sigma_B}$	P-value
One random intercept model	19452	19563	19417.8	19418	0.1733		
Two random intercept model	19431	19548	19394.6	19395	0.1608	0.1045	0.004

Where, $\boldsymbol{\sigma_W}$ and $\boldsymbol{\sigma_B}$are within and between regional standard deviation respectively, and P is the p-value of the log likelihood ratio test of the two models.

As we have seen from table- 4.6.1 the AIC of two random intercept models is reduced from 19452 to 19431, the -2loglikelihood is reduced from 19417.8 to 19394.6 & the deviance of the model is reduced from 19418 to 19395. The small p-value of the log likelihood ratio test ($P < 0.004$) also indicates that the model with two random intercept is better model.

Also when we considered a model without random effects (i.e. simply the generalized linear Model), it gives AIC value of 19631 which is large as compared to the above two models With random effects. Therefore, we conclude that, the model with two random intercepts should be used to address between and within-regional heterogeneity in the given data.

Next, the covariates for the fixed effect were assessed and the candidate covariates were Selected by removing covariates starting from with highest p-value sequentially. Then the First removable covariate is wealth status of household with the highest p-value (P = 0.5770) and Refitted the reduced model with the remaining covariates. The AIC is reduced from 19431to 19430 and the p-value of log likelihood ratio test (P = 0.1868) supports the reduced model is Preferable one. The next removable variable is Mass media (Whether mother of child heard about family planning on mass media) with p-value (P =0.4978) and refitted the reduced model. For this model, AIC is 19428.3 which is smaller than 19430 indicates the model with smaller AIC value is selected. So, the model with small number of covariates is considered to be Preferable than the other model.

Therefore, the final proposed GLMMs for number of children ever born in house hold is given as:

$$\log(\mu_i) = \beta_0 + \beta_1 \text{Age1mo} + \beta_2 \text{Age2moth} + \beta_3 \text{Agefb} + \beta_4 \text{Agefm} + \beta_5 \text{mothedu} + \beta_6 \text{husedu} + \beta_7 \text{ResidenceU} + \beta_8 \text{Religiopr} + \beta_9 \text{Religionmus} + \beta_{10} \text{ReligionOth} + \beta_{11} \text{contracepY} + \beta_{12} \text{BF} + b_{j+} u_{ij}$$

Table 4.7 parameter estimates and standard errors of the GLMM

Effects	Level	*Parameter*	Estimates (S.e)	95% conf.int	*P-value*
Intercept		$\boldsymbol{\beta_0}$	0.35326(0.08086)	(0.19476,0.51176)	0.008
	15-19(Ref)	-	-	-	-
Agemo	20-39	$\boldsymbol{\beta_1}$	1.08841(0.06936)	(0.95245, 1.22437)	<.0001
	40-49	$\boldsymbol{\beta_2}$	-1.82980(0.07312)	(-1.97312,-1.68647)	0.0001
	15-19(Ref)	-	-	-	-
Ageatfb	20-39	$\boldsymbol{\beta_3}$	-0.21247(0.01923)	(-0.25312,-0.20017)	<.0001
	15-19(Ref)	-	-	-	-
Ageatfm	20-39	$\boldsymbol{\beta_4}$	-0.08580(0.02285)	(-0.13059, -0.04102)	0.0031
	not educated(Ref)	-	-	-	-
mothedu	educated	$\boldsymbol{\beta_5}$	-0.24514(0.01913)	(-0.28265, -0.20763)	0.0001
	not educated(Ref)	-	-	-	-
husedu	educated	$\boldsymbol{\beta_6}$	-0.06137(0.01742)	(-0.09552, -0.02723)	0.0001
	Rural(Ref)	-	-	-	-
Residence	Urban	$\boldsymbol{\beta_7}$	-0.16765(0.03222)	(-0.23081,-0.02723)	0.001
	Orthodox(Ref)	-	-	-	-
	Protestant	$\boldsymbol{\beta_8}$	0.09442(0.03110)	(0.03346, 0.15538)	0.0119
Religion	Muslim	$\boldsymbol{\beta_9}$	0.04772(0.02750)	(0.00618, 0.10163)	*0.0107*
	Others	$\boldsymbol{\beta_{10}}$	0.13389(0.04954)	(0.03678, 0.23099)	0.0342
	No(Ref)	-	-	-	-
Contrace	Yes	$\boldsymbol{\beta_{11}}$	-0.06756(0.01839)	(-0.10362, -0.03150)	0.044
	not BF(Ref)	-	-	-	-
BF	BF	$\boldsymbol{\beta_{12}}$	- 0.05829(0.01722)	(-0.09204,-0.02454)	<.0001

Ref=reference category of covariates

4.4.2. PARAMETER INTERPRETATION OF GLMM

In the GLMM analysis, parameter interpretation is based on specific subjects or Cluster. The parameter interpretation is conditional on the random effects, which is common For all individual mothers in the same cluster.

Given the same random effects b_j, the estimated average number of children ever born in a family was exp (1.0884) =2.969 times higher for mothers of age group 20-39 years and exp (-1.82980) =0.1604 times lower for mothers of age group 40-49 years compared to mothers with age group 15-19 years in the same j^{th} cluster keeping constant the other fixed effect variables in the model. This implies that the average number of children ever born in a family are twice more likely for mothers whose Age group is 20-39 years than with mothers whose age group is 15-19 years in the same cluster at the given random effects. The log of average number of children ever born (CEB) is 16.04% fewer for mothers whose Age group is 40-4 9 years than with mothers whose age group is 15-19 in the same cluster at the given random effects.

In the same ways the estimated average number of children ever born is exp (-0.21247)= 0.8085 times lower for Age of mothers at first birth between age group 20-39 years compared to Age of mother at first birth between age group 15-19 years in the same j_{th} cluster keeping constant the other fixed effect variables in the model. Which implies that the log of average number of children ever born (CEB) is 23.6% times fewer for Age of mother at first birth whose age group is 20-39 years than with mothers whose age group is 15-19 years in the same cluster at the given random effects .Also the estimated number of children ever born in a family was exp(-0.08580)=0.9178 times fewer for Age of mother at first marriage between age group 20-39 years compared to Age of mother at first marriage between age group 15-19 years in the same j_{th} cluster keeping constant the other fixed effect variables in the model. i.e. the average number of children ever born in a family is 9% times lower for Age mother at first marriage whose age group is 20-39years than with mothers whose age group is 15-19 years in the same cluster at a given random effect.

In case of educational level of mother the estimated average number of children ever born in a family was exp(-0.24514)=0.7825 times fewer for educated mothers compared to uneducated mothers in the same j_{th} cluster keeping constant the other fixed effect variables in the model.

This shows that the average number of children ever born in a family is 27.8%times lower for educated mothers than uneducated mothers in the same cluster at a given random effect. Also, the average number of children ever born in a family was exp (-0.06137) =0.9405 times lower for educated husband compared to uneducated husband in the same j_{th} cluster keeping constant the other fixed effect variables in the model. This implies that the average number of children ever born in a family was 6.3% times fewer for educated husband than uneducated husband in the same cluster at a given random effect.

In the same way the estimated average of number of children ever born in a family was exp(-0.1676)=0.8457 times lower for Urban residence of mother compared to Rural mothers in the same j_{th} cluster keeping constant the other fixed effect variables in the model. This implies that the the average number of children ever born was 18.2% times fewer for urban mothers than rural mothers in the same cluster at a given random effect.

In another ways the estimated average number of children ever born in a family was exp (0.09442)=1.0990 and exp(0.04772)=1.0488 times higher for Protestant and Muslim religious mothers respectively compared with Orthodox religion mothers in the same j_{th} cluster with constant random effect in the given cluster and the other fixed effect covariates in the model are constant. i.e the average number of children ever born is 9.9% and 4.8% times higher for Protestant and Muslim religion woman respectively than orthodox religion woman

The estimated mean number of children ever born in a family was exp(-0.06756) =0.9347 times lower for mothers who used contraceptive method compared to mothers who didn't used contraceptive method in the same j^{th} cluster keeping constant the other fixed effect variables in the model. This implies the average number of children ever born in a family was 7% times fewer for mothers who used contraceptive method than who didn't use in the same cluster at a given random effect. Also the estimated average number of children ever born in a family was exp (- 0.05829) =0.943 times lower for mothers who breast feeding compared to Who didn't breast feed in the same j^{th} cluster keeping constant the other fixed effect variables in the model. Which the of log average number of children ever born in a family was 6% times lower for mothers who breast feed than who didn't breast feed in the same cluster at a given random effect.

The random effect parameters under GLMM are not estimable and then we cannot interpret it. However the estimates of within and between standard deviation of random effects are 0.1608 And 0.1045 which is larger than zero. Then, we can interpret; there is significance Heterogeneity within and between regions of level of Fertility among women of child bearing Age in Ethiopia.

4.4.3. MODEL DIAGNOSTIC FOR GLMM

The Q-Q plot from the following figure in first panel verifies that the residuals are close to Normally distributed and symmetric around zero. Thus, it meets the assumption of the Distribution of error terms. As well, to the above, the non linearity of the Q-Q plot confirms The model is not linear. Residuals versus observation "clid" number plot panel two, also Suggested that the residuals are symmetric around zero (i.e. positive and negative residuals Are almost equal). Q-Q plots for normality of random effects at regional and cluster levels are Also given in the figure at panel three and four, and illustrates that the random effects are Normally distributed with mean zero and variance covariance matrix D. Thus, the fitted GLMM model is good for the given data.

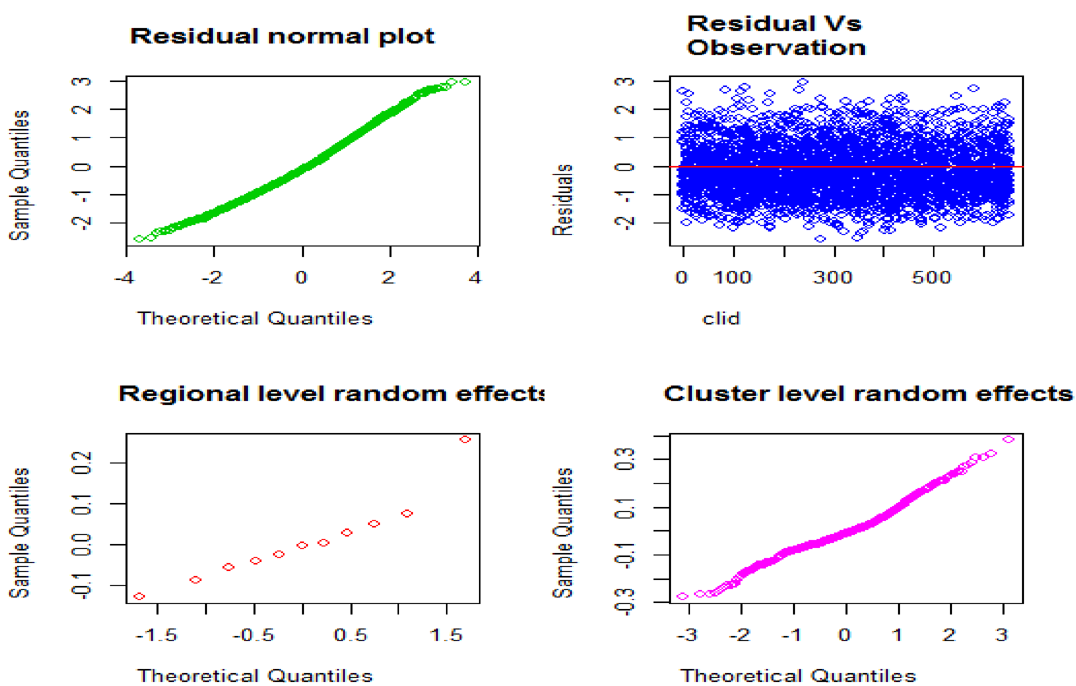

Figure 4.2. Model Diagnosis plots for the generalized linear mixed model

4.4.4. DISCUSSION

In order to effectively deal the unrestrained/Uncontrolled/ population growth and its associated problems in Ethiopia there appears need to be investigating the factors that reduce high fertility. Accordingly, this study was aimed at statistical modeling children ever born from women of 15-49 years in Ethiopia. Before any advance statistical analysis summary statistics were employed to explore the association between the response variable of interest and available covariates in the model. Thus, the analysis was extended to the statistical methods of modeling count data like Poisson regression model, Negative Binomial Regression model and Generalized Poisson regression model and Also, Generalized Linear Mixed Model.

However it was also recognized that the number of children ever born (CEB) which is count data practice often display over dispersion .i.e. a situation where the variance of response variable exceed the mean. In appropriate imposition of the Poisson May underestimates the standard error and overstate the significance of regression parameters and consequently, giving misleading inference about the regression parameter. To handling over dispersion Generalized Poisson Regression model and Negative Regression model were used.

Two different models from generalized linear models families were fitted in order to assess which model is efficiently explain the relations between response and explanatory variables. After then Generalized Poisson regression model were selected as the best model and with smallest AIC value and smallest standard error relative to Negative Binomial Model by taking account dispersion parameter. This is supported the idea explained by Mariam et al. Generalized Poisson Regression model is reasonably efficient relative to Negative Binomial Regression model (Mariam et al, 2012).

The purpose of GLMM was to evaluate within and between regional variations of number of live born children a woman has delivered in Ethiopia. Two models was fitted one with only one intercept model to assess only within regional variation and other with two random intercepts model, in order to account within and between regional variations. Additionally, generalized linear model was fitted, as the sake of comparison whether including random effects in the analysis is important. Three models were compared using the AIC value followed by likelihood ratio test and a model with two random intercept was favorable.

This demonstrates that, accounting within and between regional variations for the analysis of number of live born children a woman has delivered indicates within and between regional heterogeneity in Fertility.

All the fitted models were leads to the same conclusion that Age of mother, Age at first birth, Age at first marriage, parent's education /whether mother and husband educated/not/ ,Place of residence, contraceptive use ,Religion and Breast feeding status of mother were significantly associated with number of children ever born a woman has delivered.

Age of mother between 20-39 years has positive effect on number of children ever born woman has delivered where as for Age of mother between 40-49 years negative effect on number of live has born children a woman has delivered. This finding implies Female fertility is affected by age, After puberty, female fertility increases and then decreases, with advanced maternal age causing an increased risk of female infertility. A woman's fertility peaks in the early and mid-20s, after which it starts to, decline slowly, (Hall, Carl T.; 2002). The cessation of menstrual periods, generally occurs in the 40s and 50s and marks the cessation of fertility, although age-related infertility can occur before then (A.D.A.M.; 2011).This can be explained the fact that as Age of woman increases fertility become decline, Specially for Age of woman in mid 20s who didn't use family planning the fertility may become peaks which leads to uncontrolled fertility. This is similar with studies by (Vilaysook, 2009).

Age at first birth has significant effect on the number of children ever born alive. Age at first give birth is an important factor influencing fertility in countries like Ethiopia where level of contraception is very low. Those women who get married at early age exposed to an early sexual intercourse and early first give birth, which in turn leads to too many teen age pregnancies. A study undertaken in Ethiopia revealed a situation in which mothers with an earlier age at first birth are likely to end up in having many children (Tewdros *et al*., 2010). Apart from the negative impact it poses on women's health, this culture of early marriage has a greater likelihood of having a lot of children eventually.

Similar with the previous studies (CSA, 2006) women who marry early have on average a longer period of exposure to pregnancy and a greater number of life time births. This study

Confirmed that a significantly positive effect of Age of mother at first birth and number of children ever born. Early age at first birth in the context of marriage may in the short-term elevate a young woman's social status as she quickly proves her "value" by producing offspring. But early childbearing often means decreased mobility, less education to acquire skills that may enable the young woman to better care for her and her family and earn a wage, and fewer life opportunities in general. This in turn decrease's the young woman's decision-making power in areas related to her own reproductive health. Increasing the age at first birth not only positively impacts the health status of a young mother and child ever born; it can dramatically impact a young woman's future from an economic, social, and emotional perspective. According to (Alauddin and MacLaren, 1999) when pregnancy occurs before adolescents are not fully developed, they can be exposed too much higher risks of maternal morbidity and mortality.
i.e. The younger a woman is when she first gives birth, the longer her total child-bearing period and the more children she is likely to have which increases the risks to the life and health of both mothers and children. Similar studies states early pregnancy before girls' physiological maturity can damage their reproductive and excretal organs and can lead to increased prenatal and maternal mortality in communities where there is low coverage of maternal and child health services (Muleta et al. 2008).

According to (Boupha *et al*., 2005), Woman who live in urban areas and educated tended to have a higher age at first marriage than those who lived in the rural area and uneducated. This study Confirmed that a significantly positive effect of Age of mother at early first marriage and number of children ever born. Timing/year/ of marriage is a prime indicator of the age at first birth, but not so much an indication of fertility where there are significant number of births outside marriage (World Fertility Report, 2009). Age at first Marriage is a leading social and demographic indicator of the exposure of women to the risk of pregnancy, especially in the case of low levels of contraceptive use, and, therefore, is important for an understanding of fertility.

With the previous studies Hoem et al. (2006) find that ultimate fertility decreases somewhat with an increasing educational level. This study confirmed that a significantly negative association between education of mothers and number of children ever born in house hold.
Those educated women's have lower fertility compared with those uneducated women (Dejene, 2000; Vilaysook, 2009).

Education serves as prospect of information and knowledge regarding to the benefit of small family size and effective family planning method. This finding supported by the negative relationship is explained as an outcome from higher opportunity cost of childbearing for educated women (D‟Addio & D‟Ercole, 2005). Another finding Bledsoe and Cohen (1993) which indicate that throughout the world, formal schooling for women is the single most consistent variable correlated with their low fertility. This may be in cases where both women's and men are schooling matter; women's schooling exerts a much larger negative effect on fertility than men's schooling. It is likely that an educated family will have a better understanding and knowledge of modern family planning which implies that Education leads to better awareness of family planning a reason for this finding may be Moreover; more educated women have lower fertility due to their informed use of contraceptives in an attempt to limit family size.

As several studies, we also found that negative association between Residence of mother and the number of children ever born (DHS 2010). This finding reflects the finding of several previous studies which have reported a significantly lower number of children ever born for urban mothers compared rural mothers. The urban fertility in Sub-Saharan Africa is on average almost 30 % lower than the rural fertility (Shapiro and Tambashe 2000; Dudley and Pillet 1998). As stylized facts, mean incomes are pervasively higher in urban areas (even when adjusted for smaller household size), women tend to engage more in wage labor, paternal and maternal education levels are higher, infant mortality rates are lower. Moreover, the worldwide Demographic and Health Surveys (2010) data suggest that Ethiopia not only has one of the world’s highest fertility rates (at 5.4 children in 2005), the country also has the world’s largest rural-urban fertility differential: the projection in 2005 was that an average rural Ethiopian woman was expected to give birth to 6 children in her lifetime, relative to just 2.4 children in the country’s urban areas. Since 85% population of Ethiopia live in Rural; this reason of finding may be due to lack of awareness about family planning and contraceptive use and also they think that the children are the gift of God. They don’t care for the bad effects of over number of children.

In this study, mothers who used contraceptive method were significantly associated with decreased number of children ever born in house hold. This finding reflects the finding of several previous studies which have reported a significantly high effective use of contraceptive method for urban woman compared rural woman (Sajid et al., 2005; Azhar and Pasha, 2008).According to this finding only 10% of the mothers were used contraceptive. A first reason for this may be due to population of UN educated mothers who lives in rural area and may be UN appropriate use of contraceptive method. The second reason may be that low use of modern methods of contraception was largely due to the lack of knowledge of supply sources, and low levels of employment outside the home and unavailability of family planning.

To be able to make choices between the different types of contraceptive methods, women must first be aware of the methods that exist, their benefits and the side effects of each (Population Reports 1999; WHR 2005).

Women's educational and occupational status was found to be associated with current contraceptive practice. Kaba (2000) documented similar results. Spousal discussion about family planning and contraceptive practice has been found to be crucial for the wider acceptance of contraceptive practice and lessening partners' fertility intention in developing countries (DeRose et al. 2004; Mesfin 2002; Nagase et al. 2003; Sharan & Valente 2002). Spousal discussion about Matters related to reproduction and family planning is viewed as being successful to the extent that it directly increases the use of contraception and favorable attitudes towards contraception among couples (Sharan & Valente 2002; Toure 1996).

The husband's view on family planning also has been consistently found to be a significant factor affecting contraceptive use in several countries including Indonesia, Sub-Saharan Africa, the Philippines, India, Nepal, Pakistan, Kuwait, and Mali (Joesoef et al., 1988; Bongaarts and Bruce, 1995; Casterline and Sinding, 2000; Shah et al., 2004, Kaggwa et al., 2008).

The other factors might be that could possibly explain women's unmet needs for contraception these; insufficient supply of appropriate contraception, lack of information or misinformation about those methods and restrictive social norms governing fertility control.

Many studies have been done investigating the possible factors that could influence individual use of Contraceptives and have yielded quite consistent results. Among the most significant factors common to most studies, it is now widely agreed that in the case of poor married couples, the husband has quite a major role to play in the decision making process of child bearing and contraceptive use. The husband's desire for children, education, employment status as well as perceptions of contraceptive use is important factors in women's decisions regarding contraception and child bearing. These studies, consequently have advocated for a gender sensitive approach to family planning promotion (pilai and gupta, 2010).

This study shows that, mothers who breast feed their child have high prolonged time to get the second child than those mothers who didn't breast feed. The results confirm similar findings from a study in Ethiopia (Yohannes et al., 2004). This might be due to the fact that, while breast feeding full- time most mothers do not ovulate and do not have menstrual periods. This means that she can't pregnant, at least for a while and therefore mothers who breast feed long time have less chance of number of children ever born than those who didn't breast feed for long time.

We found that mothers who follow Muslim religious have more number of children than compared to their counterpart of Orthodox Christian mothers. The finding contradict to study Traditionally one of the indisputable generalizations in demography has been that Orthodox Christians have higher fertility rates than the Muslim (Yohannes et al., 2003).This might be due to the variation of effective use of contraceptive /family planning/ between the two religions.

However, from the previous studies, wealth index of family and exposure to mass media about family planning of mothers were significantly associated with fertility ,but these covariates doesn't significant determinant factors on this study.

5. CONCLUSION AND RECOMMENDATION

5.1. CONCLUSION

The study Analysis was the association between fertility and its possible determinant factors as well as tried to answer important issues on high level of fertility /uncontrolled fertility/of number of live born children a woman has delivered in Ethiopia by applying reasonably applicable statistical models.

Mother's Fertility was found to be very high in rural Ethiopia and it was seen that there is fertility variation between urban and rural. A total of 4976 eligible mothers from EDHS 2011 data were included in the study.

Based on the data two models; Generalized Poisson regression model and Negative binomial regression model without considering the clustering nature of the data and based on the clustering GLMM models were applicable for the appropriate analysis in this study. Here we have seen there were over dispersion and to handling the existence over dispersion in the data Generalized Poisson Regression model were selected. The primary scientific objective of GPRM is to analyze over dispersed in the case when the mean of the response variable is less than the variance of the response variable by taking account in to dispersion parameter in the model on the effects of the given factors in the study on the count response variable of interest.

For this study Generalized Poisson regression model and Negative Binomial regression have been compared for the analysis of effects of covariates on response variable and, we conclude that GPRM exhibited the best fit for this data than Negative Binomial Regression models Even if there is clustering of the data.

For Cluster specific models; GLMM was applied for the purpose of including Random effect parameters specific to clusters, which are directly used in modeling the Random variation in the dependent variable at different levels of the data.

Here GLMM, with two random intercept model was found to be appropriate for the analysis of within and between regional variations for number of live born children a woman has delivered in Ethiopia. This concluded that there is heterogeneity in fertility between and within regions. Age of mother between 20-39 years has a positive association with the number of live born children a woman has delivered and Age of mothers between 40-49 years has Negative association with Fertility which implies that the number of live born children ever born is higher for adolescent age group than elder age group.

In this study, analysis indicated that Age of mothers at first marriage has Negative effect on the number of live born children that as Age of at first marriage increases the number of children ever born in house hold becomes reduced and we can conclude that delaying Age of mothers at first marriage can reduce fertility.

Also the study conclude that Age of mothers at first birth has negative association with the number of live born children a woman has delivered and we can conclude prolonging Age at first birth reduce fertility.

In this study, analysis indicated that parental education as the most significant predictive Factors for the number of children ever-born in house hold. The number of live born children a woman has delivered decreases with educated mothers and husband. We can conclude that educated mothers have small family size than UN educated mothers.

The study concludes that mothers from rural residence have higher number of children ever born (high level of fertility). We also conclude that place of residence creates a great barrier on the number of live born children a woman has delivered in Ethiopia. The log of mean of number of children for rural woman was 24% higher compared with mother who lives in urban Ethiopia.

In addition in this study, analysis indicated that contraceptive method as the most significant predictive factors for fertility. The number of children ever born decreases for mothers who used contraceptive method. We also conclude that prolonging breast feeding reduce level of fertility in households.

5.2. RECOMMENDATION

Since uncontrolled level of fertility of number of children ever born in house hold is critical and current issue is to reduce uncontrolled level of fertility because of high population growth rates are largely the result of frequent childbearing or high fertility which is often corresponding with a large unmet need for family planning (FP).Un controlled level of fertility leads to death of mothers and children, Also; malnutrition, lack of quality social services such as education and health. Most of the researcher often interested on standard Poisson regression model and case control study in such type of count response data without considering over dispersion and clustering nature of the data in field of medical and others which leads to an appropriate parameter estimation and misleading conclusion. Therefore, it should consider over dispersion and clustering nature of the data and applied appropriate statistical model which gives relevant output and appropriate statistical inference like generalized Poisson Regression model and Negative Binomial regression model and also, cluster specific model.

This study has identified a number of important factors that influence the number of children ever born in Ethiopia. Age of mothers at first marriage and first birth are one of the most significant Predictive factors for the number of children ever born in a family. Therefore, more childbearing could be brought to a decline by delaying early marriage and giving birth at early Age especially in rural areas of Ethiopia. To discourage early marriage and to explain the perilous consequences of bearing more children among the mothers, variety of programs in this regard should be broadcasted through the electronic and print media for the long run. Also, the women and their husbands should be made more aware of the adverse effect on health, social and economic consequences of early marriage and early childbearing. Therefore, government should take all necessary steps to reduce the level of more child bearing and also reducing number of children ever born out of marriage especially in urban areas.

Parental education is appeared as a negative predictor for number of children ever born in house hold. Therefore, informal adult education for mothers and partners should be employed as an immediate intervention to provide basic education and to increase awareness about risks of high fertility and advantage of family planning. Besides, special efforts and attention to improve formal education of the girls and boys are needed in a long run specifically for rural mothers where family planning is seen as norms.

Contraceptive method also is one of the most significant predictive factors for UN wanted pregnancies and number of children ever born in house hold. Especially Mothers from rural areas and UN educated mothers were at a greater disadvantage in using contraceptive. Therefore, either formal or informal education or also vocational training for those groups of mothers may serve as an immediate strategy to improve the use contraceptive in reducing fertility. Breast feeding also appeared as a strong negative predictor for fertility. Therefore, concerning bodies including different mass Medias and health extension workers should give special attention in raising awareness about advantages of breast feeding and family planning to be able to reduce the level of fertility especially in rural areas of Ethiopia. Thus, to community conversation, women affair, health extension workers and social ritual groups to give emphasis of couple's knowledge, approval and use to family planning. Low fertility women's should encourage and about small desired children's.

We have seen there was great fertility variation between urban and rural areas and also between and within regional fertility variation. Therefore, the concerned bodies, health professionals, family and the government give emphasize on educating women especially rural areas where the level of fertility is high by changing their attitude towards delaying early marriage and contraceptive use across all family level.

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APPENDIX

############# SAS and R code#########

SAS Code for Poiss##

```
proc nlmixed data = Fertility2;
parms a0 =0.34680  a1 = 1.10941  a2 = -1.85608  a3=   -0.21439  a4 = -0.08225
a5 =  -0.24371 a6 =-0.04279   a7 = -0.19691  a8 = 0.13946 a9=  0.07810  a10=
0.18251 a11=-0.09525 a12=-0.06300;
lambda = exp(a0 + a1 * Ageofmoth1 + a2 * Ageofmoth2 + a3*Ageatfb+ a4 *Ageatfm
+ a5 * mothedu +
 a6 * husedu+ a7*Residence+
a8*Religionpr+a9*Religionmus+a10*Religionoth+a11*contrace*a12*BF);
ll = -lambda + noofchild * log(lambda) - log(fact(noofchild));
model noofchild ~ general(ll);
predict lambda out = poi_out (rename = (pred = Yhat));
run;
```

```
###SAS code for NB###
```

```
proc nlmixed data = Fertility;
parms b0=0.34680 b1= 1.10941  b2=-1.85608  b3=-0.21439 b4=-0.08225
b5=-0.24371 b6= -0.04279 b7=-0.19691 b8= 0.13946   b9= 0.07810 b10=0.18251
b11=-0.09525   b12= -0.06300;

etanb=b0+b1*Ageofmoth+b2*Ageofmoth+b3*Ageatfb+b4*Ageatfm+b5*mothedu+
b6*husedu+b7*Residence+b8*Religionpr+b9*Religionmus+b10*ReligionOth+b11*contr
ace+
b12*BF;
lambda = exp(etanb);
ll = lgamma(noofchild + 1 / k)- lgamma(noofchild + 1) -lgamma(1 / k)
        + noofchild*log(k*lambda) - ((noofchild+1/k)*log(1+k*lambda));

  model noofchild ~ general(ll);

run
```

```
/* SAS code for GPRM model*/
proc nlmixed data = Fertility;
parms b0 =  0.34680 b1 =1.10941  b2 = 1.85608 b3=  -0.21439   b4 = -0.08225
b5 =  -0.24371
 b6= -0.04279 b7 = -0.19691   b8 = 0.13946  b9=  0.07810  b10= 0.18251  b11=
-0.09525  b12=0.06300;
etagp=b0 + b1 * Ageofmoth + b2 * Ageofmoth + b3*Ageatfb+ b4 *Ageatfm + b5 *
mothedu +
b6*husedu+ b7*Residence+ b8*Religionpr+b9*Religionmus+b10*ReligionOth+
b11*contrace+b12*BF;
 lambda=exp(etagp);
ll =noofchild*log(lambda)-noofchild*log(1+k*lambda)+(noofchild-
1)*log(1+k*noofchild)
 -log(fact(noofchild))-(lambda*(1+k*noofchild))/(1+k*lambda);
ESTIMATE "inflation probability" lambda;
  model noofchild ~general(ll);
  ods output Modelfit=fit;
title2 "Generalized Poisson Regression";
run;
```

```
### R code for GLMM###
fit<-glmer.nb(noofchild~Ageofmoth+ Ageatfb+ Ageatfm +mothedu+husedu+Residence +Religion + BF+contrace+wealth+massm,data=data)
##model for random vs fexed effect)
fit1<-glmer.nb(noofchild~Ageofmoth+ Ageatfb+ Ageatfm +mothedu+husedu+Residence +Religion + contrace+ BF+wealth+massm+(1|Region)+(1|clid),data=data)
print(fit1,corr=FALSE)
summary(fit1)
fit2<-glmer.nb(noofchild~Ageofmoth+ Ageatfb+ Ageatfm +mothedu+husedu+Residence +Religion + contrace+ BF+wealth+massm+(1|clid),data=data)
print(fit2,corr=FALSE)
summary(fit2)
anova(fit2,fit1)
##best Small AIC model##
fit10<-glmer(noofchild~Ageofmoth+ Ageatfb+ Ageatfm +mothedu+husedu+Residence +Religion + contrace+ BF+wealth+massm+(1|Region)+(1|clid),family=Poisson,data=data)
print(fit10,corr=FALSE)
summary(fit10)
###insig. cov From small AIC model##
fit11<-glmer(noofchild~Ageofmoth+ Ageatfb+ Ageatfm +mothedu+husedu+Residence +Religion + contrace+ BF+massm+(1|Region)+(1|clid), family=Poisson ,data=data)
print(fit11,corr=FALSE)
summary(fit11)
anova(fit10,fit11)
fit12<-glmer(noofchild~Ageofmoth+ Ageatfb+ Ageatfm +mothedu+husedu+Residence +Religion + contrace+ BF+(1|Region)+(1|clid), family=Poisson ,data=data)
print(fit12,corr=FALSE)
summary(fit12)
fitted(fit12)
### Comparison of random effects
```

anova(fit10,fit11,fit12)

final selected model GLMM##

fitsel<-glmer(noofchild~Ageofmoth+ Ageatfb+ Ageatfm +mothedu+husedu+Residence +Religion + contrace+ BF+(1|Region)+(1|clid), family=Poisson ,data=data)

print(fitsel,corr=FALSE)

summary(fitsel)

*lower <- coef(summary(fitsel))[,1] + qnorm(.025)*coef(summary(fitsel))[,2]*

*upper <- coef(summary(fitsel))[,1] + qnorm(.975)*coef(summary(fitsel))[,2]*

cbind(coef(summary(fitsel)), lower, upper)

```
##### Diagnosis Plots######
par(mfrow=c(2,2))
qqnorm(resid(fitsel),main="Residual normal plot",col=3,adj=0.1)
plot(data$clid,resid(fitsel),xlab="clid",ylab="Residuals",main="Residual Vs
Observation",col=4,adj=0.1)
abline(h=0,col=2)
qqnorm(ranef(fitsel)$"Region"[[1]],main="Regional level random effects",col=2,adj=0.1)
qqnorm(ranef(fitsel)$"clid"[[1]],main="Cluster level random effects",col=6,adj=0.1)
```

Table 4.1. Summary of descriptive statistics for number of children ever born in house hold

Variable	**level**	**Number of children ever born (%)**												Total
		1	2	3	4	5	6	7	8	9	10	11	12	
Agemoth	15-19	136(76.4)	42(23.6)	-	-	-	-	-	-	-	-	-	-	**178**
	20-39	746(16.6)	1021(22.7)	797(17.7)	630(14)	467(10.4)	348(7.7)	227(5)	128(2.8)	83(1.8)	30(0.7)	23(0.5)	3(1)	**4503**
	40-49	-	-	5(1.7)	15(5.1)	18(6.1)	32(10.8)	59(20)	46(15.6)	66(22.4)	35(11.9)	12(4.1)	7(2.4)	**295**
Ageatfb	15-19	355(13.4)	486(18.4)	433(16.4)	374(14.2)	283(10.7)	242(9.2)	170(6.4)	120(4.5)	97(3.7)	51(1.9)	26(1.0)	5(.2)	**2642**
	20-39	527(22.6)	577(24.7)	369(15.8)	271(11.6)	202(8.7)	138(5.9)	116(5.0)	54(2.3)	52(2.2)	14(.6)	9(.4)	5(.2)	**2334**
Mothedu	noteduc	251(8.6)	486(16.7)	485(16.6)	462(15.9)	356(12.2)	298(10.2)	228(7.8)	139(4.8)	122(4.2)	54(1.9)	25(.9)	8(.3)	**2914**
	educ.	631(30.6)	577(28.0)	317(15.4)	183(8.9)	129(6.3)	82(4.0)	58(2.8)	35(1.7)	27(1.3)	11(.5)	10(.5)	2(.1)	**2062**
Massm	No	501(14.8)	674(19.9)	552(16.3)	477(14.1)	372(11.0)	284(8.4)	224(6.6)	124(3.7)	102(3.0)	46(1.4)	28(.8)	8(.2)	**3392**
	Yes	381(24.1)	389(24.6)	250(15.8)	168(10.6)	113(7.1)	96(6.1)	62(3.9)	50(3.2)	47(3.0)	19(1.2)	7(.4)	2(.1)	**1584**
Husedu	not edu	201(10.1)	350(17.5)	341(17.1)	285(14.3)	244(12.2)	192(9.6)	149(7.5)	88(4.4)	89(4.5)	38(1.9)	17(.9)	3(.2)	**1997**
	educ	681(22.9)	713(23.9)	461(15.5)	360(12.1)	241(8.1)	188(6.3)	137(4.6)	86(2.9)	60(2.0)	27(.9)	18(.6)	7(.2)	**2979**
Resid	Rural	508(13.3)	713(18.6)	627(16.4)	541(14.1)	425(11.1)	351(9.2)	262(6.8)	163(4.3)	142(3.7)	59(1.5)	30(.8)	8(.2)	**3829**
	Urban	374(32.6)	350(30.5)	175(15.3)	104(9.1)	60(5.2)	29(2.5)	24(2.1)	11(1.0)	7(.6)	6(0.5)	5(0.4)	2(.2)	**1147**
Religion	Orthodox	442(24.1)	454(24.7)	249(13.6)	201(11)	171(9.3)	117(6.4)	90(4.9)	59(3.2)	23(1.3)	21(1.1)	3(.2)	5(.3)	**1835**
	Protestant	165(13.2)	234(18.8)	205(16.4)	201(16.1)	133(10.7)	95(7.6)	86(6.9)	48(3.8)	58(4.6)	15(1.2)	7(.6)	1(.1)	**1248**
	muslim	259(14.9)	358(20.6)	324(18.6)	227(13)	163(9.4)	149(8.6)	96(5.5)	53(3)	61(3.5)	22(1.3)	25(1.4)	4(.2)	**1741**
	Others	16(10.5)	17(11.2)	24(15.8)	16(10.5)	18(11.8)	19(12.5)	14(9.2)	14(9.2)	7(4.6)	7(4.6)	-	-	**152**

Variable	**level**	**<u>Number of children ever born (%)</u>**												Total
		1	2	3	4	5	6	7	8	9	10	11	12	
	Poor	231(12)	358(18.6)	283(14.7)	275(14.3)	249(12.9)	198(10.3)	123(6.4)	83(4.3)	69(3.6)	34(1.8)	19(1)	3(.2)	**1925**
Wealth	middle	109(13.2)	160(19.4)	150(18.2)	108(13.1)	84(10.2)	74(9)	57(6.9)	38(4.6)	27(3.3)	8(1)	6(.7)	3(.4)	**824**
	Rich	542(24.3)	545(24.5)	369(16.6)	262(11.8)	152(6.8)	108(4.8)	106(4.8)	53(2.4)	53(2.4)	23(1)	10(.4)	4(.2)	**2227**
	No	496(15.1)	592(18.1)	521(15.9)	472(14.4)	346(10.6)	293(8.9)	223(6.8)	142(4.3)	115(3.5)	48(1.5)	23(.7)	8(.2)	**3279**
Contrace	Yes	386(22.7)	471(27.8)	281(16.6)	173(10.2)	139(8.2)	87(5.1)	63(3.7)	32(1.9)	34(2)	17(1)	12(.7)	2(.1)	**1697**
	not BF	365(23.8)	319(20.8)	224(14.6)	176(11.5)	142(9.3)	124(8.1)	74(4.8)	40(2.6)	43(2.8)	19(1.2)	5(.3)	4(.3)	**1535**
BF	BF	517(15)	744(21.6)	578(16.8)	469(13.6)	343(10)	256(7.4)	212(6.2)	134(3.9)	106(3.1)	46(1.3)	30(.9)	6(.2)	**3441**
	15-19	546(14.9)	749(20.4)	585(15.9)	501(13.6)	387(10.5)	305(8.3)	241(6.6)	140(3.8)	121(3.3)	56(1.5)	35(.1)	10(.3)	**3676**
Ageatfm	20-39	336(25.8)	314(24.2)	217(16.7)	144(11.1)	98(7.5)	75(5.8)	45(3.5)	34(2.6)	28(2.2)	9(.7)	-	-	**1300**

Mean of number of children ever born=3.7357315

Std Dev of number of children ever born=2.3904116

Table 4.9 Full model for variable selection of Poisson model

Coefficient	Level	Estimate	Std. Error	z value	Pr(>|z|)
Intercept		0.3575168	0.0711504	5.025	5.04e-07 ***
	15-19(Ref)	-	-	-	-
Ageofmoth	20-39	1.1060914	0.0685338	16.139	< 2e-16 ***
	40-49	-1.8543693	0.0718338	- 25.815	< 2e-16 ***
	15-19 (Ref)	-	-		
Ageatfb	20-39	-0.2137716	0.0182715	-11.700	< 2e-16 ***
	15-19(Ref)				
Ageatfm	20-39	-0.0840901	0.0217687	-3.863	0.000112 ***
	Uneducated (Ref)	-	-	-	-
Mothedu	educated	-0.2457288	0.0182106	-13.494	< 2e-16 ***
	Uneducated(Ref)	-	-	-	-
Husedu	Educated	-0.0444484	0.0165677	-2.683	0.007300 **
	No(Ref)	-	-	-	-
Massme	Yes	0.0007692	0.0174423	0.044	0.964823
	Rural(Ref)	-	-	-	-
Residence	Urban	-0.2104503	0.0247584	-8.500	< 2e-16 ***
	Orthodox(Ref)	-	-	-	-
Religion	protestant	0.1399145	0.0197283	7.092	1.32e-12 ***
	Muslim	0.0757749	0.0179709	4.217	2.48e-05 ***
	Others	0.1865659	0.0396631	4.704	2.55e-06 ***
	Poor(Ref)	-	-	-	-
Wealth	middle	-0.0419120	0.0208237	-2.013	0.044146 *
	Rich	0.0099495	0.0196100	0.507	0.611897
	No(Ref)	-	-	-	-
Contrace	Yes	-0.0957623	0.0171691	-5.578	2.44e-08 ***
	No(Ref)	-	-	-	-
BF	Yes	- 0.0635506	0.0164559	-3.862	0.000113***

Table4.10. Reduced model for variable selection of Poisson model as Reference

Coefficient	Level	Estimate	Std. Error	z value	Pr(>|z|)
Intercept		0.34680	0.07088	4.893	9.94e-07 ***
	15-19(Ref)	-	-	-	-
Ageofmoth	20-39	1.10941	0.06849	16.198	< 2e-16 ***
	40-49	- 1.85608	0.07176	-25.864	< 2e-16 ***
	15-19(Ref)	-	-	-	-
Ageatfb	20-39	-0.21439	0.01825	-11.745	< 2e-16 ***
	15-19(Ref)	-	-	-	-
Ageatfm	20-39	-0.08225	0.02175	-3.781	0.000156 ***
	Uneducated(Ref)	-	-	-	-
Mothedu	educated	-0.24371	0.01794	-13.581	< 2e-16 ***
	Uneducated(Ref)	-	-	-	-
Husedu	educated	-0.04279	0.01630	-2.625	0.008663 **
	Rural(Ref)	-	-	-	-
Residence	Urban	-0.19691	0.02263	-8.701	< 2e-16 ***
	Orthodox(Ref)	-	-	-	-
Religion	protestant	0.13946	0.01963	7.105	1.21e-12 ***
	Muslim	0.07810	0.01789	4.365	1.27e-05 ***
	Others	0.18251	0.03948	4.623	3.79e-06 ***
	No(Ref)	-	-	-	-
Contrace	Yes	-0.09525	0.01704	-5.590	2.27e-08 ***
	No(Ref)	-	-	-	-
BF	Yes	-0.06300	0.01645	-3.829	0.000129 ***

AIC=21018 **BIC**= 21135

Ref=Reference category