Common Fixed Point Results of $s - \alpha$ contraction for a pair of maps in $b-$ Dislocated Metric Spaces

A Thesis Submitted to the Department of Mathematics in Partial Fulfillment for the Requirements of the Degree of Masters of Science in Mathematics

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Declaration

I, the undersigned declare that, this research paper entitled ”Common Fixed Point Results of $s-\alpha$ contraction for a pair of maps in b - Dislocated Metric Spaces” is my own original work and it has not been submitted for the award of any academic degree or the like in any other institution or university, and that all the sources I have used or quoted have been indicated and acknowledged.

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Abstract

The purpose of this research is to construct the theorems and prove the existence and uniqueness of common fixed point results of $s-\alpha$ contraction for a pair of maps in the setting of b - dislocated metric spaces. Our results extend and generalize several well-known comparable results in the literature. The study procedure we used was that of Zoto et al., (2019). We also provided an example in support of our main result.

Contents

Chapter 1

Introduction

1.1 Background of the study

Let X be a nonempty set, a map $T : X \to X$ is said to be a self- map of X. An element x in X is called a fixed point of T if $Tx = x$. Let (X,d) be a metric space, a self-map $T : X \to X$ is said to be a contraction if there is a real number $k \in [0,1)$ such that $(Tx,Ty) \leq kd(x,y)$. Fixed point theory is an important tool in the study of nonlinear analysis as it is considered to be the key connection between pure and applied mathematics with wide applications in all branches of Mathematics, Economics, Biology, Chemistry, Physics and almost all engineering fields. The famous Banach contraction principle is one of the powerful tools in metric fixed point theory. Banach's contraction principle appeared in explicit form in Banach's (1922) thesis which states as follows: Let (X,d) be a complete metric space and let $T : X \to X$ be a contraction map, then T has a unique fixed point in X. Banach contraction principle has been extended and generalized in different directions by different researchers.

The notion of metric space has been extended, improved and generalized in many different ways. Bakhtin (1989) introduced a b - metric space as a generalization of metric space and investigated some fixed point theorem in such spaces. Hitzler (2000) introduced the notion of dislocated metric spaces. Zeyada et al. (2005) generalized the results of Hitzler (2000) and introduced the concept of complete dislocated quasi metric space.

Aage et al. (2016) proved common fixed point theorem in dislocated quasi b - metric space. Zoto et al. (2019) constructed theorems on common fixed point results on b - dislocated metric spaces and proved the existence and uniqueness. This current work extends and generalizes common fixed point results of Zoto et al. (2019) in complete dislocated b-metric space.

1.2 Statement of the Problem

In this research work, we concentrate in establishing and proving common fixed point results for a pair of maps satisfying $s-\alpha$ contraction condition in the setting b - dislocated metric spaces.

1.3 Objectives of the study

1.3.1 General objective

The general objective of this study is to establish common fixed point theorems for a pair of maps satisfying $s-\alpha$ contraction condition in the setting of b - dislocated metric spaces.

1.3.2 Specific objectives

This study has the following specific objectives

- To construct common fixed point theorems for a pair of maps satisfying $s-\alpha$ contraction condition in the setting of b - dislocated metric spaces.
- To prove the existence of common fixed points.
- To prove uniqueness of common fixed point.
- To provide an example in support of the main result.

1.4 Significance of the study

The result of this research work may have the following significance

- It may give basic research skills to the researcher.
- It may be used as a reference for any researcher who has an interest in doing research in this area.
- It may be applied in solving the existence of solution for some integral and differential equations.

1.5 Delimitation of the Study

This study focused only on establishing common fixed point theorems for a pair of self-maps satisfying $s-\alpha$ contraction condition in the setting of b - dislocated metric spaces.

Chapter 2

Review of Related literatures

Frechet (1906) introduced the notions of metric space, which is one of the cornerstones of not only mathematics but also several quantitative sciences.
Fixed point theory is one of the most dynamic research subjects in non linear analysis. In this area, the first important and significant result was proved by Banach in (1922) for a contraction maps in a complete metric space. The following results are also very important in fixed point theory.

Theorem 2.1 *(Kannan, 1968). Let* (X,d) *be a complete metric space and* $T : X \to X$ *be a self- mapping satisfying* $d(Tx,Ty) \leq a[d(x,Tx)+d(y,Ty)]$ *for all* $x,y,z \in X$ *and for all real number* $0 \leq a < 0.5$*, Then* T *has a unique fixed point.*

The mapping satisfying the above condition is called Kannan type mapping.

Theorem 2.2 *(Chatterjea, 1972). Let* (X,d) *be a complete metric space and* $T : X \to X$ *be a self- mapping satisfying* $d(Tx,Ty) \leq a[d(x,Ty)+d(y,Tx)]$ *for all* $x,y,z \in X$ *and for all real number* $0 \leq a < \frac{1}{2}$*. Then* T *has a unique fixed point.*

The mapping satisfying the above condition is called Chatterjea type mapping.
The notion of metric space has been extended, improved and generalized in many different ways. Bakhtin, (1989) introduced a b - metric space as a generalization of metric space and investigated some fixed point theorem in such spaces. Hitzler et al. (2000) introduced the notion of dislocated metric spaces. Czerwik, (1993) proved Banachs contraction theorem in so called b-metric space. Zeyada et al.,2005 generalized the results of Hitzler et al.,(2000) and introduced the concept of complete dislocated quasi metric space. The concept of quasi b-metric spaces was introduced

by Shah and Huassain et al., (2013) and obtained some fixed point results. Zoto et al.,(2014) constructed theorems on a common fixed point results on b - dislocated metric space. Chakkrid and Cholotis, (2015) introduced the concept of dislocated quasi b-metric spaces. Aage et al.,(2016) proved common fixed point theorem in dislocated quasi b - metric space.

The famous Banach's contraction principle and many other well known results in so called dislocated quasi b - metric space have been also proved.

Zoto et al.,(2019) constructed theorems on a fixed point results on b - dislocated metric spaces and proved the existence and uniqueness.

This current work extends and generalizes fixed point results of Zoto et al. (2019) in complete b - dislocated metric space.

Chapter 3

Methodology

This chapter contains study design, description of the research methodology, data collection procedures and data analysis process.

3.1 Study period and site

The study has been conducted from September 2018 to February 2020 in Jimma University under Mathematics department.

3.2 Study Design

In order to achieve the objectives stated, this study has employed analytical design.

3.3 Source of Information

This study mostly depended on document materials or secondary data. So, the available sources of information for the study were Books and published articles.

3.4 Mathematical Procedure of the Study

The mathematical procedure that the researcher followed for this research work is the following. The procedures are:

1. Establishing a theorem.

2. Constructing sequences.

3. Show that sequences are Cauchy.

4. Show the convergences of the sequences.

5. Prove the existence of common fixed points.

6. Prove the uniqueness of the common fixed points.

7. Giving applicable example for supporting the main result.

Chapter 4

Preliminaries and Main Results

4.1 Preliminaries

Throughout this thesis:
$\Re^+$ represents the set of non-negative real numbers.
N represents the set of natural numbers.
First, we remember some known definition and theorems.

Definition 4.1 *(shrivastava et al., 2012). Let X be nonempty set and a mapping $d_l : X \times X \to \Re^+$ is called a dislocated or d_l - metric if the following conditions hold:*

1. $d_l(x,y) = 0 \Rightarrow x = y$;
2. $d_l(x,y) = d_l(y,x)$;
3. $d_l(x,y) \leq d_l(x,z) + d_l(z,y)$, *for all* $x,y \in X$.

Then the pair (X,d_l) is called a d_l - metric space.

Definition 4.2 *(Kutbi et al., 2014). Let X be nonempty set and $s \geq 1$ be a real number, then a mapping $b_d : X \times X \to \Re^+$ is called b - dislocated metric if the following conditions hold:*

1. $b_d(x,y) = 0 \Rightarrow x = y$;
2. $b_d(x,y) = b_d(y,x)$;

3. $b_d(x,y) \leq s[b_d(x,z)+b_d(z,y)]$, *for all* $x,y,z \in X$.

Then the pair (X,b_d) is called a b - dislocated metric space.

Remark: The class of b - dislocated metric space is larger than that of dislocated metric space .

Definition 4.3 *(Zoto and Kumari, 2019). Let* (X,b_d) *be a complete b - dislocated metric space with parameter* $s \geq 1$. *If* $T : X \to X$ *is self-mapping that satisfy*

$$s^2 b_d(Tx,Ty) \leq \alpha \max\Big\{b_d(x,y), b_d(x,Tx), b_d(y,Ty), b_d(x,Ty), b_d(y,Tx)\Big\} \quad (4.1)$$

for all $x,y \in X$ *and* $\alpha \in [0,\frac{1}{2})$, *then* T *is called a* $s-\alpha$ *quasi-contraction.*

Lemma 4.1 *Let* (X,b_d) *be a b - dislocated metric space with parameter* $s \geq 1$. *Suppose that* $\{x_n\}$ *and* $\{y_n\}$ *are b - dislocated convergent to* $x,y \in X$ *respectively. Then we have:*

$$\frac{1}{s^2} b_d(x,y) \leq \lim_{n\to\infty} inf\, b_d(x_n,y_n) \leq \lim_{n\to\infty} Sup\, b_d(x_n,y_n) \leq s^2 b_d(x,y).$$

In particular, if $b_d(x_n,y_n) = 0$, then we have $\lim_{n\to\infty} b_d(x_n,y_n) = 0 = b_d(x,y)$.
Moreover, if each $z \in X$,we have

$$\frac{1}{s} b_d(x,z) \leq \lim_{n\to\infty} inf\, b_d(x_n,z) \leq \lim_{n\to\infty} \mathrm{Sup}\, b_d(x_n,z) \leq s b_d(x,z).$$

In particular, if $b_d(x,z) = 0$, then we have $\lim_{n\to\infty} b_d(x_n,z) = 0 = b_d(x,z)$.

Theorem 4.2 *(Zoto and kumari, 2019). Let* (X,b_d) *be complete b - dislocated metric space with parameter* $s \geq 1$. *If* $T : X \to X$ *is a self-map that is a* $s-\alpha$ *quasi contraction, then* T *has a unique fixed point in* X.

Example 4.1 *(Zoto and Kumari, 2019). Let* $X = [0,\infty)$ *and* $b_d(x,y) = (x+y)^2$ *for all* $x,y \in X$. *Then* b_d *is a b - dislocated metric on* X *with parameter* $s = 2$ *and is complete.*

Definition 4.4 *Let* (X,d) *be a metric space and* $T : X \to X$ *be a self-map, then* T *is said to be a contraction map if there exists a constant* $k \in [0,1)$ *such that* $d(Tx,Ty) \leq kd(x,y)$ *for all* $x,y \in X$.

Definition 4.5 *(Hussain et al., 2013). Let* (X, b_d) *be a b - dislocated metric space and* $\{x_n\}$ *be a sequence of points in* X*. A point* $x \in X$ *is said to be the limit of the sequence* $\{x_n\}$ *if* $\lim_{n\to\infty} d(x_n,\ x) = 0$ *and we say that the sequence* $\{x_n\}$ *is b - dislocated convergent to x and denote it by* $x_n \to x$ *as* $n \to \infty$.

Remark: (Kutbi et al., 2014). The limit of a convergent sequence in a b - dislocated metric space is unique.

Definition 4.6 *(Kutbi et al., 2014). A sequence* $\{x_n\}$ *in a b - dislocated metric space* (X, b_d) *is called a b - dislocated Cauchy sequence if and only if given* $\varepsilon > 0$*, there exists* $n_0 \in \mathbb{N}$ *such that for all* $n, m > n_0$*, we have* $b_d(x_n, x_m) < \varepsilon$ *or* $\lim_{n,m\to\infty} d(x_n\ x_m) = 0$.

Remark:(Kutbi et al., 2014). Every b - dislocated convergent sequence in b - dislocated metric space is a b - dislocated Cauchy.

Definition 4.7 *(Kutbi et al., 2014). A b - dislocated metric space* (X, b_d) *is called complete if every b - dislocated Cauchy sequence in X is a b - dislocated convergent.*

Definition 4.8 *(Sintunavarat et al., 2011). Let* $T : X \to X$ *and* $S : X \to X$ *be self-maps in* (X, b_d)*. An element* $x \in X$ *is said to be a coincidence point of* T *and* S *if and only if* $Tx = Sx = u$*. A point* $u \in X$ *is point of coincidence of* T *and* S.

Definition 4.9 *(Jungck et al., 1996). Let* T *and* S *be two self-maps on a metric space* (X, d)*. Then* T *and* S *are said to be weakly compatible if they commute at their coincident point; that is* $Tu = Su$ *for some* $u \in X$ *implies* $STu = TSu$.

Definition 4.10 *(Jungck et al., 2006). Two self-maps* $T : X \to X$ *and* $S : X \to X$ *are said to be occasionally weakly compatible (OWC) if there exists some point* $u \in X$ *such that* $Tu = Su$ *and* $STu = TSu$.

Remark: clearly weakly compatible maps are occasionally weakly compatible. However the converse is not true in general.

Definition 4.11 *(Verma et al., 2013). Let $T : X \to X$ and $S : X \to X$ be two self-maps on a metric space (X,d). Then T and S are said to satisfy the common limit in the range of S property, denoted by (CLRs) if there exists a sequence $\{x_n\} \in X$ Such that :*

$$\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = Sx,$$

for some $x \in X$.

Inspired and motivated by the result of Zoto and Kumari (2019), the purpose of this research was to extend and generalize their main theorem to common fixed point theorem involving pairs of $s-\alpha$ contraction condition in the setting of b - dislocated metric space.

4.2 Main Results

Definition 4.12 *Let* (X, b_d) *be a b - dislocated metric space with parameter* $s \geq 1$ *. If* $T, S : X \to X$ *are self-mapping that satisfy*

$$\begin{aligned} s^2 b_d(Tx, Ty) \quad \leq \quad & \alpha \max\Big\{ b_d(Sx, Sy), b_d(Sx, Tx), b_d(Sy, Ty), \\ & b_d(Sx, Ty), b_d(Sy, Tx) \Big\} \end{aligned} \tag{4.2}$$

for all $x, y \in X$ *and* $\alpha \in [0, \frac{1}{2})$*, then* T *and* S *are called an* $s - \alpha$ *contraction maps.*

Theorem 4.3 *Let* (X, b_d) *be a complete b - dislocated metric space with parameter* $s \geqslant 1$*. If* T, S *be self-maps of* X *such that: The pair* (T, S) *satisfy common limit in the range of* S *property (CLRs) in* X *and also* T *and* S *are* $s - \alpha$ *contraction maps, then:*

(i) The pair (T, S) *has a coincidence point in* X*.*

(ii) The pair (T, S) *has a unique common fixed point provided* T *and* S *are weakly compatible mapping.*

Proof: Since T and S satisfy (CLRs) property, there exists a sequence $\{x_n\} \in X$ such that:

$$\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = Su$$

for some $u \in X$. By (4.1), we have:

$$s^2 b_d(Tx, Ty) \leq \alpha \max\{b_d(Sx, Sy), b_d(Sx, Tx), b_d(Sy, Ty), b_d(Sx, Ty), b_d(Sy, Tx)\}.$$

By replacing $x = u$ and $y = x_n$ in the above condition, we obtain:

$$s^2 b_d(Tu, Tx_n) \leq \alpha \max\{b_d(Su, Sx_n), b_d(Su, Tu), b_d(Sx_n, Tx_n), b_d(Su, Tu), b_d(Sx_n, Tu)\}.$$

Taking the upper limit as $n \to \infty$ and by the above lemma, we get:

$$\begin{aligned}
s^2 b_d(Tu,Su) &\leq \alpha \max\{b_d(Su,Su), b_d(Su,Tu), b_d(Su,Su), b_d(Su,Su), b_d(Su,Tu)\} \\
&= \alpha \max\{b_d(Su,Su), b_d(Su,Tu), b_d(Su,Su), b_d(Su,Su), b_d(Su,Tu)\} \\
s^2 b_d(Tu,Su) &\leq \alpha \max\{b_d(Su,Tu), b_d(Su,Su)\}.
\end{aligned}$$

We consider the following cases:
Case 1: If $max\{b_d(Su,Tu), b_d(Su,Su)\} = b_d(Su,Tu)$,
then we have $b_d(Tu,Su) \leq \frac{\alpha}{s^2} b_d(Su,Tu)$.
Which implies $b_d(Su,Tu) = 0$, since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that $Su = Tu = z$ (say).
Case 2: If $max\{b_d(Su,Tu), b_d(Su,Su)\} = b_d(Su,Su)$, then we have:

$$\begin{aligned}
s^2 b_d(Tu,Su) &\leq \alpha\, b_d(Su,Su) \\
&\leq s\alpha[b_d(Su,Tu) + b_d(Tu,Su)] \\
&= 2s\alpha b_d(Su,Tu).
\end{aligned}$$

Then we have $b_d(Tu,Su) \leq \frac{2\alpha}{s} b_d(Su,Tu)$.
Which implies $b_d(Su,Tu) = 0$, since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that $Su = Tu = z$ (say).
Therefore, T and S have a coincidence point.
Now, by weakly compatibility property of the pair (T,S), we have:

$$Tz = T(Su) = STu = Sz.$$

Which implies that $Tz = Sz$.
Now, we show existence of a common fixed point.
First, we show that z is a fixed point of T.
By (4.2), we have:

$$s^2 b_d(Tz, Tx_n) \leq \alpha \max\{b_d(Sz, Sx_n), b_d(Sz, Tz), b_d(Sx_n, Tx_n), b_d(Sz, Tz), b_d(Sx_n, Tz)\}.$$

Taking the upper limit as $n \to \infty$, we get:

$$\begin{aligned} s^2 b_d(Tz,z) &\leq \alpha \max\{\lim_{n\to\infty} sup[b_d(Sz,Sx_n), b_d(Sz,Tz), b_d(Sx_n,Tx_n), b_d(Sz,Tx_n), b_d(Sx_n,Tz)]\} \\ &= \alpha \max\{b_d(Sz,Su), b_d(Sz,Tz), b_d(Su,Su), b_d(Sz,Su), b_d(Su,Tz)\} \end{aligned}$$

Since $Su = Tu = z$ and $Tz = Sz$, we have:

$$\begin{aligned} s^2 b_d(Tz,z) &\leq \alpha \max\{b_d(Tz,z), b_d(Sz,Sz), b_d(z,z), b_d(Tz,z), b_d(z,Tz)\} \\ &= \alpha \max\{b_d(Tz,z), b_d(Tz,Tz), b_d(z,z), b_d(Tz,z), b_d(z,Tz)\}. \end{aligned}$$

We consider the following cases:
Case 1: If $max\{b_d(Tz,z), b_d(Tz,Tz), b_d(z,z)\} = b_d(Tz,z)$,
then we have $b_d(Tz,z) \leq \frac{\alpha}{s^2} b_d(Tz,z)$.
Which implies $b_d(Tz,z) = 0$, since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that $Tz = z$.
Case 2: If $max\{b_d(Tz,z), b_d(Tz,Tz), b_d(z,z)\} = b_d(Tz,Tz)$, then we have:

$$\begin{aligned} s^2 b_d(Tz,z) &\leq \alpha\ b_d(Tz,Tz) \\ &\leq s\alpha[b_d(z,Tz) + b_d(Tz,z)] \\ &= 2s\alpha b_d(Tz,z). \end{aligned}$$

Then we have $b_d(Tz,z) \leq \frac{2\alpha}{s} b_d(Tz,z)$.
Which implies $b_d(Tz,z) = 0$, Since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that $Tz = z$.
Case 3: If $max\{b_d(Tz,z), b_d(Tz,Tz), b_d(z,z)\} = b_d(z,z)$, then we have:

$$\begin{aligned} s^2 b_d(Tz,z) &\leq \alpha\ b_d(z,z) \\ &\leq s\alpha[b_d(z,Tz) + b_d(Tz,z)] \\ &= 2s\alpha b_d(Tz,z). \end{aligned}$$

Then we have $b_d(Tz,z) \leq \frac{2\alpha}{s^2} b_d(Tz,z)$.
Which implies $b_d(Tz,z) = 0$, since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that $Tz = z$
But $Tz = Sz = z$;
Therefore, z is a common fixed point of T and S.

Uniqueness

Let z and $z\prime$ be fixed points of T and S with $z \neq z\prime$. Then by (4.2), we have:

$$\begin{aligned} s^2 b_d(Tz,Tz\prime) &\leq \alpha \max\{b_d(Sz,Sz\prime), b_d(Sz,Tz), b_d(Sz\prime,Tz\prime), b_d(Sz,Tz\prime), b_d(Sz\prime,Tz)\} \\ &= \alpha \max\{b_d(z,z\prime), b_d(z,z), b_d(z\prime,z\prime), b_d(z,z\prime), b_d(z\prime,z)\}. \end{aligned}$$

We consider the following cases:
Case 1: If $max\{b_d(z\prime,z), b_d(z,z), b_d(z\prime,z\prime)\} = b_d(z,z\prime)$.
Then we have $b_d(z\prime,z) \leq \frac{\alpha}{s^2} b_d(z,z\prime)$.
Which implies $b_d(z,z\prime) = 0$, since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that $z = z\prime$.
Case 2: If $max\{b_d(z\prime,z), b_d(z,z), b_d(z\prime,z\prime)\} = b_d(z,z)$, then we have:

$$\begin{aligned} s^2 b_d(z,z\prime) &\leq \alpha\ b_d(z,z) \\ &\leq s\alpha[b_d(z,z\prime) + b_d(z\prime,z)] \\ &= 2s\alpha b_d(z,z\prime). \end{aligned}$$

Then we have $b_d(z,z\prime) \leq \frac{2\alpha}{s} b_d(z,z\prime)$.
Which implies $b_d(z\prime,z) = 0$, since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that $z = z\prime$.
Case 3: If $max\{b_d(z,z\prime), b_d(z,z), b_d(z\prime,z\prime)\} = b_d(z\prime,z\prime)$, then we have:

$$\begin{aligned} s^2 b_d(z,z\prime) &\leq \alpha\ b_d(z\prime,z\prime) \\ &\leq s\alpha[b_d(z,z\prime) b_d(z\prime,z)] \\ &= 2s\alpha b_d(z\prime,z). \end{aligned}$$

Then we have $b_d(z\prime, z) \leq \frac{2\alpha}{s} b_d(z\prime, z\prime)$.
Which implies $b_d(z\prime, z) = 0$, since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that $z\prime = z$.
Therefore, it contradicts our assumption $z \neq z\prime$.
Hence z is a unique common fixed point of T and S. $\square$

Theorem 4.4 *Let (X, b_d) be a complete b - dislocated metric space with parameter $s \geqslant 1$. If T, S be self-maps of X such that:*

(i) The pair (T, S) satisfy occasionally weakly compatible property (OWC) in X and T and S are an $s-\alpha$ contraction maps.

Then the pair (T, S) has a unique common fixed point.

Proof: Since T and S satisfy (OWC) property, there exists a point $u \in X$ such that:

$$Tu = Su \text{ and } TSu = STu.$$

This implies that

$$TSu = TTu = STu = SSu.$$

It follows that $TTu = SSu$.
We claim that Tu is the unique common fixed point of T and S.
First, we assert that Tu is a fixed point of T.
For, if $TTu \neq Tu$, then by (4.2), we get:

$$s^2 b_d(Tu, TTu) \leq \alpha \max\Big\{b_d(Su, STu), b_d(Su, Tu), b_d(STu, TTu), b_d(Su, TTu), b_d(STu, Tu)\Big\}.$$

Since $Su = Tu$, then we have:

$$\begin{aligned} s^2 b_d(Tu, TTu) &\leq \alpha \max\Big\{b_d(Tu, TTu), b_d(Tu, Tu), b_d(TTu, TTu), b_d(Tu, TTu), b_d(TTu, Tu)\Big\} \\ &= \alpha \max\Big\{b_d(Tu, TTu), b_d(Tu, Tu), b_d(TTu, TTu)\Big\}. \end{aligned}$$

We consider the following three cases:
Case 1: If $max\{b_d(Tu, TTu), b_d(Tu, Tu), b_d(TTu, TTu)\} = b_d(Tu, TTu)$.

Then we have:
$$b_d(Tu, TTu) \leq \frac{\alpha}{s^2} b_d(Tu, TTu).$$
Which implies $b_d(Tu, TTu) = 0$, since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that $Tu = TTu$.
Case 2: If $max\{b_d(Tu, TTu), b_d(Tu, Tu), b_d(TTu, TTu)\} = b_d(Tu, Tu)$.
Then we have:

$$\begin{aligned} s^2 b_d(Tu, TTu) &\leq \alpha\, b_d(Tu, Tu) \\ &\leq s\alpha[b_d(TTu, Tu) + b_d(Tu, TTu)] \\ &= 2s\alpha b_d(TTu, Tu). \end{aligned}$$

Then we have:
$$b_d(TTu, Tu) \leq \frac{2\alpha}{s} b_d(TTu, Tu).$$
Which implies $b_d(TTu, Tu) = 0$, since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that $TTu = Tu$.
Case 3: If $max\{b_d(Tu, TTu), b_d(Tu, Tu), b_d(TTu, TTu)\} = b_d(TTu, TTu)$.
Then we have

$$\begin{aligned} s^2 b_d(Tu, TTu) &\leq \alpha\, b_d(TTu, TTu) \\ &\leq s\alpha[b_d(TTu, Tu) + b_d(Tu, TTu)] \\ &= 2s\alpha b_d(TTu, Tu). \end{aligned}$$

Then we have
$$b_d(TTu, Tu) \leq \frac{2\alpha}{s} b_d(TTu, Tu).$$
Which implies $b_d(TTu, Tu) = 0$, Since $0 \leq \alpha < \frac{1}{2}$.
This implies $Tu = TTu$ which is a contradiction with $Tu \neq TTu$.
There fore, $TTu = Tu$ and Tu is the fixed point of T.
Since $TTu = TSu = STu = Tu = SSu$, it implies $STu = Tu$.
Thus Tu is fixed point of S.
Therefore, Tu is a common fixed point of T and S.

Uniqueness

Suppose that $u, v \in X$ such that $Tu = Su = u$ and $Tv = Sv = v$ and $u \neq v$.
Then by (4.2), we get:

$$\begin{aligned}
s^2 b_d(u,v) &= s^2 b_d(Tu,Tv) \\
&\leq \alpha \max\Big\{b_d(Su,Sv), b_d(Su,Tu), b_d(Sv,Tv), b_d(Su,Tv), b_d(Sv,Tu)\Big\} \\
&= \alpha \max\Big\{b_d(u,v), b_d(u,u), b_d(v,v), b_d(u,v), b_d(v,u)\Big\} \\
&= \alpha \max\Big\{b_d(u,v), b_d(u,u), b_d(v,v)\Big\}.
\end{aligned}$$

We consider the following cases:
Case 1: If $max\{b_d(u,v), b_d(u,u), b_d(v,v)\} = b_d(v,u)$, then we have:

$$b_d(u,v) \leq \frac{\alpha}{s^2} b_d(v,u).$$

Which implies $b_d(v,u) = 0$, since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that $u = v$.
Case 2: If $max\{b_d(v,u), b_d(u,u), b_d(v,v)\} = b_d(u,u)$, then we have:

$$\begin{aligned}
s^2 b_d(v,u) &\leq \alpha\, b_d(u,u) \\
&\leq s\alpha[b_d(u,v) + b_d(u,v)] \\
&= 2s\alpha b_d(u,v).
\end{aligned}$$

Then we have:

$$b_d(v,u) \leq \frac{2\alpha}{s} b_d(v,u).$$

Which implies $b_d(v,u) = 0$, since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that $v = u$.
Case 3: If $max\{b_d(u,v), b_d(v,v), b_d(u,u)\} = b_d(v,v)$, then we have:

$$\begin{aligned}
s^2 b_d(u,v) &\leq \alpha\, b_d(v,v) \\
&\leq s\alpha[b_d(u,v) b_d(v,u)] \\
&= 2s\alpha b_d(u,v).
\end{aligned}$$

Then we have:

$$b_d(u,v) \leq \frac{2\alpha}{s} b_d(v,v).$$

Which implies $b_d(v,u) = 0$, since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that $u = v$.
Therefore, it contradicts with our assumption $u \neq v$.
Hence u is a unique common fixed point of T and S. $\square$

Theorem 4.5 *Let (X, b_d) be a complete b - dislocated metric space with parameter $s \geqslant 1$. If T, S be self-maps of X such that:*
$s^2 b_d(Tx,Ty) \leq \alpha \max\Big\{b_d(Sx,Sy), b_d(Sx,Tx), b_d(Sy,Ty), b_d(Sx,Ty), b_d(Sy,Tx)\Big\}$
for all $x, y \in X$ and $0 \leq \alpha < \frac{1}{2}$.
Then the pair (T,S) has a unique common fixed point.

Proof: Let x_0 be arbitrary given point in X. Define the sequence $y_n \in X$ such that:
$y_{2n} = Tx_{2n} = Sx_{2n+1}$, for all $n \geq 0$.
We show that $y_n \in X$ for all $n \in \mathbb{N}$.
Since $y_{2n} = Tx_{2n} = Sx_{2n+1}$, we have from (4.1) that:

$$\begin{aligned}
s^2 b_d(y_{2n}, y_{2n+1}) &= s^2 b_d(Tx_{2n}, Tx_{2n+1}) \\
&\leq \alpha \max\Big\{b_d(Sx_{2n}, Sx_{2n+1}), b_d(Sx_{2n}, Tx_{2n}), b_d(Sx_{2n+1}, Tx_{2n+1}), b_d(Sx_{2n}, Tx_{2n+1}), \\
&\quad b_d(Sx_{2n+1}, Tx_{2n}x)\Big\} \\
&= \alpha \max\Big\{b_d(y_{2n-1}, y_{2n}), b_d(y_{2n-1}, y_{2n}), b_d(y_{2n}, y_{2n+1}), b_d(y_{2n-1}, y_{2n+1}), b_d(y_{2n}, y_{2n})\Big\} \\
&\leq \alpha \max\Big\{b_d(y_{2n-1}, y_{2n}), b_d(y_{2n-1}, y_{2n}), b_d(y_{2n}, y_{2n+1}), s[b_d(y_{2n-1}, y_{2n}) + b_d(y_{2n}, y_{2n+1})], \\
&\quad s[b_d(y_{2n}, y_{2n+1} + b_d(y_{2n-1}, y_{2n})]\Big\} \\
&= \alpha \max\Big\{b_d(y_{2n-1}, y_{2n}), b_d(y_{2n-1}, y_{2n}), b_d(y_{2n}, y_{2n+1}), s[b_d(y_{2n-1}, y_{2n}) + b_d(y_{2n}, y_{2n+1})], \\
&\quad 2s[b_d(y_{2n-1}, y_{2n})]\Big\}.
\end{aligned}$$

If $b_d(y_{2n-1}, y_{2n}) \leq b_d(y_{2n}, y_{2n+1})$, for some $n \in \mathbb{N}$, then by (4.1), we have:
$s^2 b_d(y_{2n}, y_{2n+1}) \leq 2\alpha b_d(y_{2n}, y_{2n+1})$, which implies
$b_d(y_{2n}, y_{2n+1}) \leq \frac{2\alpha}{s^2} b_d(y_{2n}, y_{2n+1})$, which is not true because $\frac{2\alpha}{s^2} < 1$.

Thus $b_d(y_{2n}, y_{2n+1}) \leq b_d(y_{2n-1}, y_{2n})$ for all $n \in \mathbb{N}$.
Also, by the above inequality we get:

$$\begin{aligned} s^2 b_d(y_{2n}, y_{2n+1}) &\leq 2\alpha s b_d(y_{2n-1}, y_{2n}) \\ b_d(y_{2n-1}, y_{2n}) &\leq \frac{2\alpha}{s} b_d(y_{2n-2}, y_{2n-1}) \\ b_d(y_{2n-2}, y_{2n-1}) &\leq \frac{2\alpha}{s} b_d(y_{2n-3}, y_{2n-4}). \end{aligned}$$

Continuing like this, we have:

$$b_d(y_{2n}, y_{2n+1}) \leq c b_d(y_{2n-1}, y_{2n}) \leq c^2 b_d(y_{2n-2}, y_{2n-1}), \ldots, \leq c^n b_d(y_0, y_1),$$

where $c = \frac{2\alpha}{s}$ and $0 \leq c < 1$.
Taking the upper limit as $n \to \infty$ in inequality above, we have $b_d(y_{2n}, y_{2n+1}) \to 0$.
Now, we prove that $\{y_m\}$ is a b - dislocated Cauchy sequence where $m = 2n$.
To do this let $m, n \geq 0$ with $m > n$, we have:

$$\begin{aligned} b_d(y_n, y_m) &\leq s[b_d(y_n, y_{n+1}) + b_d(y_{n+1}, y_m)] \\ &\leq s[b_d(y_n, y_{n+1}) + s[b_d(y_{n+1}, y_{n+2}) + b_d(y_{n+2}, y_m)]] \\ &\leq s b_d(y_n, y_{n+1}) + s^2 b_d(y_{n+1}, y_{n+2}) + \ldots. + s^n b_d(y_{m-1}, y_m). \quad (4.3) \end{aligned}$$

But from inequality above, we have:

$$\begin{aligned} b_d(y_n, y_m) &= s[c^n b_d(y_0, y_1) + s c^{n+1} b_d(y_0, y_1) + \ldots] \\ &= s c^n [b_d(y_0, y_1) + s c b_d(y_0, y_1) + \ldots], \\ &= s c^n b_d(y_0, y_1)[1 + sc + (sc)^2 + \ldots], \\ b_d(y_n, y_m) &\leq \frac{s c^n}{(1 - sc)} b_d(y_0, y_1). \end{aligned}$$

Therefore, $b_d(y_n, y_m) \leq \frac{sc^n}{(1-sc)} b_d(y_0, y_1)$.
Taking the upper limit as $m, n \to \infty$, we have $b_d(y_n, y_m) \to 0$ as $sc < 1$.
Therefore, $\{ y_m\}$ is a b -dislocated Cauchy sequence in b - dislocated metric space (X, b_d).

So there is some $u \in X$ such that $\{ y_m\}$ is a b -dislocated converges to u.
Since a subsequence of a Cauchy sequence in b -dislocated metric space is a Cauchy sequence, then $\{Tx_{2n}\}$ and $\{Sx_{2n+1}\}$ are also Cauchy sequences.
If T and S are continuous mappings, we get:

$$T(u) = T(\lim_{n\to\infty} x_n) = \lim_{n\to\infty} Tx_n = S(u) = \lim_{n\to\infty} Sx_{n+1} = u.$$

Thus, u is a common fixed point of T and S.
If the self-map T is not continuous then, we consider,

$$\begin{aligned}
s^2 b_d(y_{2n}, Tu) &= s^2 b_d(Tx_{2n}, Tu) \\
&\leq \alpha \max\Big\{b_d(Sx_{2n}, Su), b_d(Sx_{2n}, Tx_{2n}), b_d(Su, Tu), b_d(Sx_{2n}, Tu), b_d(Su, Tx_{2n})\Big\} \\
&= \alpha \max\Big\{b_d(y_{2n-1}, Su), b_d(y_{2n-1}, y_{2n}), b_d(Su, Tu), b_d(y_{2n-1}, Tu), b_d(Su, y_{2n})\Big\}.
\end{aligned}$$

On taking upper limit as $n \to \infty$, we get:

$$s^2 b_d(u, Tu) \leq \alpha \max\Big\{b_d(Su, Su), b_d(Su, Tu), b_d(Su, Tu)\Big\}$$

We consider the following cases:
Case 1: If $max\{b_d(u,u), b_d(Tu,Tu), b_d(u,Tu)\} = b_d(Tu,u)$,
then we have $b_d(Tu,u) \leq \frac{\alpha}{s^2} b_d(Tu,u)$.
Which implies $b_d(Tu,u) = 0$, since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that $Tu = u$.
Case 2: If $max\{b_d(u,u), b_d(Tu,Tu), b_d(Tu,u)\} = b_d(Tu,Tu)$, then we have:

$$\begin{aligned}
s^2 b_d(Tu,u) &\leq \alpha\, b_d(Tu,Tu) \\
&\leq s\alpha[b_d(u,Tu) + b_d(Tu,u)] \\
&= 2s\alpha b_d(Tu,u).
\end{aligned}$$

Then we have $b_d(Tu,u) \leq \frac{2\alpha}{s} b_d(Tu,u)$.
Which implies $b_d(Tu,u) = 0$, since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that $Tu = u$.

Case 3: If $max\{b_d(u,u), b_d(Tu,Tu), b_d(Tu,u)\} = b_d(u,u)$, then we have:

$$\begin{aligned} s^2 b_d(Tu,u) &\leq \alpha\, b_d(u,u) \\ &\leq s\alpha[b_d(u,Tu)b_d(Tu,u)] \\ &= 2s\alpha b_d(Tu,u). \end{aligned}$$

Then we have $b_d(Tu,u) \leq \frac{2\alpha}{s} b_d(Tu,u)$.
Which implies $b_d(Tu,u) = 0$, since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that $Tu = u$.
In all cases $b_d(u,Tu) = 0$, which implies that $u = Tu$.
Thus, u is fixed point of T.
In similar cases, $b_d(Su,u) \leq \frac{2\alpha}{s} b_d(Su,u)$.
Which implies $b_d(Su,u) = 0$, since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that $Su = u$.
Thus, u is fixed point of S.
Since, $Su = u = Tu$, then u is a common fixed point of T and S.

Uniqueness

Let u and v are fixed points of T and S with $u \neq v$. Then by using 4.1, we have:

$$\begin{aligned} s^2 b_d(u,v) = s^2 b_d(Tu,Tv) &\leq \alpha \max\Big\{b_d(Su,Sv), b_d(Su,Tu), b_d(Sv,Tv), b_d(Su,Tv), b_d(Sv,Tu)\Big\} \\ &= \alpha \max\Big\{b_d(u,v), b_d(u,u), b_d(v,v), b_d(u,v), b_d(v,u)\Big\} \\ &= \alpha \max\Big\{b_d(u,v), b_d(v,v), b_d(v,u)\Big\}. \end{aligned}$$

We consider the following three cases:

Case 1: If $max\{b_d(u,v), b_d(u,u), b_d(v,v)\} = b_d(v,u)$,
then we have $b_d(u,v) \leq \frac{\alpha}{s^2} b_d(v,u)$.
Which implies $b_d(v,u) = 0$, since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that $u = v$.

Case 2: If $max\{b_d(v,u), b_d(u,u), b_d(v,v)\} = b_d(u,u)$, then we have:

$$\begin{aligned} s^2 b_d(v,u) &\leq \alpha\, b_d(u,u) \\ &\leq s\alpha[b_d(u,v) + b_d(u,v)] \\ &= 2s\alpha b_d(u,v). \end{aligned}$$

Then we have $b_d(v,u) \leq \frac{2\alpha}{s} b_d(v,u)$.
Which implies $b_d(v,u) = 0$, since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that $v = u$.
Case 3: If $max\{b_d(u,v), b_d(v,v), b_d(u,u)\} = b_d(v,v)$, then we have:

$$\begin{aligned} s^2 b_d(v,u) &\leq \alpha\, b_d(v,v) \\ &\leq s\alpha[b_d(u,v) + b_d(v,u)] \\ &= 2s\alpha b_d(u,v). \end{aligned}$$

Then we have $b_d(u,v) \leq \frac{2\alpha}{s} b_d(v,u)$.
Which implies $b_d(v,u) = 0$, since $0 \leq \alpha < \frac{1}{2}$.
Hence it follows that in all cases $u = v$.
Therefore, it contradicts with our assumption $u \neq v$.
Hence u is a unique common fixed point of T and S. □

Remark: If we take $S = I$ (I is the identity map on X) in Theorem 4.5 we set Theorem 4.1. Hence the result of this remark follows as a corollary to Theorem 4.5

Now, we give an example in support of Theorem 4.2.

Example 4.2 *Let $X=[0,1]$ and $b_d(x,y)=(x+y)^2$ for all $x,y\in X$ when $s=2$ is a b-dislocated metric on X. Then (X,b_d) is a b-dislocated metric space.*
***Solution**:*
We take the $s-\alpha$ contraction map and define the following:

$$Tx=\begin{cases}\frac{1}{25}x, \text{ if } x\in[0,1)\\ \frac{1}{30}, \text{ if } x=1\end{cases}$$

and $Sx=\frac{1}{5}x$.
Consider the sequence $\{x_n\}$ given by $X_n=\frac{1}{n}$ for all $n\in\mathbf{N}$. T and S satisfies the common limit in the range of S property.
By using (CLRs) properties, we have

$$\lim_{n\to\infty}Tx_n=\lim_{n\to\infty}Sx_n=0.$$

for $0\in Tx$ or $0\in Sx$.
We have the following cases with $\alpha=\frac{4}{25}$.
***Case 1**: For $x,y\in[0,1)$, we have:*

$$\begin{aligned}s^2b_d(Tx,Ty) &= 2^2b_d\left(\frac{1}{25}x,\frac{1}{25}y\right)\\ &= 4b_d\left(\frac{1}{25}x,\frac{1}{25}y\right)\\ &= 4\left(\frac{1}{25}x+\frac{1}{25}y\right)^2\\ &= \frac{4}{25}\left(\frac{1}{5}x+\frac{1}{5}y\right)^2\\ &\leq \frac{4}{25}b_d(Sx,Sy).\end{aligned}$$

Therefore, $s^2b_d(Tx,Ty)\leq\alpha b_d(Sx,Sy)$.

Case 2: *For $y < x$ and $x = 1$, we have:*

$$\begin{aligned}
s^2 b_d(T1, Ty) &= 2^2 b_d\left(\frac{1}{30}, \frac{1}{25}y\right) \\
&= 4 b_d\left(\frac{1}{30}, \frac{1}{25}y\right) \\
&= 4\left(\frac{1}{30} + \frac{1}{25}y\right)^2 \\
&\leq \frac{4}{25}\left(\frac{1}{5} + \frac{1}{5}y\right)^2 \\
&= \frac{4}{25} b_d\left(\frac{1}{5}, \frac{1}{5}y\right) \\
&\leq \frac{4}{25} b_d(Sx, Sy).
\end{aligned}$$

Therefore, $s^2 b_d(T1, Ty) \leq \alpha b_d(S1, Sy) = \alpha b_d(Sx, Sy)$.

Case 3: *For $x < y$ and $y = 1$, we have:*

$$\begin{aligned}
s^2 b_d(Tx, T1) &= 2^2 b_d\left(\frac{1}{25}x, \frac{1}{30}\right) \\
&= 4 b_d\left(\frac{1}{25}x, \frac{1}{30}\right) \\
&= 4\left(\frac{1}{25}x + \frac{1}{30}\right)^2 \\
&\leq \frac{4}{25}\left(\frac{1}{5}x + \frac{1}{5}\right)^2 \\
&= \frac{4}{25} b_d\left(\frac{1}{5}x, \frac{1}{5}\right) \\
&\leq \frac{4}{25} b_d(Sx, Sy).
\end{aligned}$$

Therefore, $s^2 b_d(Tx, T1) \leq \alpha b_d(Sx, S1) = \alpha b_d(Sx, Sy)$.

Case 4*: For $x = y = 1$, we have:*

$$\begin{aligned}
s^2 b_d(T1, T1) &= 2^2 b_d\left(\frac{1}{30}, \frac{1}{30}\right) \\
&= 4b_d\left(\frac{1}{30}, \frac{1}{30}\right) \\
&= 4\left(\frac{1}{30} + \frac{1}{30}\right)^2 \\
&= \frac{4}{25}\left(\frac{1}{6} + \frac{1}{6}\right)^2 \\
&\leq \frac{4}{25}\left(\frac{1}{5} + \frac{1}{5}\right)^2 \\
&= \frac{4}{25} b_d\left(\frac{1}{5}, \frac{1}{5}\right) \\
&= \frac{4}{25} b_d\left(S1, S1\right) \\
&\leq \frac{4}{25} b_d(Sx, Sy).
\end{aligned}$$

Therefore, $s^2 b_d(T1, T1) \leq \alpha b_d(S1, S1) = \alpha b_d(Sx, Sy)$.
From cases 1-4 all the conditions of theorem 4.2 are satisfied and 0 is the unique common fixed point of T and S.

Chapter 5

Conclusion and Future Scope

Zoto and Kumari, (2019) established the existence and uniqueness of fixed point for a mapping satisfying $s-\alpha$ type contraction condition in a complete dislocated metric space. In this thesis, we have explored the properties of $s-\alpha$ type contraction mapping in b - dislocated metric spaces. We established the theorem on common fixed points of two mapping satisfying $s-\alpha$ contraction condition in the setting of b - dislocated metric spaces and proved the existence and uniqueness of common fixed point for a pair of maps T and S in the setting of b - dislocated metric space. Also we provided an example in support of our main results. Our work extended fixed point result in single map to common fixed point result in a pair of maps. The presented theorem extends and generalizes several well-known comparable results in literature.

There are several published results related to existence of fixed points of self-maps defined on b - dislocated metric space. There are also many results related to the existence and uniqueness of common fixed points for a pair of maps in b -dislocated metric spaces. The researcher believes the search for the existence of common fixed points of maps satisfying $s-\alpha$ contraction conditions in b -dislocated metric space is an active area of study. So, the forthcoming postgraduate students of department of Mathematics and any researcher can exploit this opportunity and conduct their research work in this area.

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