

# THEORETICAL INTERPRETATION OF HALF LIFE OF ALPHA RADIOACTIVE NUCLEI COMPARED WITH EXPERIMENT.

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A Thesis Submitted to

The Department of Physics

PRESENTED IN FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE JIMMA UNIVERSITY JIMMA, ETHIOPIA AUGEST 2020

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# JIMMA UNIVERSITY DEPARTMENT OF PHYSICS

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Dated: <u>Augest 2020</u>

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#### JIMMA UNIVERSITY

Date: Augest 2020

Author:	By-Asefa Hadush
Title:	Theoretical Interpretation of Half Life of Alpha
	Radioactive Nuclei Compared with Experiment.
Department:	Physics
Degree: M.S	c. Convocation: Augest Year: 2020

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# **Table of Contents**

Τa	able o	of Contents	iv
$\mathbf{Li}$	st of	Tables	vi
$\mathbf{Li}$	st of	Figures	vii
$\mathbf{A}$	bstra	ıct	viii
A	cknov	wledgements	ix
1	Intr	roduction	1
	1.1	Background of the Study	1
	1.2	Statement of the Problem	2
	1.3	Objectives of the Study	3
		1.3.1 General Objectives	3
		1.3.2 Specific Objectives	3
	1.4	Significance of the Study	4
	1.5	Scope of the Study	4
	1.6	Limitation of the Study	4
<b>2</b>	Rev	view of Related Literature	<b>5</b>
	2.1	Radioactivity	5
	2.2	Alpha Decay	6
		2.2.1 Energy of Alpha Decay	7
3	Mat	terials and Methodology	9
	3.1	Materials	9
	3.2	Method	9
		3.2.1 Analytic Method	9

		3.2.2	Computational Method	10		
		3.2.3	Method of Data presentation	10		
4	Res	ults a	nd Discussion	11		
	4.1	Formu	lation of the Problem	11		
		4.1.1	Alpha Energy Dependence on Nucleon Number	11		
		4.1.2	Calculation of Half Life Dependence on Different Parameters .	17		
	4.2	Nume	rical Calculations	28		
		4.2.1	Verification of Alpha Energy Dependence on the Nucleon Num-			
			ber of Parent Nucleus	28		
		4.2.2	Verification of the calculation of Half Life Dependence on Dif-			
			ferent Parameters	30		
	4.3	Discus	ssion	34		
5	Cor	nclusio	n	36		
$\mathbf{B}_{\mathbf{i}}$	Bibliography 38					

# List of Tables

4.1	Experimental and Theoretical Values of Alpha Energy	28
4.2	Experimental and theoretical half lives of alpha radioactive nuclei	31

# List of Figures

4.1	Potential energy of alpha-particle, daughter-nucleus system as a func-	
	tion of their separation.	19
4.2	Graph of measured and observed alpha energy versus nucleon number.	30
4.3	Graph showing experimental and theoretical half lives in days against	
	radius of parent nucleus r'.	32
4.4	$Graph\ showing\ experimental\ and\ theoretical\ half\ lives\ in\ minutes\ against$	
	radius of parent nucleus, r'.	33
4.5	Graph showing experimental and theoretical half lives in years against	
	radius of parent nucleus, r'	34

# Abstract

In 1928, George Gamow used some knowledge about tunneling and the Wentzel-Kramers-Brillouin approximation(to the one-dimensional time independent Schrodinger equation) to provide the first theoretical account of alpha decay (emission of 2 protons and 2 neutrons by heavy nuclei). In this paper, it would be discussed, in first place, the WKB approximation and its application to one-dimensional potentials which act as barriers (the scattering problem). In this study, the theoretical half lives of the heavy alpha radioactive nuclei and alpha energies also have been studied. A formula for half life of alpha decay was constructed by a conventional way by considering the penetrability of a charged particle on a square coulomb barrier. In this study the effect of the centrifugal potential is ignored. The parameters in the formula are constants except for the following parameters: the radius of the parent nucleus, the proton number of the daughter nucleus and the even-odd hindrance factor,  $\aleph$ .

# Acknowledgements

Many people have supported me in countless ways in making this work a reality. Hence, I would like to take this opportunity to express my sincere gratitude to them all.First and foremost, I would like to express my profound gratitude to Dr. Teklemariam Tessema, my advisor, for his tremendous insight and scholarly support in the course of this study. His constructive feedback and suggestions added a great value to this work. My strongest thank is addressed to my Family especially to my Mother Dilay Derbew,my father Hadush Abadi and my intimate Friends. Finally I would like to thank Jimma University(department of physics) for the financial support provided and for extending facilities for carrying out this thesis.

# Chapter 1

# Introduction

## 1.1 Background of the Study

The spontaneous emission of an alpha-particle by the heavier nuclei is related with a penetration by the emitted alpha-particle through a region of very high potential energy near the nuclear surface; the region owes its existence to the repulsive potential between the nucleus and the alpha-particle and would act, in classical mechanics, as a barrier preventing emission. The penetration is a quantum-mechanical effect. Its probability depends very critically on the shape and the height of the potential energy barrier and on the kinetic energy of the alpha particle after penetration. The height is given by the nuclear radius r', since the alpha-particle is under the influence of the Coulomb repulsion without any compensating nuclear attraction when its distance from the center is larger than r'. The probability of penetration of the barrier is closely related with the lifetime of the decaying nucleus[1]

. The half lives of  ${}^{232}_{90}Th$  and  ${}^{238}_{92}U$  are greater than 10<sup>9</sup> years and the final Pb isotopes are stable. Yet the intermediate alpha decay stages have much shorter half lives. Some have less than one hour or even one second and successive stages show generally a decrease in half life and an increase in alpha decay energy as the final Pb isotope is approached. The reason that successive stage of the decay of  $^{232}_{90}Th$  and  $^{238}_{92}U$  show a decrease in half life and an increase in alpha decay energy as the final Pb isotopes are approached is that the coulomb barrier formed between the alpha particle and the daughter nucleus during alpha emission obstructs the decay. When the energy of the alpha particle increases, the probability of its penetrating the barrier increase, and so half life of the nucleus decreases. One method for estimating the decay rate or half-life for alpha decay is to use a realistic (mean-field) nuclear potential that includes deformation to calculate the penetrability of the sum of the nuclear and Coulomb potentials and then estimate the formation probability of an alpha particle. While this approach may appear promising, it has a considerable problem in that a realistic nuclear potential applicable to the whole nuclear mass region has yet to be determined in nuclear theory[2].

### **1.2** Statement of the Problem

The calculation of the absolute values of the alpha decay rate was a problem that is still not fully solved. As discussed in section 1.1, the half-life of alpha radioactive nuclei has been varied from billions of years to short period of time. In this thesis, we have investigated the main factors which play a crucial in varying the half-life of these nuclei. Therefore, investigation of the parameters was the central objective of this thesis guided by the following leading questions.

1. How the theoretical values of the alpha energies and half lives of alpha radioactive nuclei could be calculated?

- 2. What factors affect the energy of alpha particle ?
- 3. What parameters of the nucleus affect the half-life of a radioactive nuclei?
- 4. How could the theoretical value of half life of alpha radioactive nuclei agree with the experimental value?

# 1.3 Objectives of the Study

#### 1.3.1 General Objectives

The general objective of this study was to calculate the theoretical value of the halflife of alpha radioactive nuclei and find the parameters in which the half-life depends on.

#### 1.3.2 Specific Objectives

The specific objectives of this study were:

- to calculate the theoretical values of alpha energies and half-lives of radioactive nuclei.
- 2. to identify the factors that affect the energy of alpha particle.
- 3. to identify the parameters of the nucleus that affects the half-life of radioactive nuclei.
- 4. to describe how the theoretical value of the half live and alpha energies of the radioactive nuclei agree with the experimental value.

## 1.4 Significance of the Study

This study would have significant impacts in understanding of the half life of alpha radioactive nuclei. It also played a role in understanding of the alpha energies. As a result this study would have the following relevances.

- 1. The theoretical value of the half life of alpha radioactive nuclei and their energy were identified.
- 2. It was used as reference material for anyone who worked on this area.
- 3. The understanding of half life dependence on different parameters of a nucleus and the factors that affect the alpha energies was achieved.

# 1.5 Scope of the Study

The scope of this study was aimed to theoretically calculate the half life of alpha emitters.. It explores information regarding the half life of alpha radioactive nuclei in the literature.

### **1.6** Limitation of the Study

The main limitation faced to carry out this research was due to time constraints.

# Chapter 2 Review of Related Literature

The decay of alpha particle has been a source of debates for physicists for some time. According to classical theory, appositively charged alpha particle encounters a repulsive coulomb potential near the nucleus of an atom. In addition, classical physics does not explain the wide range of the half-lives of decaying particle through alpha emission, which extends from nano seconds to billions of years. On the other hand, quantum mechanics offers an alternative description; a particle partially bound within a finite potential well has a certain probability of being transmitted through the potential barrier[3].

### 2.1 Radioactivity

Decay of radioactive isotope is defined as natural disintegration of a radionuclide associated with the emission for ionizing radiation in the form of alpha particle, beta particle and gamma radiation. Radioactivity can occur both naturally and through human invention. If the composition of the nucleus deviates from the optimal range of the N: Z ratio, that is, if the nucleus has too few or many neutrons for a certain proton number (eq., in oxygen isotopes ( ${}^{14}O$ ,  ${}^{15}O$ ,  ${}^{19}O$ ,  ${}^{20}O$ ), the nucleus becomes radioactive, that is, it decays spontaneously most frequently to anther nuclei. Symbolically the process can be described as  ${}^{A}_{Z}Y \rightarrow^{A'}_{Z'}X + particles(\alpha, \beta, \gamma)$ 

Where

Y=parent nucleus

A=mass of parent nucleus

Z = charge of parent nucleus

X = daughter nucleus

A' = mass of daughter nucleus

Z' =charge of daughter nucleus

## 2.2 Alpha Decay

Alpha decay is a process, where a radioactive nucleus emits an alpha particle. This alpha particle is a 4He nucleus, consisting of two protons and two neutrons. A radioactive substance becomes more stable by alpha decay. The study of alpha decay is still one of the most reliable methods to probe on the nuclear structure by giving information on the ground state energy, ground state half-life, the nuclear spin and parity, the nuclear deformation, etc. The unknown parent nuclei can be determined by studying the alpha decay chain. The process of alpha decay is a nuclear reaction that can be written as [4]:

 ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$  Where

X = parent nucleus

Y = dauhter nucleus and  ${}_{2}^{4}He$  = helium nucleus which is alpha particle.

#### 2.2.1 Energy of Alpha Decay

Radioactive decay is a spontaneous process, caused by a system moving to lower energy state.  $\varepsilon$  value is positive for alpha decay.  $\varepsilon$  value exceeds alpha decay energy. From semi empirical mass formula, emission of an a-particle lowers Coulomb energy of nucleus and increases stability of heavy nuclei while not affecting overall binding energy per nucleon .We know that when this quantity becomes negative the alpha particle will no longer be a bound particle and will be emitted spontaneously from the initial nucleus  $\frac{A}{Z}X$  in driving some of the more important quantities on alpha decay, we can write up the  $\varepsilon$ -value equation for a decay parent nucleus.The spontaneous emission of alpha particle is given by[5]

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \alpha$$

From the law of conservation of energy we have that

$$M_X c^2 = M_Y c^2 + M_\alpha c^2 + \eta_Y + \eta_\alpha$$
 (2.2.1)

Where

 $M_X =$ mass of the parent nucleus

 $M_Y = \text{mass of daughter nucleus}$ 

 $M_{\alpha} = \text{mass of alpha particle}$ 

 $\eta_Y$  =kinetic energy of daughter nucleus

 $\eta_{\alpha} =$ kinetic energy alpha particle

Equation(2.2.1) can be rewritten as

$$(M_X - M_Y - M_\alpha)c^2 = \eta_Y + \eta_\alpha$$
 (2.2.2)

The energy of the alpha particle is also equal to the total kinetic energy given to the decay fragments.

$$\varepsilon = \eta_Y + \eta_\alpha \tag{2.2.3}$$

The initial nucleus X is at rest, then its linear momentum is zero. The daughter nucleus Y and the alpha particle moves with equal and opposite momenta.

$$P_Y = P_\alpha \tag{2.2.4}$$

The kinetic energy of the alpha particle in terms of the  $\varepsilon$  value is

$$\eta_{\alpha} = \frac{\varepsilon}{1 + \frac{M_{\alpha}}{M_{Y}}} \tag{2.2.5}$$

Because the mass ratio is small compared with 1 (recall that Y represents a heavy nucleus), it is usually sufficiently accurate to express this ratio simply as  $\frac{4}{A-4}$ , which gives, with B >> 4,

$$\eta_{\alpha} = \varepsilon (1 - \frac{4}{B}) \tag{2.2.6}$$

Where, B is the atomic mass of the parent nucleus.

# Chapter 3

# Materials and Methodology

# 3.1 Materials

- 1. Books
- 2. Web site
- 3. Published articles
- 4. Computer or Laptop
- 5. Flash
- 6. Paper and pen

# 3.2 Method

To achieve the stated objectives and problems, the following methods were used.

### 3.2.1 Analytic Method

Analytic calculation of the half lives of the alpha radioactive nuclei were defined. The equation obtained was studied about how half life depends on deferent quantities.

#### 3.2.2 Computational Method

Using the equations obtained, numerical calculation of the half lives of alpha radioactive nuclei and the alpha energy were measured.

### 3.2.3 Method of Data presentation

The data for half lives of alpha radioactive nuclei calculated theoretically were displayed in tables together with the data from the previous work. Finally the discussion were followed based on the data after each table. Then the conclusion on the relationship between the calculated and experimental half lives of alpha radioactive nuclei were drawn.

# Chapter 4 Results and Discussion

# 4.1 Formulation of the Problem

#### 4.1.1 Alpha Energy Dependence on Nucleon Number

The energy of an alpha particle emitted from a nucleus is given by the semi empirical mass formula. It gives a simple interpolation formula for ground state energy. Applying semi-empirical approach (based on experimental results) Weizsacker showed that it is possible to achieve a quantitative and more basic understanding of binding energies of nuclei. To drive the semi empirical formula, the following assumptions are taken into account[6].

- The parent nucleus, and the daughter nucleus are in their ground states before and after emission.
- Nuclear forces are saturated.
- Nucleon interactions are the same.
- Nucleus is modeled as a drop of liquid.

In the calculation of the total energy of a nucleus, different contributing factors can be considered independently.

#### 1. Volume energy term $(\Lambda_v)$ :

The nuclear volume energy assumes that all nucleons are inside the nuclear volume so that each of them has sufficient neighbors to make nuclear reaction. Thus, the nuclear volume energy is proportional to the atomic number of the parent nucleus.  $\Lambda_v \sim B$  and it is expressed as[6]

$$\Lambda_v = a_v B \tag{4.1.1}$$

Where,  $a_v$  is the volume energy coefficient which is determined by comparison with experiment and  $\Lambda_v$  is the volume energy term.

#### 2. Surface energy term $(\Lambda_{sur})$ :

The surface effect results in a decrease of the binding energy from the value of the volume term. This is because the nucleons near to the nuclear surface have reduced binding energy since they are partially surround by other nucleon. The surface term in the semi empirical mass formula is proportional to the surface area of the nucleus. The surface energy is given by [7]

$$\Lambda_{sur} = -a_{sur}B^{2/3} \tag{4.1.2}$$

Where,  $\Lambda_{sur}$  is the surface energy and  $a_{sur}$ , the surface energy coefficient.

#### 3. Asymmetry energy term $(\Lambda_{asymm})$ :

Asymmetry energy term,  $\Lambda_{asymm}$  depends on the neutron excess  $(N_n - N_P)$  and decreases the nuclear binding energy. The asymmetry energy term is negative.

Because, it reduces the energy of the binding energy of a nucleus[8]. We have neglected the quantization of energy states of individual nucleons in the nucleus and the application of the Pauli Exclusion Principle. If we put  $N_p$  protons and  $N_n$  neutrons into the nuclear energy shells, the lowest  $N_p$  energy levels are filled first. From Pauli exclusion principle, the excess  $(N_n - N_p)$  neutrons must go into previously unoccupied quantum states since the first  $N_p$  quantum states are already filled up with protons and neutrons. These  $(N_n - N_P)$  excess neutrons are occupying higher energy quantum states and are consequently less tightly bound than the first  $2N_P$  nucleons which occupy the deepest lying energy levels[9]. Thus neutron asymmetry gives rise to a disruptive term in nuclear binding energy. Excess energy per nucleon  $\propto \frac{N_n - N_p}{B}$ . Since the total number of excess neutrons is  $(N_n - N_p)$ , the total deficit in

nuclear binding energy is proportional to product of these.  $\Lambda_{asymm} = a_k \frac{(N_n - N_p)^2}{B}$ (4.1.3)

Equation(4.1.3) is equivalent with

$$\Lambda_{asymm} = a_k \frac{(2\Upsilon)^2}{B} \tag{4.1.4}$$

Thus,

$$\Lambda_{asymm} = -4a_k \frac{\Upsilon^2}{B} \tag{4.1.5}$$

Where  $a_k$  is asymmetry energy coefficient and  $\Upsilon$  is the neutron excess which is given by

$$\Upsilon = \frac{N_n - N_p}{2} \tag{4.1.6}$$

#### 4. Coulomb energy term $(\Lambda_c)$ :

The electric repulsion between each pair of proton in a nucleus also contributes

towards decreasing its binding energy [10]. The colombo energy is given by

$$\Lambda_c = \frac{1}{4\pi\epsilon_o r} \int q dq \tag{4.1.7}$$

Where  $q = N_p q_e$  is the charge of the nucleus and r is the separation between pair of protons. The charge density, $\rho$  is defined as charge per unit volume.

$$\rho = \frac{q}{\frac{4\pi r^3}{3}} \tag{4.1.8}$$

Solving for the charge q,we have

$$q = \rho \frac{4\pi r^3}{3} \tag{4.1.9}$$

Using the relation  $dq = \rho dv$  in equation(4.1.7), we have

$$\Lambda_c = \frac{\rho r^2}{3\epsilon_o} \int \rho dv \tag{4.1.10}$$

Where the volume element, dv, is given by

$$dv = r^2 sin\theta d\theta dr d\phi \tag{4.1.11}$$

If we substitute equation(4.1.11) in equation(4.1.10), we get

$$\Lambda_c = \frac{1}{3\pi\epsilon_o} \int_0^{r'} \int_0^{\pi} \int_0^{2\pi} \rho^2 r^4 \sin\theta dr d\theta d\phi \qquad (4.1.12)$$

The solution for the integral is

$$\Lambda_c = \frac{4\pi\rho^2 r'^5}{15\epsilon_o} \tag{4.1.13}$$

If we substitute equation (4.1.8) in equation (4.1.13) and using the fact that there are  $N_p(N_p - 1)$  pairs of protons, we get

$$\Lambda_c = \frac{3e^2}{20\pi\epsilon_o r'} N_p (N_p - 1)$$
(4.1.14)

Where r' is the radius of the nucleus , which is given by

$$r' = l_0 B^{1/3} \tag{4.1.15}$$

Where,  $l_o$  is a constant and its value is equal to 1.4fm. If we Substitute equation(4.1.15) in equation(4.1.14), we get

$$\Lambda_c = \frac{3e^2}{20\pi\epsilon_o l_o B^{1/3}} N_p (N_p - 1)$$
(4.1.16)

The coulomb energy becomes as follow

$$\Lambda_c = 4a_c \frac{N_p(N_p - 1)}{B^{1/3}} \tag{4.1.17}$$

The coulomb energy is negative because it arises from an effect that opposes nuclear stability. The constant  $a_c$  is related to the constant  $l_o$  in equation(4.1.16) through

$$a_c = \frac{3e^2}{20l_o\pi\epsilon_o} = 0.15Mev$$
 (4.1.18)

Putting the different terms together, we get the Weizsacker semi-empirical formula for nuclear ground state energies.

$$E_b = a_v B - 4a_k \frac{\Upsilon^2}{B} - 4a_c \frac{N_p (N_p - 1)}{B^{1/3}} - a_{sur} B^{2/3}$$
(4.1.19)

Where  $E_b$  is the nuclear binding energy. In the derivation of equation(4.1.19) we have omitted the intrinsic spin of the nucleons and shell effects. This is corrected by adding a pairing energy term,  $\Lambda_p$  to the nuclear binding energy[11]. The paring energy correction term results from pair of protons and neutrons. The even number of protons and even number of neutrons leads to most stable[12]. The odd number of protons and odd number of neutrons leads to least stable. The pairing energy correction term is given by

$$\Lambda_P = \pm \frac{a_p}{B^{3/4}} \tag{4.1.20}$$

Where,

$$\Lambda_P = \begin{cases} +ve, & \text{for even-even nuclei;} \\ -ve, & \text{for odd -odd nuclei;} \\ o, & \text{for odd -even or even-odd nuclei.} \end{cases}$$
(4.1.21)

$$E_b = a_v B - 4a_k \frac{\Upsilon^2}{B} - 4a_c \frac{N_p (N_p - 1)}{B^{1/3}} - a_{sur} B^{2/3} \pm \frac{a_p}{B^{3/4}}$$
(4.1.22)

The energy of the alpha particle is given by

.

$$\varepsilon(B, N_p) = E_b(B - 4, N_p - 2) + E_b\alpha - E_b(B, N_p)$$
 (4.1.23)

Where  $E_{b\alpha}$  is is the binding energy of the alpha particle and its value is equal to 28Mev. Adding and subtracting  $E_b(B, N_p - 2)$  to equation(4.1.23), we get

$$\varepsilon(B, N_p) = [E_b(B-4, N_p-2) - E_b(B, N_p-2)] + [E_b(B, N_p-2) - E_b(B, N_p)] + E_b\alpha$$
(4.1.24)

This is equivalent with

$$\varepsilon(B, N_p) = -4\frac{\partial E_b}{\partial B} - 2\frac{\partial E_b}{\partial N_p} + E_b\alpha \qquad (4.1.25)$$

Substituting equation(4.1.22) in equation(4.1.25) and assuming that  $B \gg 1$  and  $N_p \gg 1$  we get that

$$\varepsilon(B, N_p) = -4a_v - 16a_k \Upsilon^2 B^{-2} + 16a_c N_p B^{-1/3} - \frac{16}{3}a_c N_p^2 B^{-4/3} + \frac{8}{3}a_{sur} B^{-1/3} \pm \frac{3a_p}{B^{7/4}} + E_b \alpha$$
(4.1.26)

The values of the constants are  $a_v = 14 MeV$ ,  $a_c = 0.15 MeV$ ,  $a_k = 18.1 MeV$  and  $a_{sur} = 13 Mev$ . The value of the constant  $a_p$  is equal to 33.5 MeV. Alpha energy is written as

$$\varepsilon(B, N_p) = -4a_v - 16a_k \Upsilon^2 B^{-2} + 16a_c N_p B^{-1/3} - \frac{16}{3}a_c N_p^2 B^{-4/3} + \frac{8}{3}a_{sur} B^{-1/3} \pm \frac{3a_p}{B^{7/4}} + 28Mev$$
(4.1.27)

Equation(4.1.27) is an approximation so that we do not expect an exeat value. It is used to determine the general trend of the alpha energies. In our derivation of the formula both the parent and daughter nucleus are considered to be in their ground state.

### 4.1.2 Calculation of Half Life Dependence on Different Parameters

To calculate the half life of alpha radioactive nuclei, the following assumptions are made.

- The alpha particle before emission exists as such within the nucleus.
- It moves with a constant speed within the nucleus and collides again and again at the barrier surface is kept inside the nucleus due to the high potential barrier.
- There is a small but finite probability that the particle may penetrate when it collide with the barrier. This is due to the de-Broglie hypothesis of the wave nature of particle
- Once the particle leaves through the barrier ,it escapes from the nucleus because of its kinetic energy and coulomb repulsion force.

Since the total angular momentum for even-even nuclide is zero, and their parity is even, the ground state to ground state alpha transitions are  $0^+ \rightarrow 0^+$  transitions. The half-life of this type of decay can be theoretically calculated using the so-called onebody model of alpha particles, which assumes that the alpha particle is pre-formed (as one body) within the nucleus before penetrating the Coulomb potential barrier by the tunneling effect[13]. Tunneling effect is a quantum mechanical process by which an alpha particle can pass through a potential energy barrier that is higher than the energy of the alpha particle. According to the quantum mechanics, when alpha particle with energy  $\varepsilon$  hits a potential  $W(\varepsilon < W)$ , there is a decrease of exponential term in that region. If the potential is narrow, then it is possible for a wave function to emerge from the other side. High potential is narrow, and low potential is broad. These two parameters control the tunneling effect along with energy of the alpha particle. We will see how these parameters effect tunneling through our derivation. We consider a potential as shown in figure 4.1. Inside the potential well  $(0 \le y' \le r')$  the wave function has an oscillatory nature. In the barrier region  $(r' < y' \le a)$  the wave function decreases exponentially. When the alpha particle has escaped (y' > a), the wave function is approximately sinusoidal[14]. Inside the nuclear surface at y' = r', the potential is represented as a square well; beyond the surface, only the Coulomb repulsion operates. The alpha particle tunnels through the Coulomb barrier from r' to a[14]. The value of the coulomb potential at y' = r' is given by

$$W = \frac{N_p z_a q_e^2}{4\pi\epsilon_o r'} \tag{4.1.28}$$

Where,

W = coulomb potential

- $q_e = \text{charge of electron}$
- $z_a$  = atomic number of alpha particle

r' = the separation between the centers of the two nuclei

At larger distances the potential falls as  $\frac{1}{r'}$  according to Coulomb's Law. The barrier extends from y' = r', the nuclear radius to y' = a. The energy of the alpha particle



Figure 4.1: Potential energy of alpha-particle, daughter-nucleus system as a function of their separation.

is given by

$$\varepsilon = \frac{N_p z_a q_e^2}{4\pi\epsilon_o a} \tag{4.1.29}$$

Equation(4.1.29) shows that the energy of the alpha particle after escaping the barrier depends on the parameters such as the atomic number of the daughter nucleus and the distance at which the alpha particle escapes the barrier. The alpha energy increases with increasing atomic number of the daughter nucleus. The larger the atomic number of the daughter nucleus, the greater the alpha energy. The alpha energy also increases as the distance at which the alpha particle escapes the barrier decreases. This is because when the radius at which the alpha particle escape the barrier increases, the thickness of the potential decreases. This causes the alpha energy to decrease. Therfore , the half life of the alpha radioactive nuclei will be increase. Beyond ,a, the alphaparticle has sufficient energy to escape. According to classical mechanics, the alpha particle does not have sufficient energy to escape barrier, but it can penetrate through quantum tunneling. From equation(4.1.29) the radius ,a, at which the alpha particle leaves the barrier is

$$a = \frac{N_p z_a q_e^2}{4\pi\epsilon_o \varepsilon} \tag{4.1.30}$$

For alpha-decay, the potential inside the barrier is

$$W(y') = \frac{N_p z_a q_e^2}{4\pi\epsilon_o y'}$$
(4.1.31)

The potential, W(y') varies as  $\frac{1}{y'}$ . That means, the potential inside the potential barrier decreases when the distance, y' increases. This is due to the fact that when y' increases, the thickness of the potential well increases and the potential inside the barrier decreases. As a result the half lives of the alpha radioactive nuclei becomes small. In other words the alpha particle can no longer stay inside the nucleus. Assume that the alpha particle moves back and forth in the nucleus of radius r'. We

can estimate the radius r' of the parent nucleus as

$$r' = l_o B^{1/3} \tag{4.1.32}$$

The radius of the parent nucleus depends on its atomic mass B. Equation(4.1.32) indicates that, the radius of any nucleus increases with increasing atomic number. When the alpha particle leaves the potential barrier ,its energy is equal to its kinetic energy and the potential depth becomes zero. More energetic alpha particle will encounter barrier more often.

$$\varepsilon = E_{k\alpha} = \frac{1}{2}\tau \upsilon^2 \tag{4.1.33}$$

#### Where

 $\varepsilon = \text{energy of the alpha particle}$ 

 $\tau$ =reduced mass of the alpha particle

v = speed of the alpha particle

If we use equation (4.1.33) to find the speed of the alpha particle, we get

$$v = \sqrt{\frac{2\varepsilon}{\tau}} \tag{4.1.34}$$

We have considered that the alpha particle is none relativistic. That is, the alpha particle is not moving with the speed of light. In equation(4.1.34), we take the reduced mass because the alpha particle is moving inside the nucleus and the total angular momentum and spin of the nucleus are zero. The speed of an alpha particle depends on its energy. The larger the energy of the alpha particle, the faster the alpha particle moves inside the parent nucleus.

The frequency with which an alpha particle reaches the edge of a nucleus can be estimated as the ratio of the velocity and the distance across the nucleus.

$$F = \frac{\upsilon}{r'} \tag{4.1.35}$$

Where F is the frequency of the alpha particle.

The frequency is very large usually in the order of  $10^{20}$ . From equation(4.1.35), we can see that, the larger the speed of the alpha particle, the greater the frequency of the alpha particle.

The penetration probability of tunneling alpha particle through barrier potential are calculated within Wentzel-Kramers-Brillouin (WKB) approximation. According to the Wentzel Kramers Brillouin, for a constant potential, the wave function solutions of the schrodinger equation are the form of simple plane wave [15].

$$\varpi(y') = Ae^{\pm i\pounds(y')y'} \tag{4.1.36}$$

Where , A is the amplitude of the wave and using the normalization condition its value is found to be one and  $\pounds(y')$  is the wave vector. If the potential w(y') varies slowly with y', the solution to the schrodinger equation becomes

$$\varpi(y') = Be^{i\Omega(y')} \tag{4.1.37}$$

Where  $\Omega(y') = \pounds(y')y'$ . For a slowly varying potential,  $\Omega(y')$  varies slowly. Let  $\pounds$  can be defined as

For  $\varepsilon > W$ 

$$\pounds(y') = \sqrt{\frac{2\tau(W-\varepsilon)}{\hbar^2}}$$
(4.1.38)

For  $\varepsilon < W$ 

$$\pounds(y') = -i\sqrt{\frac{2\tau(W-\varepsilon)}{\hbar^2}}$$
(4.1.39)

Using the normalization version of equation (4.1.37), the schrodinger equation becomes

$$\frac{-\hbar^2}{2\tau}\frac{\partial^2}{\partial y'^2}\varpi(y') + W(y')\varpi(y') = \varepsilon\varpi(y')$$
(4.1.40)

Thus,

$$i\frac{\partial^2\Omega(y')}{\partial y'^2} - \left(\frac{\partial\Omega}{\partial y'}\right)^2 + \pounds^2 = 0 \tag{4.1.41}$$

Equation(4.1.41) is equivalent with

$$\left(\frac{\partial\Omega(y')}{\partial y'}\right)^2 = (\pounds(y'))^2 + i\frac{\partial^2\Omega(y')}{\partial y'^2} \tag{4.1.42}$$

From the first order Wentzel cramers Brillouin approximation, we have

$$\left(\frac{\partial\Omega(y')}{\partial y'}\right)^2 = (\pounds(y'))^2 \tag{4.1.43}$$

Equation(4.1.43) becomes

$$\frac{\partial\Omega(y')}{\partial y'} = \pm \pounds(y') \tag{4.1.44}$$

Integrating both sides of equation(4.1.44) gives

$$\Omega(y') = \pm \int \pounds(y')dy' + c_o \qquad (4.1.45)$$

Using equation(4.1.45) and equation(4.1.37) we get

$$\varpi(y') = e^{i(\pm \int \pounds(y')dy' + c_o)} \tag{4.1.46}$$

Equation(4.1.46) can be rewritten as

$$\varpi(y') = e^{i(\pm \int \pounds(y'))dy'}e^{c_o} \tag{4.1.47}$$

But, $\varpi(0) = e^{c_o}$ 

Thus, the wave function,  $\varpi(y')$  takes the form

$$\varpi(y') = \varpi(0)e^{i(\pm \int \pounds(y')dy')} \tag{4.1.48}$$

For tunneling to occur  $\varepsilon < W$ . Then, if we substitute equation (4.1.38) in equation (4.1.48), we get

$$\varpi(y') = \varpi(0)e^{i(\pm \int -i\sqrt{\frac{2\tau(W-\varepsilon)}{\hbar^2}}dy')}$$
(4.1.49)

The tunneling probability of a finite width potential barrier is given by

$$D = \frac{\varpi^*(y')\varpi(y')}{\varpi^*(0)\varpi(0)}$$
(4.1.50)

Thus we have

$$D = \frac{\varpi(0)e^{-\int \pounds(y')dy'}\varpi(0)e^{-\int \pounds(y')dy'}}{\varpi(0)e^{0}\varpi(0)e^{0}}$$
(4.1.51)

The probability becomes

$$D = e^{-2\int_{r'}^{a} \pounds dy'} \tag{4.1.52}$$

Where r' and, a, are inner and outer integration limits respectively and  $\pounds$  is wave number and it is given by

$$\pounds = \sqrt{\frac{2\tau(W-\varepsilon)}{\hbar^2}} \tag{4.1.53}$$

The Gamow factor is large which makes the transmission coefficient D extremely small[17]. Let the Gamow factor,  $\beta$  be

$$\beta = 2 \int_{r'}^{a} \pounds dy' \tag{4.1.54}$$

Using equation (4.1.52) and equation (4.1.54), alpha particle barrier penetration is given by

$$D = e^{-\beta} \tag{4.1.55}$$

If we substitute equation (4.1.53) in equation (4.1.54), we get

$$\beta = \frac{2}{\hbar} \int_{r'}^{a} \sqrt{2\tau} (W - \varepsilon)^{1/2} dy'$$
(4.1.56)

Equation(4.1.56) is equivalent with

$$\beta = \frac{2}{\hbar} \int_{r'}^{a} \sqrt{2\tau\varepsilon} \left(\frac{W}{\varepsilon} - 1\right)^{1/2} dy' \tag{4.1.57}$$

Dividing equation (4.1.28) by equation (4.1.29) gives that

$$\frac{a}{y'} = \frac{W}{\varepsilon} \tag{4.1.58}$$

Using equation (4.1.58) and equation(4.1.57), we have

$$\beta = \frac{2}{\hbar} \int_{r'}^{a} \sqrt{2\tau\varepsilon} (\frac{a}{y'} - 1)^{1/2} dy'$$
(4.1.59)

This integration is made easier using integration by substitution by letting

$$y' = asin^2\theta \tag{4.1.60}$$

Taking the first derivative with respect theta of equation (4.1.60) gives

$$dy' = 2a\cos\theta\sin\theta d\theta \tag{4.1.61}$$

Using equation(4.1.61) in equation(4.1.59), we get

$$\beta = \frac{2}{\hbar} \int_{r'}^{a} \sqrt{2\tau\varepsilon} \left(\frac{a}{a\sin^2\theta} - 1\right)^{1/2} 2a\cos\theta \sin\theta d\theta \qquad (4.1.62)$$

Equation(4.1.62) is equivalent with

$$\beta = \frac{2}{\hbar} \int_{r'}^{a} \sqrt{2\tau\varepsilon} (\cot^2\theta)^{1/2} 2a\cos\theta \sin\theta d\theta \qquad (4.1.63)$$

If we simplify equation (4.1.63), we will get

$$\beta = \frac{2}{\hbar} \sqrt{2\tau\varepsilon} a \int_{r'}^{a} ((1 + \cos 2\theta) d\theta$$
 (4.1.64)

This can be integrated to give

$$\beta = \frac{2}{\hbar} \sqrt{2\tau\varepsilon} a \left( \left(\theta + \frac{\sin 2\theta}{2}\right) \right) \tag{4.1.65}$$

The limits of integration are

When y'= 
$$r'$$
,  $sin\theta = \sqrt{\frac{r'}{a}}$  and  
When y' = a,  $\theta = \frac{\pi}{2}$ 

Equation(4.1.65) becomes

$$\beta = \frac{2}{\hbar}\sqrt{2\tau\varepsilon}a(\frac{\pi}{2} - \sin^{-1}(\frac{r'}{a})^{1/2} - (\sqrt{\frac{r'}{a}})^{1/2}(1 - \frac{r'}{a})^{1/2}))$$
(4.1.66)

For thick barriers,  $\frac{r'}{a} \ll 1$ . Then we can approximate

$$\sin(\frac{r'}{a})^{1/2} \cong (\frac{r'}{a})^{1/2}$$
 (4.1.67)

Equation (4.1.66) becomes

$$\beta = \frac{2}{\hbar} \sqrt{2\tau\varepsilon} \left( a(\frac{\pi}{2} - 2(\frac{r'}{a})^{1/2}) \right)$$
(4.1.68)

Using equation (4.1.30) and equation (4.1.33) in equation (4.1.68) we get

$$\beta = \frac{N_p z_a q_e^2}{2\epsilon_o \hbar \upsilon} - \frac{1}{\hbar} \sqrt{\frac{8 z_\alpha N_p q_e^2 \tau r'}{\pi \epsilon_o}}$$
(4.1.69)

If we replace  $z_{\alpha}$  by 2, we get that

$$\beta = \frac{N_p q_e^2}{\epsilon_o \hbar \upsilon} - \frac{4q_e}{\hbar} \sqrt{\frac{N_p \tau r'}{\pi \epsilon_o}}$$
(4.1.70)

The probability of penetrating of the alpha particle becomes

$$D = exp\{-\frac{N_p q_e^2}{\epsilon_o \hbar \upsilon} + \frac{4q_e}{\hbar} \sqrt{\frac{N_p \tau r'}{\pi \epsilon_o}}\}$$
(4.1.71)

Equation(4.1.71) indicates that the probability of penetrating increases with increasing radius of a nucleus, r'. The decay constant is defined as the product of the probability of getting through the barrier (D) by the number of attempts to make the particle go through it (given by the number of collisions with the surface in unit time[18].

$$\xi = FD \tag{4.1.72}$$

Substituting equation (4.1.71) into (4.1.72), we obtain

$$\xi = F \exp\{-\frac{N_p q_e^2}{\epsilon_o \hbar \upsilon} + \frac{4q_e}{\hbar} \sqrt{\frac{N_p \tau r'}{\pi \epsilon_o}}\}$$
(4.1.73)

Equation(4.1.73) shows that the decay constant, $\xi$  is large when both the radius and proton number of the daughter nucleus are large. The calculated emission rate is typically one order of magnitude larger than that observed, meaning that the observed half lives are longer than predicted. This has led some researchers to suggest that the probability to find a preformed alpha particle inside a heavy nucleus is on the order of 10 or less[19]. The half life of the alpha radioactive nuclei is given by

$$T = \frac{\ln 2}{\xi} \tag{4.1.74}$$

Where,  $\ln 2$  is equal to 0.693.

If we substitute equation (4.1.73) into equation (4.1.74), we get that

$$T = \frac{0.693}{F} exp\{\frac{N_p q_e^2}{\epsilon_o \hbar \upsilon} - \frac{4q_e}{\hbar} \sqrt{\frac{N_p \tau r'}{\pi \epsilon_o}}\}$$
(4.1.75)

Equation(4.1.75) only holds for even-even nuclei. Equation(4.1.75) also shows the relationship between the half life and the radius,r' from a given natural radioactive series. This relationship is useful for predicting the expected alpha decay half-lives for unknown nuclei. Odd-odd, even-odd, and odd-even nuclei have longer half-lives than predicted due to hindrance factors .The hindered alpha decay assumes existence of pre-formed alpha particles. Ground-state transition from nucleus containing odd nucleon in highest filled state can take place only if that nucleon becomes part of alpha-particle[20]. The corrected half life of the alpha emitter is given by

$$T = \frac{0.693}{F} exp\{\frac{N_p q_e^2}{\epsilon_o \hbar \upsilon} - \frac{4q_e}{\hbar} \sqrt{\frac{N_p \tau r'}{\pi \epsilon_o}}\} + \aleph$$
(4.1.76)

This is also expressed as

$$\ln T = \frac{0.693}{F} \left\{ \left\{ \frac{N_p q_e^2}{\epsilon_o \hbar \upsilon} - \frac{4q_e}{\hbar} \sqrt{\frac{N_p \tau r'}{\pi \epsilon_o}} \right\} + \aleph \right\}$$
(4.1.77)

Where,  $\aleph$  is the even-odd hindrance factor. The ratio between the measured halflife of a particular transition and that calculated using the one-particle model for an even-even nuclei at the energy of a decay is known as the hindrance factor ( $\aleph$ )[21, 22].

$$\aleph = \frac{T_{Exp.}}{T_{Theo.}} \tag{4.1.78}$$

# 4.2 Numerical Calculations

The experimental and computed numerical values of alpha energies and half lives of alpha radioactive nuclei are given below.

### 4.2.1 Verification of Alpha Energy Dependence on the Nucleon Number of Parent Nucleus

The experimental and theoretical values of the alpha energies obtained using equation(4.1.27) are shown in table 4.1. Experimental values were taken from ENSDF(Evaluated Nuclear Structure Data Files). As shown in table 4.1 the calculated values deviates less from the observed values.

Alpha radioactive element	$\varepsilon_{\alpha}(\text{Experimental})(\text{MeV})$	$\varepsilon_{\alpha}$ (Theoretical)(MeV)
$238 \\ 92 U$	4.3	4.9
$235 \\ 92 U$	4.7	5.3
$234_{92}U$	4.9	5.5
$233 \\ 92 U$	4.9	5.6
$\frac{232}{90}Th$	4.1	4.6
$\frac{230}{90}Th$	4.8	4.9
$\frac{229}{90}Th$	5.2	5.0
$\frac{228}{90}Th$	5.5	5.2
$\frac{227}{90}Th$	6.1	5.3
$\frac{226}{88}Ra$	4.9	4.3
$\frac{224}{88}Ra$	5.8	4.6
$\frac{223}{88}Ra$	5.9	4.8
$\frac{222}{86}Rn$	5.6	3.8
<sup>220</sup> <sub>86</sub> Rn	6.4	4.1
$\frac{219}{86}Rn$	7.0	4.2
$\frac{231}{91}Pa$	5.2	5.3
$\frac{227}{89}Ac$	5.0	4.8
$\frac{225}{89}Ac$	5.9	5.0
$\frac{237}{93}Np$	5.0	5.6
$\frac{213}{83}Bi$	5.7	3.3
$\frac{212}{83}Bi$	6.2	3.5
$\frac{211}{83}Bi$	6.8	3.6
$\frac{209}{83}Bi$	3.1	3.9
$\frac{218}{84}Po$	6.1	3.2
216 84 Po	6.9	3.5
$\frac{215}{84}Po$	7.5	3.6
$\frac{214}{84}Po$	7.9	3.8

Table 4.1: Experimental and Theoretical Values of Alpha Energy

Figure 4.2 indicates the plot of measured and theoretical alpha energy versus nucleon number. The vertical axis is experimental and theoretical alpha energies in Mev and the horizontal axis is the nucleon number of the parent nucleus.



Figure 4.2: Graph of measured and observed alpha energy versus nucleon number.

### 4.2.2 Verification of the calculation of Half Life Dependence on Different Parameters

Table 4.2 indicates that the observed and computed half lives of alpha emitter.Experimental values were taken from ENSDF(Evaluated Nuclear Structure Data Files).The theoretical values were obtained using equation 4.1.75 and are not exact. The calculations were performed using a nuclear radius of  $l_o = 1.4B^{\frac{1}{3}}$ (fm).

Alpha radioactive element	$\varepsilon(Mev)$	T(Experimental)	T(Theoretical)	r(fm)
$\frac{238}{92}U$	4.3	$4.5 \times 10^9 years$	$1.05 \times 10^9 years$	8.67
$^{235}_{92}U$	4.7	$7 \times 10^8$ years	$7.6 \times 10^9$ years	8.64
$\frac{234}{92}U$	4.9	$2.4 \times 10^5 years$	$1.5 \times 10^5 years$	8.63
$\frac{233}{92}U$	4.9	$1.6 \times 10^5 years$	$0.6 \times 10^5 years$	8.61
$\frac{226}{88}Ra$	4.9	1602 years	454.6 years	8.53
$\frac{224}{88}Ra$	5.8	3.6 days	3.8 days	8.50
$\frac{223}{88}Ra$	5.8	11.4 days	9.1 day	8.45
$\frac{218}{84}Po$	6.1	3.1 minutes	35.2 sec	8.4
$\frac{216}{84}Po$	6.9	158 ms	3 sec	8.40
$\frac{215}{84}Po$	7.5	1.8 ms	18sec	8.43
$\frac{214}{84}Po$	7.9	$160\mu \ s$	$231 \mu sec$	8.41
<sup>212</sup> <sub>84</sub> Po	9.0	299 ns	$1.2 \mu s$	8.34
$\frac{211}{84}Po$	7.8	516 ms	35.1sec	8.32
$\frac{210}{84}Po$	5.4	138.4 days	83.1 days	8.32
231 Pa 91	5.2	$3.3 \times 10^4 years$	$8.4 \times 10^4 years$	8.59
$\frac{227}{89}Ac$	5.0	22 years	318 years	8.54
$\frac{225}{89}Ac$	5.9	14.9 days	10 days	8.52
$\frac{219}{85}At$	8.2	0.1ms	71.4sec	8.4
$\frac{217}{85}At$	7.0	32ms	2.4sec	8.41
$\frac{222}{86}Rn$	5.6	3.8 days	4.6 days	8.48
$\frac{220}{86}$ Rn	6.4	55 sec	15 sec	8.53
$\frac{219}{86}Rn$	7.0	4 sec	22.4 sec	8.42
$\frac{237}{93}Np$	5.0	$2.1 \times 10^{6} years$	$2.3 \times 10^7 years$	8.66
$\frac{221}{87}Fr$	6.3	4.8 minutes	108 minutes	8.46
$\frac{213}{83}Bi$	5.7	45.6 minutes	17.8 minutes	8.36
$\frac{212}{83}Bi$	6.2	60.60minute	6.02 minute	8.35
$211 \\ 83 Bi$	6.8	2.1 minutes	5.8 minutes	8.33
$\frac{232}{90}Th$	4.1	$1.4 \times 10^{10} years$	$2.2 \times 10^{10} years$	8.60
$230 \\ 90 \\ Th$	4.8	$7.5 \times 10^4 years$	$2.7 \times 10^4 years$	8.57
$229 \\ 90 \\ 7h$	5.2	$7.5  imes 10^4 years$	$7.3  imes 10^4 years$	8.56
$228 \\ 90 \\ 7h$	5.5	1.9 year	218.2 days	8.55
227 Th	6.1	17 days	45 days	8.54

Table 4.2: Experimental and theoretical half lives of alpha radioactive nuclei.

Fig 4.3 shows that the plot of experimental and theoretical half lives in days versus radius of parent nucleus,r'.



Figure 4.3: Graph showing experimental and theoretical half lives in days against radius of parent nucleus r'.

Fig 4.4 shows that the plot of experimental and theoretical half lives in minutes versus radius of parent nucleus,r'.



Figure 4.4: Graph showing experimental and theoretical half lives in minutes against radius of parent nucleus, r'.

Figure 4.5 indicates the theoretical and experimental long half lives of alpha radioactive nuclei.



Figure 4.5: Graph showing experimental and theoretical half lives in years against radius of parent nucleus, r'.

### 4.3 Discussion

Equation(4.1.27) provides us rough agreement with the observed  $\epsilon_{exp.}$  values and it exactly provides  $\epsilon_{theo.} > 0$  for the heavy nuclei.Equation(4.1.27) also exactly predicts the decrease of  $\epsilon_{theo.}$  with increasing nucleon number for a sequence of isotopes such as those of uranium, although it gives too small a change of  $\epsilon_{theo.}$  with B.Figure 4.2 indicates that the experimental alpha energies are not the same. There is a gap between the experimental and theoretical values of alpha energies.The errors in the calculation of alpha energy is due to the ignorance of the spin parity and the

angular momentum. The errors also come because the formula we have derived is only an approximation. It misses many important information about the alpha decay. Table 4.2 indicates as the alpha energy increases, the half life decreases. It also indicates that small changes in alpha energy result in enormous differences in half lives. For example,  $\varepsilon$  changes by about a factor of 1, while the half-lives span about 4 orders of magnitude for the isotopes of uranium. The results obtained using equations (4.1.75) are nearly the same and are somewhat smaller than the experimental ones. The agreement is not exact, but the calculation is able to give the trend of the half-lives within 2-3 orders of magnitude. The errors in our calculation of half life come from the overestimation of the formation factor and the ignorance of the centrifugal potential. The theory presented above neglects the effects of angular momentum in that it assumes the alpha particle carries off no orbital angular momentum (l = 0). If alpha decay takes place to or from an excited state, some angular momentum may be carried off by the alpha particle with a resulting change in the decay constant. The graph also indicates the experimental values are greater than the computed values. It also indicates both the experimental and theoretical half lives are long if the radius of the parent nucleus is large[23].

# Chapter 5 Conclusion

In this study, we have studied alpha energy dependence on nucleon number and the dependence of the half lives of alpha radioactive nuclei on different parameters. The theoretical half lives are close to the experimental half lives. We have derived alternative analytic alpha energies for alpha decay processes which can also take into account the paring energy of the decaying atoms. The effect of spin parity was not considered . The effect of the centrifugal potential has been ignored. The measured half lives were compared with those computed using equation(4.1.77). The experimentally studied alpha energy and half lives of alpha emitters were found to be in good agreement with theoretical values.

Tables 4.2 summarize the experimental and theoretical half lives of alpha radioactive nuclei. More importantly, this paper also studies the effects of radius of the parent nucleus and atomic number of the daughter nucleus on alpha decay life times. Equation(4.1.77) gives half-lives that are two or three times shorter than those observed for most of the nuclei. Therefore from above analysis it may be conclude that experimental and theoretical half lives are in good agreement while for some nuclei the experimental

and theoretical half lives have large gap between them. we also conclude that the nucleus can be treated as well potential for radial distances smaller than the radius of the core and a Coulomb potential for distances larger than the nuclear radius. The model also explains the decrease in the probability of decay with increasing the radius, which have a coulomb potential term, independent on the particle angular momentum, which results in an effective potential barrier taller and wider. This result also confirms the predictions of quantum mechanics[?].

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Phys691	MSc. Thesis	6			

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Augest, 2020