PROPAGATION OF ELECTROMAGNETIC WAVES IN LEFT-HANDED MEDIA

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Abstract

In this thesis, we discussed briefly about the artificial material properties which exhibited simultaneously negative values of electric permittivity, magnetic permeability and refractive index across a common frequency band. Here, we used methods, like analytical and computational methods to achieve the targeted objectives. The proposed left-handed media are used mainly in optical and electronic devises in better way as compared to that of the conventional media. Left-handed media are applicable in electromagnetic absorbers such as, radar microwave absorbers, electrically small resonators, wave guides that can go beyond the diffraction limit, phase compensators, advancement in focussing devices(example microwave lens), and improved electrically small antennas. In this work, we have calculated about the real and imaginary parts of electric permittivity and magnetic permeability by using Drude model. Currently, left-handed media are being developed to manipulate electromagnetic radiation.

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Chapter 1

Introduction

1.1 Background of the Study

About fourteen years ago, a Russian scientist Victor Veselago had the idea that materials with negative refractive index can bend light in to the backward direction and behave in many other counterintuitive waves [1]. However, due to the absence of such materials in nature, this idea had been silent for four decades. Until ten years ago Sir J.B. Pendry proposed artificial designs that can achieve negative permeability [2]. For the first time, this brought Veselago's idea in to reality and opened up the new field of left - handed materials (LHMS). Metamaterials are a composite or structured material that exhibits properties not found in naturally occurring materials or compounds. Left-handed materials have electromagnetic properties that are distinct from any known material, and hence are examples of metamaterials. Left-handed materials (LHMs) are not naturally exiting materials that have negative electrical permittivity, negative magnetic permeability, and negative refractive index across a common frequency band. Left-handed materials (LHMS) possess electromagnetic (EM) properties not found in nature. Left-handed materials (LHMs) have attracted more attention due to their very important applications to build the perfect lens and cloaking devices. In the past few years, there has been very interesting proof for the existence of LHMs in the microwave frequency range. Recently, researchers are working hard to push the operating frequency of LHMs in to terahertz and optical regime [2]. Materials with negative index of refraction have received march attention in recent time. This is due to their unique properties of a great deal of time has been spent formulating new designs and envisioning new application for these artificial materials. Because of a negative index of refraction as a relatively new subject of study, many problems and misconceptions have arisen regarding the behavior and properties of media that exhibit this phenomenon [3].

Particularly, there has been considerable interest in metamaterials with simultaneously negative permittivity and negative permeability. It was shown in the theoretical work of Veselago that left-handed materials exhibit a number of remarkable properties, including negative index of refraction (and, hence, negative phase velocity, inverse Doppler Effect, and radiation tension instead of pressure). All these properties stem from the fact that the poynting vector in these materials as antiparallel to the wave vector, i.e, the electric field, the magnetic field and the wave vector of a plane electromagnetic wave form a left-handed system of reference [4].

Artificial electromagnetic materials with effective negative permittivity and permeability at a certain frequency band, form new electromagnetic concepts .The effective negative refractive index is an interesting electromagnetic property for a medium and provides new electromagnetic effects.

Actually, metamaterial researchers have not only demonstrated new interesting physical phenomena but have also lead to the development of new design procedures. The realization of promising new types of microwave devices and their application to mobile antennas has attracted wide spread interest.

In fact, metamaterials may significantly improve the performance of several devices and antennas. Specially, a lot attention has been drawn to use periodic structures in the design of antennas and microwave components. With the negative refractive index availability, one can dramatically improve the performance of antennas by reducing interference. Meta materials with the property are also able to reverse the Doppler Effect.

Metamaterials would open up new field for automotive electronics applications, such as scanning-beam antennas for radar and mobile communications. Surface periodic structures, with thickness much smaller the wave length, that contain new electromagnetic properties provide the development of high impedance surfaces, known as artificial magnetic conductors. These engineered surfaces can have interesting application in antenna design, because a magnetic conductor ground plane enhance the radiation of an antenna with horizontal profile.

A new generation of low profile antennas for wireless communication systems is being developed, based on the low profile micro strip patch antennas, whose lead to reduce mobile terminals. In a near future, personal mobile communications will incorporate multi-functional devices, which means that the same terminal will wave to operate in several frequencies.

Reconfigurable antenna, capable of switching the operating frequency and having low profile is promising scenario. Researchers are working in the design of radiating element above a textured surface that can be reconfigured and adapted to operate at different frequencies.

John Pendry and his co-researcher, that first suggested how materials with negative permeability could be artificially built, have shown as a negative refractive index material could be used to make perfect lens. The concept is related to focus an image with resolution not restricted by the wave length of light, which does not happen with the conventional lens.

The perfect lens would also support the evanescent waves, which result in a perfect image of object and develop high resolution of lens [5]. Several aspects of metamaterials have been published in the literature and new suggestions are being studied. This means that the march of scientific progress will lead to the future advance [6].

1.2 Statement of the Problem

Recently, researchers have realized nanostructural materials with a custom designed refractive index at optical wave lengths [7]. This work mainly focuses on the propagation of electromagnetic waves in left-handed materials and its dispersion relation can be calculated by using analytical method and graphical method. Therefore, the main aim of this study is to understand the dispersion property of electric permittivity and magnetic permeability of left-handed media.

1.3 Basic Research Questions

From different perfective point of view, some questions are raising from which the problems are round,therefore, we have answered:

What is the electric permittivity of left-handed materials?

What is the magnetic permeability of left-handed material?

How can we find the variation of real part of the relative permittivity for different plasma frequency?

How can we observe the variation of imaginary part magnetic permeability for different plasma frequencies?

1.4 Objective of the Study

1.4.1 General Objective

The main objective of this study was to understand the dispersion property of electric permittivity and magnetic permeability in left-handed media.

1.4.2 Specific Objectives

The specific objectives of the study are:

To describe the electric permittivity of left-handed materials

. To define the magnetic permeability of left-handed materials.

To find the variation of real part of relative permittivity for different plasma frequency.

To observe the variation of imaginary part of magnetic permeability for different plasma frequency.

1.5 Significance of the study

Left-handed materials become focus of progressive research and innovations in recent years. Several recent papers have exposed the usefulness of left-handed materials that produce negative indices of refraction. Left-handed materials are artificially constructed materials having electromagnetic properties not generally found in nature. Understanding the physical, chemical and the optical properties of left handed materials help to develop perfect electric and optical devices.

1.6 Scope of the Study

Because of the time constraint; the study was mainly limited to theoretical analysis and calculations for the dielectric permittivity and magnetic permeability for the given frequency range.

1.7 Limitation of the Study

The limitation of this study were time constraint in analyzing the details or main points with observational study and active web sites might be unavailable sufficiently for the study.

Chapter 2

Review of Literature

2.1 The History of Left-Handed media

In 1968, Veselago introduced the concept of left-handed media (LHM) with simultaneously negative permittivity and negative permeability, which exhibits an anti-paralleled nature of wave vectors and poynting vectors. However, such a LHM does not exist in naturally. In recent twenty years, LHM as a new artificial composite material attracted more and more attention in the field of material science, solid state physics, optics, and electromagnetism. LHM has a lot of unusual properties different from the normal right-handed materials (RHM), such as negative refractive index, reverse Doppler shift, and backward Cerenkov radiation, etc. Many theoretical and experimental investigations were made to study these electromagnetic and its applications in sub-wave length compact cavity resonator, enhanced electrically antennas and stealth effect, and so on [8].

In 1999, Pendry et al suggested that certain configurations of conducting elements would have a very strong magnetic response to applied electromagnetic fields. In particular, Pendry et al. predicted that these configurations could have a negative effective permeability over a finite frequency range. The magnetic response of naturally occurring materials known to us tails off rapidly with increasing frequency, and in particular it never found to be negative (at least without accompanying large losses). In contrast, if the conducting elements described by Pendry et al. were embedded in to some structurally robust host material (eg. Polymer or ceramic), the resulting material could be classified as a metamaterial, having a magnetic response not available by other means. Indeed, Pendry et al. suggested a variety of uses for such a metamaterial, including magnetic shielding [9].

Hence to some extent, the loss and gain,i.e., the imaginary parts of permittivity and permeability may be regarded as another two parameters for manipulating electromagnetic fields with metamaterials.

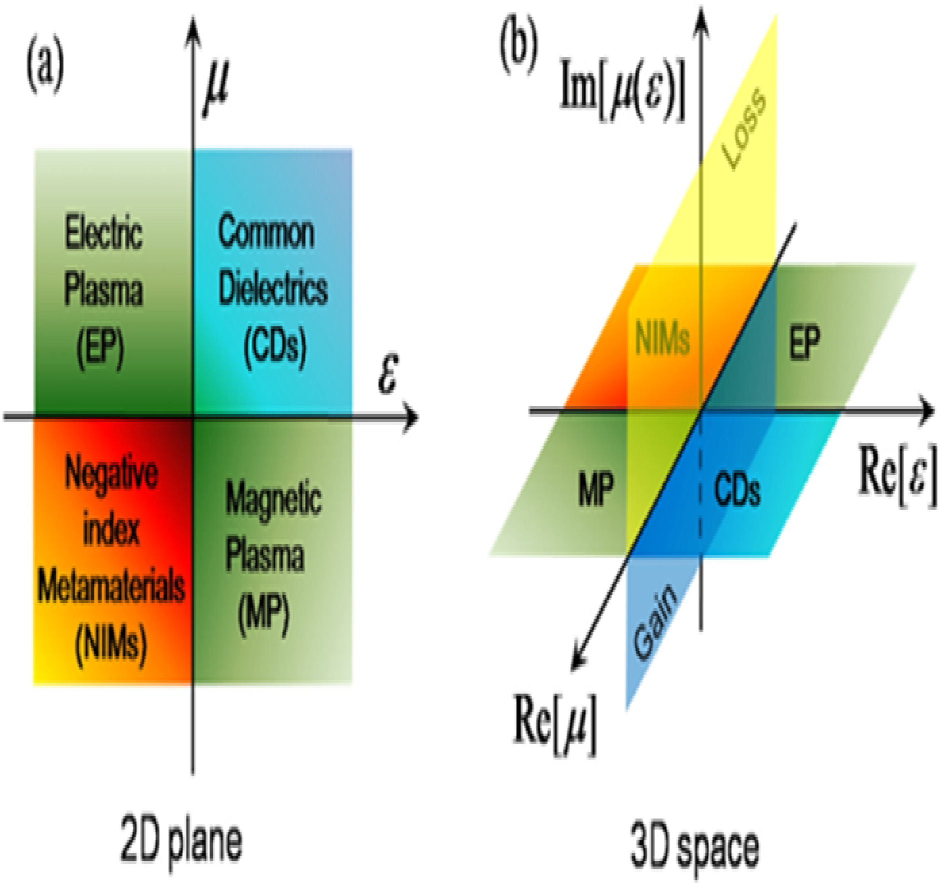


Figure 2.1: Material parameter space for (ϵ, μ) axes. (a) Two dimensional (2D) plane with two axes corresponding to the real parts of permittivity and permeability, respectively. Three dimensional (3D) space with loss or gain as new parameters. Three axes correspond to the real parts of permittivity, permeability and the imaginary part of permittivity or permeability, respectively. With regard to the parameter space, the conventional 2D plane is obviously incomplete (see fig. 2.1 a) which cannot embrace the loss or gain metamaterials. Based on new parameters, loss and gain, we suggest a three-dimensional (3D) space (see fig. 2.1 b) with three axes corresponds to the real parts of permittivity, permeability and the imaginary part of permittivity or permeability. Therein $Im\epsilon(\mu) > 0$ represent loss, while $Im\epsilon(\mu) < 0$ indicates gain [10].

2.2 The Concept of Left-Handed Media

2.2.1 Negative Refractive Index

The reflection and refraction of light at a plane interface two media of different dielectric properties are familiar phenomena. The refractive angle, the transmission, and the reflection coefficients are determined by Snell's law and Fresnel formulas. Many interesting effects happen at the interface, such as total internal reflection and the Brewster angle effect. Now, let's consider the refraction that occurs at the interface between a right-handed medium and a left-handed medium.

Applying the boundary conditions for the electric and the magnetic fields, and considering the consistency of the energy flow across the surface are obtained negative refraction.

The relation between incident angle (θ) and the refractive angle (ϕ) is determined by Snell's law [11]:

$$\frac{\sin\theta}{\sin\phi} = \frac{n_2}{n_1} = -\frac{\sqrt{\epsilon_2\mu_2}}{\sqrt{\epsilon_1\mu_1}} \tag{2.2.1}$$

here n_1 , ϵ_1 and μ_1 are the refractive index, permittivity and permeability of RHM respectively, and n_2, ϵ_2 and μ_2 are corresponding quantities for LHM.

According to the boundary condition and energy flow, we must choose the negative sign of the square root for LHM, i.e. $n_2 = -\sqrt{\epsilon_2\mu_2}$ for the case $\epsilon_2 < 0$ and $\mu_2 < 0$. The negative sign in Snell's law, in Equation (2.2.1) indicates the refraction wave undergoes the negative direction.

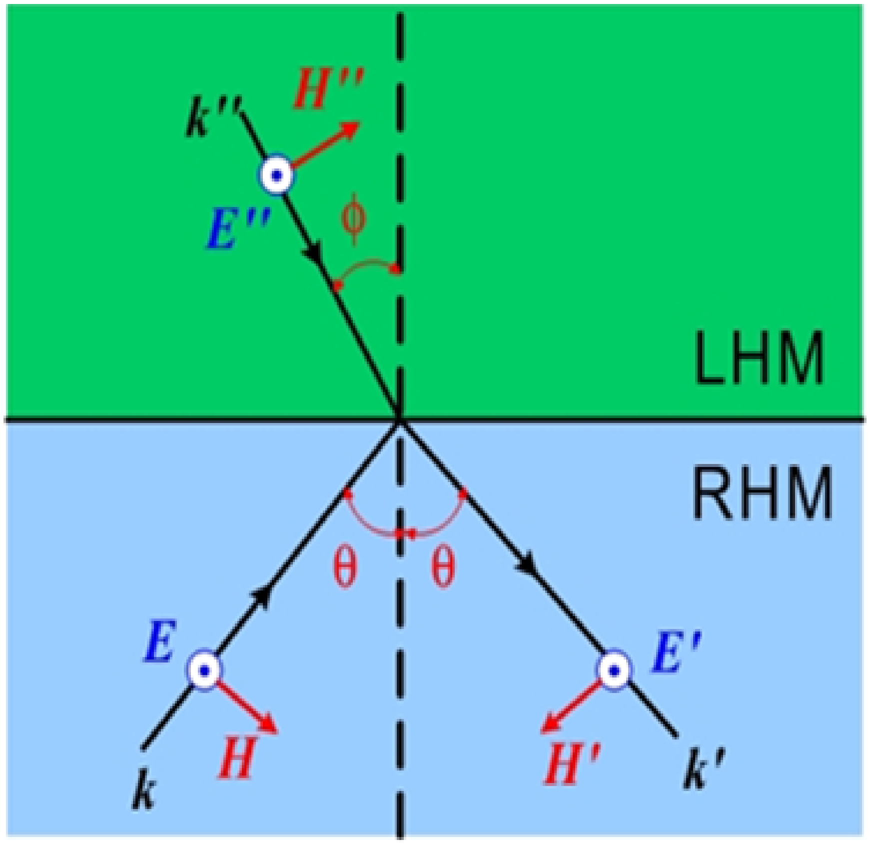


Figure 2.2: The reflection and refraction of an s-polarized (with H in the plane given by the incident rays and incident normal) EM wave at the interface between a right-handed medium and a left-handed medium. The incident EM wave, K, the reflected wave, k', and the refractive wave, k", are shown. And k" heads toward the interface, which indicates that the refractive wave travels along the backward direction on the other hand the energy flow away from the interface. Therefore, energy velocity,**Ve** is along the opposite direction of phase velocity,**Vp**.

2.2.2 Negative Permittivity and Negative Permeability Medium

Substances with one negative constitutive parameter are fond in nature. For example, the plasma medium, such as ionized gas or free electrons gas in metal, has negative permittivity all the way up to the plasma frequency, and belongs to the second quadrant.

Materials such as ferro magnets and antiferromagnetic resource and belongs to the fourth quadrant. However, left-handed materials which belong to the third quadrant do not exist in nature. In left-handed media, negative permittivity and negative permeability can be realized separately. Negative permittivity materials exist in nature. Plasma media, e.g., all the metals, have the negative permittivity up to the plasma frequency.

However, the usage of solid metal in LHM is limited by the fact that the absolute value of the negative permittivity is to large at the target frequency, i.e., from microwave to optical frequency. So, one must determine a method to scale the value of permittivity to reasonable value on the order of -1. Usually the permittivity of a metal can be described by the lossy Drude model;

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\omega_c)} \tag{2.2.2}$$

where ω_c is the damping frequency and ω_p the plasma frequency given by;

$$\omega_p^2 = \frac{ne^2}{\epsilon_0 m_{eff}} \tag{2.2.3}$$

where n is the electron density and m_{eff} is the effective mass of free electrons.

Plasma frequency, ω_p, extremely high in metals, e.g. Silver $=2\pi \times 2184 THz$ and $\omega_c = 2\pi \times 4.32 THz$. As a result the absolute value of permittivity is extremely large

, $Re\epsilon < -10^8$, and, therefore not suitable for LHM. Pendry proposed a wire array design, which can significantly decrease the plasma frequency and realize negative permittivity, $\varepsilon \approx -1$ at microwave frequencies. Moreover, the value of electric permittivity and the frequency are completely via the geometric parameters [11].

2.2.3 Maxwell's Equations and Left-Handed Media

Most electromagnetic phenomena and devices result from the interaction between wave and materials, governed by Maxwell's equations. For material impinged by a wave, its electromagnetic properties are usually determined by two material parameters, i.e., the permittivity (ε) and permeability (μ) [12].

When plane electromagnetic waves having the form $\exp[i(\mathbf{kr} - \omega t)]$ is;

$$\vec{\nabla} \times \vec{E} = -\frac{1}{C}\frac{\partial \vec{B}}{\partial t} \tag{2.2.4}$$

$$\vec{\nabla} \times \vec{H} = \frac{1}{c}\frac{\partial \vec{\vec{\vec{D}}}}{\partial t} \tag{2.2.5}$$

Together with constitutive relations

$$\vec{D} = \epsilon \vec{E} \tag{2.2.6}$$

$$\vec{B} = \mu \vec{H} \tag{2.2.7}$$

$$\vec{K} \times \vec{E} = \frac{\omega}{c}\mu \vec{H} \tag{2.2.8}$$

$$\vec{K} \times \vec{H} = -\frac{\omega}{C}\varepsilon \vec{E} \tag{2.2.9}$$

Equations (2.2.8) and (2.2.9) show that if electric permittivity ε and magnetic permeability μ are simultaneously positive, then the wave vector **K** , the electric field vector **E**, and magnetic field vector **H** form right-handed triplets. Media in which the three vectors form right-handed triplets are known as ordinary or right-handed media (RHM).

On the other hand , if ε and μ are simultaneously negative, then the vectors $\mathbf{K}, \mathbf{E}$ and $\mathbf{H}$ form left-handed triplets .Therefore, such type of media are called left-handed media (LHM).

2.2.4 The Poynting Vector (Energy Flux)

The poynting vector (energy flux) is defined as;

$$\vec{S} = \frac{c}{4\pi}(\vec{E} \times \vec{H}) \tag{2.2.10}$$

For a plane electromagnetic waves using equations, (2.2.8) and (2.2.9) the vector identity,

$\vec{a} \times \vec{b} \times \vec{c} = \vec{b}(\vec{a}.\vec{c}) - \vec{c}(\vec{a}.\vec{b})$ and $\vec{k}.\vec{E}, Equation, (2.2.8)$, takes the form;

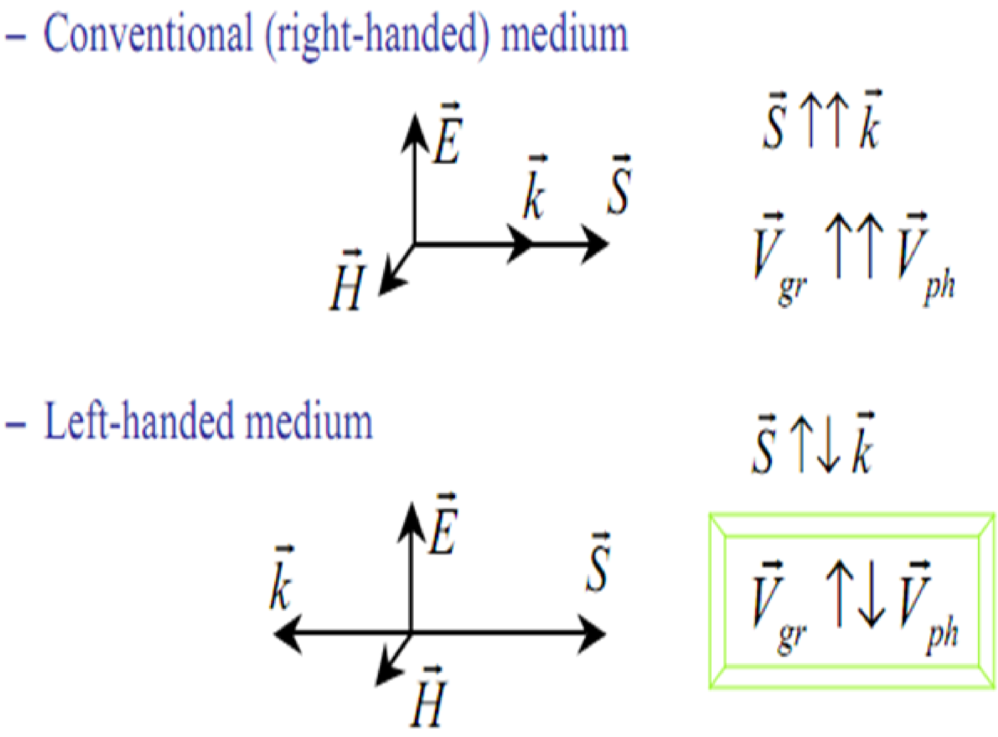
$$\vec{S} = \frac{c^2}{4\pi\omega\mu}(|\vec{E}|^2\vec{K}) \tag{2.2.11}$$

$$\vec{S} = \frac{c^2}{4\pi\omega\mu}(|\vec{H}|^2) \tag{2.2.12}$$

Note that from Equation(2. 2.10) , the quantity $(\vec{E} \times \vec{H})$ gives the direction of energy flow. For a right- handed medium, the vectors, $\vec{S}, \vec{E}$, and $\vec{H}$ form aright handed triplets.

Also from equation (2.2.11) and (2.2.12), the direction of the energy flow is in the direction of the wave vector $\vec{K}$.

In the case where ε and μ are simultaneously negative, i.e left-handed medium, the quantity still gives the direction of the energy flow ,but the wave vector K lies in opposite direction to that of the poynting vector, in contrast to what is observed in ordinary medium (RHM). In addition, with and simultaneously the vectors, and for a left-handed triplet of vectors and hence we call it left-handed medium (LHM). The following figure shows the relative orientation of the vectors and both right-handed and left-handed media.


– Conventional (right-handed) medium

$\vec{S} \uparrow\uparrow \vec{k}$

$\vec{V}_{gr} \uparrow\uparrow \vec{V}_{ph}$

– Left-handed medium

$\vec{S} \uparrow\downarrow \vec{k}$

$\boxed{\vec{V}_{gr} \uparrow\downarrow \vec{V}_{ph}}$

Figure 2.3: Relative orientations of E, H and S in right-handed medium and left-handed medium [13].

2.2.5 Frequency Dispersion Relation in Left-Handed Media

The dispersion relation in isotropic media can be expressed as follow. The plane wave solutions for these equations are given by;

$$E(\vec{r,t}) = \vec{E_0} \exp i(\vec{k}.\vec{r} - \omega t) \tag{2.2.13}$$

And from Maxwell's differential wave equations, we have

$$\nabla^2 \vec{E} = \frac{1}{\vec{v}^2} \frac{\partial^2 \vec{E}}{\partial t^2} \tag{2.2.14}$$

The dispersion relation for isotropic media is found to be;

$$\vec{k}^2 - \frac{\omega^2}{c^2} \varepsilon \mu = 0 \tag{2.2.15}$$

or

$$n^2 = \varepsilon \mu = \frac{c^2 \vec{k}^2}{\omega^2} \tag{2.2.16}$$

Further, Equation (2.2.15) may be written as;

$$n_{\pm} = \pm \sqrt{\varepsilon \mu} \tag{2.2.17}$$

Electromagnetic waves propagate in a medium if and only if n^2 is positive.

These can happen in two different ways.

1. For regular medium, where both ε and μ are simultaneously positive. The positive root, $n_+ = +\sqrt{(+\varepsilon)(+\mu)} = +\sqrt{\varepsilon \mu}$, must be taken to satisfy the requirement. This refractive index corresponds to right-handed medium (RHM).

2. For the case where both and are simultaneously negative, Maxwell's equations remains valid, provided that the negative root, $n_= - \sqrt{(-\varepsilon)(-\mu)} = -\sqrt{\varepsilon \mu}$, is taken, if the electromagnetic waves propagate in the medium is possible. This refractive

index corresponds to left-handed medium (LHM). These are justified by considering the poynting vector expressions given in equations, (2.2.11) and (2.2.12) , that is when μ becomes negative then $\vec{k}$ has to be reversed, in equation (2.2.11) or when ε becomes negative when $\vec{k}$ has to be reversed , so that the Poynting vector to retain its direction [14].

2.2.6 Dispersive and Dissipative Nature of LHMs

The negative value of ε and μ can be realized simultaneously, only if the material has frequency dispersion. This can be seen immediately from the formula of the energy density of non-dispersive media,

$$W = \varepsilon\vec{E}^2 + \mu\vec{H}^2 \tag{2.2.18}$$

If $\varepsilon < 0$ and $\mu < 0$, the energy density, W, would be negative. When there is frequency dispersion i.e.,$\varepsilon = \varepsilon(\omega)$ and $\mu = \mu(\omega)$, the total energy density is given by;

$$W = \frac{\partial(\varepsilon\omega)}{\partial\omega}\vec{E}^2 + \frac{\partial(\mu\omega)}{\partial\omega}\vec{H}^2 \tag{2.2.19}$$

The energy density is always positive,+,

$$\frac{\partial(\varepsilon\omega)}{\partial\omega} = \varepsilon + \omega\frac{\partial\varepsilon}{\partial\omega} > 0 \tag{2.2.20}$$

$$\frac{\partial(\mu\omega)}{\partial\omega} = \mu + \omega\frac{\partial\mu}{\partial\omega} > 0 \tag{2.2.21}$$

This clearly indicates that ε and μ can be simultaneously negative, given the medium is frequency dispersive and equation(2.2.8) is fulfilled. Therefore, a left-handed medium must be dispersive. Moreover, a medium with frequency dispersion is always dissipative. Following the causality principle, the real and imaginary part of permittivity, $\varepsilon(\omega) = \varepsilon'(\omega) + i\varepsilon"(\omega)$ are related by the Kramers-kronig relations.

$$\frac{\varepsilon'(\omega)}{\varepsilon_0} = 1 + \frac{2}{\pi} P \int_0^\infty \frac{\omega'\varepsilon"(\omega')}{\varepsilon_0(\omega'^2 - \omega^2)} d\omega' \tag{2.2.22}$$

$$\frac{\varepsilon"(\omega)}{\varepsilon_0} = -\frac{2\omega}{\pi} P \int_0^\infty \frac{\varepsilon"(\omega')}{(\varepsilon_0 - 1)(\omega'^2 - \omega^2} > 0 \tag{2.2.23}$$

Where, P stands for the principal value of the integration. The real and imaginary part of permeability, $\mu(\omega) = \mu'(\omega) + i\mu(\omega)$, obey the same relations. Since the imaginary part of the permittivity, the permeability and the refractive index always coexist with the real parts in the dispersive media; the left-handed material must be dissipative. A recent study shows a lower limit of electric and magnetic losses in the left handed material. If the losses are eliminated or significantly reduced for any reason, including compensated by active (gain) media, then the negative refractive will disappear [15].

2.3 Routes in Realizing Left-Handed Media

It is well known that serious investigation on left-handed media began after the pioneering theoretical work of Veselago [16] in 1968. At the time, he speculated that left-handed media with simultaneously negative values of permittivity and permeability, that may have peculiar characteristics to the normal types of RHM, are not readily available in nature.

So, he proposed the fabrication of "artificial" materials in order to realize left-handed materials. Subsequently, many scientists have carried out experiments as well as theoretical investigations to attain LHM. Pendry and Smith were the first researchers to identify a practical way to make this new material [17, 18]. Recently, the research of electromagnetic metamaterials has been reported by using chiral materials [19].

2.3.1 Isotropic Left-Handed Media

Now days, various materials that are not found in nature are being assembled artificially from naturally available materials. Such man-made structures are called metamaterials. Left-handed metamaterial were artificially constructed by Smith, et al. (2000)[20]. They successfully managed to assemble isotropic left-handed medium from two kinds of cell elements: split ring resonators (SRR) that produce negative permeability and a wire array to produce negative permittivity.

2.3.2 Negative Permittivity

Materials with negative permittivity exist in nature. The good known examples are metals, low-loss plasmas, and semiconductors at optical and infrared frequency. In 1966, Pendry proposed an artificially material which was metallic thin wire medium with negative permittivity at micro wave frequency [21]. From the available analytical models, the most common ones that are employed for the description of the effective permittivity and permeability of left handed materials are the Drude and Lorentz models [22].

These are given by;

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + ir\omega} \tag{2.3.1}$$

Which is the equation for the permittivity in the Drude model and

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2 - \omega_0^2}{\omega^2 - \omega_0^2 + ir\omega} \tag{2.3.2}$$

This is the equation for the permittivity in the Lorentz model. The parameters $\omega_p, \omega_0, \gamma$ and ω are the plasma, resonant, damping and the incident electromagnetic wave frequencies respectively.

The structure of thin metal wires demonstrates to have negative permittivity at microwave frequencies. The array of thin metal wire illustrated the effective negative permittivity . If an electric field, E is applied along the wire's axes, it induces currents through the wires which is equivalent to the generation of electric dipole moment in the metal arrays.

There are two factors affecting the electron movement in a wire of radius, r. These are ;

- The average electron density, n_eff, and
- The effective mass electrons $m - eff$ by magnetic effects [23].

$$n_{eff} = \frac{n\pi r^2}{a^2} \tag{2.3.3}$$

Where n is the density of electrons in the wire, and a is the lattice constant. From Ampere's law, a current flowing through a wire produces a magnetic field; that direction depends on the direction of current. The effective mass of an electron,m_{eff} in the wire may be expressed as,

$$m_{eff} = \frac{2\pi e^2 r^2}{c^2} \ln(a/r) \tag{2.3.4}$$

Where,e is electron charge. Also, the plasma frequency, which is associated with the fundamental oscillation frequency of the electrons while returning to the equilibrium position is given by;

$$\omega_p^2 = \frac{4\pi e^2 n_{eff}}{m_{eff}} = \frac{2\pi c^2}{a^2 \ln(a/r)} \tag{2.3.5}$$

Where,c , is the speed of light in vacuum. If the metal is lossless and the electric field is along the wire, so the case of left-handed medium, the Drude equation for the effective permittivity,ε_{eff}, can be reduced to ;

$$\varepsilon_{eff} = 1 - \frac{\omega_p^2}{\omega^2} \tag{2.3.6}$$

Where ω_p is the plasma frequency and ω is the frequency of the propagating electromagnetic wave. From equation(2.2.16) the effective permittivity is negative for $\omega_p > \omega$ and positive for $\omega_p < \omega$ [24]

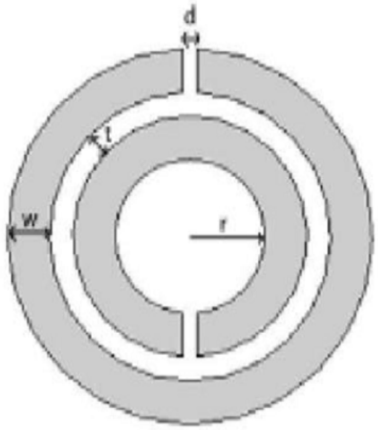


Figure 2.4: The sprit-ring resonator [17]

2.3.3 Negative Permeability

The availability of materials with negative permeability is less common in nature due to the weak magnetic interactions in most solid state materials. Only in ferromagnetic and Ferrite materials are magnetic interaction strong enough to produce regions of negative magnetic permeability [24].

In 1999, Pendry showed that the split-ring resonators (SSRs) structure possess negative permeability in the microwave frequency [20].

This artificial medium is made from two concentric (inter placed) conducting rings with one gap in each as show in$fig.$2.4. From split-ring resonator, if the magnetic field ,$\vec{H}$, is perpendicular to the planer of to ring, then the effective permeability as a function of frequency is found to be given by [20]

$$\mu_{eff}(\omega) = 1 - \frac{\omega_{pm}^2}{\omega^2 - \omega_{om}^2 + ir\omega} \tag{2.3.7}$$

In 2000, the combination of thin wire medium and SRRs were investigated by Smith, et al as potential metamaterial to attain negative permittivity and negative permeability, respectively [9]. Indeed the combined medium was used to realize Veselago's left-handed medium in certain narrow frequency domain. In 2001, Shelby, et al also achieved left-handed medium consisting metal post and SRR experimentally [18]. According to this experiment, the material constructed by combining these two media that are known to exhibit negative permittivity and negative permeability was shown to possess negative refractive index ($n < 0$) in the micro wave regions ($f \approx 5GHz$) [11].

2.4 Unique Properties of Left-Handed Media

An established slab from left-handed material with negative permittivity can be regarded in the same way as a dielectric slab. It is, however, beneficial to highlight two special properties of the left-handed slab. First, a planar lens can be investigated at the interface of the vacuum and left-handed medium due to the negative refraction (Markos and Soukoulis, 2008). Second, left-handed slab is able to amplify incoming evanescent waves which offer many applications such as perfect lenses (Koschny, Moussa, and Soukoulis, 2006).Therefore, these abnormal property make left-handed materials different in principle from any other known materials [14].

2.5 Applications of Left-Landed Materials

The unique properties of left-handed materials make them the best candidates for most applications. Left-handed materials can be used in many applications, such as cloaks, antennas, resonators, radome, sensors, absorbers and couplers etc (Gangwar, Paras and Gangwar, 2014).

These applications are required for the performance improvement of the material. Cloaking means that electromagnetic field inside the hollow cloak tends to be zero; this make the region the shells disappear. This can be achieved by guiding the electromagnet wave, in anther word transforming the coordinate system (Ergin, Stenger, Brener, Pendry and Wegener 2010).Researches concerned on the use of left-handed materials in directive antenna substrate systems (Chen, Wu, Ran, Grzegorczyk and Kong, 2006;Sui et al, 2005; Wu et al, 2005;). The problem with the designed antennas in the past is the narrow band operation.

It is well known that by Snell's law, if a source is embedded in a substrate that has a small index of refraction compared to air, its rays with be transmitted near the normal of the substrate. The light-weight property of left-handed materials added benefit to the design a wideband directive antenna (Li, Yeo, Mosig, and Martin. (2010).

Moreover, Zero-index metamaterials or left-handed materials where the effective permittivity and permeability are zero at certain frequencies can be used to achieve a wide frequency band of high directivity. To increase the gain of the antenna, a planar radome was arranged by using seven left-handed materials structures.

The fabrication of LHMs opens a door for designing sensors with specified sensitivity and enhances the resolution of sensors. These sensors are used in wide variety area such as agriculture and biomedical insruments. Conventional couplers can achieve strong forward coupling. But to obtain sufficient coupling levels, they require very long physical lengths and very tight spacing between the two lines. To avoid this drawback, LHMs induced the possibility achieving strong forward coupling with length drastically reduced (Gangwar, Paras and Gangwar, 2014) [25].

Chapter 3

Materials and Methodology

This study had been carried out by using the following procedures. These are: study site and period, method of approach, materials used, and ethical considerations.

3.1 Study Site And Period

The study had been conducted at Jimma University, department of physics from January 2019 to January 2020.

3.2 Method of Approach

To achieve the stated objectives and problems, analytical method and graphical method with computational analysis were used.

3.3 Materials

The materials used for these study were books, standard journals, published papers, thesis, and the international science conferences report (dissertation), laptop, flashes,disks,stationary materials and softwares, such as MATLAB and MATHEMATICA, computers were additional instruments used to accomplish this work based on the thesis title.

3.4 Methodology

3.4.1 Analytical Method

Important parameters, such as permittivity, permeability and their dispersion relations were solved by using analytical method.

3.4.2 Graphical Method

In this study, analytically solved problems had been described and interpreted by using MATLAB or MATHEMATICA softwares.

3.5 Ethical considerations

The University's Guidelines and Regulations were strictly followed and respected during this work. Ethical authorization, like paper citations and appropriate verification had been obtained from research review and Ethical Committee of collage of natural sciences, Jimma University. Any concerned body would informed the purpose of this study.

Chapter 4

Result and Discussion

4.1 Introduction

Left-handed materials are man made materials that have negative electric permittivity, negative magnetic permeability and negative index of refraction across a common frequency band.

Left-handed materials (LHMs) are a subset of metamaterials with an anti-parallel relation between wave propagation vector and the poynting vector, which lead to negative refraction. Metamaterials are a composite or structured material that exhibits properties not found in naturally occurring materials or compounds. Left-handed materials have electromagnetic properties that are distinct from any known material, and hence are examples of metamaterials .

Media with a negative index of refraction have received much attention in recent years. Because of their unique properties a great deal of time has been spent formulating new designs and envisioning new applications for these artificial materials. Because a negative index of refraction is a relatively new subject, many problems and misconceptions have arisen regarding the behavior and properties of media that exhibit this phenomenon.

LHMs have been studied extensively in recent years due to their unique physical properties and novel applications . Negative refraction of electromagnetic waves is the most interesting physical phenomenon exhibited by the left-handed metamaterial structures. Negative index materials (NIMs) are a subset of LHM which on top of exhibiting negative refraction, have negative values of the refractive index which may potentially lead to many novel optical devices such as perfect lensing (or super resolution) .

These systems exhibit electromagnetic properties that can not exist in nature. However the interest remind a scientific curiosity for a long time, it is recently known that these materials may have important applications, such as constructing lenses that bet the diffraction limits and engineering electromagnetic cloaks, where the induced polarization and magnetization are sufficiently large and appropriately phased, then the medium will support the formation of left-handed waves. Left-handed materials have got strong attention due to their potential applications in developing electronic and optical devices.

In such metamaterials, the electric field vector, the magnetic field vector, and the propagation vector form a left hand triad, thus the name left hand materials.

In left-handed media, the group and phase velocities are perfectly antiparallel. In these media, the group velocity and the poynting vector point in the same direction, where as the phase velocity and the wave vector are parallel and point in the same directions.

In this wok, the basic properties of electromagnetic wave propagation through homogenous left hand materials are first studied. Many of the basic properties of left hand materials are in contrast to those in right hand materials, viz , negative refraction, perfect lensing, and the inverse Doppler effect. Dispersion relations are used to study wave propagation in negative index materials.

4.2 Mathematical Formulation of the Problem

A realistic left-handed medium is considered the lossy Drude model is used for both electric permittivity and magnetic permeability.

The Drude model links with the optical and electrical properties of material with the behavior of its electrons or holes. The negative value of ε and μ can be realized simultaneously, only if the material has frequency dispersion.

The dispersion forms of the electric permittivity and the magnetic permeability are almost the same.That means:

$$\varepsilon(\omega) = \varepsilon_0 (1 - \frac{\omega_{pe}^2}{\omega(\omega + i\Gamma_e)}) \tag{4.2.1}$$

$$\mu(\omega) = \mu_0 (1 - \frac{\omega_{pm}^2}{\omega(\omega + i\Gamma_m)}) \tag{4.2.2}$$

where ω_{pe} is the electric plasma frequency, ω_{pm} the magnetic plasma frequency, Γ_e is the collision of frequency of magnetic properties. The real and imaginary parts of the dielectric constants are;

$$\varepsilon' + i\varepsilon" = \varepsilon_0 - \frac{\varepsilon_0 \omega_{pe}^2}{\omega(\omega + i\Gamma_e)} \tag{4.2.3}$$

$$\varepsilon' + i\varepsilon" = \varepsilon_0 - \frac{\varepsilon_0 \omega_{pe}^2}{\omega} (\frac{1}{\omega + i\Gamma_e} \times \frac{\omega - i\Gamma_e}{\omega - i\Gamma_e}) \tag{4.2.4}$$

$$\varepsilon' + i\varepsilon" = \varepsilon_0(1 - \frac{\omega_{pe}^2}{\omega^2 + \Gamma_e^2}) + \frac{\varepsilon_0\omega_{pe}^2 i\Gamma_e}{\omega(\omega^2 + \Gamma_e^2)} \tag{4.2.5}$$

Moreover, analysis of $(Eqn.4.2.1)$, we have the following. follow.

$$\mu' + i\mu" = \mu_0 - \frac{\mu_0\omega_{pm}^2}{\omega(\omega + i\Gamma_m)} \tag{4.2.6}$$

$$\mu' + i\mu" = \mu_0 - \frac{\mu_0\omega_{pm}^2}{\omega}(\frac{1}{\omega + i\Gamma_m} \times \frac{\omega - i\Gamma_m}{\omega - i\Gamma_m}) \tag{4.2.7}$$

$$\mu' + i\mu" = \mu_0(1 - \frac{\omega_{pm}^2}{\omega^2 + \Gamma_m^2}) + \frac{\mu_0\omega_{pm}^2 i\Gamma_m}{\omega(\omega^2 + \Gamma_m^2)} \tag{4.2.8}$$

If a lossy,left-handed medium is considered with

$$\omega_{pe} = \omega_{pm} = \omega_p \tag{4.2.9}$$

and

$$\Gamma_e = \Gamma_m = \Gamma \tag{4.2.10}$$

then, it results in:

$$\varepsilon(\omega) = \mu(\omega) \tag{4.2.11}$$

The real and imaginary part of the refractive index is given by;

$$n' = \sqrt{\varepsilon'(\omega)\mu'(\omega)} \tag{4.2.12}$$

$$n' = (\sqrt{1 - \frac{\omega_p^2}{\omega^2 + \Gamma^2}})(\sqrt{1 - \frac{\omega_p^2}{\omega^2 + \Gamma^2}}) \tag{4.2.13}$$

$$n' = 1 - \frac{\omega_p^2}{\omega^2 + \Gamma^2} \tag{4.2.14}$$

$$n" = \sqrt{\varepsilon"(\omega)\mu"(\omega)} \tag{4.2.15}$$

$$n" = (\sqrt{\frac{\Gamma\omega_p^2}{\omega^3+\omega\Gamma^2}})(\sqrt{\frac{\Gamma\omega_p^2}{\omega^3+\omega\Gamma^2}}) \tag{4.2.16}$$

$$n" = \frac{\Gamma\omega_p^2}{\omega^3+\omega\Gamma^2} \tag{4.2.17}$$

Assuming

$$\varepsilon_0 \approx \mu_0 = 1 \tag{4.2.18}$$

In case of considering low loss, the real part and the imaginary part, $\varepsilon'(\mu')$ and $\varepsilon"(\mu")$, of the electric permittivity or magnetic permeability by using the dimensionless variables:

$$z = \frac{\omega}{\omega_o} \tag{4.2.19}$$

$$\gamma = \frac{\Gamma}{\omega_o} \tag{4.2.20}$$

and

$$\nu = \frac{\omega_p}{\omega_o} \tag{4.2.21}$$

is given by;

$$\varepsilon'(\mu') = 1 - \frac{\nu^2}{z^2+\gamma^2} \tag{4.2.22}$$

$$\varepsilon"(\mu") = \frac{\gamma\nu^2}{z(z^2+\gamma^2)} \tag{4.2.23}$$

where:

- ω is any frequency $\omega_0 = 2\pi \times f_0$, where, $f_0 = 3 \times 10^{10}$ rad/sec
- $\omega_o = 2\pi \times 3 \times 10^{10} = 1.884 \times 10^{11}$ rad/sec. This frequency is a target frequency, which is employed to make the variable dimensionless and it is in the frequency range of the plasma frequency.
- z, γ and ν are dimensionless collision frequencies.

4.3 Graphical Representation of Dispersion Relation

The real part of electric permittivity or magnetic permeability for the plasma frequencies of $\omega_p = 2 \times 10^{11}$ rad/sec, $\omega_p = 4 \times 10^{11}$ rad/sec, and $\omega_p = 6 \times 10^{11}$ rad/sec is described in figure (4.1). Other parameters used are the collision frequency, $\Gamma = 10^8$ rad/sec, and the target frequency of 1.884×10^{11} rad/sec.

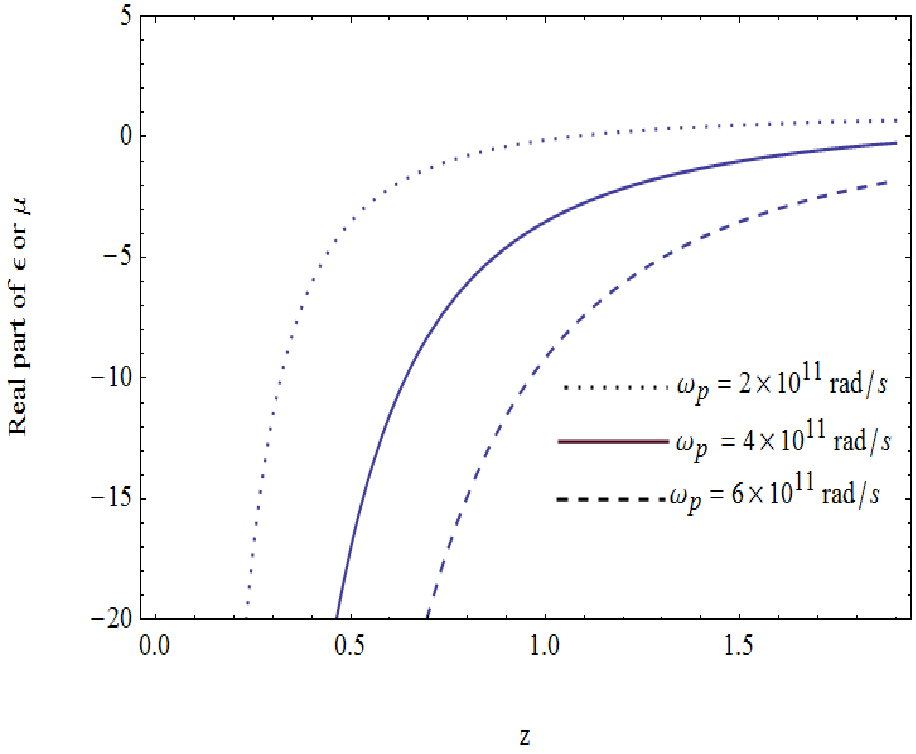


Figure 4.1: Real part of relative angular frequency against dielectric function or magnetic permeability (Drude model) for the relative angular plasma frequencies, $\omega_p = 2 \times 10^{11}$ rad/sec, $\omega_p = 4 \times 10^{11}$ rad/sec, $\omega_p = 6 \times 10^{11}$ rad/sec, which correspond to photon(radiation) energy position.

From figure (4.1), one can conclude that the real part of electric permittivity and magnetic permeability perform a negative number and the material behaves significant features of left-handed media. All the plasma frequencies, 2×10^{11} rad/sec, 4×10^{11} rad/sec and 6×10^{11} rad/sec produce negative refractive index at the target frequency. Moreover, the imaginary part of the electric permittivity or magnetic permeability are demonstrated in figure (4.2), for the same plasma frequencies and target frequency.

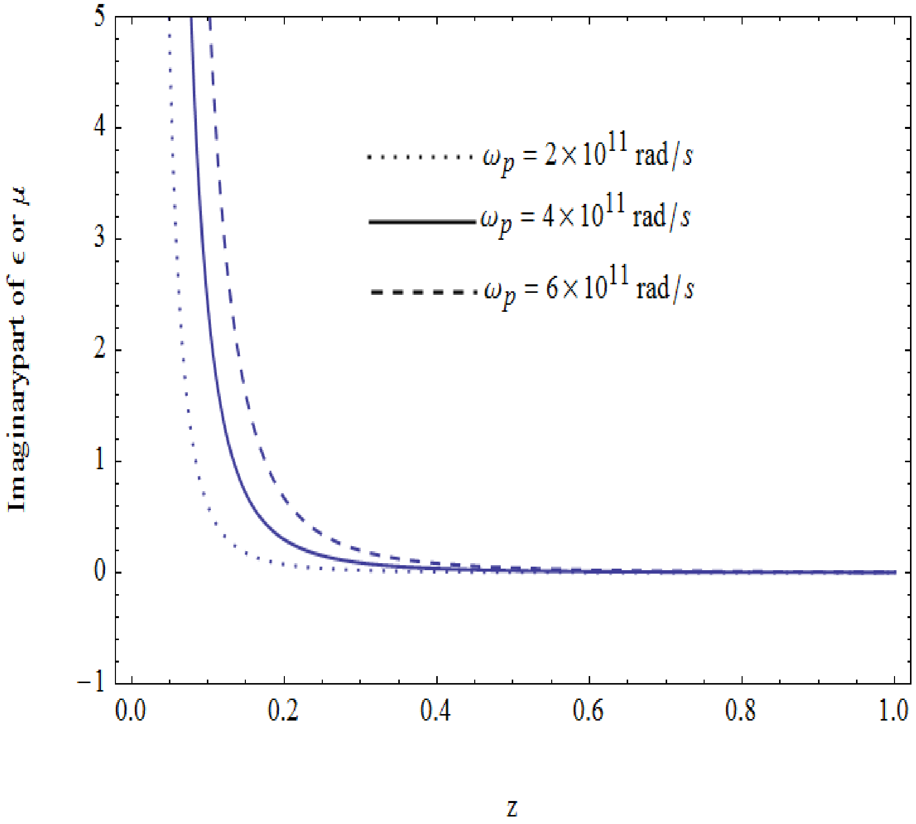


Figure 4.2: The imaginary part of relative electric permittivity or magnetic permeability against relative angular frequency for the plasma frequencies, $\omega_p = 2 \times 10^{11}$ rad/sec, $\omega_p = 4 \times 10^{11}$ rad/sec, $\omega_p = 6 \times 10^{11}$ rad/sec

The target frequency is 1.884×10^{11} rad/sec. As it is observed from the figure (4.2), the value of the imaginary part in all cases are small as compared to the real part of the permittivity or permeability.

Chapter 5

Conclusion

In this thesis, we have developed the general properties of the propagation of electromagnetic waves in left-handed media.We investigated the real and imaginary part of electric permittivity and magnetic permeability by using lossy Drude model. Various tools for understanding and characterizing left-handed materials are thereafter presented.

In this work, we have identified that the dispersion relations which gives the properties of $Re(\varepsilon) < 0$ and $Re(\mu) < 0$ gives a dispersion condition relations that allows a real wave vector in the medium, that means waves are propagating in the left handed medium. Since $\varepsilon < 0$ and $\mu < 0$, it is clear that the vectors $\vec{E}$, $\vec{H}$ and $\vec{K}$ form a left-handed media.

From figure (4.1) we can understand that the real part of the electric permittivity or magnetic permeability perform a negative number and the material behaves significant feature of left-handed media.

From figure (4.2) we can observe that the imaginary part of the electric permittivity or magnetic permeability for the relative frequency, plasma frequency and target frequency.There fore, the value of imaginary part of electric permittivity or magnetic permeability in all cases are small as compared to the real part of electric permittivity or magnetic permeability due some amount of energy loses.

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JIMMA UNIVERSITY

COLLEGE OF NATURAL SCIENCES

PERFORMANCE CERTIFICATE FOR MASTER'S DEGREE

Name of Student: **Baro Woldegebreal Yesho** ID No. **S-32835/08**

Graduate Program: **Summer, MSc.**

1. Course Work Performance

Course Code	Course Title	Cr. hr	Number Grade	Rank**	Remark
Phys699	**MSc. Thesis**	**6**			

**Excellent, Very Good, Good, Satisfactory, Fail.

Thesis Title : **Propagation of Electromagnetic Waves in Left-Handed Media**

2. Board of Examiners decision Mark × in one of the boxes. Pass ☐ Failed ☐

If failed, give reasons and indicate plans for re-examination.

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3. Approved by: Name and Signature of members of the examining Board, Department Head.

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We the undersigned, number of the Board of Examiners of the final open defense by **Baro W/Gebreal** have read and evaluated his/her thesis entitled "**Propagation of Electromagnetic waves in Left-Handed Media**" and examined the candidate. This is therefore to certify that the thesis has been accepted in partial fulfillment of the requirements for the degree Master of Science in **Physics (Condensed Matter Physics)**.

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I hereby declare that this MSc thesis is my original work and has not been presented for a degree in any other University and that all source of materials used for the dissertation have been duly acknowledged.

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This MSc thesis has been submitted for examination with my approval as University advisor.

Dr. Menberu Mengesha

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