COHERENTLY DRIVEN DEGENERATE THREE-LEVEL LASER WITH PARAMETRIC AMPLIFIER IN A VACUUM RESERVOIR

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DECLARATION

I hereby declare that this thesis is my original work and has not been presented in any other university, and that all sources of material used for the thesis have been dul acknowledged.

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Abstract

In this thesis, we have investigated the squeezing and statistical properties of light produced by coherently driven degenerate three-level laser with parametric amplifier for the cavity mode coupled to a vacuum reservoir via a single-port mirror. First we have derived the master equation in the linear approximation scheme which is used to determine stochastic differential equations. We carry out analysis applying the solutions of c-number Langevin equations associated with the normal ordering. we determined the quadrature variance and squeezing spectrum. Using the antinormal order characteristic function ,we obtain the Q-function . we have calculated the mean photon number, photon number variance and photon number distribution. We have found that a light mode is 93% squeezed below the coherent state level at steady state for combination degenerate parametric amplifier with three -level laser at $A = 3$ and it is observed that the degree of squeezing increases with the linear gain coefficient(A). The effect of parametric amplifier is to increase the intra-cavity squeezing by a maximum of 50%. The maximum intracavity squeezing is found to be 86% below coherent state level in the absence of parametric amplifier. The mean photon number increase with linear gain coefficient. The squeezing of the superposed light of single-mode light increases with linear gain coefficients with a squeezing of 95.8% below coherent state level.

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INTRODUCTION

Quantum optics deals mainly with the quantum property of light generated by various optical systems such as lasers with the effect of light on the dynamics of the atoms. Now we focused the quantum property of three-level laser coupled with degenerate parametric amplifier in vacuum reservoir.

Degenerate parametric amplifier is a typical source of squeezed light, with a maximum of 50% intracavity noise reduction [1-5]. Some authors have also established that a three-level laser under certain conditions generates squeezed light [6-7]. A squeezed state is now belonging to the selected technologies for detection of weak signals and in low noise communication [8-10]. We define a three-level laser as a quantum optical system in which three-level atoms in a cascade configuration and initially prepared in a coherent superposition of the top and bottom levels are injected at a certain rate into a cavity coupled to a vacuum reservoir via a single-port mirror see Fig.1.1. The three-level laser in which a considerable role is played by the coherent superposition of the top and bottom level of the injected atoms have been studied by different authors [7, 8, 9, 10, 12, 13, 14, 16, 17].

The squeezing in such a laser is due to the coherent superposition of the top and bottom levels. It now appears that a highly squeezed light could be generated by a combination of these two quantum optical systems. The set of energy levels of an

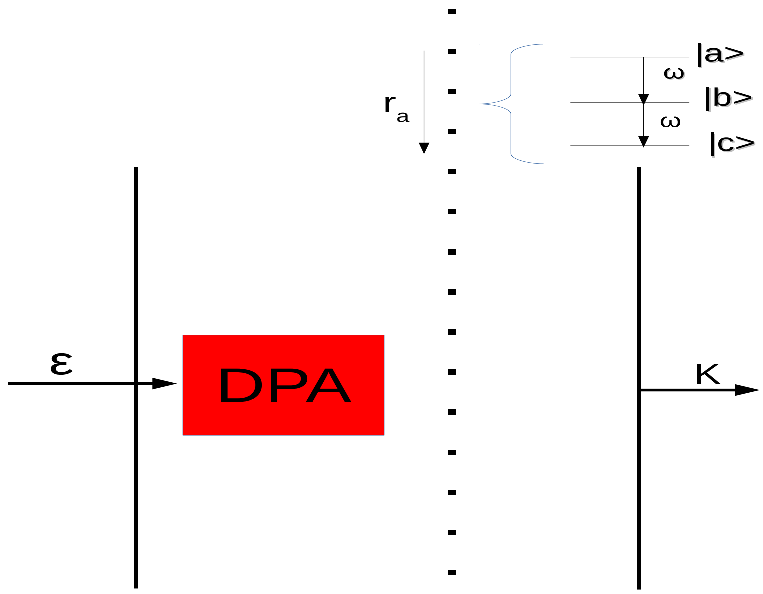

Figure 1.1: Schematic diagram of three-level laser with a degenerate parametric amplifier (DPA)

atom consists of an infinite number of discrete levels corresponding to the bound states of the electrons [11]. For a three-level atom, out of these set of energy levels only three-levels interact with electromagnetic radiation. When the three-level atom interacts with radiation, then it under goes a transition from top to bottom level via the intermediate level by emitting two photons. If the two photons generated have different frequencies, a two-mode light is generated. In this case the atom is called **non-degenerate three-level atom**. But, when the frequencies of these photons are equal, the atom generates a single-mode light. For this condition the atom is called **degenerate three-level atom**.

Ansari [7] has found the quadrature variance of degenerate three-level laser using the steady state solution of the expectation value of cavity mode variables. He found that the cavity mode is in squeezed state if the probability for the injected

atoms to be in the bottom levels is larger than the probability to be in the top levels. And almost perfect squeezing can be achieved for slightly high probability for the atoms to be in the bottom levels and for large value of linear gain coefficient.
Alebachew and Fesseha [12] have studied the squeezing properties of the cavity mode produced by a degenerate three-level laser whose cavity contains a parametric amplifier by applying the solution of the stochastic differential equations, with the top and bottom levels of injected atoms coupled by the pump mode emerging from the parametric amplifier.
In this study they showed that the optical system generates light in a squeezed state with a maximum interacavity squeezing of 93% below the coherent state level.
Recently, Misrak [13] has studied the squeezing properties of cavity mode produced by degenerate three-level laser with parametric amplifier by applying the solution of stochastic differential equations. This study showed that the quantum optical system generates squeezed light and the degree of squeezing increases with the linear gain coefficient with maximum interacavity squeezing of 96.5% below the coherent state level. K. Fesseha [15] has shown that the effect of parametric amplifier is to increase the intracavitary squeezing by a maximum of 50%.

In this thesis, we seek to analyze the squeezing and statistical properties for degenerate three-level laser whose cavity contains parametric amplifier for single light and for superposition of light beams produced by pair of degenerate three-level lasers. By making use of stochastic differential equations, we carry out the analysis applying the solutions of c-number Langevin equations associated with the normal ordering. These equations are obtained using the master equation derived in the linear approximation scheme in the good cavity limit. This is used to analyze the squeezing and statistical properties of the generated cavity radiation with the aid of the pertinent stochastic differential equations associated with the normal ordering. Imposing the requirement that the c-number equations of evolution for the first and second-order moments have the same forms as the corresponding operator [14], we obtain stochastic differential equations, associated with the normal ordering, for the dynamical variables of the cavity mode. The solutions of the resulting equations are then used to calculate the quadrature variance and the squeezing spectrum. Applying the same solutions, we also determine the antinormal ordered characteristic function with the aid of which the Q function is obtained. Finally, the Q function is used to calculate the mean photon number , the photon number variance and photon number distribution.

2

OPERATOR DYNAMICS

In this chapter we consider degenerate three-level laser driven by coherent light and with the cavity mode coupled to single mode vacuum reservoir via single port mirror. We first set up the interaction hamiltonian for degenerate three-level atom with the cavity mode and the master equation for the cavity mode containing the parametric amplifier. Finally,we obtained the solution from the master equation by using stochastic differential equations in the case.

2.1 The Hamiltonian

A three-level laser consists of a cavity in which three-level atoms in a cascade configuration are injected at a constant rate r_a and removed from the cavity after a certain time τ. We represent the top, middle, and bottom levels by $|a\rangle$, $|b\rangle$ and $|c\rangle$ respectively. In addition, we assume that the cavity mode to be at resonance with the two transitions $|a\rangle \rightarrow |b\rangle$ and $|b\rangle \rightarrow |c\rangle$,dipole allowed and with direct transition between levels $|a\rangle$ and $|c\rangle$ to be dipole forbidden. The interaction of a three-level atom with the cavity mode can be described in the interaction picture by the Hamiltonian.

$$\hat{H} = ig\bigg[\hat{a}^{\dagger}(|b\rangle\langle a| + |c\rangle\langle b|) - \hat{a}(|a\rangle\langle b| + |b\rangle\ \langle c|)\bigg], \tag{2.1}$$

where g is the coupling constant and $\hat{a}$ is the annihilation operator for the cavity mode. In this study we take the initial state of a three-level atom to be

$$|\psi(0)\rangle = C_a(0)|a\rangle + C_c(0)|c\rangle, \tag{2.2}$$

and hence the initial density operator for a single atom has the form

$$\hat{\rho}_A(0) = |\psi(0)\rangle\langle\psi(0)|, \tag{2.3}$$

from which follows

$$\hat{\rho}_A(0) = \rho_{aa}^{(0)}|a\rangle\langle a| + \rho_{ac}^{(0)}|a\rangle\langle c| + \rho_{ca}^{(0)}|c\rangle\langle a| + \rho_{cc}^{(0)}|c\rangle\langle c|, \tag{2.4}$$

where $\rho_{aa}^{(0)} = |c_a|^2$, $\rho_{ac}^{(0)} = C_a C_c^*$, $\rho_{ca}^{(0)} = C_c C_a^*$ and $\rho_{cc}^{(0)} = |c_c|^2$.

The external environment, usually referred to as a reservoir, can be thermal light, ordinary or squeezed vacuum. We are interested in the dynamics of the system and this is describable by the master equation, the Fokker-Planck equation, or quantum Langevin equations. Here, we obtain the above set of dynamical equations for a cavity mode coupled to vacuum reservoir via a single-port mirror. The resulting equations are easily adaptable to the case when the external environment is either a thermal or a vacuum reservoir. We then focus our study when the cavity mode is coupled to a vacuum reservoir. A system coupled with a vacuum reservoir can be described by the Hamiltonian

$$\hat{H} = \hat{H}_S + \hat{H}_{SR}, \tag{2.5}$$

where $\hat{H}_S$ is the Hamiltonian of the system and $\hat{H}_{SR}$ describes the interaction between the system and the reservoir. Suppose $\hat{\chi}(t)$ is the density operator for the system and the reservoir. Then the equation of evolution of this density operator is given by

$$\frac{d}{dt}\hat{\chi}(t) = -i\big[\hat{H}_S(t) + \hat{H}_{SR}, \hat{\chi}(t)\big]. \tag{2.6}$$

We are interested in the quantum dynamics of the system alone. Hence taking into account (2.6), we see that the density operator for the system, also known as the the reduced density operator,

$$\hat{\rho}(t) = Tr_R\hat{\chi}(t) \tag{2.7}$$

evolves in time according to

$$\frac{d}{dt}\hat{\rho}(t) = -i\big[\hat{H}(t), \hat{\rho}(t)\big] - iTr\big[\hat{H}_{SR}(t), \hat{\chi}(t)\big], \tag{2.8}$$

in which Tr_R indicates the trace over the reservoirs variables only. On the other hand, a formal solution of Eq. (2.6) can be written as

$$\hat{\chi}(t) = \hat{\chi}(0) - i\int_0^t \big[\hat{H}_S(t') + \hat{H}_{SR}(t'), \hat{\chi}(t')\big]dt'. \tag{2.9}$$

In order to obtain mathematically manageable that $\hat{\chi}(t')$ by some approximately valid expression. Then, in the first place, we would arrange the reservoir in such a way that its density operator $\hat{R}$ remains constant in time. This can be achieved by letting a beam of thermal light (or light in a vacuum state) of constant intensity fall continuously on the system. Moreover, we decoupled the system and reservoirs density operators, so that

$$\hat{\chi}(t') = \hat{\rho}(t')\hat{R}. \tag{2.10}$$

Therefore, with the aid of this, one can rewrite Eq. (2.9) as

$$\hat{\chi}(t') = \hat{\rho}(t')\hat{R} - \int_0^t \big[\hat{H}_S(t') + \hat{H}_{SR}(t'), \hat{\rho}(t')\hat{R}\big]dt'. \tag{2.11}$$

Now on substituting (2.11) in to (2.8) there follows

$$\begin{aligned}\frac{d}{dt}\hat{\rho}(t) &= -i\big[\hat{H}_S(t), \hat{\rho}(t)\big] - i\big[\langle\hat{H}_{SR}(t)\rangle_R, \hat{\rho}(0)\big] \\ &\quad - \int_0^t \big[\hat{R}\hat{H}_{SR}(t), \big[\hat{H}_{SR}(t'), \hat{\rho}(t')\big]\big]dt' \\ &\quad - \int_0^t Tr_R\big[\hat{H}_{SR}(t'), \big[\hat{H}_{SR}(t'), \hat{\rho}(t')\hat{R}\big]\big]dt',\end{aligned} \tag{2.12}$$

where the subscript R indicates that the expectation value is to be calculated using the reservoirs density operator $\hat{R}$. Furthermore, the master equation for a system coupled to a reservoir takes the form

$$\begin{aligned}\frac{d\hat{\rho}(t)}{dt} &= -iTr_A[\hat{H}_S, \hat{\rho}_{AR}(t,t')] - h\langle \hat{H}_{SR}^2 \hat{R}\rangle_R \hat{\rho}(t) \\ &+2hTr_R(\hat{H}_{SR}\hat{\rho}(t)\hat{R}\hat{H}_{SR}) - h\hat{\rho}(t)\langle \hat{H}_{SR}^2 \hat{R}\rangle_R, \end{aligned} \tag{2.13}$$

A light mode confined in a cavity, usually formed by two mirrors, is called a cavity mode. A commonly used cavity has a single-port mirror. One side of each cavity is a mirror through which light can enter or leave the cavity. We now proceed to obtain the equation of evolution of the reduced density operator, in short the master equation, for the atoms coupled to a two-mode vacuum reservoir via a single port-mirror. We consider the reservoirs to be composed of large number of submodes. Thus, the interaction Hamiltonian for a cavity coupled to vacuum reservoir is written as

$$\hat{H}_{SR} = i\lambda(\hat{a}^\dagger \hat{a}_{in} - \hat{a}_{in}^\dagger \hat{a}), \tag{2.14}$$

where λ is the coupling constant, $\hat{a}_{in}$ and $\hat{b}_{in}$ are the annihilation operators of a single mode vacuum reservoir. By employing Eq. (2.14), we then see that

$$hTr_R(\hat{H}_{SR}^2 \hat{R}) = hTr_R\langle (i\lambda(\hat{a}^\dagger \hat{a}_{in} - \hat{a}_{in}^\dagger \hat{a})^2\rangle. \tag{2.15}$$

This can be rewritten as

$$\begin{aligned}hTr_R(\hat{H}_{SR}^2 \hat{R}) &= -h\lambda^2 Tr_R\big[(\hat{a}^\dagger \hat{a}_{in}\hat{a}^\dagger \hat{a}_{in})_R - (\hat{a}^\dagger \hat{a}_{in}\hat{a}_{in}^\dagger \hat{a})_R \\ &-(\hat{a}_{in}^\dagger \hat{a}\hat{a}^\dagger \hat{a}_{in})_R + (\hat{a}_{in}^\dagger \hat{a}\hat{a}_{in}^\dagger \hat{a})_R\big]. \end{aligned} \tag{2.16}$$

The atomic operators with operators of the reservoir are commute to each other.

Then we observe that

$$hTr_R(\hat{H}_{SR}^2\hat{R}) = -h\lambda^2\big[\hat{a}^{\dagger 2}\langle\hat{a}_{in}^2\rangle_R - \hat{a}^\dagger\hat{a}\langle\hat{a}_{in}\hat{a}_{in}^\dagger\rangle_R - \hat{a}\hat{a}^\dagger\langle\hat{a}_{in}^\dagger\hat{a}_{in}\rangle_R + \hat{a}^{\dagger 2}\langle\hat{a}_{in}^{\dagger 2}\rangle_R\big]. \tag{2.17}$$

Now using the density operator of a single-mode vacuum reservoir

$$\hat{R} = |0\rangle\langle 0|, \tag{2.18}$$

one can easily check that

$$\langle\hat{a}_{in}^2\rangle_R = Tr_R(|0\rangle\langle 0|\hat{a}_{in}^2). \tag{2.19}$$

It then follows that

$$\langle\hat{a}_{in}^2\rangle_R = 0, \tag{2.20}$$

where $\hat{a}_{in}|0\rangle = 0$. Following the same procedure, we obtain

$$\langle\hat{a}_{in}^2\rangle = \langle\hat{a}_{in}^{\dagger 2}\rangle = \langle\hat{a}_{in}^\dagger\hat{a}_{in}\rangle = 0. \tag{2.21}$$

In addition, applying the commutation relation $[\hat{a}_{in}, \hat{a}_{in}^\dagger] = 1$, we then note that

$$\langle\hat{a}_{in}\hat{a}_{in}^\dagger\rangle = 1, \tag{2.22}$$

Hence on account of Eqs. (2.21) and (2.22) into Eq. (2.17), there follows

$$hTr_R(\hat{H}_{SR}^2\hat{R})\hat{\rho}(t) = h\lambda^2[\hat{a}^\dagger\hat{a}\hat{\rho}]. \tag{2.23}$$

In the same manner, one can readily verify that

$$h\hat{\rho}(t)Tr_R(\hat{H}_{SR}^2\hat{R}) = h\lambda^2[\hat{\rho}\hat{a}^\dagger\hat{a}]. \tag{2.24}$$

In addition, one can readily find

$$2hTr_R[\hat{H}_{SR}\hat{\rho}(t)\hat{R}\hat{H}_{SR}] = -2h\lambda^2\big[\hat{a}^\dagger\hat{\rho}\hat{a}^\dagger\langle\hat{a}_{in}^2\rangle_R - \hat{a}^\dagger\hat{\rho}\hat{a}\langle\hat{a}_{in}^\dagger\hat{a}_{in}\rangle_R - \hat{a}\hat{\rho}\hat{a}^\dagger\langle\hat{a}_{in}\hat{a}_{in}^\dagger\rangle_R + \hat{a}\hat{\rho}\hat{a}\langle\hat{a}_{in}^{\dagger 2}\rangle_R\big], \tag{2.25}$$

so that applying Eqs. Eqs.(2.21) and (2.22) in Eq. (2.25) leads to

$$2hTr_R[\hat{H}_{SR}\hat{\rho}(t)\hat{R}\hat{H}_{SR}] = 2\lambda^2 h[\hat{a}\hat{\rho}\hat{a}^\dagger]. \quad (2.26)$$

Taking into account Eq. (2.23), (2.24), and (2.26) along with (2.11), we readily obtain the master equation for a cavity mode coupled to vacuum reservoir as in the form

$$\frac{d\hat{\rho}(t)}{dt} = -\imath[\hat{H}_s, \rho(t)] + \frac{\kappa}{2}\left[2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}\right], \quad (2.27)$$

where κ=$2\lambda^2 h$ is the cavity damping constant and assuming that the cavity damping constant is taken to be the same, i.e., $\kappa_a = \kappa_b = \kappa$.

Suppose $\hat{\rho}_{AR}(t, t')$ is the density operator for single atom plus the cavity mode at a time t, with the atom injected at time t_j such that

$$(t - \tau) \leq t_j \leq t. \quad (2.28)$$

The density operator for all atoms in the cavity mode at time t can be written as

$$\frac{d}{dt}\hat{\rho}(t) = r_a \sum_j \hat{\rho}_{AR}(t, t_j)\Delta t_j, \quad (2.29)$$

where $r_a\Delta t_j$ is the number of atoms injected in to the cavity at a time Δt_j. Integrating in the limit $\Delta t_j \rightarrow 0$ results in

$$\hat{\rho}_{AR}(t) = r_a \int_{t-\tau}^{t} \hat{\rho}_{AR}(t, t')dt'. \quad (2.30)$$

Differentiating with respect to time t, we get

$$\frac{d}{dt}\hat{\rho}_{AR}(t) = r_a(\hat{\rho}_{AR}(t, t) - \hat{\rho}_{AR}(t, t - \tau) + r_a \int_{t-\tau}^{t} \frac{d}{dt}\hat{\rho}_{AR}(t, t')dt'. \quad (2.31)$$

One can write

$$\hat{\rho}_{AR}(t, t) = \hat{\rho}_A(t)\hat{\rho}(t), \quad (2.32)$$

where $\hat{\rho}(t)$ being the density operator for the cavity mode alone and $\hat{\rho}_{AR}(t, t-\tau)$ represents the density operator for an atom plus the cavity mode at a time t, with the atom being removed from the cavity at this time

$$\hat{\rho}_{AR}(t, t-\tau) = \hat{\rho}_{AR}(t, t-\tau)\hat{\rho}_{AR}(t). \tag{2.33}$$

Using Eqs. (2.32) and (2.33), one can write Eq. (2.31) as

$$\frac{d}{dt}\hat{\rho}_{AR}(t) = r_a(\hat{\rho}_A(t) - \hat{\rho}_{AR}(t, t-\tau)\hat{\rho}(t)) + r_a \int_{t-\tau}^{t} \frac{\partial}{\partial t}\hat{\rho}_{AR}(t, t')dt'. \tag{2.34}$$

In the absence of damping of the cavity mode by a vacuum reservoir, the density operator $\hat{\rho}_{AR}(t, t')$ evolves in time according to

$$\frac{\partial}{\partial t}\hat{\rho}_{AR}(t, t') = -i\big[\hat{H}, \hat{\rho}_{AR}(t, t')\big], \tag{2.35}$$

so that using this and taking in to account (2.31), one can put Eq. (2.35) in the form

$$\frac{d}{dt}\hat{\rho}_{AR}(t) = r_a\big(\hat{\rho}_A(t) - \hat{\rho}_{AR}(t, t-\tau)\hat{\rho}(t)\big) - i\big[\hat{H}, \hat{\rho}_{AR}(t, t)\big]. \tag{2.36}$$

Furthermore, tracing over the atomic variables and taking in to account the damping of the cavity mode by a vacuum reservoir and using the fact that

$$Tr\hat{\rho}_A(t) = Tr\hat{\rho}_{AR}(t-\tau) = 1, \tag{2.37}$$

Now employing interaction Hamiltonian given by Eq. (2.1) for the cavity mode in to Eq. (2.27) can be expressible as

$$\begin{aligned}\frac{d}{dt}\hat{\rho} &= -iTr\big[\big(ig\hat{a}^{\dagger}(|b\rangle\langle a| + |c\rangle\langle b|) - \hat{a}(|a\rangle\langle b| + |b\rangle\,\langle c|)\big), \hat{\rho}_{AR}(t)\big] \\ &\quad + \frac{\kappa}{2}\big[2\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger}\hat{a}\big].\end{aligned} \tag{2.38}$$

After performing the trace operation and the cyclic property, we have

$$\begin{aligned}\frac{d}{dt}\hat{\rho} &= g\big[\langle a|\hat{\rho}_{AR}\hat{a}^{\dagger}|b\rangle + \langle b|\hat{\rho}_{AR}\hat{a}^{\dagger}|c\rangle - \langle b|\hat{\rho}_{AR}\hat{a}|a\rangle - \langle c|\hat{\rho}_{AR}\hat{a}|b\rangle \\ &\quad - \hat{a}^{\dagger}\langle a|\hat{\rho}_{AR}|b\rangle - \hat{a}^{\dagger}\langle b|\hat{\rho}_{AR}|c\rangle + \hat{a}\langle b|\hat{\rho}_{AR}|a\rangle + \hat{a}\langle c|\hat{\rho}_{AR}|b\rangle\big] \\ &\quad + \frac{\kappa}{2}\big[2\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger}\hat{a}\big],\end{aligned} \tag{2.39}$$

in which the matrix element

$$\hat{\rho}_{\alpha\beta} = \langle\alpha|\hat{\rho}_{AR}|\beta\rangle, \tag{2.40}$$

with α, $\beta = a, b, c$, so that Eq. (2.40) can be written as

$$\begin{aligned} \frac{d}{dt}\hat{\rho} &= g\big[\hat{\rho}_{ab}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{\rho}_{ab} + \hat{\rho}_{bc}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{\rho}_{bc} + \hat{a}\hat{\rho}_{ba} - \hat{\rho}_{ba}\hat{a} + \hat{a}\hat{\rho}_{cb} - \hat{\rho}_{cb}\hat{a}\big] \\ &+\frac{\kappa}{2}\big[2\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger}\hat{a}\big]. \end{aligned} \tag{2.41}$$

On the other hand, from Eq. (2.40) that

$$\begin{aligned} \frac{d}{dt}\hat{\rho}_{\alpha\beta} &= r_a\big[\langle\alpha|\hat{\rho}_A(0)|\beta\rangle - \langle\alpha|\hat{\rho}(t-\tau)|\beta\rangle\big]\hat{\rho}(t) \\ &-i\big[\langle\alpha|\hat{H}_{AR}|\beta\rangle - \langle\alpha|\hat{\rho}_{AR}\hat{H}|\beta\rangle\big] - \gamma\rho_{\alpha\beta}, \end{aligned} \tag{2.42}$$

where $\gamma\rho_{\alpha\beta}$ is included to account for the decay of the atom due to spontaneous emissions and γ is considered to be the same for all the three levels, is the atomic decay rate. We assume that the atoms are removed from the cavity after they have decayed to a level other than the middle or bottom level. We then see that

$$\langle\alpha|\hat{\rho}_A(t-\tau)|\beta\rangle = 0, \tag{2.43}$$

so that Eq. (2.42) reduces to

$$\frac{d}{dt}\hat{\rho}_{\alpha\beta} = r_a(\langle\alpha|\hat{\rho}_A(0)|\beta\rangle\hat{\rho}(t) - i(\langle\alpha|\hat{H}_{AR}|\beta\rangle - \langle\alpha|\hat{\rho}_{AR}\hat{H}|\beta\rangle) - \gamma\rho_{\alpha\beta}. \tag{2.44}$$

Employing Eqs. (2.1) and (2.3) into Eq. (2.44) we have the following expressions

$$\frac{d}{dt}\hat{\rho}_{ab} = g\big[\hat{\rho}_{ac}\hat{a}^{\dagger} + \hat{a}\hat{\rho}_{bb} - \hat{\rho}_{aa}\hat{a}\big] - \gamma\hat{\rho}_{ab}, \tag{2.45}$$

$$\frac{d}{dt}\hat{\rho}_{bc} = g\big[\hat{a}\hat{\rho}_{cc} - \hat{\rho}_{bb}\hat{a} - \hat{a}^{\dagger}\rho_{ac}\big] - \gamma\hat{\rho}_{bc}, \tag{2.46}$$

$$\frac{d}{dt}\hat{\rho}_{aa} = r_a\hat{\rho}_{aa}^{(0)}\hat{\rho} + g\big[\hat{\rho}_{ab}\hat{a}^{\dagger} + \hat{a}\hat{\rho}_{ba}\big] - \gamma\hat{\rho}_{aa}, \tag{2.47}$$

$$\frac{d}{dt}\hat{\rho}_{bb} = g\big[\hat{\rho}_{bc}\hat{a}^{\dagger} + \hat{a}\hat{\rho}_{cb} - \hat{a}^{\dagger}\hat{\rho}_{ab} - \hat{\rho}_{ba}\hat{a}\big] - \gamma\hat{\rho}_{bb}, \tag{2.48}$$

$$\frac{d}{dt}\hat{\rho}_{ac} = r_a\hat{\rho}_{ac}^{(0)}\hat{\rho} + g\big[\hat{a}\hat{\rho}_{bc} - \hat{\rho}_{ab}\hat{a}\big] - \gamma\hat{\rho}_{ac}, \tag{2.49}$$

$$\frac{d}{dt}\hat{\rho}_{cc} = r_a\hat{\rho}_{cc}^{(0)}\hat{\rho} - g\big[\hat{a}^{\dagger}\hat{\rho}_{bc} + \hat{\rho}_{cb}\hat{a}\big] - \gamma\hat{\rho}_{cc}. \tag{2.50}$$

By dropping the g terms in Eqs. (2.45)-(2.50) and imposing the condition that $\kappa \ll \gamma$ (the good-cavity limit) since the atomic variable reach steady state in the relatively short period of γ^{-1}, we can take the time derivative of such variable to be zero, keeping the zero order and cavity mode variables at time t. This is termed as adiabatic approximation scheme. Then we get

$$\hat{\rho}_{aa} = \frac{r_a \rho_{aa}^{(0)}}{\gamma} \hat{\rho}, \tag{2.51}$$

$$\hat{\rho}_{bb} = 0, \tag{2.52}$$

$$\hat{\rho}_{ac} = \frac{r_a \rho_{ac}^{(0)}}{\gamma} \hat{\rho}, \tag{2.53}$$

$$\hat{\rho}_{cc} = \frac{r_a \rho_{cc}^{(0)}}{\gamma} \hat{\rho}. \tag{2.54}$$

Moreover, substituting Eqs. (2.51)-(2.54) in to Eq. (2.45) yields

$$\frac{d}{dt}\hat{\rho}_{ab} = \frac{g r_a}{\gamma}\left[\rho_{ac}^{(0)} \hat{\rho}\hat{a}^\dagger - \rho_{aa}^{(0)} \hat{\rho}\hat{a}\right] - \gamma \hat{\rho}_{ab}. \tag{2.55}$$

Now on account of Eqs. (2.51)-(2.54) in to Eq. (2.46), we see that

$$\frac{d}{dt}\hat{\rho}_{bc} = \frac{g r_a}{\gamma}\left[\rho_{cc}^{(0)} \hat{a}\hat{\rho} - \rho_{ac}^{(0)} \hat{a}^\dagger \hat{\rho}\right] - \gamma \hat{\rho}_{bc}. \tag{2.56}$$

Using once more the adiabatic approximation, we easily find

$$\hat{\rho}_{ab} = \frac{g r_a}{\gamma}\left[\rho_{ac}^{(0)} \hat{\rho}\hat{a}^\dagger - \rho_{aa}^{(0)} \hat{\rho}\hat{a}\right], \tag{2.57}$$

$$\hat{\rho}_{bc} = \frac{g r_a}{\gamma}\left[\rho_{cc}^{(0)} \hat{a}\hat{\rho} - \rho_{ac}^{(0)} \hat{a}^\dagger \hat{\rho}\right], \tag{2.58}$$

On account of these results, the master equation for the cavity mode coupled with a vacuum reservoir given by Eq. (2.41) takes the form

$$\begin{aligned} \frac{d}{dt}\hat{\rho} &= \frac{1}{2}A\rho_{aa}^{(0)}\left[2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{\rho}\hat{a}\hat{a}^\dagger - \hat{a}\hat{a}^\dagger\hat{\rho}\right] \\ &+ \frac{1}{2}(A\rho_{cc}^{(0)} + \kappa)\left[2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{\rho}\hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}\hat{\rho}\right] \\ &+ \frac{1}{2}A\rho_{ac}^{(0)}\left[\hat{\rho}\hat{a}^{\dagger 2} - \hat{a}^{\dagger 2}\hat{\rho} - 2\hat{a}^\dagger\hat{\rho}\hat{a}^\dagger\right] \\ &+ \frac{1}{2}A\rho_{ca}^{(0)}\left[\hat{\rho}\hat{a}^2 + \hat{a}^2\hat{\rho} - 2\hat{a}\hat{\rho}\hat{a}\right], \end{aligned} \tag{2.59}$$

where

$$A=\frac{2r_a g^2}{\gamma^2} \tag{2.60}$$

is the linear gain coefficient, κ is assumed to be the cavity damping constant and γ is the spontaneous atomic decay rate. Moreover, with the pump mode treated classically, a degenerate parametric amplifier is describable in the interaction picture by the Hamiltonian

$$\hat{H}=\frac{1}{2}i\varepsilon(\hat{a}^{\dagger 2}-\hat{a}^2), \tag{2.61}$$

where ε is real and constant and proportional to the amplitude of the pump mode. The master equation associated with this Hamiltonian $\hat{H}$ can be derived using the Heisnberg equation.

$$\frac{d\hat{\rho}}{dt}=-i\Big[\hat{H},\hat{\rho}\Big]. \tag{2.62}$$

and on account of Eq. (2.61), we see that

$$\frac{d\hat{\rho}}{dt}=\frac{\varepsilon}{2}\big[\hat{\rho}\hat{a}^2-\hat{a}^2\hat{\rho}+\hat{a}^{\dagger 2}\hat{\rho}-\hat{\rho}\hat{a}^{\dagger 2}\big]. \tag{2.63}$$

Therefore, Eq. (2.63) represents the master equation for the pump mode treated classically for degenerate parametric amplifier in the interaction picture.

Now taking in to account Eqs. (2.59) and (2.63), the master equation for the cavity mode of a three-level laser containing parametric amplifier can be written as

$$\begin{aligned}\frac{d\hat{\rho}}{dt} &= \frac{\varepsilon}{2}\big[\hat{\rho}\hat{a}^2-\hat{a}^2\hat{\rho}+\hat{a}^{\dagger 2}\hat{\rho}-\hat{\rho}\hat{a}^{\dagger 2}\big]\\ &+\frac{1}{2}A\rho_{aa}^{(0)}\big[2\hat{a}^\dagger\hat{\rho}\hat{a}-\hat{\rho}\hat{a}\hat{a}^\dagger-\hat{a}\hat{a}^\dagger\hat{\rho}\big]\\ &+\frac{1}{2}(A\rho_{cc}^{(0)}+\kappa)\big[2\hat{a}\hat{\rho}\hat{a}^\dagger-\hat{\rho}\hat{a}^\dagger\hat{a}-\hat{a}^\dagger\hat{a}\hat{\rho}\big]\\ &+\frac{1}{2}A\rho_{ac}^{(0)}\big[\hat{\rho}\hat{a}^{\dagger 2}+\hat{a}^{\dagger 2}\hat{\rho}-2\hat{a}^\dagger\hat{\rho}\hat{a}^\dagger\big]\\ &+\frac{1}{2}A\rho_{ca}^{(0)}\big[\hat{\rho}\hat{a}^2+\hat{a}^{\dagger 2}\hat{\rho}-2\hat{a}\hat{\rho}\hat{a}\big].\end{aligned} \tag{2.64}$$

2.2 Stochastic Differential Equations

We next seek to obtain stochastic differential equations for the cavity mode variables. To this end applying Eq. (2.64), we are able to find the time evolution of the expectation value of the annihilation operator

$$\frac{d}{dt}\langle\hat{a}\rangle = Tr\bigg(\frac{d\rho}{dt}\hat{a}\bigg), \tag{2.65}$$

from which follows

$$\begin{aligned}\frac{d}{dt}\langle\hat{a}\rangle = & Tr\big[\frac{\varepsilon}{2}\big(\hat{\rho}\hat{a}^2 - \hat{a}^2\hat{\rho} + \hat{a}^{\dagger 2}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger 2}\big)\hat{a} \\ & + \frac{1}{2}A\rho_{aa}^{(0)}\big(2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{\rho}\hat{a}\hat{a}^\dagger - \hat{a}\hat{a}^\dagger\hat{\rho}\big)\hat{a} \\ & + \frac{1}{2}(A\rho_{cc}^{(0)} + k)\big(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{\rho}\hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}\hat{\rho}\big)\hat{a} \\ & + \frac{1}{2}A\rho_{ac}^{(0)}\big(\hat{\rho}\hat{a}^{\dagger 2} + \hat{a}^{\dagger 2}\hat{\rho} - 2\hat{a}^\dagger\hat{\rho}\hat{a}^\dagger\big)\hat{a} \\ & + \frac{1}{2}A\rho_{ca}^{(0)}\big(\hat{\rho}\hat{a}^2 + \hat{a}^{\dagger 2}\hat{\rho} - 2\hat{a}\hat{\rho}\hat{a}\big)\hat{a}\big].\end{aligned} \tag{2.66}$$

Applying this master equation along with cyclic property of trace operation, and the commutation relations of

$$[a, a^\dagger] = 1, \tag{2.67}$$

$$[a, a] = [a^\dagger, a^\dagger] = 0, \tag{2.68}$$

one can readily obtains

$$\frac{d}{dt}\langle\hat{a}\rangle = -\frac{1}{2}\mu\langle\hat{a}\rangle + \varepsilon\langle\hat{a}^\dagger\rangle. \tag{2.69}$$

Following similar procedures, we have

$$\frac{d}{dt}\langle\hat{a}^2\rangle = -\mu\langle\hat{a}^2\rangle + 2\varepsilon\langle\hat{a}^\dagger\hat{a}\rangle + \varepsilon + A\hat{\rho}_{ac}^{(0)}, \tag{2.70}$$

$$\frac{d}{dt}\langle\hat{a}^\dagger\hat{a}\rangle = -\mu\langle\hat{a}^\dagger\hat{a}\rangle + \varepsilon\langle\hat{a}^2\rangle + \varepsilon\langle\hat{a}^{\dagger 2}\rangle + A\hat{\rho}_{aa}^{(0)}, \tag{2.71}$$

in which

$$\mu = \frac{1}{2}A(\hat{\rho}_{cc}^{(0)} - \hat{\rho}_{aa}^{(0)}) + \kappa. \tag{2.72}$$

The corresponding c-number of Eqs. (2.69), (2.70), and (2.71) are given as

$$\frac{d}{dt}\langle\alpha(t)\rangle = -\frac{1}{2}\mu\langle\alpha(t)\rangle + \varepsilon\langle\alpha^*(t)\rangle, \tag{2.73}$$

$$\frac{d}{dt}\langle\alpha^2(t)\rangle = -\mu\langle\alpha^2(t)\rangle + 2\varepsilon\langle\alpha^*(t)\alpha(t)\rangle + A\rho_{ac}^{(0)}, \tag{2.74}$$

$$\frac{d}{dt}\langle\alpha^*(t)\alpha(t)\rangle = -\mu\langle\alpha^*(t)\alpha(t)\rangle + \varepsilon\langle\alpha^2(t)\rangle + \varepsilon\langle\alpha^{*2}(t)\rangle + A\rho_{aa}^{(0)}. \tag{2.75}$$

On the bases of Eq. (2.73), one can write

$$\frac{d}{dt}\alpha(t) = -\frac{1}{2}\mu\alpha(t) + \varepsilon\alpha^*(t) + f(t), \tag{2.76}$$

where $f(t)$ is a noise force, the properties of which remain to be determined. We see that Eq. (2.73) and the expectation value of Eq. (2.76), will have identical forms if

$$\langle f(t)\rangle = 0. \tag{2.77}$$

Now applying the relation

$$\frac{d}{dt}\langle\alpha^2(t)\rangle = 2\langle\alpha(t)\frac{d}{dt}\alpha(t)\rangle \tag{2.78}$$

along with Eq.(2.76), one can readily get

$$\frac{d}{dt}\langle\alpha^2(t)\rangle = -\mu\langle\alpha^2(t)\rangle + 2\varepsilon\langle\alpha^*(t)\alpha(t)\rangle + 2\langle\alpha(t)f(t)\rangle, \tag{2.79}$$

and using the partial differentiation of

$$\frac{d}{dt}\langle\alpha^*(t)\alpha(t)\rangle = \langle\alpha^*(t)\frac{d}{dt}\alpha(t)\rangle + \langle\alpha(t)\frac{d}{dt}\alpha^*(t)\rangle, \tag{2.80}$$

up on inserting Eq. (2.76) and its complex conjugate, in to Eq. (2.80) we see that

$$\frac{d}{dt}\langle\alpha^*(t)\alpha(t)\rangle = -\mu\langle\alpha^*(t)\alpha(t)\rangle + \varepsilon\langle\alpha^2(t)\rangle + \varepsilon\langle\alpha^{*2}(t)\rangle$$

$$+\langle\alpha(t)f^*(t)\rangle + \langle\alpha^*(t)f(t)\rangle. \quad (2.81)$$

We note that the c-number Eqs. (2.70) and (2.79) will have the same forms if

$$\langle\alpha(t)f(t)\rangle = \frac{1}{2}(\varepsilon + A\rho_{ac}^{(0)}), \quad (2.82)$$

and similarly in view of Eqs. (2.71) and (2.81), we have

$$\langle\alpha(t)f^*(t)\rangle + \langle\alpha^*(t)f(t)\rangle = A\rho_{ac}^{(0)}. \quad (2.83)$$

A formal solution of Eq. (2.76) can be written as

$$\alpha(t) = \alpha(0)e^{-\mu t/2} + \int_0^t e^{-\mu(t-t')/2}\Big[\varepsilon\alpha^*(t') + f(t')\Big]dt'. \quad (2.84)$$

We then see that

$$\langle\alpha(t)f(t)\rangle = \langle\alpha(0)f(t)\rangle e^{-\mu t/2} + \int_0^t e^{-\mu(t-t')/2}\varepsilon\Big(\langle\alpha^*(t')f(t)\rangle\Big)dt'$$

$$+\int_0^t e^{-\mu(t-t')/2}\langle f(t)f(t')\rangle dt'. \quad (2.85)$$

Assuming that the noise force f at a time t does not affect the cavity mode variables at earlier times

$$\langle\alpha^*(t')f(t)\rangle = 0 \quad (2.86)$$

and taking in to account Eq. (2.82), we have

$$\int_0^t e^{-\mu(t-t')}\langle f(t)f(t')\rangle dt' = \frac{1}{2}(\varepsilon + A\rho_{ac}^{(0)}). \quad (2.87)$$

One can then write on the bases of this result

$$\langle f(t)f(t')\rangle = (\varepsilon + A\rho_{ac}^{(0)})\delta(t-t'). \quad (2.88)$$

It can also be established in a similar manner that

$$\langle f^*(t)f(t')\rangle = A\rho_{aa}^{(0)}\delta(t-t'). \tag{2.89}$$

It is worth mentioning that Eqs. (2.88) and (2.89) describe the correlation properties of the noise force $f(t)$ associated with the normal ordering. Now introducing a new variable defined by

$$\alpha_\pm(t) = \alpha^*(t) \pm \alpha(t), \tag{2.90}$$

on account of Eq. (2.76) one can readily write

$$\frac{d}{dt}\alpha^*(t) = -\frac{1}{2}\mu\alpha^*(t) + \varepsilon\alpha(t) + f^*(t). \tag{2.91}$$

Differentiating Eq. (2.90), one can obtain

$$\frac{d\alpha_\pm}{dt}(t) = \frac{d}{dt}\alpha^*(t) \pm \frac{d}{dt}\alpha(t). \tag{2.92}$$

Upon substituting Eq. (2.76) and (2.91) in to Eq. (2.92), we can readily get

$$\frac{d\alpha_\pm}{dt} = -\frac{1}{2}\lambda_\mp\alpha_\pm + f^*(t) \pm f(t), \tag{2.93}$$

where

$$\lambda_\mp = \mu \mp 2\varepsilon. \tag{2.94}$$

The solution of Eq. (2.93) can be written as

$$\alpha_\pm(t) = \alpha_\pm(0)e^{-\lambda_\mp t/2} + \int_0^t e^{-\lambda_\mp(t-t')/2}\Big[f^*(t') \pm f(t')\Big]dt'. \tag{2.95}$$

It then follows that

$$\alpha_+(t) = \alpha_+(0)e^{-\lambda_- t'/2} + \int_0^t e^{-\lambda_- t/2}\Big[f^*(t') + f(t')\Big]dt' \tag{2.96}$$

and

$$\alpha_-(t) = \alpha_-(0)e^{-\lambda_+ t'/2} + \int_0^t e^{-\lambda_+ t/2}\Big[f^*(t') - f(t')\Big]dt'. \tag{2.97}$$

Combining with

$$\alpha_{\pm}(t') \quad = \quad \alpha^*(t') \pm \alpha(t'), \tag{2.98}$$

which then follows

$$\alpha(t) \quad = \quad A(t)\alpha(0) + B(t)\alpha^*(0) + F(t), \tag{2.99}$$

in which

$$A(t) \quad = \quad \frac{1}{2}(e^{-\lambda_- t/2} + e^{-\lambda_+ t/2}), \tag{2.100}$$

$$B(t) \quad = \quad \frac{1}{2}(e^{-\lambda_- t/2} - e^{-\lambda_+ t/2}), \tag{2.101}$$

and

$$F(t) \quad = \quad F_+(t) + F_-(t), \tag{2.102}$$

with

$$F_{\mp}(t) \quad = \quad \int_0^1 e^{-\lambda_{\mp}(t-t')/2}\left[f(t') \pm f^*(t')\right] dt'. \tag{2.103}$$

Upon substituting Eqs. (2.100), (2.101) and (2.103) in to Eq. (2.99) we readily see that

$$\alpha(t) = \frac{1}{2}\alpha(0)\left(e^{-\lambda_- t/2} + e^{-\lambda_+ t/2}\right) + \frac{1}{2}\alpha^*(0)\left(e^{-\lambda_- t/2} - e^{-\lambda_+ t/2}\right)$$
$$+\frac{1}{2}\int_0^1 e^{-\lambda_\mp (t-t')/2}\left[f(t') \pm f^*(t')\right]dt'. \quad (2.104)$$

Rewriting Eq. (2.104), we readily get

$$\alpha(t) = \frac{1}{2}\alpha(0)e^{-\lambda_\mp t/2} + \frac{1}{2}\alpha^*(0)e^{-\lambda_\mp t/2}$$
$$+\frac{1}{2}\left[\int_0^1 e^{-\lambda_\mp (t-t')/2} f(t') \pm f^*(t')\right]dt'. \quad (2.105)$$

3

THE QUADRATURE FLUCTUATIONS

In this chapter, we seek to calculate the quadrature variances of the cavity modes as well as the squeezing spectrum of the output mode produced by a degenerate three-level laser whose cavity contains a parametric amplifier driven by coherent light and coupled to a vacuum reservoir, using the solutions of the stochastic differential equations and the correlation properties of the noise forces.

3.1 Quadrature Variance

In quantum optics the annihilation and creation operators used in describing single mode radiation can be decomposed in to two quadrature operators. For single mode radiation in any state, the product of the fluctuations in the two quadratures satisfies the uncertainty principles. The squeezing properties of single mode light are described by two quadrature operators is defined by

$$\hat{a}_+ = \hat{a}^\dagger + \hat{a} \tag{3.1}$$

and

$$\hat{a}_- = i(\hat{a}^\dagger - \hat{a}), \tag{3.2}$$

where $\hat{a}_+$ and $\hat{a}_-$ are Hermitian operators representing physical quantities called plus and minus quadratures, respectively, while $\hat{a}^\dagger$ and $\hat{a}$ are the creation and annihilation operators for light mode a respectively. With the help of Eqs. (3.1) and (3.2), we can show that the two quadrature operators satisfy the commutation relation. It is possible to express in terms of c-number variables associated with the normal ordering as

$$\Delta\alpha_{\pm}^2 \;=\; 1 \pm \langle\alpha_{\pm}(t), \alpha_{\pm}(t)\rangle, \tag{3.3}$$

in which $\alpha_{\pm}(t)$ is given by Eq. (2.90). We consider here the case for which the cavity mode is initially in the vacuum state. Hence on account of Eq. (2.95) along with Eq. (2.77), we see that

$$\langle\alpha_{\pm}(t)\rangle \;=\; 0, \tag{3.4}$$

and expression (3.3) takes the form of

$$\triangle\alpha_{\pm}^2 \;=\; 1 \pm \langle\alpha_{\pm}^2(t)\rangle. \tag{3.5}$$

Furthermore, one easily gets with the aid of Eq. (2.93) that

$$\frac{d}{dt}\langle\alpha_{\pm}^2(t)\rangle \;=\; -\lambda_{\mp}\langle\alpha_{\pm}^2(t)\rangle + 2\langle\alpha_{\pm}(t)f^*(t)\rangle \pm 2\langle\alpha_{\pm}(t)f(t)\rangle. \tag{3.6}$$

On account of Eq. (2.90) along with Eqs. (2.82) and (2.83), we note that

$$\langle\alpha_{\pm}(t)f^*(t)\rangle \;=\; \frac{1}{2}[\varepsilon + A(\rho_{ca}^{(0)} \pm \rho_{aa}^{(0)})], \tag{3.7}$$

$$\langle\alpha_{\pm}(t)f(t)\rangle \;=\; \frac{1}{2}[A\rho_{aa}^{(0)} \pm (\varepsilon + \rho_{ac}^{(0)})]. \tag{3.8}$$

Therefore, in view of this result, Eq. (3.6) can be rewritten as

$$\frac{d}{dt}\langle\alpha_{\pm}^2(t)\rangle \;=\; -\lambda_{\mp}\langle\alpha_{\pm}^2(t)\rangle + 2\varepsilon + A(\rho_{ac}^{(0)} + \rho_{ca}^{(0)} \pm 2\rho_{aa}^{(0)}). \tag{3.9}$$

With the cavity mode initially in a vacuum state, the solution of this equation has the form

$$\langle\alpha_{\pm}^2(t)\rangle \;=\; \frac{2\varepsilon + A(\rho_{ac}^{(0)} + \rho_{ca}^{(0)} \pm 2\rho_{aa}^{(0)})}{\lambda_{\mp}}[1 - e^{-\lambda_{\mp}t}]. \tag{3.10}$$

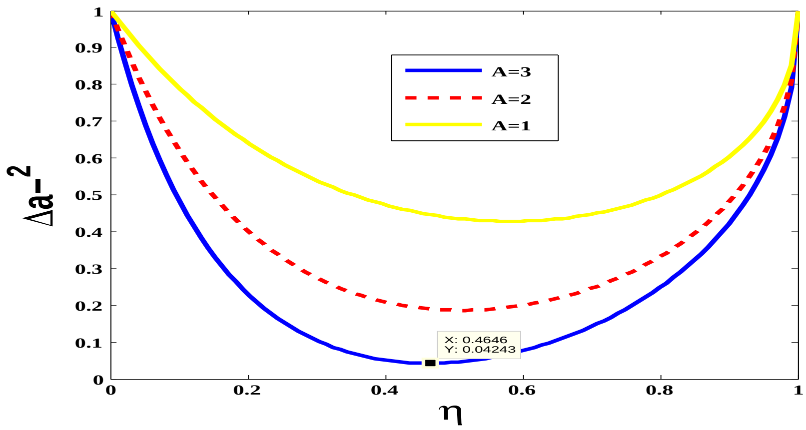

Figure 3.1: Plots of quadrature variance $(\Delta a_{-})^2$ Vs η [Eq. (3.22)] for $\kappa = 0.8$, $\theta = 0$,and $2\varepsilon = A\eta + \kappa$ and for different values of linear gain coefficient.

It proves to be more convenient to introduce a new parameter defined by

$$\rho_{aa}^{(0)} = \frac{1-\eta}{2}, \tag{3.11}$$

so that in view of the fact that

$$\rho_{aa}^{(0)} + \rho_{cc}^{(0)} = 1 \tag{3.12}$$

and

$$|\rho_{ac}^{(0)}|^2 = \rho_{aa}^{(0)}\rho_{cc}^{(0)}, \tag{3.13}$$

one easily finds that

$$\rho_{cc}^{(0)} = \frac{1+\eta}{2} \tag{3.14}$$

and

$$|\rho_{ac}^{(0)}| \;=\; \frac{1}{2}(1-\eta^2)^{1/2}. \tag{3.15}$$

Up on setting

$$\rho_{ac}^{(0)} \;=\; |\rho_{ac}^{(0)}|e^{i\theta} \tag{3.16}$$

and taking in to account of Eq. (2.94), along with Eq. (2.72), expression (3.10) can thus be put in the form

$$\langle\alpha_{\pm}^2(t)\rangle \;=\; \frac{2\varepsilon + A[(1-\eta^2)\cos\theta \pm (1-\eta)]}{A\eta+\kappa\mp 2\varepsilon}[1-e^{-(A\eta+\kappa\mp 2\varepsilon)t}]. \tag{3.17}$$

Now a combination of Eqs. (3.5) and (3.17) yields

$$\Delta\alpha_{\pm}^2(t) \;=\; 1 \pm \frac{2\varepsilon + A[(1-\eta^2)\cos\theta \pm (1-\eta)]}{A\eta+\kappa\mp 2\varepsilon}[1-e^{-(A\eta+\kappa\mp 2\varepsilon)t}], \tag{3.18}$$

so that at steady state

$$\Delta\alpha_{+}^2(t) \;=\; \frac{\kappa + A[(1+(1-\eta^2)^{1/2}\cos\theta]}{A\eta+\kappa-2\varepsilon}. \tag{3.19}$$

and

$$\Delta\alpha_{-}^2(t) \;=\; \frac{\kappa + A[(1-(1-\eta^2)^{1/2}\cos\theta]}{A\eta+\kappa+2\varepsilon}. \tag{3.20}$$

Fig3.1 is the quadrature variance of single mode light produced by degenerate three-level laser with the presence of parametric amplifier for different value of linear gain coefficient. This figure show that the degree of squeezing increases with the linear gain coefficient. It appears that almost perfect squeezing could be achieved by taking large value of linear gain coefficient(A) with maximum value A=3 for(solid line) and for small value η. More over, the minimum value of quadrature variance for $A=3$ and $\kappa=0.8$ at a steady state found 0.0424 which occurs at $\eta=0.4646$ with

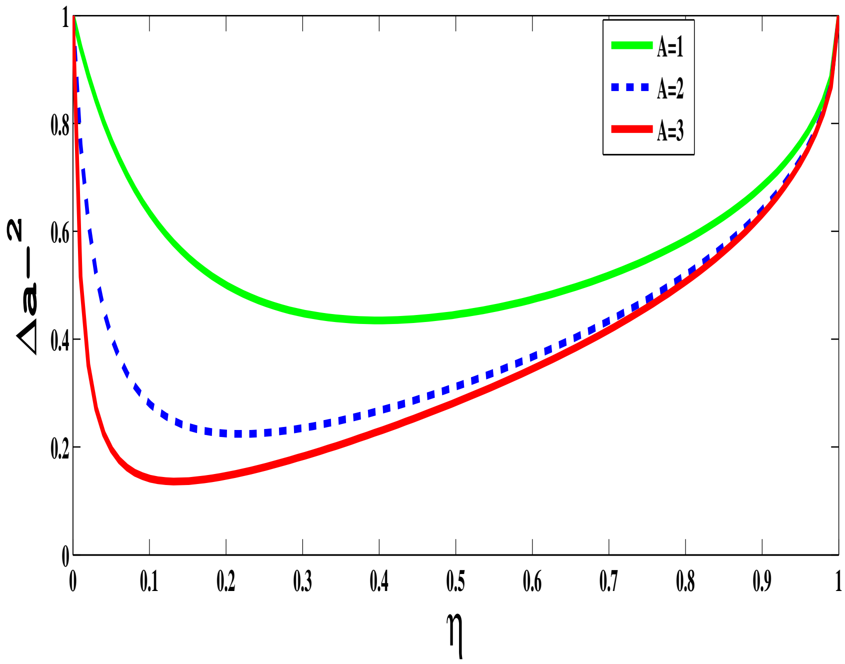

Figure 3.2: Plots of $(\Delta a_{-})^2$ for single mode [Eq. (3.23)] versus η of for $\kappa = 0.8$ and $\varepsilon = 0$ and for different values of linear gain coefficient.

maximum squeezing 95.8% below coherent state.

Fig3.2 is the quadrature variance of single mode light produced by degenerate three-level laser in the absence of parametric amplifier for different value of linear gain coefficient and $\kappa = 0.8$ at a steady state.

We have calculated the minimum value of quadrature variance is 0.1361 at $\eta = 0.1313$ with the maximum squeezing is 86% below the coherent state level for $A = 3$. In the same way for $A = 1$ the minimum value of quadrature variance at the same value of κ is found to be 0.4351 and the minimum squeezing found to be 56.49% below the coherent state level.

Since no well-behaved solution of Eq. (2.93) exists for (Aη + κ)<2ε, we interpret (Aη + κ) = 2ε as the threshold condition. Hence the solution of of this equation given by Eq. (2.95) is valid for 2ε<(Aη+κ). On the other hand, we note that from Eq. (2.62)

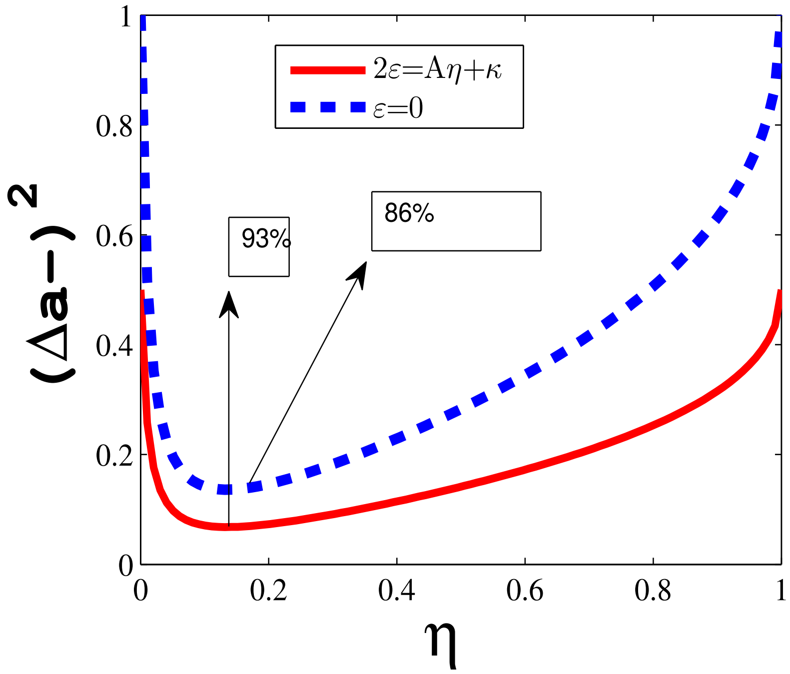

Figure 3.3: Plots of intra-cavity quadrature variance $(\Delta a_-)^2$ for single mode [Eq. (3.22) and (3.23)] versus η of for $\kappa = 0.8$, $A = 75$, and in the absence of the parametric amplifier with $\varepsilon = 0$, (dot-curve) and in the presence of the parametric amplifier with $2\varepsilon = A\eta + \kappa$, (solid- curve)

that ε is the only parameter representing the parametric amplifier. And inspection of Eq. (3.20) shows that the effect of this parameter is to decrease the value of the quadrature variance $\Delta\ a_-^2$. In addition, we see that expressions in Eqs. (3.19) and (3.20) take at threshold the form

$$\Delta\alpha_+^2 \to \infty \tag{3.21}$$

and

$$\Delta\alpha_-^2(t) = \frac{\kappa + A[(1-(1-\eta^2)^{1/2}\cos\theta]}{2(A\eta+\kappa)}. \tag{3.22}$$

Now upon setting $\varepsilon = 0$ in Eq. (3.20) it reduces to

$$\Delta\alpha_-^2(t) = \frac{\kappa + A[(1-(1-\eta^2)^{1/2}\cos\theta]}{(A\eta+\kappa)}. \tag{3.23}$$

Comparing the resulting expression (3.23) with Eq. (3.22) along with Fig.3.2, we observe that the effect of the parametric amplifier is to increase the intracavity squeezing by a maximum of 50%. The degree of squeezing increases with linear gain coefficients and it appears that almost perfect squeezing can be achieved for sufficiently large values of the linear gain coefficient.

Fig.3.3 indicates that the squeezing vanishes for η =0 and η =1 which corresponds to maximum injected atomic coherence, $\rho_{ac}^{(0)}$=1/2, and no injected atomic coherence, $\rho_{ac}^{(0)}$=0, respectively. However, as can be seen from the solid curve, the presence of the parametric amplifier leads to some degree of squeezing for η=0.

For at $\eta = 0.1313$ and $A = 75$ we have calculated the quadrature variance for $\kappa = 0.8$ at steady state to be 0.06811 with squeezing of 93% .

For the absence of parametric amplifier and the same value of A and κ the quadrature variance at steady state to be 0.1361 at $\eta = 0.1414$ with the squeezing of 86% below the coherent state level for(dot curve).

3.2 Squeezing Spectrum

The squeezing spectrum of a single-mode light is expressible in terms of c-number variables associated with the normal ordering as

$$S_{\pm}^{out}(\omega) = 1 + 2Re \int_0^{\infty} \langle \alpha_{\pm}^{out}(t), \alpha_{\pm}^{out}(t+\tau) \rangle_{ss} e^{i\omega\tau} d\tau, \tag{3.24}$$

where the subscript ”ss” stands for steady state and

$$\alpha_{\pm}^{out}(t) = \alpha_{out}^{*}(t) \pm \alpha_{out}(t). \tag{3.25}$$

We note that for a cavity mode coupled to a vacuum reservoir, the output and intra-cavity variables are related by

$$\alpha_{\pm}^{out}(t) \;=\; \sqrt{\kappa}\alpha_{\pm}(t). \tag{3.26}$$

In view of Eqs. (3.4) and (3.26), the squeezing spectrum can be put in the form

$$S_{\pm}^{out}(\omega) \;=\; 1+2\kappa Re\int_{0}^{\infty}\langle\alpha_{\pm}(t)\alpha_{\pm}(t+\tau)\rangle_{ss}e^{i\omega\tau}d\tau. \tag{3.27}$$

Furthermore, the solution of the expectation value of of Eq. (2.93)can be written as

$$\langle\alpha_{\pm}(t+\tau)\rangle \;=\; \langle\alpha_{\pm}(t)\rangle e^{-\lambda_{\mp}\tau/2}\rangle, \tag{3.28}$$

so that on account of the quantum regression theorem, have

$$\langle\alpha_{\pm}(t)\alpha_{\pm}(t+\tau)\rangle \;=\; \langle\alpha_{\pm}^{2}(t)\rangle e^{-\lambda_{\mp}\tau/2}. \tag{3.29}$$

Now with the aid of Eq. (3.29) together with Eq. (3.17), the squeezing spectrum is found to be

$$S_{\pm}^{out}(\omega) \;=\; 1\pm\frac{2\kappa\varepsilon+\kappa A[(1-\eta^{2})^{1/2}\cos\theta\pm(1-\eta)]}{\omega^{2}+[\frac{1}{2}(A\eta+\kappa\mp 2\varepsilon)]^{2}}, \tag{3.30}$$

It is easy to see that at threshold

$$S_{+}^{out}(\omega) \;=\; \frac{\omega^{2}+\kappa^{2}+\kappa A[(1+(1-\eta^{2})\cos\theta]}{\omega^{2}} \tag{3.31}$$

and

$$S_{-}^{out}(\omega) \;=\; \frac{\omega^{2}+A^{2}\eta^{2}+\kappa A[(1-(1-\eta^{2})\cos\theta]}{\omega^{2}+[A\eta+\kappa]^{2}}. \tag{3.32}$$

As shown in Fig.3.4, the squeezing spectrum of the out put cavity variable $\langle S_{-}^{out(0)}\rangle$ increases as the linear gain coefficients, A, increases.

For A=3, k=0.8, θ=0 and ω=0, the maximum squeezing spectrum gives 98.92%. For A=1 and the same values of k, θ and ω, the squeezing spectrum is 86.21% at maximum value of η=1.

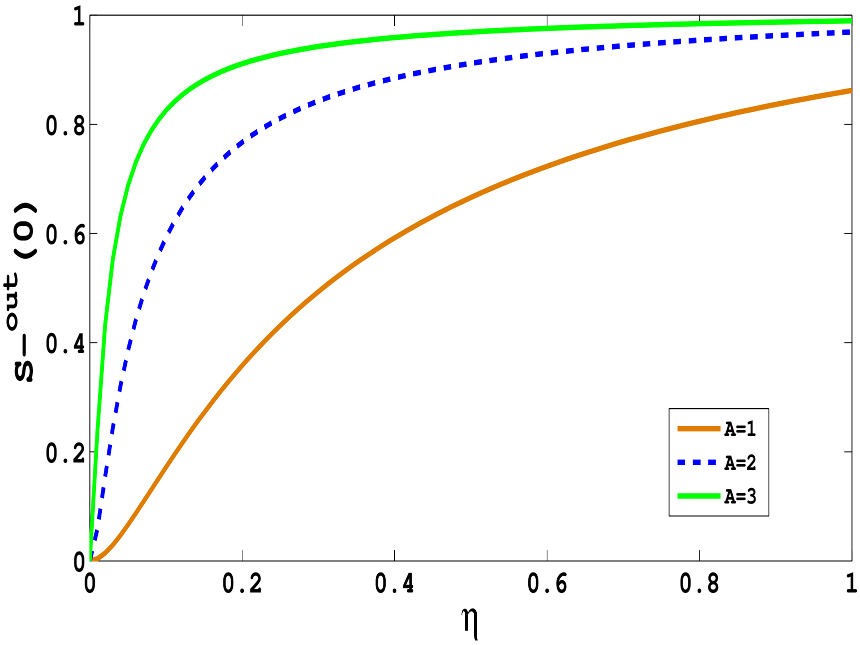

Figure 3.4: Plots of squeezing spectrum $(S_{-}^{out}(0)$ [Eq. (3.32)] versus η of for $\kappa = 0.8$, $\theta = 0.\omega = 0$ and for different values of linear gain coefficient.

4

PHOTON STATISTICS

In this chapter, with the aid of the antinormally-ordered characteristic function, we obtain the Q function. Using the Q function, we obtain the mean photon number, the photon number variance and the photon number distribution for the cavity mode.

4.1 The Q Function

The Q function is expressible in the form

$$Q(\alpha^*, \alpha, t) = \frac{1}{\pi^2}\int d^2z\phi(z^*, z, t)exp(z^*\alpha - z\alpha^*), \tag{4.1}$$

where the antinormally ordered characteristic function $\phi(z^*,z,t)$ is defined in the Heisenberg picture by

$$\phi(z^*, z, t) = Tr(\hat{\rho}(0)e^{z^*\hat{a}(t)}e^{-z^*\hat{a}^\dagger(t)}). \tag{4.2}$$

Applying the identity

$$e^Ae^B = e^Be^Ae^{[A,B]}, \tag{4.3}$$

the expression for the characteristic function can be written in terms of c-number variables associated with the normal ordering as

$$\phi(z^*, z, t) = e^{-z^*z}\langle\exp(z\alpha^* - z^*\alpha)\rangle, \tag{4.4}$$

so that employing Eq. (2.99) and assuming that $\alpha(0)$is independent of the noise force $F(t)$, we get

$$\phi(z^*,z,t) = e^{-z^*z}\langle\exp[(zA-z^*B)\alpha^*(0)+(zB-z^*A)\alpha(0)]\rangle$$

$$\langle\exp(zF^*-z^*F)\rangle. \tag{4.5}$$

Considering the cavity mode to be initially in a vacuum state, we see that

$$\langle\exp[(zA-z^*B)\alpha^*(0)+(zB-z^*A)\alpha(0)]\rangle = 1 \tag{4.6}$$

and hence

$$\phi(z^*,z,t) = e^{-z^*z}\langle\exp(-zF^*-z^*F)\rangle. \tag{4.7}$$

On account of the fact that F is Gaussian random variable, one can express Eq. (4.7) in the form [18]

$$\phi(z^*,z,t) = e^{-z^*z}\exp(\frac{1}{2}\langle[zF^*-z^*F]^2\rangle). \tag{4.8}$$

It then follows that

$$\phi(z^*,z,t) = e^{-z^*z}\exp(\frac{1}{2}\langle[z^2F^{*2}+z^{*2}F^2-2z^*zF^*F]\rangle). \tag{4.9}$$

Furthermore, from Eq. (2.102) and (2.103) one easily gets

$$\langle F^2\rangle = \langle F_+^2\rangle+\langle F_-^2\rangle+2\langle F_+F_-\rangle, \tag{4.10}$$

$$\langle F^*F\rangle = \langle F_+^2\rangle-\langle F_-^2\rangle. \tag{4.11}$$

Applying Eq. (2.103) along with Eqs. (2.880) and (2.89), it can be easily established that

$$\langle F_+^2\rangle = \frac{2\varepsilon+A(\rho_{ac}^{(0)}+\rho_{ca}^{(0)}+2\rho_{aa}^{(0)})}{4\lambda_-}[1-e^{-\lambda_-t}], \tag{4.12}$$

$$\langle F_-^2 \rangle = \frac{2\varepsilon + A(\rho_{ac}^{(0)} + \rho_{ca}^{(0)} - 2\rho_{aa}^{(0)})}{4\lambda_+}[1 - e^{-\lambda_+ t}], \tag{4.13}$$

$$\langle F_+ F_- \rangle = \frac{2\varepsilon + A(\rho_{ac}^{(0)} - \rho_{ca}^{(0)})}{4\lambda_+}[1 - e^{-\mu t}], \tag{4.14}$$

so that in view of these results, there follows

$$\begin{aligned}\langle F^2 \rangle &= \frac{2\varepsilon + A(\rho_{ac}^{(0)} + \rho_{ca}^{(0)} + 2\rho_{aa}^{(0)})}{4\lambda_-}[1 - e^{-\lambda_- t}] \\ &+ \frac{2\varepsilon + A(\rho_{ac}^{(0)} + \rho_{ac}^{(0)} - 2\rho_{aa}^{(0)})}{4\lambda_+}[1 - e^{-\lambda_+ t}] \\ &+ \frac{2\varepsilon + A(\rho_{ac}^{(0)} - \rho_{ca}^{(0)})}{4\lambda_+}[1 - e^{-\mu t}],\end{aligned} \tag{4.15}$$

$$\begin{aligned}\langle F^* F \rangle &= \frac{2\varepsilon + A(\rho_{ac}^{(0)} + \rho_{ca}^{(0)} + 2\rho_{aa}^{(0)})}{4\lambda_-}[1 - e^{-\lambda_- t}] \\ &- \frac{2\varepsilon + A(\rho_{ac}^{(0)} + \rho_{ac}^{(0)} - 2\rho_{aa}^{(0)})}{4\lambda_+}[1 - e^{-\lambda_+ t}].\end{aligned} \tag{4.16}$$

Now on account of Eqs. (4.15) and (4.16), the characteristic function (4.9) can be written as

$$\phi(z^*, z, t) = \exp[-az^*z + (bz^2 + b^*z^{*2})/2], \tag{4.17}$$

where the coefficients are expressible in terms of the parameter η as

$$\begin{aligned}a &= 1 + \frac{2\varepsilon + A[1 - \eta + (1 - \eta^2)^{1/2}\cos\theta]}{4(A\eta + \kappa - 2\varepsilon)}[1 - e^{-(A\eta + k - 2\varepsilon)t}] \\ &- \frac{2\varepsilon + A[\eta - 1 + (1 - \eta^2)^{1/2}\cos\theta]}{4(A\eta + \kappa + 2\varepsilon)}[1 - e^{-(A\eta + k + 2\varepsilon)t}]\end{aligned} \tag{4.18}$$

$$\begin{aligned}b = &\frac{2\varepsilon + A[1 - \eta + (1 - \eta^2)^{1/2}\cos\theta]}{4(A\eta + \kappa - 2\varepsilon)}[1 - e^{-(A\eta + k - 2\varepsilon)t}] \\ &+ \frac{2\varepsilon + A[\eta - 1 + (1 - \eta^2)^{1/2}\cos\theta]}{4(A\eta + \kappa + 2\varepsilon)}[1 - e^{-(A\eta + k + 2\varepsilon)t}]\end{aligned}$$

$$+\frac{iA(1-\eta^2)^{1/2}\sin\theta}{2(A\eta+\kappa)}[1-e^{-(A\eta+k)t}]. \quad (4.19)$$

Finally, introducing Eq. (4.17) in to Eq. (4.1) and carrying out the integration, the Q function for the cavity mode is found to be

$$Q(\alpha^*,\alpha,t) = \frac{[u^2-vv^*]^{1/2}}{\pi}\exp[-u\alpha^*\alpha+(u\alpha^2+v^*\alpha^{*2})/2], \quad (4.20)$$

in which

$$u = \frac{a}{a^2-bb^*}, \quad (4.21)$$

$$v = \frac{b}{a^2-bb^*}. \quad (4.22)$$

4.2 The Mean Photon Number

The mean photon number can be written employing the Q function (4.20) as

$$\langle\hat{a}^\dagger\hat{a}\rangle = -\frac{1}{\pi}[u^2-vv^*]^{1/2}\int d^2\alpha\exp[-u\alpha^*\alpha+(v^*\alpha^{*2}+v\alpha^2)]/2-1, \quad (4.23)$$

so that on performing the integration, there follows

$$\langle\hat{a}^\dagger\hat{a}\rangle = -[u^2-vv^*]^{1/2}\frac{d}{du}[\frac{1}{u^2-uu^*}]^{1/2}-1. \quad (4.24)$$

Therefore, carrying out differentiation and taking into account Eqs. (4.21) and (4.22) along with (4.18) and (4.19), one readily obtains

$$\langle\hat{a}^\dagger\hat{a}\rangle = \frac{2\varepsilon+A[1-\eta+(1-\eta^2)^{1/2}\cos\theta]}{4(A\eta+\kappa-2\varepsilon)}[1-e^{-(A\eta+k-2\varepsilon)t}]$$

$$-\frac{2\varepsilon+A[\eta-1+(1-\eta^2)^{1/2}\cos\theta]}{4(A\eta+\kappa+2\varepsilon)}[1-e^{-(A\eta+k+2\varepsilon)t}]. \quad (4.25)$$

At steady state, it is not hard to observe that the parametric amplifier contributes significantly to the mean photon number when the system is operating particularly

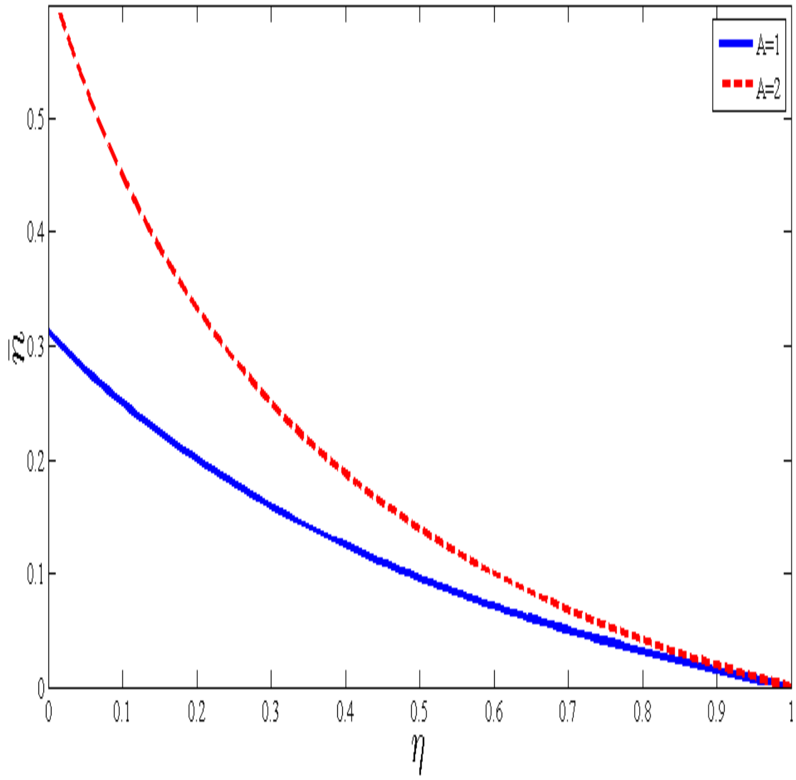

Figure 4.1: Plots of steady state of mean photon number $\bar{n}$ verses η [Eqs. (4.26)] for $\kappa = 0.8$,and different value of linear gain coefficient

near threshold. Up on direct use of steady state solutions of c-number Langevine equations,

$$\bar{n} = \frac{A(1-\eta)}{2(A\eta+\kappa)}, \tag{4.26}$$

which is identical with the expression obtained by Fesseha [8].

4.3 The variance of the photon number

In this section we determine the variance of photon number whose square root given us the uncertainty in photon number for the light beam,employing the Q-function .The variance of the photon number for light beam is expressed as [8]

$$(\triangle n)^2 = \langle \hat{n}^2 \rangle - \bar{n}^2 \tag{4.27}$$

since the mean photon number is found earlier,the only unknown is$\langle\hat{n}^2\rangle$ which can be determined using the Q-function of the laser light beam .Then we seek by using the c- number function crossponding to the operator in the antinormal- order.To do this, the number operators is expressed intermes of the annihilation and creation operators in the antinormal order using the commutation relation $\hat{a}^\dagger\hat{a} = \hat{a}\hat{a}^\dagger - 1$ as follow as

$$\begin{aligned}
\langle\hat{n}^2\rangle \quad &= \langle\hat{a}^\dagger\hat{a}\hat{a}^\dagger\hat{a}\rangle \\
&= \langle(\hat{a}\hat{a}^\dagger - 1)^2\rangle \\
&= \langle\hat{a}\hat{a}^\dagger\hat{a}\hat{a}^\dagger - 2\hat{a}\hat{a}^\dagger + 1\rangle \\
&= \langle\hat{a}^2\hat{a^\dagger}^2 - 3\hat{a}\hat{a}^\dagger + 1\rangle \\
&= \langle\hat{a}^2\hat{a^\dagger}^2 - 3\bar{n} - 2\rangle \\
&= \langle\hat{a}^2\hat{a^{\dagger\dagger}}^2\rangle - 3\bar{n} - 2
\end{aligned} \tag{4.28}$$

but

$$\langle\hat{a}^2\hat{a^\dagger}^2\rangle = \int d^2\alpha Q(\alpha,\alpha^*,t)\alpha^2\alpha^{*2} \tag{4.29}$$

Therefore, carrying out differentiation and taking into account Eqs. (4.1),(4.20), (4.21),(4.22)and (4.24)along with (4.34) one can readily obtains as

$$\langle\hat{a}^2\hat{a^\dagger}^2\rangle = \frac{2u^2+v^2}{(u^2-v^2)^{\frac{1}{2}}} \tag{4.30}$$

Applying,Eq.(4.18) ,(4.19),(4.21)and(4.22)along with in Eq.(4.35)we get the result of the variance of the photon number as

$$(\Delta n)^2 = \frac{2u^2+v^2}{(u^2-v^2)^2} - \bar{n}^2 - 3\bar{n} - 2 \tag{4.31}$$

From this we obtain the variance of the photon number as

$$(\Delta n)^2_{ss} = \frac{A(1-\eta)(A+A\eta+k)}{2(A\eta+K)^2} \tag{4.32}$$

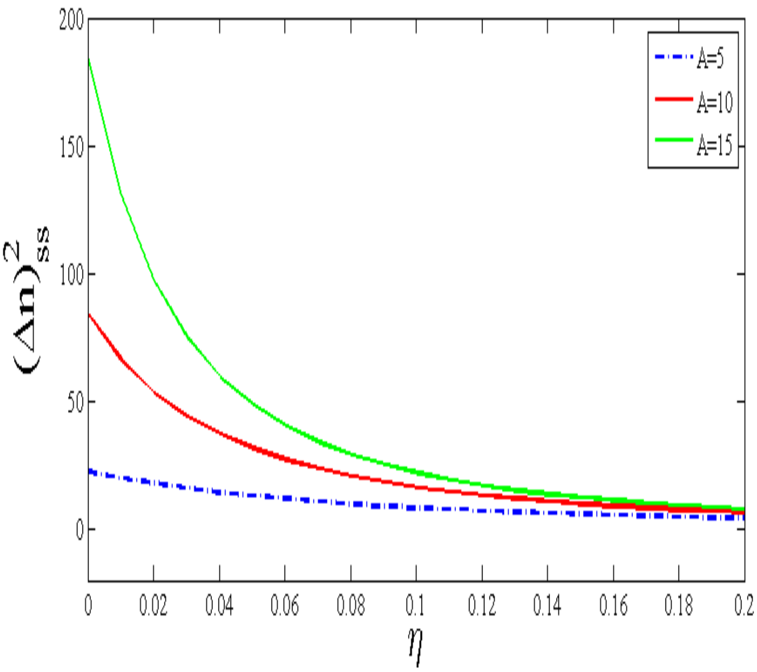

Figure 4.2: Plots of steady state of the variance of the photon number for $(\Delta n)^2_{ss}$ verses η [Eqs. (4.38)] for $\kappa = 0.8$, and different value of linear gain coefficient

Which can be expressed interms of$\bar{n}$ as

$$(\Delta n)^2_{ss} = \bar{n}(1 + \frac{A}{A\eta + k}) \tag{4.33}$$

From this we understand that the photon statistics is supper-poissonian for all values of η in the interval $0 \leq \eta \leq 1$.

4.4 Photon Number Distribution

Furthermore, the photon number distribution for a single mode light is expressible in terms of the Q function as [4,5]

$$p(n,t) = \frac{\pi}{n!}\frac{\partial^{2n}}{\partial\alpha^{*n}\partial\alpha^n}[Q(\alpha^*,\alpha,t)e^{\alpha^*\alpha}]_{\alpha=\alpha^*=0}. \tag{4.34}$$

Thus with the aid of Eqs. (4.20) and (4.27) the photon number distribution for the cavity mode can be written in the form

$$p(n,t) = \frac{1}{n!}[u^2 - vv^*]^{1/2}\frac{\partial^{2n}}{\partial\alpha^{*n}\partial\alpha^n}[\exp(1-u)\alpha^*\alpha + (v^*\alpha^{*2} + v\alpha^2)/2]. \tag{4.35}$$

Now expanding the exponential function in power series, we have

$$p(n,t) = \frac{1}{n!}[u^2 - vv^*]^{1/2} \sum_{klm} \frac{(1-u)^k v^{*l} v^m}{2^{l+m} k! l! m!} \frac{\partial^{2n}}{\partial \alpha^{*n} \partial \alpha^n}$$

$$\times [(\alpha^*)^{k+2l} \alpha^{k+2m}]_{\alpha^*=\alpha=0}, \tag{4.36}$$

so that on carrying out the differentiation and applying the condition α=$\alpha^* = 0$, there follows

$$p(n,t) = \frac{1}{n!}[u^2 - vv^*]^{1/2} \sum_{klm} \frac{(1-u)^k v^{*l} v^m (k+2l)!(k+2m)!}{2^{l+m} k! l! m! (k+2l-n)!(k+2m-n)!} \delta_{k+2l,n} \delta_{k+2m,n}. \tag{4.37}$$

Finally, on account of the result that $m = l$ and $k = n - 2l$, the photon number distribution can be written as

$$p(n,t) = \frac{1}{n!}[u^2 - vv^*]^{1/2} \sum_{l=0}^{[n]} n! \frac{(1-u)^{n-2l}(vv^*)^l}{2^{2l} l!^2 (n-2l)!}, \tag{4.38}$$

where $[n]$=n/2 for even n and $[n]$=$(n-1)$/2 for odd n. The probability of finding an even number of photons is greater than the probability of finding an odd number of photons; whether the light is produced by a three-level laser with or without a parametric amplifier. This is because the photons are always generated in pairs and the existence of some finite probability to find an odd number of photons is due to damping of the cavity mode. We also see that the probability of finding n photons, with n$\leq$4, is smaller for the light generated by the three-level laser with a parametric amplifier than for that produced without parametric amplifier, and the opposite of this holds n$\geq$5.

5

CONCLUSION

In this thesis, we have studied the squeezing and statistical properties of the light generated by degenerate three-level laser in which degenerate three-level atoms in a cascade configuration and initially prepared in a coherent superposition of the top and bottom levels are injected into a cavity coupled to vacuum reservoir via a single port-mirror. Applying the linear approximation scheme we found the master equation for a light produced by degenerate three-level laser from which we obtained the stochastic differential equations and the corresponding c-number langevine equations. Employing these solutions, we found antinormally ordered characteristic function which is used to find the Q function of the light beams. Using the Q function, we calculated the photon statistics of the light and it appears that the photon statistic is super-Poissonian while the photon number distribution decreases with photon number. We have calculated quadrature variance for $A = 3$ and $\kappa = 0.8$ at steady state to be 0.1361 with a squeezing of 86% which occurs at $\eta = 0.1313$ for the absence of parametric amplifier and the quadrature squeezing increases with the linear gain coefficient. We have shown that the effect of parametric amplifier is to increase the intracavity squeezing by maximum of 50%. Our study showed that the quantum optical system generates squeezed light and the degree of squeezing increases with the linear gain coefficient and the presence of parametric amplifier with maximum interacavity squeezing of 93% at$(\Delta a_{-})^{2} = 0.06811$ and $\eta = 0.1414$ bellow the coherent state level.

Applying the Q function derived for superposed light, we calculated the mean photon number, which becomes twice that of single light beam at steady state. This shows that the superposition of two light beams is super-Poissonian. The quadrature variance for superposed of two identical light beams decreases with linear gain coefficient having minimum value of the minus quadrature variance is 0.0424 for $A = 3$, $\kappa = 0.8$ and $\eta = 0.4646$. The result also indicate that the superposed light mode is in squeezed state with maximum squeezing of 95.8% bellow coherent state level for the same values of A, κ and η.

In this study ,we have observed that the parametric amplifier coupled with three-level laser increase the squeezing the mean photon number ,the photon number variance and photon number distribution of light which generated by it.

References

[1] G. J. Milburn and D. F. Walls, Opt. Commun. **39**, 401 (1981).

[2] L. A. Lugianto and G. Strini, Opt. Commun. **41**, 67 (1982).

[3] M. J. Collett and C. W. Gardiner, Phys. Rev. A **30**, 1386 (1984).

[4] J. Anwar and M. S. Zubairy, Phys. Rev. A **45**, 1804 (1992).

[5] B. Daniel and K. Fesseha, Opt. Commun. **151**, 384 (1998).

[6] M. O. Scully, k. Wodkiewicz, M. S. Zubairy, J. Bergou, N. Lu,and J. Meyer ter Vehn, Phys. Rev. Lett. **60**. 1832 (1988).

[7] N. A. Ansari, Phys. Rev. A **48**, 4686 (1993).

[8] Fesseha Kassahun, Fundamentals of Quantum Optics (Lulu, North Carolina, (2008).

[9] D. F. Walls, G.J Milburn, Quantum Optics (Springer,Berlin, 1995).

[10] M. O. Scully and M. S. Zubairy, Quantum Optics (Cambridge, Uk, 1997).

[11] P. Lambropoulos and D. Petrosyan, Fundamentals of Quantum Optics and Quantum Information (Springer-Verlag, Berlin Heidelberg, (2007).

[12] E. Alebachew and K. Fesseha, opt. commun. **265**, 314, (2006).

[13] Misrak Getahun, PhD Thesis, Addis Ababa University, (2009).

[14] Eyob Alebachew, Opt. Commun. **280**, 133 (2007).

[15] K. Fesseha, Phys. Rev. **63**, 033811 (2001).

[16] P. Meyste, M. Sargent III, elements of Quantum Optics, edn (Springer-Verlag, Berlin, 1991)

[17] Darge and Kassahun, Coherently driven degenerate three-level laser with

parametric amplifier. PMC Physics. B 2010 3:1

[18] L. Davidovich, Rev. Mod. Phys. **68**, 127 (1997).

[19] F. T. Arecchi, M. Asdente, and A. M. Ricca, Phys. Rev. A **14**, 383 (1976).

[20] C. Benkert and O.M. Scully, Phys. Rev. A **41**, 2756 (1990).

[21] E. Alebachew, Phys. Rev. A **76**, 023808 (2007).

[22] K. Fesseha, Opt. commun. **156**, 145 (1998).

[23] Fesseha Kassahun, e-print, arXiv:1105.1438v3.

[24] Fesseha Kassahun, *Refined Quantum Analysis of Light* (CreateSpace Independent Publishing Platform, 2014).

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