MAGNETOHYDRODAYNAMIC SLIP FLOW OF HEAT AND MASS TRANSFER OVER AN EXPONENTIALLY STRETCHING PERMEABLE SHEET EMBEDDED IN A POROUS MEDIUM WITH CHEMICAL REACTION

A THESIS SUBMITTED TO THE DEPARTMENT OF MATHEMATICS, JIMMA UNIVERSITY IN PARTIAL FULFILLMENT FOR THE REQUIREMENTS OF THE DEGREE OF MASTERS OF SCIENCE IN MATHEMATICS

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Declaration

I here submit the dissertation **magneto hydro dynamic slip flow of heat and mass transfer over an exponentially stretching permeable sheet embedded in a porous medium with chemical reaction**'' for the award of degree of Master of Science in Mathematics. I, the undersigned declare that, this study is the original and it has not been submitted to any institution elsewhere for the award of any academic degree achievement.

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Date: ____________________

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First of all, I would like to praise my God for his earnest love and grace up on me to live and work attentively and helped to become a man that I am today. Next, I would like to express my deepest appreciation to my advisor; Dr. Mitiku Daba and, Co-advisor; Mr. Habtamu Bayissa for his unreserved guidance, encouragement, and valuable suggestions and providing me related materials, advice and comment while I had prepared the thesis. I am grateful for their continual patience and constructive advice and amends to test myself and they are always in my memory.

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Abstract

In this thesis, the problem of the magneto hydrodynamic slips flow of heat and mass transfer over an exponentially stretching permeable sheet embedded in a porous medium with chemical reaction. Using a suitable similarity transformation, the governing partial differential equations are transformed in to system of none linear higher ordinary differential equations. The resulting equations are solved numerically using implicit finite difference scheme known as Keller boxmethod by implementing in math lab. The effects of flow of parameter like. Hartmann number M the permeability parameter K, Prandtl number P_r heat source Q and chemical reaction R_c are demonstrated graphically for velocity profile, temperature profile and concentration profile.

CHAPTER ONE

INTRODUCTION

1.1. Background of the study

The study of hydro magnetic electrically conducting fluid flow involving heat transfer over stretching porous sheet is of great importance in many processes as modern metallurgical and metalworking processes.

This field has attracted the attention of many researchers because of its possible applications in soil sciences, astrophysics, geophysics, nuclear power reactors etc. In cooling process of nuclear fission reactors, liquid sodium is pumped around using electromagnetic forces. In medical science, an advanced method is used for precisely delivery of medicine to cancer affected organs, in which MHD equations and finite element analysis are used to study the interaction between the magnetic fluid particles in the bloodstream and the external magnetic field. The study of fluid flow through porous medium has become predictable in the extraction of crude oil from the pores of rocks and filtration of solids from liquids. Fluid flow through porous medium also has applications in environment such as flow of ground water through soil and rocks, which is important for agriculture and pollution control. The suction/injection process has its importance in many engineering activities such as in the thermal oil recovery, designing of thrust bearing and radial diffusers. Suction is also applied to chemical processes to remove reactants. In heat pumping technology natural heat sources/sinks like air, ground, water etc. are used. This technology is used in compressors, refrigerators and air conditioners.

Heat transfer of a continuous stretching surface with suction or blowing was analyzed by(Chen and Char, 1988. Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface has been studied by (Magyari and Keller, 1999).(Elbashbeshy, 2001) considered heat transfer over an exponentially stretching continuous surface with suction. Slip flow past a stretching surface was investigated by (Andersson, 2002).(Miklavcic and Wang, 2006) analyzed viscous flow due to a shrinking sheet. Hydro magnetic flow and heat transfer adjacent to a stretching vertical sheet with prescribed surface heat flux was studied by (AmanandIshak, 2010). (Pal and Hiremath, 2010) considered computational modeling of heat transfer over an unsteady stretching surface embedded in a porous medium. Boundary layer flow and heat transfer over a stretching sheet with Newtonian heating was studied by(Salleh*et al.* 2010). (Sharma and Singh, 2010) investigated steady MHD natural convection flow with variable electrical conductivity and heat generation along an isothermal vertical plate.

The heat transfer over a stretching sheet of a hydro magnetic flow has been studied by (Chakrabarthi and Gupta, 1979). The MHD flow characteristic over a stretching sheet of a viscoelastic fluid was demonstrated by (Andersson, 1992). Later his work was extended by (Char, 1994) with mass transfer. In this paper, we have studied the effect of power index parameter, magnetic field, Prandtl number, thermal radiation. In addition to this, effect of some parameters on skin friction coefficient and surface heat transfer rate had also been investigated. The boundary layer equations governed by the partial differential equations was first transformed into a system of nonlinear ordinary differential equations using appropriate similarity transformation before being solved numerically using Keller box method (Cebeci and Bradshaw, 1984).

The governing partial differential equations representing the flow problem are reduced to nonlinear higher order ordinary differential equations by using similarity transformations. The transformed equations had been linearized and the linearized system of equations could be written in matrix form $A\delta = r$,where the elements of matrix A were block matrices of order 7×7 , δ_j and r_j are 7×1column matrices. The solution of $A\delta = r$ can be obtained using block elimination method which involves forward and backward sweeps (Chaltu et al., 2017). The calculations are repeated until convergence criterion is satisfied and calculations are stopped when $\left|\delta v_0^{(i)}\right| < \varepsilon$, where $\varepsilon = 10^{-6}$ is the desired level of accuracy.

1.2. Statement of the problem

The study had attempted to find answers for the following basic questions.

- How to study the effect of various parameters on the MHDslip flow of heat andmass transfer over an exponentially stretching sheet embedded in porous medium with chemical reaction How to apply similarity transformation to change the system of partial differential

  equations to ordinary differential equations?
- What are the parameters that affect velocity, temperature, skin friction coefficient and surface heat transfer rate?
- How to apply the Keller box method to the coupled nonlinear ordinary differential equation formed from the boundary layer equations?

1.3.Objective of the Study

1.3. 1. General objective

The general objective of this study was to analyze MHD slip flow of heat and mass transfer over an exponentially stretching sheet embedded in a porous medium with heat source/sink using Keller box method.

1.3.2. Specific objective

The study had the following specific objectives:

- ✓ To study the effect of various parameters on theHydromagnetic flow of fluid and heat and mass transfer in porous medium
- ✓ To identify the parameters that affect velocity, temperature, skin friction coefficient and surface heat transfer rate.
- ✓ To apply Keller box method to solve the coupled nonlinear ordinary differential equations formed from the boundary layer equation.

1.4. Significance of the Study

The outcomes of this study had the following importance.

- ❖ It haddeveloped the researcher knowledge on applied mathematics research.
- ❖ It might familiarize a researcher with scientific communication in applied mathematics.
- ❖ It will serve for other researchers as a useful reference for future research on this area.

1.5. Delimitation of the Study

The study is delimited to the governing partial differential equations steady flow only on the constructing Keller box method to investigate MHD slip flow of heat and mass transfer over an exponentially stretching sheet.

1.6. Definition of Important Terms

Boundary layer: is a fluid character that forms in the flow of fluid through a body of surface.
A steady Flow: Is a flow in which the various physical phenomena like velocity, pressure and density at any point do not change with time.
Stream Line: Is a path, in a steady flow field along which a given fluid particle travels.
Similarity transformations ;The transformations which reduce number of independent variables of system of partial differential equations at least one less than that of the original equations are designated similarity transformations(Hansen and Na.Y.T,1968).
Stream Function: Is a function ψ which satisfies continuity equation and defined as:

$$u = \frac{\partial\psi}{\partial y}, \quad v = \frac{-\partial\psi}{\partial x}$$

Hydro magnetic Flow: Fluid flow in the presence ofmagnetic field.
Magneto hydrodynamics: The study of the interaction between magnetic fields and electrically conducting fluids.

CHAPTER TWO

LITERATURE REVIEW

2.1. Magneto hydrodynamics (MHD)

Magnetohydrodynamics is the branch of continuum mechanics which deals with the motion of an electrically conducting fluid in the presence of a magnetic field. The word magneto hydrodynamic (MHD) is derived from: Magneto-meaning magnetic field, Hydro meaning Liquid and Dynamics which means movement. Other variants of nomenclatures are Hydromagnetic, magneto-fluid dynamics magneto-gas dynamics and so on. The concept of MHD is largely perceived to have been initiated by Faraday when he did the first quantitative observation of Magneto hydro dynamics. He did experiments with mercury as a conducting fluid flowing in a glass tube placed in magnetic field and observed that voltage was induced in direction perpendicular to both the direction of flow and magnetic field. He further showed that when an electric field is applied to a conducting fluid in the direction which is perpendicular to magnetic field, a force is exerted on the fluid in the direction perpendicular to both electric field and magnetic field. Since then a lot has been done on MHD and its related fields (Rao*etal*.,1990) studied the heat transfer in porous medium in the presence of transverse magnetic field. The effects of the heat source parameter and Nusselt number were analyzed. They discovered that the effect of increasing porous parameter is to increase the Nusselt Number.(Kinyanjui*et al* .,2003) investigated MHD Stokes problem for a vertical infinite plate in dissipative rotating fluid with Hall current as (Sigey*et al* .,2004) presented an investigation on the numerical study on natural convection turbulent heat transfer in an enclosure.

Hydromagnetic flow of Newtonian fluid and heat transfer over continuous moving flat surface with uniform suction has been studied by (Prasad*et al*., 2010). (Kumari*et al*., 1990) studied the effects of induced magnetic field and heat source/sink on flow and heat transfer characteristic over a stretching surface. (Nazar*et al*., 2004) investigated the boundary layer over a moving continuous flat plate in an electrically conducting ambient fluid with a step change in applied magnetic field.

2.2. Boundary Layer Flow of an Exponentially Stretching Sheet

MHD boundary layer flow due to an exponentially stretching sheet with radiation effect was presented by (Ishak, 2011). (Yao *et al.*,2011) studied heat transfer on a generalized stretching/shrinking wall with convective boundary condition. Heat transfer in a fluid through a porous medium over a permeable stretching surface with thermal radiation and variable thermal conductivity was analyzed by(Cortell, 2012).(Hayat, 2012) considered three-dimensional flow of a Jeffery fluid over a linearly stretching sheet. Hydromagnetic boundary layer flow over stretching surface with thermal radiation has been discussed by (Soid et al., 2012). (Mandal and Mukhopadhyay, 2013) presented, heat transfer analysis for fluid flow over an exponentially stretching porous sheet with surface heat flux in porous medium.

Slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation was shown by (Mukhopadhyay, 2013). (Norhafizah*etal.*,2013) studied numerical solution of flow and heat transfer over a stretching sheet with Newtonian heating using the Keller Box Method. (Singh and Makinde, 2015) presented a similarity solution for the combined effects of velocity slip and temperature jump on boundary layer flow over a moving surface. The MHD slip flow of a conducting Cassonnano fluid over a convectively heated stretching sheet was numerically studied by (Ibrahim and Makinde, 2016a). Other relevant papers with respect to MHD flow over a stretching sheet include (Ibrahim and Makinde, 2016b);(Khan *et al.* (2016).

CHAPTER THREE
METHODOLOGY

3.1. Study Design

The study had used mixed designs (i.e. documentary review and numerical simulation)

3.2. Study Site and Period

The study was conducted in Jimma University under the College of Natural sciences in Mathematics department from September 2018 to August 2020.

3.3. Sources of Information

Magazines, journals, different books, internet, article had assessed as a sources of information.

3.4. Mathematical Procedure of the Study

Mathematical procedure is the fundamental part the of the work in mathematical research. Hence to achieve the stated objectives the following mathematical procedures were followed

- Transforming the governed partial differential equations (PDE) to ordinary differential equations (ODE) by introducing similarity transformations.
- Reducing the given ordinary differential equations to a system of first order equations.
- Writing the reduced ordinary differential equations to finite differences.
- Linear zing the algebraic equations by using Newton's method and write them in vector matrix form.
- Solving the linear system by the block tri diagonal elimination technique.
- Finally sketches wereproduced using MATLAB.

CHAPTER FOUR

MATHEMATICAL FORMULATION, RESULT AND DISCUSSION

4.1. Mathematical formulation

Steady two dimensional laminar flow of a viscous incompressible and electrically conducting fluid past over a flat exponentially non conducting stretching porous sheet embedded in porous medium with non uniform permeability is consider the x-axis is taken in the direction along the stretching sheet and y-axis is taken normal it.

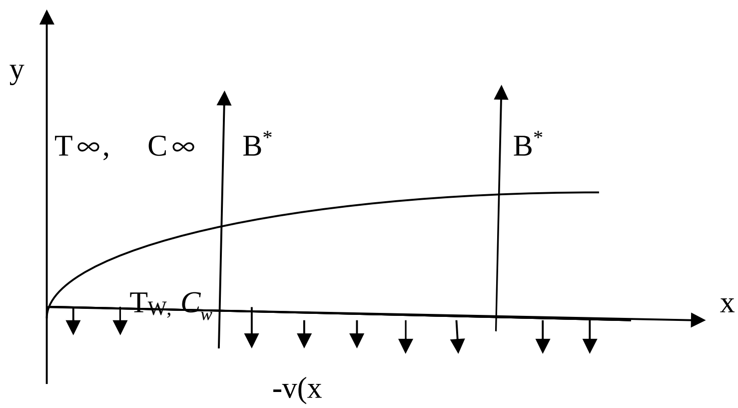

Figure 1: Physical model and coordinate system

The fluid flow confined to $y > 0$.The flow is generated by the action of two equal and opposite forces along the x-axis so that the wall is stretched keeping the origin fixed. The surface is assumedto be highly elastic and is stretched in the x-direction with the velocity $U = U_0 e^{\frac{x}{l}}$. Anon uniform magnetic field $B^* = B_0 e^{\frac{x}{2l}}$ is applied along the y-direction. The magnetic Reynolds is taken to be small and therefore the induced magnetic field is neglected. It is assumed that the temperature of the sheet T_w is variable and given by $T_w = T_\infty + T_0 e^{\frac{x}{2l}}$.

A non uniform heat source is also applied. All the fluid properties are assumed to be constant throughout the motion. Under these assumptions, the governing boundary layer equations (Bansal,1977; Bansal,1994;Schlichting and Gersten, 2003) are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \left(\frac{\sigma_e B^{*2}}{\rho} + \frac{\upsilon}{k^*}\right)u \tag{4.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q^*}{\rho c_p}(T - T_\infty) \tag{4.3}$$

$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2} - k_0(C - C_\infty) \tag{4.4}$$

where u and v are components of the velocity respectively along the x and y directions C is concentration of the fluid, T is temperature, T_∞ is temperature far from the sheet $K^* = K_0 e^{\frac{-x}{l}}$ is the non-uniform permeability of the medium and $Q^* = Q_0 e^{\frac{x}{l}}$ is the non-uniform heat source.

The appropriate boundary conditions are

$$\eta = 0,\ y = 0; u = U + L^* \frac{\partial u}{\partial y}, v = -v(x)$$

$$T = T_w + D^* \frac{\partial T}{\partial y}, C = C_w + D^* \frac{\partial C}{\partial y} \qquad (4.5)$$

$$\eta \to \infty \quad y \to \infty, u \to 0, T \to T_\infty\ C \to C_\infty$$

Where $v(x) = v_o e^{\frac{x}{2l}}$ is the section velocity at the sheet.

$L^* = l_o e^{\frac{-x}{2l}}$ *is the velocity slip factor and* $D^* = D_o e^{\frac{-x}{2l}}$

4.2. Similarity Transformations

In order to get the velocity and temperature distribution of the fluid over an exponential porous stretching sheet, the following dimensionless similarity variables(P.R.Sharma.et al, 2017).

$$\eta = y\sqrt{\frac{u_o}{2\nu l}} e^{\frac{x}{2l}} \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad \psi(x,y) = \sqrt{2\nu u_o l} e^{\frac{x}{2l}} f(\eta)\ ) \qquad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \qquad (4.6)$$

With stream functions defined by

$$u = \frac{\partial \psi}{\partial y} = u_o e^{\frac{x}{l}} f'(\eta)\ , \quad v = -\frac{\partial \psi}{\partial x} = -\sqrt{\frac{\nu u_o}{2l}} e^{\frac{x}{2l}} \left(\eta f'(\eta) + f(\eta)\right) \qquad (4.7)$$

Where f(η) is dimensionless stream functions $f'(\eta)$ is velocity profile, η is similarity variable and $\theta(\eta)$ is temperature profile also $\phi(\eta)$ is concentration profile using equation, (4.7) the continuity equation is satisfied as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\Rightarrow \frac{\partial}{\partial x}[u_o e^{\frac{x}{l}} f'(\eta)] + \frac{\partial}{\partial y}[-\sqrt{\frac{\nu u_o}{2l}} e^{\frac{x}{2l}} (\eta f'(\eta) + f(\eta))] = 0$$

$$\Rightarrow \frac{u_o}{2l} e^{\frac{x}{l}} (2f' + \eta f'') - \frac{u_o}{2l} e^{\frac{x}{l}} (2f' + \eta f'') = 0$$

Hence the continuity equation is satisfied

Again by using equation (4.6) and (4.7) equation (4.2) and (4.3) are transformed into nonlinear ordinary differential equations as follows.

$$Here: u = u_o e^{\frac{x}{l}} f'(\eta)$$

$$\Rightarrow \frac{\partial u}{\partial x} = \frac{\partial}{\partial x}[u_o e^{\frac{x}{l}} f'(\eta)] = \frac{u_o}{2l} e^{\frac{x}{l}} (2f' + \eta f'')$$

$$and \quad u\frac{\partial u}{\partial x} = [u_o e^{\frac{x}{l}} f'(\eta)][\frac{u_o}{2l} e^{\frac{x}{l}} (2f' + \eta f'')]$$

$$= \frac{(u_o e^{\frac{x}{l}})^2}{2l} (2f' + \eta f'') f'$$

$$= \frac{(u_o e^{\frac{x}{l}})^2}{2l} (2f'^2 + \eta f' f'') = \frac{u_o^2 e^{\frac{2x}{l}}}{2l} (2f'^2 + \eta f' f'') \qquad 4.8)$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}[u_o e^{\frac{x}{l}} f'(\eta)] = u_o e^{\frac{x}{l}} f'' \sqrt{\frac{u_o}{2\nu l}} e^{\frac{x}{2l}} \quad and$$

$$v\frac{\partial u}{\partial y} = [-\sqrt{\frac{\nu u_o}{2l}} e^{\frac{x}{2l}} (\eta f'(\eta) + f(\eta))\ ][u_o e^{\frac{x}{l}} f'' \sqrt{\frac{u_o}{2\nu l}} e^{\frac{x}{2l}}]$$

$$= -\frac{(u_o e^{\frac{x}{l}})^2}{2l} f''(\eta f' + f) \qquad (4.9)$$

Adding equation (4.8) and (4.9).we get

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial_y} = \frac{(u_o e^{\frac{x}{l}})^2}{2l} [(2f'^2 - ff'')\ ] \qquad (4.10)$$

$$again \ \ since \ we \ have \ \frac{\partial u}{\partial y} = u_o e^{\frac{x}{l}} f'' \sqrt{\frac{u_o}{2\nu l}} e^{\frac{x}{2l}}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{(u_o e^{\frac{x}{l}})^2}{2\nu l} f''' \ \ and$$

$$\nu\frac{\partial^2 u}{\partial y^2} = \nu\frac{(u_o e^{\frac{x}{l}})^2}{2\nu l} f'''$$

$$= \frac{(u_o e^{\frac{x}{l}})^2}{2l} f''' \qquad (4.11)$$

Substituting equation (4.10) and (4.11) into (4.2) we get

$$\frac{(u_o e^{\frac{x}{l}})^2}{2l} [(2f'^2 - ff'')\ ] = \frac{(u_o e^{\frac{x}{l}})^2}{2l} f''' - (\frac{\sigma_e \beta^{*2}}{\rho} + \frac{\nu}{k*})u$$

$$= \frac{(u_o e^{\frac{x}{l}})^2}{2l} f''' - (\frac{\sigma_e \beta_o^2 e^{\frac{x}{l}}}{\rho} + \frac{\nu}{k_o} e^{\frac{x}{l}}) u_o e^{\frac{x}{l}} f'(\eta)$$

$$(2f'^2 - ff'') = f''' - (\frac{2l\sigma_e B_o^2}{u_o\rho} + \frac{2\nu l}{u_o k_o})f'$$

$$M = \frac{2l\sigma_e B_o^2}{u_o\rho} \qquad \frac{1}{K} = \frac{2\nu l}{u_o k_o}$$

$$f''' - 2f'^2 + ff'' - (M + \frac{1}{k})f' = 0$$

Using the same procedure, we can transform equation (4.3). Here from equation (4.6) we have

$$\theta(\eta) = \frac{T - T_\infty}{Tw - T_\infty}$$

$$\Rightarrow T = [(Tw - T_\infty)\theta(\eta) + T_\infty]$$

$$\Rightarrow \frac{\partial T}{\partial x} = \theta'(\eta).\frac{\partial}{\partial x}(\eta)(Tw - T_\infty) = \frac{\theta'\eta(Tw - T_\infty)}{2l}$$

$$and \quad u\frac{\partial T}{\partial x} = [u_o e^{\frac{x}{l}} f'(\eta)][\frac{\theta'\eta(Tw - T_\infty)}{2l}]$$

$$= \frac{u_o e^{\frac{x}{l}} f'\theta'\eta(Tw - T_\infty)}{2l} \qquad (4.12)$$

$$again\,\frac{\partial T}{\partial y} = [(Tw - T_\infty)\theta'\frac{\partial}{\partial y}(\eta)] = \theta'(Tw - T_\infty)\sqrt{\frac{u_o}{2\nu l}}e^{\frac{x}{2l}}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{\partial}{\partial y}(\frac{\partial T}{\partial y}) = \frac{\partial}{\partial y}\left(\theta'(Tw - T_\infty)\sqrt{\frac{u_o}{2\nu l}}e^{\frac{x}{2l}}\right)$$

$$= \theta''(Tw - T_\infty)\frac{u_o}{2\nu l}e^{\frac{x}{l}} \qquad (4.13)$$

$$v\frac{\partial T}{\partial y}=\left(-\sqrt{\frac{\nu u_o}{2l}}e^{\frac{x}{2l}}(\eta f'(\eta)+f(\eta)\right)\left(\theta'(Tw-T_\infty)\sqrt{\frac{u_o}{2\nu l}}e^{\frac{x}{2l}}\right) \qquad (4.14)$$

$$=-\frac{u_o e^{\frac{x}{l}}(Tw-T_\infty)}{2l}\left(\eta f'\theta'+\theta' f\right)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=-\frac{u_o e^{\frac{x}{l}}(Tw-T_\infty)}{2l}\left(\theta' f\right) \qquad (4.15)$$

substituting equations (4.13) *and* (4.15) *into equation* (4.3) *we get*

$$-\frac{u_o e^{\frac{x}{l}}(Tw-T_\infty)}{2l}\left(\theta' f\right)=\frac{k}{\rho_{cp}}\left(\theta''(Tw-T_\infty)\frac{u_o}{2\nu l}e^{\frac{x}{l}}\right)+\frac{Q^*}{\rho_{cp}}(T-T_\infty)$$

$$-\frac{u_o e^{\frac{x}{l}}(Tw-T_\infty)}{2l}\left(\theta' f\right)=\frac{k}{\rho_{cp}}\left(\theta''(Tw-T_\infty)\frac{u_o}{2\nu l}e^{\frac{x}{l}}\right)+\frac{Q_o e^{\frac{x}{l}}}{\rho_{cp}}\left(\theta(T-T_\infty)\right)$$

$$\Rightarrow \theta''+\frac{\rho_{cp}}{k}\theta' f+\frac{2Q_o\nu l}{ku_o}\theta=0$$

$$\Rightarrow \theta''+\Pr\theta' f+2\Pr Q\theta=0$$

Next for equation (4)

$$\frac{\partial C}{\partial x}=\frac{\partial}{\partial x}\left[C_\infty+\phi(\eta)(C_w-C_\infty)\right]$$

$$=\phi'(\eta)(C_w-C_\infty)\eta'$$

$$=\phi'(\eta)(C_w-C_\infty)\eta\frac{1}{2l}$$

$$u\frac{\partial C}{\partial x}=[u_o e^{\frac{x}{l}}f'(\eta)]\phi'(\eta)(C_w-C_\infty)\eta\frac{1}{2l}$$

$$=[u_o e^{\frac{x}{l}}f'(\eta)]\phi'(\eta)(C_w-C_\infty)\eta\frac{1}{2l} \qquad (4.16)$$

$$\frac{\partial C}{\partial y}=\frac{\partial}{\partial y}(C_\infty+\phi(C_w-C_\infty)$$

$$=\phi'(C_w-C_\infty)\eta'$$

$$\frac{\partial C}{\partial y}=\phi'(C_w-C_\infty)e^{\frac{x}{2l}}\sqrt{\frac{u_o}{2\nu l}}$$

$$v\frac{\partial C}{\partial y}=-(\sqrt{\frac{\nu u_o}{2l}}e^{\frac{x}{2l}})(\eta f'+f)\ \phi'(C_w-C_\infty)e^{\frac{x}{2l}}\sqrt{\frac{u_o}{2\nu l}}$$

$$=-\frac{u_o}{2l}e^{\frac{x}{l}}\ \phi'(C_w-C_\infty)\quad(\eta f'+f) \qquad (4.17)$$

adding equation (16) *and* (17) *we get*

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=u_o e^{\frac{x}{l}}f'(\phi'(c_w-c_\infty)\eta(\frac{1}{2l}))+(-\frac{u_o}{2l}e^{\frac{x}{l}}\phi'(c_w-c_\infty)(\eta f'+f)$$

$$-\frac{u_o}{2l}e^{\frac{x}{l}}\phi'(c_w-c_\infty)f(c_w-c_\infty)$$

$$\frac{\partial^2 C}{\partial y^2} = \frac{\partial}{\partial y}(\frac{\partial}{\partial y}) = \frac{\partial}{\partial y}\left(\phi^{'}(C_w - C_\infty)\right)\sqrt{\frac{u_o}{2\nu l}}e^{\frac{x}{2l}}$$

$$= \phi^{''}(C_w - C_\infty)\sqrt{\frac{u_o}{2\nu l}}e^{\frac{x}{2l}}(\sqrt{\frac{u_o}{2\nu l}}e^{\frac{x}{2l}})$$

$$= \frac{u_o e^{\frac{x}{l}}}{2\nu l} \left(\phi^{''}(C_w - C_\infty)\right) \quad (4.18)$$

$$\sin ce\ we\ have\ \frac{\partial C}{\partial y} = \left(\phi^{'}(C_w - C_\infty)\right)\sqrt{\frac{u_o}{2\nu l}}e^{\frac{x}{2l}}$$

$$\Rightarrow \frac{\partial^2 C}{\partial y^2} = \frac{u_o e^{\frac{x}{l}}}{2\nu l} \left(\phi^{''}(C_w - C_\infty)\right)$$

$$= \frac{u_o e^{\frac{x}{l}}}{2\nu l} \left(\phi^{''}(C_w - C_\infty)\right) \quad (4.19)$$

equating equations (4.18) *and* (4.19) *in equation* (4.4) *be camse*

$$\frac{u_o e^{\frac{x}{l}}}{2l}\phi^{'}(C_w - C_\infty)(-f) = D\frac{u_o}{2\nu l}e^{\frac{x}{l}}\phi''(C_w - C_\infty) - k_o(C - C_\infty)$$

$$\phi^{'}(C_w - C_\infty)(-f) = \frac{D}{\nu}\phi''(C_w - C_\infty) - k_o(C - C_\infty)\frac{2l}{u_o e^{\frac{x}{l}}}$$

$$\Rightarrow \phi'' + Sc(\phi' f - R_c \phi) = 0$$

Hence the transformed nonlinear ordinary differential equation are then:

$$f''' - 2f'^2 + ff'' - (M + \frac{1}{k})f' = 0 \quad (4.20)$$

$$\theta'' + \Pr\theta' f + 2\Pr Q\theta = 0 \quad (4.21)$$

$$\phi'' + Sc(\phi' f - R_c \phi) = 0 \quad (4.22)$$

Where: primes denoted differentiation with respect to η. The Hartmann number M, thepermeability Parameter k, the Prandtl number P_r ,the heat source parameter Q, R_c is chemical reaction.

$$M = \frac{2\sigma B_o^2 l}{\rho u_o}, \quad k = \frac{u_o k_o}{2\nu l}, \quad \Pr = \frac{\mu Cp}{k}, \quad Q = \frac{2Q_o l}{\rho Cp u_o}, \quad R_c = \frac{k_o 2l}{u_o e^{\frac{x}{l}}} \quad Sc = \frac{\nu}{D}$$

Subject to the boundary condition in dim *etionless form an redue sed to ,*

$$v = -v(x)\ ,\ \sqrt{\frac{\nu u_o}{2l}}e^{\frac{x}{2l}}(\eta f' + f) = -(-v_o e^{\frac{x}{2l}}) \Rightarrow \sqrt{\frac{\nu u_o}{2l}}(f) = v_o \Rightarrow f = v\sqrt{\frac{2l}{\nu u_o}} = s$$

$$u = U + L^* \frac{\partial u}{\partial y} \Rightarrow u_o e^{\frac{x}{l}} f' = u_o e^{\frac{x}{l}} + l_o e^{\frac{-x}{2l}} \frac{\partial}{\partial y}(u_o e^{\frac{x}{l}} f') \Rightarrow u_o e^{\frac{x}{l}} f' + l_o e^{\frac{-x}{2l}} (u_o e^{\frac{x}{l}} f'')\sqrt{\frac{u_o}{2\nu l}}$$

$$f' = 1 + l_o \sqrt{\frac{u_o}{2\nu l}} f''(0) \Rightarrow f'(0) = 1 + s_v f''(0)$$

$$\eta = 0,\ f(0) = s,\ f'(0) = 1 + S_v f''(0), \theta(0) = 1 + s_T \theta'(0)\ \ \phi(0) = s_m \phi'(0)$$

$$\eta \to \infty,\ f'(\infty) \to 0\ \ \theta(\infty) \to 0, \phi(\infty) \to 0 \qquad (4.23)$$

$$s = v_o \sqrt{\frac{2l}{\nu u_o}},\quad s_v = l_o \sqrt{\frac{u_o}{2\nu l}} \quad s_T = D_o \sqrt{\frac{u_o}{2\nu l}}$$ *where s isthesuction parameter*, s_v *isthe velocity slip parameter and* s_T *isthermal slip parameter.*

Coefficient of skin friction, Nassult number and Sherworld number.

The rate of shear stress in terms of coefficient of skin friction at the sheet is given by:

skin friction

$$C_f = \frac{-2\tau_w}{\rho u_0^2}, \tau_w = \mu(\frac{\partial u}{\partial y})_{y=0} = \mu \frac{\partial}{\partial y}(u_o e^{\frac{x}{l}} f')_{y=0} = \mu(u_o e^{\frac{x}{l}} f''(0))\sqrt{\frac{u_o}{2\nu l}} e^{\frac{x}{2l}}$$

$$now \quad \tau_w = \mu(u_0 f''(0))\sqrt{\frac{u_o}{2\nu l}} e^{\frac{3x}{2l}}$$

$$C_f = \frac{-2}{\rho u_0^2}(\mu(u_0 f''(0))\sqrt{\frac{u_o}{2\nu l}} e^{\frac{3x}{2l}})$$

$$\Rightarrow \frac{-2\nu}{u_0}((f''(0))\sqrt{\frac{u_o}{2\nu l}} e^{\frac{3x}{2l}} \Rightarrow -2\nu(f''(0))\sqrt{\frac{u_o}{2\nu l}} e^{\frac{3x}{2l}} \Rightarrow -(f''(0))\sqrt{\frac{4\nu^2}{u_0 2\nu l}} e^{\frac{3x}{2l}} \Rightarrow -(f''(0))\sqrt{\frac{2\nu}{u_0 l}} e^{\frac{3x}{2l}}$$

$$U_x = u_o e^{\frac{x}{l}} \Rightarrow u_0 = \frac{U_x}{e^{\frac{x}{l}}}, C_f = -(f''(0))\sqrt{\frac{2\nu}{\frac{U_x}{e^{\frac{x}{l}}} l}} e^{\frac{3x}{2l}}$$

$$C_f = e^{\frac{-2x}{l}} f''(0)(\frac{R_{ex}}{2})^{\frac{-1}{2}}$$

$$Cf = \frac{2\tau_w}{\rho U_o^2} = \left(\frac{Re_x}{2}\right)^{\frac{-1}{2}} e^{\frac{2x}{l}} f''(0) \text{ where } \tau_w \text{ is the wall shear stress given by } \tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad (4.24)$$

rate of transfer in terms os Nusselt number at the sheet given by

$$Nu = \frac{lq_w}{k(T_w - T_\infty)} = -\left(\frac{Re_x}{2} e^{\frac{x}{l}}\right)^{\frac{1}{2}} \theta'(0) \text{ where } q_w \text{ is the rate of heat transfer given by } q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (4.25)$$

Sherworld number is defined as

$$Sh_x = xq_m \Rightarrow \sqrt{\frac{2l}{x}} \frac{1}{\sqrt{Re_x}} Sh_x = -\phi'(0)$$, where q_m is the mass flux at the surface of sheet given by:

$$q_m = -D\left(\left[\frac{\partial C}{\partial Y}\right] y = 0\right.$$

4.3.Methods of Solution

Equation (4.20) – (4.22) subject to the boundary condition (4.23) where solved numerically by Keller box method which is implemented in mat lab then let's introduce a new variable u,v,g&q such that

$$\begin{aligned} f' &= u \\ u' &= v \\ \theta' &= g \\ \phi' &= q \end{aligned} \quad (4.26)$$

So that equations (4.20) – (4.22)in terms of new variables

$$v' + fv - 2u^2 - \left(M + \frac{1}{k}\right)u = 0 \quad (4.27)$$

$$g' + \Pr gf + 2\Pr Q\theta = 0 \quad (4.28)$$

$$q' + Sc\left(qf - R_c\phi\right) = 0 \quad (4.29)$$

With the new boundary conditions

$$f(0) = S,\ u(0) = 1 + S_v v(0), u(\infty) = 0\ , \theta(\infty) = 0$$

$$\theta(0) = 1 + S_T g(0)\ , \phi(0) = 1 + S_T q(0)\ and\ \phi(\infty) = 0 \quad (4.30)$$

We know consider the net rectangle in the x-ɳ plan as shown in the figure bellow and the net points defined as follows

$$x_0 = o, x_n = x_{n-1} + k_n,\ n = 1, 2......N$$

$$\eta_0 = \eta_j = n_{j-1} + h_j\ \ , j = 1, 2.....J \quad n_j = \eta_j$$

Where k_n is the Δx- spacing and hj is the Δ ŋ spacing the here n and j are the sequence of the numbers that indicate the coordinate location.

A brief description of the method is given below (Cebeci, and Brand show, 1984) .

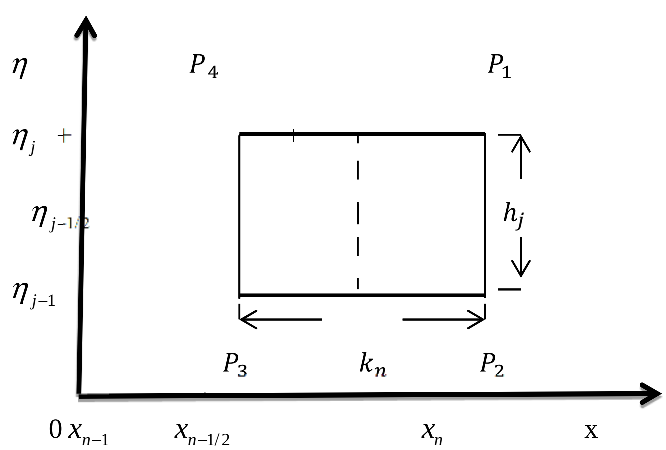

Figure 2: Finite difference grid for the box method

Now write the finite difference approximations of the ordinary differential equations

(4.23) For the mid points

$\left(x_n, \eta_{j-1/2}\right)$ *using centered difference derivatives is called centering about* $\left(x_n, \eta_{j-1/2}\right)$

$$\frac{f_j - f_{j-1}}{hj} = u_{j-1/2} \qquad \Rightarrow f_j - f_{j-1} - hj\left(u_{j-1/2}\right) = 0$$
$$\frac{u_j - u_{j-1}}{hj} = v_{j-1/2} \qquad \Rightarrow u_j - u_{j-1} - hj\left(v_{j-1/2}\right) = 0$$
$$\frac{\theta_j - \theta_{j-1}}{hj} = g_{j-1/2} \qquad \Rightarrow \theta_j - \theta_{j-1} - hj\left(g_{j-1/2}\right) = 0 \qquad (4.31)$$
$$\frac{\phi_j - \phi_{j-1}}{hj} = q_{j-1/2} \qquad \Rightarrow \phi_j - \phi_{j-1} - hj\left(q_{j-1/2}\right) = 0$$

Ordinary differential equation (4.24-4.26) are approximated by centering about the mid point $\left(x_{n-1/2}, \eta_{j-1/2}\right)$ *of the rectangle*

$$\frac{v_j - v_{j-1}}{hj} + f_{j-1/2}v_{j-1/2} - 2(u_{j-1/2})^2 - \left(M + \frac{1}{k}\right)u_{j-1/2} = 0$$

$$\frac{g_j - g_{j-1}}{hj} + \Pr(f_{j-1/2}g_{j-1/2}) + 2\Pr Q\theta_{j-1/2} = 0$$

$$\frac{q_j - q_{j-1}}{hj} + Sc(f_{j-1/2}q_{j-1/2} - R_c\phi_{j-1/2}) = 0$$

$$u_{j-1/2} = \frac{u_j + u_{j-1}}{2} \qquad etc......$$

Hence equatin (4.28 − 4.31)*becomes*

$$f_j - f_{j-1} - \frac{h_j}{2}\left(u_j + u_{j-1}\right) = 0$$
$$u_j - u_{j-1} - \frac{h_j}{2}\left(v_j + v_{j-1}\right) = 0$$
$$\theta_j - \theta_{j-1} - \frac{h_j}{2}\left(g_j + g_{j-1}\right) = 0 \qquad (4.32)$$
$$\phi_j - \phi_{j-1} - \frac{h_j}{2}\left(q_j + q_{j-1}\right) = 0$$

$$v_j - v_{j-1} + \frac{hj}{4}\left(f_j + f_{j-1}\right)\left(v_j + v_{j-1}\right) - \frac{hj}{2}\left(u_j + u_{j-1}\right)^2 - \left(M + \frac{1}{k}\right)\frac{hj}{2}\left(u_j + u_{j-1}\right) = 0 \qquad (4.\ 33)$$

$$g_j - g_{j-1} + \frac{\Pr hj}{4}\left(f_j + f_{j-1}\right)\left(g_j + g_{j-1}\right) + \frac{2\Pr Qhj}{2}\left(\theta_j + \theta_{j-1}\right) = 0 \qquad (4.34)$$

$$q_j - q_{j-1} + \frac{Schj}{4}\left[\left(f_j + f_{j-1}\right)\left(q_j + q_{j-1}\right)\right] - \frac{SchjR_c}{2}\left(\phi_j + \phi_{j-1}\right) = 0 \qquad (4.35)$$

Now linearize the non linear the system equation (4. 32-4.35) using the Newton's linearization systems, that is we assume:

$(i+1)^{th}$ *itrate*

$$f_j^{i+1} = f_j^i + \delta f_j^i \qquad etc... \qquad (4.36)$$

substitung equation (4.36) *into equations* (4.32) − (4.35) *becames*

$$f_j+\delta f_j-f_{j-1}-\delta f_{j-1}-\frac{hj}{2}\left(u_j+\delta u_j+u_{j-1}+\delta u_{j-1}\right)=0$$

$$\Rightarrow \delta f_j-\delta f_{j-1}-\frac{hj}{2}\left(\delta u_j+\delta u_{j-1}\right)=f_{j-1}-f_j+\frac{hj}{2}\left(u_j+u_{j-1}\right)$$

$$\Rightarrow \delta f_j-\delta f_{j-1}-\frac{hj}{2}\left(\delta u_j+\delta u_{j-1}\right)=\left(r_1\right)_j$$

$$where \;\; \left(r_1\right)_j=f_{j-1}-f_j+\frac{hj}{2}\left(u_j+u_{j-1}\right)$$

$$u_j+\delta u_j-u_{j-1}-\delta u_{j-1}-\frac{hj}{2}\left(v_j+\delta v_j+v_{j-1}+\delta v_{j-1}\right)=0$$

$$\Rightarrow \delta u_j-\delta u_{j-1}-\frac{hj}{2}\left(\delta v_j+\delta v_{j-1}\right)=u_{j-1}-u_j+\frac{hj}{2}\left(v_j+v_{j-1}\right)$$

$$\Rightarrow \delta u_j-\delta u_{j-1}-\frac{hj}{2}\left(\delta v_j+\delta v_{j-1}\right)=\left(r_2\right)_j$$

$$where \;\; \left(r_2\right)_j=u_{j-1}-u_j+\frac{hj}{2}\left(v_j+v_{j-1}\right)$$

$$\theta_j+\delta_j\theta-\theta_{j-1}-\delta_{j-1}\theta-\frac{hj}{2}\left(g_j+\delta g_j+g_{j-1}+\delta g_{j-1}\right)=0$$

$$\Rightarrow \delta\theta_j-\delta\theta_{j-1}-\frac{hj}{2}\left(\delta g_j+\delta g_{j-1}\right)=\theta_{j-1}-\theta_j+\frac{hj}{2}\left(g_j+g_{j-1}\right)$$

$$\Rightarrow \delta\theta_j-\delta\theta_{j-1}-\frac{hj}{2}\left(\delta g_j+\delta g_{j-1}\right)=\left(r_3\right)_j$$

$$where \;\; \left(r_3\right)_j=\theta_{j-1}-\theta_j+\frac{hj}{2}\left(g_j+g_{j-1}\right)$$

$$\phi_j+\delta\phi_j-\phi_{j-1}-\delta\phi_{j-1}-\frac{hj}{2}\left(q_j+\delta q_j+q_{j-1}+\delta q_{j-1}\right)=0$$

$$\Rightarrow \delta\phi_j-\delta\phi_{j-1}-\frac{hj}{2}\left(\delta q_j+\delta q_{j-1}\right)=\phi_{j-1}-\phi_j+\frac{hj}{2}\left(q_j+q_{j-1}\right)$$

$$\Rightarrow \delta\phi_j-\delta\phi_{j-1}-\frac{hj}{2}\left(\delta q_j+\delta q_{j-1}\right)=\left(r_4\right)_j$$

$$where \;\; \left(r_4\right)_j=\phi_{j-1}-\phi_j+\frac{hj}{2}\left(q_j+q_{j-1}\right)$$

$$v_j+\delta v_j-v_{j-1}-\delta v_{j-1}+\frac{hj}{4}\left[\left(f_j+\delta f_j+f_{j-1}+\delta f_{j-1}\right)\left(v_j+\delta v_j-v_{j-1}-\delta v_{j-1}\right)\right]-\frac{hj}{2}\left(u_j+\delta u_j+u_{j-1}+\delta u_{j-1}\right)^2$$
$$-\frac{hj}{2}\left(M+\frac{1}{k}\right)\left(u_j+\delta u_j+u_{j-1}+\delta u_{j-1}\right)=0$$

$$\Rightarrow v_j+\delta v_j-v_{j-1}-\delta v_{j-1}+\frac{hj}{4}\left[\left((f_j+f_{j-1})\delta v_j+(f_j+f_{j-1})\delta v_{j-1}\right)+\left(v_j+v_{j-1}\right)\delta f_j+\left(v_j+v_{j-1}\right)\delta f_{j-1}+(f_j+f_{j-1})(v_j+v_{j-1})\right]-$$
$$\left[\frac{hj}{2}\left(u_j+u_{j-1}\right)^2+2u_j\delta u_j+2u_j\delta u_{j-1}+2\delta u_j u_{j-1}+2u_{j-1}\delta u_{j-1}\right]-\frac{hj}{2}\left(M+\frac{1}{k}\right)\left(u_j+\delta u_j+u_{j-1}+\delta u_{j-1}\right)=0$$

Simplifying we get:

$$\left(a_1\right)_j \delta v_j + \left(a_2\right)_j \delta v_{j-1} + \left(a_3\right)_j \delta f_j + \left(a_4\right)_j \delta f_{j-1} + \left(a_5\right)_j \delta u_j + \left(a_6\right)_j \delta u_{j-1} = \left(r_5\right)_j$$

where:

$$\left(a_1\right)_j = 1 + \frac{hj}{4}(f_j + f_{j-1})$$

$$\left(a_2\right)_j = -1 + \frac{hj}{4}(f_j + f_{j-1})$$

$$\left(a_3\right)_j = \frac{hj}{4}(v_j + v_{j-1}) = \left(a_4\right)_j$$

$$\left(a_5\right)_j = -\frac{hj}{2}\left[2(u_j + u_{j-1}) - \left(M + \frac{1}{k}\right)\right] = \left(a_6\right)_j$$

$$\left(r_5\right)_j = v_{j-1} - v_j - \frac{hj}{4}\left[(f_j + f_{j-1})(v_j + v_{j-1})\right] + \frac{hj}{2}(u_j + u_{j-1})^2 + \left(M + \frac{1}{k}\right)\frac{hj}{2}(u_j + u_{j-1})$$

$$g_j + \delta g_j - g_{j-1} + \delta g_{j-1} + \frac{h_j \Pr}{4}\left((f_j + \delta f_j + f_{j-1} + \delta f_{j-1})(g_j + \delta g_j + g_{j-1} + \delta g_{j-1})\right) + h_j \Pr Q(\theta_j + \delta\theta_j + \theta_{j-1} + \delta\theta_{j-1}) = 0$$

Arranging these equations we get:

$$1 + \frac{hj\Pr}{4}(f_j + f_{j-1})\delta g_j - 1 + \frac{hj\Pr}{4}(f_j + f_{j-1})\delta g_{j-1} + \frac{hj\Pr}{4}(g_j + g_{j-1})\delta f_j + \frac{hj\Pr}{4}(g_j + g_{j-1})\delta f_{j-1} + hj\Pr Q\delta\theta_j + hj\Pr Q\delta\theta_{j-1} = g_{j-1} - g_j - \frac{hj\Pr}{4}\left(f_j + f_{j-1})(g_j + g_{j-1})\right) - hj\Pr Q\left(\theta_j + \theta_{j-1}\right)$$

then we can write this as:

$$\left(b_1\right)_j \delta g_j + \left(b_2\right)_j \delta g_{j-1} + \left(b_3\right)_j \delta f_j + \left(b_4\right)_j \delta f_{j-1} + \left(b_5\right)_j \delta\theta_j + \left(b_6\right)_j \delta\theta_{j-1} = \left(r_6\right)_j$$

where:

$$\left(b_1\right)_j = 1 + \frac{hj\Pr}{4}(f_j + f_{j-1})$$

$$\left(b_2\right)_j = -1 + \frac{hj\Pr}{4}(f_j + f_{j-1})$$

$$\left(b_3\right)_j = \frac{hj\Pr}{4}(g_j + g_{j-1}) = \left(b_4\right)_j$$

$$\left(b_5\right)_j = hj\Pr Q = \left(b_6\right)_j$$

and $\left(r_6\right)_j = g_{j-1} - g_j + \frac{hj\Pr}{4}\left(f_j + f_{j-1})(g_j + g_{j-1})\right) - hj\Pr Q\left(\theta_j + \theta_{j-1}\right)$

$$q_j + \delta q_j - q_{j-1} - \delta q_{j-1} + \frac{hjSc}{4}\left[(f_j + f_{j-1})(q_j + q_{j-1})\right] - \frac{ScR_c h_j}{2}(\phi_j + \phi_{j-1}) = 0$$

using the same techniques $R_c(\phi_j + \phi_{j-1})$

$$\delta\phi_j(R_c) + \delta\phi_{j-1}(R_c) + R_c(\phi_j + \phi_{j-1})$$

Re arranging these equations we get:

$$(c_1)_j\,\delta q_j+(c_2)_j\,\delta q_{j-1}+(c_3)_j\,\delta f_j+(c_4)_j\,\delta f_{j-1}+(c_5)_j\,\delta u_j+(c_6)_j\,\delta u_{j-1}=(r_7)_j$$

where:

$$(c_1)_j=1+\frac{hjSc}{4}(f_j+f_{j-1})$$

$$(c_2)_j=-1+\frac{hjSc}{4}(f_j+f_{j-1})$$

$$(c_3)_j=\frac{hjSc}{4}(q_j+q_{j-1})=(c_4)_j$$

$$(c_5)_j=\frac{-hjScR_c}{2}=(c_6)_j$$

$$and\quad (r_7)_j=q_{j-1}-q_j+\frac{hjSc}{4}\Big(f_j+f_{j-1})(q_j+q_{j-1})\Big)+(\phi_j+\phi_{j-1})(\frac{hjScR_c}{2})$$

In general from above equations using $(a_i)_j$, $(b_i)_j$, $(c_i)_j$ and $(r_i)_j$ we can write the system of the linear equations as:

$$\delta f_j-\delta f_{j-1}-\frac{hj}{2}\left(\delta u_j+\delta u_{j-1}\right)=(r_1)_j$$

$$\delta u_j-\delta u_{j-1}-\frac{hj}{2}\left(\delta v_j+\delta v_{j-1}\right)=(r_2)_j$$

$$\delta\theta_j-\delta\theta_{j-1}-\frac{hj}{2}\left(\delta g_j+\delta g_{j-1}\right)=(r_3)_j$$

$$\delta\phi_j-\delta\phi_{j-1}-\frac{hj}{2}\left(\delta q_j+\delta q_{j-1}\right)=(r_4)_j$$

$$(a_1)_j\,\delta v_j+(a_2)_j\,\delta v_{j-1}+(a_3)_j\,\delta f_j+(a_4)_j\,\delta f_{j-1}+(a_5)_j\,\delta u_j+(a_6)_j\,\delta u_{j-1}=(r_5)_j$$

$$(b_1)_j\,\delta g_j+(b_2)_j\,\delta g_{j-1}+(b_3)_j\,\delta f_j+(b_4)_j\,\delta f_{j-1}+(b_5)_j\,\delta\theta_j+(b_6)_j\,\delta\theta_{j-1}=(r_6)_j$$

$$(c_1)_j\,\delta q_j+(c_2)_j\,\delta q_{j-1}+(c_3)_j\,\delta f_j+(c_4)_j\,\delta f_{j-1}+(c_5)_j\,\delta\phi_j+(c_6)_j\,\delta\phi_{j-1}=(r_7)_j$$

With the boundary conditions

$$\delta u_o=1+s_v\,v_o\quad,\ \delta f_o=s\quad,\quad \delta\theta_o=1+s_T\,g_o\quad,\quad \delta\phi_o=1+Sm(0)$$

$$\delta u_j=0\qquad \delta\theta_j=0\qquad ,\ \delta\phi_j=0$$

Hence linear zed system of equations can be written in the matrix form as:

$$A\delta=r \tag{4.37}$$

$$A=\begin{bmatrix}[A_1][C_1] & & & & \\ [B_2][A_2][C_2] & & & & \\ & [B_3][A_3][C_3] & & & \\ & \ddots\quad\ddots\quad\ddots & & & \\ & & [B_{j-1}][A_{j-1}][C_{j-1}] & & \\ & & & \ddots\quad\ddots & \\ & & & & [B_j][A_j]\end{bmatrix},\quad \delta=\begin{bmatrix}\delta_1\\ \delta_2\\ \delta_3\\ \vdots\\ \vdots\\ \delta_j\end{bmatrix}\qquad r=\begin{bmatrix}r_1\\ r_2\\ r_3\\ \vdots\\ \vdots\\ r_j\end{bmatrix}$$

the elements matrix $[A]$ *are block of order* $7X7$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -d_1 & 0 & 0 & 0 & -d_1 & 0 & 0 \\ 0 & -d_1 & 0 & 0 & 0 & -d_1 & 0 \\ 0 & 0 & -d_1 & 0 & 0 & 0 & -d_1 \\ (a_2)_1 & 0 & 0 & (a_3)_1 & (a_1)_1 & 0 & 0 \\ 0 & (b_2)_1 & 0 & (b_3)_1 & 0 & (b_1)_1 & 0 \\ 0 & 0 & (c_2)_1 & (c_3)_1 & 0 & 0 & (c_1)_1 \end{bmatrix}$$

$$[A_j] = \begin{bmatrix} -d_j & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & -d_j & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & -d_j & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -d_j \\ (a_6)_j & 0 & 0 & (a_3)_j & (a_1)_j & 0 & 0 \\ 0 & (b_6)_j & 0 & (b_3)_j & 0 & (b_1)_j & 0 \\ 0 & 0 & (c_6)_j & (c_3)_j & 0 & 0 & (c_1)_j \end{bmatrix}$$

$$2 \le j \le J$$

$$[B_j] = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -d_j & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -d_j & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -d_j \\ 0 & 0 & 0 & (a_4)_j & (a_2)_j & 0 & 0 \\ 0 & 0 & 0 & (b_4)_j & 0 & (b_2)_j & 0 \\ 0 & 0 & 0 & (c_4)_j & 0 & 0 & (c_2)_j \end{bmatrix}$$

$$2 \le j \le J$$

$$[C_j] = \begin{bmatrix} -d_j & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ (a_5)_j & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (b_5)_j & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (c_5)_j & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$1 \le j \le J$$

$$where\ d_j = \frac{h_j}{2}\ and$$

$$\delta_1 = \begin{bmatrix} \delta v_o \\ \delta g_o \\ \delta q_o \\ \delta f_1 \\ \delta v_1 \\ \delta g_1 \\ \delta q_1 \end{bmatrix} \qquad \delta_j = \begin{bmatrix} \delta u_{j-1} \\ \delta \theta_{j-1} \\ \delta \phi_{j-1} \\ \delta f_j \\ \delta v_j \\ \delta g_j \\ \delta q_j \end{bmatrix} \qquad r_j = \begin{bmatrix} (r_1)_j \\ (r_2)_j \\ (r_3)_j \\ (r_4)_j \\ (r_5)_j \\ (r_6)_j \\ (r_7)_j \end{bmatrix} \qquad 1 \le j \le J$$

The solutions of equations (4.37) can be obtained byusing byblock elimination method which consist of for ward and back ward sweeps

For ward sweeps

To solve equation 37 lets de composed matrix A into product of lower triangular matrix L and upper triangular matrix U

A=LU (4.38)

Where

$$L = \begin{bmatrix} [\alpha_1] & & & \\ [\beta_1] & [\alpha_2] & & \\ \ddots & \ddots & & \\ & [\beta_{j-1}] & [\alpha_{j-1}] & \\ & & [\beta_j] & [\alpha_j] \end{bmatrix} \qquad U = \begin{bmatrix} [I] & [\Gamma_1] & & & \\ & [I] & [\Gamma_2] & & \\ & & \ddots & & \\ & & & [I] & [\Gamma_j] \\ & & & & [I] \end{bmatrix}$$

$[I]$ *is the identity matrix of order* $7X7$ *and* $[\alpha_j]$ *and* $[\Gamma_j]$ *are* $7X7$ *matrix which elements are* det *ermained by the following equation*

$$[\alpha_1] = [A_1]$$

$$[A_1][\Gamma_1] = [C_1]$$

$$[\alpha_j] = [\beta_j][\Gamma_{j-1}] = [A_j] \qquad j = 2,3...J$$

$$[A_1][\Gamma_1] = [C_1] \qquad j = 2,3....J$$

Back ward sweep

Equation (4.38)can be substituting into equation (4.37) and becomes

$$LU\delta = r \qquad (4.39)$$

if we define

$$U\delta = W \qquad (4.40)$$

$$where\ W=\left[w_1,w_2,w_3,..............w_{j-1}w_J\right]^T \qquad (4.41)$$

w_j *are the elements of* $7X1$ *columen matrix*

the elements of W *can be found by solving equation* (4.39)

$$[\alpha_1][w_1]=[r_1]$$

$$\left[\alpha_j\right]\left[w_j\right]=\left[r_j\right]-\left[\beta_j\right]\left[w_{j-1}\right] \qquad ,2\le j\le J$$

Once the element of W are found we can find the solution of equation 41 using recurrent relations.

$$[\alpha_1][w_1]=[r_1]$$

$$\left[\delta_j\right]=\left[w_j\right]$$

$$\left[\delta_j\right]=\left[w_j\right]-\left[r_j\right]\left[\delta_{j+1}\right] \quad , \qquad 1\le j\le J_{j-1}$$

4.4. Numerical Results

The following tables show comparison of earlier works on skin friction coefficient and surface heat transfer rate. From the tables it is shown that the present result agrees with the previous results done by Sharma et al.(2017).

Table 1 Numerical values of $-\boldsymbol{f''}(\mathbf{0})$ for different values of physical parameter Previous and (Present Result)

K	**M**	**S**	$\boldsymbol{S_v}$	Sharma et al.(2017)	**Present Result**
1	0.5	3	0.1	2.7051005	2.70465801
2	0,5	3	0.1	2.6221080	2.62123082
3	0.5	3	0.1	2.8681380	2.86805734
1	2	3	0.1	3.1313106	3.13131440

Table 2. Numerical values of $-\boldsymbol{\theta'}(\mathbf{0})$ for different values of physical parameter Previous and Preset result

K	$\boldsymbol{P_r}$	**Q**	**M**	**S**	$\boldsymbol{S_v}$	**S$_t$**	Sharma et al. (2017)	**Present Result**
1	0.71	0.5	0.5	3	0.1	0.1	1.677319	1.67808160
2	0.71	0.5	0.5	3	0.1	0.1	-	1.67963622
3	0.71	0.5	0.5	3	0.1	0.1	1.682538	1.68018416
1	0.71	0.5	2	3	0.1	0.1	1.667540	1.67963622
1	1	0.5	0.5	3	0.1	0.1	2.241582	2.19440021
1	1	1	0.5	3	0.1	0.1	1.383828	1.37765499

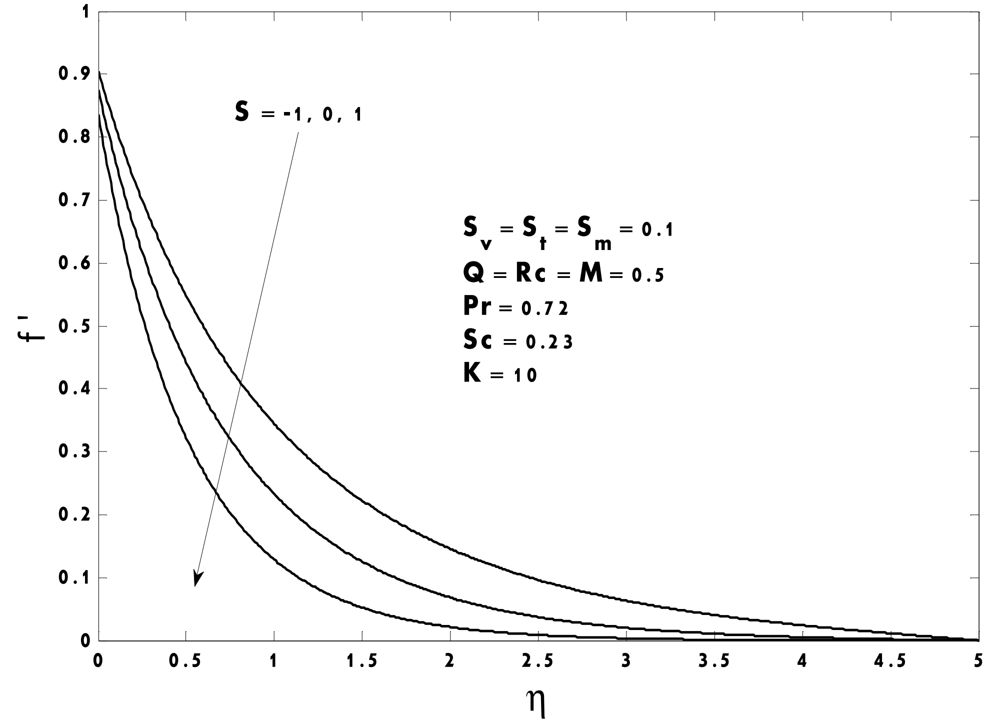

Figure 3 Effect of suction/injection parameter s on velocity profile

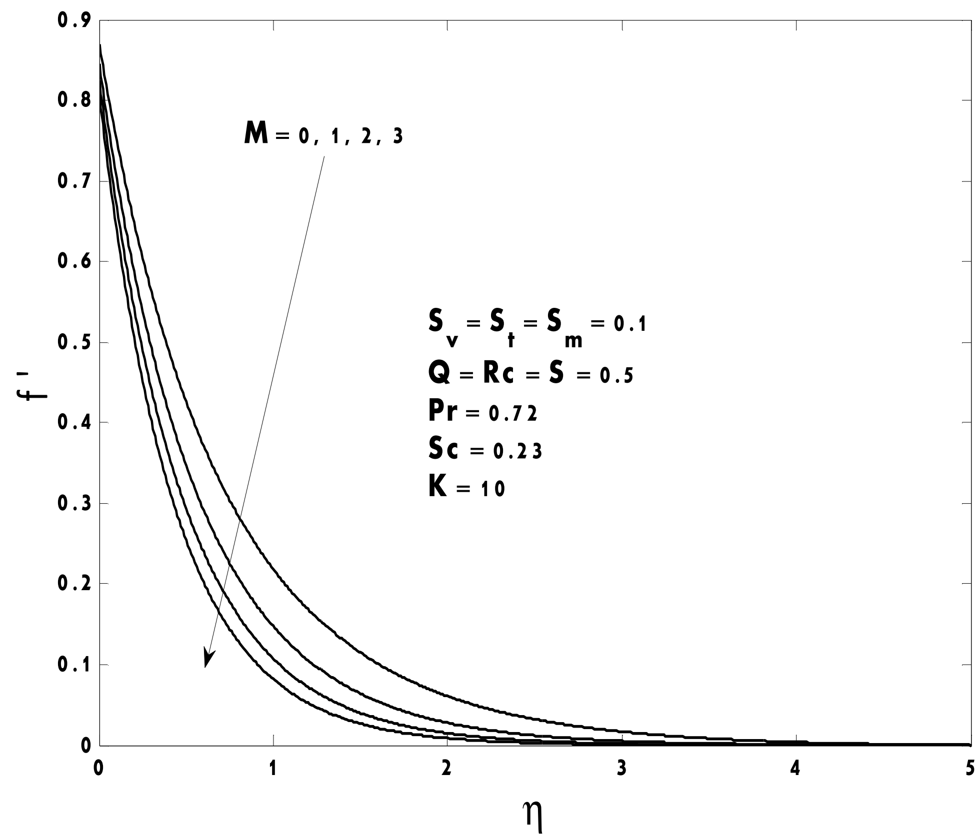

Figure 4. Effect of Hartmann parameter M on velocity profile

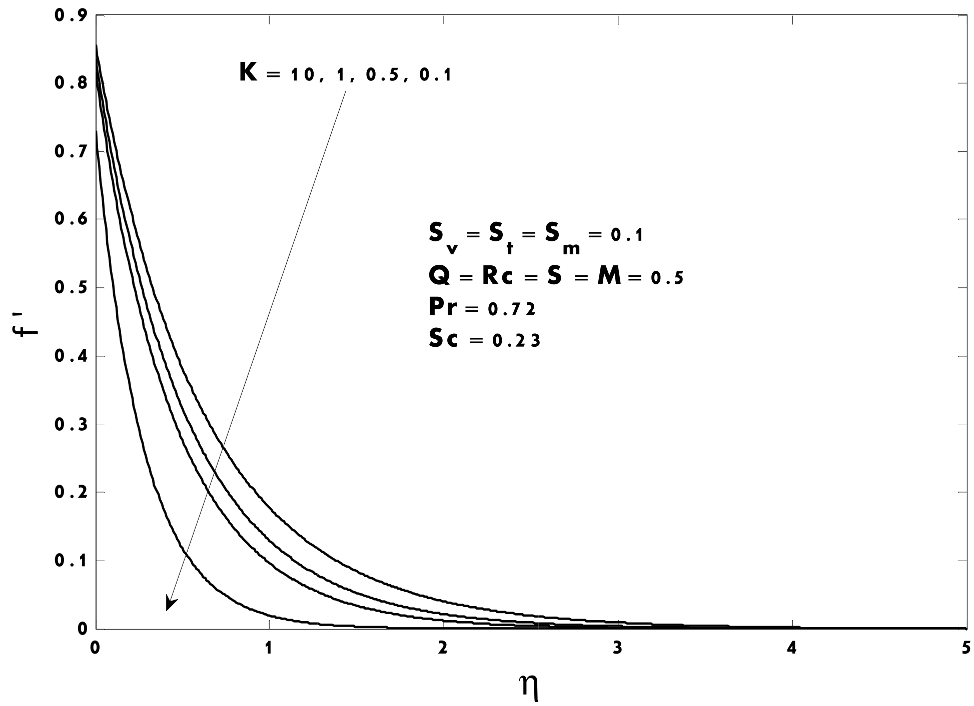

Figure 5. Effect of permeability parameter K on velocity profile

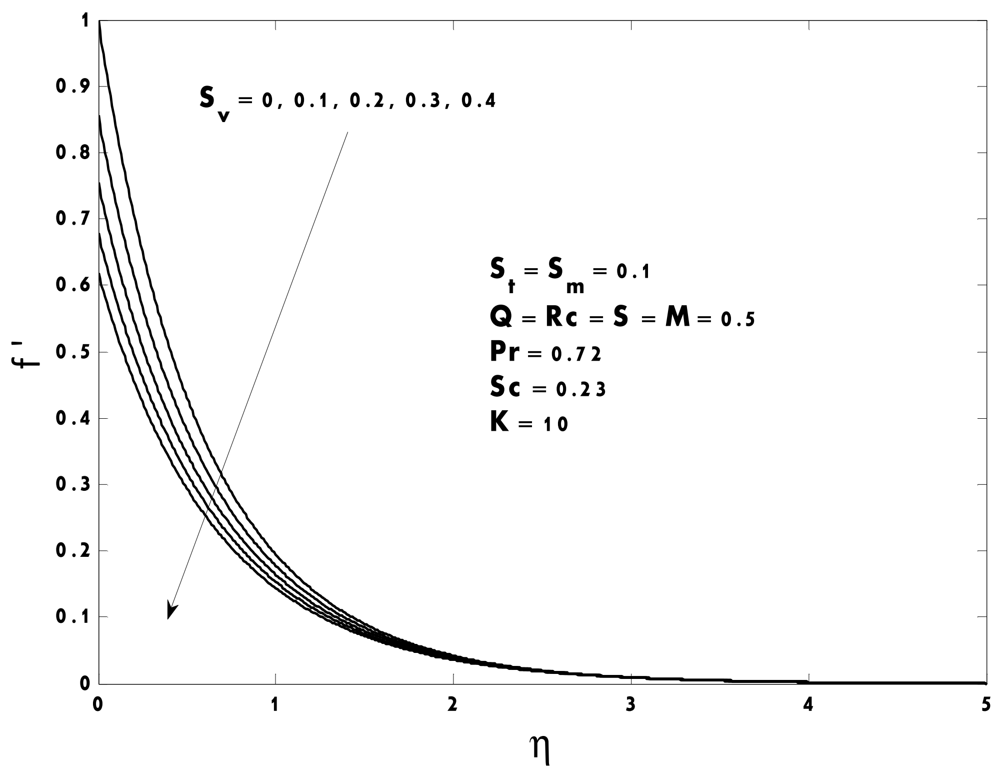

Figure 6. Effect of velocity slip parameter s_v on velocity profile

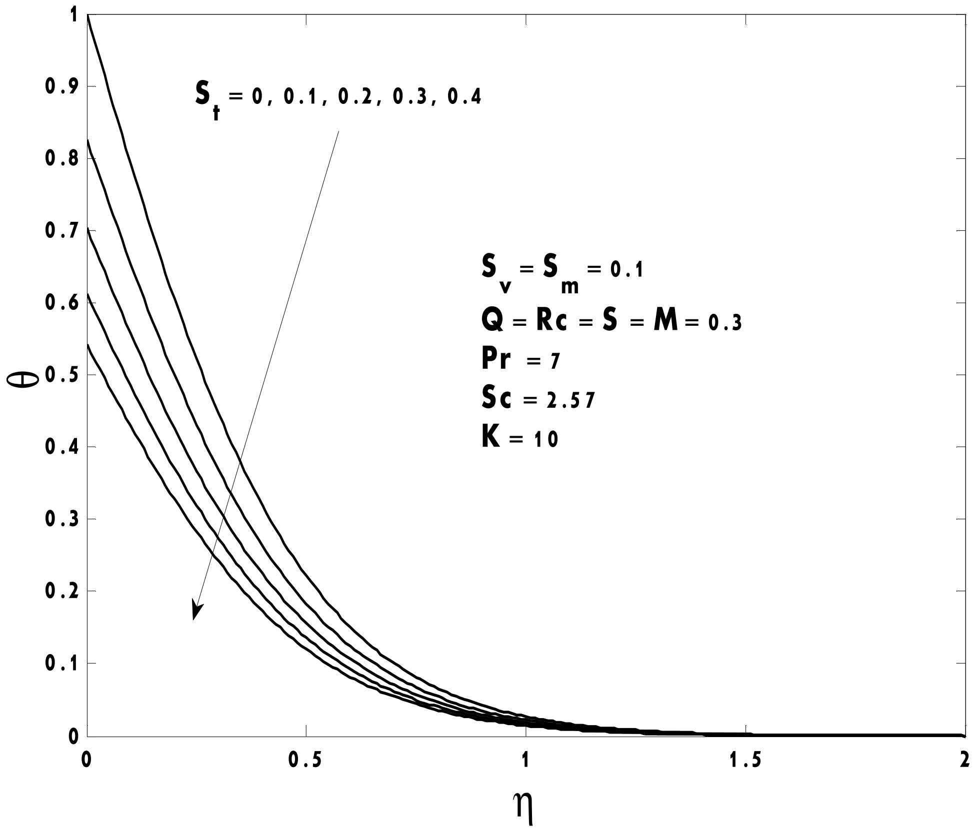

Figure 7. Effect of thermal slip parameter S_t on temperature profile

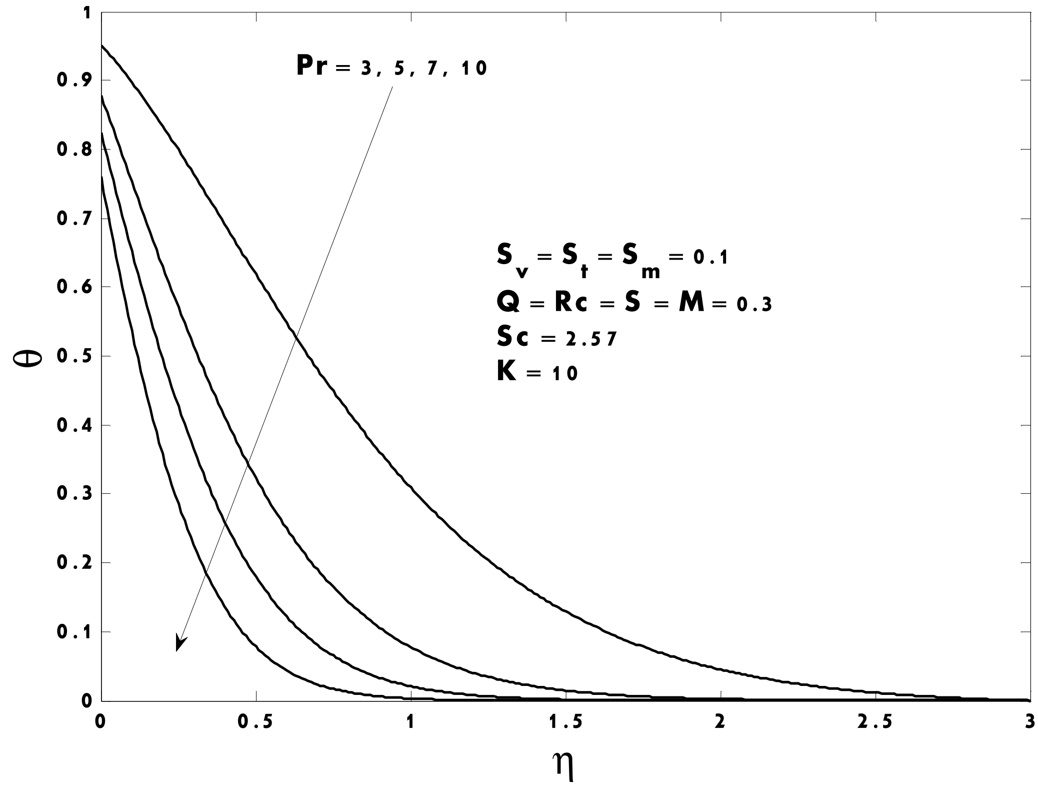

Figure 8. Effect of Prandtl number P_r on temperature profile

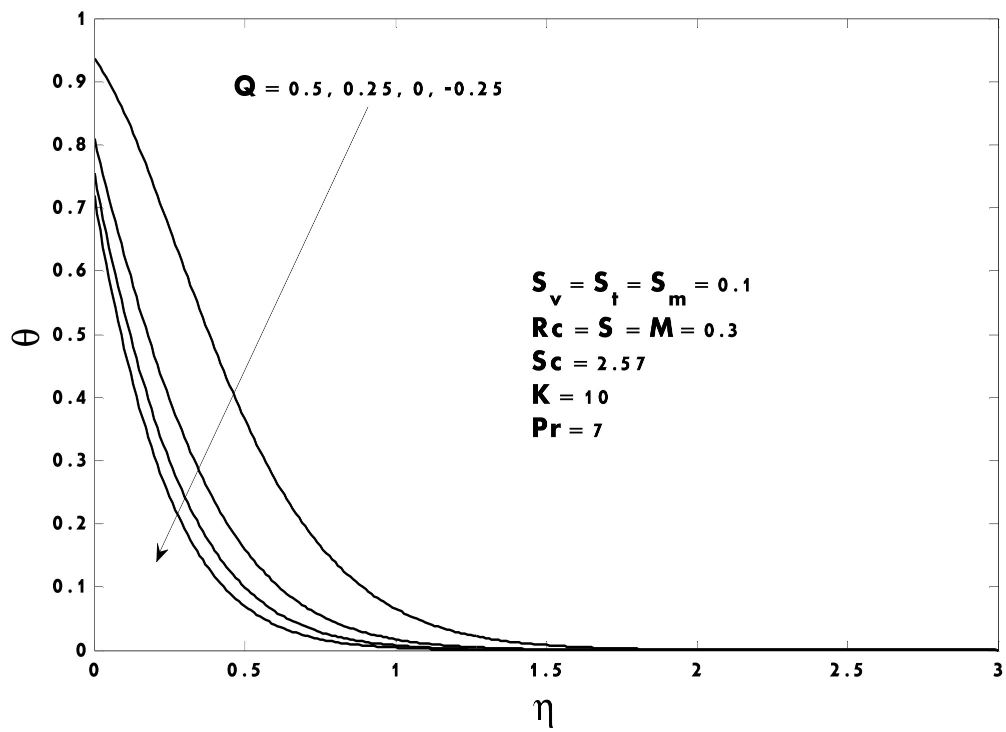

Figure 9. Effect of heat source parameter Q on Temperature profile

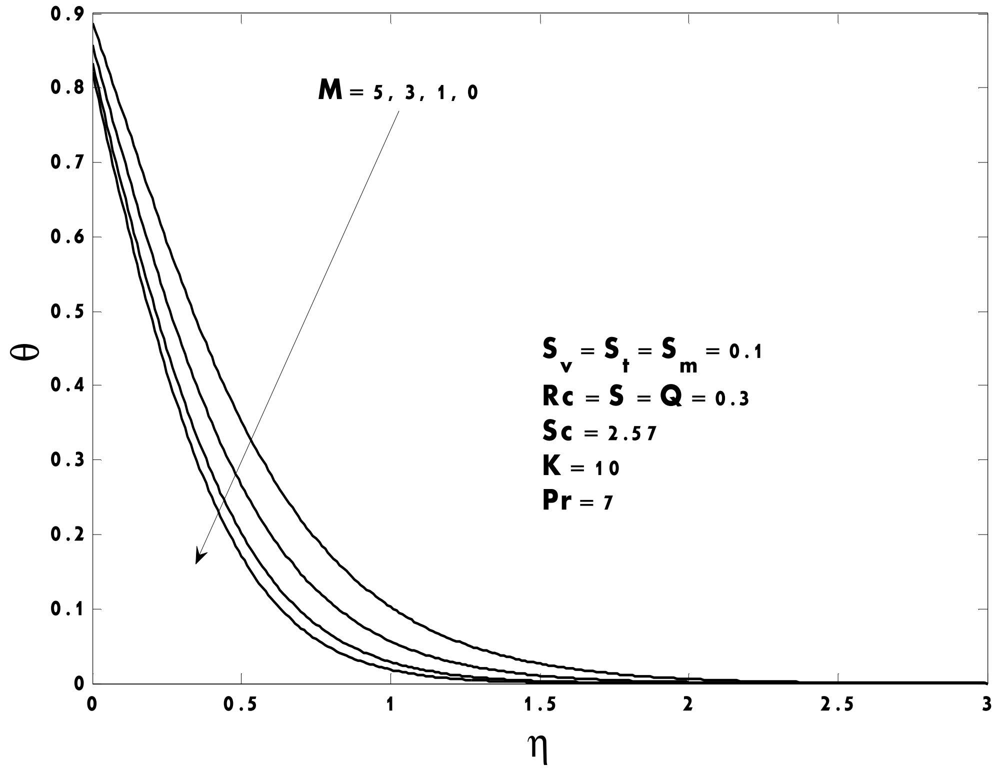

Figure 10. Effect of Harthmann number parameter M on Temperature profile

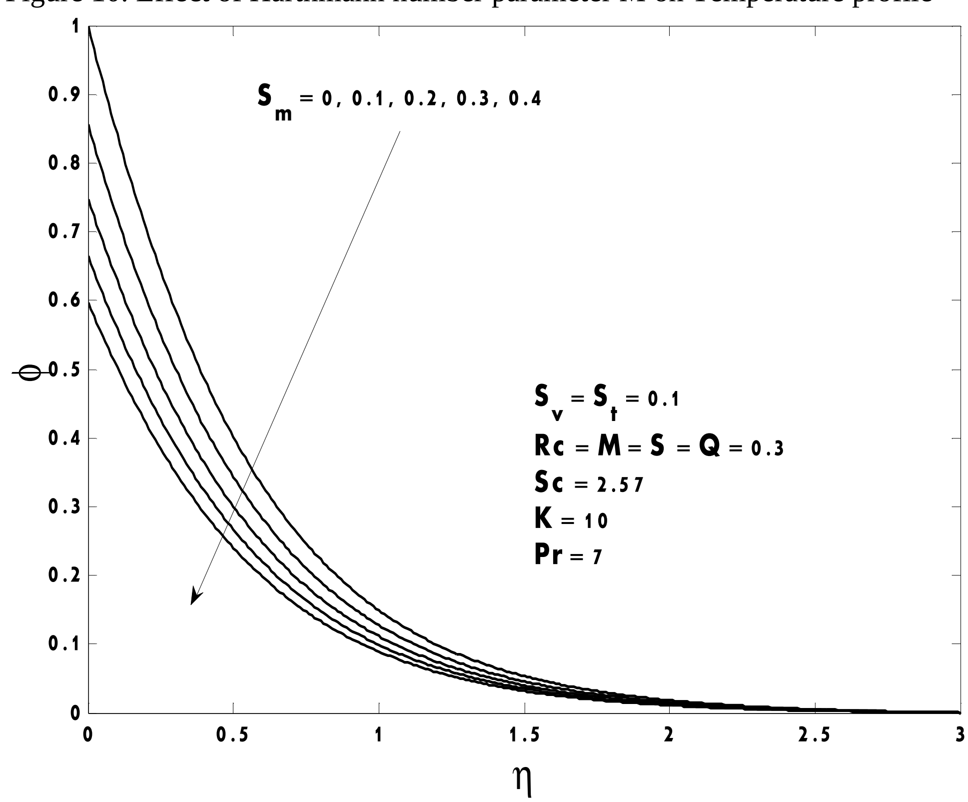

Figure 11. Effect of concentration slip parameter S_m on concentration profile

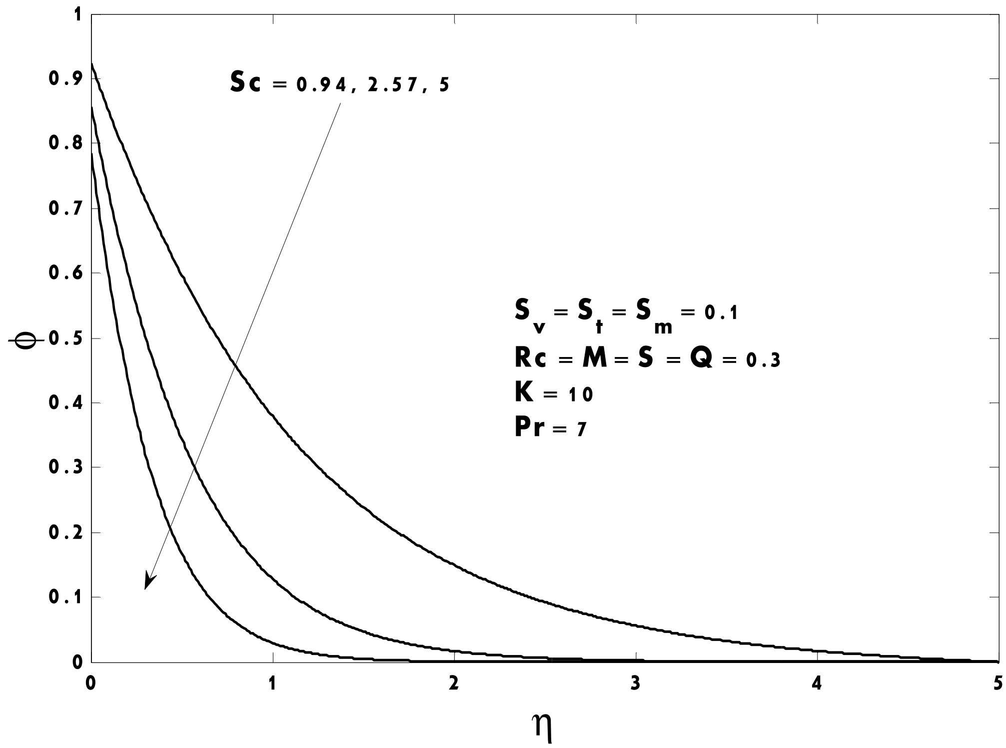

Figure 12. Effect of Schmidt number parameter S_c on concentration profile

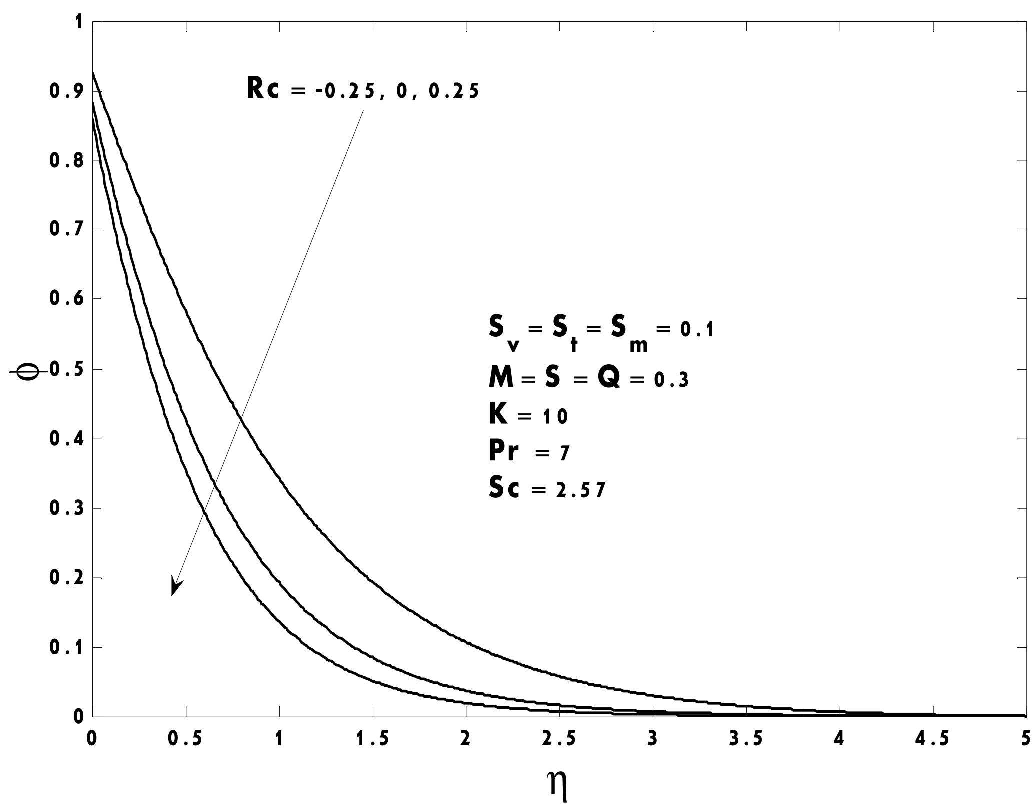

Figure 13. Effect of chemical reaction parameter Rc on concentration profile

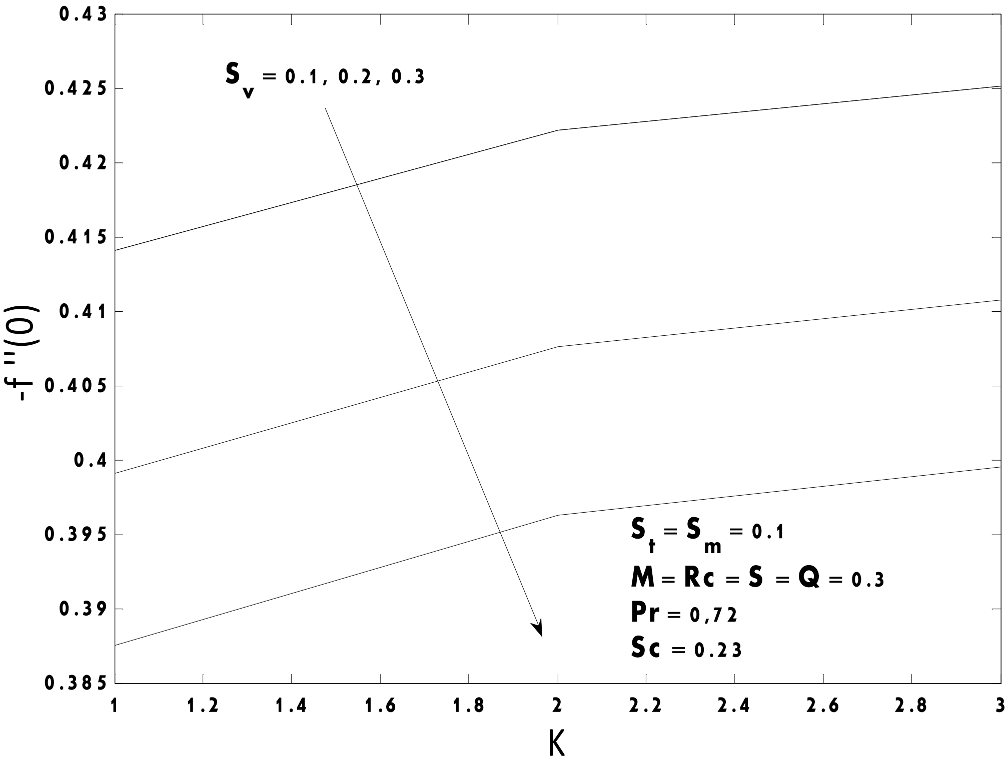

Figure 14. Effect of velocity slip parameter S_v and permeability parameter K on skin friction

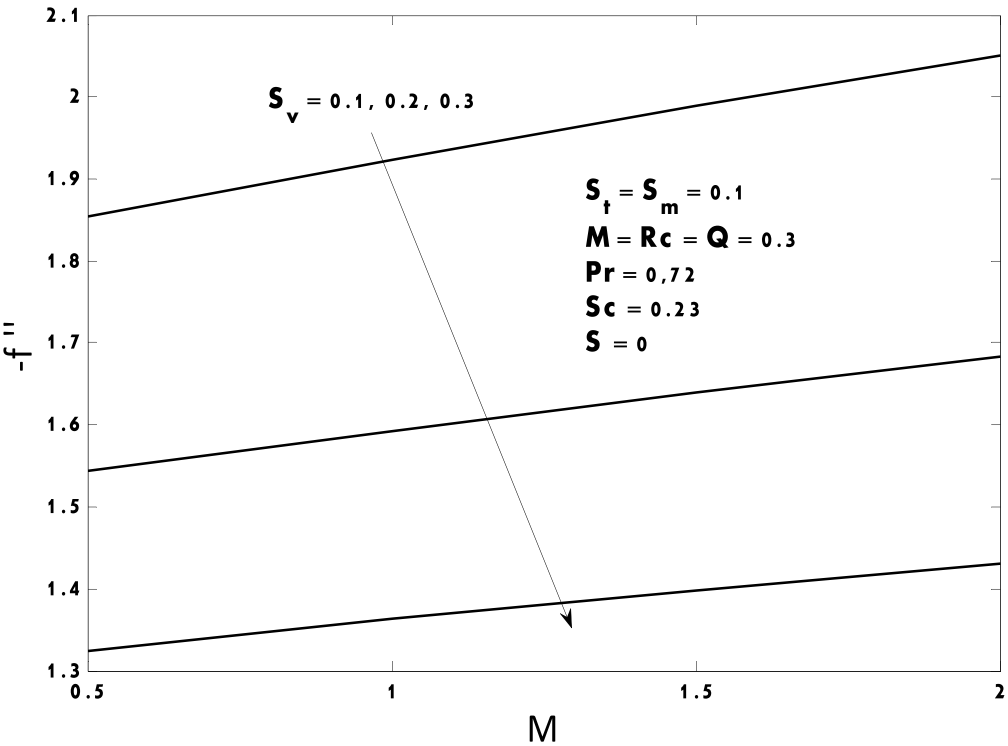

Figure15. Effect of velocity slip parameter S_v and Hartmann number on skin friction

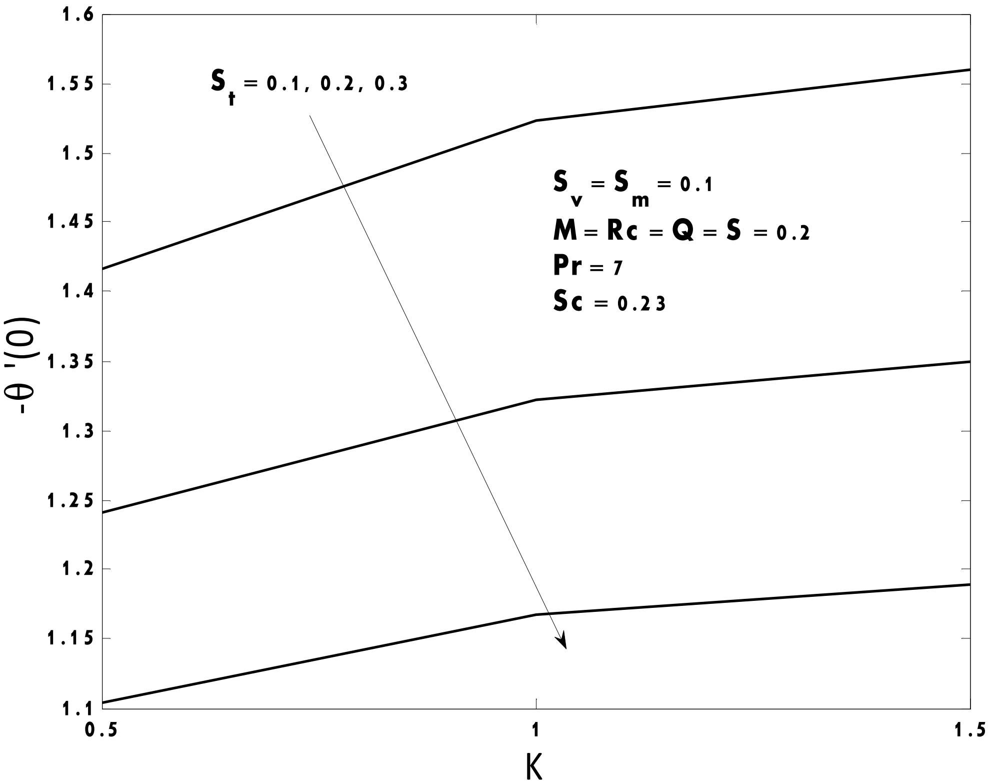

Figure 16. Effect of thermal slip parameter S_t and permeability parameter K on heat transfer rate

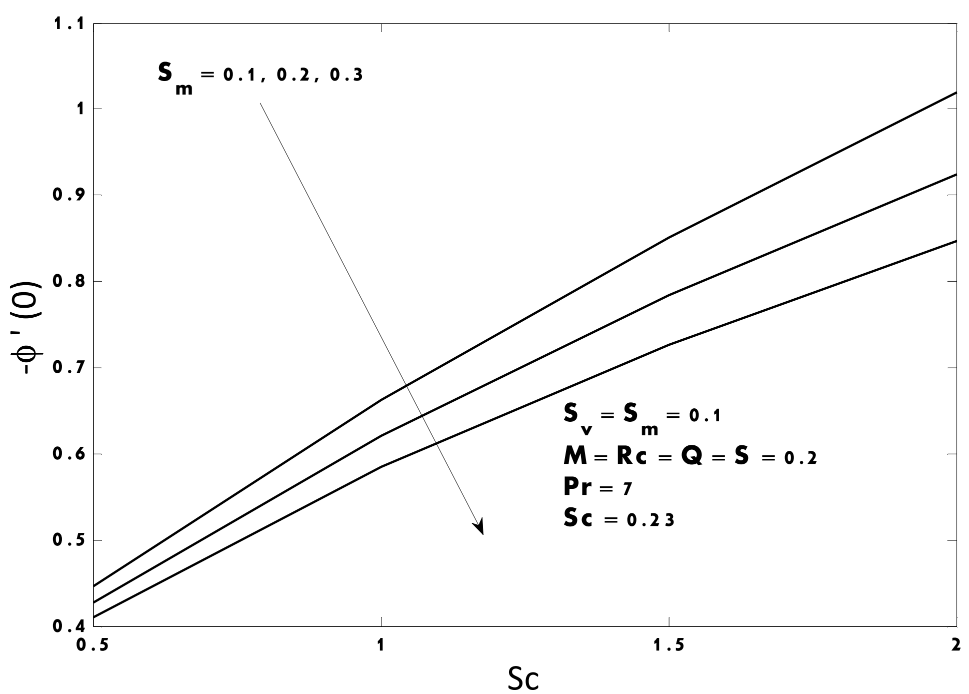

Figure 17. Effect of concentration slip parameter Sm and Schmidt number on mass transfer rate.

4.5. Discussion

The system of nonlinear higher ordinary differential equations (4.20), (4,21)and (4.22) with boundary condition (4.23) were solved numerically using Keller box Method. The effect of different parameters like Hart Mann numbers, the permeability parameter , suction/injection parameter, Prandtl number, heat source and chemical reaction parameter on velocity, temperature, concentration profile, skin friction coefficient, heat and mass transfer rates have been analyzed.

Figures 3, 4, 5 and 6 show the effect of suction/injection parameter s, Hartman number M permeability parameter K and velocity slip parameter Sv on velocity profile respectively, while the other parameters are constant. We observed from the figures that increasing suction/injection parameter, Hartman number, and velocity slip parameter is decreasing in velocity while increasing permeability parameter raises the velocity profile of the fluid. Figures 7, 8, 9 and 10 show the effect of thermal slip parameter, Prandlt number, heat source/sink parameter and Hartman number on temperature profile, respectively. The figures reveal that increasing thermal slip parameter and Prandtl number tends to decrease temperature profile and increasing heat source/sink parameter and Hartmann number rises the temperature. Figures 11, 12 and 13 shows the effect of concentration slip parameter, Schmidt number and chemical reaction parameter on concentration profile respectively. From the figures one can observe that increasing each of the parameters diminishes species concentration. Effect of velocity slip parameter and permeability parameter on skin friction coefficient is displayed in figure 14. The figure depicts that reducing the velocity slip parameter and an increase in permeability parameter increase the skin friction coefficient. Figure 15 shows the effects of velocity slip parameter and Hartman number on skin friction coefficient. It is observed from the figure that a decrease in the velocity slip parameter and an increase in Hartmann number increase the skin friction coefficient.

Figure 16 shows the effect of thermal slip parameter and permeability parameter surface heat transfer rate. From the figure one cam conclude that decreasing the thermal slip parameter and an increase in permeability parameter raises the surface heat transfer rate. Figure 17 exhibits effect of concentration slip parameter and Schmidt number on surface mass transfer rate. The figure reveals that a decrease in concentration slip parameter and an increase in Schmidt number increases surface mass transfer rate.

CHAPTER FIVE

CONCLUSION AND SCOPE FOR THE FUTURE WORK

5.1. Conclusion

In this study, the effect of different parameters like Hart Mann number, the permeability parameter, Prandtl number heat source//sink parameter and chemical reaction parameter on velocity, temperature, concentration profile, skin friction coefficient, surface heat and mass transfer rates have been analyzed by Keller box method. Briefly the above discussion can be summarized as follows:

- ❖ Increasing suction/injection parameter, Hartman number, and velocity slip parameter is decreasing in velocity while increasing permeability parameter raises the velocity profile of the fluid.
- ❖ Increasing thermal slip parameter and Prandtl number tends to decrease temperature profile and increasing heat source/sink parameter and Hartmann number rises the temperature.
- ❖ Increasing concentration slip parameter, Schmidt number and chemical reaction parameter diminishes species concentration.
- ❖ Reducing the velocity slip parameter and an increase in permeability parameter increase the skin friction coefficient.
- ❖ Decrease in the velocity slip parameter and an increase in Hartmann number increase the skin friction coefficient.
- ❖ Decreasing the thermal slip parameter and an increase in permeability parameter raises the surface heat transfer rate.
- ❖ Decreasing concentration slip parameter and an increase in Schmidt number increases surface mass transfer rate.

5.2. Scope for the future work

In the present thesis, numerical solution obtained for hydro magnetic incompressible laminar fluid flow over nonlinear stretching sheet in presence of heat source/sink by Keller box method. So, one can find the solution for the problem of unsteady incompressible fluid flow over nonlinear stretching/shrinking sheet.

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