THE RATE OF STAR FORMATION IN GALAXIES

By

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Abstract

The physics of star formation that includes the processing agents and environmental conditions are fundamental issues in astrophysics. Observational data of nearby galaxies, over broad range of galactic environments and metallicities indicate that, star formation occurs mainly in the molecular phase of the interstellar medium (ISM). This star forming environmental correlation is currently being used in modeling variety of statistical techniques to estimate physical parameters.

However, recent progress in the field also shows new observational discoveries including the dark matter & dark energy correlation to the phenomena. So there are more and more complications in understanding the large-scale star formation, one which incorporates our new understanding of the star formation within clouds and the ensemble properties of star formation in any galaxy including our own Milk-Way galaxy. So, with this motivation we raised the current 3-component interaction model of star forming system: Molecular cloud - Atomic gas - Active stellar evolution system.

It is a simple realistic model that considers the physical interaction of the system that considers the standard physics we have at hand. With the recently raised computational concept we want for further analytical models and solutions how transfer process occurs between the system and how it influence the rate of formation.

Comparing the theoretically generated result with that of observation, we found an encouraging result that shall be further investigated and adopted to progress the development of the field in the future.

Keywords: Star formation-molecular cloud-atomic gas-stellar evolution-star forming interaction model.

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General Introduction

I. Literature review and background

Stars are formed in molecular clouds in the interstellar medium, which consist mostly of molecular hydrogen (primordial elements made a few minutes after the beginning of the universe and dust). The dust originates from the cool surfaces of super giant, massive clouds. The estimated clouds size ranges from less than a light year to several hundred light years across and range in mass from 10 to 10 million on solar masses. Gravity plays great role for the core collapse of these star forming molecular clouds [3].

The gravity driven collapsing molecular cloud breaks in to piece in a hierarchical manner until the fragment reach stellar mass. In each of these fragments the collapsing gas radiates away the energy gained by the release of gravitational potential energy. As the density increases the fragment become opaque and thus limits radiation. In effect, this raises the temperature of the cloud and inhibits further fragmentation. The fragment condenses in to rotating spheroids of gas that serve as stellar embryos. Complicating this picture of a collapsing cloud are the effect of turbulence, macroscopic flows, rotation, magnetic fields and the cloud geometry. Both rotational and magnetic fields can hinder the collapse of a cloud. Turbulence is instrumental in causing fragmentation of the cloud and, the smallest scale it promotes collapse [17].

During the collapse, the density of the cloud increases towards the center and thus the middle region became optically opaque first at density about $10^{-13} gcm^{-3}$. This region is considered as the first hydrostatic core, forms where the collapse is essentially halted. It continues to increase in temperature as determined by the virial theorem. The gas falling toward this opaque region collides with it and creates shock waves that heat the core.

When the core temperature reaches about 2000k, the thermal energy dissociates the H_2 molecule, followed by the ionization of the hydrogen and helium atom. The estimated density of in falling material is about $10^{8} gcm^{-3}$, where it is sufficiently transparent to allow energy radiated by the protostar to escape. The combination of convection within the protostar and radiation from its exterior allow the star to contract further. The belief is that the process continues until the gas is hot enough for the internal pressure to support the protostar against further gravitational collapse. When the accretion rate is nearly completed the resulting object is considered as protostar [19].

In general, the star formation process and relevant parameters is a complicated issue. It needs diverse fields for accurate theory. Environmentally, the effects of local and global activities need careful modeling. There is strong environmental correlation to the formation and evolution scenario. Today the evolution of dark matter is considered to describe in shaping the large scale structures of the universe. The physics of baryons, including star formation, is much more difficult to understand and to implement in models of galaxy formation and evolution. Stars are believed to form from the collapse of giant molecular clouds(GMCs) the gas loss pressure, flows towards the center of halo potential well while its density increase. Once the gas density exceeds the density of the dark matter, the gas continuous to collapse under its own gravitational potential. In the presence of efficient cooling, collapse continues until matter becomes dense enough to enable the formation of stars.

Observationally, the measured spectrum of a galaxy contains, in principle, information concerning the physical processes that led to its formation and evolution. The amount of gas transformed into stars, the metallicity of that gas and the dust produced in it at a given time all affect the integrated light of a galaxy. The nearby spectra is considered to contain a precious fossil record concerning the conditions of the interstellar medium in the past, and can be compared with methods based on measurements of recent star formation activity, measured at different redshifts. But there are challenges of recovery all the information contained in the spectrum. This limits the amount of information that can be extracted and also introduces artificial degeneracies among the parameters that could be lifted using all information in the spectrum.

However, within this limit there is a great deal of progress by way of finding constraints from observations by measuring a reliable star formation rate (SFR) at various scales and red shifts, and studying the main drivers of its variations. The SFR and stellar masses M_{star} are the main parameters estimated from large samples of galaxies. A large number of works found a tight relation between the SFR and M_{star} both at low and high red shift, often called **the Main Sequence (MS) of galaxies**. Major progress has been made in the measure of the SFR inside the disk of nearby galaxies, with a strong relation found between the SFR and the molecular gas content, and measurement of the star formation efficiency in galaxies is now possible up to large redshifts. Besides these studies aimed at understanding the process of star formation, the global amount of star formation in the universe is measured by building statistical samples, with observables related to the recent star formation [15].

Molecular clouds are shaped in to a complex filamentary structure by supersonic turbulence, with only a small fraction of the clouds mass channeled in to collapsing protostar over a free-fall of the system. In recent years the physics of supersonic turbulence has been widely explored with computer simulations, leading to statistical models of this fragmentation process and to the prediction of the star formation rate, as a function of fundamental physical parameters molecular clouds ,such as the virial parameters, the rms much number, the compressive fraction of the turbulent driver, and the ratio of gas to magnetic pressure. Infrared space telescopes, as well as ground based observations have provided un precedented probes of the filamentary structure of molecular clouds and the location of star within them [4]. The recent recognition of massive near by clouds with little star formation rates in even near by clouds of similar mass can vary considerably as much as an order of magnitude [12]. Therefore, systematic and comparative observational studies of the physical properties of local molecular clouds and this relation of these properties to the varying levels of star formation activity within them could lead to new insights concerning the underlying physics controlling the star formation rates in molecular gas.

II. Statement of the problem

The physics of star formation that includes the processing agents and environmental conditions are fundamental issues in astrophysics. Observational data of nearby galaxies, over broad range of galactic environments and metallicities indicate that, star formation occurs mainly in the molecular phase of the interstellar medium (ISM).

This star forming environmental correlation is currently being used in modeling variety of statistical techniques to estimate physical parameters. Theoretical models are in progress in diverse approaches attempting to address the observation. There is an encouraging astrophysical work on star formation, both observationally and theoretically. On the other hand, new observational discoveries open up the complication of large-scale star formation, one which incorporates our new understanding of the star formation within clouds and the ensemble properties of star formation in our own MW into our more limited but broader understanding of star formation in external galaxies. So, though the subject itself is growing and transforming rapidly it is also relatively immature field.

Research questions

- Where, when and how star formation in molecular clouds are being stimulated?
- How is star formation related to nearby stellar evolution?
- In what way does cool atomic cloud determine star formation?

III. Objectives

a) General objective

To study the rate of star formation in galaxies.

b) Specific objectives

- To derive relevant factors that are correlated to star formation stimuli in molecular clouds.

- To find the impact of nearby stellar evolution on cool molecular cloud in the process of star formation.
- To determine the interaction effect of cool atomic cloud on molecular cloud in star formation.

IV. Methodology

The 3-component star forming interaction system: Molecular cloud - Atomic gas - Active stellar evolution system is being considered. Mainly the model is so considered due to its simplicity but realistic from point standard physics we have at hand. In modeling of the system we assumed the rate at which a star forming molecular cloud accretes from its surrounding and then we do derive the proportionality constant of the accretion rate in terms of the total initial masses of the interacting system, the characteristic oscillation frequency of the molecule and the coupling constants. Then, we generate theoretical data using MATHEMATIC 11 for analysis of the result being compared with observation for its validity.

Chapter 1

Basic definitions of stars and star formation theory

1.1 Basic Definition of Star

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A star is an object that radiates energy from an internal source and is bound by its own gravity. On the other hand star is type of astronomical object consisting of a luminous spheroid of plasma held together by its own gravity and it is a huge glowing ball of hot gas, mainly hydrogen and helium. The nearest star to Earth is the Sun [1].

1.2 What are stars made of ?

Almost all stars, including our own Sun, are primarily composed of the two lightest elements on the periodic table, hydrogen (73%) and helium (25%), while the remaining (2%) consist of other heavier elements. It makes sense that stars would contain significant amounts of hydrogen as it is the most abundant element in the entire Universe. Logically, helium is the second most abundant as it is created from hydrogen fusion. Interestingly, this pattern does not continue for the heavier elements with oxygen being the third most abundant. In all, the answer to " What are stars made of? " can simply be answered by: hydrogen, helium, and a small percent of everything else [9].

1.3 Theory of star formation

Star formation is the process by which dense regions within molecular clouds in stellar space, sometimes referred to as stellar nurseries or star forming regions collapse to form stars [17].

Stars are born in the molecular interstellar medium, the raw material for star formation and lives for a certain amount of time on its internal energy supply, and eventually dies when this supply is exhausted. The definition imposes that stars can have only a limited range of masses, between $\sim 0.1 M_\odot$ and $\sim 1000 M_\odot$. In the Milky Way, all star formation essentially takes place in molecular clouds and most star formation takes place in giant molecular clouds(GMC) with mass $M > 10^5 M_\odot$ and not the diffuse neutral ISM dominated by atomic hydrogen (extended HI gas disk) [13].

The star formation rate (SFR) of molecular clouds can be estimated from the far-infrared (FIR) luminosity emitted by the warm dust heated by embedded high-mass OB stars . All strong high-mass star formation regions are associated with GMCs, especially the cores of GMCs. The ratio of FIR luminosity to the CO luminosity, or to the cloud mass, a measure of the SFR per solar mass of the cloud and an indicator of star formation efficiency (SFE), ranges over a factor of 100 for different clouds, and over a factor of 1000 from clouds to the cores of GMCs [13].

An understanding of the physical conditions in GMCs and their relation to galactic dynamics is a prerequisite to the understanding of the star formation process, the SFR in galaxies and star bursts [10].

Star formation in galaxies is closely tied up with the local gas density although the important component is the molecular gas. Globally, the SFR correlates with the molecular gas content in galaxies, as traced (found) by CO emission, including luminous and ultra luminous infrared galaxies (LIGs and ULIGs1). In Galactic star-forming regions, active high-mass star formation is intimately related to the very dense molecular gas in the cores.

Stars are considered to be isolated in space, so that their structure and evolution depend only on intrinsic properties (mass and composition). For most single stars in the galaxy this condition is satisfied to a high degree (compare for instance the radius of the Sun with the distance to its nearest neighbor Proxima Centauri). However, for stars in dense clusters, or in binary systems, the evolution can be influenced by interaction with neighboring stars. Also, stars are formed with a homogeneous composition, a reasonable assumption since the molecular clouds out of which they form are well-mixed [13].

In practice there is relatively little variation in composition from star to star, so that the initial mass is the most important parameter that determines the evolution of a star. Moreover, spherical symmetry, which is promoted by self-gravity a good approximation for most stars. Deviations from spherical symmetry can arise if non-central forces become important relative to gravity, in particular rotation and magnetic fields. Although many stars are observed to have magnetic fields, the field strength (even in highly magnetized neutron stars) is always negligible compared to gravity. Finally, understanding the structure and evolution of stars, and their observational properties, requires laws of physics involving different areas (e.g. thermodynamics, nuclear physics, electrodynamics, plasma physics) [6].

Star formation occurs as a result of the action of gravity on a wide range of scales.On galactic scale the tendency of interstellar matter to condense under gravity in to star forming clouds is countered by galactic tidal forces, and star formation can occur only where the gas become dense enough for its self gravity to overcome these tidal force, for example in spiral arms. On the intermediate scales of star forming giant molecular clouds (GMCs),turbulence and magnetic fields may be the most important effects counteracting gravity,and star formation may involve the dissipation of turbulence and magnetic fields [1].

1.4 Molecular Clouds

Molecular clouds are over dense regions in the Milky Way disk predominantly composed of molecular H_2 and CO and dust. Because they are dense, their dust and gas is self shielding the cloud from stellar optical and UV-light from the outside. For this reason the molecular clouds can not be seen in the visual except for the fact that they obscure the background objects. The dark irregular bands of absorption in the Milky Way are due to these absorbing clouds. The best way to see molecular clouds are CO line observations, e.g. at $\lambda = 2.6mm$,in the radio range. The following types of molecular clouds are distinguished [13]:

-Bok globules: They are the opaque clouds of dense gas and dust which are small, isolated, gravitational bound molecular clouds of $\leq 100M_{\odot}$ in which at most a few stars are born.

-Molecular clouds have masses of $10^3 M_\odot - 10^4 M_\odot$ distributed in irregular structures with dimensions of $\approx$ 10 pc consisting of clumps, filaments bubbles and containing usually hundreds of new-born stars,

-Giant molecular clouds are just larger than normal molecular clouds with a total mass in the range $10^5 M_\odot - 10^7 M_\odot$,dimensions up to 100 pc, and thousands of young stars.

-**Molecular clouds in the solar neighborhood**: The sun resides in a hot ($10^6 K$), low density bubble with a diameter of $\approx$ 50 pc. The nearest star forming clouds are located at about 140 pc and because of their proximity (being proximate or closeness) they are important regions for detailed investigations of the star and planet formation process. Well studied regions are [6]:

-The Taurus molecular cloud at a distance of about 140 pc is a large, about 30 pc wide, loose association of many molecular cores with a total mass of about $\approx 10^4 M_\odot$ and several hundred young stars. Because of its proximity there are many well known prototype objects, like T Tau or AB Aur in this star forming region [9].

-The ρ cloud at a distance of 130 pc has a denser gas concentration than Taurus with a main core and several additional smaller clouds and about 500 young stars with an average age of about 0.2 Myr. The total gas mass is about $\approx 10^4 M_\odot$.

-**The orion molecular cloud complex**: has a distance of about 400 pc and a diameter of 30 pc. Orion is the nearest high mass star forming region with in total about 10000 young stars with an age less than 15 Myr. The Orion molecular cloud complex includes the Orion nebula M42 (HII region), reflection nebulae, dark nebulae (Horse head nebula), an OB associations mainly located in the Belt and Sword of the Orion constellation. The Orion nebula is ionized by the brightest star in the Trapezium cluster [18].

1.5 Turbulence in molecular clouds

One of the major problems in star-formation theory has been to understand why star formation is so inefficient. Only about (1%)of gas in molecular clouds is converted to stars. The updated star formation law presented by the authors relies on the recent consensus that turbulence - random particle motions - is crucial for preventing too much gas from collapsing into stars. Check out this Astrobite for a video that shows how important turbulence is for regulating star formation and take a look at this article for an artistic treatment of turbulence. One way we define just how turbulent a gas is to use the Mach number of the gas. The Mach number is a measure of the turbulent motions in the gas compared to the speed of sound, and can be determined by observing the width of emission lines from molecules such as carbon monoxide. The higher the Mach number, the more turbulent the gas, and the greater range in gas densities in the cloud. Therefore, a higher Mach number means more gas will have sufficient density to collapse into stars, resulting in a higher star formation rate. This relation between Mach number and density variance forms the basis for a new star formation law.

1.6 Cloud formation

Since molecular clouds are transient (temporary) features, it follows that they are constantly being formed and destroyed. It is necessary to understand the process by which they are continually being reassembled from more dispersed gas [14].

The rate at which interstellar gas is presently being collected in to star forming molecular clouds in our galaxy is related to the star formation rate and it can be estimated empirically from the observed star formation rate and the efficiency of star formation in molecular clouds. The total rate of star formation in our galaxy is of the order of $3M_{\odot}$ per year. Since only about few percent or less of the mass of a typical molecular cloud converted in to stars; it implies that at least $150M_{\odot}$ of gas per year is being turned in to star forming molecular clouds. The total amount of gas in our galaxy is about 5×10^9M [17]. Two possible formation mechanisms for molecular clouds that have been considered are;

(1) cloud growth by random collisions and coalescence and

(2) gravitational instability or swing amplification.

1.7 Cloud collapse

An interstellar cloud of gas will remain in hydrostatic equilibrium as long as the kinetic energy of the gas pressure is in balance with the potential energy of the internal gravitational force. **Mathematically this is expressed using the viral theorem, which states that, to maintain equilibrium, the gravitational potential energy must equal twice the internal thermal energy**(which we will see in chapter 2). If a cloud is massive enough that the gas pressure is insufficient to support it, the cloud will undergo gravitational collapse. The mass above which a cloud will undergo such collapse is called the Jeans mass. The Jeans mass depends on the temperature and density of the cloud, but is typically thousands to tens of thousands of solar masses. This coincides with the typical mass of an open cluster of stars, which is the end product of a collapsing cloud [10].

As it collapses, a molecular cloud breaks into smaller and smaller pieces in a hierarchical manner, until the fragments reach stellar mass. In each of these fragments, the collapsing gas radiates away the energy gained by the release of gravitational potential energy. As the density increases, the fragments become opaque and are thus less efficient at radiating away their energy. This raises the temperature of the cloud and inhibits further fragmentation. The fragments now condense into rotating spheres of gas that serve as stellar embryos [2]. **Cases;**

1.If the collapsing cloud is too low: If the cloud has $M < 0.8M_{\odot}$, it will contract, heat up, but the central temperature will never reach the 10,000,000K limit required to start the conversion of H to He. The outer layers get warm, enough to appear similar to cool, dim stars for a few million years, but after that they steadily fade away. Such objects are called brown dwarfs.

2.If the collapsing cloud is too large: If the mass of the cloud exceeds about $100M_{\odot}$, it will collapse and heat up very quickly. Nuclear reactions occur so rapidly that the star becomes very luminous and blows itself apart - either catastrophically or more gently by blowing off only the outer layers.

1.8 Cloud Collapse and Fragmentation

A giant molecular cloud must begin forming stars soon after the cloud itself has formed, since relatively few of the largest molecular clouds are not forming star. Even if as many as half of all molecular clouds are not forming stars, the time delay between the formation of a molecular cloud and the onset of star formation in it cannot exceed the subsequent duration of the star formation activity which is of the order of 10Myr and comparable to the internal dynamical time scale. Since it takes somewhat longer than this to build large molecular clouds it is likely that star formation begin already in molecular clouds while they are still being assembled; moreover, star formation must begin within a time not much longer than the dynamical or free-fall time of such a cloud. Collapse and star formation can occur in the densest part of a cloud even if the cloud as a whole is not collapsing, and this must in fact be what usually happens because there is no evidence that most star forming clouds are undergoing any rapid over all collapse, and there is even evidence that many of them are being dispersed [12].

Star formation involves the collapse of a cloud or part of a cloud under gravity and the association fragmentation of the cloud in to smaller and smaller bound clumps. This is expected to occur because molecular clouds typically contain many times the **"Jeans mass"**, which is the minimum mass for gravitational bound fragments.

The near-constant low temperature across molecular clouds is an important feature of the star formation process because of its influence on the Jeans mass, and it is what makes possible the collapse of pre stellar cloud cores with masses as small as one solar mass. The Jeans mass is the critical mass at which a cloud becomes unstable and starts to collapse, as it possesses insufficient pressure support to balance the force of gravity. In the absence of pressure or other support, gravitational collapse of such a cloud will occur in a free fall time:

$$\tau_{ff} = \sqrt{\frac{3\pi}{32G\rho}} \tag{1.8.1}$$

The star formation process in molecular clouds appears to be fast. Once the collapse of a cloud region sets in, it rapidly forms an entire cluster of stars with in 10^6 years or less. The resulting stellar population is widely dispersed throughout the cloud and, since collapsing clumps are frequently destroyed by shock interaction, the overall star formation rate is low [4].

Observations show that the gas (which contains mainly molecular hydrogen, H2) is highly clumpy, and virtually all molecular gas distribute in GMCs [11].

The molecular cloud always rotates due to deferential in the disk in which they are formed. If a collapse conserves angular momentum, this would imply a notation period of well below 1 sec of the emerging star. This means that angular momentum has to be transferred during the collapse.

Further more potential energy of the clouds $E_P \propto -\frac{GM^2}{r}$ is released during the collapse. This energy therefore must be radiated or transported away, despite the high opacity of the surrounding medium. Present-day star formation in our galaxy is observed to take place in cold molecular clouds which appear to be in a state of highly compressible magneto hydrodynamic (MHD) turbulent on large scales from hundreds to thousands of parsec (Braun 1999).

An interstellar cloud of gas will remain in hydrostatic equilibrium as long as the kinetic energy of the gas pressure is in balance potential energy of the internal gravitational force. Mathematically this is expressed using the virial theorem which state that to maintain equilibrium the gravitational potential energy must equal twice the internal thermal energy which is given by:

$$2K + U = 0 \tag{1.8.2}$$

If $2K < |U|$,the cloud will collapse under the force of gravity.Then the gravitational potential energy can be written as:

$$U \simeq \frac{3}{5}(\frac{GM_C^2}{R_C}) \tag{1.8.3}$$

where M_C and R_C are respectively the mass and the radius of the cloud.

The average kinetic energy per particle is $K = \frac{3}{2}\kappa\mathrm{T}$ where κ is Boltzman constant. Thus, the total internal kinetic energy of the cloud is just $K = \frac{3}{2}\mathrm{N}\kappa\mathrm{T}$, where N is the total number of particles. We can write N in terms of the mass and the mean molecular weight as:

$$N = \frac{M_C}{\mu m_H} \tag{1.8.4}$$

We can therefore write the condition for gravitational collapse $(2K < |U|)$.

$$\frac{3M_C}{\rho m_H}KT < \frac{3GM_C^2}{5R_C} \tag{1.8.5}$$

$$R_C = (\frac{3M_C}{4\pi\rho_O})^{1/3} \tag{1.8.6}$$

where ρ_0 is the initial density of the cloud prior to collapse with the assumption that the cloud is a sphere of constant density. By substituting we obtain the important concept of the Jeans mass.

$$M_C \simeq (\frac{5KT}{Gm_H})^{\frac{3}{2}}(\frac{3}{4\pi\rho_0})^{1/2} \tag{1.8.7}$$

If a cloud is massive enough that the gas pressure is insufficient to support it, the cloud will undergo gravitational collapse. The mass above which a cloud will undergo such collapse is the Jeans mass. The Jeans mass depends on the temperature and density of the cloud, but is typically thousands to tens of thousands of solar mass [9].

$$M_J = 3 \times 10^4 \sqrt{\frac{T^3}{M}} M_\odot \tag{1.8.8}$$

where M_J is Jeans mass, $M_\odot$ is solar mass, M is cloud mass and T is temperature. In triggered star formation one of several events might occur to compress a molecular cloud and initiate its gravitational collapse. Molecular cloud may collide with each other or a nearby supernova explosion can be a trigger sending shocked matter in to the cloud at very high speed.The resulting new stars may themselves soon produce supernovae,producing self-propagating star formation. Alternatively, galactic collisions can trigger massive starbursts of star formation as the gas clouds in each galaxy are compressed and agitated by tidal force. The latter mechanism may be responsible for the formation of globular clusters [12].

1.9 Initial Mass Function and Star Formation

The stellar initial mass function (IMF) is defined as the distribution of stellar masses that form in a given region in one star burst event. Apart from being interesting in its own right, the IMF has important consequences for the observable properties of galaxies. The relative abundance of stars in different mass ranges affects various measurable quantities: most of the stellar mass is contributed by low-mass stars, the luminosity is mainly due to massive stars, while intermediate-mass and massive stars are responsible for enriching the interstellar medium (ISM) with metals.

Understanding the physics of star formation and the form of the IMF is therefore crucial for correct interpretation of observations of galaxies. In particular, we would like to have an understanding of the masses of forming stars as a function of the physical conditions in the star formation region, the efficiency of star formation, and its rate [13].

Collapsing interstellar clouds form stars in the mass range from 0.1 to $100M_\odot$. The initial mass function (IMF) describes the mass distribution for the formed stars. According to the classical work of Salpeter (1955), this distribution can be described for stars of about solar mass and above with a potential law of the form:

$$\frac{dN_S}{dM} \propto M^{-2/3} \tag{1.9.1}$$

for $M > M_\odot$

This relation is often given as a logarithmic power law of the form

$$\frac{dN_S}{dlogM} \propto M^{-1.35}$$

because

$$\frac{dN_S}{dM} = \frac{dN_S}{dlogM}\frac{dlogM}{dM} = \frac{1}{M}\frac{dN_S}{dlogM} \tag{1.9.2}$$

This law indicates, that the number of newly formed stars with a mass between 1 and $2M_\odot$ is about 20 times larger than the stars with masses between 10 and $20M_\odot$. If we consider the gas mass of the molecular cloud, then about twice as much gas ends up in stars between 1 and $2M_\odot$ when compared to stars with masses between 10 and $20M_\odot$. The initial mass function seems to be valid for many regions in the Universe, for the star formation in small molecular clouds, larger cloud complexes, and the largest star forming regions in the local Universe. Up to now no star forming regions have been found for which the Salpeter IMF is a bad description [18].

For low mass stars the mass distribution shows a turn over. Since M-stars $M < 0.5M_{\odot}$ have a main-sequence life time which is longer than the age of the universe we can just use as first approximation the frequency of stars with different spectral types as rough description for the IMF of low mass stars.

Considering the complex physics involved in the star formation process is surprising that the initial mass function is such an universal law which seems to be valid everywhere in the Universe. There must be one essential process which dominates the outcome of the stellar mass distribution. This could be the fragmentation process. Further it seems to be clear that there are different regimes of formation between stars and planets. The low frequency of sub stellar object in the mass range 0.01-0.1$M_{\odot}$ indicates that such objects are not easily formed via the normal star forming process, perhaps because the formation of small fragments or their survival in molecular clouds is rather unlikely. On the other side the planets are very frequent but seem to form predominantly around stars. This indicates that there exists a bimodal formation mechanism of hydrostatic astronomical objects.

- Stars are formed by the collapse and fragmentation of clouds,
- The Planets are the result of a formation process in circum stellar disks.

1.9.1 Low mass and high mass star formation

Stars of different masses are thought to form by slightly different mechanisms. The theory of low-mass star formation, which is well-supported by a plethora of observations, suggests that low-mass stars form by the gravitational collapse of rotating density enhancements within molecular clouds. The collapse of a rotating cloud of gas and dust leads to the formation of an accretion disk through which matter is channeled onto a central proto star. For stars with masses higher than about $8M_{\odot}$, however, the mechanism of star formation is not well understood [3].

Massive stars emit copious quantities of radiation which pushes against in falling material. In the past, it was thought that this radiation pressure might be substantial enough to halt accretion onto the massive protostar and prevent the formation of stars with masses more than a few tens of solar masses. Recent theoretical work has shown that the production of a jet and outflow clears a cavity through which much of the radiation from a massive protostar can escape without hindering accretion through the disk and onto the protostar. Present thinking is that massive stars may therefore be able to form by a mechanism similar to that by which low mass stars form. Another theory of massive star formation suggests that massive stars may form by the coalescence of two or more stars of lower mass [17].

Chapter 2

States and dynamics of galaxy in star formation

2.1 Definition of Galaxy

A galaxy is a gravitationally bound system of stars, stellar remnants, interstellar gas, dust, and dark matter.

They usually contain several million to over a trillion stars and can range in size from a few thousand to several hundred thousand light years across. There are hundreds of billions of galaxies in the Universe and they are vary in size, structure, and luminosity, and,like stars, are found alone, in pairs, or in clusters [12].

2.2 Star Formation in Active and Normal Galaxies

Normal Galaxies:A galaxy that has a lens shaped central portion with two arms that begins to coil in the same plane and in the same fashion immediately up on emerging from opposite sides of it [8].

2.2.1 Spiral Galaxies

Spiral galaxies get their name from the shape of their disks, in which stars, gas and dust are concentrated in spiral arms that extend outward from the central nucleus of the galaxies. They are divided into three main types according to how tightly wound the spiral arms are. These are; Sa, Sb and Sc. Sa galaxies have very tightly wound arms around a larger central nucleus. Sc galaxies have very loosely wound arms around a smaller nucleus. Sbs are between, having moderately wound arms around an average sized nucleus.

Our Milky Way galaxy is an example of a spiral galaxy. Spiral galaxies are rich in gas and dust and have a high rate of star formation. Since spirals contain a high fraction of hot, young stars, they are often among the brightest galaxies in the universe [10].

2.2.2 Elliptical Galaxies

Elliptical galaxies are elliptical in shape and are divided into eight subgroups: E0-E7 depending on their elongation. E0 elliptical are nearly circular, while E7s are highly elongated. Elliptical galaxies contain primarily old stars, and do not have much gas and dust. There is very little new star formation in these galaxies.That means most elliptical contain older, low-mass stars, and because they lack a great deal of star-making gas and dust clouds, there is little new star formation occurring in them. Ellipticals can have as few as a hundred million to perhaps a hundred trillion stars, and they can range in size from a few thousand light-years across to more than a few hundred thousand [7].

2.2.3 Irregular Galaxies

Irregular galaxies have no particular shape. They are among the smallest galaxies and they contain a vast amount of gas and dust. As a result they have a very high rate of star formation. The Large and Small Magellanic Clouds are examples of irregular galaxies.

The infrared emission from galaxies comes primarily from three sources: stars, interstellar gas, and dust. The emission from stars peaks in the near-infrared (1-3 micrometers). Emission from atoms and molecules in interstellar gas makes up only a few percent of the infrared output of galaxies. The primary source of infrared radiation beyond 3 micrometers is thermal emission from dust grains heated by starlight. The brightest infrared galaxies are usually the ones which have a lot of dust (from star-forming regions for example). Spiral galaxies which are rich in gas and dust are strong infrared sources and are still forming new stars. About half of the luminosity of an average spiral galaxy is radiated at far-infrared wavelengths. Elliptical galaxies are faint in the infrared because they do not have much gas and dust [2].

2.3 Active galaxies:

They are galaxies that have a small core of emission embedded at the center of an otherwise typical galaxy. This core is typically highly variable and very bright compared to the rest of the galaxies [14].

It was determined that active galaxies have higher stellar masses (SMs) within the central kilo parsec radius than normal galaxies do independent of the Hubble types of the host galaxies; but both active and normal galaxies exhibit similar specific star formation rates (SSFRs),ranging between $10^{-10.5}$ and $10^{-9.5}$ yr^{-1}. The central SM surface density might be used as an indicator to identify AGNs. It also discovered that certain AGNs exhibit substantial inner stellar structures in the IR images; most of the AGNs with inner structures are Seyferts, whereas only a few LINERs exhibit inner structures. We note that the AGNs with inner structures show a positive correlation between the radio activity of the AGNs and the SFRs of the host galaxies, but the sources without inner structures show a negative correlation between the radio power and the SFRs [19].

2.4 Star formation triggers in galaxies

Trigger in star formation is a process that promotes the formation of dense cold clouds in the interstellar media of galaxies and these processes are reviewed [10].

Those that involve background stellar mass include two-fluid instabilities, spiral density wave shocking, and bar accretion. Young stellar pressures trigger gas accumulation on the periphery of cleared cavities, which often take the form of rings by the time new stars form. Stellar pressures also trigger star formation in bright-rim structures, directly squeezing the pre-existing clumps in nearby clouds and clearing out the lower density gas between them. Observations of these processes are common. How they fit into the empirical star formation laws, which relate the star formation rate primarily to the gas density, is unclear. Most likely, star formation follows directly from the formation of cold dense gas, whatever the origin of that gas. If the average pressure from the weight of the gas layer is large enough to produce a high molecular fraction in the ambient medium, then star formation should follow from a variety of processes that combine and lose their distinctive origins. Pressurized triggering might have more influence on the star formation rate in regions with low average molecular fraction. This implies, for example, that the arm/inter arm ratio of star formation efficiency should be higher in the outer regions of galaxies than in the main disks [7].

2.4.1 Galaxy interaction induced star formation at different red shifts

Galaxies form their stars with a pretty similar fashion.Galaxies with a higher gas surface density will obviously form stars at a higher rate, but we see that they do so along a well defined sequence. Isolated galaxies lie on the Sequence of Disks. However some galaxies are seen above this sequence, which means that they are forming stars at a much faster rate than expected from their gas distribution [9].

In the local Universe, detailed observations of these star bursting galaxies showed that they are all undergoing an interaction with another similar galaxy. These interactions will eventually lead to a merging of the two galaxies into a single system, the merger remnant.

Accurate measurements of the star formation rate (SFR) of galaxies up to very high-red shift have shown that the fraction of galaxies in the star bursting mode of star formation stays very close to 2%, with no clear trend with red shift [6].

However, both observations and cosmological simulations show that the fraction of galaxies undergoing a star burst per unit time increases with red shift.

Therefore, mergers and interactions look much less efficient to trigger this star bursting mode of star formation at high red shift. The principle difference between low and high-red shift massive and star forming galactic disks is the importance of their gas component with respect to their total mass, which can amount to 60% while it is only around 10% for local galaxy. It results in a highly turbulent disk with a clumpy morphology.

The simulations show that the star formation rate of the low gas fraction galaxy is strongly enhanced during the interaction, as is expected from observations of local merging systems. However, the star formation rate of the gas-rich case is only weakly enhanced by the interaction.The analysis shows that the physical mechanisms responsible for the star bursting mode of star formation in local galaxies are indeed less triggered by the interaction when the gas fraction is high [19].

2.5 Starburst galaxy

A star burst galaxy is a galaxy undergoing an exceptionally high rate of star formation, as compared to the long-term average rate of star formation in the galaxy or the star formation rate observed in most other galaxies. For example, the star formation rate of the Milky Way galaxy is approximately $3M_{\odot}$(three times sola mass), however, star burst galaxies can experience star formation rates that are more than a factor of 100 times greater [12]. In a star burst galaxy, the rate of star formation is so large that the galaxy will consume its entire gas reservoir, from which the stars are forming, on a timescale much shorter than the age of the galaxy. As such, the star burst nature of a galaxy is a phase, and one that typically occupies a brief period of a galaxy's evolution. The majorities of starburst galaxies are in the midst of a merger or close encounter with another galaxy. Star burst galaxies include M82,NGC 4038/NGC 4039(the Antennae Galaxies), and IC10 [19].

2.6 Basic stellar evolutionary equations

The basic theory of stellar structure assumes spherical symmetry, so that all variables depend on only one thing, the distance (r) from the center of the star.On spherical shells of radius r, all physical variables (temperature, density, pressure chemical com position) are assumes to be uniform. The principle variables of stellar structure are pressure (P), temperature(T), density (ρ), luminosity (L) through a shell at r L(r)and mass interior to r M(r). For an isolated static, spherically symmetric star four basic laws/equations needed to describe structure [14] .
All physical quantities depend on the distance from the center of the star alone.
The fundamental hydrodynamic equations are being derived from the Boltzman transport equations. For ordinary stellar evolutions and formations the classical Maxwell Boltzmann distribution is considered.

2.6.1 Homogeneous Boltzmann Transport Equation

In stellar astrophysical, modeling gas flows around stars or in interstellar space, the ideal gas assumption is very much accurate. Therefore, in our analysis of the stellar evolution including magnetic field dynamism we apply the classical Boltzmann statistical distributions and derive the dynamic equations from Boltzmann transport equations [2].
The Boltzmann transport equation in six dimensional position-velocity phase space basically expresses the change in the phase density within a differential volume, in

terms of the flow through these faces, and the creation or destruction of particles within that volume. In the canonical position-momentum coordinate system, the Boltzmann transport equation (BTE) is given by [2]:

$$\sum_{i=1}^{3}\left[\dot{X}_i\frac{\partial f}{\partial X_i}+\dot{P}_i\frac{\partial f}{\partial p_{\,i}}\right]+\frac{\partial f}{\partial t} \quad = \quad s \Longrightarrow BTE \tag{2.6.1}$$

where $f \;=\; f(x,\dot{x},t)$is the number density distribution functions, s is the rate of particle creation /destraction,

$$\dot{x}_i \quad = \quad \frac{\partial x_i}{\partial t} \tag{2.6.2}$$

$$and \tag{2.6.3}$$

$$\dot{p}_i \quad = \quad \frac{\partial p_i}{\partial t} \tag{2.6.4}$$

This equation 2.6.1 can be recast in vector notation as

$$\frac{\partial f}{\partial t}+\overrightarrow{v}\cdot\overrightarrow{\nabla}_f+\overrightarrow{F}\cdot\quad\overrightarrow{\nabla}_p f= \quad S \tag{2.6.5}$$

where $\overrightarrow{F}$ is force and $\overrightarrow{\nabla}_i$ is the momentum gradient.

In conservative field system since $\overrightarrow{F}=-\overrightarrow{\nabla}\Phi$ where Φ is a scalar potential (example: gravitational scalar potential), then BTE will be given as :

$$\frac{\partial f}{\partial t}+\overrightarrow{v}\cdot\overrightarrow{\nabla}f-\frac{1}{m}\nabla\Phi\cdot\nabla_f \quad = \quad S \tag{2.6.6}$$

The potential gradient $\nabla\Phi$ has replaced the momentum time derivative while ∇_v is a gradient with respect to velocity. The quantity m is the mass of a typical particle. It is also not unusual to find the BTE written in terms of the total stokes time derivative

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \overrightarrow{v} \cdot \nabla \tag{2.6.7}$$

where $\overrightarrow{v}$is the flow velocity and $\frac{\partial}{\partial t}$ is the Eulerian time derivative. If we take ∇ to be a six-dimensional velocity and r to be a six-dimensional gradient the BTE becomes

$$\frac{Df}{Dt} = S \tag{2.6.8}$$

If the creation/destruction rate of particles is zero (S = 0), we will obtain the homogeneous Boltzmann Transport Equation (BTE) given as

$$\frac{\partial f}{\partial t} + \overrightarrow{v} \cdot \nabla f - \frac{1}{m}\nabla\Phi \cdot \nabla_v f = 0 \tag{2.6.9}$$

This is Liouvilles theorem. The physical interpretation of Liouville Equation is the 6N-dimensional analogue of the equation of continuity of an incompressible fluid. It implies that the phase points of the ensemble are neither created nor destroyed. In Astrophysics it is called the Vlasov equation, or sometimes the Collision less Boltz mann Equation. It is used to describe the evolution of a large number of collision less particles moving in a gravitational potential. In the case of classical statistical me chanics, the number of particles N is very large, (of the order of Avogadros number, for a laboratory-scale system) [6].

Setting $\frac{\partial \rho}{\partial t} = 0$ gives an equation for the stationary states of the system and can be used to find the density of microstates accessible in a given statistical ensemble. For example in an equilibrium of the Maxwell-Boltzmann statistical distribution ρ is given as

$\frac{H}{\rho\alpha\varrho(^{K_B T})}$

where H is the Hamiltonian, T is the temperature and K_B is the Boltzmann constant. The right-hand side of the BTE is a measure of the rate at which particles are created or destructed in the phase space volume.

Note that creation or destruction in phase space includes a good deal more than the conventional spatial creation or destruction of particles. To be sure, that type of change is included, but in addition processes which change a particles position in momentum space may move a particle in or out of such a volume. From BTE the right-hand side is zero or the creation or destruction rate of particles is zero, this is known as Homogeneous Boltzmann Transport Equation Liouvilles theorem of statistical mechanics.

2.6.2 Moments of the Boltzmann Transport Equation and Conservation of Laws

By the moment of a function we mean the integral of some properly of interest, weighted by its distribution function, over the space for which the distribution function is defined. The mean of a distribution function is simply the first moment of the distribution function, and the variance can be simply related to the second moment. In general, if the distribution function is analytic, all the information contained in the function is also contained in the moments of that function. The complete solution to the BTE is, in general, extremely difficult and usually would contain much more information about the system than we wish to know. The process of integrating the function over its defined space to obtain a specific moment removes or averages out much of the information about the function. This is a standard trick of mathematical physics and one which is employed over and over throughout this. Almost every instance of this type carries with it the name of some distinguished scientist or is identified with some fundamental conservation laws, but the process of its formulation and its origin are basically the same [8].

2.6.3 The Zero moment of Boltzmann Transport Equation and Conservation of Matter

To derive conservation laws and energy balance we start from; n_{th} functions of BTE, we have

$$M_n[f(x)] = \int x^n f(x)dx \tag{2.6.10}$$

The local spatial density is given as

$$\rho = m\int_{-\infty}^{+\infty} f(x,\overrightarrow{v})d\overrightarrow{v} \tag{2.6.11}$$

The related BTE is

$$\int_{-\infty}^{+\infty}(\frac{\partial f}{\partial t}+\sum_{i=1}^{3}v_i\frac{\partial f}{\partial x_i}+\sum_{i=1}^{3}v_i\frac{\dot{\partial} f}{\partial v_i}dv_i) = \int_{-\infty}^{+\infty} Sd\overrightarrow{v} \tag{2.6.12}$$

The integral of the creation rate S over all velocity space becomes simply the creation rate for particles in physical space, which we called $\Im$.

$$\frac{\partial}{\partial t}(\int_{-\infty}^{+\infty} fd\overrightarrow{v}+\int_{-\infty}^{+\infty}(\overrightarrow{v}\cdot\nabla f dv)+\int_{-\infty}^{+\infty}(\dot{\overrightarrow{v}}\cdot\nabla_v f))d\overrightarrow{v} = \Im \tag{2.6.13}$$

From this equation, the second term can be define by the vector identity

$$\overrightarrow{v} \cdot \overrightarrow{\nabla} f \quad = \quad \nabla \cdot (f \overrightarrow{v}) - f \nabla \cdot \overrightarrow{v} \tag{2.6.14}$$

Where $\overrightarrow{v} \cdot \overrightarrow{\nabla} f = \nabla \cdot (f \overrightarrow{v})$

Again the third term of the above equation is :

$$\dot{\overrightarrow{v}} \cdot \overrightarrow{\nabla_v} f \quad = \quad -\frac{\overrightarrow{\nabla}\Phi}{m} \cdot \overrightarrow{\nabla} f \tag{2.6.15}$$

Now combining the above equation we have:

$$\frac{1}{m}\frac{\partial \rho}{\partial t} + \int_{-\infty}^{+\infty} (\dot{\overrightarrow{v}} \cdot f \overrightarrow{v} d\overrightarrow{v}) - \int_{-\infty}^{+\infty} (\frac{\nabla\Phi}{m} \cdot \overrightarrow{\nabla_v f}) d\overrightarrow{v} \quad = \quad \Im \tag{2.6.16}$$

$$\frac{\partial \rho}{\partial t} + m\overrightarrow{\nabla} \cdot [\int_{-\infty}^{+\infty} \overrightarrow{v} f d\overrightarrow{v}] - \overrightarrow{\nabla}\Phi \cdot \int_{-\infty}^{+\infty} \overrightarrow{\nabla_v} f d\overrightarrow{v} \quad = \quad \Im m \tag{2.6.17}$$

From the above equation 2.6.17 no particle with infinite velocity,then the last integral will be vanish.

$$\frac{\partial \rho}{\partial t} + m\overrightarrow{\nabla} \cdot [\int_{-\infty}^{+\infty} \overrightarrow{v} f d\overrightarrow{v}] \quad = \quad \Im m \tag{2.6.18}$$

$$M_1[f(v)] \quad = \quad \int f(v) dv$$

The mean flow velocity U is a measure of the mean flow rate of the material,for a normalization scale

$$\overrightarrow{U} \quad = \quad \frac{\int_{-\infty}^{+\infty} \overrightarrow{v} f d\overrightarrow{v}}{\int_{-\infty}^{+\infty} f d\overrightarrow{v}} \tag{2.6.19}$$

From equation above,

$$\frac{\rho}{m} = \int_{-\infty}^{+\infty} f(x, \overrightarrow{v}) d\overrightarrow{v} \tag{2.6.20}$$

Using equations(2.6.19),(2.1.20) in (2.6.18) we get continuity equation as

$$\frac{\partial \rho}{\partial t} + \overrightarrow{\nabla} \cdot (\rho \overrightarrow{U}) = \overrightarrow{\Im} m \tag{2.6.21}$$

The First Moment of Boltzmann Transport Equation - Euler-Lagrange Equations and Conservation of Momentum

To produce an expression of the conservation of momentum let as multiply the Boltz man transport equation by the local particle velocity $\overrightarrow{v}$

$$\overrightarrow{v}[\int_{-\infty}^{+\infty} \frac{\partial f}{\partial t} d\overrightarrow{v} + \int_{-\infty}^{+\infty} \overrightarrow{v} \cdot \nabla f d\overrightarrow{v} + \int_{-\infty}^{+\infty} (\dot{\overrightarrow{v}} \nabla_v f) d\overrightarrow{v}] = \int_{-\infty}^{+\infty} \overrightarrow{v} S d\overrightarrow{v} \tag{2.6.22}$$

From this equation (2.6.22) the first term is,

$$\int_{-\infty}^{+\infty} \overrightarrow{v} \frac{\partial f}{\partial t} d\overrightarrow{v} = \frac{\partial}{\partial t} \int_{-\infty}^{+\infty} \overrightarrow{v} f d\overrightarrow{v} = \frac{\partial}{\partial t} [\int_{-\infty}^{+\infty} f(v) d\overrightarrow{v}] \tag{2.6.23}$$

$$\frac{\int_{-\infty}^{+\infty} \overrightarrow{v} f(v) d\overrightarrow{v}}{\int_{-\infty}^{+\infty} f(v) d\overrightarrow{v}} = \frac{\partial}{\partial t} [\frac{\rho \overrightarrow{U}}{m}] = \frac{\partial}{\partial t} (n \overrightarrow{U})$$

The second term from equation 2.6.21 is;

$$\int_{-\infty}^{+\infty} \overrightarrow{v} \cdot \nabla f d\overrightarrow{v} = \int_{-\infty}^{+\infty} \overrightarrow{v} (\dot{\overrightarrow{v}} \cdot \nabla f d\overrightarrow{v}) \tag{2.6.24}$$

where

$$\dot{\vec{v}} = \frac{-\nabla\Phi}{m} \tag{2.6.25}$$

$$\int_{-\infty}^{+\infty} \vec{v}(\vec{v}\cdot\vec{\nabla}_v f)d\vec{v} = \int_{-\infty}^{+\infty} \vec{v}(\frac{-\nabla\Phi}{m}\cdot\vec{\nabla}_v f)d\vec{v} \tag{2.6.26}$$

$$\int_{-\infty}^{+\infty} \vec{v}(\vec{v}\cdot\vec{\nabla}_v f)d\vec{v} = \frac{-\nabla\Phi}{m}\int_{-\infty}^{+\infty} \vec{v}\nabla f d\vec{v} \tag{2.6.27}$$

$$(\vec{\nabla}_v f)\vec{v} = \vec{\nabla}_v(f\vec{v}) - f(\vec{\nabla}_v\vec{v}) = \vec{\nabla}_v(f\vec{v}) - fI \tag{2.6.28}$$

where I is the identity matrix.

$$\int_{-\infty}^{+\infty} \vec{v}(\vec{v}\cdot\vec{\nabla}_v f)d\vec{v} = \frac{-\nabla\Phi}{m}\cdot\int_{-\infty}^{+\infty} \vec{\nabla}(f\vec{v})d\vec{v} + \frac{\vec{\nabla}\Phi}{m}\cdot\int_{-\infty}^{+\infty} fd\vec{v} \tag{2.6.29}$$

$$\int_{-\infty}^{+\infty} \vec{v}(\vec{v}\cdot\vec{\nabla}_v f)d\vec{v} = \frac{-\nabla\Phi}{m}\cdot\int_{-\infty}^{+\infty} f\vec{v}d\vec{v} \tag{2.6.30}$$

$$= n\frac{\nabla\Phi}{m} \tag{2.6.31}$$

$$\frac{\partial}{\partial t}(n\vec{u}) + \int_{-\infty}^{+\infty} \vec{v}(\vec{\nabla}\cdot(\vec{v}f))d\vec{v} + n\frac{\nabla\Phi}{m} = \int_{-\infty}^{+\infty} Sd\vec{v} \tag{2.6.32}$$

$$\frac{\partial}{\partial t}(n\vec{u}) = u\frac{\partial\vec{n}}{\partial t} + n\frac{\partial\vec{u}}{\partial t} \tag{2.6.33}$$

$$\frac{\partial n}{\partial t} = -\nabla\cdot(n\vec{u}) + \int_{-\infty}^{+\infty} Sd\vec{v} = -(\nabla\vec{v}\cdot\vec{\nabla}_n + \vec{\nabla}_n\vec{v})\int_{-\infty}^{+\infty} Sd\vec{v} \tag{2.6.34}$$

From the continuity equation

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\vec{u}) = \int_{-\infty}^{+\infty} Sd\vec{v} \tag{2.6.35}$$

Then from equation 2.6.33 and 2.6.34 we get

$$\frac{\partial}{\partial t}(n\overrightarrow{u}) \;=\; U\frac{\partial n}{\partial t} + n\frac{\partial \overrightarrow{v}}{\partial t} - (\overrightarrow{v}\nabla_n + n\nabla\cdot\overrightarrow{v})\overrightarrow{v}\int_{-\infty}^{+\infty}\overrightarrow{v}Sd\overrightarrow{v} \qquad (2.6.36)$$

Using the equation 2.6.30 and 2.6.36

$$n\frac{\partial \overrightarrow{v}}{\partial t} - (\overrightarrow{v}\cdot\overrightarrow{\nabla}_n + \overrightarrow{\nabla}_n\cdot\overrightarrow{v})\overrightarrow{v} + \int_{-\infty}^{+\infty}\overrightarrow{v}(\overrightarrow{v}\cdot(\overrightarrow{v}f)d\overrightarrow{v} + n\frac{\nabla\Phi}{m} \;=\; \int_{-\infty}^{+\infty}S(\overrightarrow{\nu}-\overrightarrow{v})d\overrightarrow{v}$$

The velocity tensor u is given as

$$\overrightarrow{U} = \frac{\int_{-\infty}^{+\infty}\overrightarrow{v}fd\overrightarrow{v}}{\int_{-\infty}^{+\infty}fd\overrightarrow{v}} \qquad (2.6.37)$$

$$\rho\frac{\partial v}{\partial t} + \rho(\overrightarrow{v}\cdot\overrightarrow{\nabla})\overrightarrow{v} + \overrightarrow{\nabla}\cdot(\rho(\overrightarrow{v} - \overrightarrow{v}\,\overrightarrow{v})) + n\nabla\Phi \;=\; \int_{-\infty}^{+\infty}ms(\overrightarrow{\nu}-\overrightarrow{v})d\overrightarrow{\nu}$$

The quantity $(\overrightarrow{v} - \overrightarrow{v}\,\overrightarrow{v})$ is the pressure tensor. The pressure tensor the second moment of $f(\overrightarrow{\nu})$ is $\overrightarrow{p}$ equal to

$$\frac{\int_{-\infty}^{+\infty}f(\nu)(\overrightarrow{\nu}-\overrightarrow{v})(\overrightarrow{\nu}-\overrightarrow{v})d(\nu)}{\int_{-\infty}^{+\infty}f(\nu)d(\nu)} \qquad (2.6.38)$$

It describes the difference between the local flow $\overrightarrow{\nu}$ and the mean flow $\overrightarrow{v}$. The first velocity moment of the BTE becomes

$$\frac{\partial \overrightarrow{v}}{\partial t} + (\overrightarrow{U}\cdot\overrightarrow{\nabla})\overrightarrow{v} \;=\; -\nabla\Phi - \frac{1}{\rho}\nabla p + \frac{1}{\rho}\int_{-\infty}^{+\infty}ms(\overrightarrow{\nu}-\overrightarrow{v})d\overrightarrow{\nu} \qquad (2.6.39)$$

This set of vector equations is **the Euler-Lagrange equations of hydrodynamic flow.** This assumption of local velocity leads to the simpler and more familiar expression for hydrodynamic flow

$$\frac{\partial \overrightarrow{v}}{\partial t} + (\overrightarrow{v}\cdot\nabla)\overrightarrow{v} \;=\; -\nabla\Phi - \frac{\nabla p}{\rho} \qquad (2.6.40)$$

Under the assumption of a nearly isotropic velocity field, P will be ρ and an expression known as an equation of state. From equation (2.6.40)the left-hand side is zero. The Euler-Lagrange equations of hydrodynamic flow is

$$\nabla P = -\rho \nabla \Phi \tag{2.6.41}$$

which is known as the equation of hydrostatic equilibrium: This equation is usually an expression of the conservation of linear momentum. The zeroth moment of the BTE results in the conservation of matter, where as the first velocity moment equations which represent the conservation of linear momentum. The second velocity moment represent an expression for the conservation of energy [13].

2.7 The Second Moment of Boltzmann Transport Equation - Ergodic equations and Energy Conservation

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The Euler-Lagrange equations of hydrodynamic flow, represent the first velocity moment of the transport equation. These are vector equations we obtain a scalar result by taking the scalar product of a position vector with the flow equations and integrating over all space with the system. The origin of the position vector is important only in the interpretation of some of the terms which will arise in the expression. The left-hand side of equation (2.6.39) is the total time derivative of the flow velocity $\overrightarrow{U}$ then, the first spatial moment equation becomes

$$\int_{\nu} \rho \overrightarrow{r} \frac{d\overrightarrow{v}}{dt} dv + \int_{\nu} \rho \overrightarrow{r} \cdot \nabla \Phi dv + \int_{r} \vec{r} \cdot \nabla p dv = 0 \tag{2.7.1}$$

$$\int_{\nu} \rho \frac{dQ}{dt} dv = \frac{d}{dt} \int_{\nu} \rho Q dv \tag{2.7.2}$$

Since $\overrightarrow{v}$ is the time rate of change of position,

$$\overrightarrow{v} = \frac{d\overrightarrow{r}}{dt} \tag{2.7.3}$$

$$\begin{aligned}
\overrightarrow{r} \cdot \frac{d\overrightarrow{v}}{dt} &= \frac{d}{dt}(\overrightarrow{r} \cdot \overrightarrow{v}) - \frac{d\overrightarrow{r}}{dt} \cdot \overrightarrow{v} \\
&= \frac{d}{dt}(\overrightarrow{r} \cdot \overrightarrow{v}) - \overrightarrow{v}\,\overrightarrow{v} \\
&= \frac{d}{dt}(\overrightarrow{r} \cdot \overrightarrow{v}) - u^2 \\
&= \frac{d}{dt}\overrightarrow{r} \cdot \frac{d\overrightarrow{r}}{dt} - u^2 \\
&= \frac{1}{2}\frac{d}{dt}(\frac{d}{dt}(\overrightarrow{r} \cdot \overrightarrow{r}))
\end{aligned}$$

$$\overrightarrow{r} \cdot \frac{d\overrightarrow{v}}{dt} = \frac{1}{2}\frac{d^2}{dt^2}r - u^2 \tag{2.7.4}$$

The first integral of equation (2.7.1) by using equation (2.7.4)

$$\int_v \rho\overrightarrow{r}\frac{d\overrightarrow{u}}{dt}dv = \frac{1}{2}\int_v \rho\frac{d^2\vec{r}}{dt^2}dv - \int_v \rho u^2 dv \tag{2.7.5}$$

$$\int_v \rho\overrightarrow{r}\frac{d\overrightarrow{u}}{dt} = \frac{1}{2}\frac{d^2}{dt^2}\int_v r^2\rho dv - u^2\int_v \rho dv \tag{2.7.6}$$

Where I is the moment of inertia is given by:

$$I = \int_V r^2 p dv \tag{2.7.7}$$

And also the kinetic energy in bulk motion

$$T = \frac{1}{2}\int_v \rho U^2 dv \tag{2.7.8}$$

This implies that the mass is given as

$$m = \int_v \rho dv \tag{2.7.9}$$

$$\frac{1}{2}\frac{d^2I}{dt^2} = mu^2 \tag{2.7.10}$$

But $mu^2 = T$

$$\int_v \rho\vec{r}\cdot\frac{d\vec{v}}{dt} = \frac{1}{2}\frac{d^2I}{dt^2} - 2T \tag{2.7.11}$$

The third integral of equation(2.7.1)

$$\int_v \vec{r}\cdot\vec{\nabla}\rho dv = \int_v \vec{r}\cdot\vec{\vec{\nabla}}(\vec{r}p)dv - \int_v p(\bar{\nabla}\cdot\vec{r})dv = \oint_s P_s\vec{r}\hat{n}dA - 3\int_s Pdv = \oint_s P_s\vec{r}\hat{n}dA - 3U$$

where

$$\vec{\nabla}\cdot\vec{r} = (\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial Z})\cdot(\hat{i}x + \hat{j}y + \hat{k}z) = 1 + 1 + 1 = 3 \tag{2.7.12}$$

Then internal kinetic energy density of an ideal gas is

$$\epsilon = \frac{3}{2}\frac{\rho\kappa T}{\mu m_H} \tag{2.7.13}$$

We can replace the pressure **P**

$$P = \frac{2}{3}\epsilon \tag{2.7.14}$$

The integral then yields twice the total internal kinetic energy of the system ,and the moment of equation becomes

$$\frac{1}{2}\frac{d^2I}{dt^2} = 2(T + U) - \int_v \rho\vec{v}\cdot\vec{\nabla}\Phi dv \tag{2.7.15}$$

The last term on the right-hand side of the equation (2.7.15) is called the **total potential energy**. Also the expression is called Lagranges identity and is also called the non-averaged form of the virial theorem.

$$\frac{1}{2}\frac{d^2I}{dt^2} - 2T - 2U + \int_v \rho\vec{v}\cdot\vec{\nabla}\Phi dv = 0 \tag{2.7.16}$$

A system in equilibrium, so that the time average of equation(2.7.16)remove the accelerative changes of the moment of inertia ($< \frac{1}{2}\frac{d^2I}{dt^2} >= 0$).

$$2 < T > +2 < U > + < \Omega > \;=\; 0 \tag{2.7.17}$$

The theorem which permits is the Ergodic theorem .

2.8 The summarized Boltzmann Transport and Hydrodynamic equation

From zero moment of Boltzmann transport equation conservation of matter we can have continuity equation.

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{U}) \;=\; \Im m \tag{2.8.1}$$

From first moment of Boltzmann transport equation conservation of linear momentum we have hydrodynamic equation

$$\frac{\partial \vec{U}}{\partial t} + (\vec{U} \cdot \nabla)\vec{U} \;=\; -\nabla\Phi - \frac{\nabla P}{\rho} \tag{2.8.2}$$

Second moment of Boltzmann transport equation conservation of energy

$$\frac{1}{2}\frac{d^2I}{dt^2} - 2T - 2U + \int_v \rho\vec{v} \cdot \vec{\nabla}\Phi dv \;=\; 0 \tag{2.8.3}$$

A system in equilibrium, so that the time average of equation(2.8.3)remove the accelerative changes of the moment of inertia ($< \frac{1}{2}\frac{d^2I}{dt^2} >= 0$).

$$2 < T > +2 < U > + < \Omega > \;=\; 0 \tag{2.8.4}$$

In astronomy this theorem is called Ergodic theorem [6]. By now we have the basic mathematical tools to apply in stellar evolution. All evolutionary equations can be derived by applying appropriate boundary conditions to the BTEs.

2.9 Equation of state of an ideal gas

In thermodynamics, an equation of state provides the mathematical relation among variables such as temperature, pressure, density, and internal energy. Equations of state (EOS) are useful in describing the properties of fluids, mixtures of fluids, solids, and even the interiors of stars. **For stars, the state usually describe the relation among pressure(P), temperature(T),density (n: number of density of particles or ρ:mass density).** Formulation of the Boltzmann Transport Equation (BTE) also provides an ideal setting for the formulation of the equation of state for a gas under wide-ranging conditions. The relationship between the pressure is given by the pressure tensor and the state variables (p,T,ρ) of the distribution function. The pressure tensor is p($\vec{v} - \vec{v}\vec{v}$). If f($\vec{\nu}$) is symmetric in ($\vec{\nu}$) ,then ($\vec{v}$) must be zero (or there exist an inertial coordinate system in which ($\vec{v}$) is zero, and the divergence of the pressure can be replaced by the gradient of a scalar which we call the gas pressure and will be given by

$$\vec{P} = \rho \frac{\int_{-\infty}^{+\infty} f(\nu) d\nu}{\int_{-\infty}^{+\infty} f(\nu) d\nu} \tag{2.9.1}$$

From the Maxwell-Boltzmann statics the distribution function of particles in terms of their velocity is given by

$$f(\nu) = constant = exponent(\tfrac{-mv^2}{2kT}) \tag{2.9.2}$$

The mean pressure is

$$\vec{P} = c\rho \frac{\int_{-\infty}^{+\infty} v^2 exponent(\frac{-mv^2}{2kT})}{c \int exponent^{(\frac{-mv^2}{2kT})}} \tag{2.9.3}$$

where $\alpha = \frac{m}{2kT}$,the integral of the function is

$$\int_{-\infty}^{+\infty} U^2 exp(-\alpha v^2) dv = \frac{1}{4}\sqrt{\prod}\alpha^{\frac{-3}{2}} \tag{2.9.4}$$

$$= \sqrt{\frac{\prod}{\alpha}} \tag{2.9.5}$$

then,

$$\bar{P} = \frac{\rho\frac{1}{4}\sqrt{\prod}\alpha^{\frac{-3}{2}}}{\sqrt{\frac{\prod}{\alpha}}} \tag{2.9.6}$$

$$= \rho\frac{\frac{\sqrt{\prod}\alpha^{\frac{-3}{2}}}{4}}{2\sqrt{\prod}\alpha^{\frac{-1}{2}}} \tag{2.9.7}$$

$$= \rho\frac{\alpha^{-1}}{2} = \frac{\rho}{2\alpha} \tag{2.9.8}$$

but ,$\alpha = \frac{m}{2kT}$. Then the mean pressure is

$$\bar{P} = \frac{\rho}{\frac{2m}{2kT}} = \frac{\rho kT}{m} \tag{2.9.9}$$

$$= nkT \tag{2.9.10}$$

2.10 Fundamental Equations of Stellar Structure

(1) Conservation of mass: For a spherical shell of thickness dr is

$$\frac{dM_r}{dr} = 4\pi r^2 \rho(_r) \tag{2.10.1}$$

written in terms of integral, this is

$$M_r = \int_0^r 4\pi r^2 \rho_r dr \tag{2.10.2}$$

However, over its lifetime a stars radius will change by many orders of magnitude while its mass will remain relatively constant. Moreover, the amount of nuclear reactions occurring inside a star depends on the density and temperature not where it is in the star. A better and more natural way to treat stellar structure is therefore to use mass as the independent parameter, rather than r. Thus

$$dr = \frac{1}{4\pi r^2 \rho} \tag{2.10.3}$$

This is the Lagrangian form of the equation (rather than the Eulerian form). All the equations of stellar structure will be expressed in the Lagrangian form, and most of the parameters will be expressed in per unit mass, rather than per unit size or volume.

(2)Conservation of Energy (at each radius, the change in the energy flux equals the local rate of energy of release).Consider the net energy per second passing outward through a shell at radius r.If no energy is created in the shell, then the amount of energy in equals the amount of energy out, and $\frac{dL}{dr} = 0$ However, if additional energy is released or absorbed within the shell, then $\frac{dL}{dr}$ will be non-zero. Lets define ϵ as the energy released per second by a unit mass of matter. Then:

$$\frac{dL}{dr} = 4\pi r^2 \rho \epsilon \tag{2.10.4}$$

or, in the Lagrangian

$$\frac{dL}{dr} = \epsilon_n + \epsilon_v + \epsilon_g \tag{2.10.5}$$

Note that ϵ has three components.

1.ϵ_n the total energy created by nuclear reactions.

2.ϵ_v the energy in to neutrinos, and

3.ϵ_g the energy produced or lost by gravitational expansion or contraction .Thus

$$\frac{dL}{dr} = \epsilon_n + \epsilon_v + \epsilon_g \tag{2.10.6}$$

(3) Equation of Energy Transport(relation between the energy flux and the local gradient of temperature). Assume that the star is in thermal equilibrium at each radius the gas is neither heating up nor cooling down with time. The transport equation also describes how energy is transported through the layers of the star, i.e. how the gas affects the radiation as it travels through. Depends on local density, opacity and temperature gradient. Let the rate of energy generation per unit mass be ϵ Then:

$$dL = 4\pi r^2 \rho dr \times q\frac{dL}{dr} = 4\pi r^2 \rho \epsilon \tag{2.10.7}$$

(4) Equation of Hydrostatic Equilibrium. The force of gravity pulls the stellar material towards the center. It is resisted by the pressure force due to the thermal motions of the gas molecules. The first equilibrium condition is that these forces in equilibrium. Radial forces acting on the element:

$$Gravityinward : F_g = \frac{-GM\Delta m}{r^2} \tag{2.10.8}$$

Pressure (net force due to difference in pressure between upper and lower face):

$$F_p = P_r ds - P(r+dr)ds - [p(_r) + \frac{dp}{dr}]ds = -\frac{dp}{dr}drds \tag{2.10.9}$$

Mass of element $\Delta m = \rho drds$

Applying Newtons second law (F = ma)

$$\Delta m\ddot{r} = F_g + F_p = -\frac{Gm\Delta m}{r^2} - \frac{dp}{dr}drds \tag{2.10.10}$$

Acceleration =0 every where if star static. Setting acceleration to zero,and substituting for Δm:

$$0 = -\frac{Gm\rho drds}{r^2} - \frac{dp}{dr}drds \tag{2.10.11}$$

$$\frac{dp}{dr} = -\frac{Gm}{r^2}\rho \tag{2.10.12}$$

Basic equations are supplemented by Equations of State (pressure of a gas as a function of density and temperature) and opacity (how transparent it is to radiation).

2.11 Nuclear Energy Generation Rate as f $(\rho; T)$ Equation of State In Stars

Interior of a star contains a mixture of ions, electrons and radiation (photons). For most stars (exception very low mass stars and stellar remnants) the ions and electrons can be treated as an ideal gas and quantum effects can be neglected .

Total pressure $p = p_i + p_e + p_r = p_{gas} + p_r$

where

p_i is the pressure of the ions

p_e is the pressure of the electron

p_r is the radiation pressure

The equation of state for the ideal gas is :

$$p_{gas} = nkT \tag{2.11.1}$$

where n is the number of particles per unit volume; $n = n_i + n_e$, where n_i and n_e are the number of densities of ions and electrons.

In terms of the mass density ρ :

$$p_{gas} = \frac{\rho}{\mu m_H} kT \tag{2.11.2}$$

where m_H is the mass of hydrogen and μ is the average mass of particle in units of m_H.

The ideal gas constant is :

$$R = \frac{k}{m_H} \Longrightarrow p_{gas} = \frac{R}{\mu} KT \tag{2.11.3}$$

Radiation pressure:For black body radiation

$$p_{gas} = \frac{1}{3} a T^4 \tag{2.11.4}$$

where a is the radiation constant.

$$a = \frac{8\pi^5 k^4}{15 c^3 h^3} \tag{2.11.5}$$

Gas pressure is most important in **low-mass stars**.

Radiation pressure is most important in **high mass stars**.

2.12 Time scale of stellar evolution

There are three important time scales in the life of star.

2.12.1 The Nuclear(evolutionary)Time Scale

The time in which a star radiates away all the energy that can be released by nuclear reactions. An estimate of this time can be obtained if one calculates the time in which all available hydrogen is turned into helium. On the basis of theoretical considerations and evolutionary computations it is known that only just over 10 percent of the total mass of hydrogen in the star can be consumed before other, more rapid evolutionary mechanisms set in. Since 0.7 percent of the rest mass is turned into energy in hydrogen burning, the nuclear time scale will be [14];

$$T_n \sim \frac{\kappa_n M c^2}{L} \tag{2.12.1}$$

where k_n is just the fraction of the rest mass available to a particular nuclear process, M is rest mass, L is stellar luminosity and c is speed of light.

$$\Longrightarrow T_n = \frac{E_{nuclear}}{L} \tag{2.12.2}$$

$$T_n \approx \frac{0.007 \times 0.01 M c^2}{L} \tag{2.12.3}$$

For the Sun one obtains the nuclear time scale 10^{10} years, and thus

$$T_n \approx \left(\frac{M}{M_{SUN}} \div \frac{L}{L_{SUN}}\right) \times 10^{10} a = \left(\frac{M}{M_{SUN}}\right)\left(\frac{L_{SUN}}{L}\right) \times 10^{10} a \tag{2.12.4}$$

2.12.2 Dynamical time scale

Measure of the time scale on which a star would expand or contract if the balance between pressure gradient and gravity was suddenly disrupted (some as free -fall time scale)

$$\tau_{dyn} = \frac{characteristicradius}{characteristicvelocity} = \frac{R}{V_{esk}} \tag{2.12.5}$$

Escape velocity from the surface of the star:

$$V_{esk} = \sqrt{\frac{2GM}{R}} \tag{2.12.6}$$

$$\tau_{dyn} = \sqrt{\frac{R^3}{2GM}} \tag{2.12.7}$$

In terms of mean density:

$$\tau_{dyn} = \frac{1}{\sqrt{G\bar{\rho}}} \tag{2.12.8}$$

where $\bar{\rho}$ is the mean density of the star (molecular cloud).

2.12.3 Thermal (Kelvin-Helmholtz) Time scale

Kelvin-Helmholtz Time scale is the time in which a star would radiate away all its thermal energy if the nuclear energy production where suddenly turned off.

$$\tau_{\kappa H} = \frac{U}{L} \tag{2.12.9}$$

Virial theorem: the thermal energy U is roughly equal to the gravitational potential energy.

$$\tau_{\kappa H} = \frac{GM^2}{RL} \tag{2.12.10}$$

Important time scale:Determines how quickly a star contracts before nuclear fusion starts,i.e.sets roughly the pre-main-sequence lifetime. Most stars, most of the time in hydrostatic and thermal equilibrium, with slow changes in structure and composition occurring on the (long) time scale τ_{nuc} fusion occurs [6].

Dynamical time scale: The third and shortest time scale is the time it would take a star to collapse if the pressure supporting it against gravity were suddenly removed.It can be estimated from the time it would take for a particle to fall freely from the stellar surface to the center.This is half of the period given by Keplers third law, where the semi major axis of the orbit corresponds to half the stellar radius R:

$$t_d = \frac{2}{\pi}\sqrt{\frac{(R/2)^3}{GM}} \approx \sqrt{\frac{R^3}{GM}} \tag{2.12.11}$$

Chapter 3

The Rate of Star Formation in Galaxy

3.1 Star formation in molecular clouds and accretion rate

Once a molecular cloud is being fragmented and starts to form a progenitor star (the "baby-star"), it growth up by accreting matter from its environment. The rate at which it accretes is given by [5].

$$\frac{dM}{M^{3/2}} = \gamma dt \tag{3.1.1}$$

where γ is a proportionality constant related to radial oscillation of the spherically collapsing cloud.

3.2 A three-component model for star formation

Following the work of [16] here we assume a three-component star forming interacting systems.

The model contains three active components: (1) cool atomic clouds, (2) cool molecular clouds, and (3)active young stars. Each of these components may interact with the other components or with the external activating systems like galaxies, etc. To this end, the model is an open system connected with two mass reservoirs outside the system.

i) Cool atomic cloud : Its main component is neutral hydrogen, the most abundant chemical element in the Galaxy .The density varies over a large range, but direct star formation in these clouds seems not to occur. The cooling capacity of these clouds is not large enough to allow a sufficient condensation. This component is connected to an unlimited reservoir of new atomic gas outside the system.

ii) Cool molecular cloud : Mainly consist of molecular hydrogen HII. The densities mast enormously and are generally much higher than in neutral clouds The temperature in such cloud decreases as its density increases as a consequence of large cooling capacity of the CO molecules. They have smaller dimensions than neutral clouds.

iii) Young, active stars : Mostly accompanied by hot ionized HII gas.These stars strongly affect the surrounding gas clouds and are responsible for shock waves in these clouds. In this way, new condensation regions may be formed in the molecular clouds of the system.The presence of young stars therefore has a positive effect on the stellar birth rate.The capacity of influencing the other components ends when these young stars evolve to neutron stars. Although these remnants are still physically present in the system, their masses have stopped playing an active role in the star formation process. We therefore say that this mass has left the active star formation system. The second reservoir is hence a waste reservoir containing the stellar remnants.

3.3 Interaction of the system

The three mass components S for the total mass of active stars, M for the total mass of molecular clouds, and A for the total mass of atomic clouds. It is assumed that the total mass of the system remains constant; thus, we assume that the amount of mass lost by stellar evolution is exactly replaced by fresh atomic clouds entering the star formation region from the external sources. Now calling the total mass of the system T, we write

$$T = A + M + S \tag{3.3.1}$$

There are three kinds of interaction for the atomic cloud component:

a) First, there is a constant replenishment by new atomic clouds in an amount equal to the amount of mass leaving the active system by stellar evolution.The amount of new gas may therefore be considered as proportional to the amount of stellar mass S.We will call the proportional constant of this process K_1.

b) Secondly, the atomic component is increased as young, active stars lose mass by stellar wind. This process is also proportional to the number of stars and therefore to the amount of stellar mass. The proportional constant for this process is K_2.

c) The third interaction is the transformation of atomic into molecular clouds. This process is clearly proportional to the amount of atomic gas A, but since the transformation becomes more and more effective with the cooling capacity of the cloud, and since this capacity increases with the square of the density of molecular content, we assume that the transformation of atomic into molecular gas is proportional to the square of the molecular mass. This third processes loss of atomic gas and therefore is written with a minus sign in the differential equation and a proportional constant K_3

$$\frac{dA}{dt} = K_1 S + K_2 S - K_3 A M^2 \tag{3.3.2}$$

Or

$$\frac{dA}{dt} = K_{12} S - K_3 A M^2 \tag{3.3.3}$$

where K_{12} is the coupled constants K_1 & K_2. In fact a further analysis shows that this is a typical oscillation frequency of the couple.

The rate of the star formation may be considered as being proportional to the n^{th} power of the density of molecular cloud.Values of n can be selected between 0.5 and 3.5 or between 1 and 2. It is assumed that the presence of other young stars is a necessary condition for star formation since they will perturb the molecular cloud and provoke condensations. In this way we may also state that the star formation rate is proportional to the number of active stars already present. Let us call the proportional constant K_4. This process increases the mass of stellar component. Two other process decrease it: stellar evolution, for which we may use K_1, and mass loss by stellar wind, for which we again use K_2 Both processes are proportional to the amount of stellar mass. Thus, the equation describing the variation of stellar mass in the system is

$$\frac{dS}{dt} = K_4 S M^n - K_1 S - K_2 S \tag{3.3.4}$$

Or

$$\frac{dS}{dt} = K_4 S M^n - K_{12} S \tag{3.3.5}$$

Transformation of atomic into molecular gas, which increases for the variable M, and stellar formation, which decreases the amount of molecular mass. The equation for M will be

$$\frac{dM}{dt} = K_3 A M^2 - K_4 S M^n \tag{3.3.6}$$

Note that, basically eqn. 3.1.1 & eqn. 3.3.6 should represent the same physics. Consequently, it helps us to impose an additional condition in integrating the coupled system of differential equations.

The constant parameters $K_1,\ K_2,\ K_3$ and K_4 can be further transformed by introducing the dimensionless parameter constants k_1 & K_2 given by

$$k_1 = \frac{K_3 T^2}{K_{12}} \tag{3.3.7}$$

$$k_2 = \frac{K_4 T^n}{K_{12}} \tag{3.3.8}$$

The solution of $M(t), S(t),$ and $A(t)$

$$M(t) = \frac{M_0}{(1+\frac{1}{2}\sqrt{M_0}\gamma t)^2} \tag{3.3.9}$$

$$S(t) = \frac{S_0 Exp\left(\frac{2k_2 K_{12}}{(2n-1)\gamma} T^{-n} M_0^{(2n-1)/2}\right)}{Exp\left(\frac{2k_2 K_{12}}{(2n-1)\gamma}\frac{T^{-n} M_0^{(2n-1)/2}}{(1+\frac{1}{2}\sqrt{M_0}\gamma t)^{2n-1}} + K_{12} t\right)} \tag{3.3.10}$$

$$A(t) = \frac{k_1}{k_2} M^{n-2} + \frac{\gamma M^{-\frac{1}{2}}}{k_2 K_{12}} \tag{3.3.11}$$

On the other hand $A(t)$ can also be determined from eqn. 3.3.1. Then, between this and eqn. 3.3.11 we can express the constant parameter γ in terms of the characteristic oscillation frequency K_{12} worked out at $t = 0$.

$$\gamma = \frac{K_{12}}{\sqrt{M_0}}[k_1 a_0 - k_2 s_0 m_0^{n-1}] \tag{3.3.12}$$

. Following [3], we introduce the dimensionless mass ratio parameters $m(t), s(t),$ and $a(t)$ corresponding respectively to $M(t), S(t),$ and $A(t)$ as

$$a = \frac{A}{T}, \quad m = \frac{M}{T}, \quad s = \frac{S}{T} \tag{3.3.13}$$

where,

$$a(t) + m(t) + s(t) = 1 \tag{3.3.14}$$

Now γ in terms of the dimensionless mass ratios being evaluated at the initial condition, the total mass, the characteristic frequency and the two dimensionless k-constants is given as

$$\gamma = \frac{\sigma K_{12}}{\sqrt{m_0 T}} \tag{3.3.15}$$

where

$$\sigma = k_1 a_0 - k_2 s_0 m_0^{n-1} \tag{3.3.16}$$

Now, the complete solutions of the ratio masses of the system evolving in time are given by

$$m(t) = \frac{m_0}{(1 + \frac{1}{2}\sigma K_{12} t)^2} \tag{3.3.17}$$

$$s(t) = s_0 \exp\left(\frac{2k_2 m_0^n \left(1 - (500K_{12}\sigma t + 1)^{1-2n}\right)}{(2n-1)\sigma} - 1000K_{12}t\right) \tag{3.3.18}$$

$$a(t) = 1 - [m + s](t) \tag{3.3.19}$$

Chapter 4

Result and Discussion

In this work we assumed the rate at which a star forming molecular cloud accretes from its surrounding is being given by equation 3.1.1. Then we did derive the proportionality constant of the accretion rate in terms of the total initial masses of the interacting system, the characteristic oscillation frequency of the molecule and the coupling constants.

Following the 3-component interacting system in star forming active region, such as dense spiral arms we have derived a complete analytical solutions of the mass transfers of the system and as well as the rate at which the masses being transferred.

Then we have generated numerical data computationally using MATHEMATICA11 for the evolution of the masses and the rate at which the masses being flow out or into each component in time. The plots of the results are as shown in Fig.4.1 and Fig. 4.2. In plotting the graphs we used the k-parameters, the initial mass parameters and the mass index respectively as: $k_1 = 12$; $k2 = 20$; $m_0 = 0.7$; $a_0 = 0.2$; $s_0 = 0.1$; & $n = 2.5$. The characteristic couple oscillation frequency is between 0 & 1 cycles per the order of the time of evolution during the formation, where t is in unit of second. For the plots we have used $K_{12} = 10^{-2.4}$.

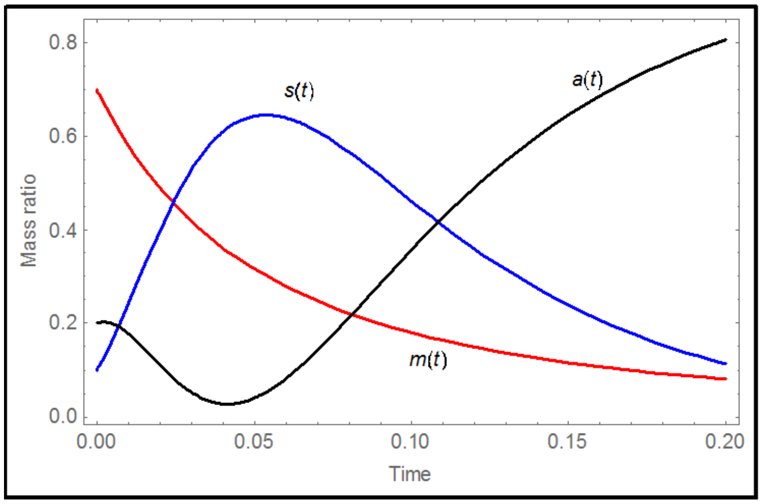

Figure 4.1: The evolution of the three-component mass system: $m(t)$-molecular mass, red; $s(t)$ - stellar mass - blue; $a(t)$ - atomic mass - black

As we observe from the plots at the beginning both the stellar mass and the atomic mass decrease. While the stellar mass increases. But after a sufficient time the stellar mass stops for a moment and begins to decrease while the molecular gas continues decrease. On the other hand, the atomic gas turns to increase at the expense of the decrease in the other two. This, is true as one expects from m the standard theory of formation.

The rate of mass transfers of the three components of the systems are all in different characteristics. The molecular gas ever continues to decrease asymptotically until it gets extinct, or the rate of mass transfer $|dm(t)/dt|$ decreases asymptotically. The stellar rate of transfer at the beginning radically decreases, stops for a while and gradually increases till it comes to stops. On the other hand, the atomic gas behaves in three ways. First for a short period of time it decreases radically, then increases for a longer period of time relatively and then gradually continues to decrease until it comes to halt.

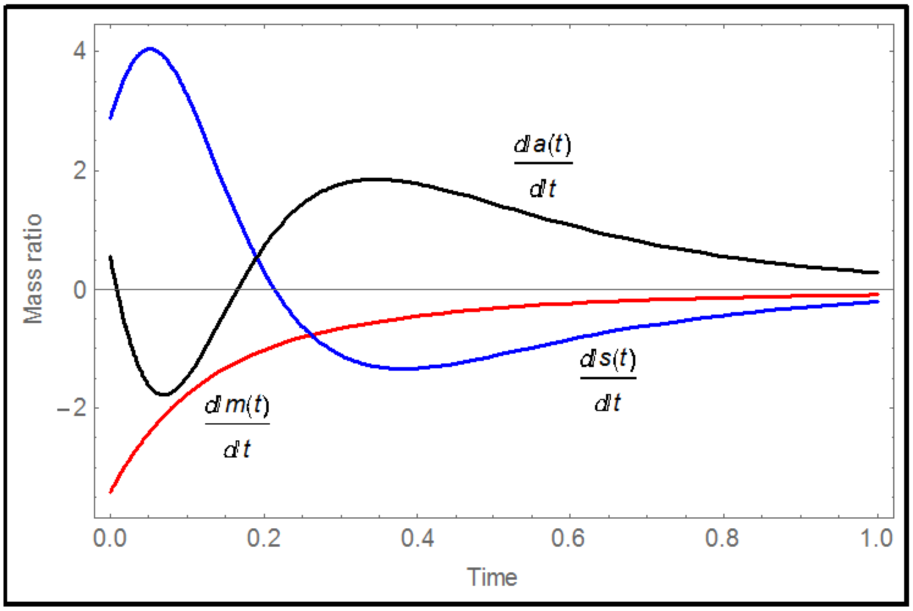

Figure 4.2: The evolution of mass transfer rates of the three-component system: $\frac{dm(t)}{dt}$-molecular mass rate, red; $\frac{ds(t)}{dt}$-stellar mass rate - blue; $\frac{da(t)}{dt}$ - atomic mass rate - black

Fig.4.3 shows the validity of our work compared to observation.

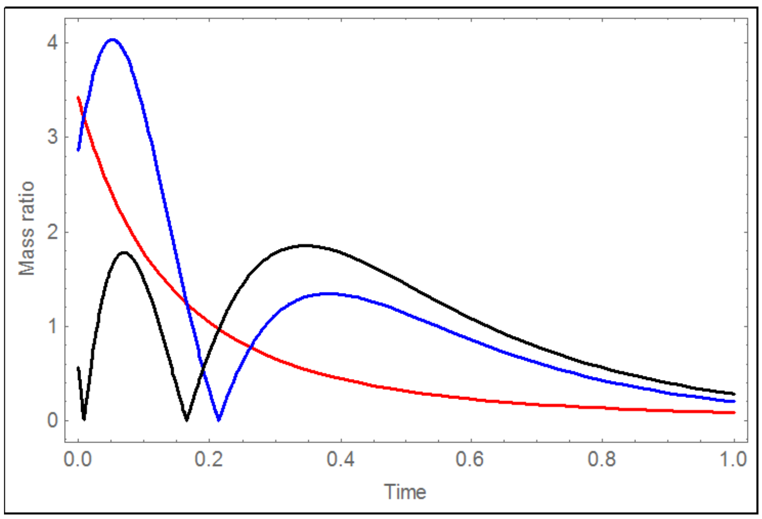

Figure 4.3: The rate of malformation as a function of time. Theoretical work being compared with observation. The left panel is our theoretical result while the right panel is observational result

Chapter 5

summary and conclusion

Star formation occur as a result of the action of gravity on a wide range of scales. On galactic scale the tendency of interstellar matter to condense under gravity in to star forming clouds is countered by galactic tidal forces, and star formation can occur only where the gas become dense enough for its self gravity to over come these tidal force,for example in spiral arms. On the intermediate scales of star forming giant molecular clouds (GMCs), turbulence and magnetic fields may be the most important effects counteracting gravity, and star formation may involve the dissipation of turbulence and magnetic fields. On top of these, there is external agents that will trigger the system to begin the formation such as shock waves external to the system.

Due to the complex system and complicated theories needed to work out in stellar formation and evolution, we have worked out the three-component interaction system of formation. As we have worked out in chapter 3 and discussed the results in chapter 4 we have successfully derived analytical solutions to the dynamical evolution of the masses of the interacting system including their rate of evolution. However, we believe that this work needs further development and inputs to give more meanings to the parameters therein introduced.

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