Degenerate Squeezed Three Level Laser and Sub-harmonic Light Coupled to Vacuum Reservoir

A Thesis Submitted to

Department of Physics

In Fulfillment of the

Requirements for the Degree of

Masters of Science (Quantum Optics)

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Jimma, Oct. 2016

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SCHOOL OF GRADUATE STUDIES

JIMMA UNIVERSITY

October, 2016

Author: **Gebreigziabher Abreha**

Title: **Degenerate Squeezed Three Level Laser and Sub-harmonic Light Coupled to Vacuum Reservoir** . Department: **Physics**

Degree : **M.Sc**. Convocation: October **Year:2016**

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DECLARATION

I hereby declare that this MSC thesis is my original work and has not been presented for a degree in any other universities, and that all sources of material used for the dissertation have been duly acknowledged.

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Jimma University

Oct. 2016

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COLLEGE OF NATURAL SCIENCES

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Name of Student: **Gebreigziabher Abreha Desta** ID No. **SSMSc 01945/05**

Graduate Program: **Summer, MSc.**

1. Course Work Performance

Course Code	Course Title	Cr. hr	Number Grade	Rank **	Remark
Phys 799	**MSc. Thesis**	**6**	**73.5**	**Good**	

** Excellent, Very Good, Good, Satisfactory, Fail.

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Abstract

In this thesis, we study the squeezing and statistical properties of light produced by degenerate squeezed three level laser and one mode subharmonic generator. We first obtain, the master equation, with the aid of the master equation, we determine c-number Langevin equations for a degenerate squeezed three level laser and one-mode subharmonic generator. With the help of solutions of c-number Langevin equations, we also obtain the antinormally ordered characteristic function defined in the Heisenberg picture. The resulting characteristic function is used to determine the Q-function of the light beams produced by degenerate squeezed three level laser and a one-mode sub-harmonic generator. Finally we superposed the two Q functions. Employing the resulting Q-function, we calculated the photon statistis and quadrature fluctuation.And we have seen that squeezing is enhanced for the superposed light beams.

Acknowledgements

Above all, I would like to thank the almighty God, for letting me alive to accomplish this cycle. My strongest thank goes to Dr. Solomon Getahun (my advisor and instructor) for his tireless advice, supervision and contribution of valuable suggestions and also I would like to thank to my family for their consistent support and motivation. Finally, I would like to thank to all of my friends and colleagues for their cooperation and all the good and bad times we had together.

Contents

1

Introduction

There has been a considerable interest in the analysis of the squeezing and statistical properties of the light generated by three-level lasers [1,2,3]. A three-level laser may be defined as a quantum optical system in which three-level atoms in a cascade configuration, initially prepared in a coherent superposition of the top and bottom levels, are injected in to a cavity coupled to a vacuum reservoir via a single-port. One interesting features of a three level laser involves the coupling of the top and bottom levels of the atoms by injecting a strong coherent light in to the cavity. When a three-level atom in a cascade configuration makes a transition from the top to the bottom level via the intermediate level, two photons are generated. If the two photons have the same frequency, then the three-level atom is called degenerate three-level atom otherwise it is called non degenerate. Three-level lasers in which the crucial role is played by the coherent superposition of the top and bottom levels of the injected atoms have been studied by several authors [1, 6]. These studies show that this quantum optical system can generate light in a squeezed state under certain conditions. A three-level laser with the top and bottom levels of the atoms injected in to the cavity coupled by a strong coherent light can also generate light in a squeezed state [7 ,8]. Ansari et al. [1]. Here we consider a degenerate three level laser in which the injected atoms are initially prepared in the top level and with the top and bottom levels of the atoms coupled by a strong coherent light.

Squeezing is one of the interesting non classical feature of light that has been attracting attention and studied by many authors [1,11]. In squeezed light the noise in one quadrature is below the vacuum coherent level at the expense of enhanced fluctuations in the other quadrature, with the product of the uncertainties in the two quadratures satisfying the uncertainty relation. Squeezed lights have potential applications in low noise communications and precision measurements [13, 14]. Squeezed light can be generated by quantum optical processes such as parametric oscillation [1-10], second harmonic generation [9,12,15], and four-wave mixing [9,15].

The concept of laser has been studied by different authors. TM Mainman of higher research laboratory scientist was first to experimentally demonstrate laser by flashing light through a ruby crystal in 1960. Three level lasers produced by the superposition of the top and bottom levels of the injected atoms have been studied by several authors [1, 6]. These studies show that this quantum optical system can generate light in a squeezed state under certain conditions. Ansari [1] has calculated the quadrature variance of the cavity mode for a degenerate three level laser employing the steady state solutions of the equations of evolution of the expectation values of the cavity mode variables. He has found that the cavity mode is in a squeezed state if the probability for the injected atoms to be in bottom level is larger than the probability for the atoms in the top level.

And almost perfect squeezing can be obtained for slightly more probable for the atoms to be in the bottom level and for large value of linear gain coefficient. Using c-number langavin equation, Fisseha K. [16] has shown that the light produced by a degenerate three level laser is a squeezed state when the probability for the injected atoms to be the bottom level is greater than that of the upper level. And the degree of squeezing increases with the linear gain coefficient. A subharmonic generator has been considered as an important source of squeezed light. It is one of the most interesting and well characterized optical devices in quantum optics. In this device a pumped photon interacts with a nonlinear crystal inside a cavity and is down converted into two highly correlated photons.If these photons have the same frequency, the device is called a one mode subharmonic generator, otherwise it is called a two mode subharmonic generator. The quadrature squeezing and photon statistics of the signal mode produced by one mode subharmonic generator coupled to a squeezed vacuum reservoir have been analyzed by a number of authors [11, 12, 14]. One mode sub harmonic generation is one of the most interesting and widely studied quantum optical process. In this process a pump photon of frequency 2ω is down converted into a pair of signal photons each of frequency ω. A theoretical analysis of the statistical and squeezing properties of the signal mode produced by one-mode subharmonic generation has been made by a number of authors [11, 12, 14, 15]. Among other things, it has been predicted that the signal mode has a maximum squeezing of 50% below the vacuum state level [4-7].

In this thesis, we study the photon statistics and quadrature squeezing of the light produced by degenerate squeezed and subharmonic light.

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2. Degenerate Squeezed Three Level Laser

In this chapter we seek to find the master equation and c-number Langavin equation for the cavity mode produced by degenerate three level laser. Using the solution of c-number Langavin equation,we find the antinormally ordered characterstic function. Employing the antinormally ordered characterstic function, we obtain the Q-function. Then applying the Q-function, we calculate the mean photon number, the variance of the photon number, the quadrature variance and quadrature squeezing.

2.1 The master equation

The interaction of a three level atom with the cavity mode in the rotating wave approxi- mation and in the interaction picture can be described by the the Hamiltonian [18, 20].

$$\hat{H} \;=\; ig\bigg((|a\rangle\langle b|+|b\rangle\langle c|)a - \hat{a}^{\dagger}(|b\rangle\langle a|+|c\rangle\langle b|)\bigg), \tag{2.1}$$

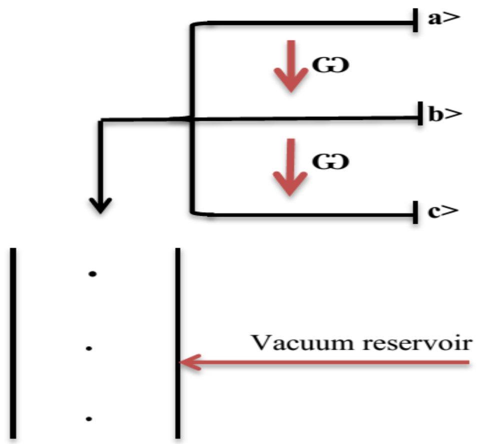

Figure 2.1: Schematic diagram of degenerate squeezed three level laser

where g is the coupling constant and $\hat{a}$ is the annihilation operator for the cavity mode. We take the initial state of a three level atom to be

$$|\psi_A(0)\rangle = C_a(0)|a\rangle + C_c(0)|c\rangle, \tag{2.2}$$

where $C_a(0)$ and $C_c(0)$ are probability amplitudes for the three level atom to be in the upper and bottom levels respectively. The initial density operator for a single atom has the form

$$\hat{\rho}_A(0) = \hat{\rho}^{(0)}_{aa}|a\rangle\langle a| + \hat{\rho}^{(0)}_{ac}|a\rangle\langle c| + \hat{\rho}^{(0)}_{ca}|c\rangle\langle a| + \hat{\rho}^{(0)}_{cc}|c\rangle\langle c|, \tag{2.3}$$

where $\hat{\rho}^{(0)}_{aa} = |C_a|^2, \hat{\rho}^{(0)}_{ac} = C_a C_c^*, \hat{\rho}^{(0)}_{ca} = C_c C_a^*, \hat{\rho}^{(0)}_{cc} = |C|^2$. Suppose $\hat{\rho}_{AR}(t, t_j)$ is the density operator for a single atom plus the cavity mode at a time t, with the atom injected at a time t_j such that $(t - \tau) \leq t_j \leq t$. The density operator for all atoms in the cavity plus the cavity mode at time t can then be written as

$$\hat{\rho}_{AR}(t) = r_a \sum_j \hat{\rho}_{AR}(t, t_j)\Delta t_j, \tag{2.4}$$

where r_a the rate atoms are injected into the cavity. Now converting the summation over integration as $\Delta t_j \longrightarrow 0$,we have

$$\hat{\rho}_{AR}(t) \ = r_a \int_{t-\tau}^{t} \hat{\rho}_{AR}(t,t^{'})dt^{'}, \tag{2.5}$$

and on differentiating with respect to t, there follows

$$\frac{d}{dt}\hat{\rho}_{AR}(t) = r_a(\hat{\rho}_{AR}(t,t) - \hat{\rho}_{AR}(t,t-\tau)) + r_a \int_{t-\tau}^{t} \frac{\partial}{\partial t}\hat{\rho}_{AR}(t,t^{'})dt^{'}. \tag{2.6}$$

We observe that $\hat{\rho}_{AR}(t,t)$ is the density operator for the cavity mode plus the atom injected at time t. This density operator can thus be expressed as

$$\hat{\rho}_{AR}(t,t) = \hat{\rho}_A(t)\rho(t), \tag{2.7}$$

with $\hat{\rho}(t)$ being the density operator for a cavity mode alone. We also note that $\hat{\rho}_{AR}(t,t-\tau)$ represents the density operator for an atom plus the cavity mode at a time t, with being removed from the cavity at this time.This operator can also put in the form

$$\hat{\rho}_{AR}(t,t-\tau) = \hat{\rho}_A(t-\tau)\rho(t). \tag{2.8}$$

Now in view of Eqs.(2.7) and (2.8), one can write Eq.(2.6) as

$$\frac{d}{dt}\hat{\rho}_{AR} \ = \ r_a(\hat{\rho}_A(t) - \hat{\rho}_A(t-\tau))\hat{\rho}(t) + r_a \int_{t-\tau}^{t} \frac{\partial}{\partial t}\hat{\rho}_{AR}(t,t^{'})dt^{'}. \tag{2.9}$$

In the absence of damping of the cavity mode by vacuum reservoir, the density operator $\hat{\rho}_{AR}(t,t^{'})$ evolves in time according to

$$\frac{\partial}{\partial t}\hat{\rho}_{AR}(t,t^{'}) = -i[\hat{H},\hat{\rho}_{AR}(t,t^{'})], \tag{2.10}$$

so that using this and taking in to account Eq.(2.5), one can put Eq.(2.9) in the form

$$\frac{d}{dt}\hat{\rho}_{AR}(t) = r_a(\hat{\rho}_A(t) - \hat{\rho}_A(t-\tau))\hat{\rho}_{(t)} - i[\hat{H},\hat{\rho}_{AR}(t,t^{'})]. \tag{2.11}$$

Furthermore, tracing over the atomic variables and taking into account the damping of the cavity mode by a vacuum reservoir, we have

$$\frac{d\hat{\rho}}{dt} = -iTr_A[\hat{H},\hat{\rho}_{AR}(t)] + \frac{1}{2}\kappa(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{\rho}\hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}\hat{\rho}), \tag{2.12}$$

we have used the fact that

$$Tr\hat{\rho}_A(t) = Tr\hat{\rho}_{AR}(t-\tau) = 1. \tag{2.13}$$

Employing Eq.(2.1) the master equation for the cavity mode can be put in the form

$$\begin{aligned}\frac{d}{dt}\hat{\rho} &= g(\hat{\rho}_{ab}\hat{a}^\dagger - \hat{a}^\dagger\hat{\rho}_{ab} + \hat{\rho}_{bc}\hat{a}^\dagger - a^\dagger\hat{\rho}_{bc} + \hat{a}\hat{\rho}_{ba} - \hat{\rho}_{ba}\hat{a} + \hat{a}\hat{\rho}_{cb} - \hat{\rho}_{cb}\hat{a} \\ &\quad + \frac{1}{2}\kappa(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{\rho}\hat{a}^\dagger\hat{a} - a^\dagger\hat{a}\hat{\rho}),\end{aligned} \tag{2.14}$$

in which the matrix element $\hat{\rho}_{\alpha\beta}$ is defined by

$$\hat{\rho}_{\alpha\beta} = \langle\alpha|\hat{\rho}_{AR}|\beta\rangle, \tag{2.15}$$

with $\alpha, \beta = a, b, c$. On the other hand, we see from Eq.(2.11) that

$$\begin{aligned}\frac{d}{dt}\hat{\rho}_{\alpha\beta} &= r_a(\langle\alpha|\hat{\rho}_A(0)|\beta\rangle - \langle\alpha\hat{\rho}_A(t-\tau)|\beta\rangle)\hat{\rho}(t) \\ &\quad -i(\langle\alpha\hat{H}\hat{\rho}_{AR}|\beta\rangle - \langle\alpha\hat{\rho}_{AR}\hat{H}|\beta\rangle) - \gamma\hat{\rho}_{\alpha\beta},\end{aligned} \tag{2.16}$$

where the last term is included to account for the decay of the atoms due to spontaneous emission. Here γ, considered to be the same for all three levels,is the atomic decay rate. We assume that the atoms are removed from the cavity after they have decayed to a level other than the middle or bottom level.We then see that

$$\langle\alpha|\hat{\rho}_A(t-\tau)|\beta\rangle = 0, \tag{2.17}$$

hence Eq.(2.16) reduces to

$$\frac{d}{dt}\hat{\rho}_{\alpha\beta} = r_a\langle\alpha|\hat{\rho}_A(0)|\beta\rangle\hat{\rho}(t) - i(\langle\alpha\hat{H}\hat{\rho}_{AR}|\beta\rangle - \langle\alpha\hat{\rho}_{AR}\hat{H}|\beta\rangle) - \gamma\hat{\rho}_{\alpha\beta}. \tag{2.18}$$

Appliying Eq.(2.28) and taking in to account Eqs. (2.1) and (2.3), one readly obtains

$$\frac{d}{dt}\hat{\rho}_{ab} = g(\hat{\rho}_{ac}\hat{a}^\dagger + \hat{a}\hat{\rho}_{bb}\hat{a} - \hat{\rho}_{aa}\hat{a}) - \gamma\hat{\rho}_{ab}, \tag{2.19}$$

$$\frac{d}{dt}\hat{\rho}_{bc} = g(\hat{a}\hat{\rho}_{cc} - \hat{\rho}_{bb}\hat{a} - \hat{a}^\dagger\hat{\rho}_{ac}) - \gamma\hat{\rho}_{bc}, \tag{2.20}$$

$$\frac{d}{dt}\hat{\rho}_{aa} = r_a\hat{\rho}_{aa}^{(0)}\hat{\rho} + g(\hat{\rho}_{ab}\hat{a}^\dagger + \hat{a}\hat{\rho}_{ba}) - \gamma\hat{\rho}_{aa}, \tag{2.21}$$

$$\frac{d}{dt}\hat{\rho}_{ac} = r_a\hat{\rho}_{ac}^{(0)}\hat{\rho} + g(\hat{a}\hat{\rho}_{bc} - \hat{\rho}_{ba}\hat{a}) - \gamma\hat{\rho}_{ac}, \tag{2.22}$$

$$\frac{d}{dt}\hat{\rho}_{bb} = g(\hat{\rho}_{bc}\hat{a}^\dagger + \hat{a}\hat{\rho}_{cb} - \hat{a}^\dagger\hat{\rho}_{ab} - \hat{\rho}_{ba}\hat{a}) - \gamma\hat{\rho}_{bb}, \tag{2.23}$$

$$\frac{d}{dt}\hat{\rho}_{cc} = r_a\hat{\rho}_{cc}^{(0)}\hat{\rho}g(\hat{a}^\dagger\hat{\rho}_{bc} + \hat{\rho}_{cb}\hat{a}) - \gamma\hat{\rho}_{cc}. \tag{2.24}$$

Upon dropping the g-terms and applying the adiabatic approximation scheme, we get from Eqs.(2.21), (2.22), (2.23) and (2.24) that

$$\hat{\rho}_{aa} = \frac{r_a\hat{\rho}_{aa}^{(0)}\hat{\rho}}{\gamma}, \tag{2.25}$$

$$\hat{\rho}_{bb} = 0, \tag{2.26}$$

$$\hat{\rho}_{ac} = \frac{r_a\hat{\rho}_{ac}^{(0)}\hat{\rho}}{\gamma}, \tag{2.27}$$

$$\hat{\rho}_{cc} = \frac{r_a\hat{\rho}_{cc}^{(0)}\hat{\rho}}{\gamma}. \tag{2.28}$$

Now combination of Eqs. (2.19), (2.25), (2.26) and (2.27) as well as (2.20), (2.26), (2.27) and (2.28) leads to

$$\frac{d}{dt}\hat{\rho}_{ab} = \frac{r_a g}{dt}(\hat{\rho}_{cc}^{(0)}\hat{\rho}\hat{a}^\dagger - \hat{\rho}_{aa}^{(0)}\hat{\rho}\hat{a}) - \gamma\hat{\rho}_{ab}, \tag{2.29}$$

$$\frac{d}{dt}\hat{\rho}_{bc} = \frac{r_a g}{\gamma}(\hat{\rho}_{cc}\hat{a}\hat{\rho} - \hat{\rho}_{ac}\hat{a}^\dagger\hat{\rho}) - \gamma\hat{\rho}_{bc}. \tag{2.30}$$

Finally, on account of Eqs. (2.29) and (2.30), the master equation for the cavity mode given by Eq.(2.14) takes the form

$$\begin{aligned}\frac{d}{dt}\hat{\rho} \;=\; & \frac{1}{2}A\hat{\rho}_{aa}^{(0)}(2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{\rho}\hat{a}\hat{a}^\dagger - \hat{a}\hat{a}^\dagger\hat{\rho}) \\ & +\frac{1}{2}(A\hat{\rho}_{cc} + \kappa)(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{\rho}\hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}\hat{\rho}) \\ & +\frac{\hat{\rho}_{ac}^{(0)}A}{2}(\hat{\rho}\hat{a}^{\dagger 2} + \hat{a}^{\dagger 2}\hat{\rho} - 2\hat{a}^\dagger\hat{\rho}a^\dagger) \\ & +\frac{\hat{\rho}_{ca}^{(0)}A}{2}(\hat{\rho}\hat{a}^{\dagger 2} + \hat{a}^2\hat{\rho} - 2\hat{a}\hat{\rho}\hat{a}),\end{aligned} \tag{2.31}$$

where $A = \frac{2r_a g^2}{\gamma^2}$ is the linear gain coefficient. Next we wish to find the approximate solution of the c-number Langavin equation for the degenerate three level laser.

C-number Langavin equation

Employing relations

$$\tfrac{d}{dt}\langle A\rangle = Tr(\tfrac{d}{dt}\rho A),$$

with the aid of Eq.(2.31) it can be readly verified that

$$\begin{aligned}\frac{d}{dt}\langle\hat{a}(t)\rangle \;=\; & \frac{A}{2}\hat{\rho}_{aa}^{(0)}Tr(2\hat{a}^\dagger(t)\hat{\rho}\hat{a}(t) - \hat{\rho(a)}\hat{a}^\dagger(t) - \hat{a}(t)\hat{a}^\dagger(t)\hat{\rho}) \\ & +\frac{1}{2}(A\hat{\rho}_{cc} + \kappa)Tr(2\hat{a}(t)\hat{\rho}\hat{a}^\dagger(t) - \hat{\rho}\hat{a}^\dagger(t)\hat{a}(t) - \hat{a}^\dagger(t)\hat{a}(t)\hat{\rho}) \\ & +\frac{A\hat{\rho}_{ac}^{(0)}}{2}Tr(\hat{\rho}\hat{a}^{\dagger 2}(t) + \hat{a}^{\dagger 2}(t)\hat{\rho} - 2\hat{a}^\dagger(t)\hat{\rho}a\dagger(t)) \\ & +\frac{A\hat{\rho}_{ca}^{(0)}}{2}Tr(\hat{\rho}\hat{a}^{\dagger 2}(t) + \hat{a}^2(t)\hat{\rho} - 2\hat{a}(t)\hat{\rho}\hat{a}(t)),\end{aligned} \tag{2.32}$$

$$\frac{d}{dt}\langle\hat{a}\rangle = -\frac{1}{2}\mu\langle\hat{a}\rangle, \tag{2.33}$$

$$\frac{d}{dt}\langle\hat{a}(t)\hat{a}(t)\rangle = -\frac{\mu}{2}\langle\hat{a}^2(t)\rangle + A\rho_{ac}^{(0)}, \tag{2.34}$$

$$\frac{d}{dt}\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle = -\mu\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle + A\rho_{aa}^{(0)} \tag{2.35}$$

where $\mu = A(\hat{\rho}_{ca} - \hat{\rho}_{aa} + \kappa)$.

We see that the c-number Langavin equation corresponding to Eqs. (2.32), (2.33), (2.34) and (2.35) is given by

$$\frac{d}{dt}\langle\alpha(t)\rangle = -\frac{\mu}{2}\langle\alpha(t)\rangle, \tag{2.36}$$

$$\frac{d}{dt}\langle\alpha(t)\alpha(t)\rangle = \frac{-\mu}{2}\langle\alpha(t)\alpha(t)\rangle + A\rho_{ac}^{(0)}, \tag{2.37}$$

$$\frac{d}{dt}\langle\alpha^*(t)\alpha(t)\rangle = -\mu\langle\alpha^*(t)\alpha(t)\rangle + A\rho_{aa}^{(0)}, \tag{2.38}$$

At steady state, solutions of Eqs.(2.36), (2.37) and (2.38) have the form

$$\langle\alpha(t)\rangle_{ss} = 0,$$

$$\langle\alpha^2(t)\rangle_{ss} = \frac{A\rho_{ac}^{(0)}}{\mu},$$

and

$$\langle\alpha^*(t)\alpha(t)\rangle_{ss} = \frac{A\rho_{aa}^{(0)}}{\mu}.$$

On the basis of Eq. (2.36), we can write

$$\frac{d}{dt}\alpha(t) = -\frac{\mu}{2}\alpha(t) + f(t), \tag{2.39}$$

where $f(t)$ is a noise force vanishing mean. It can be readily established that

$$\langle f(t)f(t^{'})\rangle \;=\; A\hat{\rho}^{(0)}_{ac}\delta(t-t^{'}), \tag{2.40}$$

and

$$\langle f^{*}(t)f(t^{'})\rangle \;=\; A\hat{\rho}^{(0)}_{aa}\delta(t-t^{'}). \tag{2.41}$$

Now we introduce a new variable

$$\alpha_{\pm}(t) \;=\; \alpha^{*}(t)\pm\alpha(t). \tag{2.42}$$

This implies that

$$\frac{d}{dt}\alpha_{\pm}(t) \;=\; -\mu/2\alpha\pm(t)+f^{*}(t)\pm f(t). \tag{2.43}$$

We observe that this equation has no well behaved soluion for

$\kappa < A(\hat{\rho}_{aa}-\hat{\rho}_{cc})$. Therefore we can write the solution for $\kappa > A(\hat{\rho}_{aa}-\hat{\rho}_{cc})$ by setting $\kappa = A(\hat{\rho}_{aa}-\hat{\rho}_{cc})$ as a threshold condition. The formal solution of Eq.(2.43) can be written as

$$\alpha_{\pm}(t) \;=\; \alpha_{\pm}(0)e^{-\mu/2}+\int_{0}^{t}e^{-\mu(t-t^{'})/2}\Big(f^{*}(t)\pm f(t)\Big)dt^{'}. \tag{2.44}$$

In a view of Eq.(2.42), we find

$$\alpha(t) \;=\; A_{+}(t)\alpha(0)+B_{+}(t)-B_{-}(t), \tag{2.45}$$

$$A_{+}(t)=e^{-}\mu/2,$$

$$B_{\pm}(t)=1/2\int_{0}^{t}e^{-\mu(t-t^{'})/2}\Big(f^{*}(t)\pm f(t)\Big)dt^{'}.$$

Assuming the cavity mode initially to be in a vacuum state, the expectation value of Eq.(2.45) can be written as

$$\begin{aligned}\langle \alpha(t) \rangle &= \langle B_+(t) \rangle - \langle B_-(t) \rangle \\ &= \int_0^t e^{-\mu(t-t')/2} \langle f(t) \rangle dt'. \end{aligned} \tag{2.46}$$

In a view of Eqs.(2.36),(2.39) along with Eq.(2.46),it can be easily verified that

$$\langle \alpha(t) \rangle = 0. \tag{2.47}$$

We see that $\alpha(t)$ is a Gaussian variable with zero mean.

2.2 The Q function

Now we seek to obtain the Q-function for a light produced by degenerate three level laser. The Q-function can be expressed by using the anti-normally ordered characteristic function as

$$Q(\alpha^*, \alpha, t) = \frac{1}{\pi^2} \int d^2 z \phi_a(z^*, z, t) e^{z^* \alpha - z\alpha *}, \tag{2.48}$$

where $\phi_a(z^*, z, t)$ is the anti-normally ordered characteristic function. It can be defined as

$$\phi_a(z^*, z, t) = Tr(\hat{\rho} e^{-z^* \hat{a}} e^{z \hat{a}^\dagger}).$$

Employing the identity

$$e^{\hat{A}} e^{\hat{B}} = e^{\hat{B}} e^{\hat{A}} e^{[\hat{A}, \hat{B}]},$$

we obtain

$$\phi_a(z^*, z, t) = e^{-zz^*} Tr(\hat{\rho} e^{z\hat{a}^\dagger} e^{-z^* \hat{a}}). \tag{2.49}$$

Since α is a Gaussian variable with zero mean Eq. (49) can be written in the form of

$$\phi_a(z^*, z, t) \;=\; e^{-zz^*} exp(\langle\alpha{*}^2\rangle z^2/2 + \langle\alpha^2\rangle z{*}^2/2 - zz^*\langle\alpha\alpha^*\rangle). \tag{2.50}$$

Now we define a new parameter as follows

$$\hat{\rho}_{aa}^{(0)} = 1 - \eta/2$$

$$\hat{\rho}_{aa}^{(0)} + \hat{\rho}_{cc}^{(0)} = 1,$$

and

$$|\hat{\rho}_{ac}^{(0)}|^2 = \hat{\rho}_{aa}^{(0)}\hat{\rho}_{cc}^{(0)},$$

one easily find,

$$\hat{\rho}_{cc}^{(0)} = 1 + \eta/2.$$

and

$$|\hat{\rho}_{ac}^{(0)}| = \sqrt{1-\eta^2}/2,$$

up on setting $\hat{\rho}_{ac}^{(0)} = |\hat{\rho}_{ac}^{(0)}|e^{i\theta}$. By taking into account the above relations,we see that

$$\phi_a(z^*, z, t) \;=\; exp[-azz^* + (z^2b^* + z^{*2}b)/2], \tag{2.51}$$

$$a_1 = 1 + \frac{A(1-\eta)}{2(A\eta+\kappa)}, \tag{2.52}$$

and

$$b_1 = \frac{A(1-\eta)}{2(A\eta+\kappa)} e^{i\theta}. \tag{2.53}$$

Hence, introducing Eq. (51) into Eq. (48) and carrying out the integration, the Q-function for the cavity mode formed at a steady state to be

$$Q(\alpha^*, \alpha, t) = \frac{(u^2 - vv^*)^{1/2}}{\pi} exp(-u\alpha\alpha^* + (\alpha^2 v^* + \alpha^{*2} v)/2), \tag{2.54}$$

in which

$$u = \frac{a_1}{a_1^2 - b_1 b_1^*} \tag{2.55}$$

and

$$v = \frac{b_1}{a_1^2 - b_1 b_1^*} \tag{2.56}$$

By setting $\theta = 0^0$, then $v^* = v$, hence we see

$$Q(\alpha^*, \alpha, t) = \frac{(u^2 - v^2)^{1/2}}{\pi} exp(-u\alpha^*\alpha - \frac{v}{2}(\alpha^2 + \alpha^{*2}), \tag{2.57}$$

where

$$v = \frac{b_1}{a_1^2 - b_1^2}, \tag{2.58}$$

with $b_1 = b_1^*$

2.3 Photon Statistics

The statistical properties of a light beam is described in-terms of the mean photon number,the variance photon number,quadrature variance and the photon number distribution.Hence, we calculate the mean photon number, the variace photon number of the light generated by degenerate squeezed three level laser employing the Q- function [18].

2.3.1 The mean Photon number

The mean photon number for the degenerate three level laser in-terms of Q-function can be defined as

$$\overline{n} = \int d^2\alpha Q(\alpha^*, \alpha)\bigg(\alpha^*\alpha - 1\bigg). \tag{2.59}$$

Performing integration over Eq. (2.59) using the identity

$$\int \frac{d^2z}{\pi} exp\bigg(azz^* + bz + cz^* + A^{'}z^2 + B^{'}z^{*2}\bigg)$$
$$= \bigg[\frac{1}{a^2 - 4A'B'}\bigg]^{\frac{1}{2}} exp\bigg[\frac{abc + A^{'}c^2 + B^{'}b^2}{a^2 - 4A'B'}\bigg], a > 0 \tag{2.60}$$

we readly obtain

$$\overline{n} = \frac{u}{u^2 - v^2} - 1. \tag{2.61}$$

The mean photon number at steady state turns out to be

$$\overline{n} = \frac{A(1-\eta)}{2(A\eta + \kappa)} \tag{2.62}$$

2.3.2 The Variance of photon number

The variance of the photon number can be given by

$$(\Delta n)^2 = \langle \hat{n}^2 \rangle - \overline{n}^2. \tag{2.63}$$

It then follows

$$(\Delta n)^2 = \overline{n} + \overline{n}^2 + \langle \alpha(t) \rangle_{ss} \langle \alpha(t) \rangle_{ss}, \tag{2.64}$$

so that using and its complex conjugate, the variance the photon number at steady state can be written as

$$(\Delta n)^2_{ss} = \frac{A(1-\eta)(A + A\eta + \kappa)}{2(A\eta + \kappa)^2} \tag{2.65}$$

$$(\Delta n)^2_{ss} = \bar{n}(1 + \frac{A}{A\eta + \kappa}) \tag{2.66}$$

2.4 Quadrature fluctuation

Next we calculate the quadrature variance and squeezing.

2.4.1 Quadrature variance

The quadrature variance of single mode light is described by the minus and plus quadrature operator as,

$$\hat{a}_+ = \hat{a} + \hat{a}^\dagger \tag{2.67}$$

$$\hat{a}_- = i(\hat{a}^\dagger - \hat{a}), \tag{2.68}$$

where $\hat{a}_+$ and $\hat{a}_-$ are Hermitian operators representing the physical quantities called plus and minus quadratures, respectively.Therefore the quadrature can be expressed interms of the quadrature operators as

$$(\Delta\hat{a}_\pm)^2 = \langle\hat{a}^2_\pm\rangle - \langle\hat{a}_\pm\rangle^2. \tag{2.69}$$

Then tends to

$$(\Delta\hat{a}_+)^2 = 1 + \langle\hat{a}^2\rangle + \langle\hat{a}^{\dagger 2}\rangle + 2\langle\hat{a}^\dagger\hat{a}\rangle, \tag{2.70}$$

and

$$(\Delta\hat{a}_-)^2 = 1 + 2\langle\hat{a}^\dagger\hat{a}\rangle + \langle\hat{a}^{\dagger 2}\rangle - \langle\hat{a}^2\rangle. \tag{2.71}$$

Then we can evaluate the expectation value of $\hat{a}^2$ using the Q-function of Light beam as

$$\langle \hat{a}^2 \rangle = \int d^2\alpha Q(\alpha, \alpha^*)\alpha^2,$$

where α^2 is the c-number variable corresponding to the operator $\hat{a}^2$. Using Eq.(2.54),we have

$$\langle \hat{a}^2 \rangle \;=\; (u^2 - v^2)^{\frac{1}{2}} \int \frac{d^2\alpha}{\pi} exp\bigg(- u\alpha\alpha^* + v\frac{(\alpha^2 + \alpha^{*2})}{2}\bigg)\alpha^2. \tag{2.72}$$

Upon carrying the integration, we readily get

$$\langle \hat{a}^2 \rangle \;=\; \frac{v}{(u^2 - v)^2}. \tag{2.73}$$

By substituting Eqs. (2.69),(2.70) and (2.71) into (2.67),the quadrature variance becomes

$$(\Delta a_{\pm})^2 \;=\; 1 + 2\overline{n} + \frac{2v}{(u^2 - v^2)^2}. \tag{2.74}$$

Hence

$$(\Delta a_{\pm})^2 \;=\; \frac{\kappa + A\bigg(1 \pm (1 - \eta^2)^{\frac{1}{2}} \cos\theta\bigg)}{A\eta + \kappa}\bigg[1 - e^{(A\eta+\kappa)t}\bigg]. \tag{2.75}$$

2.4.2 Quadrature squeezing

One can obtain the squeezing as

$$S_+ = 1 - (\Delta a_+)^2. \tag{2.76}$$

In a view of Eq.(2.74), the squeezing can be written as

$$S_+ = 1 - \frac{\kappa + A\bigg(1 \pm (1 - \eta^2)^{\frac{1}{2}} \cos\theta\bigg)}{A\eta + \kappa}\bigg[1 - e^{(A\eta+\kappa)t}\bigg]. \tag{2.77}$$

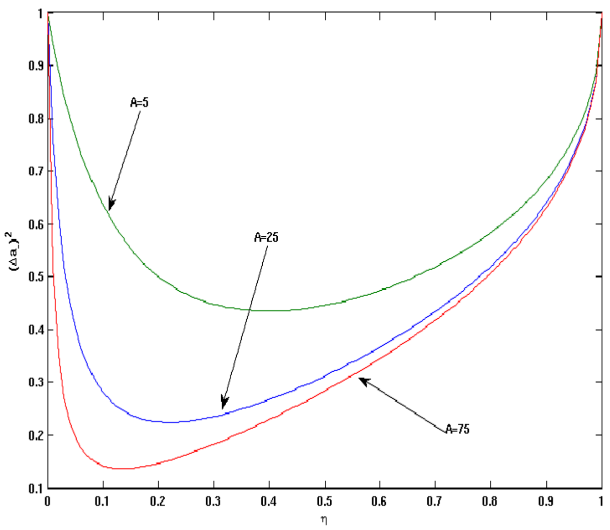

Figure 2.2: Plots of $(\Delta a_{\pm})^2$ at steady state versus η for $\kappa = 0.8$, $\theta = 0$, and for $A = 5, 25, 75$

.

At a steady state it turns out

$$S_{+} = 2 + 2\overline{n} + \frac{2v}{(u^2 - v^2)^2}. \tag{2.78}$$

3

One Mode Sub-harmonic Light

We first describe the Hamiltonian of the system. Using the Hamiltonian, we calculate c-number Langavin equation and the Q-function for the signal mode produced by one mode sub-harmonic generator coupled to a vacuum reservoir via a single port mirror.

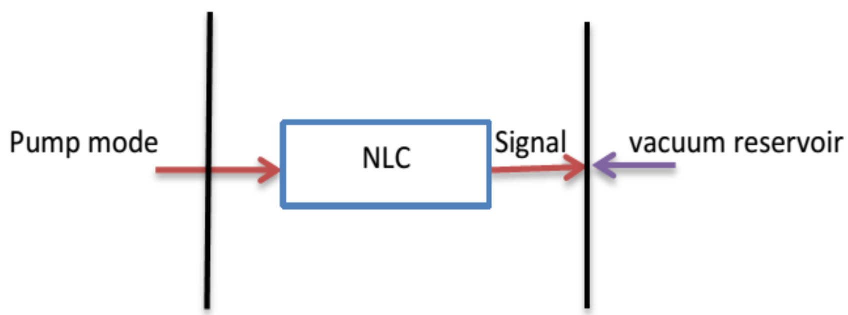

Figure 3.1: Schematic diagram of one mode subharmonic generator

3.1 C-number Langavin equation

In one mode sub-harmonic generator, a pump photon of frequency 2ω is down converted into a pair of single photons each of frequency ω [19,20].

$$\hat{H} = i\frac{\epsilon}{2}(\hat{a}^2 - \hat{a}^{\dagger 2}), \tag{3.1}$$

where $\epsilon = \lambda\beta$ and $\hat{a}$ is the annihilation operator for the signal mode and λ is coupling constant between signal mode and driving mode. On the other hand the master equation for a one mode subharmonic generator coupled to vacuum reservoir can be put in the form [17].

$$\begin{aligned} \frac{d}{dt}\hat{\rho}_{(t)} &= \frac{\epsilon}{2}(\hat{a}^{\dagger 2}\hat{\rho}(t) - \hat{a}^2\hat{\rho}(t) - \hat{\rho}(t)\hat{a}^{\dagger 2} + \hat{\rho}_{(t)}\hat{a}^2) \\ &+ \frac{k}{2}(2\hat{a}\hat{\rho}(t)\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho}(t) - \hat{\rho}(t)\hat{a}^\dagger\hat{a}). \end{aligned} \tag{3.2}$$

Employing the relation

$$\frac{d}{dt}\langle\hat{A}(t)\rangle = Tr(\frac{d}{dt}\hat{\rho}(t)\hat{A}), \tag{3.3}$$

with the commutation relation

$$[\hat{a}, \hat{a}^\dagger] = 1, \tag{3.4}$$

we readily find

$$\frac{d}{dt}\langle\hat{a}(t)\rangle = -\frac{\kappa}{2}\langle\hat{a}(t)\rangle - \epsilon\langle\hat{a}(t)^\dagger\rangle, \tag{3.5}$$

and

$$\frac{d}{dt}\langle\hat{a}(t)\hat{a}(t)\rangle = -\kappa\langle\hat{a}^2\rangle - 2\epsilon(\langle\hat{a}(t)^\dagger\hat{a}(t)\rangle) - \epsilon. \tag{3.6}$$

$$\frac{d}{dt}\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle = -\epsilon\langle\hat{a}^2(t)\rangle - \epsilon\langle\hat{a}^{\dagger 2}(t)\rangle - \kappa\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle. \tag{3.7}$$

The c-number Langavin equation corresponding to operators in normal ordering can be put in the form

$$\frac{d}{dt}\langle\alpha(t)\rangle \;=\; -\frac{\kappa}{2}\langle\alpha(t)\rangle - \epsilon\langle\alpha(t)^*\rangle, \tag{3.8}$$

$$\frac{d}{dt}\langle\alpha(t)\alpha(t)\rangle \;=\; -\kappa\langle\alpha(t)^2\rangle - 2\epsilon\langle\alpha(t)^*\alpha(t)\rangle - \epsilon, \tag{3.9}$$

$$\frac{d}{dt}\langle\alpha(t)^*\alpha(t)\rangle \;=\; -\kappa\langle\alpha(t)^*\alpha(t)\rangle - \epsilon\langle\alpha(t)*^2\alpha\rangle - \epsilon\langle\alpha(t)^2\rangle, \tag{3.10}$$

On the basis of Eq. (3.7), one can write

$$\frac{d}{dt}\alpha(t) \;=\; -\frac{\kappa}{2}\alpha(t) - \epsilon\alpha^*(t) + f(t), \tag{3.11}$$

where f(t) is a noise force, the properties of which remains to be determined.We note that Eq.(3.7) and the expectation value of Eq.(3.10) will have identical form if

$$\langle f(t)\rangle \;=\; 0 \tag{3.12}$$

applying the relation

$$\frac{d}{dt}\langle A(t)A(t)\rangle \;=\; \langle A(t)\frac{d}{dt}A(t)\rangle + \langle\frac{d}{dt}A(t)A(t)\rangle, \tag{3.13}$$

we have

$$\frac{d}{dt}\langle\alpha(t)\alpha(t)\rangle \;=\; -\kappa\langle\alpha(t)^2\rangle - 2\epsilon\langle\alpha(t)^*\alpha(t)\rangle + 2\langle\alpha(t)(f(t))\rangle. \tag{3.14}$$

Compression of Eqs.(3.8) and (3.13) shows that

$$\langle\alpha(t)f(t)\rangle \;=\; -\frac{\epsilon}{2}. \tag{3.15}$$

The formal solution of Eq.(3.10)

$$\alpha(t) \;=\; \alpha(0)e^{-\kappa t/2} + \int_0^{(t)} e^{-\kappa(t-t')}/2\Big(-\epsilon\alpha^*(t) + f(t')\Big)dt'. \tag{3.16}$$

Multiplying from the left at both sides by f(t), we find

$$\langle\alpha(t)f(t)\rangle \;=\; \int_0^t e^{\frac{-k(t-t^{'})}{2}}\Big(\langle f(t)f(t^{'})\rangle\Big)dt^{'} = (\frac{-\epsilon}{2}). \tag{3.17}$$

$$\int_0^t e^a(t-t^{'})[\langle f(t)g(t^{'})\rangle]dt^{'} \;=\; D, \tag{3.18}$$

we assert that

$$\langle f(t)g(t^{'})\rangle \;=\; 2D\delta(t-t'), \tag{3.19}$$

where D is a constant or some function of time t, we see that

$$\langle f(t)f(t^{'})\rangle \;=\; -\epsilon\delta(t-t'). \tag{3.20}$$

Following the same procedure, one can readily obtain the correlation properties of the noise force as in the form

$$\langle f(t)f^*(t^{'})\rangle \;=\; \langle f^*(t)f(t^{'})\rangle = 0 \tag{3.21}$$

It worth that Eqs.(3.11),(3.17) and (3.19) describe the correlation properties of the noise force f(t) associated with the normal ordering . In order to obtain the formal solution of Eq.(3.10) we introduce a new variable defined by

$$\alpha_+(t) \;=\; \alpha(t)^* \pm \alpha(t). \tag{3.22}$$

It can then be verified employing Eq.(3.10) and its complex conjugate

$$\frac{d}{dt}\alpha_{\pm(t)} \;=\; -\frac{\lambda\pm}{2}\alpha\pm(t) + f^*(t)\pm f(t), \tag{3.23}$$

Where

$$\lambda\pm \;=\; \kappa\pm 2\epsilon. \tag{3.24}$$

According to Eq.(3.23) and (3.24),the equation of evolution of α_- does not have a well-behaved solution for $k < 2\epsilon$. We then identify $k = 2\epsilon$ as the threshold condition. For $2\epsilon < k$, the solution of Eq.(3.23), can be written as

$$\alpha_{\pm(t)} \;=\; \alpha_\pm(0)e^{-1/2\lambda\pm t} + \int_0^t e^{\lambda\pm(t-t')/2}\Big(f^*(t') \pm f(t')\Big)dt^{'}. \tag{3.25}$$

Now with the aid of Eqs.(3.22) and (3.25) we readily get

$$\alpha(t) \;=\; A_+(t)\alpha(0) + A_-(t)\alpha(0)^* + B_+(t) - B_-(t), \tag{3.26}$$

in which

$$A_\pm(t) \;=\; \frac{1}{2}\Big(e^{\frac{-1}{2}\lambda_+ t} \pm e^{\frac{-1}{2}\lambda_- t}\Big) \tag{3.27}$$

$$B_\pm(t) \;=\; \frac{1}{2}\int_0^t e^{\frac{-1}{2}\lambda\pm(t-t')}\Big[f^*(t') - f(t')\Big]dt', \tag{3.28}$$

3.2 The Q function

We now proceed to calculate the Q function for the signal mode applying the anti-normally ordered characteristic function in the Heisenberg picture.

$$Q(\alpha^*, \alpha, t) \;=\; \frac{1}{\pi^2}\int d_z^2\phi_a(z^*, z, t)e^{z^*\alpha - z\alpha*}, \tag{3.29}$$

The characteristic function is defined as

$$\phi_a(z,t) = Tr\big(\hat{\rho}_{(0)}e^{z^*\hat{a}(t)}e^{z\hat{a}^\dagger(t)}\big). \tag{3.30}$$

By applying the identity

$$e^{\hat{A}}e^{\hat{B}} = \quad e^{\hat{A}}e^{\hat{B}}e^{[\hat{A},\hat{B}]}, \tag{3.31}$$

we find

$$\phi_a(z,t) \;=\; e^{z^*z}Tr\Big(\hat{\rho}(0)e^{z^*\hat{a}(t)}e^{z\hat{a}^\dagger(t)}\Big). \tag{3.32}$$

Replacing the cavity mode operators by a c-number function in Eq.(3.32), we obtain

$$\phi_a(z,t) \;=\; e^{z^*z}\Big\langle exp(z\alpha^* - z^*\alpha)\Big\rangle. \tag{3.33}$$

We have shown that $\alpha(t)$ is a Gaussian variable with zero mean. One can easily establish

$$\phi_a(z,t) \;=\; e^{z^*z}ex(\frac{1}{2}\Big\langle(z\alpha^* - z^*\alpha)^2\Big\rangle). \tag{3.34}$$

It can be verified that

$$\langle\alpha^2\rangle = \langle\beta^2\rangle$$

$$\langle\alpha^{*2}\rangle = \langle\beta^{*2}\rangle$$

$$\langle\alpha^{*2}\alpha\rangle = \langle\beta^{*2}\beta\rangle,$$

$$\beta \;=\; \beta_+ - \beta_-, \tag{3.35}$$

$$\langle\beta^2\rangle \;=\; \langle\beta_+^2\rangle + \langle\beta_-^2\rangle - 2\langle\beta_+\beta_-\rangle, \tag{3.36}$$

$$\langle\beta^{*2}\rangle \;=\; \langle\beta_+^2\rangle + \langle\beta_-^2\rangle + 2\langle\beta_+\beta_-\rangle, \tag{3.37}$$

$$\langle\beta^*\beta\rangle = \langle\beta_+^2\rangle - \langle\beta_-^2\rangle,$$

In a view of Eq.(3.25) one can check that

$$\langle\beta\pm\rangle = 1/2\int_0^t e^{\lambda/2\pm}(t-t')/2[f^*(t') \pm f(t')dt']$$

$$\langle\beta_+^2\rangle = 1/4\int_0^t e^{-\lambda/2+(t-t^{'})/2}dt^{'}dt^{''}[\langle f^*(t^{'})f^*(t^{''})\rangle$$

$$+\langle f^*(t^{'})f(t^{''})\rangle + f(t^{''})f(t^{'}) + f^*(t^{'})^*(t^{''}) + \langle f(t^{''})f(t^{'})\rangle]$$

$$\langle\beta_+^2\rangle = 1/4\int_0^t e^{-\lambda/2+(t-t^{'})/2}[\langle f^*(t^{'})f^*(t^{''})\rangle$$

$$+\langle f(t^{'})f(t^{''})\rangle]dt^{'}dt^{''}$$

$$\langle\beta_+^2\rangle = 1/4\int_0^t e^{-\lambda/2+(t-t^{'})/2}[-\epsilon\delta(t-t^{'}) - \epsilon\delta(t-t^{'})]dt^{'}dt^{''}$$

Further more integrating by using the identity

$$\int d\alpha\delta(x-y)e^y$$

This implies that

$$\langle\beta_+^2\rangle = \tfrac{-\epsilon}{2\lambda_+}(1-e^{\lambda-t})$$

Similarly

$$\langle\beta_-^2\rangle = \tfrac{-\epsilon}{2\lambda_-}(1-e^{\lambda-t})$$

$$\langle\beta_+\beta_-\rangle = 0$$

We recall that

$$\langle\beta^2\rangle = \langle\beta^{*2}\rangle = \langle\beta_+^2\rangle + \langle\beta_-^2\rangle - 2\langle\beta_+\beta_-\rangle \tag{3.38}$$

$$\langle\beta^2\rangle = \tfrac{-\epsilon}{2\lambda_+}(1-e^{\lambda-t}) - \tfrac{-\epsilon}{2\lambda_-}(1-e^{\lambda-t})$$

$$\langle\beta^*\beta\rangle = \langle\beta_+^2\rangle + \langle\beta_-^2\rangle$$

$$\langle\beta^*\beta\rangle = \frac{-\epsilon}{2\lambda_+}(1-e^{\lambda-t}) + \frac{-\epsilon}{2\lambda_-}(1-e^{\lambda-t}). \tag{3.39}$$

In a view of Eqs. (3.38) and (3.39), Eq. (3.30) takes the form

$$\begin{aligned}\phi_a(z,t) \ &= \ e^{-z^*z}exp[\frac{1}{2}z^2\frac{-\epsilon}{2\lambda_+}(1-e^{\lambda-t}) - \frac{-\epsilon}{2\lambda_-}(1-e^{\lambda-t}))\\ &+z^{*2}\frac{-\epsilon}{2\lambda_+}(1-e^{\lambda-t}) - \frac{-\epsilon}{2\lambda_-}(1-e^{\lambda-t}))\\ &-2z^*z\alpha^*(\frac{-\epsilon}{2\lambda_+}(1-e^{\lambda-t}) + \frac{-\epsilon}{2\lambda_-}(1-e^{\lambda-t})).\end{aligned} \tag{3.40}$$

This expression leads to

$$\phi_a(z,t) = exp[-a_2z^*z + \frac{b_2(z^2+z^{*2})}{2}], \tag{3.41}$$

$$a_2 = 1 - \frac{\epsilon}{2\lambda_+}(1-e^{\lambda-t}) + \frac{-\epsilon}{2\lambda_-}(1-e^{\lambda-t}) \tag{3.42}$$

$$b_2 = -(\frac{-\epsilon}{2\lambda_+}(1-e^{\lambda-t}) + \frac{-\epsilon}{2\lambda_-}(1-e^{\lambda-t})) \tag{3.43}$$

Finally, substituting Eq.(3.41) into Eq.(3.29) and carrying out the integration, the Q-function for the signal mode turns out

$$Q(\alpha^*,\alpha,t) \ = \ \frac{1}{\pi}[l^2-m^2]^{\frac{1}{2}}exp(\frac{l\alpha\alpha^* + m(\alpha^2+\alpha^{*2})}{2}), \tag{3.44}$$

where

$$l \ = \ \frac{a_2}{(a_2^2-b_2^2)}, \tag{3.45}$$

$$m \ = \ \frac{b_2}{(a_2^2-b_2^2)}. \tag{3.46}$$

3.3 Photon statistics

Here we wish to calculate the mean photon number and quadrature variance for single light beam produced by one mode sbharmonic generator.

3.3.1 The mean photon number

The mean photon number of the signal mode is can be defined as

$$\langle \hat{n} \rangle \;=\; \int d^2\alpha Q(\alpha^*,\alpha)\hat{n}(\alpha^*,\alpha). \tag{3.47}$$

where

$$\hat{n}_a(\alpha^*,\alpha) \;=\; \alpha^*,\alpha - 1, \tag{3.48}$$

is the c-number function corresponding to the operator function $\hat{n}(\hat{a}^\dagger,\hat{a})$ in the anti-normal order. On account of Eqs (3.44) and (3.48) Eq. (3.47) can be written as

$$\overline{n} \;=\; \big(l^2-m^2\big)^{\frac{1}{2}}\int \frac{d^2\alpha}{\pi} exp\big[-l\alpha\alpha^* + m(\alpha^{*2}+\alpha^2)/2\big](\alpha^*,\alpha-1). \tag{3.49}$$

This can be put in the form

$$\overline{n} \;=\; \big(l^2-m^2\big)^{\frac{1}{2}}\frac{d^2}{d_n d_m}\int \frac{d^2\alpha}{\pi}$$
$$exp\big[-l\alpha\alpha^* + n\alpha + m\alpha^* + m(\alpha^{*2}+\alpha^2)/2\big]_{n=m=0} - 1, \tag{3.50}$$

Therefore, carrying out the integration using Eq.(2.60) and appling the condition $n=m=0$, we get

$$\overline{n} \;=\; \frac{l}{l^2-m^2} - 1. \tag{3.51}$$

Now on account Eq.(3.45)and Eq.(3.46), we easily find

$$\overline{n} \;=\; a - 1 \tag{3.52}$$

and in view of Eq. (3.42), the mean photon number takes the form

$$\overline{n} \;=\; -\frac{\epsilon}{2\lambda_{+}}\big(1-e^{\lambda_{+}t}\big)+\frac{\epsilon}{2\lambda_{-}}\big(1-e^{\lambda_{-}t}\big). \qquad (3.53)$$

Thus at steady state, we see that

$$\overline{n} \;=\; \frac{2\varepsilon^2}{k^2-4\varepsilon^2}. \qquad (3.54)$$

3.3.2 The variance of the photon number

The variance of photon number for the signal mode can be defined as

$$(\triangle n)^2 \;=\; \langle(\hat{a}^{\dagger}\hat{a})^2\rangle-\overline{n}^2. \qquad (3.55)$$

Using the Commutation relation, we see that

$$(\triangle n)^2 \;=\; \langle\hat{a}^2\hat{a}^{\dagger 2}\rangle-\overline{n}^2-3\overline{n}-2. \qquad (3.56)$$

So that employing the Q-function, we have

$$\langle\hat{a}^2\hat{a}^{\dagger 2}\rangle \;=\; \int d^2\alpha Q(\alpha,t)\alpha^{*2}\alpha^2. \qquad (3.57)$$

This can be put in the form

$$\langle\hat{a}^2\hat{a}^{\dagger 2}\rangle \;=\; \big(l^2-m^2\big)^{\frac{1}{2}}\frac{d^4}{d_{p^2}d_{q^2}}\int\frac{d^2\alpha}{\pi}$$
$$exp\big[-l\alpha\alpha^*+p\alpha+q\alpha^*+m(\alpha^{*2}+\alpha^2)/2\big]_{p=q=0}-1. \qquad (3.58)$$

Upon carrying out the integration using Eq.(2.60),we get

$$\langle\hat{a}^2\hat{a}^{\dagger 2}\rangle \;=\; \frac{d^4}{d_{p^2}d_{q^2}}exp\big[\frac{-lpq+m(p^2+q^2)/2}{l^2-m^2}\big]_{p=q=0}-1. \qquad (3.59)$$

Performing the differentiation and applying the condition z=η=0, one easily finds

$$\langle\hat{a}^2\hat{a}^{\dagger 2}\rangle \;=\; \frac{2l^2+m^2}{(l^2-m^2)^2}. \qquad (3.60)$$

Thus taking into account Eqs.(3.45) and (3.46), we readily find

$$\langle \hat{a}^2 \hat{a}^{\dagger 2} \rangle \;=\; 2a^2 + b^2. \tag{3.61}$$

Now with the aid of Eqs.(3.52) and (3.61), Eq.(3.56) can be put in the form

$$(\triangle n)^2 \;=\; a^2 + b^2 - a. \tag{3.62}$$

Finally, applying Eqs.(3.42)and (3.43), the variance of the photon number turns out to be

$$\begin{aligned}(\triangle n)^2 \;=\; & \frac{\epsilon^2}{2\lambda_+^2}\big[1 - e^{-\lambda_+ t}\big]^2 + \frac{\epsilon^2}{2\lambda_-^2}\big[1 - e^{-\lambda_- t}\big]^2 \\ & + \frac{\epsilon}{2\lambda_-}\big(1 - e^{\lambda_- t}\big) - \frac{\epsilon}{2\lambda_+}\big(1 - e^{\lambda_+ t}\big).\end{aligned} \tag{3.63}$$

At a steady state takes the form

$$(\triangle n)^2 \;=\; \big(\frac{\varepsilon k}{k^2 - 4\varepsilon^2}\big)^2 + \overline{n}^2 + \overline{n}. \tag{3.64}$$

We assert that the photon statistics of the light produced by one mode subharmonic generator is superpoissonian.

3.4 Quadrature fluctuation

In this section we will calculate the quadrature variance and quadrature squeezing for a single light mode.

3.4.1 Quadrature variance

Now we proceed to calculate the quadrature variance for the signal mode produced by one mode subharmonic generator.

$$(\Delta a_\pm)^2 \;=\; 1 + \langle : (\hat{a} \pm (_{(}t), (\hat{a} \pm (_{(}t)) : \rangle. \tag{3.65}$$

where :: stands for normal ordering. The two quadratures are given by

$$\hat{a}_{+} \;=\; (\hat{a}^{\dagger} + \hat{a}) \tag{3.66}$$

and

$$\hat{a}_{-} \;=\; i(\hat{a}^{\dagger} - \hat{a}). \tag{3.67}$$

This quadrature variance can be written in-terms of c-number variables associated with normal ordering as

$$(\triangle a_{\pm})^2 \;=\; 1 \underline{+} \langle \alpha_{\pm}(t), \alpha_{\pm}(t)\rangle, \tag{3.68}$$

in which

$$\alpha_{\pm}(t) \;=\; \alpha^* \underline{+} \alpha. \tag{3.69}$$

Eq.(3.68) can be written as

$$(\triangle a_{\pm})^2 \;=\; 1 \underline{+} \langle \alpha_{\pm}^2\rangle \underline{+} \langle \alpha_{\pm}\rangle^2. \tag{3.70}$$

We note that

$$\langle \alpha_{\pm}(t)\rangle. \;=\; 0. \tag{3.71}$$

Hence Eq.(3.70) can be put in the form

$$(\triangle a_{\pm})^2 \;=\; 1 \underline{+} \langle \alpha_{\pm}^2\rangle. \tag{3.72}$$

In addition, using Eq. (3.23),we easily see

$$\frac{d}{dt}\langle \alpha_{\pm(t)}^2\rangle \;=\; -\lambda_{\pm}\langle \alpha_{\pm(t)}^2\rangle + 2\langle \alpha_{\pm(t)} f^*(t)\rangle \underline{+} 2\langle \alpha_{\pm(t)} f(t)\rangle. \tag{3.73}$$

In view of Eqs. (3.12), (3.15) and (3.15), we note that

$$\langle \alpha(t) f^*(t)\rangle \;=\; \langle \alpha(t)^* f(t)\rangle = 0. \tag{3.74}$$

Now with the aid of Eq.(3.71) along with Eqs. (3.15) and (3.74), we readily find

$$\langle \alpha_{\pm(t)} f^*(t) \rangle = -\frac{\varepsilon}{2} \tag{3.75}$$

and

$$\langle \alpha_{\pm(t)} f(t) \rangle \;=\; \mp\frac{\varepsilon}{2}. \tag{3.76}$$

Therefore, in view of the results Eq. (3.73) can be rewritten as

$$\frac{d}{dt}\langle \alpha_{+}^2(t) \rangle \;=\; -\lambda_{\pm}\langle \alpha_{+}^2(t) \rangle - 2\varepsilon. \tag{3.77}$$

A formal solution of this can be written as

$$\langle \alpha_{+}^2(t) \rangle \;=\; \langle \alpha_{+}^2(0) \rangle e^{-\lambda_{+}}t + \int_0^t e^{-\lambda_{\pm}(t-t')}[-2\varepsilon]dt', \tag{3.78}$$

with the cavity mode initially in a vacuum state, this equation turns out to be

$$\langle \alpha_{+}(t) \rangle \;=\; -\frac{2\varepsilon}{\lambda_{\pm}}[1 - e^{-\lambda_{\pm}t}]. \tag{3.79}$$

Now combination of Eq.(3.72) and Eq. (3.79) yields

$$(\triangle a_{\pm}(t))^2 \;=\; 1\pm\frac{2\varepsilon}{\lambda_{\pm}}[1 - e^{-\lambda_{\pm}t}]. \tag{3.80}$$

Moreover, taking into account Eq.(3.76),at steady state and at threshold, we see that

$$(\triangle a_{\pm}(t))^2 \;=\; \frac{1}{2}. \tag{3.81}$$

3.4.2 Quadrature squeezing

Quadrature squeezing is defined by

$$S_{+} = 1 - (\Delta a_{+})^2. \tag{3.82}$$

In a view of equations (3.81) and (3.82), we have

$$S_+ = 1 - \frac{1}{2}. \tag{3.83}$$

We observe that the signal mode is in squeezed state and the squeezing occurs in the plus quadrature. The squeezing increases with time and reaches its maximum value at steady state.

We note that at steady state and at threshold there is a 50% squeezing of the signal mode

4

The Superposition of Degenerate Squeezed Three Laser with Subharmonic Light

Here we seek to study the statistical and squeezing properties of the superposed light mode produced by degenerate three level laser and one-mode subharmonic generator. To this end, first we determine the Q function for this light mode. With the aid of the resulting Q function, we calculate the photon Statistics and quadrature squeezing.

4.1 The Q function

The Q-function is used to describe the superposition of the two light beams with the same frequency. The Q function, defined by

$$Q(\alpha^*, \alpha, t) \;=\; \frac{1}{\pi}\langle\alpha|\hat{\rho}_2|\alpha\rangle, \tag{4.1}$$

where Q is the c-number function corresponding to normally ordered density operator divided by π. Suppose $\hat{\rho}(\hat{a}^\dagger, \hat{a})$ is the density operator for a certain light beams. Then upon expanding it in the normal order and applying the completeness relation for a coherent states, one easily finds[20].

$$\hat{\rho}' = \frac{1}{\pi}\int d^2\beta \sum_{kl} C_{kl}\beta^{*k}|\beta\rangle\langle\beta|a^l, \tag{4.2}$$

where

$$|\beta\rangle\langle\beta|a^l = (\beta + \frac{\partial}{\partial\beta^*})^l|\beta\rangle\langle\beta|. \tag{4.3}$$

There follows

$$\hat{\rho}_1 = \frac{1}{\pi}\int d^2\beta \sum_{kl} C_{kl}\beta^{*k}(\beta + \frac{\partial}{\partial\beta^*})^l|\beta\rangle\langle\beta|. \tag{4.4}$$

This expression for the density operator can be put in the form

$$\hat{\rho}_1 = \frac{1}{\pi}\int d^2\beta \sum_{kl} C_{kl}\beta^{*k}(\beta + \frac{\partial}{\partial\beta^*})^l\hat{D}(\beta)|0\rangle\langle 0|\hat{D}(-\beta). \tag{4.5}$$

We now realize that the density operator for the superposition of the first light beam and another one is expressible as

$$\hat{\rho}_2 = \frac{1}{\pi}\int d^2\gamma \sum_{mn} \gamma^{*m}(\gamma + \frac{\partial}{\partial\gamma^*})^n\hat{D}(\gamma)\hat{\rho}_1^0\hat{D}(-\gamma), \tag{4.6}$$

so that in a view of Eq. (4.5),we have

$$\hat{\rho}_2 = \frac{1}{\pi}\int d^2\beta d^2\gamma \sum_{kl} C_{kl}\beta^{*k}(\beta + \frac{\partial}{\partial\beta^*})^l \sum_{mn}\gamma^{*m}(\gamma + \frac{\partial}{\partial\gamma^*})^n \times e^{(\alpha-\beta-\gamma)}$$
$$\times e^{-\alpha^*\alpha+\alpha^*\beta+\alpha^*\gamma}. \tag{4.7}$$

Introducing Eq. (4.7) into (4.1) and carrying uot the integration, the Q function for a pair of superposed single-mode beams can be given by

$$\begin{aligned} Q(\alpha^*,\alpha) = & \frac{1}{\pi}\int d^2\beta d^2\gamma Q(\beta^*,\alpha-\gamma)Q(\gamma^*,\alpha-\beta), \\ & exp[-\alpha^*\alpha - \beta^*\beta - \gamma^*\gamma + \alpha^*\beta + \alpha\beta^* + \alpha^*\gamma + \\ & \alpha\gamma^* - \beta^*\gamma - \beta\gamma^*]. \end{aligned} \tag{4.8}$$

where

$$Q'(\beta^*, \alpha - \gamma, t) = \frac{1}{\pi} \sum_{kl} C_{kl} \beta^{*k} (\alpha - \gamma)^l \tag{4.9}$$

and

$$Q''(\gamma^*, \alpha - \beta, t) = \frac{1}{\pi} \sum_{mn} C_{mn} \beta^{*m} (\alpha - \gamma)^n. \tag{4.10}$$

Up on setting $\theta = 0$ into Eq. (2.53), we note that $v^* = v$. Therefore the Q-fuction for the degenerate squeezed three level laser can be written as

$$Q(\beta^*, \alpha - \gamma) = \frac{[u^2 - v^2]^{1/2}}{\pi} exp\Big[- u\beta^*\alpha + u\beta^*\gamma - \frac{v}{2}\alpha^2 + v\alpha\gamma - \frac{v}{2}\gamma^2 - \frac{v}{2}\beta^{*}\Big] \tag{4.11}$$

On the hand, the Q function for the one mode sub harmonic light is written as

$$Q(\gamma^*, \alpha - \beta) = \frac{[l^2 - m^2]^{1/2}}{\pi} exp\Big[- l\gamma^*\alpha + l\gamma^*\beta - \frac{m}{2}\alpha^2 + m\alpha\beta - \frac{m}{2}\beta^2 - \frac{m}{2}\gamma^{*2}\Big]. \tag{4.12}$$

Now introducing Eqs. (4.11), (4.12) in to Eq. (4.8), we have

$$Q(\alpha^*, \alpha, t) = \frac{[u^2 - vv^*]^{1/2}[l^2 - m^2]^{1/2}}{\pi} exp\Big[\alpha^*\alpha + \frac{v}{2}\alpha^2 - \frac{v}{2}\alpha^{*2}\Big] \times I_1 \times I_2, \tag{4.13}$$

in which

$$I_1 = \int \frac{d^2\beta}{\pi} exp\Big[- \beta^*\beta - u\alpha\beta^* + \frac{v}{2}\beta^{*2} - \frac{m}{2}\beta^2 + m\alpha\beta + \alpha\beta^* + \alpha^*\beta\Big] \tag{4.14}$$

and

$$I_2 = \int \frac{d^2\gamma}{\pi} exp\Big[- \gamma^*\gamma + \frac{v^*}{2}\gamma^2 + u\beta^*\gamma - v^*\alpha\gamma - l\alpha\gamma^* + l\beta\gamma^* - \frac{m}{2}\gamma^{*2} + \alpha^*\gamma + \alpha\gamma^* - \beta^*\gamma - \beta\gamma^*\Big]. \tag{4.15}$$

Thus performing the integration over γ, we have

$$Q(\alpha^*,\alpha,t) = \left[\frac{(u^2-v^2)^{1/2}(l^2-m^2)^{1/2}a}{\pi}\right]exp\left[-la\alpha^*\alpha+(vm^2-m+l^2v)\frac{a\alpha^2}{2}-\frac{1}{2}ma\alpha\alpha^2\right] \times I_3, \tag{4.16}$$

where

$$\begin{aligned} I_3 &= \int\frac{d^2\beta}{\pi}exp\bigg[-(u+l-ul+vm)a\beta^*\beta+\Big((-l^2v+m+lv+vm^2)\alpha a\beta+(l+vm)\alpha^*a\beta\Big) \\ &+\Big((l-ul)\alpha a\beta^*+(m-um)\alpha^*a\beta^*\Big)+\Big(2vl-vm^2vl^2+v-m)a\beta^2\Big) \\ &+\Big(-2mu-mv^2-mu^2+v-m\Big)a\beta^2\bigg], \end{aligned} \tag{4.17}$$

where

$$a = exp\left[\frac{1}{(1-vm)}\right]. \tag{4.18}$$

Upon carrying out the integration over β, we readly find

$$Q(\alpha^*,\alpha,t) = \frac{(u^2-v^2)^{1/2}(m^2-l^2)^{1/2}}{\pi r(t)^{1/2}}exp\left[\frac{-p(t)}{r(t)}\alpha^*\alpha-\frac{q(t)}{2r(t)}\alpha^2-\frac{q(t)}{2r(t)}\alpha^{*2}\right], \tag{4.19}$$

$$p(t) = u^2(l^2-l-m^2)+u(m^2-l^2)+v^2(l-l^2+m^2), \tag{4.20}$$

$$q(t) = v^2m+m^2v-l^2v-u^2m \tag{4.21}$$

$$\begin{aligned} r(t) &= v^2-2lv^2+l^2(v^2-1)+2vm+m^2-v^2m^2 \\ &+2u(l^2-l-m^2)+u^2(2l-l^2+m^2-1). \end{aligned} \tag{4.22}$$

4.2 Photon Statistics

In this section we calculate the mean photon number and the variance of the photon number for the superposition of degenerate three level laser and one mode subharmonic light beams.

4.2.1 The mean photon number

Next we seek to calculate the mean photon number for the superposition of degenerate squeezed three level laser and one mode sub-harmonic light beams. To this end, the mean photon number is expressible interms of the Q function as

$$\langle \hat{a}^{\dagger}\hat{a}\rangle = \frac{1}{\pi}\int d^2\alpha Q(\alpha^*,\alpha,t)[\alpha^*,\alpha-1]. \tag{4.23}$$

Upon substituting Eq. (4.19) Eq.(4.23), we find

$$\begin{aligned}\langle \hat{a}^{\dagger}\hat{a}\rangle = \frac{(u^2-v^2)^{1/2}(m^2-l^2)^{1/2}}{r^{1/2}}\Bigg[\frac{d}{dz}\frac{d}{dw}\int d^2exp\bigg(\frac{-p}{r}\alpha^*\alpha+\frac{q}{2r}\alpha^2 \\ -\frac{q}{2r}\alpha^{*2}+w\alpha+z\alpha^*\bigg)-1\Bigg]\end{aligned} \tag{4.24}$$

Upon performing the integration over α, we find

$$\langle \hat{a}^{\dagger}\hat{a}\rangle = \frac{d}{dw}\frac{d}{dz}exp\left[\frac{\frac{p}{r}wz-\frac{q}{2r}w^2-\frac{q}{2r}z^2}{\frac{p^2}{r^2}-\frac{q^2}{r^2}}\right], \tag{4.25}$$

in which

$$\left[\frac{(u^2-v^2)(m^2-l^2)r}{p^2-q^2}\right]^{1/2} = 1, \tag{4.26}$$

is the normalization constant which is equal to unity

Upon carrying out the differentiation with respect to w and z we readly obtain

$$\langle \hat{a}^{\dagger}\hat{a}\rangle = \frac{rp}{p^2-q^2}-1. \tag{4.27}$$

Substituting Eqs. (4.20), (4.21) and (4.22) into (4.27), we see that

$$\langle \hat{a}^{\dagger}\hat{a}\rangle = \left[\frac{u}{u^2-v^2}+\frac{l}{l^2-m^2}-2\right] \tag{4.28}$$

Then introducing Eqs. (2.52) and (3.44), we get

$$\langle \hat{a}^\dagger \hat{a} \rangle \;=\; \Big[a_1 - 1 + a_2 - 1\Big]. \tag{4.29}$$

Finally, putting the values of a_1 and a_2 Eq. (4.29) the mean photon number tuns out as

$$\bar{n} \;=\; \frac{A(1-\eta)}{2(A\eta+\kappa)} + \frac{2\epsilon^2}{\kappa^2 - 4\epsilon^2}. \tag{4.30}$$

From this we see that the superposed mean photon number is the sum of the mean numbers.

4.2.2 The variance of the photon number

Next we proceed to obtain the variance of the photon number for the light mode produced by degenerate three level laser and one-mode subharmonic generator.

$$(\Delta n)^2 \;=\; \langle(\hat{c}^\dagger \hat{c})^2\rangle - \langle \hat{c}^\dagger \hat{c}\rangle^2. \tag{4.31}$$

Employing the commutation relation

$$[\hat{c}, \hat{c}^\dagger] \;=\; 2, \tag{4.32}$$

we have

$$(\Delta n)^2 \;=\; \langle \hat{c}^{\dagger 2}\hat{c}^2\rangle + 2\langle \hat{c}^\dagger \hat{c}\rangle - \langle \hat{c}^\dagger \hat{c}\rangle^2. \tag{4.33}$$

$$(\Delta n)^2 \;=\; \langle \hat{c}^{\dagger 2}\rangle\langle \hat{c}^2\rangle + 2\langle \hat{c}^\dagger \hat{c}\rangle - \langle \hat{c}^\dagger \hat{c}\rangle^2. \tag{4.34}$$

$$\langle \hat{c}^{\dagger 2}\rangle\langle \hat{c}^2\rangle \;=\; 4\langle\hat{c}_1^\dagger c_2^\dagger)\rangle\langle(\hat{c}_1\hat{c}_2)\rangle \tag{4.35}$$

$$\langle \hat{c}_1^\dagger c_2^\dagger)\rangle\langle(\hat{c}_1\hat{c}_2)\rangle = \langle(\hat{c}_1^\dagger + c_2^\dagger)^2\rangle\langle(\hat{c}_1 + \hat{c}_2)^2\rangle, \tag{4.36}$$

where

$$c^\dagger = c_1^\dagger + c_2^\dagger \tag{4.37}$$

$$c = c_1 + c_2 \tag{4.38}$$

Employing the the Q function, one can write

$$\langle \hat{c}_1^\dagger c_2^\dagger)\rangle = \left[\frac{(u_2 - v^2)(l^2 - m^2)}{r}\right]^{1/2} \int \frac{d^2\alpha d^2\beta}{\pi} exp(\frac{-p}{r}\alpha^*\alpha - \frac{q}{2r}(\alpha^2 + \alpha^2$$
$$-\alpha^*\alpha - \beta^*\beta + \alpha^*\beta + \alpha\beta^*)\alpha^*\beta^* \tag{4.39}$$

Integrating Eq. (4.40)with respect to β, we obtain

$$\langle \hat{c}_1^\dagger c_2^\dagger)\rangle = \left[\frac{(u_2 - v^2)(l^2 - m^2)}{r}\right]^{1/2} \frac{d^2}{dxdy} \int exp\Bigg[\frac{-p}{r}\alpha^*\alpha$$
$$+\alpha^*\alpha + y\alpha^* - \frac{q}{2r}(\alpha^2 + \alpha^{*2} - \alpha^*\alpha - x\alpha^*)\Bigg]_{x=y=0}. \tag{4.40}$$

Upon integrating Eq. (4.40) with respect to α , we see that

$$\langle \hat{c}_1^\dagger c_2^\dagger)\rangle = \left[\frac{(u_2 - v^2)(l^2 - m^2)}{r}\right]^{1/2} (\frac{r^2}{p^2 + q^2})^{1/2} \frac{d^2}{dxdy} exp\left[\frac{-rq(x+y)}{p^2 + q^2}\right] \tag{4.41}$$

Taking out differentiation, we readily get

$$\langle \hat{c}_1^\dagger c_2^\dagger)\rangle = \left[\frac{(u_2 - v^2)(l^2 - m^2)r}{p^2 - q^2}\right]^{1/2} \frac{-2rq}{p^2 + q^2} \tag{4.42}$$

In a view of Eq. (4.26) Eq.(4.42) can be written as

$$\langle(\hat{c}_1^\dagger c_2^\dagger)\rangle = \frac{-2rq}{p^2 + q^2}. \tag{4.43}$$

Employing Eqs. (4.20), (4.21) and (4.22), we see that

$$\langle \hat{c}_1^\dagger c_2^\dagger)\rangle = \frac{2m_1}{l_1^2 - m1^2} + \frac{2m_2}{l_2^2 - m_2^2}. \tag{4.44}$$

Using Eqs. (2.55), (256),(3.45) and (3.46) yields

$$\langle \hat{c}_1^\dagger c_2^\dagger)\rangle \;=\; 8(a_1 + a_2) \tag{4.45}$$

By substituting the value of a_1 and a_2, the variance of the photon number turns out

$$\begin{aligned}(\Delta n)^2 \;=\; & 8\left[\frac{A(1-\eta)}{2(A\eta+\kappa)} + \frac{2\epsilon^2}{\kappa^2 - 4\epsilon^2}\right]^2 + 2\left[\frac{A(1-\eta)}{2(A\eta+\kappa)} + \frac{2\epsilon^2}{\kappa^2 - 4\epsilon^2}\right] \\ & + \left[\frac{A(1-\eta)}{2(A\eta+\kappa)} + \frac{2\epsilon^2}{\kappa^2 - 4\epsilon^2}\right]^2 \end{aligned} \tag{4.46}$$

Unlike the mean photon number, the variance of the photon number is not the sum of the mean photon number of the individual light beams.

4.3 Quadrature Fluctuation

Employing the Q function, we seek to calculate, the quadrature variances and squeezing.

4.3.1 The quadrature variance

The quadrature variance is given by

$$(\Delta c_\pm)^2 \;=\; \langle \hat{c}_\pm^2, \hat{c}_\pm\rangle, \tag{4.47}$$

where

$$\hat{c}_+ \;=\; \hat{c}^+ + \hat{c} \tag{4.48}$$

and

$$\hat{c}_- \;=\; i(\hat{c}^+ - \hat{c}), \tag{4.49}$$

in which $\hat{c}$ is the annihilation operator for the superposed twin light modes. Since $\hat{a}$ and $\hat{b}$ are Gaussian variables with zero mean , $\hat{c}$ is also a Gaussian variables with zero mean.Therefore, we readily find

$$\langle \hat{c}_{\pm} \rangle = 0. \tag{4.50}$$

Upon introducing Eq.(4.50) into Eq.(4.47), we get

$$(\Delta c_{\pm})^2 = \langle \hat{c}_{\pm}^2 \rangle, \tag{4.51}$$

then we see that

$$(\Delta c_{\pm})^2 = 2 + 2\langle \hat{c}^\dagger \hat{c} \rangle \pm 2\langle \hat{c}^{\dagger 2} \rangle. \tag{4.52}$$

where

$$\langle \hat{c}^{\dagger 2} \rangle = \langle \hat{c}^2 \rangle \tag{4.53}$$

and

$$[\hat{c}, \hat{c}^\dagger] = 2. \tag{4.54}$$

Substituting Eqs. (4.37) and (4.38), we have

$$\begin{aligned} (\Delta c_{\pm})^2 = & \frac{\kappa + A(1 \pm (1-\eta^2)^{1/2})}{A\eta + \kappa} \\ & +1 \mp \frac{2\epsilon}{\kappa \pm 2\epsilon} \end{aligned} \tag{4.55}$$

4.4.2 The quadrature squeezing

Quadrature squeezing for a superposed light beams can be defined as

$$S_+ = 1 - (\Delta c_+)^2 \tag{4.56}$$

In a view of Eqs. (4.55) and (4.56) We obtain

$$\begin{aligned} S_{+} &= \frac{A\eta - A(1 + (1 - \eta^2)^{1/2})}{A\eta + \kappa} \\ &+1 - \frac{2\epsilon}{\kappa + 2\epsilon} \end{aligned} \tag{4.57}$$

Here we note that, squeezing occurs at the plus quadrature.

5

Conclussion

In this thesis we have considered degenerate squeeezed three-level laser and one mode Subharmonic Light. Taking into account the interaction of degenerate three level atoms with cavity mode and the damping of the cavity mode by a vacuum reservoir, we obtained the equations of evolution of cavity operators. We use the master equation for a light produced by degenerate squeezed three-level laser and one mode Sub-harmonic Light from which we obtained the of c-number Langevin equations. Employing these solutions of c-number Langevin equations, we found the antinormally ordered characteristic function which was used to find the Q-function of a light beam generated by degenerate squeeezed three-level laser and one mode Subharmonic Light Coupled to vacuum reservoir

Employing this Q function, we calculated the mean photon number,the variance of the photon number and the quadrature variance. We have carried out our analysis by putting the vacuum noise operators in normally order and applying the adiabatic approximation scheme, which establishs nearly exact relation for systems operating close to steady state.

Finally, we superposed the two Q-functions. Applying the superposed Q-function, we calculated photon statistics and quadrature squeezing of the superposed light beams.From the result we have seen that the squeezing occurs in the plus quadrature and squeezing is enhanced.

6

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DECLARATION

I hereby declare that this MSC thesis is my original work and has not been presented for a degree in any other universities, and that all sources of material used for the dissertation have been duly acknowledged.

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Place and date of submission:

Department of Physics

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Oct. 2016