

JIMMA UNIVERSITY SCHOOL OF GRADUATE STUDIES JIMMA INSTITUTE OF TECHNOLOGY FACULTY OF CIVIL AND ENVIRONMENTAL ENGINEERING STRUCTURAL ENGINEERING STREAM

Finite Element Analysis on Flexural and Lateral Torsional Buckling

Behavior of Steel I-Beam with Web Opening

A Research Thesis Submitted to School of Graduate Studies of Jimma University in Partial Fulfillment of the Requirements for the Degree of Master of Science in Structural Engineering

By: Yidnekachew Ali

January, 2020 Jimma, Ethiopia

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JIMMA UNIVERSITY

SCHOOL OF GRADUATE STUDIES JIMMA INSTITUTE OF TECHNOLOGY FACULTY OF CIVIL AND ENVIRONMENTAL ENGINEERING STRUCTURAL ENGINEERING CHAIR

FINITE ELEMENT ANALYSIS OF FLEXURAL AND TORSIONAL BUCKLING **BEHAVIOR OF STEEL I-BEAM WITH OPENINGS**

YIDNEKACHEW ALI

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DECLARATION

This research is my or	riginal work and has n	ot been presented for dea	gree in any other
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Web Opening

ABSTRACT

A steel I-beam with different types of web openings makes in different structures, where in the different technical utilities such as ventilation ducts, electric cables; water carrying pipes and others can be incorporated. But the provision of web openings in I-steel beams introduces typical failure mode. Among that flexure and lateral torsional buckling are a typical failure that occurs in steel I-beam.

The main objective of this research was to investigate the behavior of flexure and lateral torsional buckling of steel I-beam with web opening and its ultimate load carrying capacity and buckling moment resistance using nonlinear finite element analysis, Both geometric and material nonlinearities are included in the finite element model

The research focused on the effect of shape of opening, total depth to opening depth (D/Do) ratio and spacing to opening depth (S/Do) ratio on the flexural and lateral torsional behavior of steel castellated beam with web opening. Total 62 samples of steel I beam with web opening studied using non-linear finite element analysis with ABAQUS Software package under concentrated load at mid-span of the beam. The FE models are validated with existing experimental data from literature for flexural behavior and numerical calculation by using EBCS-3 code for lateral torsional buckling to examine the accuracy of the simulations. Values from FE model agree with the test and numerical calculation results.

In conclusion, the analysis result showed that, Shape of opening has a significant effect on the ultimate load capacity and buckling moment resistance value. Ultimate load-carrying capacity of Castellated beam with diamond, circular, hexagon opening become bigger 28.5%, 22%, 7.8% compared with the beam without web opening (original beam), but reduct 26.82 % compared square opening with original beam, and also Buckling moments resistance value with diamond, circular, hexagon opening become bigger 17.6%, 14.2%, 8.2% compared with the beam without web opening (original beam), but reduct 16.3% compared square opening with original beam. D/DO and S/DO has a significant effect on flexural behavior and D/Do ratio has slightly effect on the buckling moment resistance value, but S/Do ratio has a significant effect on buckling moment resistance value. Beam with depth of opening 0.7 times its overall depth with S/Do=1.4 is the optimize section which has high ultimate load carrying capacity for circular, hexagon and square opening and also beam with depth of opening 0.78 times its overall depth with S/Do=1.4 is the optimize section which has high ultimate load carrying capacity for diamond opening.

Key words: Steel I-beam, castellated beam, Flexure, Lateral torsional buckling, web opening, Finite element analysis

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ACRONYMS

FEA	Finite Element Analysis
EBCS	Ethiopian Building code standards
ASIC	American Institute of Steel Construction
EC	Euro Code
D	Overall depth of section
t _w	Thickness of web
Do	Depth of opening
В	Width of flange
UB	Universal beam
LTB	Lateral torsional buckling
$\mathbf{f}_{\mathbf{y}}$	Yield stress
$\mathbf{f}_{\mathbf{u}}$	Ultimate stress
Е	Elastic modulus of beam
e	Spacing between two hole end
S	Spacing between a centers of opening
L	Length of the beam
It	Torsional constant
I_{w}	Warping constant
Iz	Second moment of area about the minor axis
M _{cr}	Elastic critical moment
Mb, _{Rd}	Buckling resistance moment
Lu	Unbranced length of beam
L _p	Minimum unbranced length for attaining yield of members
C _b	Moment gradient factor
M_n	Nominal moment capacity
M_p	Plastic moment
M_{u}	Moment derived from classical solution
r _{ts}	Effective radius of gyration
r _y	Radius of gyration about y-axis

S _x	Elastic section modulus about x-axis
W_y	Section modulus of a section
Z _x	Plastic section modulus
M _{max}	Absolute value of maximum moment
Ma	Absolute value of first quarter moment
M_b	Absolute value of second quarter moment
M_{c}	Absolute value of third quarter moment
J	Saint-venant tortional constant
G	Shear modulus of elasticity
λ_{LT}^{-}	Non dimensional slenderness parameter
$\Phi_{_{LT}}$	Dimensionless parameter
$lpha_{\scriptscriptstyle LT}$	Imperfection factor
λ	Modified slenderness ratio
β	End moment ratio
ϕ	Resistance factor

CHAPTER ONE INTRODUCTION

1.1. Background of the study

Since 1940s, many attempts have been made by structural engineers to find new ways to reduce the cost of steel structures. Open web expanded steel beams was initially used in structures during World War II to decrease the cost of steel structures. Due to limitations on maximum allowable deflections, the high strength properties of structural steel cannot always be utilized to the best advantage. As a result, several new methods have been aimed at increasing the stiffness of steel members without any increase in the weight of steel required. Hence, steel beams with web openings [1].

In modern buildings, openings are frequently required to be provided in structural members so that building services may be incorporated into structural zones for simplified layout and installation. The presence of large web openings may have a severe penalty on the load carrying capacities of floor beams, depending on the shapes, the dimensions and the locations of the openings [2].

The web openings complicate the failure behaviour of steel I- beams. It introduces typical failure modes, these failure modes are: flexural mechanism; web post buckling due to shear force; Web post buckling due to compression force; rupture of welded joints; local web buckling, lateral torsional buckling.Steel I beams with web openings made from built-up section or welded wide flange beam and fabricated from standard hot rolled I-sections which are castellated beam are commonly used in recent constructions to cover long spans with shapes of opening like hexagonal, circular and rectangular [3].

Structural stability problems have substantial effects on the design steps of steel structures. Flexure and lateral torsional buckling are the most important stability problems that occur in steel I-beam. Lateral torsional buckling can be defined as a combination of lateral displacement and twisting due to an application of load on an unsupported beam and may often be a controlling factor in steel beam design [4].

1.2. Statement of the problem

A steel I-beam with different types of web openings makes in different structures globally,

where in the different technical utilities like ventilation ducts, electric cables, water carrying pipes and other can be incorporated. The provision of web openings in I-steel beams introduces typical failure modes depending on geometry of the beams, size of web openings, spacing between openings, web slenderness, type of loading, quality of welding, and lateral restraint conditions and other.[3].

Flexure and lateral torsional buckling are the most important stability problem and typical failure that occurs in steel I-beam and there is a limitation of study on flexural and lateral torsional buckling of steel I-beam with different shape of opening.

Therefore it becomes necessary to investigate the behaviour of flexural and lateral torsional buckling of steel I- beams with opening by considering different parameters such as various shape and size of opening, location of opening and spacing between opening.

1.3. Research question

The research mainly focused to answer the following research questions:

- 1. What is the effect of various shapes of web opening on flexure and lateral torsional buckling of steel I- beam?
- 2. What is the effect of total beam depth to opening depth (D/Do) ratio and the spacing and opening depth(S/ Do) ratio on flexure and lateral torsional buckling of steel I-beam?
- 3. Which depth to opening depth (D/Do) ratio and the spacing and opening depth(S/Do) ratio has high load carrying capacity and buckling moment resistance for different shape of opening?

1.4. Objective of study

1.4.1. General objective

The main objective of this research was to investigate the behavior of flexural and lateral torsional buckling of steel I-beam with web opening using finite element analysis.

1.4.2. Specific objective

To achieve the main objective of the research have the following specific objectives

1. To investigate the effect of various shape of web opening on flexure and lateral torsional buckling behavior of steel I-beam.

- 2. To investigate the effect of total beam depth to opening depth D/Do) ratio and the spacing and opening depth(S/ Do) ratio on flexure and lateral torsional buckling of steel I-beam.
- 3. To assess the optimized total depth (D/Do) ratio and the spacing and opening depth(S/ Do) ratio that has high load carrying and buckling moment resistance for different shape of opening.

1.5. Significance of the study

Recently steel I- beam with web opening is made and provided in different structure such as large building, steel girder plate and other. Therefore the output of this research can give and add information about the effect and behaviour of web opening on flexure and lateral torsional buckling of I-steel beam.

The manufacturer and designer can use the result of this research as a guidance for the design and construction of steel beam, and also It can allow the researcher to assess the effect of the web opening on the steel I-beam with regarding to flexure and lateral torsional buckling, built academic knowledge and provide base for further career improvement.

It can also benefit Jimma Institute of Technology in attaining its objective as a centre of academic excellence and accelerate the national development through provision of problem solving research output to the policy and decision makers.

1.6. Scope and limitation of the study

- ✓ The study focused on the behaviour of flexural and lateral torsional buckling of steel I beam with web opening.
- ✓ In this study four types of opening will be adopted, which square, hexagon, circular, diamond.
- ✓ Use homogeneous and isotropic material.
- ✓ The beam is simply supported and also a type of loading is concentrated load at the mid span.

CHAPTER TWO RELATED LITERATURE REVIEW

2.1. General

Beams are frequently used in many structures in various shapes and sizes mainly because of its capability of withstanding loads by resisting bending and shear. Different types of steel sections are being produced and practiced by the designers since many years; however, I-sections steel beam are, due to their load bearing properties, most popular and widely used in the construction industry of different structure [5].

The web openings complicate the failure behavior of steel I- beams. As local failure modes around the openings, web post buckling or Vierendeel mechanism are generally found. However, the already existing failure modes for regular I-section members should also be considered, such as flexural failure and lateral–torsional buckling failure [6].

2.2. Use of steel I-beam with web opening

Steel I-beam with web opening used in different large structure such as industrial building, parking garages, warehouses, hospital, schools etc. Provision of openings in steel beams create flexibility in the use of the floor area and roof system, where in the different technical utilities and services such as ventilation ducts, electric cables, water carrying pipes etc. And also allow reducing the height of buildings to fit a required number of floors, otherwise fitting more floors in a given height limit having a significant economic impact to the whole structure's budget, and they have scene of beauty as well [7].



Figure 2. 1: Web openings steel beam applications [8].

2.3. Review on steel I-beam with web opening

A large amount of research efforts on the structural behavior of steel and composite beams with web openings have been reported in the literature over the last three decades. Most of the work has examined the effects of web openings on steel beams. Problems associated with cutting openings in webs of steel beams have been considered by Redwood and Cho [9]. They resulted in approximate method for the design of steel beams with web openings.

One of the issues raised since the steel structure was introduced in the construction industry is how to reduce the weight and cost of the component parts such as girder and beams. Cold formed built-up steel members are widely employed in steel construction because they are lighter and more efficient than hot-rolled ones. Nowadays have easy availability and accessible cost of high-strength low-alloy steels, weathering steels, and zinc-coated steels [10].

Srinath and Shanmugarajan [10] have investigated the impact of web opening on the flexural behavior of Cold formed built-up I section under two point loading for the simply supported end conditions. They carried out experimental investigation on 8 specimens by varying the thickness and depth of the built-up beam.

Erdal and Saka [11] Studied the load carrying capacity of optimally designed castellated beam with various number of holes and spacing. Finite element analysis of same beams is also carried out under the application of centrally applied point load and failure patterns are studied and verified using ANSYS. Study shows that, even though the members are relatively of shorter spans, lateral supports are governing factor for the analysis of beams due to torsional buckling. It is concluded that, the beam fails in Vierendeel mode when the load is applied above the openings while it fails in web post buckling when load is applied in between space of the openings.

Recently steel beam with web opening made from standard hot-rolled I-section that is called as castellated beams. The castellated or perforated web beam is the beam which has perforation or openings in its web portion. Generally, the openings are with hexagonal or square or circular in shapes. The beams with circular openings are called as cellular beams. The advantage of using such beams is that it causes reduction in total weight of the structure and hence requires less quantity of steel. Use of castellated beam with hexagonal opening is very common in recent years because of the simplicity in its fabrication. Castellated beams are fabricated by cutting flange of a hot rolled steel I beam along its centerline and then welding the two halves so that the overall beam depth gets increased for more efficient structural performance against bending [12]. Non-composite castellated beams are more susceptible to lateral torsional buckling than composite beams due to lack of lateral support to the compression flange [9].

The presence of web openings significantly influences the structural performance of the beams, which in particular is dependent on the geometry (shape, diameter, and critical opening length), location (shear-moment interaction), and spacing (closely and widely spaced) between perforations [13].

Load carrying capacity of simply supported Castellated steel beams susceptible to web post bucking is studied. FEA method is used to evaluate the load carrying capacity castellated beam. The parameter studies are also carried out in order to assess the cross section classification to compare the ultimate load behavior. Among the main features of these beams can be pointed to architectural features and height which resulting in greater strength and stiffness of the beams without the added weight of the beams. In this paper, the load carrying capacity of castellated beam is reviewed. The unit member with fillet corner opening has a higher load carrying capacity as compared with those rectangular openings when they have the same opening height, but lower than that with circular opening [14].

The failure modes mainly depend on area of openings, location of opening, length of the teesection above and below the opening, opening depth and type of opening, type of loading. The experimental testing on steel beams with web opening of various shapes and sizes is conducted. Six potential failure modes associated with castellated beams are [15].

A. Formation of Flexure Mechanism

This mode of failure can occur when a section is subject to pure bending. The span subjected to pure bending moment, the tee sections above and below the holes yielded in a manner similar to that of a plain webbed beam, although the spread of yield towards the central axis was stopped by the presence of the holes by which time the two throat sections had become completely plastic in compression and in tension.

B. Lateral-Torsional Buckling

Non-composite castellated beams are more susceptible to lateral-torsional buckling than composite beams due to lack of lateral support to the compression flange. The lateral torsional buckling behaviour of castellated beams is similar to that of plain webbed beams. The holes had a significant influence on lateral-torsional buckling behavior.

C. Formation of Vierendeel Mechanism

Vierendeel bending is caused by the need to transfer the shear force across the opening to be consistent with the rate of change of bending moment, in the absence of local or overall instability, hexagonal castellated beams have two basic modes of plastic collapse, depending on the opening geometry. The failure is dependent on the presence of a shear force of high magnitude in the holes through span.

D. Rupture of the Welded Joint in a Web Post

Rupture of a welded joint in a web-post can result when the width of the web-post or length of welded joint is small. This mode of failure is caused by the action of the horizontal shearing force in the web-post, which is needed to balance the shear forces applied at the points of contra flexure at the ends of the upper I section.

E. Shear Bucking of a Web Post

The horizontal shear force in the web-post is associated with double curvature bending over the height of the post. In castellated beam one inclined edge of the opening will be stressed in tension, and the opposite edge in compression and buckling will cause a twisting effect of the web post along its height.

Steel sections are generally safe in strength requirement but the difficulty is that the section have to satisfy serviceability requirement i.e. deflection criteria in safety check. Hence, it is essential to use beams with more depth in order to satisfy this requirement. Using perforated web or open web beams is the best solution in order to overcome this difficulty. Perforated web beam is the beam which has perforation or openings in its web portion. One more advantage of using beams with perforated web is that a reduction in total weight of the structure is possible and hence lesser quantity of steel is required [16].

2.4. Flexural behavior of steel beam

Wakchaure, M.R and Sagade, A.V [17] studied the flexural behavior of castellated beams.

Beams were modelled with increase in depth of web openings. Analysis is carried out on beam with two point load and simply supported support condition. The deflection at center of beam and study of various failure patterns are studied. The beams with increase in depth are then compared with each other and with parent section for various parameters and for serviceability criteria. From the analysis results, it is concluded that, castellated beams have proved to be efficient for moderately loaded longer spans where the design is controlled by deflection.

A number of common and practical web openings are considered in the study on steel Ibeam. As height of castellated beam will get increase it gives high bending and shear strength as section modulus of castellated beam will get increase. As a result load carrying capacity will get increase and such type of beams also allows to structural work [18]. Shan et al. [19] have investigated bending and shear behavior of web elements with openings for coldformed steel beams.

Seetha,D [20] conducted the experimental and analytical study for deflection calculation of different section of beams with different support condition and different loadings with web openings. As a result several new methods have been aimed at increasing the stiffness of the steel members without any increase in weight of the steel required. Beam with web opening have proved to be efficient for moderately loaded longer span where the design is controlled by moment capacity or deflection. The deflection pattern at the Center distance of the beam is studied for different parametric condition by same depth of web opening to the depth of beam ratio and also for various combinations of shapes of opening.

2.5. Lateral torsional buckling of steel I-beam

Lateral torsional buckling is a behavior which is one of the instability conditions induced by the compressed flange of unrestrained beam subjected to bending around the major axis. If a beam reaches the critical moment value under the applied load or moment, this beam may expose to lateral torsional buckling failure. The critical moment is a function of lateral and torsional stiffness. This is affected by the boundary conditions; un braced length, material nonlinearities, load pattern and dimensions of the member cross section. If a beam is under the influence of lateral torsional buckling, it experiences simultaneous in-plane displacement, lateral displacement and twisting because of bending. Lateral displacement and twisting of the simply supported beam under the bending moment considering lateral torsional buckling behavior [19].

Lateral-torsional buckling can be avoided by properly spaced and designed lateral bracing. Bracing is usually assumed to be elastic and characterized by elastic stiffness. It is well known that an elastic lateral brace restricts partially the lateral buckling of slender beams and increase the elastic buckling moment. Accordingly, the effect of elastic lateral bracing stiffness on the inelastic flexural torsional buckling of simply supported castellated beams with an elastic lateral restraint under pure bending is investigated by previous researcher. The effect of bracing depends not only on the stiffness of the restraint but also on the modified slenderness of the beam [20].

Beams are frequently used in many structures in various shapes and sizes mainly because of its capability of withstanding loads by resisting bending and shear; Usually flexural member such as beams and girders have much greater strength about the major axis compared to minor axis. As a result of this, laterally unsupported beams and girders might fail by lateral-torsional buckling before the attainment of its full in-plane capacity. Thus, lateral torsional buckling (LTB) can be considered as a limit state of structural design where the deformation changes suddenly from in-plane bending to combined lateral deflection and twisting . The final failure pattern involves lateral deflection and twisting in combination with various extents of yielding and flange and/or web local buckling depending on the specific member characteristics. The consequences of such kind of premature failure is devastating particularly if it occurs during the construction phase .Therefore, it is very important to understand and investigate the behaviour of structures and ensuring the structural stability of its members as a whole[21].

Since the mid-nineteenth century, research has been performed intensively on lateral torsional buckling of beams and reported in several text books . However, the analytical procedure of obtaining LTB strength are complex and only for the simplest cases the closed form solutions can be found. Depending on this length behaviour of LTB can be divided into three parts such as (1) elastic buckling, (2) inelastic buckling and (3) plastic behaviour. The relationship between critical moment (Mcr) and unbraced length (L) for lateral-torsional



buckling can be presented graphically as shown in Figure 2.2[5].

Figure 2. 2: Different mode of LTB of beam

Different structural steel design standards provide different algebraic equations for Estimating the LTB resistance. However, in a general sense, all of them use similar approach: staring from the calculation of elastic LTB resistance M_u and followed by a reduction of this theoretical resistance by considering various factors such as geometric imperfections, local and/or distorsional buckling, residual stress etc. Depending on the variables considered, nominal resistance for LTB varies considerably from statndard to standard. Another important difference between Eurocode 3 and other standards is Eurocode 3 provides two different strength curves for rolled and welded section. However, American standards AISC simplicitly assume that the beam has no initial out-of-straightness for long members that fail by elastic LTB . Moreover, AISC make no distinction between rolled and welded beams[21].

2.5.1 Design approach of different standards

Usually, flexural members such as beams and girders have a much greater strength about the major axis compared to the minor axis. As a result of this, laterally unsupported beams and girders might fail by lateral-torsional buckling before the attainment of their full in-plane capacity.

Therefore, lateral torsional buckling can be considered as a limit state of structural design and is governed by one of three possible modes of failure. These are 1) elastic lateral torsional buckling, 2) inelastic lateral torsional buckling, and 3) yielding of the cross-section. The standards studied in this section follow different methods to determine the resistance of a beam under each failure mode.

ANSI/AISC 360-10

ANSI/AISC 360-10 provides a resistance calculation method by which the LTB resistance curve is clearly divided into three distinct part. It uses two limiting spans L_p and L_r to fix the failure mode where the span L_r

$$L_{r} = 1.95r_{ts} \frac{E}{0.75F_{y}} \sqrt{\frac{J}{S_{x}h_{o}} + \sqrt{\left(\frac{J}{S_{x}h_{o}}\right)^{2} + 6.76\left(\frac{0.7Fy}{E}\right)^{2}}}$$
[2.1]

where h_o is centre to centre distance between flange and r_{ts} is the effective radius of gyration which can be calculated by the following expression,

$$r_{ts} = \frac{\sqrt{I_y c_w}}{s_x}$$
[2.2]

The minimum unbraced length for attaining yielding of a member, L_p is determined by

$$L_p = 1.76r_y \sqrt{\frac{E}{F_y}}$$
[2.3]

in which r_y is the radius of gyration about the y-axis.

When $L_u > L_r$, where L_u is the un braced length of the section, the failure mode of buckling is termed as elastic lateral torsional buckling which is calculated by the expression as follows.

$$M_{n} = C_{b}M_{u} = C_{b}\frac{\Pi}{L_{U}}\sqrt{EI_{y}GJ + \left(\frac{\Pi E}{L_{U}}\right)^{2}I_{y}C_{w}} \le M_{p}$$

$$[2.4]$$

where, C_b is the ANSI/AISC 360-10 moment gradient factor given by,

$$C_{b} = \frac{12.5M_{\text{max}}}{2.5M_{\text{max}} + 3M_{A} + 4M_{B} + 3M_{C}} \le 3.0$$
[2.5]

When $L_p < L_u < L_r$, the mode of failure is governed by inelastic lateral torsional buckling and the nominal moment capacity of beam is determined using the following equation

$$M_{n} = C_{b} \left[M_{p} - (M_{p} - 0.7F_{Y}S_{x}) \left(\frac{L_{u} - L_{p}}{L_{r} - L_{p}} \right) \right] \le M_{p}$$
[2.6]

When $L_u < L_p$, the mode of failure is termed as fully yielded and the nominal moment capacity is determined by $M_n = Z_x F_y$ for compact section while $M_n = S_x F_y$ for non-compact sections.

Eurocode 3 (EN 1993-1-1:2005)

Eurocode 3 provides two methods, general method and alternative method, for determining the lateral buckling resistance of a beam. The code requires a reduction factor (X_{LT}) to be applied to the moment resistance of the cross section to obtain the lateral torsional buckling moment resistance. The corresponding nominal flexure resistance of a laterally unsupported beam is calculated as:

$$Mn = X_{LT} W_{v} F_{v}$$
[2.7]

where W_y is either taken as Z_x , plastic section modulus for Class 1 and 2 section or S_x , elastic section modulus for Class 3 section. The factored resistance, M_r , is then obtained by dividing the nominal resistance by the partial safety factor for resistance of members to instability M γ_1 , which is taken as 1.0.

Eurocode 3: General method

In the general method, X_{LT} can be obtained as:

$$X_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \le 1.0$$
[2.8]

Where
$$\Phi_{LT} = 0.5 \left[1 + \alpha_{LT} (\dot{\lambda_{LT}} - 0.2) + \bar{\lambda}_{LT}^2 \right]$$

 α_{LT} is an imperfection factor, values of which are given in Eurocode 3 for different lateral torsional buckling curves. Eurocode provides distinct buckling curves (a, b, c, and d) based on the height to width ratio of the I-shaped sections.

 λ_{LT} , non-dimensional slenderness is given as

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y F_y}{M_{Cr}}}$$
[2.9]

where M_{cr} is elastic critical moment for lateral torsional buckling. The following formula can be used to calculate elastic critical moment.

$$M_{cr} = C_1 \frac{\pi^2 E I_Z}{L^2} \left[\frac{I_W}{I_Z} + \frac{L^2 G I_t}{\pi^2 E I_Z} \right]^{0.5}$$
[2.10]

C1 is a moment gradient factor for which no expression is given in this standard rather specified values for both end moment and transverse loading or any combination of loading.

Eurocode 3: Alternative method

This method is designed for lateral torsional buckling of rolled or equivalent welded sections. The reduction factor X_{LT} may be determined from:

$$X_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \beta \bar{\lambda}_{LT}^2}} \le 1.0; also, X_{LT} \le \frac{1}{\bar{\lambda}_{LT}^2}$$

$$\Phi_{LT} = 0.5 \left[1 + \alpha_{LT} (\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta \bar{\lambda}_{LT}^2 \right]$$
[2.11]

where the coefficient $\lambda_{LT,0}$ corresponds to the plateau length of the LTB curves for rolled and welded sections and it is taken as 0.4 for the alternative method; β is correction factor for the LTB curves for rolled and welded sections and it is taken as 0.75 for the alternative method; α_{LT} is an imperfection factor, value of which is given in Eurocode 3 for different lateral torsional buckling curves. The values of α_{LT} for different buckling curves are given in the Eurocode.

2.5.2 Factors affecting lateral torsional buckling

There are several contributing factors that can affect the resistance of a member due to lateral torsional buckling. These are residual stress, initial out-of-straightness, moment gradient effect, load-height effect, support and restraint, etc. In this section, only the effects of residual stress and initial out-of-straightness are briefly described[22].

Residual stress

During the fabrication of welded I-shape beams, the steel is subjected highly localized thermal expansion at the weld zones, and as the subsequent cooling is not uniform throughout the element, self-equilibrating internal stress patterns are formed, known as residual stresses. Both the distribution and magnitude of residual stresses depend on the manufacturing process, for instance hot rolling, or welding of plates that can be either rolled or flame cut. The residual stresses due to cold straightening are usually present only in some segments of the member and thus it can be neglected in most of the cases.

Initial Imperfection

Geometric imperfection influences the LTB resistance capacity of a beam. This effect is even more significant for thin-walled structure. Imperfections are unintentionally introduced due to the mistake in the manufacturing process and may significantly decrease the load-carrying capacity of a structure. Both lateral deflection and twist of an imperfect beam tend to increase as soon as the commencement of loading and continue until the applied moment turns into the critical moment, Mcr. After this, both of this quantity decrease sharply due to the considerable reduction in stiffness which indicates that the beam is no longer able to carry any load further. So, additional deformations caused by the presence of imperfection incorporates other stresses and thus it affects the stability of a member by minimizing the load carrying capacity of it.

2.6. Process of castellated of steel I beam with web opening

Castellated beams with steel with hexagonal, diamond, square openings that are made by cutting a saw tooth pattern along its centerline in the web of a rolled I-beam section along the

length of its span. The two parts of original beam are then welded together to produce a beam of greater depth with halves of opening holes in the steel section as shown below in figure 2.3 [23].





Castellated beam with circular web openings that are made by twice cutting an original rolled beams web in a half circular pattern along its centerline, separating two tee parts and rewelding these two halves of hot rolled steel sections as shown in Figure 2.3



Figure 2.3: Process of castellation of steel I beam with circular web opening [23]

CHAPTER THREE RESEARCH METHODOLOGY

3.1 General

The main purpose of this research was to study the flexural and lateral torsional buckling behavior of steel I beam with web opening that is castellated beam. To achieve this, a finite element analysis was conducted with all appropriate parameters considered. The finite element method is a numerical analysis technique for obtaining approximate solutions to a wide variety of engineering problems. Finite element analysis is a powerful computer method of analysis that can be used to obtain solutions to a wide range of structural problems involving the use of ordinary or partial differential equations. FE solvers can either use linear or non-linear analysis.

3.2 Research design

The research design is an inductive process that used to assess the effect of web opening on flexure and lateral torsional buckling behavior of steel I-beam.figure 3.1 showed the work flow used in this research.



Figure 3. 1: Work Flow

3.3. Study variables

3.3.1. Dependent variables

The dependent variables, which are to be observed and measured to determine the effect of the independent variables, are listed below.

- Ultimate load capacity of a beam
- Lateral and vertical deflection
- Buckling moment resistance

3.3.2 Independent variables

The independent variables, which are to be measured and manipulated to determine its relationship to observed phenomena, are selected and listed below.

- The ratio of steel I-beam depth to opening depth (D/Do) and
- the ratio of spacing and opening depth(S/Do).
- Shape of opening

3.4 Method of analysis

In this paper, the analysis was conducted numerically, using the finite element analysis. Finite element analysis is chosen due to the complexity of geometry of the castellated steel beam, and was executed with the Abaqus/CAE 6.14 software. The commercial finite element software Abaqus is the most widely used software in the academic research of material and geometric nonlinear analysis due to the flexibility that it provides for the users with numerous options for materials models, analysis and solutions techniques. Abaqus is a powerful engineering simulation programs, based on the finite element method that can solve problems ranging from relatively simple linear analyses to the most challenging nonlinear simulations.

3.5 Modeling

3.5.1 Steel I beam section and material properties

The Castellated beams are prepared from hot rolled steel I sections. The web of I beam is cut in zigzag pattern along the center line in desired opening shape, then re-joining the two halves on one another by means of welding. In case of castellation process the total depth of

original beam increase by half of the opening depth, the process of castellation is illustrated in Fig below,





Figure 3. 2: Process of castellation of steel I beam with opening

In this research, an UB 127 x 76 x 13 was chosen as original beam with 2000 mm length. The original beam was modified into a castellated steel I beam with circular, diamond, hexagon and square shaped openings, All model of the castellated beams in this research have been derived from the UB 127 x 76 x 13.Steel section dimension specification for hot rolled I section BS 5950:2000 code of practice used in this research. The dimension of UB 127 x 76 x 13 is as follows

Table 3. 1: Steel section dimension of UB 127 x 76 x 13

Н	В	t_{f}	t _w
127mm	76mm	7.6mm	4mm

Grades of steel according to EBCS 3

Nominal steel grades	Thickness(mm)				
	t≤40mm		40mm < t ≤100mm		
	F _y (Mpa)	F _u (Mpa)	F _y (Mpa)	F _u (Mpa)	
F _e 360	235	360	215	340	
F _e 430	275	430	255	410	
F _e 510	355	510	335	490	

The grade of steel that used in this research was $F_e 510$ and generally the material properties used in the analysis summarized as follows;

Steel properties	Magnitude
Modulus of elasticity	210000 Mpa
Poison's ratio(v)	0.3
Yield stress (fy)	355 Mpa
Ultimate stress (fu)	510 Mpa

Table 3.3: Steel material properties used in the analysis

3.5.2 Parameters for parametric study

The parameter considered for this study is D/Do ratios, S/Do ratios of the opening and also shape of opening to investigate the effect of opening on flexural and lateral torsional buckling behavior of steel castellated I beam.

The Euro code (BS-5950) gives the design guidelines for the limits of web perforations

a) 1.08 < S/ Do < 1.5

b) 1.25 < D/ Do< 1.75

For this study the spacing ratio is varied from 1.2 to 1.4 and opening ratio varied from 1.28 to 1.65 as shown Table 3. As mentioned on section 3.1 all models of the castellated beams in this research have been derived from the UB 127 x 76 x 13. The below table shown the variation of parameters in each shape of opening. Total 62 castellated beams are modeled and

analyzed for flexural and lateral torsional buckling behavior and the original beam is also analyzed for comparison. Total depth opening equals to 127 plus half of opening depth.

MODEL	Do	D	D/D _o	S/D _o	S	e
1	110	182	1.65	1.4	154	44
2	125	189.5	1.52	1.4	175	50
3	140	197	1.41	1.4	196	56
4	155	204.5	1.32	1.4	217	62
5	163	208.5	1.28	1.4	228.2	65.2
6	110	182	1.65	1.3	143	33
7	125	189.5	1.52	1.3	162.5	37.5
8	140	197	1.41	1.3	182	42
9	155	204.5	1.32	1.3	210.5	46.5
10	163	212	1.28	1.3	211.9	48.9
11	110	182	1.65	1.2	132	22
12	125	189.5	1.52	1.2	150	25
13	140	197	1.41	1.2	168	28
14	155	204.5	1.32	1.2	186	31
15	163	212	1.28	1.2	195.6	32.6

Table 3.4: Parameters considered for castellated beam for each opening shape

3.6. Finite Element Modeling

3.6.1 Model description

In this research, simply supported steel beam beams were modeled using shell elements because it is suitable for incorporating both geometric and material nonlinearities .Besides, shell elements are sufficiently capable of predicting the effect of geometric imperfections i.e.

initial out of-straightness . Also, effects of loading and support conditions can be investigated in greater detail. The subsequent sections will present the description of the model adopted for flexural and lateral torsional buckling analysis in this study in detail.

3.6.2 Flexural Analysis

In this section will present the description of the model adopted for flexural analysis

3.6.2.1 Loading and boundary condition

Displacement boundary conditions are needed to constrain the model to get a unique solution. To achieve this ,As shown fig the translations at the nodes (UY,UZ) are restrained in left end side in order to obtain a roller joint and the translations at the nodes (UX,UY,UZ) are restrained in the right side in order to obtain the hinged joint. The step static general used and the force F, a gradually increasing load in the downward direction is applied at the center of the beam until failure.



Figure 3. 3: loading and boundary condition for flexural analysis

4.6.2.2 Element and mesh configuration

As an initial step, the finite element analysis requires meshing of the model. Hence, the model is divided into a number of small elements. After the application of the load, the stress

and the strain are calculated at integration points of these elements. An important step in finite element modeling is the selection of the mesh density. For this particular research a 4 noded shell element without reduced integration, element S4 in ABAQUS, and 15mm mesh size was selected for flexural FE modelling analysis.



Figure 3. 4: Mesh model for flexural analysis

3.6.3 Lateral torsional buckling analysis

3.6.3.1 Analysis type

Two types of analysis i.e. elastic buckling analysis and non-linear load-deformation analysis were conducted to estimate the ultimate load carrying capacity and ultimate moment resistance of steel I beams. First, an eigenvalue analysis is performed for elastic buckling analysis in which eigenvalues and corresponding Eigen modes are requested using the linear perturbation buckling analysis. These Eigen modes are used to introduce initial imperfections in the FE models.

A Riks analysis method which is based on the arc-length method is selected for the nonlinear post-buckling analysis since this technique is usually suitable for predicting the instability as

well as for obtaining the non-linear behavior of geometric collapse. The Riks method is a form of Newton-Raphson iteration method, in which load proportionality factor is introduced to provide solutions concurrently for load and displacement.

3.6.3.2 Material Properties

A bilinear elastoplastic stress versus strain curve is assumed for all the models. In addition, a nonlinear isotropic strain hardening of 2% of the elastic stiffness was considered for all analyses. The yield stress for both web and flanges is taken as 355 MPa and ultimate stress is taken 510 mpa and also The modulus of elasticity and Poisson's ratio are taken as 210 000 MPa and 0.3, respectively.

3.6.3.3 Geometric Imperfection

It is known to all that the global stability of steel beams can be significantly affected by the magnitude of initial geometric imperfections. In the numerical analysis, the initial geometric imperfections of beams were determined by using an initial elastic eigenvalue analysis with relevant eigen mode. Thus, the first buckling mode shape derived by an eigenvalue buckling analysis was introduced into finite element model, with the magnitudes of the initial imperfection recommend in different research's as L/1000,L is the laterally un braced length .In this study the length of the beam is 2000mm,therefore 2000/1000=2mm, incorporated in the non-linear finite element analysis by using ABQUES key words.

3.6.3.4 Loading and Boundary Conditions

A. Boundary condition

The most basic case for lateral-torsional buckling is a simply supported beam with a constant moment over the laterally unsupported length. Idealized simply supported boundary conditions that allow for major and minor axis rotations and warping displacements while Preventing in-plane and out-of-plane deflections and twists, were used at the supported end of the member. These boundary conditions have been incorporated into the FE model by the following criteria.

Z- axis, Y-axis, X-axis in the longitudinal, transverse, lateral direction respectively. The centroid of the web or the half height of web ends at the left endes was assumed to have hinge support, where displacement at x,y,z directions (Ux,Uy,Uz) and the rotation about the z axis(URz) were restrained. While the right end web mid height was assumed to have roller

support ,where the displacement in directions x,y (Ux,Uy) and rotation about the z axis(URz) are restrained. Lastly the left and right end of web restrained against x displacement(Ux), and the left and right end of flange restrained against y displacement(Uy).

B. Load application condition

In the finite element models, end moments were applied as two force couples with a concentrated force at the four corner points of the end cross-sections as shown in Fig. 3.4. The magnitude of the concentrated forces is determined to ensure an end moment of 1kN·m. Similar methods of applying end moments was used by several researchers [25] and also The force F, a gradually increasing load in the downward direction is applied at the centre of the beam until failure.



Figure 3.5: Way of applying end moment on ABAQUES



Figure 3.6: Load and boundary condition for lateral torsional buckling analysis

3.6.3.5 Element and mesh configuration

In order to investigate the non-linear lateral torsional buckling failure, shell elements are considered as a promising modelling building block as they can provide required degrees of freedom to capture the real buckling deformations and spread of plasticity effects .In this analysis 8-noded shell element with reduced integration, element S8R in ABAQUS, and 15mm mesh size was selected for lateral torsional buckling FE modelling analysis because it's give accurate value when compared FEM critical buckling moment result with EBCS -3 code analytical calculation (Chapter -4 section 4.2.2).



Figure 3.7: Mesh model for lateral torsional buckling analysis

CHAPTER FOUR RESULTS AND DISCUSSIONS

4.1. General

This chapter presents the results obtained using the finite element analysis method through the use of the program, ABAQUS to study the effect of opening on flexural and lateral torsional buckling behavior of steel I beam .The study was undertaken by considering the effect of different (D/Do) and (S/Do) ratio and also shape of opening on flexural and lateral torsional buckling behavior of steel I beam.

4.2. Model Validation

In order to validate the proposed model, for flexural used experimental results done by Erdal, F[24] and numerical calculation by using EBCS-3 code for lateral torsional buckling was made.

4.2.1. flexural behavior validation

The model validation was done by the comparison of load-deflection data by Erdal, F[24]

4.2.1.1. General description on the genesis of data used in experiment

A. The material properties of the specimen used in the experiment

Table 4.1: Material properties of the specimen used in experiment Erdal, F[24]

Steel properties	Magnitude
Modulus of elasticity	190000 Mpa
Poison's ratio(v)	0.3
Yield stress (fy)	390 Mpa

Table 4.2: Steel section dimension of the specimen used in experiment by Erdal, F[24]

Н	В	t _f	t _w	Do	S	L
355.6mm	106mm	8.7mm	251mm	251mm	94mm	2846mm

B. Test set up and loading

The hydraulic universal testing machine with capacity 1000KN was used to test the steel beam. The steel beam with web opening was loaded under monotonically increasing

concentrated load acting at a mid-span of a beam until collapse. The ultimate load and the mid span deflection readings were recorded at some interval of load increments. The loading and test setup is shown in Figure 4.1.



Figure 4. 1: Experimental set up by Erdal, F[24]

C. Comparison of the Results

The load deflection curve for both experimental and finite element analysis (ABAQUS Software package) has been presented in figure 4.2. The comparison showed a good agreement between experimental and finite element result.



Figure 4. 2: Comparison between experimental and finite element analysis result

The beam tested by Erdal, F[24] obtained an ultimate load 286.4 kN and the ultimate load obtained from the finite element model was of 298.75 kN. Comparing those values

Difference(%) =
$$\frac{(298.75 - 286.4)}{298.75} * 100 = 4.1\%$$

Therefore, the finite element model estimated an ultimate load that is 4.1 % higher than that achieved by Erdal, F[24] .Therefore it can be concluded that the model created was reliable and conservative in predicting the ultimate load.

4.2.2 Lateral torsional buckling behavior validation

To verify the accuracy of the finite element model of the beam, the model is used to carry on an eigenvalue analysis of the beam to identify its critical moment values that correspond to a lateral torsional buckling mode of failure. This verification done by evaluating the critical buckling moment of simply supported beams without hole and subjected to uniform end moment and then compared with to the analytical solution of the elastic critical moment for lateral torsional buckling (Mcr) according EBCS-3, section 4.6.3.2 by using the following formula (APPENDIX -A)

$$M_{cr} = \frac{\pi^{2} E I_{Z}}{L^{2}} \left[\frac{I_{W}}{I_{Z}} + \frac{L^{2} G I_{t}}{\pi^{2} E I_{Z}} \right]^{0.5}$$

Critical buckling moment calculated from code by using equation for the selected UB 127*76*13 (Appendix -A) was 31 KN-m and the critical buckling moment obtained from the finite element model from eigen value analysis for mode shape one 29.5 kN as shown fig 4.3 Comparing those values

Difference(%) =
$$\frac{(31 - 29.5)}{31} * 100 = 4.83\%$$

Figure 4.3 shows that the critical buckling moment for the selected UB 127 x 76 x 13 by using finite element analysis



Figure 4. 3: Finite element result of critical buckling moment for UB 127 x 76 x 13

Therefore, the finite element model estimated an critical buckling moment that is not exceed 4.83 % when compare to analytical calculation using a code. Therefore it can be concluded that the model created was reliable and conservative in predicting the ultimate moment resistance of the beam by considering material and geometric imperfection for non-linear finite element analysis.

4.3. Parametric studies

4.3.1. Effect of D/DO and S/DO ratio on ultimate load carrying capacity

In order to study the effect of different D/Do and S/Do ratio, and to find aspect ratio that satisfactory in respect of high load carry capacity. As mentioned on chapter three 1.65,1.52,1.41,1.32,,1.28 D/Do ratio in other words the depth of opening 0.6D, 0.65D, 0.7D,0.75D ,0,78D with 1.2, 1.3, 1.4 S/DO ratio are modeled in ABAQUES ,6.14 for circular , diamond ,hexagon, and square web opening.

A figure (4.4), (4.5),(4.6) and(4.7) shows the numerical results of the finite element analyses of ultimate load versus depth of opening, with different S/Do ratio for circular, diamond, square, and hexagon opening respectively. A depth of opening indicate or correspond to D/Do ratio, for different S/Do ratio as mentioned on chapter three, table 3.4.



Figure 4. 4: ultimate load versus depth of opening, with S/Do ratio for circular web opening From the figure (4.4), it is observed that the beam with depth of 140mm and S/Do ratio of 1.4 gives more satisfying results than the other in respect of ultimate load carrying capacity for circular, opening .In the other words the depth of 140mm refers to D/Do =1.41 or 0.7D, Therefore beam with depth of opening 0.7 times its overall depth with S/Do=1.4 is the optimize section which has high ultimate load carrying capacity for circular opening.



Figure 4.5: Ultimate load versus depth of opening, with S/Do ratio for diamond opening From the figure (4.5), it is observed that for diamond opening, the beam with depth of 163mm and S/Do ratio of 1.4 gives more satisfying results than the other in respect of ultimate load carrying .In the other words the depth of 163mm refers to D/Do =1.28 or 0.78D, Therefore beam with depth of opening 0.78 times its overall depth with S/Do=1.4 is the optimize section which has high ultimate load carrying capacity for diamond opening.





From the figure (4.6) it is observed that the beam with depth of 140mm and S/Do ratio of 1.4 gives more satisfying results than the other in respect of ultimate load carrying capacity for square opening .In the other words the depth of 140mm refers to D/Do =1.41 or 0.7D, Therefore beam with depth of opening 0.7 times its overall depth with S/Do=1.4 is the optimize section which has high ultimate load carrying capacity for and square opening.





From the figure (4.7) it is observed that the beam with depth of 140mm and S/Do ratio of 1.4 gives more satisfying results than the other in respect of ultimate load carrying capacity for hexagon opening .In the other words the depth of 140mm refers to D/Do =1.41 or 0.7D, Therefore beam with depth of opening 0.7 times its overall depth with S/Do=1.4 is the optimize section which has high ultimate load carrying capacity for hexagon opening.

In general from the figure, each figure reveal that, D/Do and S/Do has a significant effect on the ultimate load carrying capacity (flexural resistance) of a steel beam.

From the figure (4.4),(4.6),(4.7) it is observed that the beam with depth of 140mm and S/Do ratio of 1.4 gives more satisfying results than the other in respect of ultimate load carrying capacity for circular, hexagon and square opening .In the other words the depth of 140mm refers to D/Do =1.41 or 0.7D, Therefore beam with depth of opening 0.7 times its overall depth with S/Do=1.4 is the optimize section which has high ultimate load carrying capacity for circular, hexagon and square opening.

From the figure (4.5), it is observed that for diamond opening, the beam with depth of 163mm and S/Do ratio of 1.4 gives more satisfying results than the other in respect of ultimate load carrying .In the other words the depth of 163mm refers to D/Do =1.28 or 0.78D, Therefore beam with depth of opening 0.78 times its overall depth with S/Do=1.4 is the optimize section which has high ultimate load carrying capacity for diamond opening.

4.3.2 Effect of shape of opening on ultimate load carrying capacity

To investigate the effect of shape of opening on ultimate load carrying capacity of the beam, D/Do=1.41 or 0.7D and S/Do=1.4 are selected for comparison, because it has high loads capacity compared to other ratio and also analysed original beam (steel I-beam without web opening) to study the effect of castellated beam compared with original beam. Figure 4.8, ultimate load versus deflection of a beam with different web opening and without opening

From the analysis the ultimate load result was 88.58 KN, 81.82 KN, 68.73 KN,46.35 KN, 63.337 KN for diamond, circular, hexagon, square and original beam respectively, this results show that the ultimate load-carrying capacity of diamond, circular, hexagon opening become bigger 28.5%, 22%,7.8% respectively compared with the beam without web opening, but about 26.82 % reduction in ultimate load carrying capacity was observed when compared square opening with without web opening. Square openings has high stress concentration around the corner regions and also its has large deformation and low load-carrying capacity compared to the other type of web openings.



Figure 4.8: Ultimate load versus deflection of a beam with web opening and without opening Therefore its shows that the shape of opening has a significant effect on the load carrying capacity of the beam value. It was noted that diamond opening has the highest load-carrying capacity compared to other web opening shapes and original beam and its show that the ultimate load-carrying capacity and the stiffness decrease with increase in opening area. Therefore castellated beam with diamond, circular, hexagon opening shape has high ultimate load-carrying capacity compared to original beam.

4.3.3. Effect of D/DO and S/DO ratio on buckling resistance moment

In order to study the effect of different D/Do and S/Do ratio on buckling moment resistance 1.65,1.52,1.41,1.32,1.28 D/Do ratio in other words depth of opening 0.6D, 0.65D, 0.7D, 0.75D,0,78D with 1.2, 1.3, 1.4 S/DO ratio are modeled for circular , diamond ,hexagon, and square web opening. A figure (4.9), (4.10),(4.11) and(4.12) shows the numerical results of the finite element analyses of buckling resistance moment versus depth of opening, with different S/Do ratio for circular, diamond, square, and hexagon opening respectively.



Figure 4.9: Buckling moment resistance versus depth of opening, with S/Do ratio for circular web opening



Figure 4.10: Buckling moment resistance versus depth of opening, with S/Do ratio for diamond web opening



Figure 4. 11: Buckling moment resistance versus depth of opening, with S/Do ratio for square web opening



Figure 4. 12: Buckling moment resistance versus depth of opening, with different S/Do ratio for hexagon web opening

When compared the buckling moment capacity of each depth of opening, its corresponding to D/Do aspect ratio, the value was almost straight line or it's has a little difference as shown the above all figure, especially for S/Do=1.4 and 1.3. Therefore its shows that D/DO ratio has slightly effect on the buckling moment resistance value .But when compared the buckling moment value for S/Do ratio ,significant reduction is seen in buckling moments values from S/D0=1.4 to S/Do =1.2. Therefore, spacing-to-diameter ratio (S/Do) significantly affected the lateral torsional buckling moment capacity and also S/Do=1.4 has high values of the buckling moment capacity when compared to 1.3 and 1.2.

4.3.4 Effect of shape of opening on buckling resistance moment

To investigate the effect of shape of opening on buckling resistance moment of the beam, D/Do=1.32 and S/Do=1.4 are selected for comparison and analysed original beam to study the effect of castellated beam compared with original beam.



Figure 4. 13: Buckling moment resistance versus lateral displacement of a beam with web opening and without opening

From the analysis the buckling moment resistance result was 26.1 KN-m, 25.1 KN-m, 23.4 KN-m,18.02 KN-m, 21.53 KN-m for diamond, circular, hexagon, square and original beam respectively, from this analysis result, buckling moments resistance value of diamond, circular, hexagon opening become bigger 17.5%,14.2%,8.0% compared with the beam without web opening, but about 16.3% reduction in buckling moment resistance was observed when compared square opening with without web opening. Therefore its shows that the shape of opening has a significant effect on the buckling moment resistance value. It was noted that diamond opening has the highest buckling moment compared to other web opening shapes and original beam. Besides that, the differences in buckling moment values decrease when the opening becomes larger in size. Therefore castellated beam with diamond, circular, hexagon opening shape has high buckling moment resistance compared to original beam.

CHAPTER FIVE CONCLUSIONS AND RECOMMENDATION

5.1 CONCLUSIONS

The main objective of this research is to study the flexural and lateral torsional buckling behavior of steel I beam with web opening with different shape of opening, total depth to opening depth (D/Do) ratio and spacing to opening (S/Do) ratio. This research included a nonlinear finite element analysis by (ABAQUS Software package) in order to predict the ultimate load carrying capacity and buckling moment resistance of steel I beam with opening. Based on these research analytical evidences, the following conclusions are drawn.

- 1. Ultimate load-carrying capacity of Castellated beam with diamond, circular, hexagon opening become bigger 28.5%, 22%,7.8% compared with the beam without web opening, but about 26.82 % reduction in ultimate load carrying capacity was observed when compared square opening with without web opening.
- 2. D/Do and S/Do has a significant effect on the ultimate load carrying capacity (flexural resistance) of a steel beam. Beam with depth of opening 0.7 times its overall depth with S/Do=1.4 is the optimize section which has high ultimate load carrying capacity for circular, hexagon and square opening
- 3. Beam with depth of opening 0.78 times its overall depth with S/Do=1.4 is the optimize section which has high ultimate load carrying capacity for diamond opening.
- 4. Buckling moments resistance value of Castellated beam with diamond, circular, hexagon opening become bigger 17.5%,14.2%,8.0% compared with the beam without web opening, but about 16.3% reduction in buckling moment resistance was observed when compared square opening with without web opening.
- 5. D/DO ratio has slightly effect on the buckling moment resistance value, But spacingto-diameter ratio (S/Do) significantly affected the lateral torsional buckling moment capacity and also S/Do=1.4 has high values of the buckling moment capacity when compared to 1.3 and 1.2.
- 6. Shape of opening has a significant effect on the ultimate load capacity and buckling moment resistance value. Diamond opening has the highest ultimate load capacity

and buckling moment resistance compared to other web opening shapes and original beam.

 Square openings has high stress concentration around the corner regions and also its has large deformation and low load-carrying capacity compared to the other type of web openings and original beam.

5.2. RECOMMENDATION

The following recommendations are suggested for future research studies:

- The present study was on a numerical method of analyzing the behavior of flexural and lateral torsional buckling of steel I beam with web opening. Further study should be done on the shear behavior of steel I-beam with web opening
- 2. More investigation should be done on local buckling and Vierendeel mechanism behavior of steel I-beam with web opening.
- 3. Further study should be done on the behavior of flexural and lateral torsional buckling steel I-beam with web opening with different types of loading.

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APPENDIX-A

Elastic critical moment using EBCS-3 section 4.6.3.2

Calculation of Elastic critical moment for lateral torsional buckling according to EBCS-3, section 4.6.3.2 for an UB 127 x 76 x 13 by using the following formula for validation purpose

$$M_{cr} = \frac{\pi^{2} E I_{Z}}{L^{2}} \left[\frac{I_{W}}{I_{Z}} + \frac{L^{2} G I_{t}}{\pi^{2} E I_{Z}} \right]^{0.5}$$

The relevant section properties of UB 127 x 76 x 13 are:

H= 127mm	$I_{y=}473 \text{cm}^4$	$w_{el,y=}$ 74.6cm ³
B= 76mm	$I_{z=}55.7 \text{cm}^4$	$W_{pl,y=}$ 84.2cm ³

tw=4 $I_{w=} 0.00199*10^6 \text{ cm}^6$

tf= 7.6 $I_{t=} 0.85 cm^4$

use G= 80 Gpa , E=210000 Mpa

$$M_{cr} = \frac{\pi^2 * 210000^* 55.7 * 10^4}{2000^2} \left[\frac{0.00199 * 10^{12}}{55.7 * 10^4} + \frac{2000^2 * 80000^* 2.85 * 10^4}{\pi^2 * 210000^* 55.7 * 10^4} \right]^{0.5}$$
$$M_{cr} = 31KN - m$$

APPENDIX-B

Ultimate load verses mid-span deflection

Table B. 1 FEA results of castellated beam with circular openings

Model	Do	D	D/D _o	S/D _o	S	e	Ultimate load (KN)	Deflection (mm)
1	110	182	1.65	1.4	154	44	70.83	13.8
2	125	189.5	1.52	1.4	175	50	71.6	10.89
3	140	197	1.41	1.4	196	56	81.2	14.68
4	155	204.5	1.32	1.4	217	62	66.76	10.27
5	163	208.5	1.28	1.4	228.2	65.2	78.13	17.65
6	110	182	1.65	1.3	143	33	69.74	63.75
7	125	189.5	1.52	1.3	162.5	37.5	75.19	48.43
8	140	197	1.41	1.3	182	42	65.1	18.48
9	155	204.5	1.32	1.3	210.5	46.5	70.68	41.06
10	163	212	1.28	1.3	211.9	48.9	61.53	10.89
11	110	182	1.65	1.2	132	22	52.32	38.73
12	125	189.5	1.52	1.2	150	25	56.86	58.14
13	140	197	1.41	1.2	168	28	54.86	110.61
14	155	204.5	1.32	1.2	186	31	43.77	70.73
15	163	212	1.28	1.2	195.6	32.6	47.68	51.56

Model	Do	D	D/D _o	S/Do	S	e	Ultimate load (KN)	Deflection (mm)
1	110	182	1.65	1.4	154	44	8135	11.60
2	125	189.5	1.52	1.4	175	50	82.89	11.62
3	140	197	1.41	1.4	196	56	88.59	12.40
4	155	204.5	1.32	1.4	217	62	85.04	8.66
5	163	208.5	1.28	1.4	228.2	65.2	90.95	14.80
6	110	182	1.65	1.3	143	33	81.08	11.61
7	125	189.5	1.52	1.3	162.5	37.5	84.52	12.0
8	140	197	1.41	1.3	182	42	83.38	11.68
9	155	204.5	1.32	1.3	210.5	46.5	89.68	15.2
10	163	212	1.28	1.3	211.9	48.9	84.41	9.47
11	110	182	1.65	1.2	132	22	78.12	14.68
12	125	189.5	1.52	1.2	150	25	79.86	14.80
13	140	197	1.41	1.2	168	28	83.72	13.6
14	155	204.5	1.32	1.2	186	31	76.81	21.64
15	163	212	1.28	1.2	195.6	32.6	74.2	23.2

Table B. 2 FEA results of castellated beam with diamond openings

Model	Do	D	D/D _o	S/D _o	S	e	Ultimate load (KN)	Deflection (mm)
1	110	182	1.65	1.4	154	44	40.48	132.9
2	125	189.5	1.52	1.4	175	50	45.56	61.56
3	140	197	1.41	1.4	196	56	46.35	68.46
4	155	204.5	1.32	1.4	217	62	39.24	29.19
5	163	208.5	1.28	1.4	228.2	65.2	42.94	43.85
6	110	182	1.65	1.3	143	33	37.19	207.24
7	125	189.5	1.52	1.3	162.5	37.5	38.34	237.24
8	140	197	1.41	1.3	182	42	28.25	206.6
9	155	204.5	1.32	1.3	210.5	46.5	31.70	268.141
10	163	212	1.28	1.3	211.9	48.9	27.3	252.62
11	110	182	1.65	1.2	132	22	26.33	249.972
12	125	189.5	1.52	1.2	150	25	28.4	259.32
13	140	197	1.41	1.2	168	28	30.3	264.15
14	155	204.5	1.32	1.2	186	31	27.4	253.72
15	163	212	1.28	1.2	195.6	32.6	28.1	262.43

Table B. 3 FEA results of castellated beam with square openings

Model	Do	D	D/D _o	S/D _o	S	e	Ultimate load (KN)	Deflection (mm)
1	110	182	1.65	1.4	154	44	65.59	12.31
2	125	189.5	1.52	1.4	175	50	63.55	11.84
3	140	197	1.41	1.4	196	56	68.73	16.58
4	155	204.5	1.32	1.4	217	62	53.81	12.26
5	163	208.5	1.28	1.4	228.2	65.2	53.9	23.13
6	110	182	1.65	1.3	143	33	66.98	65.13
7	125	189.5	1.52	1.3	162.5	37.5	62.3	44.6
8	140	197	1.41	1.3	182	42	56.73	16.58
9	155	204.5	1.32	1.3	210.5	46.5	61.57	22.28
10	163	212	1.28	1.3	211.9	48.9	53.6	32.3
11	110	182	1.65	1.2	132	22	52.81	74.61
12	125	189.5	1.52	1.2	150	25	57.3	88.63
13	140	197	1.41	1.2	168	28	52.33	90.59
14	155	204.5	1.32	1.2	186	31	47.8	68.1
15	163	212	1.28	1.2	195.6	32.6	49.84	71.594

Table B. 4 FEA results of castellated beam with hexagon openings

APPENDIX-C

Buckling moment resistance verses lateral deflection

Table C. 1 FEA results of castellated beam with circular openings

							Buckling moment	Lateral
Model	Do	D	D/D _o	S/D _o	S	e	resistance, Mb, _{Rd}	Deflection
							(KN-m)	(mm)
1	110	182	1.65	1.4	154	44	23.795	8.34
2	125	189.5	1.52	1.4	175	50	24.325	11.54
3	140	197	1.41	1.4	196	56	24.67	12.15
4	155	204.5	1.32	1.4	217	62	25.1	10.12
5	163	208.5	1.28	1.4	228.2	65.2	25.15	10.53
6	110	182	1.65	1.3	143	33	23.67	8.76
7	125	189.5	1.52	1.3	162.5	37.5	24.16	10.96
8	140	197	1.41	1.3	182	42	24.17	10.97
9	155	204.5	1.32	1.3	210.5	46.5	24.78	10.94
10	163	208.5	1.28	1.3	211.9	48.9	24.55	10.97
11	110	182	1.65	1.2	132	22	21.995	8.7
12	125	189.5	1.52	1.2	150	25	23.7	8.66
13	140	197	1.41	1.2	168	28	20.755	5.14
14	155	204.5	1.32	1.2	186	31	18.405	3.84
15	163	208.5	1.28	1.2	195.6	32.6	19.06	3.93

							Buckling moment	Lateral
Model	Do	D	D/D _o	S/Do	S	e	resistance, Mb, _{Rd}	Deflection
							(KN-m)	(mm)
1	110	182	1.65	1.4	154	44	24.45	10.85
2	125	189.5	1.52	1.4	175	50	25.1	10.99
3	140	197	1.41	1.4	196	56	25.5	9.90
4	155	204.5	1.32	1.4	217	62	26.145	9.97
5	163	208.5	1.28	1.4	228.2	65.2	25.87	11.63
6	110	182	1.65	1.3	143	33	24.295	10.99
7	125	189.5	1.52	1.3	162.5	37.5	24.91	10.40
8	140	197	1.41	1.3	182	42	25.42	10.05
9	155	204.5	1.32	1.3	210.5	46.5	25.795	11.60
10	163	208.5	1.28	1.3	211.9	48.9	26.37	9.76
11	110	182	1.65	1.2	132	22	24.205	11.02
12	125	189.5	1.52	1.2	150	25	24.79	5.318
13	140	197	1.41	1.2	168	28	25.18	10.31
14	155	204.5	1.32	1.2	186	31	25.425	10.57
15	163	208.5	1.28	1.2	195.6	32.6	24.6	7.9

Table C. 2 FEA results of castellated beam with diamond openings

Model	Do	D	D/D _o	S/D _o	S	e	Buckling moment resistance, Mb, _{Rd} (KN-m)	Lateral Deflection (mm)
1	110	182	1.65	1.4	154	44	17.975	4.74
2	125	189.5	1.52	1.4	175	50	20.31	7.00
3	140	197	1.41	1.4	196	56	18.41	10.84
4	155	204.5	1.32	1.4	217	62	18.02	4.72
5	163	208.5	1.28	1.4	228.2	65.2	19.135	9.68
6	110	182	1.65	1.3	143	33	13	5.33
7	125	189.5	1.52	1.3	162.5	37.5	14.795	3.85
8	140	197	1.41	1.3	182	42	12.48	3.30
9	155	204.5	1.32	1.3	210.5	46.5	11.835	3.71
10	163	208.5	1.28	1.3	211.9	48.9	12.1	4.97
11	110	182	1.65	1.2	132	22	8.815	4.00
12	125	189.5	1.52	1.2	150	25	9.8	5.2
13	140	197	1.41	1.2	168	28	9.05	6.3
14	155	204.5	1.32	1.2	186	31	8.95	3.8
15	163	208.5	1.28	1.2	195.6	32.6	9.4	4.86

Table C. 3 FEA results of castellated beam with square openings

Model	Do	D	D/D _o	S/D _o	S	e	Buckling moment resistance, Mb, _{Rd} (KN-m)	Lateral Deflection (mm)
1	110	182	1.65	1.4	154	44	23.6	9.2
2	125	189.5	1.52	1.4	175	50	23.915	8.69
3	140	197	1.41	1.4	196	56	24.435	7.84
4	155	204.5	1.32	1.4	217	62	23.455	8.73
5	163	208.5	1.28	1.4	228.2	65.2	23.305	9.68
6	110	182	1.65	1.3	143	33	23.49	7.88
7	125	189.5	1.52	1.3	162.5	37.5	24.43	6.72
8	140	197	1.41	1.3	182	42	23.43	8.08
9	155	204.5	1.32	1.3	210.5	46.5	23.865	10.09
10	163	208.5	1.28	1.3	211.9	48.9	23.6	9.1
11	110	182	1.65	1.2	132	22	23.475	8.50
12	125	189.5	1.52	1.2	150	25	23.65	4.91
13	140	197	1.41	1.2	168	28	21.535	5.18
14	155	204.5	1.32	1.2	186	31	19.4	3.93
15	163	208.5	1.28	1.2	195.6	32.6	18.185	2.72

Table C. 4 FEA results of c	astellated beam with	hexagon openings
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APPENDIX-D

A. Sample of modeling



Figure D-1 Modeling of circular opening







Figure D-3 Modeling of diamond opening

















Figure D-7 Analysis of diamond opening



