DEFLECTION OF LIGHT BY MASSIVE OBJECTS

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Contents

List of Tables

List of Figures

Acronyms

Abbrevations and their representations

- GR - General theory of relativity
- ΛCDM-Lambda Cold Dark Matter
- EFE-Einstein Field Equation
- WMAP - Wilkinson Microwave Anisotropy Probe
- CCD-Charge Coupled Device

Abstract

The discovery of gravitational lensing made the deflection of light as an important tool for astronomy and cosmology. Since Einstein introduced general theory of relativity for the expanding universe, the cosmological constant (Λ) is still a part of studies upto now. And in this work our main objective is in order to study the deflection of light by massive objects. Specifically we derived the lens equation in the presence of cosmological constant. Then we evaluated the contribution of cosmological constant to image size or image magnification. And finally we estimated the effect of cosmological constant to angular distances and image positions. The Einstein Field equation (EFE) is implemented to derive the lens equation in the presence of Λ. And using this lens equation for point mass system, we calculated image sizes, angular distances and image positions for the selected Einstein ringed systems. We expressed the results for the selected Einstein ringed objects by tables. As we expressed at the end of this work, the calculated image size or image magnification is a negative value and this implies that the image formed by gravitational lensing is de-magnified. And also we have seen the effect of cosmological constant to image position. The contribution of Λ for the Einstein correction part is 2% and this matches with other previous works.

Keywords

Deflection of light, cosmological constant, image magnification, GR

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Chapter 1

Introduction

Deflection of light by gravity was predicted by General Relativity and observationally confirmed in 1919. In the following decades various aspects of the gravitational lens effect were explored theoretically. Among them the possibility of multiple or ring-like images of background sources, the use of lensing as a gravitational telescope on very faint and distant objects, and the possibility to determine Hubble's constant with lensing. Currently gravitational lensing became an observational science after the discovery of the first doubly imaged quasar in 1979[2].

Although the deflection of light at the solar limb was very successfully hailed as the first experiment to confirm a prediction of Einstein's theory of General Relativity in 1919, it took more than half a century to establish this phenomenon observationally in some other environment. By now different realizations of lensing are known and observed[2].

Within the last 20 years gravitational lensing has changed from being considered a geometric curiosity to a helpful and in some ways unique tool of modern astrophysics. And most importantly a number of astrophysical problems makes it an attractive tool in many branches of astronomy[3].

Gravitational lensing is a deflection of light from a background source due to the space-time curvature caused by massive objects along the line of sight (lenses). Gravitational lensing both magnifies (or de-magnifies) the flux and distorts the shape of the lensed source[3].

The formalism of gravitational lensing treats the propagation of light in the limit of geometrical optics, meaning that light rays travel in straight lines until they are eventually deflected by a gravitational lens. This is a simplification of the full framework of general relativity, in which light follows the curvature of spacetime on null geodesics of the metric. The approximation is valid where gravitation is weak enough to be linearised. Then it is possible to separate the equations into a smooth background metric, in the following always assumed to be a flat ΛCDM cosmology, and any number of perturbations which act as deflectors or lenses. The simplest such deflector is a point mass M[2].

The ΛCDM or Lambda cold dark matter model is a standard cosmological model in which cosmological issues of the universe we live in is estimated. The universe contains a cosmological constant, which can be taken as fluid denoted by Lambda (Greek Λ), associated with dark energy, and cold dark matter (abbreviated CDM)[4]. Nowadays, the cosmological constant is most often discussed in the context of Universes with the flat Euclidean geometry, $\kappa = 0$

1.1 Statement of the problem

For many years the cosmological constant was almost universally considered to be 0. However, recent improved astronomical techniques have found that a positive value of Λ is needed to explain the accelerating universe. Einstein field equations have already initiated researchers on cosmological issues. Nowadays gravitational lensing paved a way for astronoical studies and many researchers did on this area. However,still there are several issues on discussion such as deflection of light by massive objects,time delay of Radar echoes etc.

Research questions

– How does cosmological constant modify lens equation?
– What is the contribution of cosmological constant to image size?
– How does Λ affect angular distances and image positions?

1.2 Objectives

1.2.1 General objectives

To study deflection of light by massive objects.

1.2.2 Specific objectives

- to derive the lens equation in the presence of cosmological constant..
- to identify the effect of cosmological constant to image size.
- to calculate the angular distances and image positions in presence of Λ .

1.3 Methodology

Einstein field equations (EFE) are implemented to derive the lens equation in the presence of cosmological constant. Then the derived lens equation is used to estimate angular distances and image sizes for selected ringed objects.Mathematica-11 is used for the semi- analytical analysis.

Chapter 2

Theory of general relativity

2.1 Introduction

General relativity is the geometric theory of gravitation. One of Einsteins great insights was to make general relativity a geometric theory of gravitation. In special relativity, space-time is the arena for physics. Spacetime consists of events, which require four numbers for their complete specification: three numbers to give the spatial location with respect to some chosen coordinate grid, and one number to give the time.Geometrically, space-time is represented by a four-dimensional manifold (surface), each point in the manifold corresponding to an event in spacetime.The general theory of relativity is a classical field theory of gravitation in which all variables are assumed to be continuous and are uniquely specified. The basic philosophy of general relativity is to relate the geometry of space time, which determines the motion of matter, to the density of matter-energy, known as the stress energy tensor. This relation is accomplished through the Einstein field equations. The geometry of space-time is dictated by the metric tensor which defines the properties of that geometry and basically describes how travel in one coordinate involves another coordinate, so that

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{2.1.1}$$

The elements of the metric tensor are dimensionless; for ordinary Euclidean space they are all unity if $\mu = \nu$ and zero otherwise. General relativity is defined on a four dimensional Riemannian manifold[3]. Coordinates in this non-Euclidian space are denoted by $x_\mu = (x^0, x^1, x^2, x^3)$.

Now the field equations relate second derivatives of the metric tensor to the properties of the local matter-energy density expressed in terms of the stress-energy tensor. Specifically the Einstein field equations are

$$G_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu} \tag{2.1.2}$$

where,

$G_{\mu\nu}$ -is known as the Einstein tensor and
$T_{\mu\nu}$ - is the stress energy tensor in physical units (say grams per cubic centimeter).

The quantity G/c^2 is a very small number in any common system of units, which shows that the departure from Euclidean space is small unless the stress-energy is exceptionally large.

2.2 Einstein field equation

The Einstein field equations (EFE; also known as Einsteins equations) comprise the set of equations in Albert Einsteins general theory of relativity that describe the fundamental interaction of gravitation as a result of space-time being curved by mass and energy. Similar to the way that electromagnetic fields are determined using charges and currents via Maxwells equations. The EFE are used to determine the space-time geometry resulting from the presence of mass energy and linear momentum, i.e; they determine the metric tensor of space-time for a given arrangement of stress energy in the space-time.

Einstein's equation tells us how the presence of matter curves space-time, and so we need to describe the matter under consideration. The Einstein field equations (EFE) may be written in the form

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{2.2.1}$$

where,

- $R_{\mu\nu}$ - is the Ricci curvature tensor
- R -is the scalar curvature
- $g_{\mu\nu}$ -is the metric tensor
- Λ-is the cosmological constant
- G -is Newton's gravitational constant
- c -is the speed of light in vacuum, and
- $T_{\mu\nu}$ -is the stress energy tensor.

The EFE is a tensor equation relating a set of symmetric 4 x 4 tensors. Each tensor has 10 independent components. The four Bianchi identities reduce the number of independent equations from 10 to 6, leaving the metric with four gauge fixing degrees of freedom, which correspond to the freedom to choose a coordinate system.

In fact, when fully written out, the EFE are a system of ten coupled, non-linear, hyperbolic-elliptic partial differential equations.One can write the EFE in a more compact form by defining the Einstein tensor.

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \tag{2.2.2}$$

Which is a symmetric second-rank tensor that is a function of the metric. The EFE can then be written as

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{C^4}T_{\mu\nu} \tag{2.2.3}$$

In standard units, each term on the left has units of $1/length^2$. With this choice of Einstein constant as $8\pi G/c^4$, then the stress-energy tensor on the right side of the equation must be written with each component in units of energy-density (i.e., energy per volume = pressure). Using geometrized units where $G = c = 1$, this can be rewritten as

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{2.2.4}$$

The expression on the left represents the curvature of space-time as determined by the metric; the expression on the right represents the matter/energy content of space-time. The EFE can then be interpreted as a set of equations dictating how matter/energy determines the curvature of spacetime.

2.3 Cosmological constant

Albert Einstein modified his original field equations to include a cosmological constant term Λ proportional to the metric

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{2.3.1}$$

The energy conservation law is not affected because Λ is constant.

Einstein originally introduced the cosmological constant term.Despite Einstein's motivation for introducing the cosmological constant term, there is nothing inconsistent with the presence of such a term in the equations. For many years the cosmological constant was almost universally considered to be 0. However, recent improved astronomical techniques have found that a positive value of Λ is needed to explain the accelerating universe. However, the cosmological constant is negligible at the scale of a galaxy or smaller.

Einstein thought of the cosmological constant as an independent parameter, but its term in the field equation can also be moved algebraically to the other side, written as part of the stressenergy tensor

$$T^{(vac)}_{\mu\nu} = -\frac{\Lambda c^4}{8\pi G} g_{\mu\nu} \tag{2.3.2}$$

Then the resulting vacuum energy density is a constant and we have

$$\rho^{(vac)}_{\mu\nu} = \frac{\Lambda c^2}{8\pi G} \tag{2.3.3}$$

The existence of a cosmological constant is thus equivalent to the existence of a non-zero vacuum energy. Thus, the terms "cosmological constant" and "vacuum energy" are now used interchangeably in general relativity.

A positive vacuum energy density resulting from a cosmological constant implies a negative pressure, and vice versa. If the energy density is positive, the associated negative pressure will drive an accelerated expansion of the universe.

Positive Value of Cosmological Constant

Observations announced in 1998 of distance redshift relation for Type Ia supernovae (Supernova Cosmology Project (Perlmutter et al. (1999)) indicated that the expansion of the universe is accelerating. When combined with measurements of the cosmic microwave background radiation these implied a value of $\Omega_\Lambda \sim 0.7$(Baker et al. (1999)) a result which has been supported and refined by more recent measurements. There are other possible causes of an accelerating universe, such as quintessence, but the cosmological constant is in most respects the simplest solution. Thus, the current standard model of cosmology, the Lambda-CDM model, includes the cosmological constant, which is measured to be on the order of $10^{-52}m^{-2}$, in metric units. It is often expressed as $10^{-35}s^{-2}$ or 10^{-122} (Barrow and Shaw (2011)) in other unit systems. The value is based on recent measurements of vacuum energy density, $\rho_{vacuum} = 5.96 \times 10^{-27}kg/m^3$ or $10^{-47}GeV^4$,in other unit systems. As was only recently seen, by works of 't Hooft, Susskind and others, a positive cosmological constant has surprising consequences, such as a finite maximum entropy of the observable universe[3].

In addition to the above when formulating general relativity, Einstein believed that the Universe was static, but found that his theory of general relativity did not permit it. This is simply because all matter attracts gravitationally. None of the solutions we have found correspond to a static Universe with constant a. In order to arrange a static Universe, he proposed a change to the equations, something he would later famously call his "greatest blunder". That was the introduction of a cosmological constant.

The introduction of such a tenn is permitted by general relativity, and although Einstein's original motivation has long since faded, it is currently seen as one of the most important and enigmatic objects in cosmology. The cosmological constant Λ appears in the Friedmann equation as an extra term, giving

$$H^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2} + \frac{\Lambda}{3} \tag{2.3.4}$$

In principle, Λ can be positive or negative, though the positive case is much more commonly considered. Einstein's original idea was to balance curvature, Λ and ρ to get $H(t) = 0$ and hence a static Universe. In fact, this idea was rather misguided, since such a balance proves to be unstable to small perturbations, and hence presumably couldn't arise in practice. Nowadays, the cosmological constant is most often discussed in the context of Universes with the flat Euclidean geometry, $\kappa = 0$

The effect of Λ can be seen more directly from the acceleration equation. By using the Friedmann equation as given above, gives

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda}{3} \tag{2.3.5}$$

A positive cosmological constant gives a positive contribution to $\ddot{a}$, and so acts effectively as a repulsive force. In particular, if the cosmological constant is sufficiently large, it can overcome the gravitational attraction represented by the first term and lead to an accelerating Universe [3].

Chapter 3

Gravitational lensing

3.1 Introduction to gravitational lensing

Gravitational light deflection is determined by the gravitational field through which light propagates. It is essential to realize that this simple fact implies that gravitational light deflection is independent of the nature of the matter and of its state. Lensing is equally sensitive to dark and luminous matter, and to matter in equilibrium or far out of it. On the negative side, this implies that lensing alone cannot distinguish between these forms of matter, but on the positive side, it also cannot miss one of these matter forms. Hence, lensing is an ideal tool for measuring the total mass of astronomical bodies, dark and luminous.(Schneider 2006)

The deflection of light by the Sun can be measured during a total solar eclipse when it is possible to observe stars projected near the Solar surface. Light deflection then slightly changes their positions. A measurement of the deflection in 1919, with a sufficient accuracy to distinguish between the 'Newtonian'and the GR value, provided a tremendous success for Einstein's new theory of gravity.(Schneider)

Soon thereafter, Lodge (1919) used the term 'lens' in the context of gravitational light deflection, but noted that 'it has no focal length'. Chwolson (1924) considered a source perfectly coaligned with a foreground mass, concluding that the source should be imaged as a ring around the lens. In fact,only fairly recently did it become known that Einstein made some unpublished notes on this effect in 1912 (Renn et al. 1997). Hence, calling them 'Einstein rings' is indeed appropriate. If the alignment is not perfect, two images of the background source would be visible, one on either side of the foreground star. Einstein, in 1936, after being approached by the Czech engineer Rudi Mandl, wrote a paper where he considered this lensing effect by a star, including both the image positions, their separation, and their magnifications. He concluded that the angular separation between the two images would be far too small (of order milli-arcseconds) to be resolvable, so that "there is no great chance of observing this phenomenon" (Einstein 1936)

3.1.1 The early years, before general relativity

The Newtonian theory of gravitation predicts that the gravitational force F on a particle of mass m is proportional to m, so that the gravitational acceleration $a = F/m$ is independent of m. Therefore, the trajectory of a test particle in a gravitational field is independent of its mass but depends, for a given initial position and direction, only on the velocity of the test particle. About 200 years ago, several physicists and astronomers speculated that, if light could be treated like a particle, light rays may be influenced in a gravitational field as well. John Mitchell in 1784, in a letter to Henry Cavendish, and later Johann von Soldner in 1804, mentioned the possibility that light propagating in the field of a spherical mass M (like a star) would be deflected by an angle $\hat{a}_N = 2GM/c^2\xi$, where G and c are Newton constant of gravity and the speed of light, respectively, and ξ is the impact parameter of the incoming light ray. At roughly the same time, Pierre-Simon Laplace in 1795 noted "that the gravitational force of a heavenly body could be so large, that light couldnot flow out of it" (Laplace 1975), i.e. , the escape velocity $v_e = \sqrt{2GM/R}$ from the surface of a spherical mass M of radius R becomes the velocity of light, which happens if $R = R_s \equiv 2GM/c^2$, now a days called the Schwarzschild radius of a mass M[5].

3.1.2 Gravitational light deflection in GR

In November 1915, Albert Einstein (Nobel Prize in Physics 1921) presented his theory of gravity, which he nicknamed General Relativity (GR) Einstein(1915), an extension of his theory of special relativity. This was one of the greatest achievements in the history of science, a modern milestone. It was based on the Equivalence Principle, which states that the gravitational mass of a body is the same as its inertial mass. You cannot distinguish gravity from acceleration! Einstein had already checked that this could explain the precession of the perihelion of Mercury, a problem of Newtonian mechanics. The new insight was that gravity is really geometric in nature and that the curving of space and time, space-time, makes bodies move as if they were affected by a force. The crucial physical parameters are the metric of spacetime, a matrix that allows us to compute infinitesimal distances (actually infinitesimal line elements or proper times in the language of special relativity.) It became immediately clear that Einstein's theory could be applied to cosmological situations, and Karl Schwarzschild very soon found the general solution for the metric around a massive body such as the Sun or a star[6].

In 1917, Einstein applied the GR equations to the entire Universe[7], making the implicit assumption that the Universe is homogenous. If we consider cosmological scales large enough such that local clusters of matter are evened out. He argued that this assumption fit well with his theory and he was not bothered by the fact that the observations at the time did not really substantiate his conjecture. Remarkably, the solutions of the equations indicated that the Universe could not be stable. This was contrary to all the thinking of the time and bothered Einstein. He soon found a solution, however. His theory of 1915 was not the most general one consistent with the Equivalence Principle. He could also introduce a cosmological constant, a constant energy density component of the Universe. With this Einstein could balance the Universe to make it static.

In the beginning of the 1920's, the Russian mathematician and physicist Alexander Friedmann studied the problem of the dynamics of the Universe using essentially the same assumptions as Einstein, and found in 1922 that Einstein's steady state solution was really unstable [8]. Any small perturbation would make the Universe non-static. At first Einstein did not believe Friedmann's results and submitted his criticism to Zeitschrift für Physik, where Friedmann's paper had been published. However, a year later Einstein found that he had made a mistake and submitted a new letter to the journal acknowledging this fact. Even so, Einstein did not like the concept of an expanding Universe and is said to have found the idea "abominable". In 1924, Friedmann presented his full equations [9], but after he died in 1925 his work remained essentially neglected or unknown, even though it had been published in a prestigious journal.

We have to remember that a true revolution was going on in physics during these years with the advent of the new quantum mechanics, and most physicists were busy with this process. In 1927, the Belgian priest and physicist Georges Lemaitre working independently from Friedmann performed similar calculations based on GR and arrived at the same results [10, 11]. Unfortunately, Lemaître's paper was published in a local Belgian journal and again the results did not spread far, even though Einstein knew of them and discussed them with Lemaitre.

In the beginning of the 20^{th} century it was generally believed that the entire Universe only consisted of our galaxy, the Milky Way. Many nebulae which had been found in the sky were thought to be merely gas clouds in distant parts of the Milky Way. In 1912, Vesto Slipher[12, 13],while working at the Lowell Observatory, pioneered measurements of the shifts towards red of the light from the brightest of these spiral nebulae. The redshift of an object depends on its velocity radially away from us, and Slipher found that the nebulae seemed to move faster than the Milky Way escape velocity.

In the following years, the nature of the spiral nebulae was intensely debated. Could there be more than one galaxy? This question was finally settled in the 1920s with Edwin Hubble as a key figure. Using the new 100-inch telescope at Mt Wilson, Hubble was able to resolve individual stars in the Andromeda nebula and some other spiral nebulae, discovering that some of these stars were Cepheids, dimming and brightening with a regular period [14].

The Cepheids are pulsating giants with a characteristic relation between luminosity and the time interval between peaks in brightness, discovered by the American astronomer Henrietta Leavitt in 1912.

Hubble used Leavitt's relation to estimate the distance to the spiral nebulae, concluding that they were much too distant to be part of the Milky Way and hence must be galaxies of their own. Combining his own measurements and those of other astronomers he was able to plot the distances to 46 galaxies and found a rough proportionality of an object's distance with its redshift. In 1929, he published what is today known as 'Hubble's law'. A galaxy's distance is proportional to its radial recession velocity [15].

Even though Hubble's data were quite rough and not as precise as the modern ones, the law became generally accepted, and Einstein had to admit that the Universe is indeed expanding. It is said, that he called the introduction of the cosmological constant his "greatest mistake". From this time on, the importance of the cosmological constant faded, although it reappeared from time to time.

Hubble's and others' results from 1926 to 1934, even though not very precise, were encouraging indications of a homogeneous Universe and most scientists were quick to accept the notion. The concept of a homogeneous and isotropic Universe is called the Cosmological Principle. This goes back to Copernicus, who stated that the Earth is in no special, favoured place in the Universe. In modern language it is assumed that the Universe looks the same on cosmological scales to all observers, independent of their location and independent of in which direction they look in. The assumption of the Cosmological Principle was inherent in the work of Friedmann and Lemaitre but virtually unknown in large parts of the scientific society. Thanks to the work of Howard Robertson in 1935-1936 [16] and Arthur Walker in 1936 it became well known.

The evolution of the Universe is described by Einstein's theory of general relativity. In relativistic field theories, the vacuum energy contribution is given by an expression mathematically similar to the famous cosmological constant in Einstein's theory. The question of whether the vacuum energy term is truly time independent like the cosmological constant, or varies with time, is currently a very hot research topic.

Gravitational lensing ("lensing") occurs when a gravitating mass distorts a space-time and anything in it. The paths followed by electromagnetic radiation from a star, galaxy, or other source are bent as well. This can be seen directly from equations of motion for photons. Light can be modeled as a massless point particle following a worldline in the "geometric optics approximation" [17]

Lensing provided the first experimental verification of GR through observations of starlight bending around the Sun during an eclipse in 1919 [18] and continues to be a major source of insight into gravitation [19]. Lensing magnifies the image relative to the source, modifies the time it takes light to reach its destination, and distorts the image. The bending of light by massive bodies was anticipated as early as the 18^{th} century by Henry Cavendish [20].

Early calculations of light bending relied on the assumption that light was massive and therefore attracted to a massive body viewing light as reacting to a force. In 1911, Einstein re imagined light bending as a result of his principle of equivalence viewing light as traveling on a null geodesic in a curved spacetime. In recent years, lensing has become a powerful probe of many astrophysical and cosmological questions. Strong lensing, or systems in which multiple images of a single source are detectable or in which an Einstein ring or part of one (an arc) is visible, can inform us of the Hubble constant and other cosmological parameters [21].

Statistical measurements of lensing where the light deflection is too weak to detect in a single background image, or weak lensing, provides a powerful probe of the matter distribution in the universe [22]. Weak lensing is a particularly important probe of dark matter and has been proposed as a tool to distinguish GR from alternative theories [23]. Microlensing, where a transient lens causes a source to temporarily brighten, has provided a way to search for massive compact halo objects (MACHOs) and extrasolar planets[23]. In short, gravitational lensing has proven to be a versatile tool for examining a wide variety of questions.

3.1.3 Deflection of light near elliptical galaxies or stars

Newtonian gravity predicted deflection of light passing around a massive object[24]. In 1911, Einstein calculated the value of bending of light for a spherically symmetric a massive object

[24]. In 1916, when Schwarzschild calculated the gravitational field around a scal object such as galaxy, star or a planet,then the first measurement was made by Eddington, Dayson and Davidson , during the solar eclipse in 1919. In 1924, Chwolson mentioned fictitious double star and the mirror-reversed nature of the secondary image. He also mentioned the symmetric case of a star exactly behind star, resulting in a circular image. In 1936, Einstein reported about the appearance of a luminous circle which today is called as Einstein-ring. Einstein and Fritz Zwicky pointed out that galaxies are much more likely to be gravitationally lensed than stars and that one can use the gravitational lens effect as a natural telescope. In 1988, Grossman and Narayan studied simulation of lensing total of 101 randomly generated cluster and Bodenner and Will calculated the deflection of light in a spherically symmetric body to the second order. They used three different types of, Schwarzschild, Isotropic and Harmonic coordinates systems. The gravitational field, and thus the deflection angle,depend neither on the nature of the matter nor on its physical state but is depend to shape and geometry of object[24, 25].

3.2 Forms of gravitational lensing

Gravitational lensing was first proposed by Albert Einstein during the preparation of his theory of general relativity. He noted that because massive objects curve space-time, the path of light passing near those massive objects will bend light around them. In fact, it was gravitational lensing that made Einstein a household name as the deflection of light around the sun was used to test his theory during a solar eclipse. Since that time, gravitational lensing has become an indispensable tool for astronomers.

3.2.1 Strong gravitational lensing

The first strong gravitational lens, discovered in 1979, was indeed linked to a quasar ($QSO0957+561$[26], and although the phenomenon was expected on theoretical grounds, it left the astronomers surprise. The existence of two objects separated by about 6̋(6 arcsec) and characterized by an identical spectrum led to the conclusion that they were the doubled image of the same quasar, clearly showing that Zwicky was perfectly right and that galaxies may act as gravitational lenses.

Afterwards, also the lens galaxy was identified, and it was established that its dynamical mass, responsible for the light deflection, was at least ten-times larger than the visible mass.

This double quasar was also the first object for which the time delay (about 420 days) between the two images [26], due to the different paths of the photons forming the two images, has been measured. This has also allowed obtaining an independent estimate of the lens galaxy dynamical mass. Observations can also show four images of the same quasar, as in the case of the so-called Einstein Cross, or when the lens and the source are closely aligned, one can observe the Einstein ring.

The macroscopic effect of multiple images formation is generally called strong lensing, which also consists of the formation of arcs, as those clearly visible in the deep sky field images by the Sloan Digital Sky Survey (SDSS). The sources of strong lensing events are often quasars, galaxies, galaxy clusters and supernovae, whereas the lenses are usually galaxies or galaxy

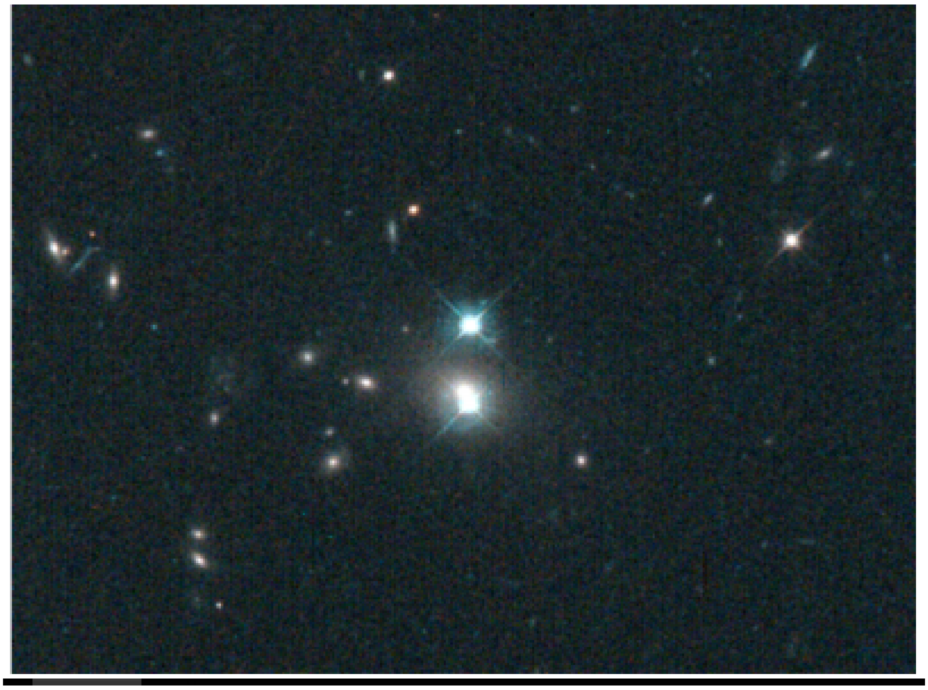

Figure 3.1: HST image of QSO 1957 +561(soucail et al.1987a)

clusters. The image separation is generally larger than a few tenths of an arcsec, often up to a few arcsecs.

Strong gravitational lensing is nowadays a powerful tool for investigation in astrophysics[27]. Strong lensing gives a unique opportunity to measure the dynamical mass of the lens object using, for example, the mass estimator$M(< R_E) = \pi \Sigma_{cr} \theta_E^2$.

Light rays leaving a source in different directions are focused on the same point by the intervening galaxy or cluster of galaxies. These are called strong lenses.

The first strong lensing observation was of the doubly imaged quasar $Q0957 + 561$ by Walsh,Carswell, and Weymann (1979). An optical image of $QSO0957 + 561$ taken by $HSTs$ WFPCII camera is shown in Figure 2.1. The magnification produced by strong lensing affects the observable properties of active galaxies, quasars, and any other lensed sources. Strong lensing also may provide information for cosmology. For example, the time delay among the multiple images of a quasar can be used to measure the Hubble constant.

Figure3.1

The first large luminous arc produced by strong lensing (Figure 3.2) was found in the massive nearby cluster, Abell 370, in 1986 by Lynds and Petrosian (1986) at Kitt Peak National Observatory (KPNO) and by Soucail et al. (1987a) at the Canada France Hawaii Telescope (CFHT). Giant arcs are due to the lensing effect of rich clusters of galaxies on background galaxies, with huge magnifications that can distort the galaxy shapes into long arcs around the clusters cores. The cluster Abell 2218 contains the most famous example of gravitationally lensed arcs (Figure 3.3). Until recently, the most massive galaxies and galaxy clusters have been the object of gravitational lensing studies. Galaxy groups are comprised of a lower density of galaxies than clusters, making them more difficult to detect.

After some controversy regarding whether ΛCDM (cold dark matter plus Cosmological Constant) simulations predict enough dark matter substructures to account for the observations, some indication is found of an excess of massive galaxy satellites), more recent analysis, taking also into account the uncertainty in the lens system ellipticity, finds results consistent with those predicted by the standard cosmological model. Three properties make strong gravitational lensing a most useful tool to measure and understand the universe. Figure3.2

- Firstly,strong lensing observable - such as relative positions, flux ratios, and time delays between multiple images - depend on the gravitational potential of the foreground galaxy (lens or deflector) and its derivatives.

- Secondly, the lensing observable also depend on the overall geometry of the universe via angular diameter distances between observer, deflector, and source.

- Thirdly, the background source often appears magnified to the observer, sometimes by more than an order of magnitude.
  As a result, gravitational lensing can be used to address three major astrophysical issues:

- Understanding the spatial distribution of mass at kpc and sub-kpc scale where baryons and DM interact to shape galaxies as we see them.

Figure 3.3

- Determining the overall geometry, content, and kinematics of the universe.

- Studying galaxies, black holes, and active nuclei that are too small or too faint to be resolved or detected with current instrumentation.

Strong lensing is characterized by a lens creating very substantial image distortions culminating in multiple images, large luminous arcs, and occasionally Einstein rings. These image distortions can be seen through telescopes. Figure3.4 shows a particularly clear example of a large arc nearly forming an Einstein ring taken by the Hubble Telescope[28].

3.2.2 Weak lensing

In the deep field surveys of the sky, also arclets (i.e., single distorted images with an elliptical shape) and weakly distorted images of galaxies, with an almost invisible individual elongation,

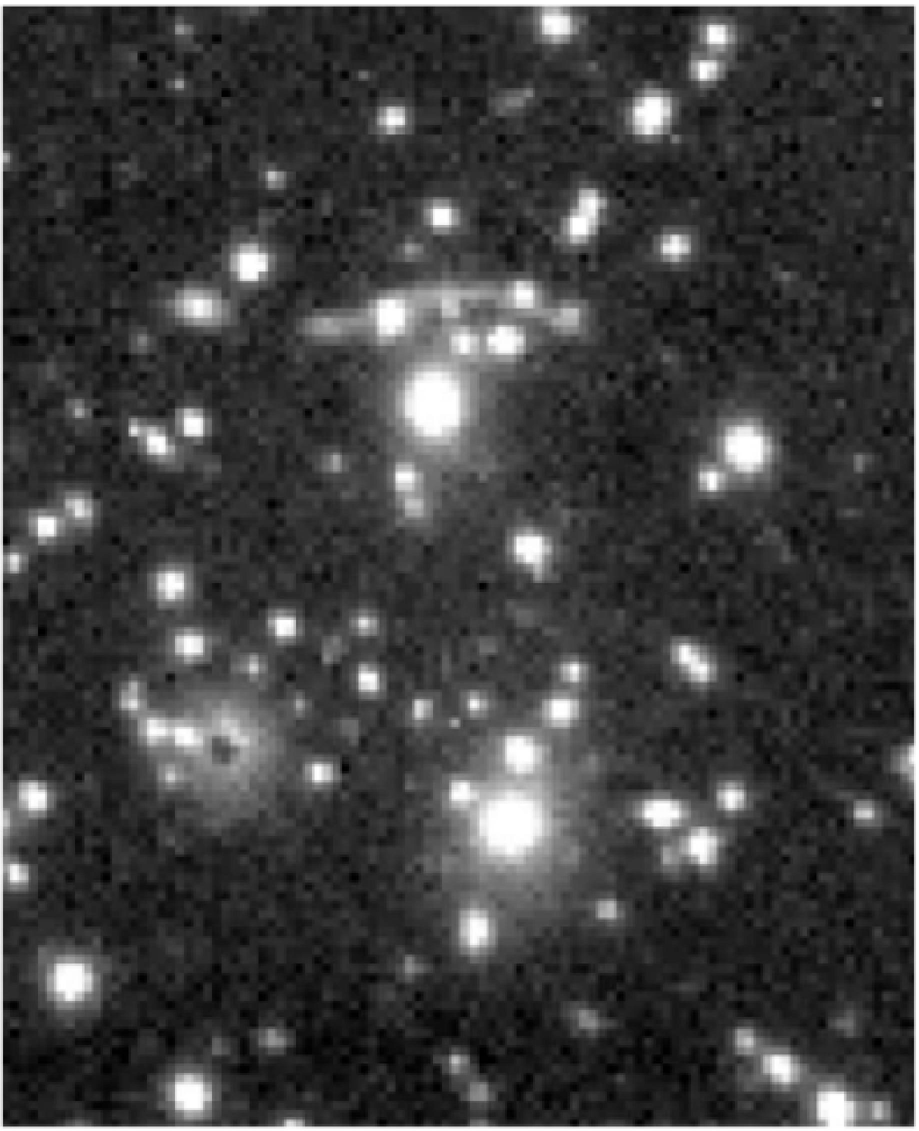

Figure 3.2: First observed giant gravitational arc(soucail et al.1987a)

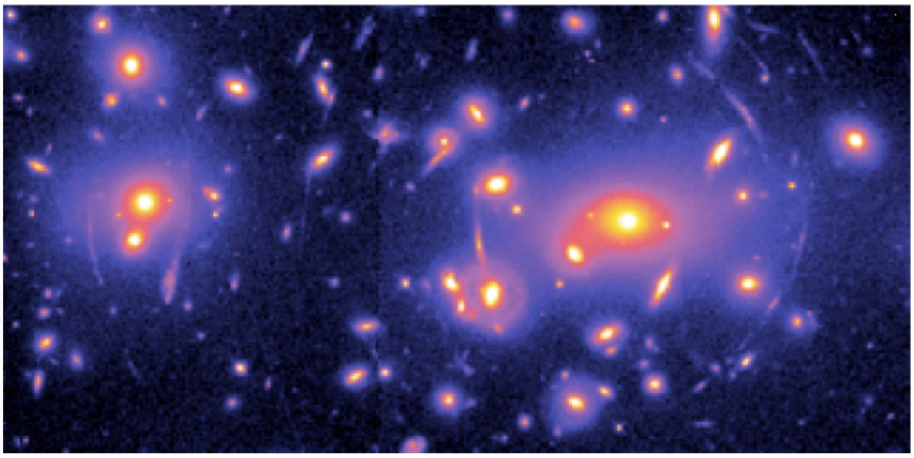

Figure 3.3: Foreground galaxies in the cluster Abell 2218 distort the images of background galaxies.Giant elliptical arcs surround the centeral region of the cluster at right

Figure 3.4: A Hubble image of a gravitational lens. A foreground galaxy lenses a background galaxy resulting in a large luminous arc around the lens.

have been detected. This effect is known as weak lensing and is playing an increasingly important role in cosmology.

The weak lensings main feature is the shape deformation of background galaxies, whose light crosses a mass distribution (e.g., a galaxy or a galaxy cluster) that acts as a gravitational lens. Actually, gravitational lensing gives rise to two distinct effects on a source image: convergence, which is isotropic, and shear, which is anisotropic. In the weak lensing regime, the observer makes use of the shear, that is the image deformation (sometimes related to the galaxy orientation), while the convergence effect is not used, since the intrinsic luminosity and the size of the lensed objects are unknown.

The first weak lensing event was detected in 1990 as statistical tangential alignment of galaxies behind massive clusters , but only in 2000, coherent galaxy distortions were measured in blind fields, showing the existence of the cosmic shear. The weak lensing cannot be measured by a single galaxy, but its observation relies on the statistical analysis of the shape and alignment of a large number of galaxies in a certain direction.

There are at least two major issues in weak lensing studies, one mainly relying on the theory, the other one on observations: the former concerns finding the best way to reconstruct the intervening mass distribution from the shear field $\gamma = (\gamma_1, \gamma_2)$, the latter with looking for the best way to determine the true ellipticity of a faint galaxy, which is smeared out by the instrumental point spread function PSF). To solve these issues, several approaches have been proposed, which can be distinguished into two broad families: direct and inverse methods. On the theoretical side, the direct approaches are: the integral method, which consists of expressing the projected mass density distribution as the convolution of by a kernel, and the local inversion method, which instead starts from the gradient of ϕ (e.g., under and the references there in). The inverse approaches work on the lensing potential, and they include the use of the maximum likelihood or the maximum entropy methods to determine the most likely projected mass distribution that reproduces the shear field. The inverse methods are particularly useful since they make it possible to quantify the errors in the resultant lensing mass estimates, as, for instance, errors deriving from the assumption of a spherical mass model when fitting a non-spherical system.

The inverse methods allow one also to derive constraints from external observations, such as X-ray data on galaxy clusters strong lensing or CMB lensing. In particular, one can compare mass measurements from weak lensing and X-ray observations for large samples of galaxy clusters.

weak lensing observations showed that the mass was largely concentrated around the galaxies themselves, and this enabled a clear, independent measurement of the amount of dark matter.

Weak lensing is characterized by small deviations in the image of background galaxies and galaxy clusters. The lensed images of background sources are still resolvable, however statistical analysis is necessary to determine if gravitational lensing is taking place. This is because the images are not distorted enough to differentiate between gravitational lensing and the regular orientation of a galaxy.

3.2.3 Microlensing

The Microlensing lensing is the phenomenon that occurs when θ_E is smaller than the typical telescope angular resolution, as in the case of stars lensing the light from background stars. If the source and the lens are aligned (first panel on the left), the circular symmetry of the problem leads to the formation of a luminous annulus having radius θ_E around the lens position. Otherwise, increasing the θ_S value, the secondary image gets closer to the lens position, while the primary image drifts apart from it, and in the limit of $\theta_s \gg \theta_E$, the microlensing phenomenon tends to disappear. However, observing multiple images during a microlensing event is practically impossible with the present technology.(Jean Surdej and Jean-Franois Claeskens, 2007) For instance,in the case in which the phenomenon is maximized, corresponding to the perfect alignment.

Gravitational lensing due to the close alignment of a foreground lens and a background source star, what we now call microlensing, was first published by Einstein in 1936[29]. However, he dismissed the practicality of microlensing, stating that "there is no great chance of observing this phenomenon." Of course, he is correct in that the probability of close alignment of two stars within our galaxy is on the order of 10^{-6} . Thus, the field of gravitational microlensing lay dormant until the publication of Paczynski's paper on the subject[30]. Paczynski recognized that the advent of CCDs and the high speed computing required to analyze their images made it possible to observe a large number of stars simultaneously. He concluded that such a survey would make it possible, and likely, that microlensing could indeed be observed in modern times.

Microlensing was first used to search for Massive Compact Halo Objects (MACHO),which are dark stars in the outer ring of our galaxy[31]. At the time, MACHOs were thought to be a significant contribution to dark matter within our galaxy. Large scale surveys were conducted and thousands of microlensing events have since been observed, although the idea that MACHOs contribute to dark matter has largely been discredited.

Even before the first microlensing events were observed, Mao and Paczynski suggested that microlensing could be used to exoplanets orbiting around lens[32]. A planet has a characteristic effect on the overall amplification of a source star and thus could be detected using similar techniques to that of the MACHO search. Gould and Loeb considered this and developed a "two tier" procedure for detecting planetary microlensing events[33]. First, a single survey monitors a large number of stars in the galactic bulge, searching for the signature amplification due to the primary lens. Second, an alert is put out to a large number of observatories to monitor the event continuously for many days. Since the implementation of this procedure, 44 planets have been detected by microlensing.

The final regime, microlensing, is characterized by very small deviations in the path of light rays. The images formed are similar to that of strong lensing, but are too small to resolve with current generation of telescopes. This doesn't exclude microlensing from being incredibly useful. The microlensing effects that can be measured, namely the amplification of a source star's brightness, is used to determine information about the lens such as mass and distance from the earth. While strong and weak lensing focus primarily on the images formed by gravitational lensing, microlensing focuses exclusively on the amplification of the background source.

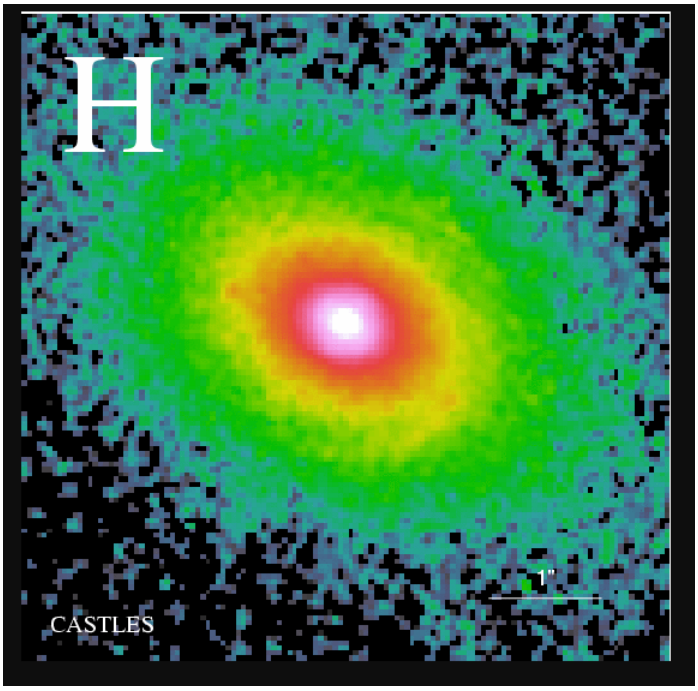

Figure 3.5: Q0047-2808 forming extended Einstein ring (source:castles[1])

3.3 Applications of gravitational lensing

3.3.1 Measure mass and mass distribution

Gravitational light deflection is determined by the gravitational field through which light propagates. This in turn is related to the mass distribution via the Poisson equation (or its general relativity generalization). It is essential to realize that this simple fact implies that gravitational light deflection is independent of the nature of the matter and of its state. Lensing is equally sensitive to dark and luminous matter, and to matter in equilibrium or far out of it. On the negative side, this implies that lensing alone cannot distinguish between these forms of matter, but on the positive side, it also cannot miss one of these matter forms. Hence, lensing is an ideal tool for measuring the total mass of astronomical bodies, dark and luminous.

From the Einstein deflection law (schneider 2006), it is obvious that characteristic image separations scale with the lens mass like$M^{1/2}$. Hence, the observation of multiple images and rings immediately allows an estimate of the mass of the lensing galaxy or more precisely, the mass within a cylinder with a diameter of the image separation or the ring diameter, centered on the lens. More detailed modeling, and additional observables, such as flux ratios, can yield very precise mass estimates. Indeed, accurate mass estimates within galaxies, with an uncertainty of a few percent, have been achieved by far the most precise mass determinations in (extragalactic) astronomy. Similarly, from the locations of giant arcs in clusters, the masses of the central parts of clusters can be determined . With the advent of HST imaging and the discovery of multiple image systems in some strong lensing clusters, detailed mass models have been obtained, which led to very precise mass estimates in those clusters (needless to say, they confirm the dominance of dark matter in clusters).

Weak lensing studies of clusters estimate the mass distribution to much larger radii than the strong lensing regime, and, like strong lensing effects, probe for asymmetries and substructures in the cluster mass. For example, already the strong lensing properties of the cluster A2218 reveals the bimodal nature of the mass distribution. In fact, substructure in the mass distribution of lens galaxies has been detected, thereby confirming one of the robust predictions of the Cold Dark Matter model for our Universe. In addition, the mass distribution of galaxies at large radii, where one runs out of local dynamical tracers, can be studied statistically using an effect called galaxy–galaxy lensing (schneider 2006).

3.3.2 Constraining the number density of mass concentrations

The probability for a lensing event to occur (e.g., the fraction of high-redshift sources that are multiply imaged, or the fraction of stars undergoing micro-lensing) depends on the projected number density of potential lenses. Hence, by investigating statistically well-defined samples of sources and their lensed fraction, we can infer the number density of lenses. Examples of such studies are estimates of the number density of compact objects in the dark halo of our Galaxy, the redshift evolution of the number density of galaxies acting as strong lenses, and the number density of clusters producing strong and weak lensing signals. Upper limits on the number of lensing events can also be translated into upper bounds on the number density of putative lenses: e.g., the fact that nearly all multiply-imaged sources have a visible lens galaxy puts strong upper bounds on the number density of dark lenses (they can at most provide a few percent of the galaxy-mass objects), and the non-detection of lens systems with image separations of tens of milli-arcseconds provides bounds on the number density of compact galaxies with masses $\sim 10^9 M_\odot$. In fact, by now lensing has put stringent constraints on the population of compact massive objects in the Universe over an extremely broad range of mass scales, from $\sim 10^{-3} M_\odot$ (from upper limits on the variability of distant quasars) to $\sim 10^{16} M_\odot$ (from the absence of very wide pairs of quasars), with only a few mass gaps within this range. Even lower-mass objects ($\sim 10^{-6} M_\odot$) can be ruled out as significant contributors to the dark matter in our Milky Way (schneider 2006).

3.3.3 Providing estimates of cosmological parameters

Following Refsdal's idea, the Hubble constant can be obtained from the time delay in multiple image systems. This method has the advantage of being independent of the usual distance ladder used in determinations of H_0 , and it also measures the Hubble constant on a truly cosmic scale, in contrast to the quite local measurements based on Cepheid distances. Despite the determination of time delays in a number of systems, values for H_0 by lensing are burdened with the uncertainties of the lens models. However, there is a trend toward slightly lower values of the Hubble constant than obtained from Cepheids. Other cosmological parameters can

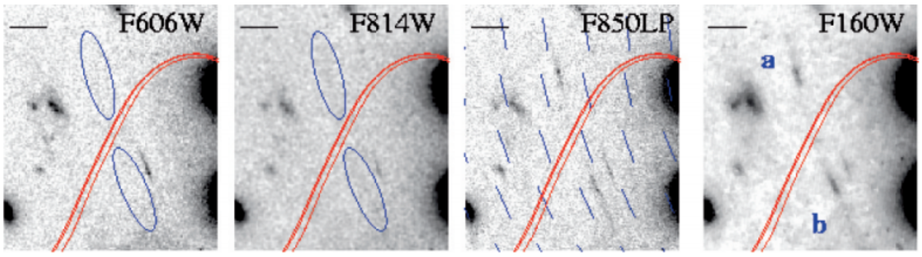

Figure 3.6: Example for the use of a gravitational lens as a natural telescope. In a search for very high redshift objects, deep multi-band HST images are taken near the critical curves of clusters

also be obtained from lensing. For example, the fraction of lensed high-redshift quasars when combined with the distribution of image separations can be used to estimate the cosmological model. Weak lensing by the large-scale structure is sensitive to the matter density parameter and the normalization of the density fluctuations, and significant constraints on these parameters have been obtained. In particular in combination with results from the anisotropy of the cosmic microwave background, future cosmic shear studies will provide an invaluable probe of the equation of state of the dark energy. Weak lensing has also successfully been used to determine the bias parameter, which describes the relation between the statistical distribution of galaxies and the underlying dark matter, and for which only few alternative methods are available (schneider 2006).

3.3.4 Lenses as natural telescopes

Since a lens can magnify background sources, these appear brighter than they would without a lens. This makes it easier to investigate these sources in detail, e.g. through spectroscopic observations. In some cases, this magnification is even essential to detect the sources in the first place, provided their lensed brightness just exceeds the detection threshold of a survey or of the current instrumental sensitivity. This magnification effect has in fact yielded spectacular results, such as very detailed spectra of very distant galaxies, the detection of some of the highest redshift galaxies behind cluster lenses, and the detection of very faint sub-millimeter sources in cluster fields. In fact,the lens magnification can be very large in some rare cases, but these rare cases truly stick out. Some of the most extreme sources, with regards to their apparent luminosity, are strongly magnified such as the spectacular IRAS galaxy F10214.

3.3.5 Searches for planets

The light curves of Galactic microlensing events are affected by companions of the main lens. For example, light curves of binary stars are readily identified as such, provided their separation falls into a favorable range determined by the geometry of the lens system. Because of that, even planets will leave an observable trace in the microlensing light curves if they are situated at the right radius from the star and at the right orbital phase. Although these traces can be quite subtle, and last for a short time only, current observing campaigns aimed at the search for planets have the sensitivity for their detection, and several candidate events for the detection of planetary signals in microlensing light curves have been reported. Indeed, microlensing is considered to be the simplest (and cheapest) possibility to detect the presence of low-mass planets around distant stars.

Chapter 4

Gravitational Lensing in the standard ΛCDM Cosmology

4.1 Introduction

In the past during the 1980s, most research focused on cold dark matter with critical density in matter,around 95%CDM and 5% baryons. These showed success at forming galaxies and clusters of galaxies, but problems remained. Notably, the model required a Hubble constant lower than preferred by observations, and observations around 1988-1990 showed more large-scale galaxy clustering than predicted. These difficulties sharpened with the discovery of CMB anisotropy by COBE in 1992, and several modified CDM models, including ΛCDM and mixed cold and hot dark matter, came under active consideration through the mid-1990s[34, 3].

The ΛCDM model then became the leading model following the observations of accelerating expansion in 1998, and was quickly supported by other observations. In 2000, the BOOMER and microwave background experiment measured the total (matter-energy) density to be close to 100% of critical,whereas in 2001 the 2nd FGRS galaxy redshift survey measured the matter density to be near 25%. The large difference between these values supports a positive Λ or dark energy. Much more precise spacecraft measurements of the microwave background from WMAP in 2003 - 2010 and Planck in 2013 - 2015 have continued to support the model and pin down the parameter values, most of which are now constrained below 1% uncertainty[3, 2].

Nowadays, gravitational lenses are much more than just an interesting general relativistic phenomenon. Now that a significant number of lens systems has been identified, lensing is used more and more as an observation tool, allowing us to answer astrophysical as well as cosmological questions, from estimates of the amount of dark matter contained in the lens mass to the determination of fundamental parameters of the big bang models[3, 4].

The thesis was mainly adopted by Considering GR in the presence of positive cosmological constant to derive gravitational lensing equation and effect of cosmological constant through

vacuum fluid approach with the simple point Mass source model.

All the angular distances also consider the cosmological constant and the expanding universe scenario by way of the transformation between the static and co-moving coordinates. The lensing equation assumes the deflectors (the lens) and the sources positions angular distances through the observed redshifts by Hubble law. Then, the analytically derived Lens equations are being used to generate some numerical values to compare with observation, For the computation Mathematica 11 is used.

4.2 General lens system

The basic setup for a gravitational lens scenario is displayed in figure (4.1).The three ingridients in such a lensing situation are the source S,the lens L,and the observer O. In this scenario light rays emitted by the source are deflected by the lens which will produce two images,S_1 and S_2.

Assuming a spherical-symmetric lens,the underlying spacetime around the lens is well described by the Schwarzchild metric:

$$ds^2 = \left(1 - \frac{2m}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{2m}{r}} - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \tag{4.2.1}$$

4.3 The deflection angle and effect of cosmological constant in gravitational lensing through vacuum fluid approach

4.3.1 Deflection angle in terms of matter and Cosmological constant

In [3],the total deflection angle is given by

$$\frac{\partial n}{\partial r} = \alpha dr \tag{4.3.1}$$

where ,

- n-index of refraction

$$\alpha = -\int \frac{1}{n}\frac{\partial n}{\partial r} dr \tag{4.3.2}$$

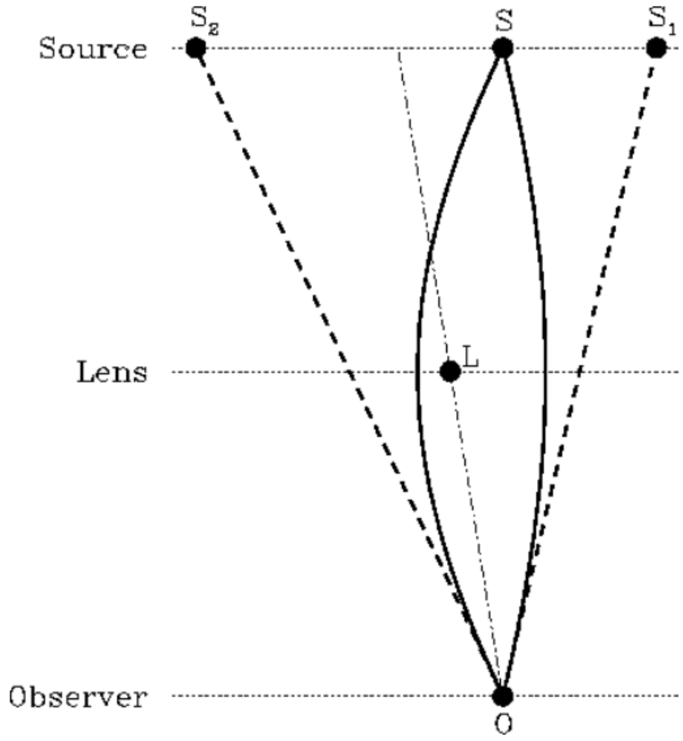

Figure 4.1: Set up of gravitational lens

Now from the above total deflection angle, the deflection angle interms of matter and Cosmological constant Λ for refractive index $\left(n = 1 + \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{3}\right)$ is

$$\begin{aligned} \alpha &= -\int_{-D_{LS}}^{D_L} \nabla \left(\frac{2GM}{c^2 r} - \frac{\Lambda r^2}{3} \right) \\ &= -\int_{-D_{LS}}^{D_L} \frac{1}{r} \left(\frac{2GM}{c^2 r^2} - \frac{2\Lambda r}{3} \right) \\ \alpha &= -\int_{-D_{LS}}^{D_L} \frac{1}{r} (\frac{2GM}{c^2 r^3} - \frac{2\Lambda}{3}) \end{aligned} \tag{4.3.3}$$

$$\alpha(\Lambda, m) = -\int_{-D_{LS}}^{D_L} \left(\frac{2GM}{c^2 r^3} - \frac{2}{3}\Lambda \right) y dx \tag{4.3.4}$$

The integration limit, for the origin of the plane of the lens located at the center of the lens, is all right. But the integration limit in the final result for the correction term is switched only from the source to the plane of the lens.This effect neglects the lensing from the plane of the lens to the observer, and so contradicts the starting assumption of the lensing system being considered with varying refractive index from its center outwards or the reverse.

So with this comment we will have the following improved approximation on the effect of the cosmological constant on lensing.

4.3.2 Deflection angle contribution due to matter

The first integral term of eqn.4.3.4 is represented by (m). Of course it is the deflection angle contribution due to matter.

$$\alpha(m) = -\int_{-D_{LS}}^{D_L} \left(\frac{2GM}{c^2 r^3} \right) y dx \tag{4.3.5}$$

As D_L and D_{LS} get very large it is possible to replace the limit of integration from $-\infty$ to $+\infty$. So

$$\alpha(m) = -\int_{-\infty}^{+\infty} \left(\frac{2GM}{c^2 r^3} \right) y dx \tag{4.3.6}$$

$$\alpha(m) = \frac{2GM}{c^2 y} \int_{-\infty}^{+\infty} \left(\frac{1}{r^3} \right) dr \tag{4.3.7}$$

For our Spacetime is spherical symmetry r has the r,θ and φ components

$$r = \begin{bmatrix} r\sin\theta\cos\theta \\ r\sin\theta\sin\phi \\ r\cos\theta \end{bmatrix}$$

then

$$\frac{\partial r}{\partial r} = \begin{vmatrix} r\sin\theta\cos\theta \\ r\sin\theta\sin\phi \\ r\cos\theta \end{vmatrix} = \left|\tfrac{\partial r}{\partial r}\right| = 1$$

$$\frac{\partial r}{\partial \theta} = \begin{bmatrix} r\cos\theta\cos\phi \\ r\cos\theta\sin\phi \\ -r\sin\theta \end{bmatrix} = \left|\frac{\partial r}{\partial \theta}\right| = r$$

$$\frac{\partial r}{\partial \phi} = \begin{bmatrix} -r\sin\theta\sin\phi \\ r\sin\theta\cos\phi \\ 0 \end{bmatrix} = \left|\frac{\partial r}{\partial \phi}\right| = r\sin\theta$$

The surface element spaning from θ to $\theta + d\theta$ and $\varphi to + d\varphi$ at constant spherical surface r

$$\left|\frac{|\partial r}{\partial \theta}\hat{r} \times \frac{\partial r}{\partial \phi}\hat{r}\right| |d\theta d\phi = r^2 \sin\theta d\theta d\phi$$

Hence, by integrating by part

$$\int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi} \sin\theta d\theta \int_{r=0}^{\infty} \frac{1}{r^3} dr = 4 \tag{4.3.8}$$

then we have

$$\alpha(m) = \frac{2GM}{c^2 y} \int_{-\infty}^{+\infty} \left(\frac{1}{r^3}\right) dr$$

$$\alpha(m) = \frac{2GM}{c^2 y} \left(\int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi} sin\theta d\theta \int_{r=0}^{\infty} \frac{1}{r^3} dr\right)$$

This is easily integrated to give us

$$\alpha(m) = \frac{4GM}{c^2 y} \tag{4.3.9}$$

It seems that the integral depends on y, but the matter contribution is just within its strong field. Hence for effective matter contribution of Einstein photon deflection

$$\alpha(m) = \frac{4GM}{c^2 b} \tag{4.3.10}$$

where b is the closest distance by the photon to the lensing. Of course it is possible to have some additional terms from second to other higher order terms in GM/c^2 , which revives the Robertson - Walker metric expansion form.

4.3.3 Deflection angle due to cosmological Constant contribution

The integral of the cosmological effect part as in equation 4.3.4 is trivially integrated to give us

$$\alpha(\Lambda) = \frac{2}{3}\Lambda y \int_{-D_{LS}}^{D_L} dx = \frac{2}{3}\Lambda y D_S \tag{4.3.11}$$

Here we note that the effect is vacuum dominance and therefore one cannot treat the effect in similar manner as that of the matter. With this understanding the deflection angle

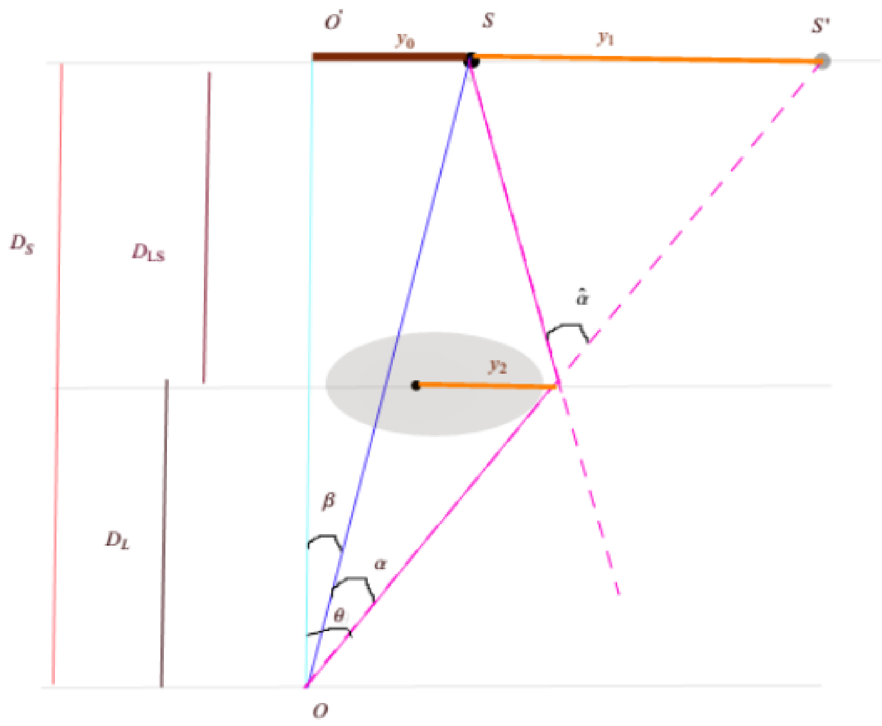

Figure 4.2: A gravitational-lens system of point mass

arising due to cosmology varies with y over the whole space extended along the path of the photon. So in equation 4.3.11 the value of y averaged over all the path length of the photon must be used (fig.4.2). Though, it still needs further analysis (future work), we can reasonably approximate the average value of y as in the following manner;

Let y_1 is the average value of y along the path of photon from the source to the plane of the lens. That means it varies from βD_S to the closest distance b or θD_L . Then for order of cosmological distances it is quite reasonable to average it as,

$$y_1 = \frac{1}{2}\left(y_0 + y_I\right) \tag{4.3.12}$$

$$\tan\beta = \frac{y_0}{D_s}$$

$$y_0 = \beta y D_S$$

$$\tan\theta = \frac{y_I}{D_L}$$

$$y_I = \theta D_L$$

Since D_{LS} and D_L are nearly the same order of magnitudes we can use D_L as D_{LS} and for very small angle $\tan\beta \approx \beta, \tan\theta \approx \theta$. Then for y_1 is the average of y_0 and y_I

$$y_1 = \frac{1}{2}(\theta D_L + \beta D_s) \tag{4.3.13}$$

In a similar way we define y_2 as the average of y over the path of the photon travel from the plane of the lens to the observer given by

$$y_2 = \frac{1}{2}\theta D_L \tag{4.3.14}$$

Since D_L and D_{LS} are nearly the same order of magnitudes, we can once again reasonably average y over y_1 and y_2 to obtain

$$y_{av} = \frac{1}{2}(\theta D_L + \frac{1}{2}\beta D_S) \tag{4.3.15}$$

So, the contribution of cosmological constant to the deflection of light in the vicinity of eqs. 4.3.11, 4.3.15 is given by

$$\begin{aligned} \alpha(\Lambda) &= \int_{-D_{LS}}^{D_L} -\frac{2}{3}\Lambda y dx \\ &= -\frac{2}{3}\Lambda y \int_{-D_L S}^{D_L} dx \\ &= -\frac{2}{3}\Lambda y x|_{-D_{LS}}^{D_L} \\ &= \frac{2}{3}\Lambda y(D_L - (-D_{LS})) \\ \alpha(\Lambda) &= -\frac{2}{3}\Lambda y D_S \end{aligned} \tag{4.3.16}$$

From the curved space-time background of vacuum fluid source the angular distance is not additive i.e;

$$D_S \neq D_L + D_{LS} \tag{4.3.17}$$

From the curved space-time background of vacuum fluid source the angular distance is not additive instead we use the distance in terms of redshift as follows[3]

$$D_S(1+z_s) = D_L(1+z_L) + D_{LS}(1+z_s) \tag{4.3.18}$$

$$\alpha(\Lambda) = -\frac{2}{3}\Lambda(\frac{1}{2}(\theta D_L + \frac{1}{2}\beta D_S)) \tag{4.3.19}$$

$$\alpha(\Lambda) = -\frac{2}{3}\Lambda \times \frac{1}{2}(\theta D_L + \frac{1}{2}\beta D_S)D_S \tag{4.3.20}$$

$$\alpha(\Lambda) = -\frac{1}{3}\Lambda\left(\theta D_L + \frac{1}{2}\beta D_S\right)D_S \tag{4.3.21}$$

by using eqs. 4.3.13 ,4.3.21 the angle of deflection for lensing through vacuum fluid approach is given by

$$\alpha(\Lambda, m) = \frac{4GM}{c^2 b} - \frac{1}{3}\Lambda\left(\theta D_L + \frac{1}{2}\beta D_S\right)yD_s \tag{4.3.22}$$

Or

$$\alpha(\Lambda, m) = \frac{4GM}{c^2 D_L \theta} - \frac{1}{3}\Lambda(\beta D_s)d_s \tag{4.3.23}$$

Where,$b = \theta D_L$

4.3.4 The Lens Equation

Assuming spherical spacetime, From fig.4.3 below let

$$\widehat{O'S} = \beta D_s$$

$$\widehat{O'S'} = \theta y D_S$$

$$\widehat{SS'} = \alpha D_s$$

$$\widehat{SS'} = \hat{a} D_{LS}$$

Then we have

$$\alpha D_s = \hat{a} D_{LS}$$

$$\hat{a} = \frac{aD_s}{D_{LS}}$$

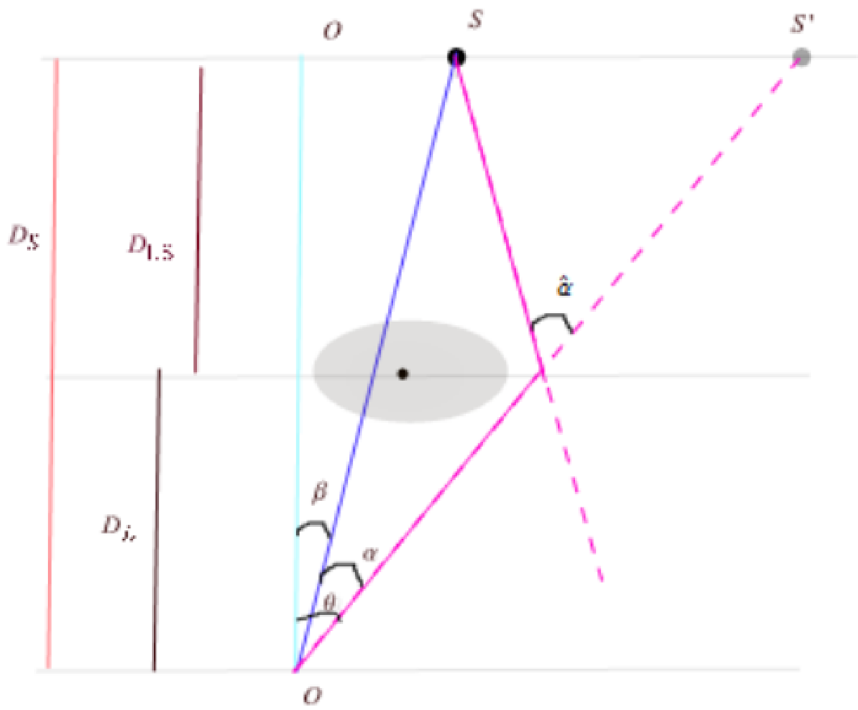

Figure 4.3: A gravitational-lens system of point mass: The optical axis runs from the observer O through the centre of the lens to O'. The angle between the source S and the optical axis O' is β, the angle between the image S' and the optical axis O' is θ. The light ray towards the image is bent by the deflection angle $\hat{a}$, measured at the lens. The reduced deflection angle α is measured at the observer

1

Then now the lens equation measured at observer is derived as follows

$$\widehat{O'S'} = \widehat{O'S} + \widehat{SS'} \tag{4.3.24}$$

$$\widehat{O'S'} = \beta D_s + \hat{a} D_{LS} \tag{4.3.25}$$

$$\widehat{O'S'} = \theta D_s = \beta D_s + \left(\frac{\alpha D_s}{D_{LS}}\right) D_{LS} \tag{4.3.26}$$

$$\vec{\theta} = \vec{\beta} + \vec{\alpha} \tag{4.3.27}$$

Let's write in terms of θ

$$\vec{a}(\theta) = \vec{\theta} - \vec{\beta}(\theta) \tag{4.3.28}$$

And also

$$\widehat{O'S} = \widehat{O'S'} - \widehat{SS'} \tag{4.3.29}$$

$$\beta D_s = \theta D_s - \hat{a} D_{LS} \tag{4.3.30}$$

Then we have the lens equation as follows

$$\beta = \theta - \hat{a}\frac{D_{LS}}{D_S} \tag{4.3.31}$$

For small angles and with the angle expressed in radians, the point of nearest approach y at an angle α for the lens L on a distance D_L is given by $y = \theta D_L$, For a source right behind the lens, $\theta D_L = 0$, and the lens equation for a point mass gives a characteristic value for θ that is called the Einstein radius, denoted θ_E . Putting $\beta D_S = 0$ and solving for θ gives the Einstein radius for a point mass provides a convenient linear scale to make dimensionless lensing variables. The Einstein radius most prominent for a lens typically halfway between the source and the observer.

Then substituting eq. 4.3.23 in the lens equation 4.3.31 we get lens equation with the effect of Cosmological constant;

$$\beta = \theta - \frac{D_{LS}}{D_S}\left(\frac{4GM}{c^2 D_L \theta} - \frac{1}{3}\Lambda\left(\theta D_L + \frac{1}{2}\beta D_S\right) D_S\right) \tag{4.3.32}$$

This is fundamental lens equation in the presence of cosmological constant.

Alignment

When the source is exactly behind the lens, the angular position of the source(S) and the Optical sight(O) becomes Aligned;i.e $\beta = 0$ then

$$\beta = \theta - \frac{D_{LS}}{D_S}\left(\frac{4GM}{c^2 \theta D_L} - \frac{1}{3}\Lambda\left(\theta D_L + \frac{1}{2}\beta D_S\right) D_S\right) \tag{4.3.33}$$

$$0 = \theta - \frac{D_{LS}}{D_S}\left(\frac{4GM D_S}{c^2 \theta D_L} - \frac{1}{3}\Lambda \theta D_L D_S - 0\right) \tag{4.3.34}$$

$$\theta^2(1 + \frac{1}{3}\Lambda D_L D_{LS}) = \theta_E^2 \quad (4.3.35)$$

Now by representing the Einsteins ring radius $\theta_{E\Lambda}^2$ with cosmological correction in Schwarzschild de Sitter metric in terms of the purely Schwarzschild metric θ_E as

$$\theta_{E\Lambda}^2 = \frac{\theta_E^2}{1 + F_\Lambda} \quad (4.3.36)$$

Where θ_E is given by

$$\theta_E^2 = \frac{4GM}{C^2}\frac{D_{LS}}{D_L D_S} \quad (4.3.37)$$

And F_Λ is the correction factor to Einstein ring radius due to cosmological constant given by

$$F_\Lambda = \frac{1}{3}\Lambda D_L D_{LS} \quad (4.3.38)$$

In conclusion we observe that the Einstein radius is affected by the factor

$$\frac{1}{1 + F_\Lambda} \quad (4.3.39)$$

This implies that the deflection of light due to the presence of cosmological constant decreases.

4.4 Image magnification and distortion

4.4.1 Image magnification

The ratio between the angular area of the image in the observer sky and the angular area of the source in absence of lensing gives the (signed) amplification of the image,

$$\mu = \frac{\sin\theta}{\sin\beta}\frac{d\theta}{d\beta} \quad (4.4.1)$$

The magnification of the apparent luminosity is given by correcting such a geometrical amplification for the standard redshift factor. The derivative in Eq. (4.4.1) can be computed through the chain rule by deriving the coordinate position of the source φ with respect to either β or θ and then combining the results suitably. After introducing the scaled angular variables, the result can be rearranged as a series in ε,

$$\mu = \mu_0 + \mu_{1\varepsilon} + \mu_{2\varepsilon^2} + O(\varepsilon^3) \quad (4.4.2)$$

The first coefficients of the above expansion series are like pure Schwarzschild lensing,

$$\mu_0 = \frac{\theta_0^4}{\theta_0^4 - 1}, \quad (4.4.3)$$

and

$$\mu_1 = -\frac{15\pi\theta_0^3}{16\left(\theta_0^2+1\right)^3} \tag{4.4.4}$$

The Λ correction shows up at the next order,

$$\mu_2 = \frac{8\theta_0^2}{\left(1-\theta_0^2\right)\left(1+\theta_0^2\right)^3}\left[\theta_0^4\left(4+2\theta_0^2-\frac{675\pi^2}{1024\left(1+\theta_0^2\right)^2}\right.\right.$$
$$\left.\left.+D\theta_0^2(9-10\theta_0^2-5\theta_0^4\right)-\frac{D^2}{3}\left(1+16\theta_0^2-23\theta_0^4-12\theta_0^6\right)+\frac{\theta_0^2}{4r_{\Lambda\varepsilon}^2}\right] \tag{4.4.5}$$

Let us consider the microlening case when the two images can not be resolved and the observable is the total magnification $\mu_{tot} = |\mu_+| + |\mu_-|$. Using the above results,μ_{tot} can be written in terms of the unlensed source position as

$$\mu_{tot} \simeq \frac{\beta^2+2}{\beta\sqrt{\beta^2+4}} - \frac{15\pi\varepsilon}{8\left(\beta^2+4\right)^{3/2}} - \frac{4\varepsilon^2}{\beta\left(\beta^2+4\right)^{3/2}} \times \left[\frac{1}{r_{\Lambda\varepsilon}^2} + 4\left(6+6\beta^2+\beta^4\right)\right.$$
$$\left. - \frac{675\pi^2}{256\left(\beta^2+4\right)} - 2D\left(12+30\beta^2+5\beta^4\right) + \frac{4D^2}{3}\left(18+35\beta^2+6\beta^4\right)\right. \tag{4.4.6}$$

The contribution of Λ to the total magnification is negative so that images are slightly de-amplified. The cosmological constant is isotropic and does not perturb the spherical symmetry of the lens. The caustic surface is still a line coincident with the optical axis behind the lens. The tangential critical circle corresponding to the point-like caustics is a perturbed Einstein ring with angular radius

$$\theta_t \simeq \theta_E\left[1+\frac{15\pi}{32}\varepsilon+\left(4-\frac{4D^2}{3}-\frac{675\pi^2}{2048}-\frac{1}{2r_{\Lambda\varepsilon}^2}\right)\varepsilon^2\right] \tag{4.4.7}$$

Due to Λ the area of the Einstein ring slightly decreases.

4.4.2 Image distortion

One of the main features of gravitational lensing is the distortion which it introduces into the shape of the sources. This is particularly evident when the source has no negligible apparent size. For example, background galaxies can appear as very long arcs in galaxy clusters.

The distortion arises because light bundles are deflected differentially. Ideally the shape of the images can be determined by solving the lens equation for all the points within the extended source. In particular, if the source is much smaller than the angular size on which the physical properties of the lens change.

4.5 Angular distance measures in gravitational lensing

In cosmology (or to be more specific, cosmography, the measurement of the Universe) there are many ways to specify the distance between two points, because in the expanding Universe, the distances between comoving objects are constantly changing, and Earth-bound observers look back in time as they look out in distance. The unifying aspect is that all distance measures somehow measure the separation between events on radial null trajectories, ie,trajectories of photons which terminate at the observer.

4.5.1 Cosmographic parameters

The Hubble constant (H $_0$)

The Hubble constant is the constant of proportionality between recession speed v and distance d in the expanding Universe;

$$v = H_0 d \tag{4.5.1}$$

The subscripted "0" refers to the present epoch because in general H changes with time. The dimensions of H_0 are inverse time, but it is usually written

$$H_0 = 100hkms^{-1}Mpc^{-1} \tag{4.5.2}$$

where h is a dimensionless number parameterizing our ignorance. (Word on the street is that $0.6 < h < 0.9$.) The inverse of the Hubble constant is the Hubble time t_H

$$t_H \equiv \frac{1}{H_0} = 9.79h^{-1}yr = 3.09 \times 10^{17}h^-1s \tag{4.5.3}$$

and the speed of light c times the Hubble time is the Hubble distance D_H

$$D_H \equiv \frac{c}{H_0} = \frac{c}{H_0(1+z)} \tag{4.5.4}$$

where,Z is the redshift,for gravitational lensing at source Z_S,at LensZ_L These quantities set the scale of the Universe, and often cosmologists work in geometric units with $c = t_H = D_H = 1$.

The mass density ρ of the Universe and the value of the cosmological constant Λ

These are dynamical properties of the Universe, affecting the time evolution of the metric, but in these we will treat them as purely kinematic parameters. They can be made into dimensionless density parameters Ω_M and Ω_Λ by

$$\Omega_M \equiv \frac{8\pi G\rho_0}{3H_0^2} \tag{4.5.5}$$

$$\Omega_\Lambda \equiv \frac{\Lambda c^2}{3H_0^2} \tag{4.5.6}$$

where the subscripted "0"s indicate that the quantities (which in general evolve with time) are to be evaluated at the present epoch. A third density parameter Ω_κ measures the "curvature of space" and can be defined by the relation

$$\Omega_M + \Omega_\Lambda + \Omega_\kappa = 1 \tag{4.5.7}$$

Assuming the observed Flatness $\Omega_\kappa = 0, \Omega_M = 0.3, \Omega_\Lambda = 0.7$

4.5.2 Comoving distance (line-of-sight)

A small comoving distance δD_C between two nearby objects in the Universe is the distance between them which remains constant with epoch if the two objects are moving with the Hubble flow. In other words, it is the distance between them which would be measured with rulers at the time they are being observed (the proper distance) divided by the ratio of the scale factor of the Universe then to now; it is the proper distance multiplied by $(1+z)$. The total line-of-sight comoving distance D_C from us to a distant object is computed by integrating the infinitesimal δD_C contributions between nearby events along the radial ray from $z = 0$ to the object.As defined in[35],

$$E(z) \equiv \sqrt{\Omega_M(1+z)^3 + \Omega_\kappa(1+z)^2 + \Omega_\Lambda} \tag{4.5.8}$$

which is proportional to the time derivative of the logarithm of the scale factor $(ie, a(t)/a(t))$, $\dot{}$with$z$ redshift and Ω_M , Ω_κ and Ω_Λ the three density parameters defined above. (For this reason, $H(z) = H_0E(z)$ is the Hubble constant as measured by a hypothetical astronomer working at redshift z.) Since $dz = da, dz/E(z)$ is proportional to the time-of-flight of a photon traveling across the redshift interval dz, divided by the scale factor at that time. Since the speed of light is constant, this is a proper distance divided by the scale factor, which is the definition of a comoving distance[35]. The total line-of-sight comoving distance is then given by integrating these contributions, or

$$D_c = D_H \int_0^z \frac{dz'}{E(z')} \tag{4.5.9}$$

where D_H is the Hubble distance as shown in equation 4.5.4

In some sense the line-of-sight comoving distance is the fundamental distance measure in cosmography since, as will be seen below, all others are quite simply derived in terms of it [35]. The line-of-sight comoving distance between two nearby events (ie, close in redshift or distance) is the distance which we would measure locally between the events today if those two points were locked into the Hubble flow. It is the correct distance measure for measuring aspects of large-scale structure imprinted on the Hubble flow, eg, distances between "walls,

4.5.3 Angular diameter distance

The angular diameter distance D_A is defined as the ratio of an object's physical transverse size to its angular size (in radians). It is used to convert angular separations in telescope images into proper separations at the source. It is famous for not increasing indefinitely as $z \to \infty$; it turns over at $z \sim 1$ and thereafter more distant objects actually appear larger

$$D_A = \frac{D_M}{1+z} \tag{4.5.10}$$

There is also an angular diameter distance D_A between two objects at redshifts source and observer at redshifts Z_S and the observer and lens at Z_L are frequently used in gravitational lensing[35]. It is not found by subtracting the two individual angular diameter distances.

Then by using equation 4.5.9,

The angular distance from observer to lens $D_L$is

$$D_L = \frac{c}{H_0(1+z_L)} \int_o^{z_L} \frac{dz'}{E(z')} \tag{4.5.11}$$

The angular distance from observer to source D_S is

$$D_S = \frac{c}{H_0(1+z_S)} \int_o^{z_S} \frac{dz'}{E(z')} \tag{4.5.12}$$

And the angular distance from Lens to source D_{LS} is

$$D_{LS} = \frac{c}{H_0(1+z_S)} \int_{Z_L}^{Z_S} \frac{dz'}{E(z')} \tag{4.5.13}$$

Where,

$$D_H = \frac{c}{H_0(1+Z)} = \text{the Hubble distance}$$

$$c = \text{speed of light}$$

$$H_0 = \text{Hubble constant}$$

Chapter 5

Result and discussion

5.1 Lens equation in the presence of cosmological constant

Data source;www.cfa.harvard.edu/castles
By using the lens equation with cosmological constant(i.e,eq.4.3.32) and by considering the point mass assumption we will see the effect of cosmological constant (Λ).The data selection is based on the images that form einstein ring.

5.2 Data analysis for the four(4)Einstein's ring extracted from observation

Then using equations (4.5.11,4.5.12,4.5.13) we can get the correction part as

$$F_\Lambda = \frac{\Omega_\Lambda}{3} \frac{1}{(1+Z_S)(1+Z_L)} \int_0^{Z_L} \frac{dz'}{E(z')} \int_{Z_L}^{Z_S} \frac{dz'}{E(z')} \tag{5.2.1}$$

Additionally,the Einstein ring is determined by the velocity dispersion given by

$$\theta_E = \frac{4\pi\sigma^2}{c^2} \frac{D_{LS}}{D_S} \tag{5.2.2}$$

5.3 Calculating angular distances

By using mathemtica-11,we calculated the angular distances in giga parsec(Gpc) for the four Einstein rings extracted from the data. To calculate the following tables we used the following constants

$$c = 3.0 \times 10^8 m/s$$

$$\Omega_\Lambda = 0.7$$

$$\Omega_m = 0.3$$
$$\Lambda = 1 \times 10^{-52} m^{-2}$$
$$H_0 = 71 \times 10^3 m/s/Mpc$$

Data of selected ring of the lens system and the angular distances

No.	Lens System	Z_S	Z_L	$D_S(Gpc)$	$D_L(Gpc))$	$D_{LS}(Gpc)$
1	Q0047-2808	3.60	0.48	1.47469	1.2151	1.08375
2	HST15433+5332	2.092	0.497	1.6943	1.23822	1.09481
3	MG1549+3047	1.17	0.11	1.68121	0.40815	1.47243
4	CFRS03+1077	2.941	0.938	1.57676	1.60374	0.788119

Table 5.1: The angular distances in giga parsec (source:castles[1])

Data of selected ring of the lens system

No.	Lens System	Z_S	Z_L	$\sigma(m/s)$	F_Λ	$\theta_E(arcsec)$	$\theta_{E\Lambda}(arcsec)$	%
1	$Q0047-2808$	3.6	0.48	229000	0.0418035	1.10991	1.08742	2.02684
2	$HST1543+5332$	2.092	0.497	108000	0.0430336	0.217064	0.212539	2.08463
3	$MG1549+3047$	1.17	0.11	227000	0.0190776	1.29974	1.28752	0.940447
4	$CFRS03+1077$	2.941	0.938	256000	0.0401232	0.943405	0.92503	1.94774

Table 5.2: The image position in arcsec (source:castles[1])

For this table we considered a flat cosmological model defined by the parameters $\Omega_\Lambda = 0.7, \Omega_M = 0.3$ and $H_0 = 71 \times 10^3 m/s/Mpc$. From this table 5.2,the effect of Λ on image position is 2%. This result matches with previous works of others[3, 4].

5.4 Image position and magnifications

In the symmetric case by using lens equation (4.3.36 and 4.3.37 the image magnification can be written as

$$\mu = \left(1 - \left[\frac{\theta_E}{\theta_{E\Lambda}}\right]^4\right)^{-1} \tag{5.4.1}$$

Where,μ -is image magnification
θ_E-is the Einstein radius
The magnification of one image (the one inside the Einstein radius) is negative. For $\beta \to 0$ the magnification diverges. In the limit of geometrical optics the Einstein ring of a point source has infinite magnification .

5.5 Contribution of cosmological constant to image magnification

After this by using the tables (table 5.1 and 5.2) lets investigate the effect of the cosmological constant for the image magnification(μ). Now by applying eq.(5.4.1) as
The contribution of Λ to the total magnification is negative so that the images are slightly de-

Data of selected ring of the lens system to calculate the magnification

No.	Lens System	θ_E	$\theta_{E\Lambda}$	μ
1	$Q0047-2808$	1.10991	1.08742	-11.7192
2	$HST1543+5332$	0.217064	0.212539	-11.3741
3	$MG1549+3047$	1.29974	1.28752	-0.3842
4	$CFRS03+1077$	0.943405	0.92502	-12.2166

Table 5.3: The calculated image magnification (source:castles[1])

amplified.

Chapter 6

Summary and Conclusion

- By using simple point mass models for the Lens, we have derived the lens equation in the presence of cosmological constant, i.e
- $\beta = \theta - \frac{D_{LS}}{D_S}\left(\frac{4GM}{c^2 D_L \theta} - \frac{1}{3}\Lambda\left(\theta D_L + \frac{1}{2}\beta D_S\right) D_S\right)$
- $\frac{1}{3}\Lambda D_{LS} D_L$ is the cosmological constant contribution to the deflection of light and it is known as the correction factor, and it Contributes about 2% in my result.
- The contribution of Λ is completely involved in the form of the angular diameter distance D_A .
- In the symmetric case as we calculated in table 5.3, the magnification of the image is negative.
- This implies that the images are de-magnified.
- The Λ correction for the magnification of the image does not exist for the zeroth and first order, but it shows up on the second order.
- Generally,the angular distances, image sizes and image positions are affected by the cosmological constant.

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