JIMMA UNIVERSITY

JIMMA INSTITUTE OF TECHNOLOGY

SCHOOL OF GRADUATE STUDIES

FACULTY OF ELECTRICAL AND COMPUTER ENGINEERING

Modeling and Performance analysis of FOPID controller for Interacting Coupled Tank System

A Thesis Submitted to Jimma Institute of Technology, School of Graduate Studies, Jimma University

Thesis on the partial fulfillment of the requirements for the degree of Master of Science in Electrical and Computer Engineering (Control and Instrumentation Engineering)

By:

Muluken Teka

December, 2021

Jimma, Ethiopia

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APPROVAL LETTER

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Approved by Board of Examiners

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DECLARATION

I, the undersigned, declare that this thesis is my original work, and has not been presented for a degree in this or other universities, and all sources of materials used for this thesis work have been fully acknowledged.

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This thesis has been submitted for examination with my approval as a university advisor

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LIST OF SYMBOLS

λ……………………………….Fractional order lambda

μ……………………………….Fractional order miu

P……………………………….Proportional

I……………………………….Integral

D……………………………….Derivative

k_p……………………………...Proportional gain

k_i……………………………...Integral gain

k_d……………………….........Derivative gain

k………………………… …...A gain of second-order plant

e(t)……………………………..Error signal

r(t)……………………………….Input

G(s)……………………………...Controller transfer function(s-domain)

$G_p(s)$……………………………The transfer function of the plant

S……………………………….Shaped or oscillatory open-loop dynamics

m……………………………….Relative dominance

$G_a(j\omega)$…………………………...The transfer function of the frequency domain

T_p……………………………...Peak time

T_r……………………………...Rise time

T_s……………………………...Settling time

ξ………………………………..Damping ratio of the second-order plant

ω_n ……………………………The natural frequency of the second-order plant

α ……………………………….Relative dominance

$Q_{in}(t)$……………………………The flow of liquid into tanks

$Q_{out}(t)$……………………………..Flow of liquid

Im………………………………...Imaginary

Re………………………………..Real

Q………………………………...Weighting matrix

R…………………………………Scalar quantity

K………………………………...State feedback gain

A……………………………….. (nxn) square state matrix

B………………………………...(nxr) input matrix

C……………………………….. (mxr) direct feedback matrix

P…………………………………Riccati matrix

h(t)………………………………Height of the tank

A…………………………………Area of tank

$\frac{dx}{dt}$...The derivative of x concerning the time

R………………………………….Resistance flow

cm………………………………..Centimeter

ω_b…………………………………Low frequency

ω_h…………………………………High transition frequency

T_s …………………………………Sampling time

I…………………………………..Identity matrix

LIST OF ABBREVIATION

ARE……………………………………...Algebraic Riccati equation

A/D………………………………………Analog to digital conversion

BIBO……………………………………Bounded input bounded output

CTS………………………………………Coupled tank system

CV…………………………………..........Control variable

CRONE………………………………..…Commande Robuste d'ordere Non Entire

CL………………………………………..Closed loop

DE……………………………………….Differential equations

DPID……………………………………..Digital proportional-integral-derivative

DSP………………………………………Digital signal processing

D/A………………………………………Digital to analog conversion

FOPDT…………………………...……First-order plus dead time

FOTF…………………………............Fractional-order transfer function

FOC……………………………………...Fractional order controller

FOMCON…………………………… …. Fractional-order modeling control

FOPID…………………………………...Fractional order proportional integral derivative

FODE……………………………………Fractional order differential equation

FOTF……………………………………Fractional order transfer function

IFOTD…………………………………...Investigation of fractional order time domain

IOPID………………………………….…...Integer order proportional integral derivative

LTI………………………………………Linear time-invariant

MV………………………………………Manipulated variable

MATLAB………………………………...Matrix laboratory

MIMO……………………………………..Mult input multi-output

ODE……………………………………...Ordinary differential equation

PI………………………………………..Proportional plus integral

PD…………………………………….....Proportional plus derivative

PV……………………………………….. Process variable

PID………………………………………Proportional plus integral plus derivative

SP……………………………………….....Set-point

SISO…………………………………….Single input single output

TID……………………………………… Tilt-Integral Derivative

TF……………………………………….Transfer function

ZOH………………………………….……...Zero order hold

ABSTRACT

Process control is essential in the industrial process because it guarantees the safety and optimization in a process. Additionally, process control is a useful tool to satisfy the environmental procedure and product quality necessities. In industries, one of the controlling process variables are liquid level, the liquid level controllers are a significant concern popular process and collective illustrative also real-world in engineering methods. Liquid level coupled tank system can be arranged into two fashionable forms of interacting and non- interacting form. In this thesis work only focus on interacting coupled tank control systems, there are many problems which affected the liquid level like nonlinearity of the system, modeling uncertainties and complex analysis, so to overcome those problems, and obtain constant stable output and fast response various controllers are required.

Industrial control engineering requires fluid moved and reserved in a container. This thesis work presents the performance analysis of fractional order-PID controller for governing a liquid level of the tank system. Also for more validation of this work test other method of controllers. Various types of fractional-order (FOPID, TID, CRONE, and FOLL) controller and from those fractional order controller techniques FOPID and TID control will be tested to get compare the performance of interacting coupled tank system.

The output response is conducted within MATLAB®/Simulink® situation to verify the performances of the system in terms of rising Time (T_r), settling time (T_s), delay time (T_d), peak time (T_p), undershoot and Overshoot (Os). The simulation result shows that controlling interacting CTS without disturbance the response is good, but including external disturbance on the second tank the controllers tested on this work is show week response except FOPID controller. The reason is FOPID controller has two more adjust parameter those increase the robustness of the system. From the controllers tested in this thesis work the fractional order proportional integral derivative controller (FOPID) has good performance compare to PID, TID, and digital-PID controllers that tested on this thesis. The performance specification of FOPID controller O_s is 14.368*%*, T_p is 1.014*sec*, T_d is 0.338*sec*, T_s is 3.581*sec* and T_r is 336.485*msec*.

Keywords: - Process control, Level process, coupled tank, PID, FOPID, and Digital-PID

CHAPTER ONE

INTRODUCTION

1.1. Background

Many industrial and scientific processing needs an understanding of the amount of the contents in the tank and other containers. In many cases, it is not conceivable or not real-world to directly view the inner. The more understandable industrial application includes tank level gauging of milk, beer, wine in the industry, and level gauging of acid, oil, water treatment, and solvent vessels in chemical plants; level checking of liquid in reservoirs. All of these industries use coupled tank system. The arrangement of coupled tank system is two fashionable forms those are interacting and non-interacting. This research focus only on interacting coupled tank arrangement.

Process control is essential in the industrial process because it guarantees the safety and optimization in a process. Additionally, process control is a useful tool to satisfy the environmental procedure and product quality necessities. For chemical engineers, process control is widely appropriate in the industrial process. In many processes, such as petrochemical, paper, and water treatment engineering is using the container to govern the level of the fluid. An equitable of the controller in the level control is to maintain at a given level of the set-point and to receive a recent set-point. The proceeding industry necessitates liquid prospect inflated, deposited containers, then inflate to an alternative reservoir, the fluids handled via fraternization behavior fashionable a container, nevertheless continually the level of fluid a reservoir essential organized, besides the movement a fluid commitment remain controlled. Height and outflow governor chamber is by the side of core industries' environments. Interacting coupled tank regulation of system manufacturing is an inspiring assignment for frequent sources outstanding to non-linearity. Controls the fluid level in the coupled container are the most important concern in the system. The level of the fluid is too high for various disappointed response stability, cause destruction. When the liquid height is low, it might require unstable significances for the system. The industrial process presents many challenging control problems due to their non-linear dynamics behavior. Nonlinear models are used where accuracy over a wider range of operations is required where they can be directly incorporated into the controller algorithm. Because of the characteristic nonlinearity, most of the chemical process industries require control techniques, those to control such systems used fractional order controls. The nonlinear system taken up for the study is the coupled tanks. The engineering application of liquid level control is tremendous, specifically in chemical process industries.

1.2. Type of industrial tank systems

To analysis the mathematical model of the coupled tank systems first you have to know what types of tank systems are used. Related to tank type the mathematical model also differs. There are many variation of level process which contains: - spherical tank, cylindrical tank, canonical tank, and rectangular tank which show in Figure 1.1. Usually controlled based on the error signal and have a least upstream or downstream the control valve. When the inlet flow q_i and the outflow q_o of the system has controlling the level of the tank. From those tank in this work focus on rectangular coupled tank system;

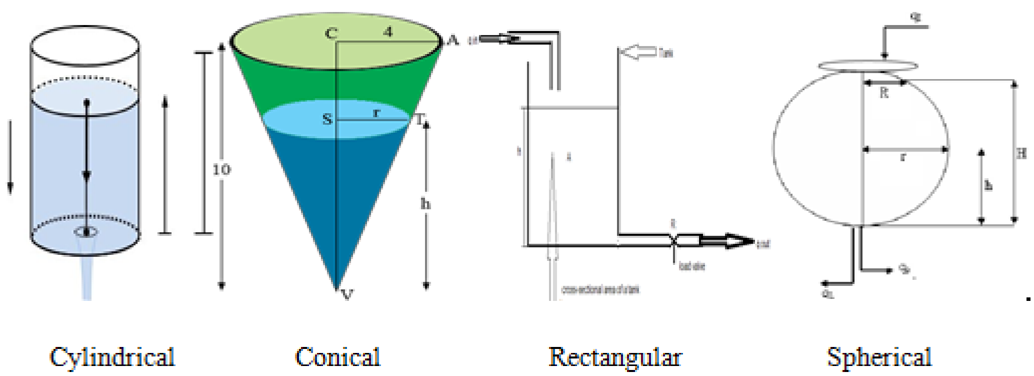

Figure 1.1:- Type of tank systems [1]

In earlier industry, people controlled the liquid level done a fixed liquid height switch. When the liquid is up to a convinced level, the switch is mechanically closed or open to control the level. With the continuous progress of mechanization, continuous control of the liquid level, that is, perceiving the fluid level at all a time is required in the industry.

Liquid level is one of the vital variables used in industries like power plants, chemical reaction control, petrochemical process, food processing plant, paint manufacturing plant, and waste water treatment tanks. Most of these world industries widely use PID controller for regulating the level process of the tank and managing industrial processes, in over 90% cases, industrial

PID controllers are used. PID controller has been used successfully for regulating processes in industry for more than 70 years and they are often referred to as standard controllers [2]. It is a simple and effective control method and it can be easily implemented for industrial control applications in linear systems control. But it is challengeable to use for nonlinear systems and do not get the desired response, on the other hand the simplicity of these controllers can also represent their weakness, because there are controlled objects (objects with significant nonlinearity), which require the application of complex control laws in order to achieve satisfactory performance [`2].

The modern digital control systems require more and more strong and fastest calculation components. Today, digital controllers are being used in many large and small-scale control systems, replacing the analog controllers. It is now a common practice to implement PID controllers in its digital version, i.e. they operate in discrete time domain and deal with analog signals quantized in a limited number of levels. Moreover, in such controller we do not need much space and they are not expensive. A lower bound for the sample period is the computing time of a whole cycle of the digital-PID. However, the digital-PID control algorithm is not advisable for the complex and nonlinear system, one of the ways to improve the product and also for the effectiveness of traditional PID controllers is to use fractional order controllers with non-integer derivation and integration parts [3].

Controlling of level process regulator is the main problem in a process variable along with a mutual demonstrative and concrete in engineering practices. The preceding manufacturing, human-contained the fluid level terminated a permanent fluid level adjustment while a liquefied corresponding with a convinced level; now this thesis work paper, the success of fractional-order PID controller to control the process level concerning the interacting coupled tank is analyzed. The multitude of industries processes such as petrochemical, water treatment, and beer is with a container to governor the height. Level of the fluid is necessarily governed within appropriate controllers. A key purpose of a controller in the systems keeps a height at the chosen set point and adaptable to the new set-point [4].

The IOPID controls are normally recycled in controlling the height of the container in industries since governing the liquid in many tanks and undertaking between the tanks is a straightforward complication in the industries. Proportional controllers are appropriate to lead industrial controllers and for that reason are issues of steady exertion for the progress of their performance and robustness. One of the prospects to increase proportional integral derivative controls are to practice $PI^{\lambda}D^{\mu}$ among fractional-order integer portions. The FO controllers are derived from the integer order by adding the fractional powers in integral and derivative terms. It permits us to adjust the derivative Miu (μ) and integral lambda (λ) order in addition to the k_p, k_i and k_d constants where the values of μ and λ invention between zero and one [4]. The fractional calculus is getting much more attention for such kind of situation in the field of control system engineering due to its potential and significant importance. The controllers, making use of fractional-order derivatives and integrals, give improved results compared to the classical controller IOPID in terms of robustness. Fractional-order (FO) controllers are usually expressed by fractional-order differential equations [5].

The process industries needed liquid to stand pumped and stored fashionable the tanks and formerly pump it to another tank. Repeatedly the level of the tank must be controlled. In furthermost industries, chemical processes present many stimulating outstanding to their non-linear dynamics behavior. Various industrial processes are non-linear in nature and designing the controller for such type of process is very tedious [6]. From the time when the non-linearity, the controller develops a challenging task to attain reasonable performance using different shapes of tanks.

The primary task of the controls is to retain a scheme at a set point alternatively, from a FOC, FOPID controller which is widely applied in feedback control of the industrial system. These controllers are defined with their simple construction and method. FOPID controllers are also enabled to provide expected performance for systems [7]. This thesis work is intended to analyze the achievement of the FOPID controller on process level control. This study proposes a FOPID controller for level process control in the coupled tank system.

1.3. Statement of problem

The statement of problem is to investigate the execution analysis of fractional order PID controller for process-level control in a coupled tank in an instance of the interacting arrangement. The liquid position control system has the characteristics of higher nonlinearity, complexity, and also integral-order PID or conventional controller may not track the required output under all circumstances of the system property. Those entire problems obtain in a coupled tank system, so to overcome the problem or a particular explanation to succeed in the achievement of the system is by apply the fractional order-PID controller.

1.4. Objective

1.4.1. General objective

The main objective of the study is to analysis the performance of fractional order-PID controller for process-level controls in the face of interacting coupled tank systems.

1.4.2. Specific objectives

The specific objectives in order to address the general objective of the study are:

- ✓ To develop a mathematical model of the interacting coupled tank system and numerical analysis of the system.
- ✓ To develop the model of FOPID for level process control systems in case of the interacting coupled tank system.
- ✓ To check the stability of coupled tank system by using Lyapunov quadratic stability method.
- ✓ To simulate the overall system using MATLAB®/SIMULINK®.
- ✓ To evaluate the performance and comparative analysis of all the proposed controllers.

1.5. Relevance of the study

In the modern industrial process controlling system is the concentration on its performance. Due to the problematic of numerous factories that interrupt the system, so to reduce the disruption, increase the quality of the product, and reduce operating cost it is beneficial to analyze the attainment of a system by using fractional order controller for level process control in case of interacting coupled tank system. In this study, fractional order PID controllers are proposed. A technique using a FOPID controller and also the evaluation the controller performance is investigated with MATLAB®/SIMULINK® simulation.

1.6. Scope of the study

The thesis is deal about controlling of interacting CTS using fractional order PID controller. Modeling of CTS with external disturbance on tank 2 only; all of the models are verify through simulation using MATLAB®/SIMULINK® and also need to add the FOMCON toolbox into MATLAB®/SIMULINK® for fractional order controller. The effectiveness of controllers is studied popular detail of the transit response within a chosen value of input value.

1.7. Motivation

The improvement of the control process for coupled tank systems is complex and more stimulating because of dynamics or non-linear by nature which shows maximum point behavior. The interacting containers are a challenge as a result of the following issues.

- ✓ Non-linear & maximum stage system.
- ✓ Multi-variable description effects collaboration within a coupled tank extremely liquid could move either two directions.
- ✓ A level characterized by the coupled tanks has to be conserved at the chosen set point.

Performance analysis of the process has a substantial problem in control as well as increasingly accepted as an area of importance in many processes industry. This problem has individual importance among the engineering community, because it offers a solution to control the processes that cannot be designated by a SISO model. This study aims to assessment of the FOPID controllers in order to get the process level to regulate when the system is interacting.

1.8. Methodology

The methodology of this thesis is engaged to carry out the analysis start with reviewing of literature, which includes reading of books, publication papers and thesis works to get essential information and ideas that will assist me to focus my work. The central part of the paper undertaking is mathematical modeling of the coupled tank, and finding transfer function based on physical parameters of coupled tank system. Then a difficulty problem arising from here is higher nonlinearity, complexity and conventional controller may not track the required output under all circumstances so to overcome the system FOPID controllers are used. Therefore, $PI^{\lambda}D^{\mu}$ controller is designed for controlling coupled tank system and performance analysis and interpretations is done with simulation results; a good performance is observed in fractional order PID controller.

A controller design is based on using MATLAB/SIMULINK. At the end also add another controller to improve system performance significantly. Hence, in this paper the conventional PID, digital-PID, FOPID, and TID controller simulation results will be compared using time domain specifications for instance overshoot, rise time, peak time, delay time, and settling time.

1.9. Organization of the thesis

The thesis is organized in to six parts:

- ✓ Chapter one- Introduction
- ✓ Chapter two- Literature review
- ✓ Chapter three- System modeling
- ✓ Chapter four- Controlling system and design
- ✓ Chapter five- Simulation result and discussions
- ✓ Chapter six- Conclusion and future works

CHAPTER TWO

LITERATURE REVIEW

2.1. Introduction

This chapter will discuss about the article that referred for these IOPID, digital and fractional order-PID and TID controllers and the plant that has proposed. Controlling the level process is the main issue that rises in the process industry. Advanced systems cover a wide range of technologies related to hard sciences, such as modeling, control theory and soft sciences. It is very important to keep the process working probably and safely in the industry, for environmental issues and for the quality of the product being processed. There are so many papers done around those control systems.

2.2. Related work

Some important concepts and previously accomplished works related to this thesis are reviewed as follows.

Varikuntla Rama koteswara rao et al (2021) studied; Coupled Tank System is one of the applications in industrial production. The process control especially controlling liquid level is important and widely applied in various fields such as liquid storage tank, a feeding tank, a product tank, the intermediate buffer containers and water tanks. In CTS, the overall process need liquids to be pumped, stored in the tank and pumped again to another tank for certain desired level. The liquid is required to be maintained in a specific height or certain range. Efficient and effective controls of these processes have immense economical advantage and its success depends on the type of control strategy. CTS are a typical representative of the process control. It has nonlinear and complex characteristics [8].

Dung Vuong Quoc et al (2021) studied PID controllers have been used in industrial control applications for a long time. This is due to its simplicity, low-cost design and robust performance in a wide range of operating conditions. 90% of the controllers used in industries are PID controllers. By adding a derivative term into PI controller, it improves the stability of control loop. The combined effect of Proportional and Derivative (D) controllers introduces a predictive capability in the controller since D term is able to react to foreseeable future control errors. The proportional term works as a fast-acting correction which will produce a change in the output as quickly as the error arises. The integral action takes a period of finite time to act but has the capability to make the steady zero state error. Unfortunately, tuning the PID controllers is tedious and it might be difficult to tune the PID gains properly due to the nonlinearity and the high complexity of the system [9].

O.T. Makki and L S Mohammad, (2021), studied PID controllers are widely used in multiple industries because of their simple mechanisms and structures. Usually, such controllers are tuned manually to obtain suitable values for the relevant parameters, which are Kp, Ki, and Kd. Choosing values for these parameters depends on maximizing the transfer function within the plant to develop a stable, controlled system that can handle uncertainties and perturbations. Sometimes, the derivative gain may be omitted because it is challenging to choose the appropriate value for this; however, incorrect values for PID coefficients may lead to unstable behavior [10].

Alok Prakash et al (2016) studied Design of IMC based PID Controller for Coupled Tank System. The performance of the PID controllers commonly used in process control industries depends upon the controller tuning parameters. An IMC based controller is designed and presented here for a coupled tank level control system which is of non-interacting type. The obtained transfer function is approximated into FOPDT model for the estimation of the IMC-PID controller tuning parameters in terms closed-loop time constant [11].

Daniel Y. Abramovitch (2016) studied a unified framework for considering analog and digital PID controllers by tying them to second order linear models. The uniform treatment allows a designer to read different PID specifications in the literature and easily relate them in a common way. These relationships also allow: better design of continuous time PID controllers by extracting the gains from standard controller forms, better implementation of discrete PID controllers through a better understanding of their relationships to the continuous forms, and simplified linear analysis of PID controllers [12].

Kartik Sharma1 et al (2017) described digital-PID controller design for DC/DC converters using re-design approach. To obtain desired characteristics in terms of loop gain, crossover-frequency and phase-margin, digital filter direct form based PID controller is designed. Conventionally analog methods were popular for control of these converters. The developed controller is mapped from continuous domain to discrete domain by using backward Euler methods, the digital-PID have greater performance compare to conventional one [13].

Dariusz Horla et al (2020) studied currently; fractional-order control is increasingly used to achieve better performance in control systems in comparison with classic integer-order control. It also becomes more and more popular in industrial applications. Tuning of FO controllers is challenging, since, in a general framework of a FOPID controller there are five parameters to tune while with a PID integer-order controller there are only three parameters [14].

Rohit Sharma, Sumit Mohanty et al (2016) performed designing of digital-PID controller which controls the blood glucose level of diabetic patient. Therefore PID designed with Cohencoon is implemented here and converted into digital domain using various transformation techniques. It can further be implemented on FPGA or any other programmable logic device [15].

Adrian-Josue Guel-Cortez et al (2021) studied, These fractional-order PID or $PI^{\lambda}D^{\mu}$ controllers include a derivative D^{μ} and an integral I^{λ} of non-integer orders $\mu, \lambda > 0 \in \mathbb{R}$. The values μ and λ add more degrees of freedom to the controller, which creates a more flexible controller in comparison with the classical PID controller. Additionally it has been shown that $PI^{\lambda}D^{\mu}$ controllers provide better results when being applied to fractional-order systems [16].

Aiswarya Lakshmi Sasidharan Nair et al (2017) performed the control of the fluid level in the tank and movement in between the tanks is a fundamental issue in industries. The liquid process requires be pumping, reserving in the tank. The liquids will be handled many times in the industries but continuously the level of fluids in the container must be adjusted. It is essential to understand how the tank is controlled and how the level control problems are solved [17].

Sruthi V. J and Dr. Binu L. S (2016) studied FOPID Controller for level control in a spherical tank. Fractional order mathematical phenomena can be used to describe and model a real object more accurately than the classical integer methods. It is a generalization of classical integer order control theory with differentiation and integration of non-integer orders. Due to different rates of inlet and outlet streams the spherical tank height of stored liquid changes with time. The level of liquid varies nonlinearly as the area of cross section varies with height [18].

Marwan Nafea et al (2018) studied liquid flow and level control are essential requirements in various industries, controlling the liquids flow and levels in industries is challenging due to the existence of nonlinearity and modeling uncertainties of the plants. A time-varying nonlinear dynamic model is developed and the corresponding linearized perturbation models are derived from the nonlinear model. Controlling the level of liquids in coupled tanks and the flow between them is a challenging issue for the industries. Designing a controller for such systems usually requires deriving complicated mathematical models of these systems, which are obtained from advanced physics and chemistry laws. In addition, the presence of nonlinearities and modeling uncertainties add more challenges to this type of process control [19].

Muhammad Awais et al (2018) studied the control of liquid level in tanks and flow between tanks is a problem in the process technologies. PID controllers are easy to implement and robust in nature. But the main disadvantage is that PID cannot handle the constraints and tuning PID controllers is rather very difficult. In this paper, model predictive control is designed for a simple two coupled tanks comprising many tool of control, it is clear that the MPC control is very suitable for nonlinear processes (by using linearizing processes). Therefore allows for basic a good disturbance rejection and good robustness to model errors. Thus, we can design other models of process control level as the method of generalized predictive control GPC which may include disturbances and noise on the inputs and outputs [20].

Jitendra Kumar Goyal et al (2019) those researcher studies, the earlier approaches to designing a linear controller for a nonlinear system are two-fold. The first considers linearizing the plant around some operating point, there by ignoring the dynamics posed by higher-order terms while the second approach is to represent the system nonlinearities in the form of model uncertainties, without any approximation of the higher order terms. Since the system is governed by a highly nonlinear dynamics, it requires linearization of the system for analysis using a linear control technique. Design a PI controller and an intelligent fuzzy controller for good tracking performance of the liquid levels [21].

Trinh Luong Mien (2019) presented a case study where the basic PID controller is combined with the fuzzy logic calculator for the nonlinear model of the liquid level of CTS. The simulation results suggest that the fuzzy-PID proposed controller can be applied to the liquid level control process in the chemical industry, where noise is always presented. The fuzzy-PID controller can improve quality of the liquid level coupled-tank control system, increase the process efficiency and bring economic benefit to end-user [22].

P. Gabriel Grace Keerthana et al (2016) studied; the level control in tanks is an ultimate issue in process control. It is important to pump up fluid and after that hold them inside tanks and after that exchange it to some other tank. Commonly the liquid should be prepared by chemical reactions or blending treatment within the tanks, in somewhat situation, dependably the level of liquid should be controlled. A very high might disturb response equilibria, result in some harm, or generate leakage of important or hazardous material. A very low level might produce terrible outcomes for the consecutive processes. Accordingly, liquid level control is a vital task in process industries. Conical tanks have applications in many industrial processes, to be specific processing of food, blending and treatment of waste water. Its shape allows better flow of solids, slurries and viscous fluids. So, level control in this tank is a very difficult issue because it is nonlinear and it always shows change in cross sectional area for that case to design linear quadratic regulator and also compare the performance with PID controller [23].

2.4. Summary of the literature

This chapter is about the explanation for some article that will refer to gets the information or some knowledge that will apply to make the thesis run successfully. In the above literatures observe that, the proposed controller was got a better performance for different plant than conventional controller. Some paper has lack of controller design and also the coupled tank system is a very sensitive area in industry.

Generally, when analysis the gap between the reviewed paper and performance of FOPID considering the fluid of control in the coupled tank system is; modeling of control the level process of an interacting CTS having high nonlinearity occurs on effect of external disturbance on the 2nd tank. Also to require a good gain of controller proposed pole placement method of tuning. Stability checked by advanced Lyapunov stability method. And for comparative and validation purpose use three controllers (IOPID, DPID and TID).

CHAPTER THREE

SYSTEM MODELING

3.1. Introduction

This section will discuss about the model that has been used to complete this thesis. The coupled tank liquid level system will be used in this thesis as a plant. Before to control the plant by proposed controllers must get to be know the modeling of the plant. For controlling any process, we need to do its mathematical modeling. Mathematical modeling of level processes is characterized by large time constants and/or time delays (dead time). In the instance of constant flow, in the stationary regime, the input and output quantities are equals. Mathematical models are needed for these processes to determine the mechanism of the process, simulate the behavior, establish the control structures, and establish the control strategies. They are simulated to get the expected process behavior with a proposed control system with particular set of tuning Parameters. Level process is one of the most common processes faced in industries. Owing to safety or process requirement, the level of the process liquid must be maintained at a certain level in spite of the disturbances.

3.2. Modeling of interacting coupled tank system

In this case, two tanks connected to the form of interacting form; here the level of the first tank depends on the level of the second tank. The straightforward principle of control interacting two containers is continued the height of the tank steady after the flow of fluid into the tank and flow out of the tank. Level of fluid of the tank is the main parameter used to govern the linked tank system in process industries [24]. The flow input rate is adjusted to maintain and govern the liquid level at an indicated value. The interacting coupled tank in figure 3.1 shows the schematic representation of a coupled tank system.Tank1 and Tank 2 were connected using a small pipe with the help of a valve. And there is an outlet connected to tank 2. The liquid used in the plant is accumulating to be non-viscous, incompressible and steady [25].

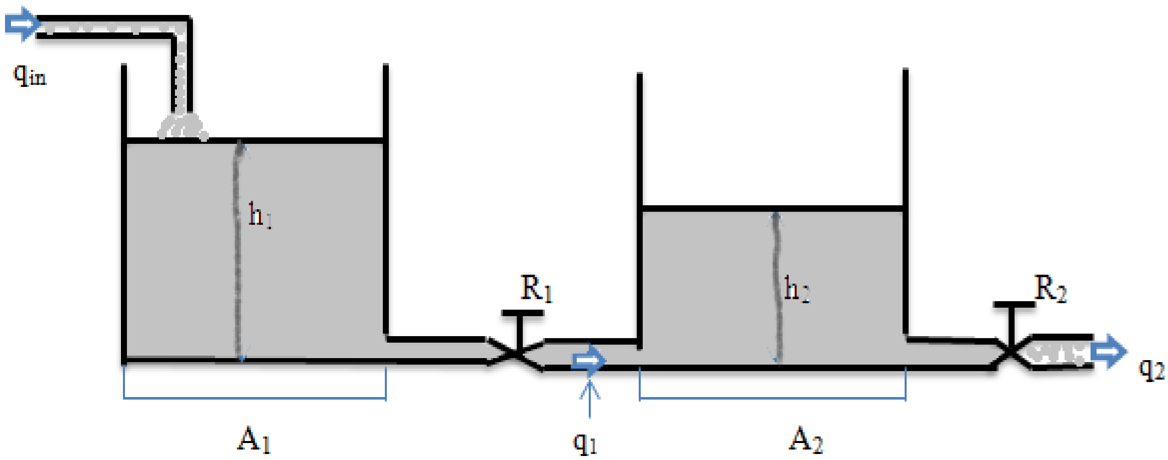

Figure 3.1:- Schematic diagram of interacting coupled tank system

The main parameters of the above system is,

- Inlet flow (q_{in})
- Out late flow (q_0)
- Cross-sectional areas (A)
- Heights of liquid in the tank (h)
- Valve/ pipe/resistance (R)

Where,

q_{in}=Volumetric flow rate input (cm^3/sec)

q_1=Volumetric flow rate from tank 1 to tank 2 (cm^3/sec)

q_2= Volumetric flow rate from tank output (cm^3/sec)

h_1 =Height of the liquid level input (cm)

h_2 =Height of the liquid level in tank 2 (cm)

A_1= Cross sectional area of tank 1 (cm^2)

A_2= Cross sectional area of tank 2 (cm^2)

R_1=Linear resistance of flow from tank 1 through valve 1 (cm^{-2}sec)

R_2=Linear resistance of flow from tank 2 through valve 2 (cm^{-2}sec)

Consider the flow of the liquid through a pipe from the pump to the process tank and another pipe from the process tank to another process tank. The resistance R for liquid-flow in such a container is defined as the change in the level difference to unit change in flow rate, that is

$$R = \frac{\text{change in level difference, cm}}{\text{change in flow rate, } {cm^3}/{sec}} \tag{3.1}$$

Changes in the 2nd system will effect on the first system. The liquid level in 2nd tank will affect the level on the 1st system (interacting system). The differential equations of tank 1 and tank 2 are obtained using the flow balance equation and given in (3.2) and (3.8).

For tank 1:-

Mass balance equation can be written as

$$A_1 \frac{dh_1}{dt} = q_{in} - q_1 \tag{3.2}$$

From the definition of resistance, the relationship between h_1, h_2 and q_1 is given by

$$q_1 = \frac{h_1 - h_2}{R_1} \tag{3.3}$$

Substitute (3.3) in to (3.2) we get,

$$A_1 \frac{dh_1}{dt} = q_{in} - \frac{h_1 - h_2}{R_1} \tag{3.4}$$

$$R_1 A_1 \frac{dh_1}{dt} = R_1 q_{in} - h_1 + h_2 \tag{3.5}$$

By taking Laplace transform

Note that R_1A_1 is the time constant of the system, Taking the LT of both sides of (3.5), we get

$$R_1 A_1 s h_1(s) = R_1 q_{in}(s) - h_1(s) + h_2(2) \tag{3.6}$$

$$h_1(s) = \frac{R_1 q_{in}(s) + h_2(s)}{1 + R_1 A_1 s} \tag{3.7}$$

When $\tau_1 = R_1 A_1$

$$h_1(s) = \frac{R_1 q_{in}(s) + h_2(s)}{1 + \tau_1 s} \tag{3.8}$$

For tank2:-

The mass balance equation is

$$A_1 \frac{dh_2}{dt} = q_1 - q_2 \tag{3.9}$$

$$q_2 = \frac{h_2}{R_2} \tag{3.10}$$

$$\left.\begin{aligned} A_2 \frac{dh_2}{dt} &= q_1 + q_2 \\ A_2 \frac{dh_2}{dt} &= \frac{h_1 - h_2}{R_1} - \frac{h_2}{R_2} \end{aligned}\right\} \tag{3.11}$$

From (3.11) we get,

$$R_2 R_1 A_2 \frac{dh_2}{dt} = R_2 h_1 - R_2 h_1 - h_2 R_1 \tag{3.12}$$

On dividing by R_1 and taking Laplace transform

$$A_2 R_2 s h_2(s) + \frac{R_2}{R_1} h_2(s) + h_2(s) = \frac{R_2}{R_1} h_1(s) \tag{3.13}$$

$$h_2(s)(\tau_2 s + \frac{R_2}{R_1} + 1) = \frac{R_2}{R_1} h_1(s) \tag{3.14}$$

Where $\tau_2 = R_2 A_2$

Substitute (3.8) in to (3.14) and we get,

$$h_2(s)(\tau_2 s + \frac{R_2}{R_1} + 1) = \frac{R_2}{R_1} \frac{R_1 q_{in}(s) + h_2(s)}{1 + \tau_1 s} \tag{3.15}$$

$$h_2(s)(\tau_2 s + \frac{R_2}{R_1} + 1)(R_1 + R_2\tau_1 s) = R_2 R_1 q_{in}(s) + R_2 h_2(s)$$

$$h_2(s)(\tau_2 s + \frac{R_2}{R_1} + 1)(R_1 + R_2\tau_1 s) - R_2 h_2(s) = R_2 R_1 q_{in}(s) \tag{3.16}$$

Therefore the transfer function of the two tank interacting system is expressed as

$$\frac{h_2(s)}{q_{in}(s)} = \frac{R_1 R_2}{R_1\tau_1\tau_2 s^2 + (R_1\tau_1 + R_1\tau_2 + R_2\tau_1)s + R_1} \tag{3.17}$$

$$\therefore \frac{h_2(s)}{q_{in}(s)} = \frac{R_2}{\tau_1\tau_2 s^2 + (\tau_1 + \tau_2 + R_2 A_1)s + 1} \tag{3.18}$$

3.3. Modeling of coupled tank systems with disturbance

Disturbance analysis for an interacting container of fluid level control systems can be as follows:-

Where, D_{in} – disturbance of the system

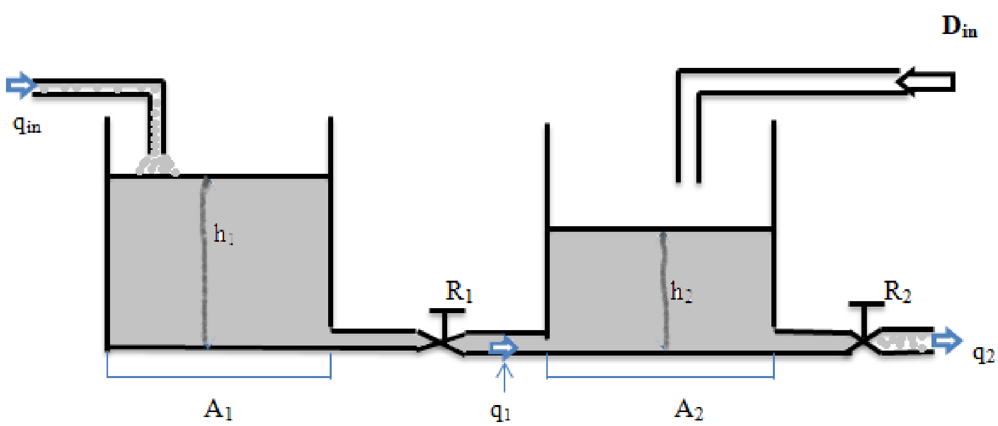

Figure 3.2:- Schematic diagram of the interacting coupled tank with a disturbance

The disturbance transfer function analysis as below

$$A_2 \frac{dh_2}{dt} = D_{in} - q_2 \tag{3.19}$$

Since from (3.39) $q_2 = \frac{h_2}{R_2}$ substitute into (3.19) and we get,

$$A_2 \frac{dh_2}{dt} = D_{in} - \frac{h_2}{R_2} \tag{3.20}$$

The LT of (3.20) we get,

$$\frac{h_2(s)}{D_{in}(s)} = \frac{R_2}{A_2 R_2 s + 1} \tag{3.21}$$

The expression of external disturbance is similar to the above analysis.

$$\frac{h_1(s)}{D_{ex}(s)} = \frac{R_1}{A_1 R_1 s + 1} \tag{3.22}$$

CHAPTER FOUR

CONTROL METHODS AND DESIGN

4.1. Introduction

A control structure is a system or set of devices, that manages the signal or regulates they presence of additional processes or systems to accomplish the chosen set-point. In other words, the definition of the control method can be simplified as a system, which controls other systems.

4.2. Type of controllers framework

There are two categories of control framework in control building such as open loop control framework and closed loop control framework. If the actuating signal (input of the plant/system) depends only on the reference signal and independent of the plant output which is called open loop control system while if the actuating signal depends both the reference signal and output of plant a control system is called feedback control (closed loop) system. The open loop control system is not used in practical since a plant/system is easily affected by parameter variations, noise and disturbance, but the feedback control system is most widely used in practical because, it can reduce the effect of parameter variations, disturbance and suppress noise. Open loop control methods are where in a control accomplishment is self-regulating the response the processes but a CLS in which the response affects the input insure that the input will regulate itself based on the response generated [26].

If the process does not happened the desired performance specification, Controllers are used and also connected either in series with or parallel to the plant depending upon the specification. Show in figure 4.1 feedback control system with a controller.

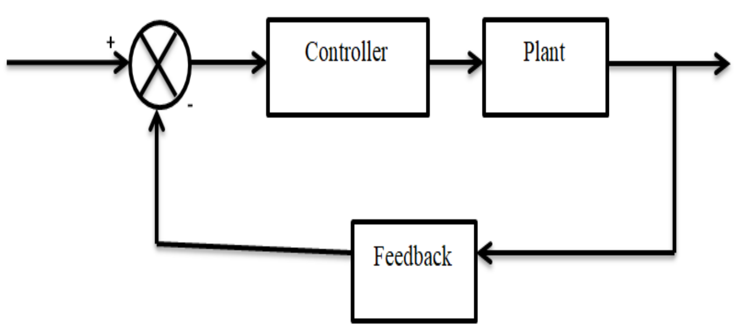

Figure 4.1:- Block diagram of closed loop control system

As shown figure 4.1, control signals generated which are the change among the set-point signal and the feedback signal. The control signal resolves the scale through whatever an output response turns against the desired point. The controller is staying to keeping the controlled variable such as liquid level, motor speed; robot joint angle at a set-point in addition to the input signal is applied to plant which will give good results. Feedback control is an action in which a measured variable is compared to its desired value to produce an error signal which is acted upon in such as to decrease the amplitude of the error. For a plant with MIMO, the situation desires numerous regulators. For a SISO, need a single controller for managing.

4.3. Introduction to PID controller

PID controller used as a linear controller for the control process with a good response. A PID controller is commonly used in the process and applying accurately the grouping of three types of corrective actions to the error signal, which characterizes how far or near is the desired set point from the actual output. As widely known, these three control actions are proportional, integral and derivative The key feature when tuning PID controllers is in determining how to best syndicate those three terms to accomplish the most well-organized instruction of the process variable for the well-thought-out problem. As well known, the most understandable way is to use a simple weighted sum where each term is multiplied by a tuning constant or gain [27] PID's working principle is that it calculates an error value from the processed measured value and the desired reference point. The work of the PID-controller is to reduce the error by changing the inputs of the system. Proportional-integral derivative control is the basic control scheme of the classical control system [19]. The assessment of a process could be improved using the proper value of gain k_p, k_i and k_d.

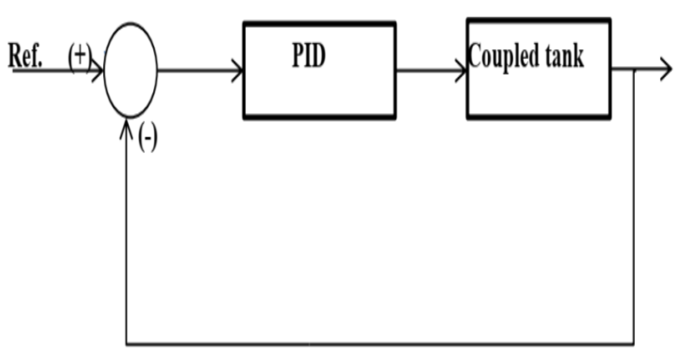

Figure 4.2:- Block diagram of proportional-integral-derivative controller

Proportional gain k_p- improve the structure to create it protected a capacity disorder. k_i- Supports to decrease errors of steady-state. K_d- Supports to improve the establishment of a closed loop [28]. By calculating the value of three constants in the PID controller, it can offer a control achievement designed for specific activities.

Now to design the proportional-integral-derivative controller a certain dominant pole placement method is used. The transfer function of PID controller need to Control 2^{nd} order plant ($G_p(s)$) (with slows designed or oscillatory open-loop changing aspects).

4.4.1. Tuning of the controller parameters

Before going to model the controller we need to design the tuning parameter, tuning method is pole placement design. The pole placement design method simply attempts to find a controller that gives desired closed loop poles [29].

First to find the value of the controller parameter so start from IOPID or conventional PID controller, suppose that the process is characterized by the second- order model.

$$G_p(s) = \frac{k}{s^2 + 2\xi\omega_n s + \omega_n^2} \tag{4.1}$$

Where,

ξ - Damping ratio of a second order plant

ω_n – The natural frequency of a second order plant

k – Constant of a second order plant

This model has four variables. We accept that the method is controlled by a PID controller parameterized as,

$$G_c(s) = \left[k_p + \frac{k_i}{s} + k_d s\right] = \frac{k_d s^2 + k_p s + k_i}{s} \tag{4.2}$$

Where,

$$\left.\begin{array}{r} k_p - \text{proportional gain} \\ k_i - \text{integral gain} \\ k_d - \text{derivative gain} \end{array}\right\} \text{ of the PID gain}$$

Then, the closed loop system transfer function becomes,

$$G_{cl}(s) = \frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)} \tag{4.3}$$

Substitute for $G_c(s)$ and $G_p(s)$ from above (4.1) and (4.2) respectively into equation (4.3) and has the characteristic equation give as,

$$G_{cl}(s) = \frac{k(k_d s^2 k_p s + k_i)}{s^3 + (2\xi\omega_n + kk_d)s^2 + (\omega_n^2 + kk_p)s + kk_i} \tag{4.4}$$

as considered [30] currently, if the chosen CL calculation of the second order scheme can be specified as the qualifications on the ξ^{cl} (damping ratio of the CL scheme) and ω_n^{cl} (natural frequency of the CL system) as in one can simply substitute the location of the real zero (α) by (-$\alpha\xi^{cl}\omega_n^{cl}$), providing that s = -$\alpha\xi^{cl}\omega_n^{cl}$ is chosen to be huge sufficient regarding (-$\alpha\xi^{cl}\omega_n^{cl}$). Consequently, selecting an appropriate of comparative governance (α), the third order CL system (3rd) [31] determination accomplish like second order classification consuming the user – specified CL damping ratio ξ^{cl} (percentage of maximum overshoot) and closed loop natural frequency ω_n^{cl}(rise time). In this situation, the distinguishing polynomial is inscribed as:

$$(s+\alpha\xi^{ci}\omega_n^{cl})(s^2 + 2\xi^{cl}\omega_n^{cl} + (\omega_n^{cl})^2) = 0 \tag{4.5}$$

After multiplication of the above (4.5) we get,

$$s^3 + (2+\alpha)\xi^{cl}\omega_n^{cl}s^2 + (1+2(\xi^{cl})^2(\omega_n^{cl})^2)s + \alpha\xi^{cl}(\omega_n^{cl})^3) = 0 \tag{4.6}$$

Relating the coefficients of (4.6) with the denominator of (4.4), mathematical modeling of PID controller gain can be calculated as follow.

$$\omega_n^2 + kk_p = 1 + 2\alpha(\xi^{cl})^2(\omega_n^{cl})^2 \tag{4.7}$$

$$k_p = \frac{(1 + 2\alpha(\xi^{cl})^2(\omega_n{}^{cl})^2) - \omega_n{}^2}{k} \tag{4.8}$$

$$kk_i = \alpha\xi^{cl}(\omega_n^{cl})^3 \tag{4.9}$$

$k_i = \frac{\alpha\xi^{cl}(\omega_n^{cl})^3}{k}$ (4.10)

$2\xi\omega_n + kk_d = (2+\alpha)\xi^{cl}\omega_n^{cl}$ (4.11)

$kk_d = (2+\alpha)\xi^{cl}\omega_n^{cl} - 2\xi\omega_n$ (4.12)

$k_d = \frac{(2+\alpha)\xi^{cl}\omega_n^{cl} - 2\xi\omega_n}{k}$ (4.13)

The overall parameters of PID controller (4.8), (4.10), and (4.13) from the above get in this way,

$$\left.\begin{aligned} k_p &= \frac{(1+2\alpha({\xi^{cl}}^2)({\omega_n^{cl}}^2)) - \omega_n^2}{k} \\ k_i &= \frac{\alpha\xi^{cl}(\omega_n^{cl})^3)}{k} \\ k_d &= \frac{(2+\alpha)\xi^{cl}\omega_n^{cl} - 2\xi\omega_n}{k} \end{aligned}\right\} \quad (4.14)$$

4.5. Introduction to Digital controller

In most modern engineering systems, it is necessary to control the evolution with time of one or more of the system variables. Controllers are required to ensure satisfactory transient and steady-state behavior for these engineering systems. To guarantee satisfactory performance in the presence of disturbances and model uncertainty, most controllers in use today employ some form of negative feedback [32].

The controller that manipulates the error signal to determine the desired control action has classically been an analog system, which includes electrical, fluid, pneumatic, or mechanical components. These systems all have analog inputs and outputs (i.e., their input and output signals are defined over a continuous time interval and have values that are defined over a continuous range of amplitudes). In the past few decades, analog controllers have often been replaced by digital controllers whose inputs and outputs are defined at discrete time instances. The digital controllers are in the form of digital circuits, digital computers, or microprocessors [33].

The reason to use the digital controller;

- ✓ We can handle delays and loss of information
- ✓ They don't cost much
- ✓ Flexibility to change later
- ✓ More robust to environment disturbance

4.5.1. Digital-PID controller

The discrete time PID controller is developed from analog counterpart using different methods [34]. Digital control systems employ a computer as a fundamental component in the controller. The computer typically receives a measurement of the controlled variable, also often receives the reference input, and produces its output using an algorithm. This output is usually converted to an analog signal using a D/A converter, then amplified by a power amplifier to drive the plant. A block diagram of a typical digital control system is shown in Figure 4.3.

Previous work on continuous time PID controller design in parameter space, corresponding results are missing for digital-PID controllers. While it is always possible to design a continuous time PID controller and then discretize it for a digital implementation, it is preferable to directly design the digital-PID in the z-domain especially in the presence of a sampling time that is not too small which is typical for automotive control systems that rely on measurements from the CAN bus [35]. The modern digital control systems require more and more strong and fastest calculation components. field, digital-PID controller can not only use the software to realize the PID control algorithm, but also can use the logic function of the computer to make the PID control more flexible. At present, digital-PID controller has been widely used in mechanical, electromechanical, metallurgy, chemical industry and so on [36].

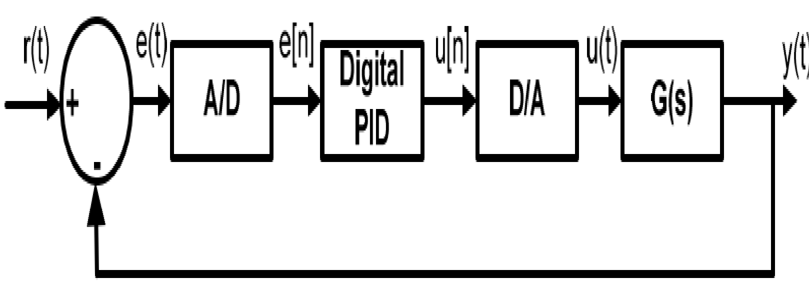

Figure 4.3:- Block diagram of Digital-PID controller [36]

4.5.2. Method of conversion of conventional controller

There are different method to convert continues transfer function into discrete function. Those are,

- ✓ 'zoh'- Zero order hold on the inputs
- ✓ 'foh'- linear interpolation inputs
- ✓ 'impulse'- impulse invariant discretization
- ✓ Back ward Euler approximation to the derivative
- ✓ For ward Euler approximation
- ✓ 'tustin'- bilinear (Tustin) approximation or trapezoidal method
- ✓ 'Matched'- matched pole zero method (for SISO system only).

Some of the most popular techniques are briefly listed in table 4.1

Table 4.1:- Different way of conversion process [37]

Method	**Approximation**
Forward difference method	$S \longleftarrow \frac{Z-1}{T}$
Backward difference method	$S \longleftarrow \frac{Z-1}{TZ}$
Trapezoidal/Tustin transformation method	$S \longleftarrow \frac{2}{T}(\frac{Z-1}{Z+1}) = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$

The trapezoid-rule substitution is additionally known, particularly in digital and sampled-data control circles, as Tustin's method. The transformation is also called the bilinear transformation from consideration of its mathematical form. The design strategy can be summarized by expressing the rule as show: Given a continuous transfer function (channel), H(s), a discrete equivalent can be found by the substitution.

$$H_T(z) = H(s)|_s = \frac{2}{T}\left[\frac{(z-1)}{(z+1)}\right] \quad (4.15)$$

Controllers are required to ensure satisfactory transient and steady state behavior for the engineering systems. To guarantee satisfactory performance in the presence of disturbance and model uncertainty, most controllers in use today employ some form of negative feedback. Today the analog controllers have been replaced by digital controllers to achieve better control [39].

The digital-PID controller design based on analog PID is converted via bilinear transformation. This transformation is given by:

$$s = \frac{2}{T}\left[\frac{(z-1)}{(z+1)}\right] \tag{4.16}$$

Where T is the sample time interval,

In this particular power system T is small enough to results in a higher sample rate, the digital PID controller gives the same result as the analog counterpart as expected. Moreover the digital computer in the loop yields many advantages such as reduction of cost, flexibility of the design change, and importantly it immunes to noise. Any future change in the system is replaced by a simple software update over an expansive hardware replacement. Using a digital-PID controller achieves a balance in both rising time and overshoot. In the particular case, the lesser over shoot and smaller rising time was obtained simultaneously, which implies that the finer the tuner [38]. Consider (4.2), and take conversion process by using trapezoidal integration (Tustin or Bilinear transform method and substitute (4.16) we get,

$$G_c(z) = k_p + \frac{K_i}{\frac{2}{T}\left[\frac{(z-1)}{(z+1)}\right]} + k_d\frac{2}{T}\left[\frac{(z-1)}{(z+1)}\right] \tag{4.17}$$

$$G_c(z) = \frac{k_p[2T(z-1)(z+1)] + k_i[T^2(z+1)(z+1)] + k_d[4(z+1)(z-1)]}{2T(z-1)(z+1)} \tag{4.18}$$

$$= \frac{k_p z^2 - k_p + \frac{T}{2}k_i z^2 + k_i Tz + k_i\frac{T}{2} + \frac{2}{T}z^2 k_d - \frac{2}{T}k_d}{z^2 - 1} \tag{4.19}$$

Let assume k_1, k_2 and k_3 for assigning the complex expression of (4.20) and we get,

$$\left.\begin{aligned} k_1 &= k_p + \frac{T}{2}k_i + \frac{2}{T}k_d \\ k_2 &= k_i T \\ k_3 &= -k_p + k_i\frac{T}{2} - \frac{2}{T}k_d \end{aligned}\right\} \tag{4.20}$$

$$\therefore G_c(z) = \frac{k_1 + k_2 z^{-1} + k_3 z^{-2}}{1 - z^{-2}} \tag{4.21}$$

Selection of the sampling rate:

- ✓ The proper choice of the sampling interval is very important. Too low sampling frequency may lose so much information that the control performance deteriorates and the system dynamics is lost.
- ✓ Too high sampling rate increases the burden of the processor; also, it may lead to discrete representation with bad numerical properties.
- ✓ For oscillating systems the sampling interval is often tied to the frequency of the dominating oscillation. For damped systems the sampling interval is usually chosen to be in relation to the time constant.

4.6. Basic concepts of Fractional order calculus

Fractional calculus is an old accurate topic since the 17th century. The sectional-order calculus is a zone of mathematics that expresses through derivatives and integrals since non-integer guidelines. In additional verses, it is an oversimplification of the old fashioned calculus that indicates equivalent conceptions and apparatuses but through considerably larger appropriates. Happening the last two periods, fractional calculus has remained recollected by scientists and engineers and applied in an accumulative number of grounds, specifically in the area of regulator concept. Fractional calculus benefits to estimate$\frac{d^\alpha}{dt^\alpha}$, α - fold integrals where α is fractional, irrational, or complex. Fractional order systems, α is measured to be fractional. These mathematical occurrences permit telling an actual purpose additional correctly than conventional integer order approaches. The foremost purpose of expending the integers order simulations remained the interval of an explanation aimed at sectional differential equivalences. The success of fractional order controllers is absolute with a lot of success due to the development of effective methods in differentiation and integration of non-integer order equations. There are dissimilar explanations of Fractional-order-differentiations and integrations. Some of the definitions range directly from integer-order calculus. The well-established definitions include the Grunwald-Letnikov definition, the Cauchy integral formula, the Caputo definition, and the

Riemann-Liouville. Classification determination stands brief as follows and formerly their belongings determination remains particular.

The FOC is based on generalization of differentiation and integration to an arbitrary order, which can be rational, irrational or even complex. This generalization has led to the introduction of basic continuous differ integral operator [39].

Cauchy's describe sectional Order integration. Characterization is an all-purpose the per request of the integer-order Cauchy formulation

$$D^{\gamma} f(t) = \frac{\Gamma(\gamma+1)}{2\pi j} \int_{C} \frac{g(\tau)}{(\tau-t)^{\gamma+1}} d\tau \tag{4.22}$$

Wherever C is the smooth curve neighboring the single-valued purpose g (t),

Grunwald-Letnikov's explanation is perhaps the best known due to its most suitability for the realization of discrete control algorithms. The Grunwald-Letnikov definition is expresses as [35].

$$a^{D_t^{\alpha}} f(t) = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{\frac{t-a}{h}} (-1)^j \binom{\alpha}{j} f(t - jh) \tag{4.23}$$

Where $W_j^{\alpha} = (-1)^j \binom{\alpha}{j}$ denotes the factors of the polynomial $(1\text{-}z)^{\alpha}$ the factors can likewise remain developed recursively beginning

$W_0^{\alpha} = 1$, $W_j^{\alpha} = (1- \frac{\alpha+1}{j} W_{j-1}^{\alpha}$, when j = 1, 2, 3 … … … … ..

Riemann–Liouville defines Fractional Order differentiation the Fractional Order integration is defined as [40]

$$a^{D_t^{-\beta}} f(t) = \frac{1}{\Gamma(\beta)} \int_{b}^{t} (t-\tau)^{\beta-1} f(\tau) d\tau \tag{4.24}$$

Where 0<β<1and b - the leading interval design, repeatedly supposed towards remain nothing, i.e., b = 0. The separation is formally designated as $a^{D_t^{-\beta}}$ f(t), The explanation exceeding formula is the nethermost repeatedly recycled classification in fractional- order calculus. The indexes on collected edges of D characterize the lower and upper restrictions in the combination.

Caputo's describe sectional Order separation). Caputo's classification is specified by

$$ {}_{o}D_{t}^{\gamma}\, y(t) = \frac{1}{Γ(1-\gamma)}\int_{0}^{t}\frac{y^{(m+1)}(\tau)}{(t-\tau)^{\gamma}}\, d\tau \quad (4.25)$$

Where α=m+ γ, m is an integer, and 0 < γ ≤ 1. Similarly, Caputo's sectional Order addition is well-defined as

$$ {}_{o}D_{t}^{\gamma}\, y(t) = \frac{1}{Γ(1-\gamma)}\int_{0}^{t}\frac{y(\tau)}{(t-\tau)^{\gamma+1}}\, d\tau,\ \gamma<0 \quad (4.26)$$

4.7. Types of fractional order controller

A FOPID regulator is the extension of the classical PID-controller based on fractional-calculus. For several years, engineering PID controllers have been presenting very commonly used in process industries. The significance of PID controls, incessant efforts is existence ended to develop robustness. An automatic regulator method, FOC is oversimplification integer-order controllers would regulate the robust control achievement and also more accurate. The FOC which stands the overview of conventional IOC would main to additional detailed and robust governor performance. However, it is practical; the fractional-order simulations involve the FOC toward accomplishing the better assessments. The FOC is functional to rectilinear or non-rectilinear dynamic forces to improved performances of a scheme. Generally, there are four main categories of FOC [41].

- ✓ Fractional order PID-controller (FOPID)
- ✓ Tilted proportional and integer controller (TID)
- ✓ Commande Robuste d'ordere Non Entire controller (CRONE)
- ✓ Fractional lead-lag compensator (FLL)

4.7.1. $PI^{\lambda}D^{\mu}$ controller

The FOPID controller has two more adaptable parameters than the PID controller, and the order of the controller can be chosen randomly, the FOPID controller owns more flexibility. In addition to that, the FOPID controller has great adaptability to the parameter variation of the control system. When the constraint of the control system change contained by a certain range, the system characteristics remain unchanged. So the FOPID controller has the characteristic of strong robustness. Control techniques including feedback control, optimal control, predictive control, neural network control, fuzzy logic control, and so on, have been established meaningfully.

Proportional integral derivative controllers are linear and symmetric and they have complications in the occurrence of non-linearity. This problem can be solved by using a FOPID controller. A regulator through integrator of actual order λ and differentiator actual order μ is a more common cause of the classical PID-controller which delivers more flexibility and robustness in tuning and control by an additional two degree of freedom in the system.

Table 4.2:- Special property of FOPID controller [41].

Controller	**Special identification of the controller**
FOPID	Has two more adaptable parameters
	Controller has the characteristic of strong robustness
	More flexibility
	Great adaptability to the parameter variation of the control system

These degrees of freedom are related to the orders of the integral and derivative parts which are extended to non-integer (fractional) values. The FOPID control suggested by Podlubny is a simplification of a traditional proportional-integral-derivative regulator expending the fractional multiplication. The TF is well-defined by (4.28).

A FOPID control is characterized by five constraints, i.e. the proportional gain (Kp) the integral gain (Ki), the derivative gain (Kd), integral order (λ), and derivative order (μ) [42]. The FOPID controller is the increase of the conventional PID controller based on fractional mathematics. Due to the dominant importance of PID controllers develop their features and robustness [41].

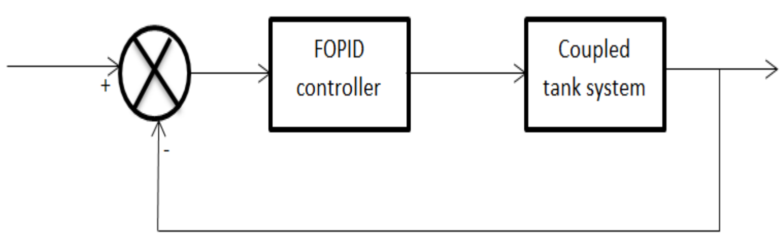

Figure 4.4:- Block diagram of coupled tank system with FOPID controller

Fractional-order controllers have received significant responsiveness in the last years both from the academic and industrial point of view. Podlubny suggested an overall form of the IOPID control, is called $PI^{\lambda}D^{\mu}$ control, where the values of λ and μ lie between 0 and 1. Related with the PID controller, the FOPID controller has more two adjustable parameters, which makes the parameter tuning more flexible, it is very essential significative for successful the control accuracy, therefore, the FOPID a controller applied for managing a level process in interacting two-tank [47]. The differential equation of $PI^{\lambda}D^{\mu}$ the controller is shown as

$$U(t) = k_p e(t) + k_i Dt^{-\lambda} e(t) + k_d Dt^{\mu} e(t) \quad (4.27)$$

Relating Laplace transforms to this equation with zero initial conditions, the TF of the controller can be expressed by:

$$G_{fc}(s) = \frac{U(s)}{E(s)} = k_p + k_i s^{-\lambda} + k_d s^{\mu}, \qquad (\lambda>0, \mu>0) \quad (4.28)$$

Taking λ= 1 and μ = 1 it is the conventional PID controller, if λ = 1 and μ = 0 it is the conventional PI controller, if λ= 0 and μ = 1 it is the unadventurous PD controller and if both μ and λ = 0 it is P controller, it $PI^{\lambda}D^{\mu}$ can be seen that the adjustable range of the fractional $PI^{\lambda}D^{\mu}$ the controller is wider than the conventional-PID controller, therefore, the guideline performance of the FOC is greater than the IOPID controls.

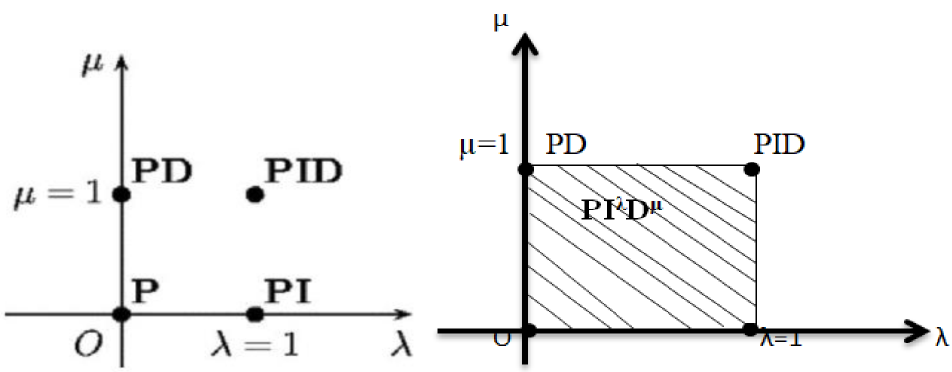

Figure 4.5:- Graphs of PID controller [4]

4.7.2. Tilted proportional and integer (TID) controller

Tilted-Proportional and Integral (TID) Controller is to provide an enhanced feed-back loop controller gains of the classical Proportional integral controller. In TID structure the proportional compensating unit is exchanged with a compensator having a transfer function characterized by $\frac{1}{s^{1/n}}$ or $\frac{1}{s^{-\frac{1}{n}}}$. This compensator is herein referred to by way of a “Tilt” compensator, as it delivers a feedback gain as a function of frequency which is tilted or formed for the gain/frequency of a conservative or positional recompense unit. The perfect controller is herein referred to as a Tilt-Integral Derivative (TID) controller. It contains three components tunable feedback loop control system which contains a proportional integral derivative controller. The individual change from the conventional controller is that the corresponding recompensing part of the framework is replaced with a more reasonable compensator which is having a transfer function. The term “Tilt” suggests that it can provide a feedback improvement as a frequency purpose that is shaped or tilted to improvement frequency of predictable reparation entity. A transfer function of TID can be written:-

$$G(s) = k_t(1/s)^{1/n} + \frac{k_i}{s} + sk_d \tag{4.29}$$

Where n is a non-zero real number, the above transfer function is shown in the figure 4.6 as:

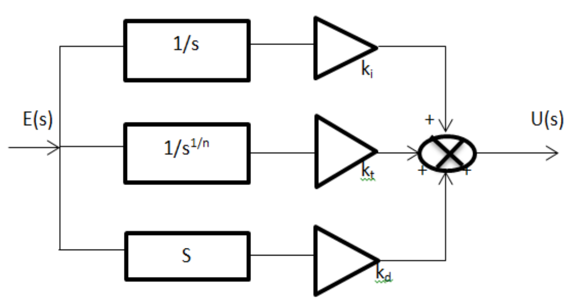

Figure 4.6:- Tilt-integral derivative controller [43]

The effects of TID controller can be summarized as:

- ✓ Simple Tuning

- ✓ Feedback control is improved
- ✓ Disturbance rejection ratio is improved
- ✓ System parameter variation has less influence on closed-loop response

The main parameter for tuning in the TID controller is the tunable coefficient "n" and the other parameters (k_i and k_d) are the same as we used in the conventional PID controller. Thus, the most optimum value of "n" is obtained by this technique for the lower percent of under/overshoot, settling time, and steady-state error [43].

4.8. MATLAB toolboxes for analyzing fractional order systems

In recent years, as fractional calculus develops more and more broadly used across different academic disciplines, there are growing difficulties for the numerical tools for the calculation of fractional integration/differentiation or the simulation of fractional-order systems.

A) @fotf

@fotf (fractional-order transfer- function) is a control tool-box for fractional-order systems developed by Xue. Most of the FOTF exclusive are extended from the MATLAB built in-functions. @fotf tool-box uses the overload programming method to enable the related methods of the Matlab built-in functions to covenant through fractional-order models. The FOTF objects created from it can be collaborating utilizing generated from the MATLAB transfer-function course. Conversely, the over-loading of related functions such as impulse (), step (), etc. @fotf toolbox supports the time delay in the transfer function. It doesn't straight support the TF matrix; hence, MIMO systems can't be simulated. Nevertheless, meanwhile, it delivers Simulink-block - encapsulation of the complicated function @fotf (), MIMO correlation recognized via physically adding round interfaces. A draw-back of the fraction order transfer function is that the sample time has a moderately big influence on accurateness [44].

B) Ninteger

Ninteger, a non-integer MATLAB control toolbox, is a toolbox proposed to help with emerging FOC and evaluating their performance. Fractional order Simulink block diagram are involving the following function, such as nid, nipid, fotf, etc. [45].

C) CRONE

The crone-tool-box, established meanwhile 19th by a crone group, is a Matlab and Simulink toolbox devoted to presentations of non-integer derivatives in industrial skill [45]. It developed beginning the inventive script version of the remaining programming. A respectable article of the toolbox is certain of the approaches are appreciated for MIMO fractional transfer functions. Numerous extra toolboxes are encouraged by CRONE, e.g Ninteger and FOMCON. The drawbacks of the crone toolbox are interval suspension can't be combined interested in the engendered FOTF. Crone a MATLAB toolbox consider more dominant than just simulating FOS.

D) FOMCON

The FOMCON (Fractional- Order-Modeling, and Control) *Toolbox* is established by Aleksei [46]. Its kernel works systems in @fotf, ninteger, and Crone. Summarizes the particular main function of individuals three–fractional-order (FOMCON) toolboxes, the relative FOMCON within three-toolboxes is obtainable in figure 4.7 particular distinguished deviations of the unique @fotf are

- ✓ New fotf() practices string parser to allow operators to input-transfer- function as a string
- ✓ tf2ss() are loaded and foss() are additional, which varieties the adaptation between fotf() entity and foss() article. The crone MATLAB-toolbox is consistently intelligent to do a task; contrariwise, an index is encoded in Matlab P-code format.

In this thesis, the FOMCON Matlab toolbox is used because it contains the main functionalities of fotf, Ninteger, and Crone toolboxes [44].

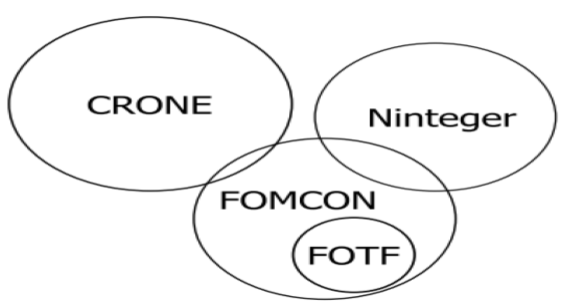

Figure 4.7:- FOMCON relation to other fractional-order MATLAB toolboxes [44]

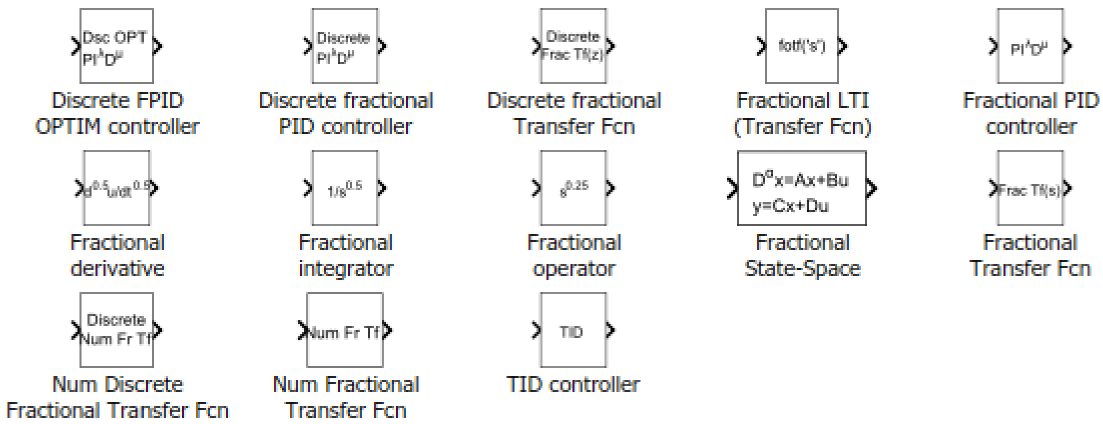

Figure 4.8:- FOMCON Simulink library

4.9. Dynamic behavior of the plant

To understand and control complex systems, we must obtain their quantitative mathematical models by analyzing the relationships between the variables of the system and employing Laplace transform or other mathematical tools [47]. A set of linear –differential-equation remained formulated to obtain a model transfer function that described the physical system of the plant.

4.10. Performance measures criteria

To test our model, we choose to use performance measures. These measures will be used in analyzing the performances of Fractional order proportional- integral-derivative controllers.

Transient-response: - one of the furthermost essential characteristics of control is the reply of a system as a function of time.

The speed of the response as represented by the rise-time (T_r) and The closeness of the response to the desired response as represented by the overshoot (O_s) and its receipts for the system to touch to its steady-state is settling-time (Ts) [48].

4.11. Parameters of level control system

The achievement of a process-level control system in interacting coupled tank for and compare the reply of the system with IOPID, digital-PID, TID and fractional-order PID controller, the selected parameter of the level tank is specified in table 4.3.

However, in order to achieve a good control performance the physical parameter values of the plant should be estimated. The difficulty with the parameter estimation for this system is due to the nonlinear dependence on the parameters, we consider the physical parameter's value from the following table 4.3 the effects of the environmental changes on the plant and the measurement noise [32].

Table 4.3:- Physical parameters of coupled tank system [32]

No	Description	Parameter	Value	Units
1	Area of tank1	A_1	250	cm^2
2	Area of tank2	A_2	250	cm^2
3	Resistance of tank1	R_1	0.01	sec/cm^2
4	Resistance of tank2	R_2	0.01	sec/cm^2
5	Height of tank1	h_1	30	cm
6	Height of tank2	h_2	15	cm

Since the open-loop transfer function of the system is given above by (3.18) and with (4.1) and the unknown open-loop system parameters, ξ, and ω_n are obtained. Using the parameters given in table 4.3

$$\left.\begin{aligned} k &= \frac{R_2}{\tau_1 * \tau_2} \\ \omega_n &= \sqrt{\frac{1}{\tau_1 * \tau_2}} \\ \xi &= \frac{\tau_1 + \tau_2 + R_2 A_1}{2 * \tau_1 * \tau_2 * \omega_n} \end{aligned}\right\} \tag{4.30}$$

$$k = \frac{R_2}{\tau_1 * \tau_2} = \frac{0.01}{(250*0.01)*(250*0.01)} = \frac{0.01}{6.25} = 0.0016 \tag{4.31}$$

$$\omega_n = \sqrt{\frac{1}{\tau_1 * \tau_2}} = \sqrt{\frac{1}{6.25}} = \sqrt{0.16} = 0.4 \tag{4.32}$$

$$\xi = \frac{\tau_1 + \tau_2 + R_2 A_1}{2 * \tau_1 * \tau_2 * \omega_n} = \frac{250*0.01+250*0.01+0.01*250}{2*250*0.01*250*0.01*0.4} = \frac{7.5}{5} = 1.5 \tag{4.33}$$

$$\left.\begin{array}{l} k_p = \dfrac{(1+2\alpha({\xi^{cl}}^2)({\omega_n^{cl}}^2)) - \omega_n^2}{k} \\ k_i = \dfrac{\alpha\xi^{cl}(\omega_n^{cl})^3)}{k} \\ k_d = \dfrac{(2+\alpha)\xi^{cl}\omega_n^{cl} - 2\xi\omega_n}{k} \end{array}\right\}$$

$$K_p = \frac{(1+2\alpha(\xi^{cl})^2(\omega_n{}^{cl})^2) - \omega_n{}^2}{k} = \frac{8.06}{0.0016} = 5037.5 \tag{4.34}$$

$$k_i = \frac{\alpha\xi^{cl}(\omega_n^{cl})^3}{k} = \frac{7.6}{0.0016} = 4750 \tag{4.35}$$

$$k_d = \frac{(2+\alpha)\xi^{cl}\omega_n^{cl} - 2\xi\omega_n}{k} = \frac{4.5}{0.0016} = 2812.5 \tag{4.36}$$

The closed loop analysis of proportional-integral-derivative controller with a plant of interacting coupled tank system as follows,

$$G_p(s) = \frac{0.0016}{S^2+1.2S+0.16} \tag{4.37}$$

The disturbance that consider $\frac{h_2(s)}{D_{in}(s)} = \frac{R_2}{A_2R_2s+1} = \frac{0.01}{2.5s+1}$ (4.38)

For modeling of coupled tank system with disturbance, use the plant of CTS cascade with disturbance that considered on (4.38) then control the nonlinear system using FOPID controller.

The selecting the desired parameters of a closed system and using the relation in (3.18), the gain of the classical-PID controller gain is considered which produce particular pole placement at specific- damping and frequency, providing relative dominance (α) in selected iteratively by examining the accurateness of system response. Here specific considerations of closed loop scheme preferred are $\xi^{cl} = 0.95$ and $\omega_n^{cl} = 2\frac{rad}{sec}$ selecting the appropriate value of the relative dominance by trial and error by checking the accuracy of system response.

4.12. Stability of the coupled tank system

The stability of the system can be determined by using Quadratic form of Lyapunov stability criterion. This type of stability is the more general type of stability that a means of checking the system whether stable or not. There are different method of checking stability in side Lyapunov the most common are Direct, Indirect, Quadratic form, and Popov stability. From these methods we use Quadratic form of Lyapunov stability,

Consider the function f(x) = x^TQx, where x = $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $x^T = [x_1 \quad x_2]$ and Q = I (Identity matrix). This is a case of a general form. We perform multiplication to get

$$x^TQx = [x_1 \quad x_2]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= [x_1 \quad x_2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1^2 + x_2^2$$

$V(x_1, x_2) = x_1^2 + x_2^2$, according to the property the system is positive definite of quadratic form. By apply Sylvester's theorem v(x) = x^TPx, in a Quadratic form P is a symmetric matrix because of $P_{nm}= P_{mn}$, that mean $P_{12}=P_{21}$. The Lyapunov equation is $A^TP + PA= -Q$.

In case of nonlinear systems, the behavior for small derivations about the equilibrium point is different from that for large deviations. Hence local stability for such systems does not indicate the overall stability in the state space. Also the nonlinear systems having multiple equilibrium states, the trajectories move from one equilibrium point and tend to other with time. Thus stability in case of nonlinear system is always referred to equilibrium state instead of global term stability which is the total stability of the system.

To check the stability of the system described by equation $G_p(s)$ we will solve the equation of $A^TP + PA= -Q$ where for any symmetric, positive, definite real symmetric matrix Q. Let us select Q=I, where I is identity matrix.

Accordingly, effecting state-space equation of a coupled tank system can be imitated from the (4.42)

$$\left.\begin{aligned} \frac{dx}{dt} &= Ax + Bu \\ y &= Cx + Du \end{aligned}\right\} \tag{4.39}$$

$$A = \begin{bmatrix} 0 & 1 \\ -0.16 & -1.2 \end{bmatrix} \tag{4.40}$$

$$B = \begin{bmatrix} 0 \\ 0.0016 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}, D = 0 \tag{4.41}$$

$$\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -0.16 & -1.2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.0016 \end{bmatrix}\begin{bmatrix} q_{in} \end{bmatrix} \tag{4.42}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{4.43}$$

We can find out matrix P for any arbitrary choice of positive,

$$A^T P + PA = -Q \tag{4.44}$$

Let us;

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

$$A^T P + PA = -Q$$

$$\begin{bmatrix} 0 & -0.16 \\ 1 & -1.2 \end{bmatrix}\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} + \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -0.16 & -1.2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -0.16p_{21} & -0.16p_{22} \\ p_{11} - 1.2p_{21} & p_{12} - 1.2p_{22} \end{bmatrix} + \begin{bmatrix} -0.16p_{12} & p_{11} - 1.2p_{12} \\ -0.16p_{22} & p_{21} - 1.2p_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -0.16p_{21} - -0.16p_{12} & -0.16p_{22} + p_{11} - 1.2p_{12} \\ p_{11} - 1.2p_{21} - 0.16p_{22} & p_{12} - 1.2p_{22} + p_{21} - 1.2p_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

But P is symmetric matrix $\therefore p_{12} = p_{21}$

$$\begin{bmatrix} -0.32p_{12} & -0.16p_{22} + p_{11} - 1.2p_{12} \\ p_{11} - 1.2p_{21} - 0.16p_{22} & 2p_{12} - 2.4p_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \tag{4.45}$$

$-0.32p_{12} = -1$ And $2p_{12} - 2.4p_{22} = -1$

$-0.16p_{22} + p_{11} - 1.2p_{12} = 0$ And $p_{11} - 1.2p_{21} - 0.16p_{22} = 0$

$p_{12} = 3.125$, substitute into $2p_{12} - 2.4p_{22} = -1$ and we get $p_{22} = 3.02$ next substitute p_{12} and p_{22} on $p_{11} - 1.2p_{21} - 0.16p_{22} = 0$ and we get $p_{11} = 4.2332$

$$p = \begin{bmatrix} 4.2332 & 3.125 \\ 3.126 & 3.02 \end{bmatrix} \quad (4.46)$$

Using Sylvester's criterion it can be seen that P is positive definiteness

$$p = \begin{bmatrix} 4.2332 & 3.125 \\ 3.126 & 3.02 \end{bmatrix} = 12.78 - 9.76 = 3.02 > 0$$

$\therefore$ p is positive definite. Hence the equilibrium state at origin is asymptotically stable in the large.

CHAPTER FIVE

SIMULATION RESULTS AND DISCUSSION

5.1. Introduction

This chapter presents the simulation result using fractional order-PID controller for the coupled tank system. And also consider other three controllers like IOPID, digital-PID and TID controller for comparative and verification purpose of the thesis. The response performances of each controller are evaluated based on transient response criteria.

5.2. Closed loop performance of interacting coupled tank system

The closed loop performance of the interacting coupled tank system without any disturbance consideration is investigated using its MATLAB/Simulink model, the parameters used to carry out the simulation studies are provided in Table 4.3.

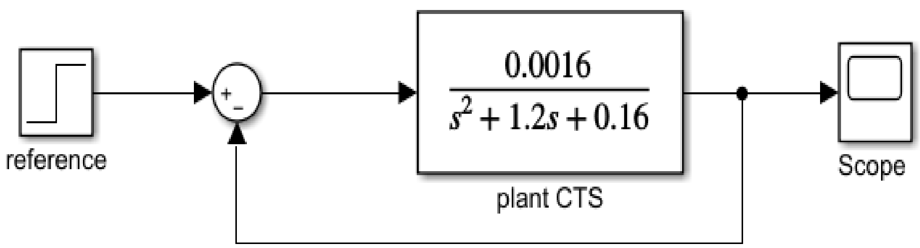

Figure 5.1:- Simulink model of closed loop CTS

The Simulink model of closed loop coupled tank system simulation result shown in figure 5.2.

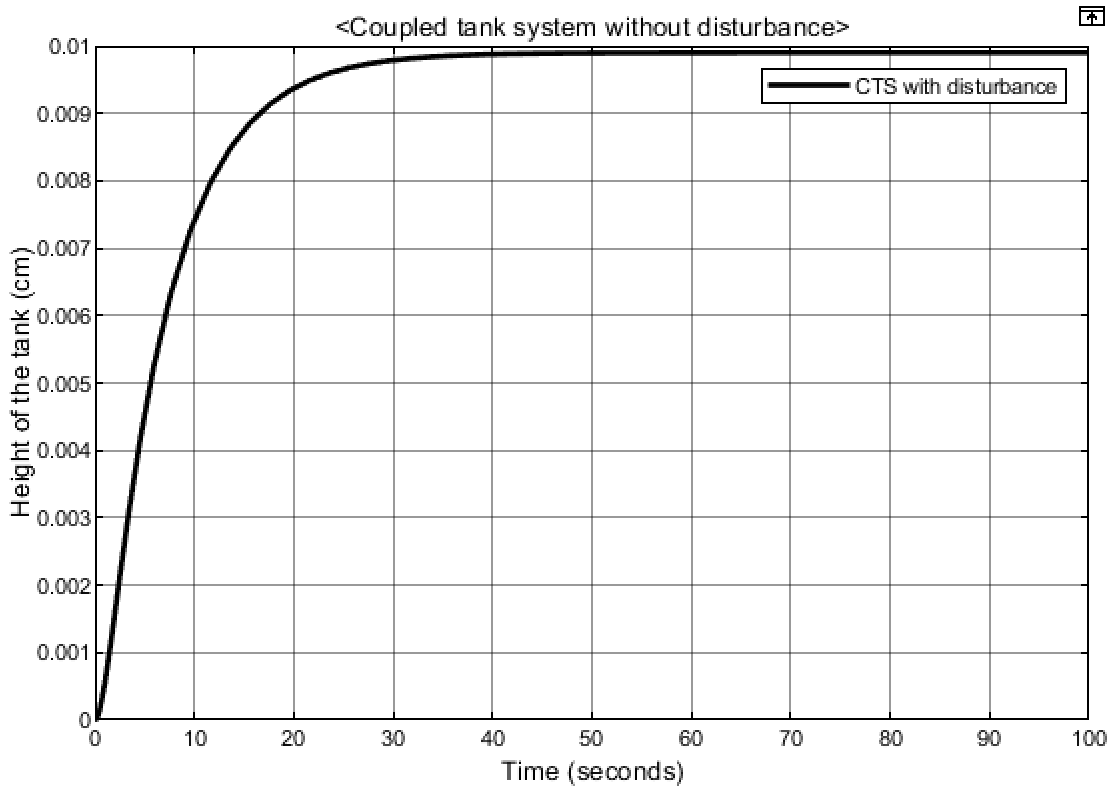

Figure 5.2:- Simulation result of interacting CTS

The closed loop performance of the interacting coupled tank system is investigated using its MATLAB/Simulink model with considering external disturbance, the parameters used to carry out the simulation studies are provided in Table 4.3.

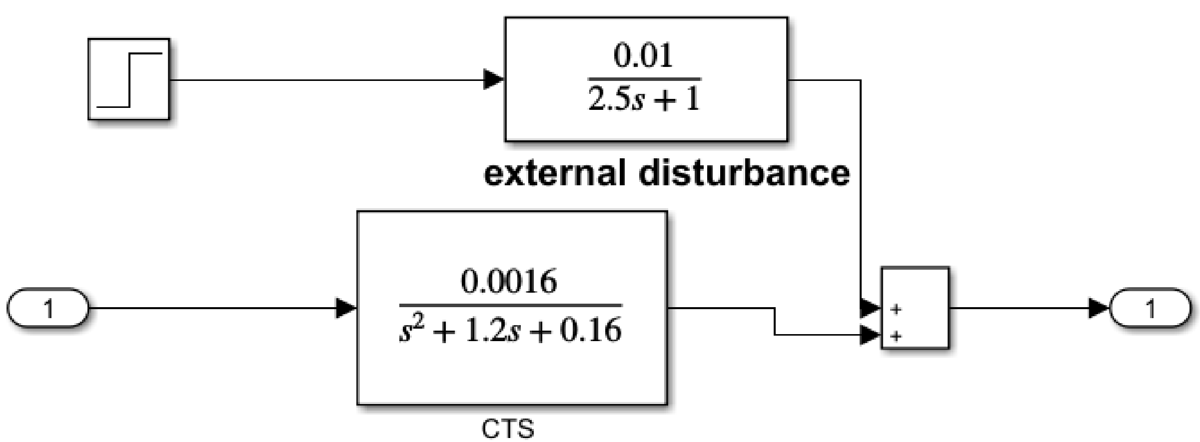

Figure 5.3:- Simulink model of closed loop CTS with disturbance

The simulation result of the Simulink model of closed loop performance of interacting coupled tank system is as shown in figure 5.3. The disturbance it may be moisture or any external noise that affect the system or in other word system to be high nonlinear in case of those disturbance.

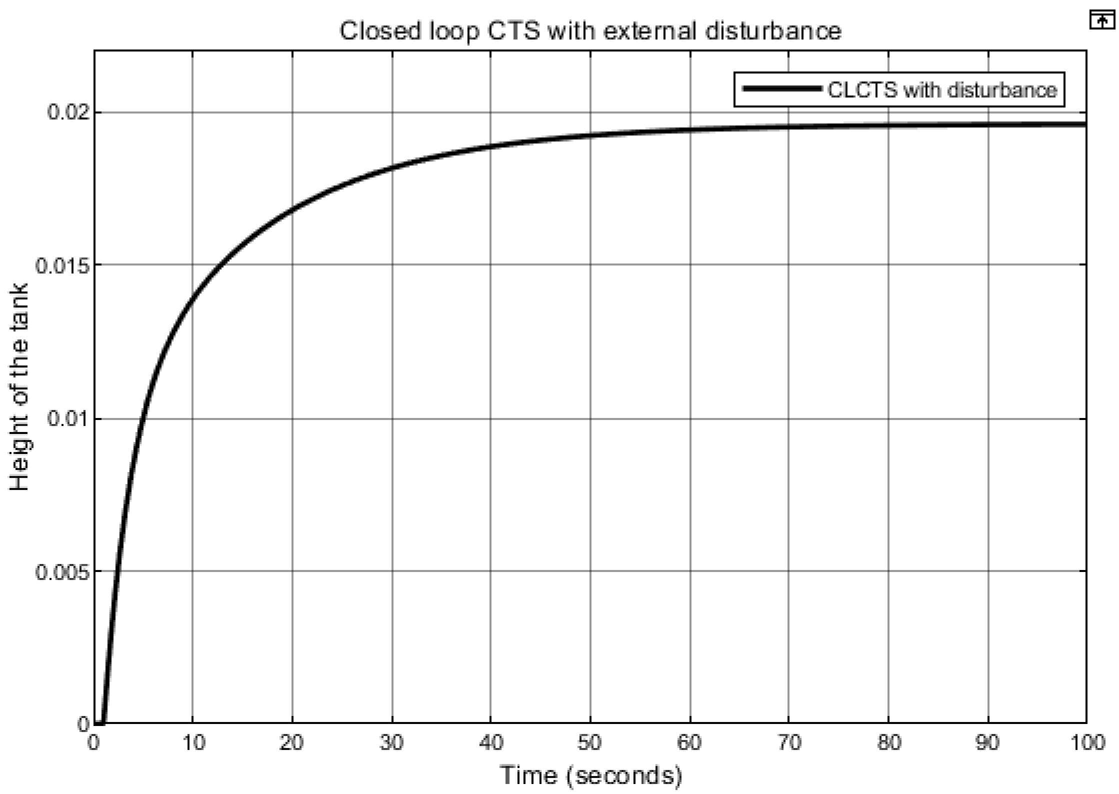

Figure 5.4:- Simulation result of interacting CTS with disturbance

The result shows that, a very week response. This shows that necessity of controller for regulating of the system.

5.3. Performance of interacting coupled tank with conventional-PID controller

To demonstrate the performance of the interacting coupled tank with a PID controller using in MATLAB/Simulink, in this Simulink the interacting coupled tank system without external disturbance done. The parameter of the level control is given in table 4.3 a Simulink model of the interacting coupled-tank with the PID controller is given in figure 5.5.

And then compare the performance of interacting coupled tank system based PID controller with and without disturbance.

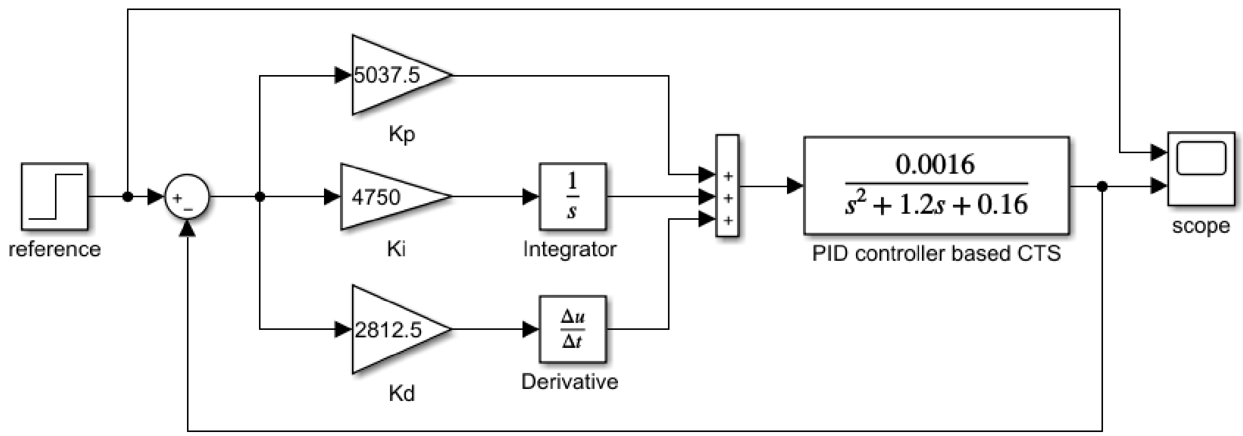

Figure 5.5:- Simulink model of CTS based PID controller

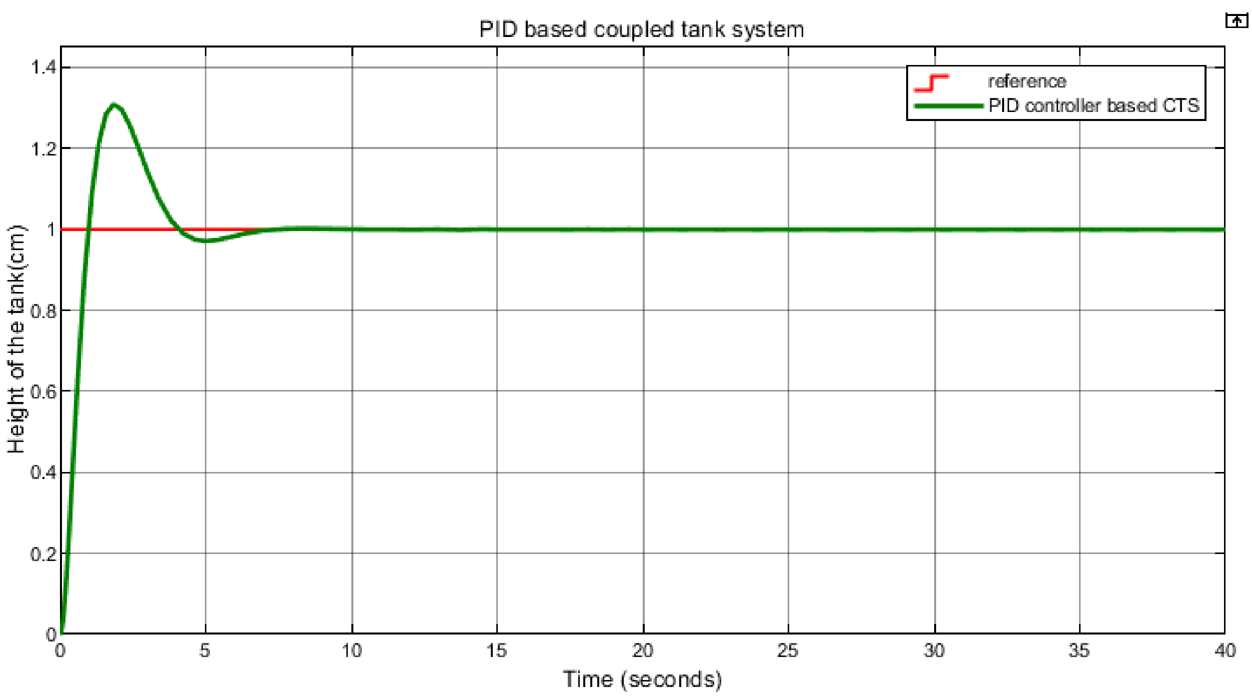

Figure 5.6:- Simulation result of interacting CTS with PID controller

The simulation result of interacting coupled tank system using PID controller is obtain week response compare to the desired output. The system is considering some disturbance (that means the system to be nonlinear) the output response is seeing in figure 5.8.

To demonstrate the performance of the interacting coupled tank with a PID controller using in MATLAB/Simulink, the parameter of the level control is given in table 4.3 a Simulink model of the interacting coupled tank based PID controller with internal and external disturbance and is given in figure 5.7. The external disturbance connected by adder with plant.

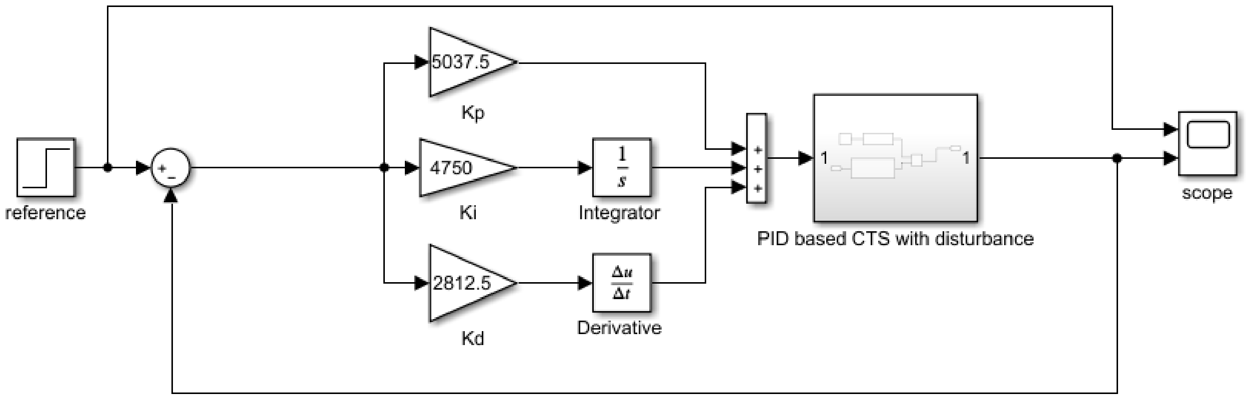

Figure 5.7:- Simulink model of CTS based PID controller with disturbance

In the figure 5.7 shows that the Simulink block of PID controller based CTS with external disturbance contain plant block and disturbance block, the response obtained from the Simulink model is shown in figure 5.8.

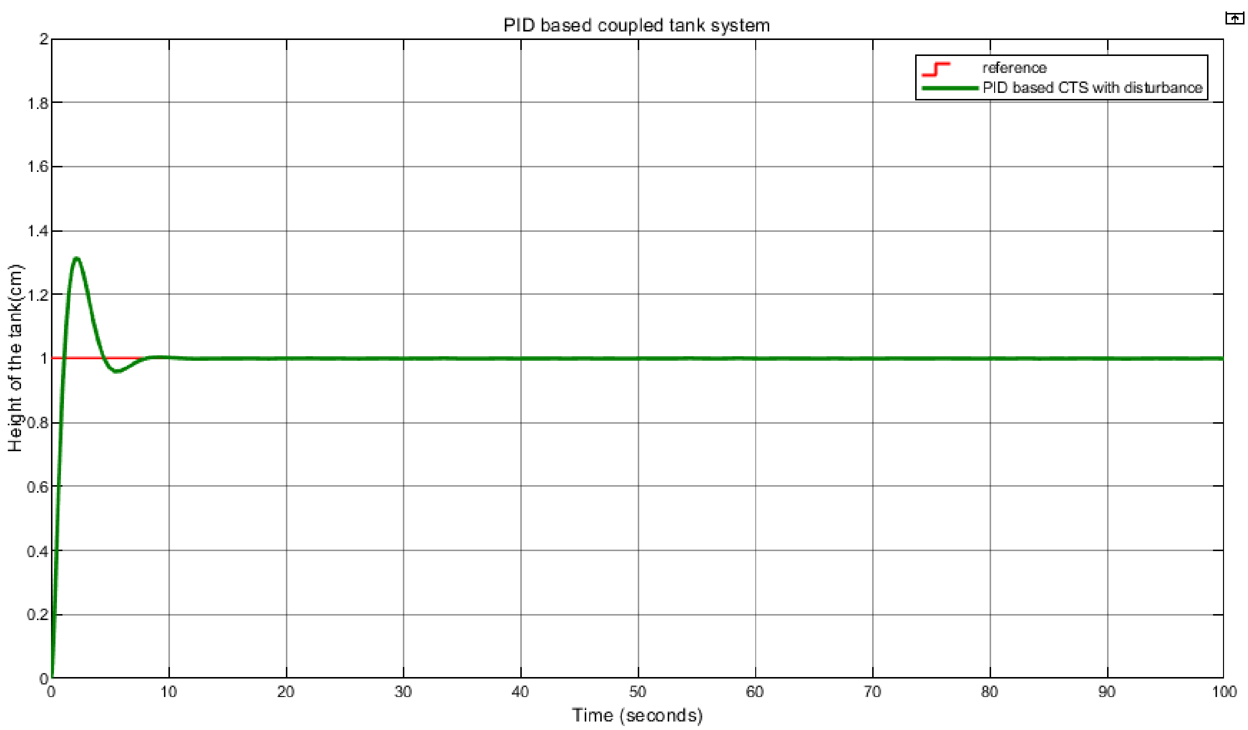

Figure 5.8:- Simulation result of interacting CTS based PID controller with disturbance

As a result, obtained in figure 5.8 using the PID controls for the coupled interacting tank with external disturbance, the overshoot is 30.925% and rise time 790.510*msec*.

5.4. Performance of interacting coupled tank with Digital-PID controller

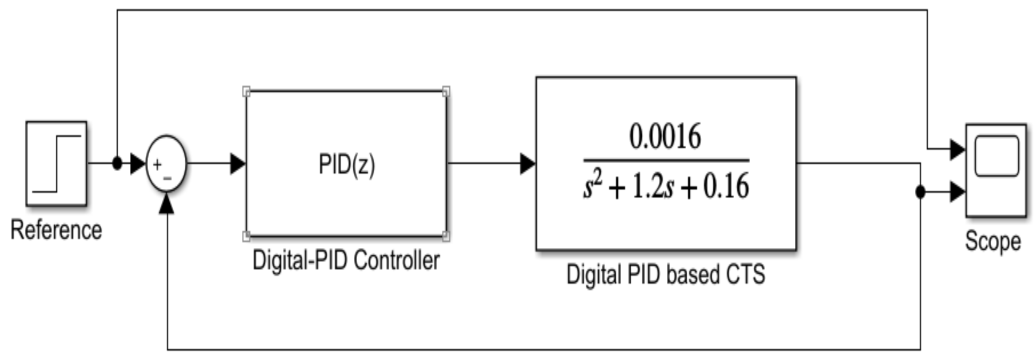

Figure 5.9:- Simulink model of CTS based Digital-PID controller

Steps to digitalize process are seeing the step one by one using Simulink diagram. First click the block diagram of PID control block on MATLAB Library and double click block then select the discrete time. Next goes to discrete time setting and adjust the sampling time (T_s) for better performance use 0.1*sec* then select the parallel form and also select types of method that convert analog to digitalize and filtered method, select the trapezoidal integration method. The reason to use this method is trapezoidal or bilinear transformation by its own convert continues to discrete and also convert the response into continues one. Then goes to controller parameters to tune the parameters and insert the parameter values internally proportional gain (k), integral gain (I), and derivative gain (D). Finally check the overall settings to click apply and then OK.

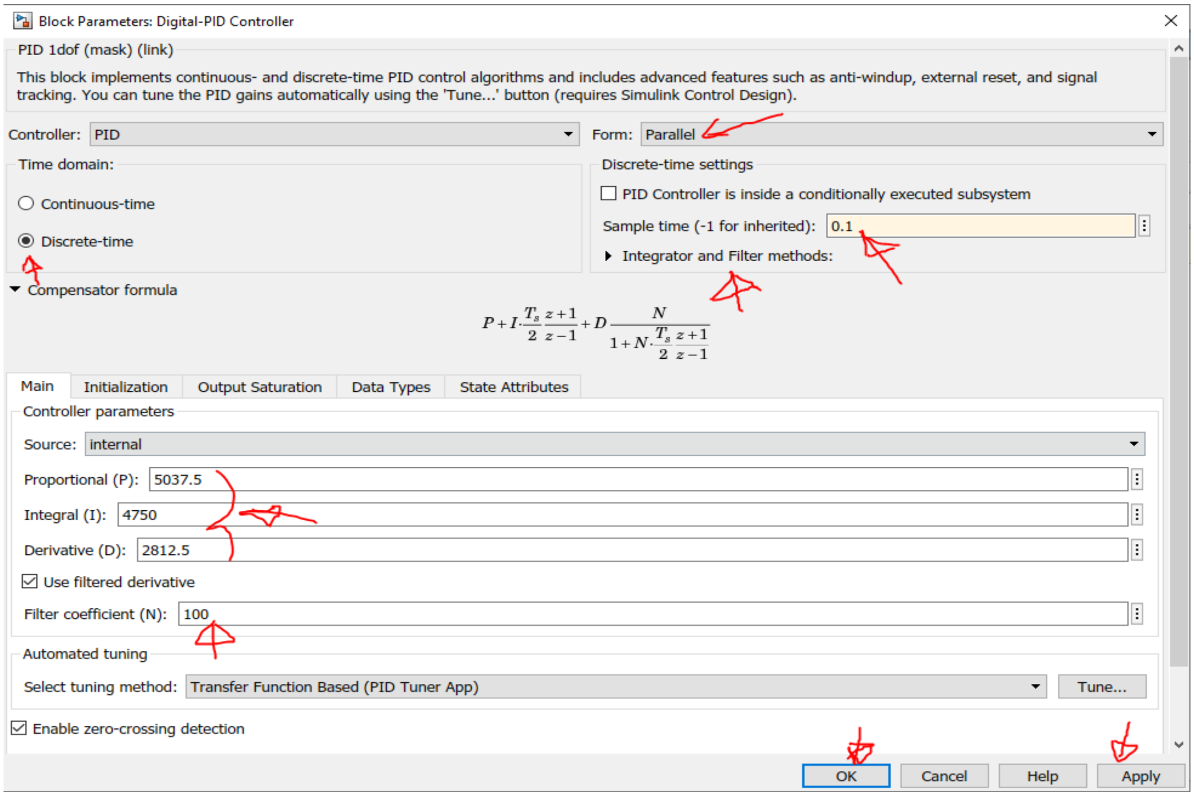

Figure 5.10:- Step to conversion process using Simulink block diagram

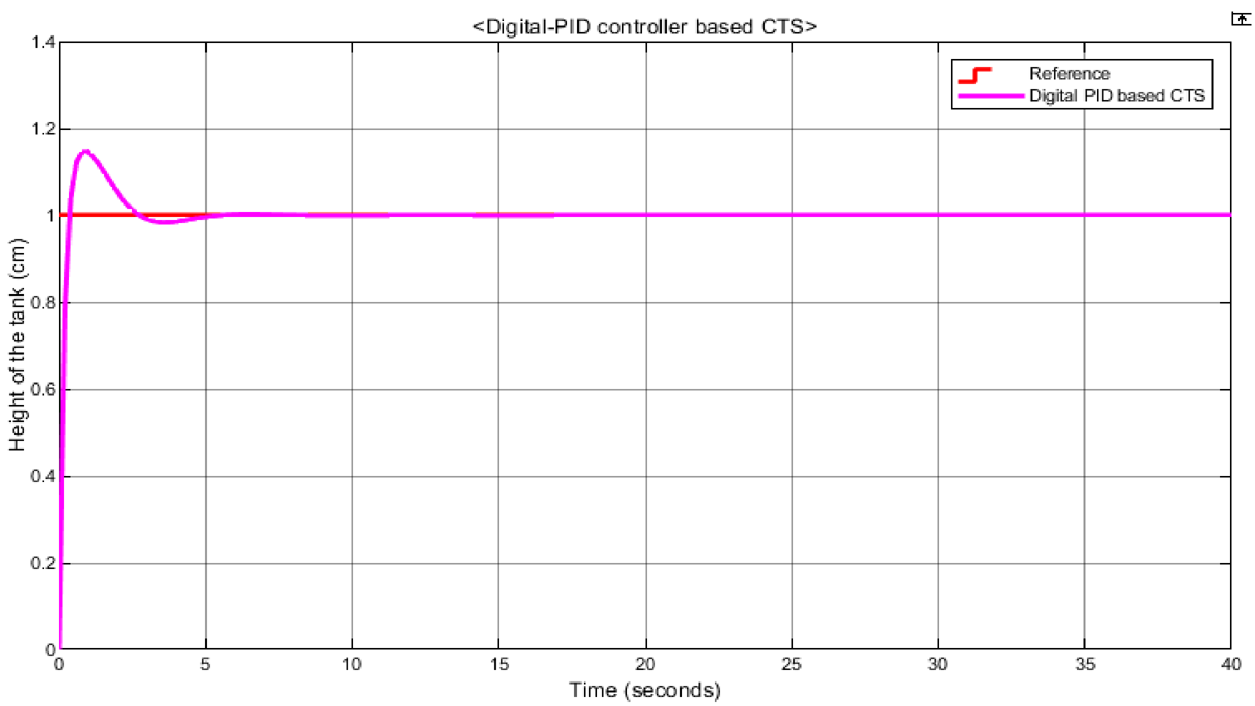

Figure 5.11:- Simulation result of CTS based Digital-PID controller

The simulation result of interacting coupled tank system using digital-PID controller without disturbance get good response compare to PID controller but this result is obtain only in linear system otherwise the system is consider external disturbance (that mean the system to be high nonlinear) the output response is seeing in figure 5.13.

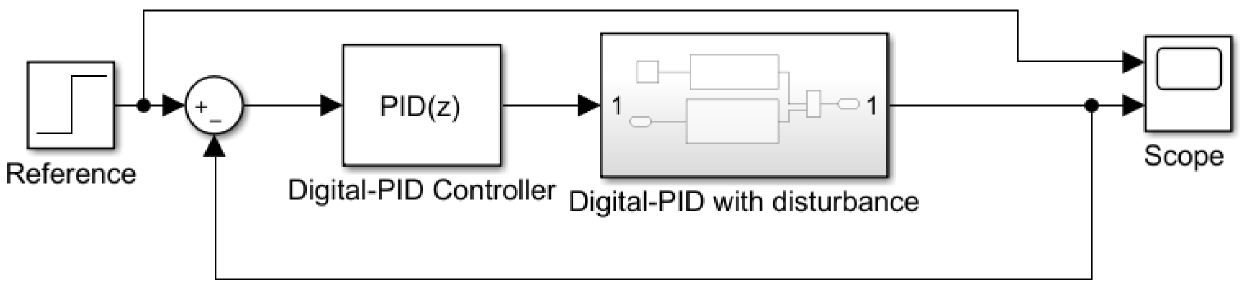

Figure 5.12:- Simulink model of CTS based Digital-PID controller with disturbance

In figure 5.12 the CTS based digital-PID block contains the plant, external disturbance. The digital-PID controller based CTS with disturbance the simulation result shows oscillated signal. But the response doesn't show the desired output.

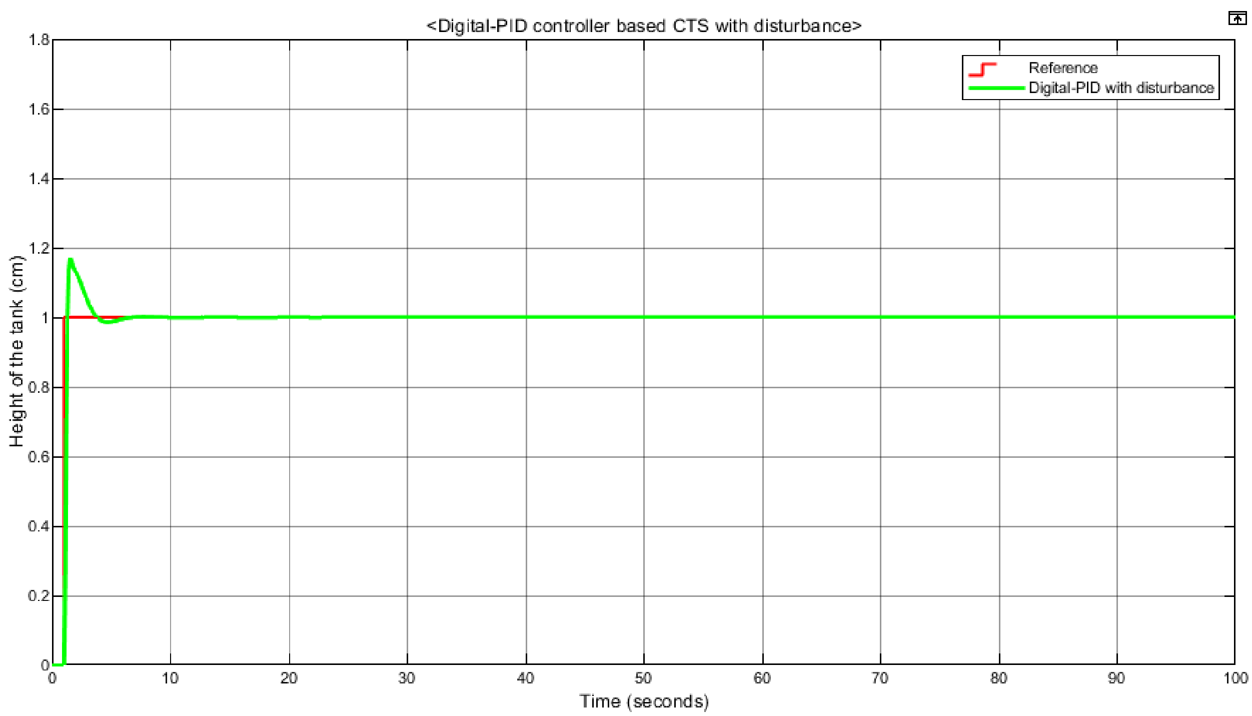

Figure 5.13:- Simulation result of CTS based Digital-PID controller with disturbance

When the sampling time change the simulation result of Digital-PID controller various output shows. This mean you have to select the appropriate value to get better response. So the appropriate value of the sampling time for digital-PID controller is 0.1*sec* this get by checking the accuracy of the response. As a result obtained the overshoot is 17.059*%* and rise time 223.066*msec*.

5.5. Performance of interacting coupled tank with TID controller

Tilt-integral-derivative (TID) compensator is one of the factional order controllers, this control structure most like conventional PID controller with three tunable parameters. But in TID the proportional behavior replaced by $s^{(1/n)}$ or $s^{(-1/n)}$, n is nonzero real number and for this plant prefer 5. To demonstrate the performance of coupled interacting tanks with the TID controller in MATLAB/Simulink, the parameters of the level control are the same as given in table 4.3. The MATLAB/Simulink model of interacting tanks with TID control is shown in figure 5.14,

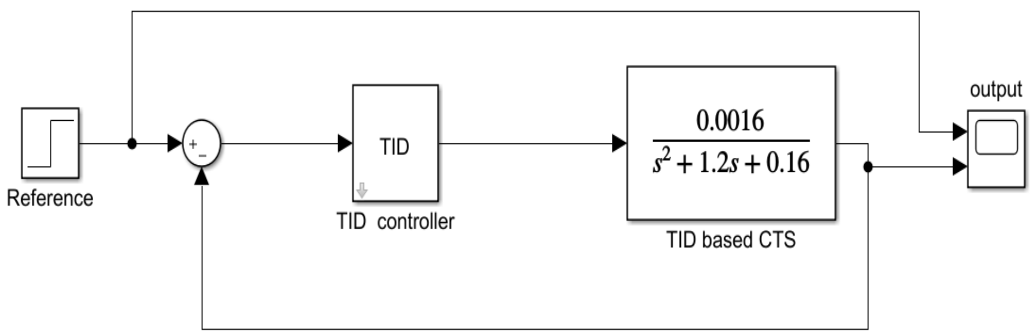

Figure 5.14:- Simulink model of CTS based TID controller

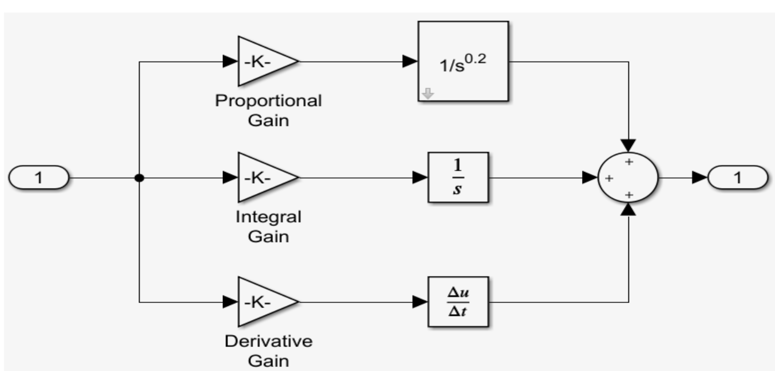

Figure 5.15:- Internal of tilted integral derivative (TID) controller

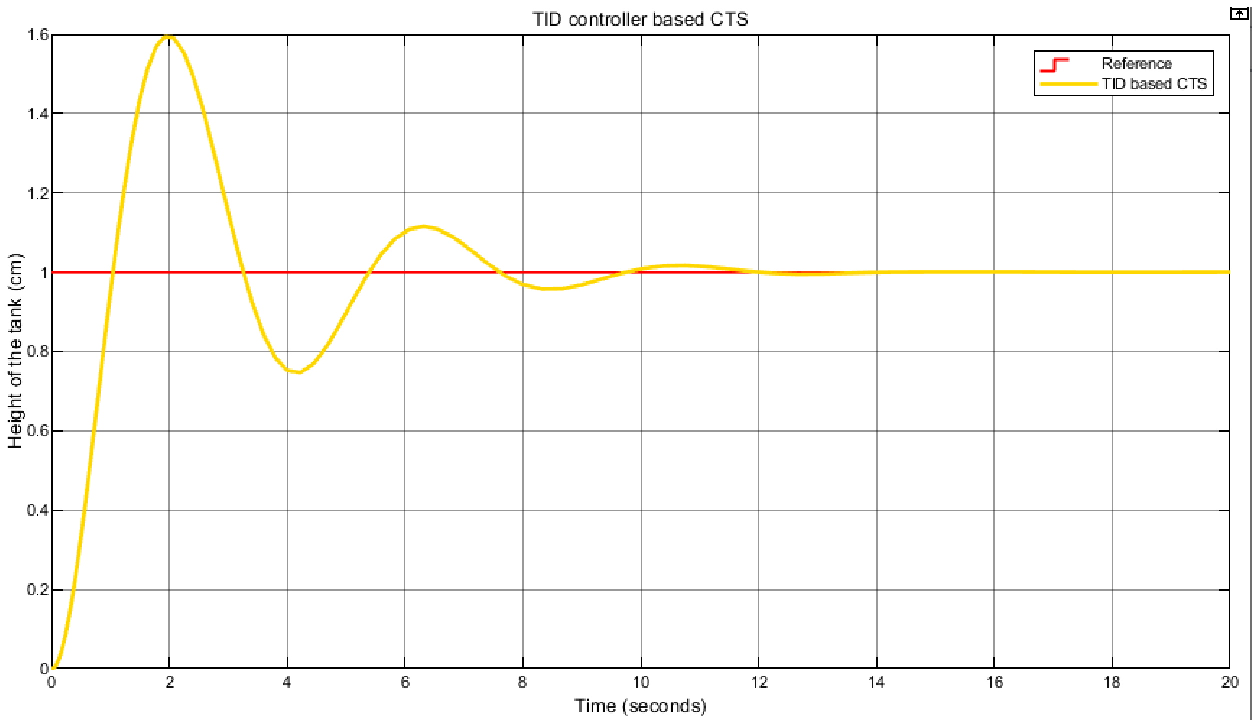

Figure 5.16:- Simulation result of CTS based TID controller with disturbance

The simulation result shows that, TID controller also not obtain the desired response for linear system. For nonlinear system (considering external disturbance) the TID controller based CTS shows in figure 5.17.

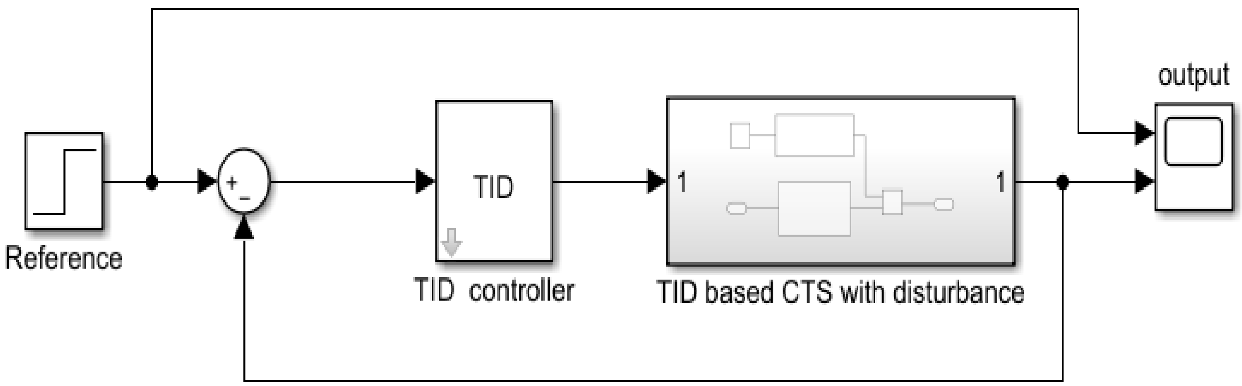

Figure 5.17:- Simulink model of CTS based TID controller with disturbance

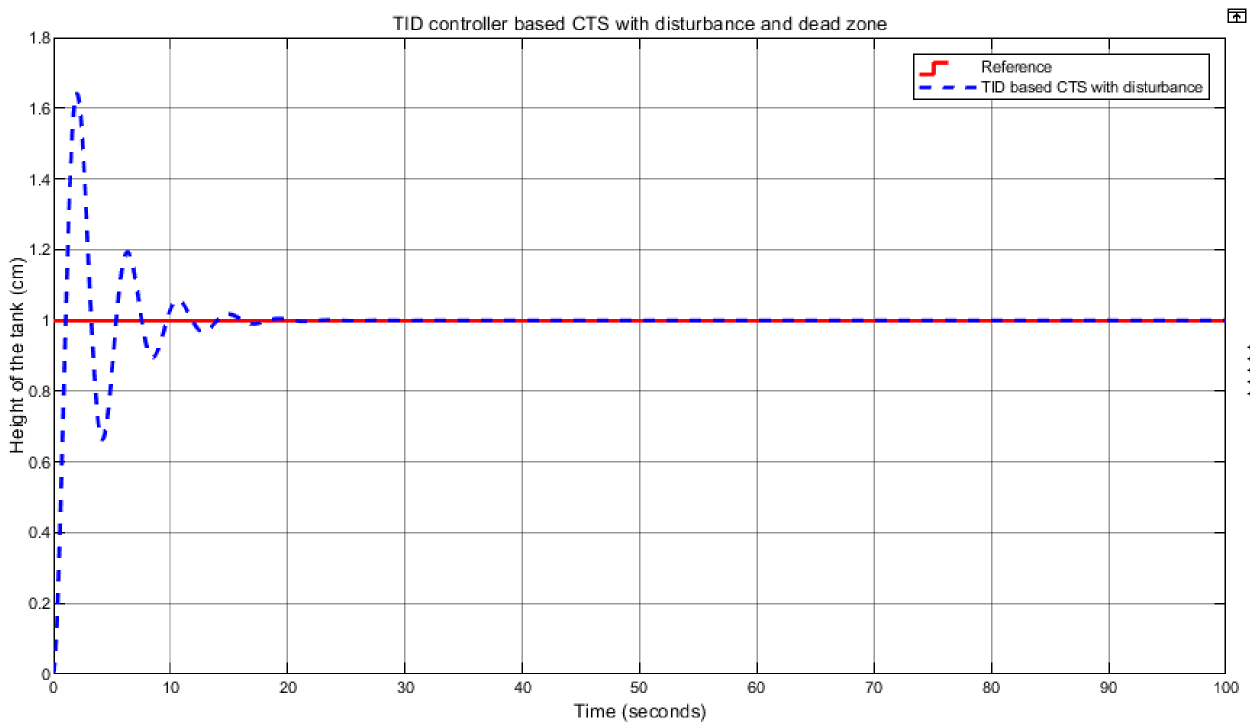

Figure 5.18:- Simulation result of CTS based TID controller with disturbance

From the result obtained in Figure 5.18, the output response of a system below the set point when the value of n is an increase, but if the value of n equals to five the overshoot is 115.965% the desired output of the interacting level tank is obtained at the rise time 488.654*msec*. This shows that TID controller is not suitable controller for nonlinear interacting coupled tank system.

5.6. Performance of interacting coupled tank with fractional-order PID controller

To demonstrate the performance of coupled interacting tank with fractional order PID controller in MATLAB/Simulink, the parameters of the level control are the same as given in table 4.3. FOPID controller combines PID controllers gain and in addition to two parameters those are lambda (λ) with integral gain and Miu (μ) with derivative gain. A simulated model for the fractional-order PID control system is as shown in figure 5.19.

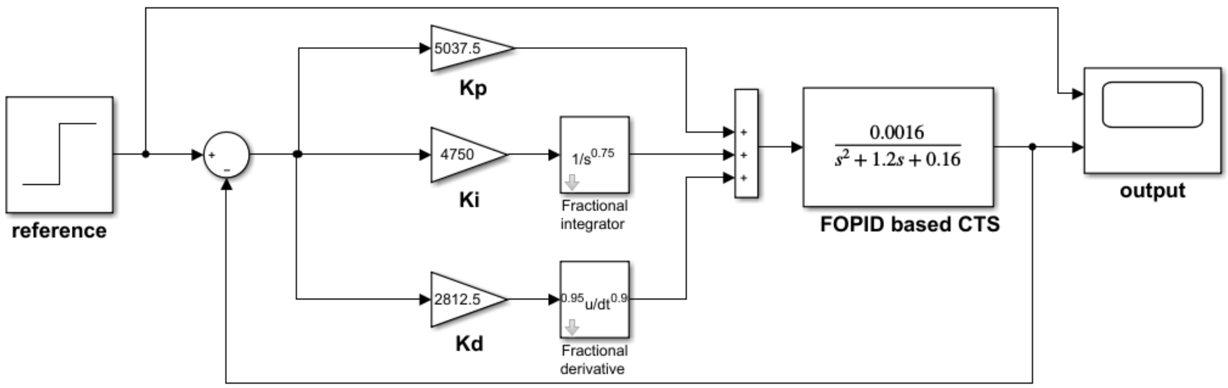

Figure 5.19:- Simulink model of CTS based FOPID controller

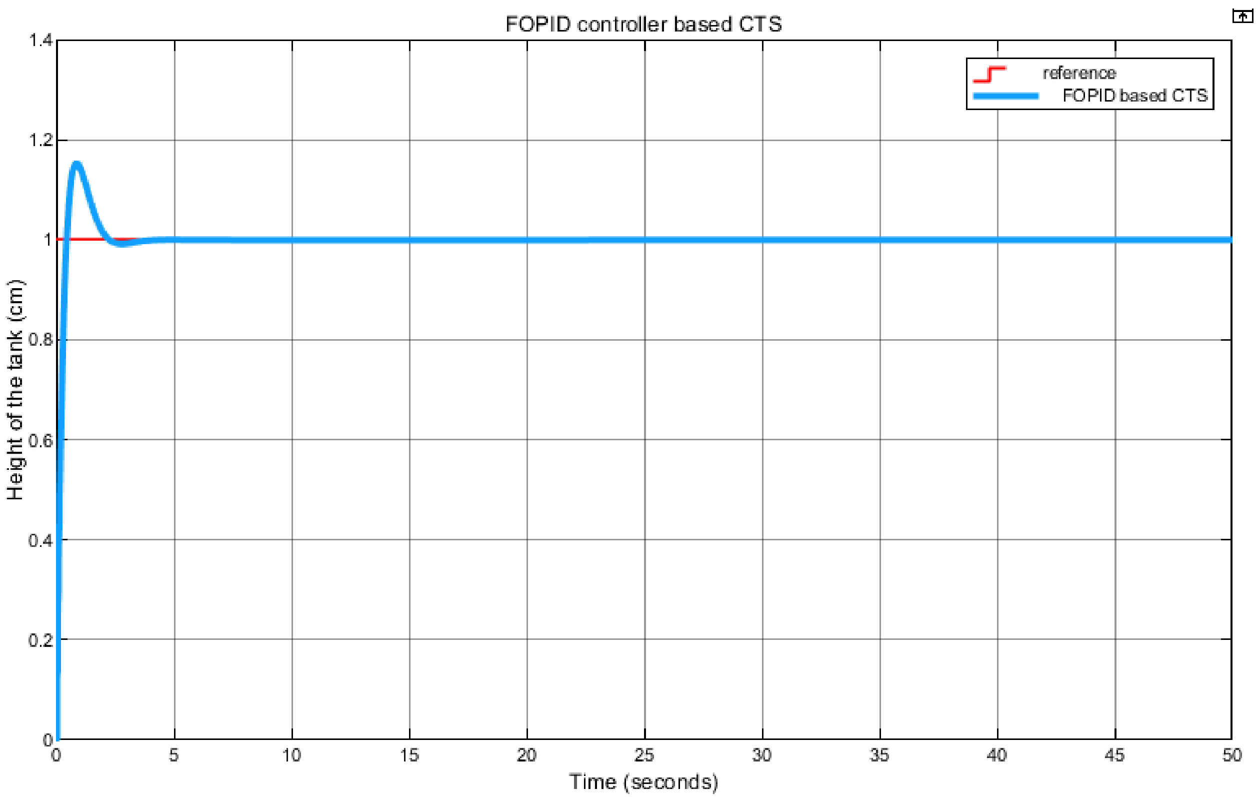

Figure 5.20:- Simulation result of CTS based FOPID controller

The overall comparative analysis of the simulation result of the entire controller based CTS without any disturbance the fractional order-PID controller shows the better response for linear system. In high nonlinear system of CTS shown in figure 5.21

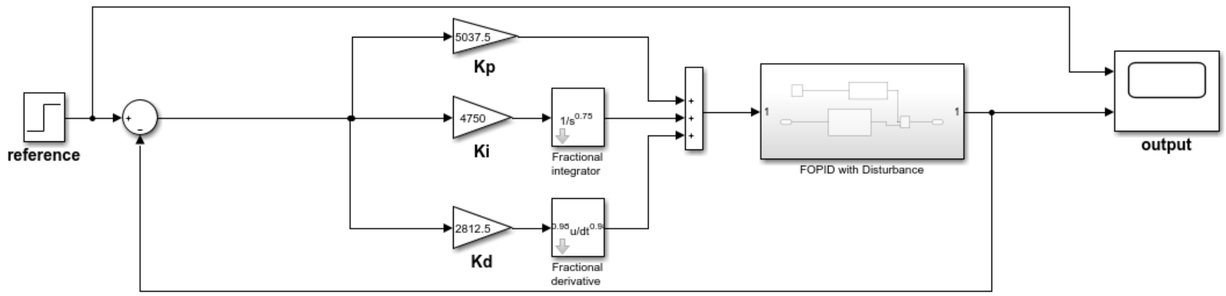

Figure 5.21:- Simulink model of CTS based FOPID controller with disturbance

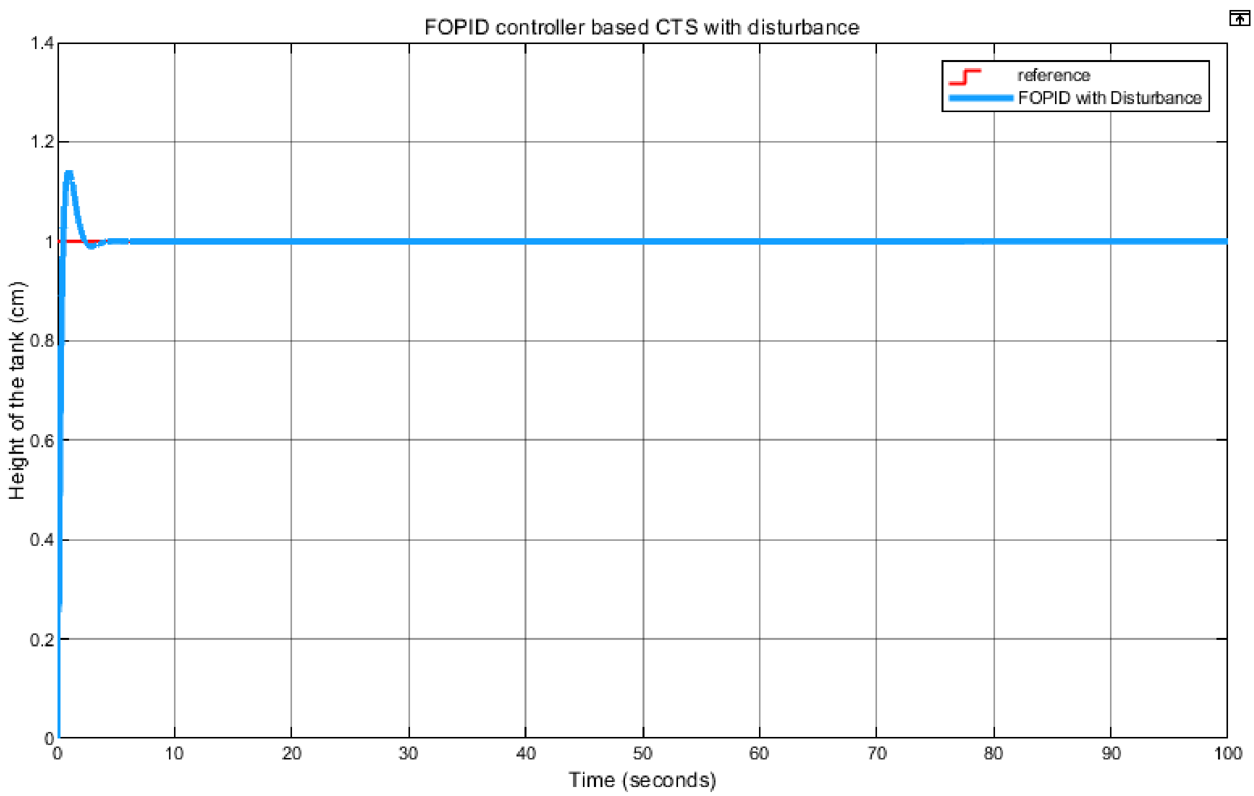

Figure 5.22:- Simulation result of CTS based FOPID controller with disturbance

As we observed from figure 5.21, the result obtained in Figure 5.22 using fractional order PID controller for the coupled interacting tank. The range of integral and derivative order is [0, 1]. By checking the accuracy of the response the parameter value selected is λ and μ, is 0.75 and 0.95 respectively. We don't have to need to add dead zone for FOPID because the controller by its own reduce the time delay and noise. The overshoot is 14.368%, the desired output of the interacting level tank is obtained rise time 335.485*msec*. This result shows the good response compare to tested controllers.

The overall performance comparison of interacting coupled tank system with conventional-PID, Digital-PID, TID, and FOPID is show in figure 5.23 and figure 5.24.

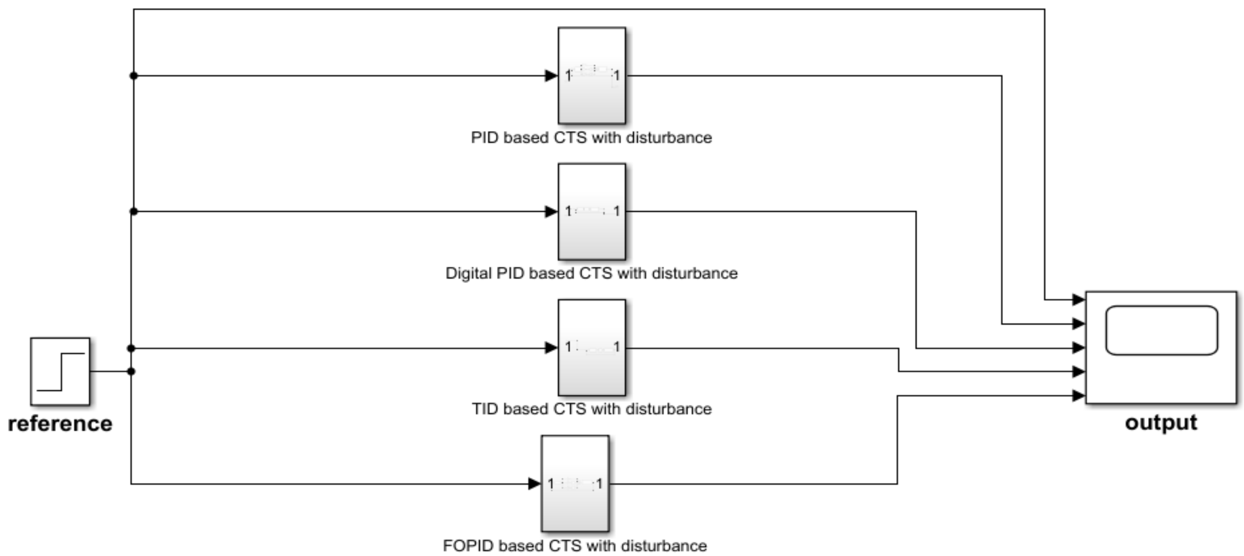

Figure 5.23:- Simulink model of Comparative performance analysis of CTS with PID, digital-PID, TID, and FOPID controller

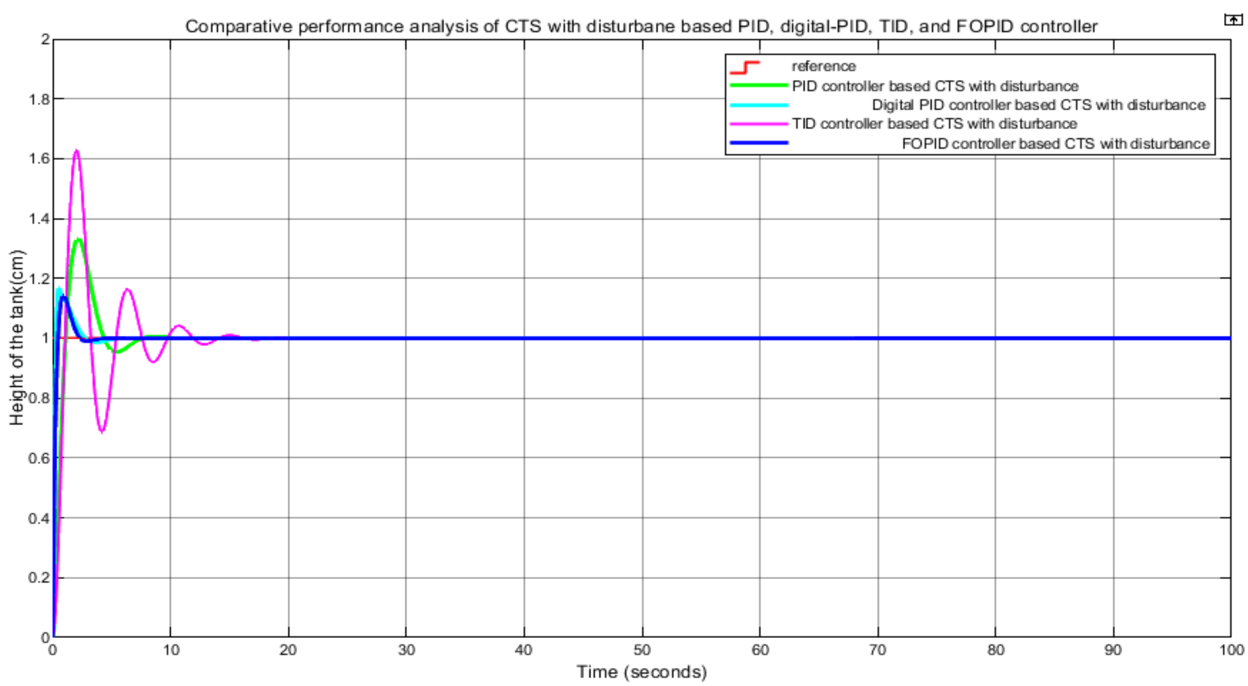

Figure 5.24:- Simulation result of comparative performance analysis of CTS with PID, digital-PID, TID, and FOPID controller

The overall comparison of all the method of controllers use in this work and the specific values of each result shown in table 5.1,

Table 5.1:- Comparative results of all method of controllers with considering disturbance

Method of controllers	**Performance Criteria's**						Integral and derivative order	
	Overshoot O_s (%)	Undershoot (%)	Rise time T_r (*sec*)	Delay time T_d (*sec*)	Peak time T_p (*sec*)	Settling time T_s (*sec*)	**λ**	**μ**
IOPID	30.925	1.789	0.790	1.013	2.302	8.195	1	1
Digital-PID	17.059	-0.824	0.223	1.359	1.540	6.612	-	-
TID	115.965	2.562	0.488	1.034	2.162	15.508	-	-
FOPID	14.368	1.969	0.336	0.338	1.014	3.581	0.75	0.95

CHAPTER SIX

CONCLUSION AND FUTURE WORKS

6.1. Conclusion

Process control is essential in the industrial process because it guarantees the safety and optimization is a process. Additionally, process control is a useful tool to satisfy the environmental procedure and product quality necessities. For chemical engineers, process control is widely appropriate in the manufacturing system. Governing the height of the container in an industrial process is the main issue, for that case needed to design the appropriate controller. In this thesis work performance analysis FOPID control for process-level in case of the coupled tank system.

The FOPID controller is the enlargement of the conventional-PID controller based on fractional calculus. For numerous years, in industries, an IOPID control has been common in the function of industrial control. The greatness involves the clarity of design and its best achievement, such as a low percentage of overshoot and small rise time in slow industrial processes which are essential. FOPID Controller is to deliver a better feedback loop controller having the asset of the other tested controllers, but if and only if a response which is nearer to the academically optimal response.

FOPID controller with integral order and derivative order has a better robustness compared to the other types of controller like TID, Digital-PID and integer-order-proportional-integral-derivative controllers that tested on this work. The simulation result shows that controllers used in linear case of CTS the performance is good, but including external disturbance on the second tank the controllers tested on this work is don’t show better response except FOPID controller. The reason is FOPID controller has two more adjust degree of freedom those increase the robustness of the system.

Generally the overall this thesis work shows that the FOPID controller is the suitable controller to monitor the nonlinear system, model uncertainty, reduction of disturbance, and handling of time delay compares to different tested controllers. Therefore, the robustness of FOPID controller is better achievement compare to the tested controller on this work.

6.2. Future works

In this work the considered disturbance of the system is external disturbance on the second tank, this came the system to nonlinear property, but also there may be a probability of external disturbance occur on the first tank, disturbance occur in measurement error, the researcher suggests those occurrences as a future research ideas.

Moreover, I suggest testing another optimal control techniques and algorithm than the used prescribed convergence algorithm. Analyze and evaluate the performance of optimal control techniques designed using different algorithms could lead to new insight.

REFERENCES

[1] R. F. Hassan, “Optimized Structure of Digital Pid Controller,” no. April, 2008.

[2] T. Uzunović, E. Žunić, A. Badnjević, I. Mioković, and S. Konjicija, “Implementation of digital PID controller,” *MIPRO 2010 - 33rd Int. Conv. Inf. Commun. Technol. Electron. Microelectron. Proc.*, no. March 2016, pp. 1357–1361, 2010.

[3] K. Arab and A. Mp, “PID Control Theory,” *Introd. to PID Control. - Theory, Tuning Appl. to Front. Areas*, no. February, 2012, doi: 10.5772/34364.

[4] S. Karad, S. Chatterji, and P. Suryawanshi, “Performance Analysis of Fractional Order PID Controller with the Conventional PID Controller for Bioreactor Control,” *Int. J. Sci. Eng. Res.*, vol. 3, no. 6, pp. 1–6, 2012.

[5] H. R. Patel and V. A. Shah, “Comparative study between Fractional Order P I λ D µ and Integer Order PID Controller : A case study of coupled conical tank system with actuator faults,” *2019 4th Conf. Control Fault Toler. Syst.*, pp. 390–396, 2019.

[6] S.-E. Oltean, M. Dulau, and A.-V. Duka, “Model Reference Adaptive Control Design for Slow Processes. A Case Study on Level Process Control,” *Procedia Technol.*, vol. 22, no. March, pp. 629–636, 2016, doi: 10.1016/j.protcy.2016.01.131.

[7] W. Li, H. Ding, and K. Cheng, “An investigation on the design and performance assessment of double-PID and LQR controllers for the inverted pendulum,” *Proc. 2012 UKACC Int. Conf. Control. Control 2012*, no. September, pp. 190–196, 2012, doi: 10.1109/CONTROL.2012.6334628.

[8] N. Viswanadhapalli, “Design and Optimization of A PID Controller for Controlling Desired Level of A Design and Optimization of A PID Controller for Controlling Desired Level of A Coupled Tank System,” no. February, 2021.

[9] Q. Dung Vuong, T. huy Tran, and T. Loi Nguyen, “Tuning PID Controller Bases on Random Search Algorithms,” *Int. J. Eng. Res.*, vol. V10, no. 01, pp. 73–77, 2021, doi: 10.17577/ijertv10is010038.

[10] H. Jaafar, S. Hussien, and N. Selamat, "Development of PID Controller for Controlling Desired Level of Coupled Tank System," *Int. J. Innov. Technol. Explor. Eng.*, vol. 3, no. 9, pp. 32–36, 2014, [Online]. Available: http://ijitee.org/attachments/File/v3i9/I1469023914.pdf.

[11] A. Prakash, A. P. Yadav, and R. Kumar, "Design of IMC based PID Controller for Coupled Tank System," *Int. J. Eng. Res. Technol.*, vol. 4, no. 15, pp. 1–8, 2016.

[12] D. Y. Abramovitch, "A unified framework for analog and digital PID controllers," *2015 IEEE Conf. Control Appl. CCA 2015 - Proc.*, no. September 2015, pp. 1492–1497, 2015, doi: 10.1109/CCA.2015.7320822.

[13] K. Sharma and D. K. Palwalia, "Control of DC-DC Converters," 2017.

[14] D. Horla and T. Sadalla, "Optimal tuning of fractional-order controllers based on Fibonacci-search method," *ISA Trans.*, vol. 104, no. September, pp. 287–298, 2020, doi: 10.1016/j.isatra.2020.05.022.

[15] R. Sharma, S. Mohanty, and A. Basu, "Tuning of digital PID controller for blood glucose level of diabetic patient," *2016 IEEE Int. Conf. Recent Trends Electron. Inf. Commun. Technol. RTEICT 2016 - Proc.*, no. May, pp. 332–336, 2017, doi: 10.1109/RTEICT.2016.7807837.

[16] A. J. Guel-Cortez, C. F. Méndez-Barrios, E. jin Kim, and M. Sen, "Fractional-order controllers for irrational systems," *IET Control Theory Appl.*, vol. 15, no. 7, pp. 965–977, 2021, doi: 10.1049/cth2.12095.

[17] A. L. S. Nair, S. A. J. Mary, and J. A. Linsely, "Modeling and control of level control process - A comparative study," *2017 Innov. Power Adv. Comput. Technol. i-PACT 2017*, vol. 2017-January, pp. 1–4, 2017, doi: 10.1109/IPACT.2017.8245161.

[18] V. J. Sruthi and L. S. Binu, "Fractional Order PID Controller for Level Control in a Spherical Tank," vol. 4, no. 17, pp. 1–5, 2016.

[19] M. Nafea, A. R. M. Ali, J. Baliah, and M. S. M. Ali, "Metamodel-based optimization of a

PID controller parameters for a coupled-tank system," *Telkomnika (Telecommunication Comput. Electron. Control.*, vol. 16, no. 4, pp. 1590–1596, 2018, doi: 10.12928/TELKOMNIKA.v16i4.9069.

[20] M. Awais, H. Alam, and A. Muhammad, "Model Predictive Control Design for coupled Tank System," *Int. Conf. Qual. Time 2017*, no. January, 2018, [Online]. Available: https://www.researchgate.net/publication/322750799_Model_Predictive_Control_Design_for_coupled_Tank_System.

[21] J. K. Goyal, S. Aggarwal, S. Ghosh, S. Kamal, and U. Vuyyuru, "R∞ Based PI Controller Design for Coupled Tank System through Polytopic Modeling," *IECON Proc. (Industrial Electron. Conf.*, vol. 2019-October, no. December, pp. 383–388, 2019, doi: 10.1109/IECON.2019.8927571.

[22] A. Kumar, M. Vashishth, and L. Rai, "Liquid level control of coupled tank system using Fractional PID controller," *Int. J. Emerg. Trends Electr. Electron.*, vol. 3, no. 1, pp. 61–64, 2013.

[23] P. G. G. Keerthana, J. Gnanasoundharam, and K. Latha, "Design of Model Reference Adaptive Controller for Two Tank Conical Interacting System," *2016 Int. Conf. Comput. Power, Energy, Inf. Commun. ICCPEIC 2016*, no. 2, pp. 255–258, 2016, doi: 10.1109/ICCPEIC.2016.7557205.

[24] L. Samet, N. Masmoudi, M. W. Kharrat, and L. Kamoun, "A digital PID controller for real time and multi loop control: A comparative study," *Proc. IEEE Int. Conf. Electron. Circuits, Syst.*, vol. 1, pp. 291–296, 1998, doi: 10.1109/ICECS.1998.813324.

[25] A. R. Gopisetti, B. A. Reddy, K. Anusha, and R. P. Kumar, "Performance Evaluation of Second Order Sliding Mode Control Strategies for a Coupled Tank System," *Proc. 2018 Int. Conf. Curr. Trends Towar. Converging Technol. ICCTCT 2018*, pp. 1–7, 2018, doi: 10.1109/ICCTCT.2018.8551080.

[26] P. N. Paraskevopoulos, *Modern control engineering*. 2017.

[27] I. Tejado, B. M. Vinagre, J. E. Traver, J. Prieto-Arranz, and C. Nuevo-Gallardo, "Back to

basics: Meaning of the parameters of fractional order PID controllers," *Mathematics*, vol. 7, no. 6, pp. 1–16, 2019, doi: 10.3390/MATH7060530.

[28] B. J. Parvat, S. A. Deo, and B. Kaduc, "Mathematical Modeling of Interacting and Non Interacting Tank System," *Int. J. Appl. or Innov. Eng. Manag.*, vol. 4, no. 1, pp. 86–92, 2015.

[29] T. L. Mien, "Liquid Level Control of Coupled-Tank System Using Fuzzy-Pid Controller," *Int. J. Eng. Res. Technol.*, vol. 6, no. 11, pp. 459–464, 2019, [Online]. Available: www.ijert.org.

[30] Q.-G. Wang, Z. Zhang, K. J. Astrom, Y. Zhang, and Y. Zhang, *Guaranteed Dominant Pole Placement with PID Controllers*, vol. 41, no. 2. IFAC, 2008.

[31] P. Aplcarian and H. D. Tuan, "Parameterized LMIs In Control Theory - Decision and Control, 1998. Proceedings of the 37th IEEE Conference on," no. December, pp. 152–157, 1998.

[32] M. G. Stohy, H. S. Abbas, A.-H. M. El-Sayed, and A. G. Abo El-maged, "Parameter Estimation and Pi Control for a Water Coupled Tank System," *J. Adv. Eng. Trends*, vol. 38, no. 2, pp. 147–159, 2020, doi: 10.21608/jaet.2020.73062.

[33] M. S. Fadali and A. Visioli, *Digital Control Engineering Analysis and Design Second Edition*. 2012.

[34] P. Oziablo, D. Mozyrska, and M. Wyrwas, "A digital PID controller based on Grünwald-Letnikov fractional-, variable-order operator," *2019 24th Int. Conf. Methods Model. Autom. Robot. MMAR 2019*, pp. 460–465, 2019, doi: 10.1109/MMAR.2019.8864688.

[35] H. Wang, S. Y. Gelbal, and L. Guvenc, "Multi-Objective Digital PID Controller Design in Parameter Space and its Application to Automated Path following," *IEEE Access*, vol. 9, pp. 46874–46885, 2021, doi: 10.1109/ACCESS.2021.3066925.

[36] W. Jing and M. Zhou, "Digital PID controller designed and realized based on state machine," *Proc. 2016 IEEE Adv. Inf. Manag. Commun. Electron. Autom. Control Conf.*

IMCEC 2016, pp. 1873–1877, 2017, doi: 10.1109/IMCEC.2016.7867543.

[37] X. Dingyu and C. Yangquan, *Solving applied mathematical problems with MATLAB*. 2009.

[38] S. Sim, W. So, and H. G. Yeh, "SSR alleviation via analog and digital PID controller," *2016 IEEE Green Energy Syst. Conf. IGSEC 2016*, 2016, doi: 10.1109/IGESC.2016.7790072.

[39] R. E. Gutiérrez, J. M. Rosário, and J. Tenreiro MacHado, "Fractional order calculus: Basic concepts and engineering applications," *Math. Probl. Eng.*, vol. 2010, 2010, doi: 10.1155/2010/375858.

[40] D. Kumar and D. Baleanu, *Editorial: Fractional calculus and its applications in physics*, vol. 7, no. JUN. 2019.

[41] A. Sandhya, R. Sandhya, and M. Prameela, "An overview of Fractional order PID Controllers and its Industrial applications," *Int. J. Innov. Eng. Technol.*, vol. 6, no. 4, pp. 534–546, 2016.

[42] P. Kumar, S. Chatterjee, D. Shah, U. K. Saha, and S. Chatterjee, "On comparison of tuning method of FOPID controller for controlling field controlled DC servo motor," *Cogent Eng.*, vol. 4, no. 1, 2017, doi: 10.1080/23311916.2017.1357875.

[43] E. Edet and R. Katebi, "On Fractional-order PID Controllers," *IFAC-PapersOnLine*, vol. 51, no. 4, pp. 739–744, 2018, doi: 10.1016/j.ifacol.2018.06.208.

[44] A. Tepljakov, E. Petlenkov, and J. Belikov, "FOMCON : a MATLAB Toolbox for Fractional-order System Identification and Control," *Int. J. Microelectron. Comput. Sci.*, vol. 2, no. 2, pp. 51–62, 2011.

[45] A. Oustaloup, P. Melchior, P. Lanusse, O. Cois, and F. Dancla, "The CRONE toolbox for Matlab," no. March 2014, pp. 190–195, 2002, doi: 10.1109/cacsd.2000.900210.

[46] I. Petras, "Stability of Fractional-Order Systems with Rational Orders," pp. 1–25, 2008, [Online]. Available: http://arxiv.org/abs/0811.4102.

[47] B. Roland s, "Advanced Control Engineering," *Adv. Control Eng.*, 2001, doi: 10.1016/b978-0-7506-5100-4.x5000-1.

[48] E. Natsheh and K. A. Buragga, "Comparison between Conventional and Fuzzy Logic PID Controllers for Controlling DC Motors," *IJCSI Int. J. Comput. Sci. Issues ISSN*, vol. 7, no. 5, pp. 1694–814, 2010, [Online]. Available: www.IJCSI.org.

APPENDICES

Appendix A: Analytical equations

The controller parameters on (4.30), (4.38) and (4.39) the numerical analysis is below;

$$\left.\begin{aligned} k &= \frac{R_2}{\tau_1 * \tau_2} \\ \omega_n &= \sqrt{\frac{1}{\tau_1 * \tau_2}} \\ \xi &= \frac{\tau_1 + \tau_2 + R_2 A_1}{2 * \tau_1 * \tau_2 * \omega_n} \end{aligned}\right\}$$

$$k = \frac{R_2}{\tau_1 * \tau_2} = \frac{0.01}{(250*0.01)*(250*0.01)} = \frac{0.01}{6.25} = 0.0016$$

$$\omega_n = \sqrt{\frac{1}{\tau_1 * \tau_2}} = \sqrt{\frac{1}{6.25}} = \sqrt{0.16} = 0.4$$

$$\xi = \frac{\tau_1 + \tau_2 + R_2 A_1}{2 * \tau_1 * \tau_2 * \omega_n} = \frac{250*0.01+250*0.01+0.01*250}{2*250*0.01*250*0.01*0.4} = \frac{7.5}{5} = 1.5$$

$$\left.\begin{aligned} kp &= \frac{(1 + 2\alpha(\xi^{cl})^2(\omega_n{}^{cl})^2) - \omega_n{}^2}{k} \\ ki &= \frac{\alpha \xi^{cl} (\omega_n^{cl})^3}{k} \\ kd &= \frac{(2+\alpha)\xi^{cl}\omega_n^{cl} - 2\xi\omega_n}{k} \end{aligned}\right\}$$

$$K_p = \frac{(1 + 2\alpha(\xi^{cl})^2(\omega_n{}^{cl})^2) - \omega_n{}^2}{k} = \frac{8.06}{0.0016} = 5037.5$$

$$k_i = \frac{\alpha \xi^{cl} (\omega_n^{cl})^3}{k} = \frac{7.6}{0.0016} = 4750$$

$$k_d = \frac{(2+\alpha)\xi^{cl}\omega_n^{cl} - 2\xi\omega_n}{k} = \frac{4.5}{0.0016} = 2812.5$$

The closed loop analysis of proportional-integral-derivative controller with a plant of interacting coupled tank system as follows,

$$G_p(s) = \frac{0.0016}{S^2 + 1.2S + 0.16}$$

$$G_c(s) = k_p + \frac{k_i}{s} + k_d s \quad \frac{k_d s^2 + k_i s + k_p}{s}$$

$$G_{cl\ pid}(s) = \frac{Gp(s)Gc(s)}{1+Gp(s)Gc(s)}$$

$$= \frac{4.5s^2+11.136s+7.6}{2812.5s^3+6964s^2+4761.136s+7.6}$$

The closed loop analysis of digital-proportional-integral-derivative controller with a plant of interacting coupled tank system as follows,

$$G_p(s) = \frac{0.0016}{S^2+1.2S+0.16}$$

$$G_{dpid}(z) = \frac{k_p z^2 - k_p + \frac{T}{2}k_i z^2 + k_i T z + k_i \frac{T}{2} + \frac{2}{T}z^2 k_d - \frac{2}{T}k_d}{z^2-1}$$

$$G_{cl\ pid}(s) = \frac{Gp(s)Gc(s)}{1+Gp(s)Gc(s)}$$

The closed loop analysis of fractional order-proportional-integral-derivative controller with a plant of interacting coupled tank system when the value of $\mu=\lambda= 0.5$ as follows,

$$G(s)_{fcl} = \frac{4.5s+8.06s^{0.5}+7.616}{s^{2.5}+1.2s^{1.5}+4.5s+8.22s^{0.5}+7.616}$$

For more robustness and better performance use the lambda and Miu values 0.125 and 1.125 respectively.

Appendix B: Programing with MATLAB Code

```
>> clear
>> clear all;
% to create the transfer function of coupled tank can be used
% A1=250cm^2;A2=250cm^2;R1=0.01sec/cm^2;R2=0.01sec/cm^2;
>> num=[0.0016]; % numrator of the transfer function of coupled tank system
>> den=[1 1.2 0.16]; % denumerator of the transfer function of coupled tank system
% from the above physical parameter values I get the transfer function of the plant
>> plant=tf(num,den); % gives continues-time open loop transfer function of coupled tank system
>> kp=5037.5;
>> ki=4750;
>> kd=2812.5;
% the parameter of the controller values get using pole placement method
% manually calculated
>> sys=pid(kp,ki,kd)% the conventional PID controller
>> sys1=plant*sys
>> PID=feedback(sys1,1) % gives continues-time closed loop transfer function of coupled tank system
                     %with PID controller
>> step(PID) % Step Response of coupled tank system with PID
>> hold on
>> Ts=0.3; % sampling time
For better performance use 0.1sec of sampling time.
>> DPID=c2d(PID,Ts,'tustin')% conversion of conventional to digital controller using trapezoidal integration method
```

```
 % and gives discrete-time closed loop transfer function of coupled tank system
                         %with digital-PID controller
>> step(DPID)   % Step Response of coupled tank system with digital-PID
>> hold on
>> s=fotf('s') % to call the fractional order transfer function
>> foc=kp+ki/s^0.35+kd*s^0.75  % transfer function of FOPID controller
>> sysfoc=foc*plant
>> FOPID=feedback(sysfoc,1) % and gives fractional order closed loop transfer function of coupled tank system
                         %with FOPID controller
>> step(FOPID) % Step Response of coupled tank system with FOPID
>> hold on
>> n=5;
>> TID=kp*(1/s)^1/n+ki/s+kd*s;
>> sysTID=TID*plant;
>> TIDCL=feedback(sysTID,1);
>> step(TIDCL)
```

For reduced the overshoot, settling time, rise time response take value of λ=0.75 and μ=0.95 for TID controller the value of n=5;and for digital-PID controller to get better performance use the sampling time T_s=0.1;