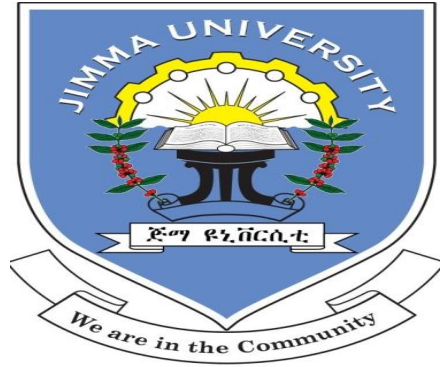


JIMMA UNIVERSITY  
SCHOOL OF GRADUATE STUDIES  
JIMMA INSTITUTE OF TECHNOLOGY  
FACULTY OF CIVIL AND ENVIRONMENTAL ENGINEERING  
STRUCTURAL ENGINEERING STREAM  
OPTIMIZATION OF PRESTRESSED CONCRETE GIRDERS FOR BRIDGE DESIGN

BY  
WUBISHET JEMANEH ABEBE

A Thesis Submitted to the School of Graduate Studies of Jimma University in  
Partial Fulfilment of the Requirements for the Degree of Master of Science in  
Civil Engineering (Structural Engineering)

JUNE 2018  
JIMMA, ETHIOPIA



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APPROVED BY BOARD OF EXAMINERS

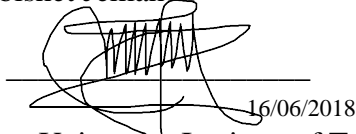
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DECLARATION

Here the undersigned declare that all the works done in this study originates from my own work and that all secondary sources referred to have been duly acknowledged and cited.

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## Abstract

*Structural concrete bridge girders forms almost major portions of total cost of superstructure and they appear deep particularly for large span bridges as compared to ordinary beams to meet the required stiffness and stability. Enlarged size of the girders makes the overall cost of the bridge be costlier and to cope with this, great cost saving was possible to achieve through the use of structural design optimization.*

*In this research, design optimization was carried out by taking total material cost of girders as an objective function and all requirements of strength, stability, serviceability, fatigue and geometric restrictions as constraint functions. A straight girder system bridge with a total width of 9.9m and supporting dual lanes of traffic with standard width of 3.65m each and 1.3m wide overhang both sides was used. It was subjected to three main load cases, the action of dead, live and prestressing loads. Dead load includes self-weight of bridge deck components, railings, girders, diaphragms and wearing surface. Live load was the design vehicular live load of AASHTO LRFD HL-93. Prestressing force was based on maximum tensile prestress at the top fiber and minimum compressive prestress at the bottom fiber. Other load effects like impact factor and multiple presence factor were also taken into account. Linear static method of analysis was used. A program was developed for design optimization of prestressed concrete girders in MATLAB R2017a software.*

*In this study, effects of construction materials, grades of concrete, girder spacing, bridge length on the optimum cost were investigated. The results of optimization indicates that reinforced concrete (RC) T girder was economical up to a span of 40m and for a span longer than 40m prestressed concrete (PC) box girder was better. It was observed that as grades of concrete increases depth of the girders reduces, for bridge supporting dual lane of traffic, an optimum girder spacing was found to be 2.5m. Optimum design of prestressed bridge girders could reduce cost of material with 38% for prestressed concrete T girder and 25% for prestressed concrete box girder as compared to the cost of conventional design approach.*

**Key words:** Partially prestressed Concrete, Reinforced Concrete, T Girder, Box Girder, Design Optimization, Post tensioning, Genetic Algorithm, Girder Spacing, Grade of Concrete.

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## **Abbreviations**

AASHTO	America Association of State highway and Transportation Officials
ACI	American Concrete Institute
ALGA	Augmented Lagrangian Genetic Algorithm
ASCE	American Society of Civil engineers
ASTM	American Society for Testing Materials
CEB	Comite Euro International dubeton
EA	Evolutionary algorithm
ETB	Ethiopian Birr
FIP	Federation International de la Precontrainte
GA	Genetic algorithm
IM	Impact factor
LRFD	Load and resistance factor design
ODOT	Ohio department of transportation
PC	Prestressed Concrete
RC	Reinforced Concrete
SLP	Sequential linear programming
SUMT	Sequential unconstrained minimization technique

## **Notations**

All notation have been defined where they first used. These notations are summarized below:

$\epsilon_{cp}$	Tensile strain in the concrete at the level of the tendon at decompression stage.
$\epsilon_o$	Compressive strain at the extreme top fiber at service load stage
$\epsilon_{oc}$	Compressive strain in the concrete at the level of the tendon
$\epsilon_s$	Tensile strain in the reinforcing steel at working loads
A	Cross sectional area of concrete (mm <sup>2</sup> )
a	Depth of equivalent rectangular stress block (mm)
a'	Distance from the left support to the point of truck load for which deflection is to be computed.
A <sub>c</sub>	Area of concrete cross section (mm <sup>2</sup> )
A <sub>ct</sub>	Area of cracked transformed section under service limit state (mm <sup>2</sup> )
A <sub>t</sub>	Effective tension area of concrete surrounding one bar (mm <sup>2</sup> )
A <sub>p</sub>	Area of prestressing steel (mm <sup>2</sup> )
A <sub>s</sub>	Area of nonprestressed steel tension reinforcement (mm <sup>2</sup> )
A <sub>s</sub> '	Area of nonprestressed steel compression zone reinforcement (mm <sup>2</sup> )
A <sub>v</sub>	Cross sectional area of shear reinforcement within a distance S (mm <sup>2</sup> )
b <sub>e</sub>	Width of compression face of the section of exterior girder (mm)
b <sub>i</sub>	Width of compression face of the section of interior girder (mm)
b <sub>w</sub>	Web width of the cross section (mm)
C	Resultant compressive force in compression zone of concrete (N)
c	Depth of the neutral axis (mm)

$C_c$	Unit cost of concrete per cubic millimeter (ETB/mm <sup>3</sup> )
$C_n$	Compressive force in compression zone of concrete used to reduce the resultant Compressive force C when NA depth exceeds flange thickness (N)
$C_p$	Unit cost of pre-stressing tendons per ton (ETB/ton)
$C_s$	Unit cost of reinforcement steel per ton (ETB/ton)
$d$	Distance from extreme compression fiber to centroid of nonprestressed tension reinforcement (mm)
$d_c$	Thickness of concrete cover measured from extreme tension fiber to centroid of closest bar ther to (mm)
$d_e$	Depth from extreme compression fiber to centroid of tensile force (mm)
$d_p$	Depth from extreme compression fiber to centroid of prestressing steel (mm)
$d_s'$	Distance from extreme compression fiber to centroid of nonprestressed compression zone reinforcement (mm)
$d_v$	Effective depth of shearing force (N)
$d_z$	Depth from extreme compression fiber to centroid of resultant compression force C (mm)
$d_{zn}$	Depth from extreme compression fiber to centroid of compression force $C_n$ (mm)
$e$	Eccentricity of prestressing force from the centroid of the section (mm)
$E_c$	Modulus of elastic of concrete (N/mm <sup>2</sup> )
$E_p$	Modulus of elastic of prestressing steel (N/mm <sup>2</sup> )
$E_s$	Modulus of elastic of reinforcing steel (N/mm <sup>2</sup> )
$f_{br}$	Stress range at the extreme bottom fiber (N/mm <sup>2</sup> )
$f_c'$	Specified cylindrical compressive strength of concrete (N/mm <sup>2</sup> )
$f_{cpe}$	Compressive stress in concrete due to effective prestress forces only (N/mm <sup>2</sup> )
$f_{ct}$	Maximum allowable compressive stress in concrete at initial prestress (N/mm <sup>2</sup> )

$f_{cw}$	Maximum allowable compressive stress in concrete at service load (N/mm <sup>2</sup> )
$f_{fp}$	Stress range in prestressing steel due to fatigue load (N/mm <sup>2</sup> )
$f_{fs}$	Stress range in reinforcing steel due to fatigue load (N/mm <sup>2</sup> )
$f_{inf}$	Stress at the extreme bottom fiber for a given eccentricity $e$ (N/mm <sup>2</sup> )
$f_{min}$	Minimum live load stress where there is stress reversal (N/mm <sup>2</sup> )
$f_p$	Total stress in prestressing tendons at the application of service loads (N/mm <sup>2</sup> )
$f_{pe}$	Effective stress in prestressing steel (N/mm <sup>2</sup> )
$f_{ps}$	Average stress in prestressing steel (N/mm <sup>2</sup> )
$f_{pu}$	Ultimate tensile strength of prestressing steel (N/mm <sup>2</sup> )
$f_{py}$	Yield strength of prestressing steel (N/mm <sup>2</sup> )
$f_r$	Modulus of rupture (N/mm <sup>2</sup> )
$f_s$	Stress in nonprestressed steel reinforcement at the application of service loads (N/mm <sup>2</sup> )
$f_{tr}$	Stress range at the extreme top fiber (N/mm <sup>2</sup> )
$f_{tt}$	Maximum allowable tensile stress in concrete at initial prestress (N/mm <sup>2</sup> )
$f_{tw}$	Maximum allowable tensile stress in concrete at service load (N/mm <sup>2</sup> )
$F_x$	Forces acting in the horizontal direction (N)
$f_y$	Yield strength of non prestressed steel tension reinforcement (N/mm <sup>2</sup> )
$f_y'$	Yield strength of non prestressed steel compression zone reinforcement (N/mm <sup>2</sup> )
$\gamma$	Unit weight of steel reinforcement bars and prestressing tendons (ton/mm <sup>3</sup> )
$g_s$	Girder spacing (mm)
$h$	Height of the deformation (mm)
$\eta$	Prestress loss factor

$h$	Over all depth of the section (mm)
$h_1$	Distance from centroid of tensile steel to NA depth (mm)
$h_2$	Depth from extreme compression fiber to depth of NA (mm)
$h_f$	Thickness of the flange (mm)
$I$	Second moment of area or moment of inertia of concrete cross section ( $\text{mm}^4$ )
$I_{ct}$	Moment of inertia of cracked transformed section under service limit state ( $\text{mm}^4$ )
$I_e$	Effective moment of inertia of the section ( $\text{mm}^4$ )
$L$	Span length of the girder (mm)
$M_3$	Working moment at service limit state III (Nmm)
$M_{cr}$	Cracking moment (Nmm)
$M_d$	Ultimate factored design moment due to all loads (Nmm)
$M_f$	Maximum fatigue load moment (Nmm)
$M_g$	Total unfactored dead load moment (Nmm)
$M_{min}$	Minimum moment due to self weight or during handling of the member (Nmm)
$M_n$	Nominal moment of resistance (Nmm)
$M_r$	Total factored moment of resistance of the section (Nmm)
$M_w$	Working moment at service limit state I (Nmm)
$n_p$	Modular ratio of prestressing steel
$n_s$	Modular ratio of reinforcing steel
$P$	Prestressing force (N)
$r$	Base radius of the deformation (mm) and
$S$	Spacing of stirrups (mm)
$T_p$	Tension force in the prestressing steel at service limit state (N)
$T_s$	Tension force in the reinforcing steel at service limit state (N)



$V_c$	Shear resisting force due to tensile stress in the concrete (N)
$V_n$	Nominal shear resistance (N)
$V_p$	Component of prestressing force in the direction of shearing force (N)
$V_s$	Shear resisting force due to tensile stress in traverse reinforcement (N)
$V_u$	Factored design shearing force d distance from face of support (N)
$w_{oh}$	Width of overhang (mm)
$W_{str}$	Weight of stirrups (ton)
$x$	Distance from left support to a point at which maximum service load moment occurs.
$y$	NA depth of the cracked section under service limit state (mm)
$y_b$	Depth from extreme bottom fiber to centroid of the section (mm)
$y_{ct}$	Depth from extreme compression fiber to centroid of cracked section (mm)
$y_t$	Depth from extreme top fiber to centroid of the section (mm)
$Z_b$	Section modulus of the extreme bottom fiber (mm <sup>3</sup> )
$Z_c$	Section modulus for the extreme fiber of the composite section where tensile stress is caused by externally applied loads (mm <sup>3</sup> )
$Z_{nc}$	Section modulus for the extreme fiber of monolithic or noncomposite section where tensile stress is caused by externally applied loads (mm <sup>3</sup> ) that is $Z_b$
$Z_t$	Section modulus of the extreme top fiber (mm <sup>3</sup> ).
$\Delta_{all}$	Allowable deflection for live load (mm)
$\Delta_d$	Total long term deflection due to dead load (mm)
$\Delta_{di}$	Immediate deflection due to dead load (mm)
$\Delta_{kl}$	Deflection due to truck load (mm)
$\Delta_{LL}$	Deflection due to live load (mm)
$\Delta_{Ln}$	Deflection due to design lane load (mm)
$\Delta_p$	Upward deflection due to prestress force (mm)
$\Phi$	Resistance factor
$\beta_1$	Stress block factor
$\rho_s$	Density of reinforcement steel

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## **CHAPTER ONE**

### **INTRODUCTION**

#### **1.1 Background**

Nowadays, a rapid growth of computer performance enables and encourages new developments in science and engineering. Particularly, within the field of structural mechanics, modeling of materials and therefore the prediction of structural response is more accurate than in past decades. These are new challenges that we want to discover, but there are also several problems, that must be solved. For instance, the research within applied optimization is mainly lead by automotive and aerospace industries. Therefore, the emphasis is put mainly on the computational fluid dynamics domain and structural optimization area, especially on the shape optimization. Because Civil Engineering problems are dominantly connected with static problems and topology and/or size optimization, there is a gap between current researches and the application of new methods into the discipline of Civil Engineering [1].

In bridge construction, the cost of materials is often a factor in end of the project cost deliverables which is mostly expensive especially in overdesign structural members. Engineers most jump in to conclusion of increasing the section area and adding reinforcement on the design for fear of durability issue of the end product which is the structures itself. Structural optimization of members is often neglected during the design process for it may take another time from the tedious analysis and design of structures.

Therefore, the goal of this thesis was to review and enhance the current state of art regarding to structural design optimization and to show possibilities of this methods in cost

optimization of structural concrete that is reinforced concrete and prestressed concrete bridge girders.

## **1.2 Statement of the problem**

The use of traditional method of design may leads to oversize structural members and as a result inefficient use of limited resources like construction materials and other are included in the practice. Design optimization is not only cross sectional sizing of the members but also it is looking for the optimal path (feasible direction) or possible combinations of alternatives that drive us to achieve the optimal cost or it may be selecting the best thing or material which brings optimal cost keeping all the requirements being satisfied.

On the other hand, although a significant amount of research has been published in the field of structural optimization since the 1960s, little of the research effort has been utilized in structural design practice. One reason for this is that only a small portion of the research targets real-world applications.

Therefore there is a need to conduct research on cost optimization of structures, particularly structural concrete bridges girders where significant cost savings may be possible. In this research cost optimization of bridge girders was studied to bridge the gap of oversizing member of bridge construction

## **1.3 Objectives of the Research**

### **General objective**

The main objective of the research was to optimize cost of structural concrete bridge girders using suitable optimization program.

## **Specific objectives**

To support the above general objective, the following specific objective was sought:

- To compare the effect of construction materials on the optimum cost of the girders.
- To study the effect characteristic strength of concrete on optimum cost of the girder.
- To compare the cost of optimum design with the conventional design approach.
- To investigate the optimal girder spacing.
- To compare cost ratio of concrete and steel over specific length of bridge.
- To investigate optimum cross sectional dimensions of bridge girders.
- To compare commonly used structural optimization algorithms.

### **1.4 Significances of the Study**

The research targeted on cost optimization of prestressed concrete (PC) girder structures to facilitate the use of optimization methods in structural design practice. This was preferably be carried out for real-world projects to close the gap between theory and practice. The purpose of this thesis was to contribute to the closing of this gap by implementing cost optimization to practical problems. Ideally, the structure to optimize should both be common and large enough to allow for significant cost savings. Prestressed and reinforced concrete bridge girders meet both of these requirements and was therefore selected as the type of structure to optimize.

It is expected that developing practical implementations such as this will facilitate the usage of optimization methods by practicing engineers. To further promote this, the thesis puts emphasis on technical details of the implementation, and highlights the potential cost

savings by comparing optimized bridge girders with conventionally designed bridge girders.

The research may benefit designers and bridge owners towards the use of partially prestressed concrete structures which is rarely used in Ethiopia. It may also be used to guide decision makers to compare and select economical bridge cross section and type at the phase feasibility study.

### **1.5 Scope of the Study**

In this research, simply supported bridge with straight girder system which has a total width 9.9m with variable length was considered. Commonly used tee and box girder sections made up of reinforced concrete and post tensioned partially prestressed concrete were considered case by case and subjected to routine iterations of optimization by genetic algorithm (GA) to find the optimum cost of materials. Girders were spaced apart with a range of 1.5m to 4m within the road width to determine the optimum girder spacing. For this research, grades of concrete with specified characteristic cylindrical compressive strength of 30Mpa to 70Mpa, non prestressed reinforcement bars of grade 420 of diameter 12mm to 32mm , and commonly available prestressing strands of grade 270 (1860) low relaxation 7-wire strands of diameter 9.53mm to 15.24mm were used.

### **1.6 Research Questions**

In this section the relevant research questions that bear in to the mind of the researcher were stated.

1. Which section is economical from commonly used cast insitu tee and box girder sections?

2. For a given range of bridge length, what material is economical for bridge girder construction?
3. What is the effect of grades of concrete on the optimum cost and depth of prestressed concrete girders?
4. How much is the optimum girder spacing for bridge supporting dual lanes of traffic?
5. With what amount could design optimization reduce the cost of conventionally designed prestressed concrete bridge girders?
6. In comparison of cost of concrete and reinforcement steel, which material cost could govern for a given bridge span?
7. What is suitable structural design optimization program?



## **CHAPTER TWO**

### **RELATED LITERATURE REVIEW**

Recent advances in the field of computational intelligence have led to a number of promising optimization algorithms. These algorithms have the potential to find optimal or near-optimal solutions to complex problems within a reasonable time frame. Structural optimization is a research field where such algorithms are applied to optimally design structures. It is essentially a combination of two research fields: structural mechanics and computational intelligence [2].

Optimization is a process of making things better. Life is full of optimization problems which all of us are solving many of them each day in our life activities. Which route is closer to school? Which bread is better to buy having the lowest cost while giving good energy? Optimization is fine-tuning the inputs of a process, function or device to find the maximum or minimum output(s). The inputs are the variables, the process or function called objective function, cost function or fitness value (function) and the output(s) is fitness or cost [3].

The primary aim of structural optimization is to determine the most suitable combination of design variables, so to achieve satisfactory performance of the structure subject to the behavioral and geometric constraints imposed, with the goal of optimality being defined by the objective function for specified loading or environmental conditions. In this thesis cost minimization of bridge girders is tackled using genetic algorithm. Basically, the process of optimum design of prestressed concrete structures may be looked upon as a mathematical programming problem in which the total cost or consumption of materials is minimized, subject to certain functional constraints, such as the limitation of stresses, deflections and crack widths at serviceability limit states and flexure and shear strength requirements at the limit state of collapse [4].

Generally, optimization problems involve long and tedious computations and as such manual computations are limited to simple problems comprising a few design variables. However, the development of high speed electronic digital computers has revived the

interest in optimization problems and significant advances have been made in the field of structural optimization. In fact, the real impetus to the growth of interest in optimum seeking methods came only after the pioneering work of Dantzig, who developed the simplex algorithm for the solution of linear programming problems [5].

## **2.1 Optimization Techniques**

In using the mathematical programming methods, the process of optimization begins with an acceptable design point. A new point is selected suitably so as to minimize the objective function. The search for another new point is continued from the previous point until the optimum point is reached. There are several well established techniques for selecting a new point and to proceed towards the optimum point, depending upon the nature of the problem, such as linear and nonlinear programming.

Linear programming methods were used by Kirsch to optimize indeterminate prestressed concrete beams with prismatic cross sections through a “bounding procedure”. In linear programming problem, the objective function and constraints are linear functions of the design variables and the solution is based on the elementary properties of systems of linear equations. The properties of systems of proportionality, additivity, divisibility and deterministic features are utilized in the mathematical formulation of the linear programming problem. A linear function in three dimensional space is a plane representing the locus all design points. In n dimensional space, the surface so defined is a hyper plane. In these cases, the intersections of the constraints give solutions which are the simultaneous solutions of the constraint equations meeting at that point. Due to linearity, the optimum solution should be any one of the intersections of the constraints [6].

Linear programming problems can be conveniently solved by the revised simplex method. The simplex algorithm for solving the general linear programming problem is an iterative procedure which yields an exact optimal solution in a finite number of steps. One of the most powerful techniques for solving nonlinear programming problems is to transform the problem by some means in to a form which permits the application of the simplex

algorithm. Thus, the simplex method turns out to be one of the most powerful computational devices for solving linear as well as nonlinear programming problems [7].

Cohn and MacRae studied simply supported reinforced, fully prestressed (pretensioned and post-tensioned), and partially prestressed concrete I-beams with fixed cross sectional geometry subjected to serviceability and ultimate limit states constraints using a nonlinear programming technique. In nonlinear programming problems, the objective function and the constraints are nonlinear functions of the design variables. Since the boundaries of the feasible regions or the contours of equal values of the merit function are straight lines, the optimum solution need not necessarily be at an intersection of the constraints [8].

Over the years, several techniques have been developed for the solution of nonlinear programming problems. Some of the prominent techniques are [9]:

1. Method of feasible directions
2. Sequential unconstrained minimization technique (SUMT)
3. Sequential linear programming (SLP)
4. Dynamic programming.

*The method of feasible direction* can be grouped under the direct methods of approach on general nonlinear inequality constrained optimization problems. Two well-known procedures which embody the philosophy of the method of feasible directions are Rosen's gradient projection algorithm [10] and Zoutendijk's procedure [11]. This method was probably the first nonlinear programming procedure to be used in structural optimization problems by Schmit in 1960 [12]. In this method, starting from an initial feasible point, the nearest boundary is reached and a new feasible direction is found. An appropriate step is taken along this feasible direction to get the new design point. The procedure is repeated until the optimum design point is reached.

*In sequential unconstrained minimization technique*, the constrained minimization problem is converted in to an unconstrained one by introducing an interior or exterior penalty function. This method has proved to be highly advantageous in practical structural design problems.

*In sequential linear programming*, the nonlinear objective function and constraints are linearized in the vicinity of the starting point and a new design point is obtained by solving the linear programming problem. The sequence of linearizing in the neighborhood and solving by linear programming is continued from the new point till the optimum is reached.

*Dynamic programming* which widely applied in operations research and economics, is basically a mathematical approach for multi stage decision problems. This approach is well suited to the optimal design of certain kinds of structure, in general those in which the interaction between different parts is rather simple. The main limitation of dynamic programming is that it does not lend itself to the construction of general purpose computer programs suitable for a wide range of distinct problems.

## **2.2 Forms of Structural Optimization**

### **2.2.1 Shape Optimization**

In this form of optimization the topology of structure is known a-priori but there can be some part and/or detail of the structure, in which, for instance, high stresses can produce problems. Therefore the objective is usually to find the best shape that will result in the most suitable stress distribution. Parameters of shapes are dimensions of the optimized parts or a set of variables describing the shape, e.g. coefficients of spline functions. Examples for the reinforced concrete area herein can be finding the proper shape of holes within plate members [13].

### **2.2.2 Size Optimization**

In this form of an optimization a structure is defined by a set of sizes, dimensions or cross-sections. These are combined to achieve the desired optimality criteria.

In the case of steel structures in particular, nearly all possible optimization problems have been subjected to some form of investigation. To list a few successfully solved problems, optimization of nonlinear steel frames with semi-rigid connections [14], optimization against buckling [15] or a finding minimum weight in connection with a minimum number of steel profiles used in a design [16] and cost optimization of prestressed I girder [17] can be found in the corresponding literature.

As a consequence of the definitions introduced above, we can distinguish one additional form of structural optimization. If a design variable - the size of a member or the material property - can reach zero value, i.e. it is not necessary in the structure and can be removed. The cornerstone of this approach is the so-called ground structure, which defines all possible positions of nodes and the set of all possible members/connections among these nodes. Then the goal is the removal of inefficient members to obtain an optimal structure. If coordinates of nodes are also unknown, this form becomes part of topology optimization. Therefore the layout optimization can be seen as the connection point between the previously cited two kinds of optimization [18].

An interesting feature in solving this form of optimization is the possibility of failure of hard-kill methods. In some cases a weak member is removed although it is necessary for the efficiency of the static scheme [19].

### **2.2.3 Topology Optimization**

By topology optimization we understand finding a structure without knowing its final form beforehand. Only the environment, optimality criteria and constraints are known. The major Civil Engineering representatives serve as a decision tool in selecting an appropriate static scheme of a desired structure. They are mostly applied to the pin-jointed structures, where the nodal coordinates of joints are optimization variables. Based on the position of supports and objective functions, several historically well-known schemes can be discovered. The typical example of this optimization form within the reinforced concrete area is placement of steel reinforcing bars into a concrete block. In other words, we search for the most suitable strut-and-tie model [20].

This form of optimization is the least investigated part of structural optimization. Here you can find the search for a proper shape for shell, membrane or tent like structures. Only few papers on this topic can be found in the literature, e.g. [21] or [22], with even fewer dealing with reinforced concrete structures. And finally, the Mathematical Programming methods are known as the only efficient solutions for this type of optimization problems.

### **2.3 Genetic Algorithm**

Genetic Algorithm (GA) is global optimization technique developed by John Holland in 1975. It belongs to the family of evolutionary algorithms that search for solutions to optimization problems by "evolving" better and better solutions. A genetic algorithm begins with a "population" of solutions and then chooses "parents" to reproduce. During reproduction, each parent is copied, and then parents may combine in an analog to natural crossbreeding, or the copies may be modified, in an analog to genetic mutation. The new solutions are evaluated and added to the population, and low quality solutions are deleted from the population to make room for new solutions. As this process of parent selection, copying, crossbreeding, and mutation is repeated, the members of the population tends to

get better. When the algorithm is halted, the best member of the current population is taken as the solution to the problem posed. Then, the genetic algorithm loops over an iteration process to make the population evolve [23].

A single objective decision problem given an  $n$ -dimensional decision variable vector  $x = \{x_1, \dots, x_n\}$  in the population space  $X$ , find a vector  $\mathbf{x}^*$  that minimizes the objective function to the value  $f^*(x)$ . The solution space  $X$  is generally restricted by a series of constraints, such as  $g_i^*(x) = b_j$  for  $j = 1, \dots, m$  and bounds on the decision variables. A solution is said to be *Pareto optimal* if it is not dominated by any other solution in the solution space. A Pareto optimal solution cannot be improved with respect to any objective without worsening at least one other objective. The set of all feasible non dominated solutions in  $X$  is referred to as the *Pareto optimal set*, and for a given Pareto optimal set, the corresponding objective function values in the objective space is called the *Pareto front*. For many problems, the number of Pareto optimal solutions is enormous (may be infinite).

In GA terminology, a solution vector  $x \in X$  is called an individual or a *chromosome*. Chromosomes are made of discrete units called *genes*. Each gene controls one or more features of the chromosome. In the original implementation of GA by Holland, genes are assumed to be binary numbers. In later implementations, more varied gene types have been introduced. Normally, a chromosome corresponds to a unique solution  $x$  in the solution space. This requires a mapping mechanism between the solution space and the chromosomes. This mapping is called an encoding. In fact, GA works on the encoding of a problem, not on the problem itself.

GA operates with a collection of chromosomes, called a *population*. The population is normally randomly initialized. As the search evolves, the population includes fitter and fitter solutions, and eventually it converges, meaning that it is dominated by a single solution. Holland also presented a proof of convergence (the schema theorem) to the global optimum where chromosomes are binary vectors.

During the run of GA algorithm, a selection of parents for reproduction and recombination for creating offspring is essential. These aspects are called GA's operators [24].

*Selection:* the first step consists of selecting individuals for reproduction. This selection is done randomly with a probability depending on the relative fitness of the individuals so that best ones are often chosen for reproduction than poor ones.

*Reproduction:* in the second step, offspring is bred by the selected individuals. For generating new chromosomes, the algorithm can use both recombination and mutation. GA use two operators to generate new solutions from existing ones: *crossover* and *mutation*. The crossover operator is the most important operator of GA. In crossover, generally two chromosomes, called *parents*, are combined together to form new chromosomes, called *offspring*. The parents are selected among existing chromosomes in the population with preference towards fitness so that offspring is expected to inherit good genes which make the parents fitter. By iteratively applying the crossover operator, genes of good chromosomes are expected to appear more frequently in the population, eventually leading to convergence an overall good solution.

The *mutation* operator introduces random changes into characteristics of chromosomes. Mutation is generally applied at the gene level. In typical GA implementation, the mutation rate (probability of changing the properties of a gene) is very small, typically less than 1%. Therefore, the new chromosome produced by mutation will not be very different from the original one. Mutation plays a critical role in GA. As discussed earlier, crossover leads the population to converge by making the chromosomes in the population alike. Mutation reintroduces genetic diversity back into the population and assists the search escape from local optima.

*Reproduction:* during the last step, individuals from the old population are killed and replaced by the new ones which involves selection of chromosomes for the next generation. In the most general case, the fitness of an individual determines the probability of its survival for the next generation. There are different selection procedures in GA depending on how the fitness values are used. Proportional selection, ranking, and tournament selection are the most popular selection procedures.

*Evaluation:* then the fitness of the new chromosomes is evaluated. The algorithm is stopped when the population converges toward the optimal solution.



The three basic features of the structural optimization problem are;

1. The design variables
2. The objective function
3. The constraints

### **2.3.1 Objective Function**

In the structural design problem, there should be a well-defined criterion by which the performance or cost of the structure can be judged under different combinations of design variables. This index is generally referred to as the objective function may comprise the cost of concrete, steel and prestressing tendons in the member.

### **2.3.2 Design Variables**

The design variables are generally grouped under the following categories:

- (a) Dimensional variables represented by the member sizes, such as the depth of a girder, cross sectional areas of a member and moment of inertia of a flexural member.
- (b) Configuration or geometric variables, represented by the coordinates of element joints.
- (c) Variables involving modulus of elasticity.
- (d) A majority of the structural optimization problems are concerned with the selection of member sizes because of the relative simplicity of the problem and, in many of the practical problems of structural design, the geometry and material properties are pre-assigned and hence considered as fixed.

### **2.3.3 Design Constraints**

Constraint is a limitation or restriction imposed directly on a variable or group of variables in order that the design is acceptable. They are expressed in the equality or inequality form and are divided into the following groups.

- (a) *Side constraints* are specified limitations (minimum or maximum imposed on a design variable and are usually explicit in form).

(b) *Behavior constraints* are those imposed on the structural response. Typical explicit behavior constraints are given by formulae presented in design specifications. Behavior constraints are generally nonlinear functions of design variables and are implicitly related to design variables. In structural designs, behavior constraints are usually imposed on stresses and displacements. The displacement constraints prescribe the global rigidity of the structure.

## **2.4 Nonlinear Constraint Solver Algorithms**

### **2.4.1 Augmented Lagrangian Genetic Algorithm (ALGA)**

Augmented Lagrangian Genetic Algorithm by default, the genetic algorithm uses the Augmented Lagrangian Genetic Algorithm (ALGA) to solve nonlinear constraint problems without integer constraints. The optimization problem formulated by Samir El Mourabit [2] given below can be solved by the ALGA algorithm.

$$\min_x f(x) \text{ such that}$$

$$g_i(x) \leq 0, i = 1 \dots m$$

$$g_{eq_i}(x) \leq 0, i = m + 1 \dots mt$$

$$A \cdot x \leq b$$

$$A_{eq} \cdot x \leq b_{eq}$$

$$lb \leq x_i \leq ub,$$

Where  $f(x)$  stands for the objective function,  $g(x)$  represents the nonlinear inequality constraints,  $g_{eq}(x)$  represents the equality constraints,  $m$  is the number of nonlinear inequality constraints, and  $mt$  is the total number of nonlinear constraints. The Augmented Lagrangian Genetic Algorithm (ALGA) attempts to solve a nonlinear optimization problem with nonlinear constraints, linear constraints, and bounds. In this approach, bounds and linear constraints are handled separately from nonlinear constraints. A sub problem is formulated by combining the fitness function and nonlinear constraint function using the Lagrangian and the penalty parameters. A sequence of such optimization problems are approximately minimized using the genetic algorithm such that the linear

constraints and bounds are satisfied. A sub-problem formulation is defined as follows according to Deb [25].

$$\phi(x, \lambda, s, \rho) = f(x) - \sum_{i=1}^m \lambda_i s_i \log(s_i - c_i(x)) + \sum_{i=m+1}^{mt} \lambda_i c_{eq}(x) + \frac{\rho}{2} \sum_{i=m+1}^{mt} \lambda_i c_{eq}(x)^2 \quad (3.1)$$

Where

The components  $\lambda_i$  of the vector  $\lambda$  are nonnegative and are known as Lagrange multiplier estimates

The elements  $s_i$  of the vector  $s$  are nonnegative shifts

$\rho$  is the positive penalty parameter.

The algorithm begins by using an initial value for the penalty parameter (Initial Penalty). The genetic algorithm minimizes a sequence of sub problems, each of which is an approximation of the original problem. Each sub problem has a fixed value of  $\lambda$ ,  $s$ , and  $\rho$ . When the sub problem is minimized to a required accuracy and satisfies feasibility conditions, the Lagrangian estimates are updated. Otherwise, the penalty parameter is increased by a penalty factor (Penalty Factor). This results in a new sub problem formulation and minimization problem. These steps are repeated until the stopping criteria are met [26].

Each sub problem solution represents one generation. The number of function evaluations per generation is therefore much higher when using nonlinear constraints than otherwise. Choose the Augmented Lagrangian algorithm by setting the Nonlinear Constraint Algorithm option to 'auglag' using optimoptions.

### **2.4.2 Penalty Algorithm**

The penalty algorithm is similar to the Integer GA Algorithm. In its evaluation of the fitness of an individual, GA computes a penalty value as follows:

If the individual is feasible, the penalty function is the fitness function.

If the individual is infeasible, the penalty function is the maximum fitness function among feasible members of the population, plus a sum of the constraint violations of the (infeasible) individual [25].

## **2.5 Advantages of Partial Prestressing**

Prestressing system is imposition of internal stresses into a structure in opposite action of stresses caused by service or working loads. So, in concrete structures, prestressing provides a pre-compressive axial force to eliminate or greatly reduce internal tensile stresses along service time of structure. The application of prestressing on concrete structures, thus concrete bridges, leads to considerable advantages such as smaller sections, longer spans, minimum deflections and increased durability due less or free from cracks. However, the disadvantages of prestressing are cost of some special equipment, expert supervision to ensure closer quality control in manufacture and losses in initial prestressing forces [27].

Generally, prestressing tendon is used to obtain full prestressed concrete (PC) structures. Sometimes, prestressing tendon may be used in combination with conventional reinforcing steel to obtain partial prestressed concrete (PPC), which in between full prestressed concrete (PC) and reinforced concrete (RC). Partial prestressed concrete (PPC) allows some tension and cracking under full service load while ensuring sufficient ultimate strength. Therefore, it is used to control camber and deflection, increase ductility and save costs.

## **CHAPTER THREE**

### **RESEARCH METHODOLOGY**

In this research design optimization problems were handled with the use of evolutionary or genetic algorithm (GA) after it has been tested under simple manually solved optimization problems and its performance was compared with other programs.

Recent advances in the field of computational intelligence led to a number of promising optimization algorithms. These algorithms have the potential to find optimal or nearly optimal solutions to complex problems within a reasonable time frame. Structural optimization is a research field where algorithms are applied to optimally design structures. It is essentially combination of two research fields: structural mechanics and computational intelligence.

In answering the thesis objectives, the commonly used cast insitu bridge girder cross sections tee and box sections have been considered case by case so as to compare their cost efficiency and recommend for practical use.

#### **3.1 Method of Structural Analysis**

Linear static method of structural analysis with the use of simplified load distribution factors for distributing the loads among internal and external girders was used. Structural analysis software SAP2000 and excel spread sheet were used.

#### **3.2 Method of Design Optimization**

In this research the worst load effects are investigated under applicable load combinations and then the results input into GA optimization code prepared within the built in MATLAB R2017a software. After that, the code was run to generate the out puts and the validity of the result was verified by exporting into excel spreadsheet. If the results were satisfactory, then it was used as an optimum results.

Genetic algorithm (GA) based optimization basically depends on three important aspects:

- 1) Coding of design variables

- 2) Evaluation of fitness of each solution string
- 3) Application of genetic parameters (selection, cross over and mutation) to generate the next generation of solution strings.

Beside the GA other optimization solvers such as fmincon, simulated annealing, and pattern search were also available in MATLAB R2017a software.

### **3.3 Materials Used**

Structural concrete bridge construction materials such as concrete with grades of specified characteristic cylindrical compressive strength of 30Mpa to 70Mpa, grade G 420 deformed reinforcement bars with diameter of 12mm to 32mm as shown in Figure 3.1a, and for post tensioning system grade G 270 (1860) low relaxation 7 wire strands with a diameter ranging from 9.53mm to 15.24mm as per ASTM A 416/A 416 M designation, given in Figure 3.1b below. Tendons were assumed to be extended at the intermediate using couplers (if necessary) and at the end secured to end anchorage system. Parabolic tendon profile which used to provide shear resistance due to prestress was used and this layout was kept in position with the use of harping devices. Partially prestressed post tensioning system of prestressing was used in this study



(a)



(b)

Figure 3.1 Reinforcement Bars and Prestressing 7-Wire Strands  
(Source: <http://www.Henan Prestressing Equipment Co., Ltd.com>)

ASTM A416 Grade 1860 (270) Low relaxation strands were given in the following table.

Table 3.1 ASTM Standard Strands Designation

<i>Strand designation No</i>	<i>Diam. of strand (mm)</i>	<i>Area of strand (mm<sup>2</sup>)</i>	<i>Minimum breaking strength (kN)</i>
9	9.53	54.80	102.30
11	11.11	74.20	137.90
13	12.70	98.70	183.70
13a	13.20	107.70	200.20
14	14.29	123.90	230.00
15	15.24	140.00	260.70
18	17.78	189.70	353.20

Ducts for tendons were rigid or semi rigid either galvanized ferrous metal or polyethylene. For post tensioning system used in this design optimization, the strands were in closed within ducts whose inside cross sectional area of be at least 2.0 times the net area of the prestressing steel for multiple strand tendons used here with one exception where tendons are to be placed by the pull-through method, the duct area shall be at least 2.5 times the net

area of the prestressing steel as stated in AASHTO LRFD Article 5.4.6.2. Tendons were anchored at the end supports by using end anchorage devices. These devices have a standard number of holes in which strands are secured like the one given in Table 3.2 below. This number determines the number of strands per tendon.

Table 3.2 Commonly Used Anchorage Devices

(Source: <http://www.cclint.com>)

<i>Anchorage Designation</i>	<i>No. of strands for diameter 9.53 to 13.2 (mm)</i>	<i>No. of strands for diameter 14.29 to 17.78 (mm)</i>	<i>Inside diameter of duct (mm)</i>	<i>Outside diameter of duct (mm)</i>	<i>Dist. b/n c.g duct &amp; c.g strands, Z (mm)</i>
XM-45	19	13	80	85	20
XM-50	22	15	90	95	20
XM-55	25	17	100	105	20
XM-60	27	19	100	105	20
XM-70	31	22	100	105	20
XM-75	37	25	115	120	25
XM-80	40	27	115	120	25
XM-85	46	31	125	130	25
XM-90	51	35	140	145	25
XM-100	55	37	140	145	25

### **3.4 Optimization Procedure with GA in Matlab**

Procedures involved in design optimization by genetic algorithm (GA) in Matlab was given by in following steps.

*Step 1.* Define fitness function. Open Matlab and from HOME menu click New Script button >> the new script edit field is displayed under the EDITOR menu. Use % symbol to write a comment for readers in which Matlab could not read if % appears before any statement. The new script start to type the fitness function and give it a name you want, in this case let it be 'Tpcintgirderfun' to denote the fitness function of prestressed concrete interior T girder.



```
Editor - Untitled8*
Untitled8* x +
1 % This is fitness function
2 function f = Tpcintgirderfun(x)
3
```

Enter other cost parameters before declaring the fitness function as follows

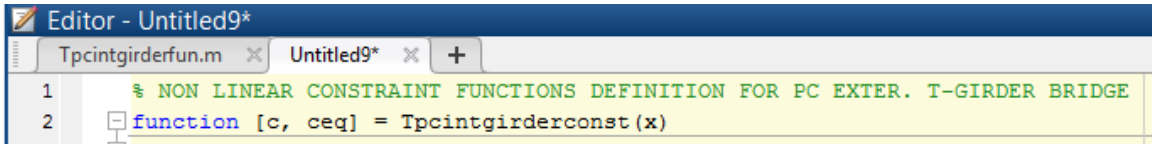
```
Editor - Untitled8*
Untitled8* x +
1 % This is fitness function
2 function f = Tpcintgirderfun(x)
3 % Cost parameters
4 Cc = 2840e-9; % unit rate of fc'= 30 concrete (ETB/m3)
5 Cs = 27940; % unit rate of reinforcing steel (ETB/ton)
6 Cp = 46450; % unit rate of prestressing 7-wire strands (ETB/ton)
7 L = 50000; % span length (mm)
8 NL = 4; % number of legs of vertical stirrups
9 dsh = 12; % diam. of shear rebar (mm)
10 av = NL*pi*dsh^2/4; % area of f12mm for shear reinforcement within a distance S (mm2)
11 density = 7.850e-9; % density of steel prestressing strands and reinforcing bars (ton/mm3)
12 Ag = x(1)*x(2); % concrete cross sectional area of the girder (mm2)
13 Wstr = density*av*(L/x(6)+1)*2*(x(2)/2+2*(x(1)-280)); % weight of stirrups (ton)
```

Now define the cost function as given below

```
Editor - Untitled8*
Untitled8* x +
1 % This is fitness function
2 function f = Tpcintgirderfun(x)
3 % Cost parameters
4 Cc = 2840e-9; % unit rate of fc'= 30 concrete (ETB/m3)
5 Cs = 27940; % unit rate of reinforcing steel (ETB/ton)
6 Cp = 46450; % unit rate of prestressing 7-wire strands (ETB/ton)
7 L = 50000; % span length (mm)
8 NL = 4; % number of legs of vertical stirrups
9 dsh = 12; % diam. of shear rebar (mm)
10 av = NL*pi*dsh^2/4; % area of f12mm for shear reinforcement within a distance S (mm2)
11 density = 7.850e-9; % density of steel prestressing strands and reinforcing bars (ton/mm3)
12 Ag = x(1)*x(2); % concrete cross sectional area of the girder (mm2)
13 Wstr = density*av*(L/x(6)+1)*2*(x(2)/2+2*(x(1)-280)); % weight of stirrups (ton)
14 % Cost cost function prestressed exterior T-girder
15 % z = Cc*(Ag-As-Ap)*L-Wstr/density)+ Cs*(density*As*L+Wstr)+Cp*(density*Ap*L)
16 f = Cc*(Ag -x(4) - x(5))*L-Wstr/density)+...
17 Cs*(density*x(4)*L + Wstr)+ Cp*density*x(5)*L;
```

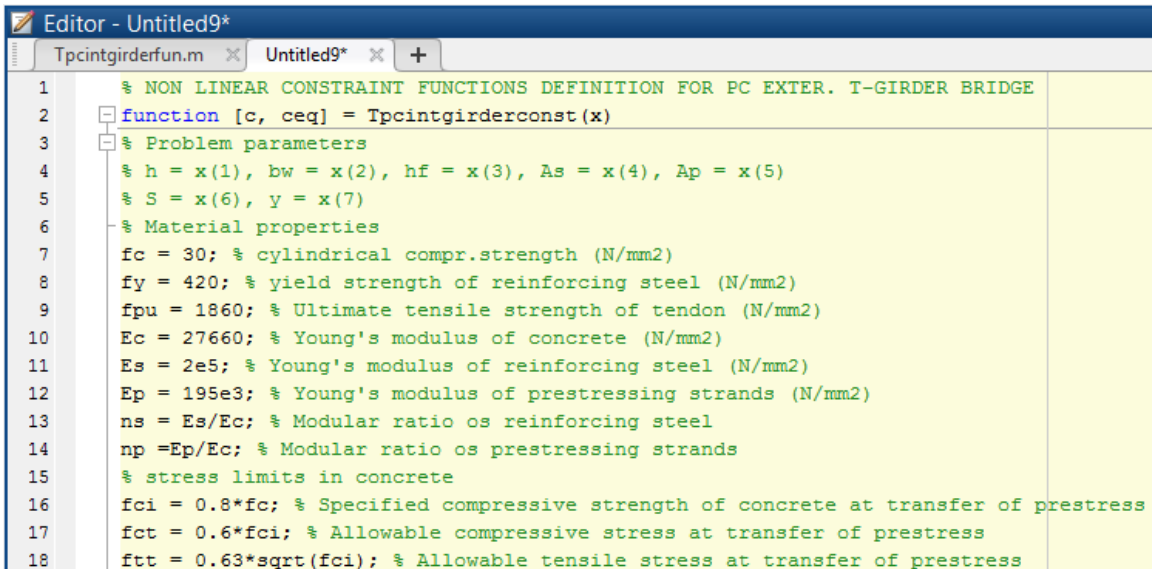
And save it suppressing “CTRL+S” to file folder you want, do not change the name it gives ‘Tpcintgirderfun’ which is the name of fitness function you define earlier.

Step 2. Define the constraint function. Click the plus sign to add a new script and type the constraint functions as follows.



```
Editor - Untitled9*
Tpcintgirderfun.m x Untitled9* x +
1      % NON LINEAR CONSTRAINT FUNCTIONS DEFINITION FOR PC EXTER. T-GIRDER BRIDGE
2      function [c, ceq] = Tpcintgirderconst(x)
```

Next enter parameters of constrained functions. Note  $c$  and  $c_{eq}$  stands for nonlinear inequality and equality constrained functions respectively.



```
Editor - Untitled9*
Tpcintgirderfun.m x Untitled9* x +
1      % NON LINEAR CONSTRAINT FUNCTIONS DEFINITION FOR PC EXTER. T-GIRDER BRIDGE
2      function [c, ceq] = Tpcintgirderconst(x)
3      % Problem parameters
4      % h = x(1), bw = x(2), hf = x(3), As = x(4), Ap = x(5)
5      % S = x(6), y = x(7)
6      % Material properties
7      fc = 30; % cylindrical compr.strength (N/mm2)
8      fy = 420; % yield strength of reinforcing steel (N/mm2)
9      fpu = 1860; % Ultimate tensile strength of tendon (N/mm2)
10     Ec = 27660; % Young's modulus of concrete (N/mm2)
11     Es = 2e5; % Young's modulus of reinforcing steel (N/mm2)
12     Ep = 195e3; % Young's modulus of prestressing strands (N/mm2)
13     ns = Es/Ec; % Modular ratio os reinforcing steel
14     np =Ep/Ec; % Modular ratio os prestressing strands
15     % stress limits in concrete
16     fci = 0.8*fc; % Specified compressive strength of concrete at transfer of prestress
17     fct = 0.6*fci; % Allowable compressive stress at transfer of prestress
18     ftt = 0.63*sqrt(fci); % Allowable tensile stress at transfer of prestress
```

After you define all parameters in terms of design variables, next state the constraint functions as follows.

```

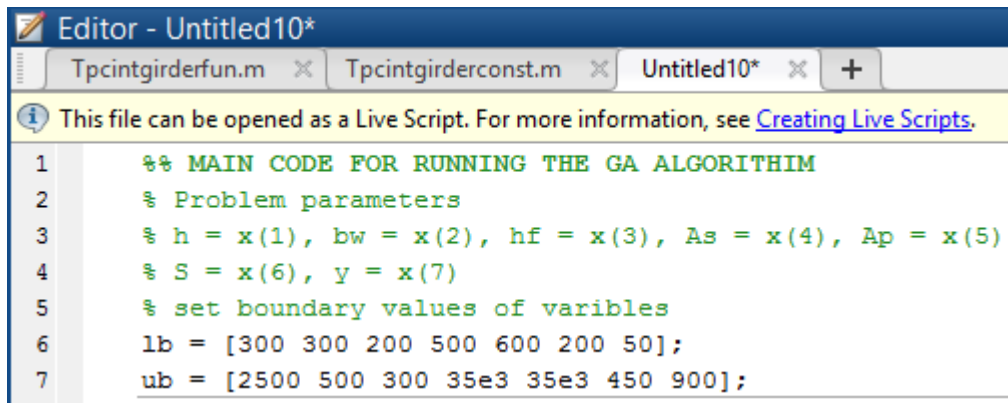
Editor - Untitled9*
Tpcintgirderfun.m x Untitled9* x +
176 %% Non linear inequality constraints [c] written of the form gi(xi)<= 0
177 g1 = ftt-P*(1/Ac+e/Zt)-Mg/Zt;
178 g2 = P*(1/Ac+e/Zb)-Mg/Zb-fct;
179 g3 = 0.85*P*(1/Ac-e/Zt)+Mw/Zt-fcw;
180 g4 = ftw-0.85*P*(1/Ac+e/Zb)+M3/Zb;
181 g5 = Md-0.9*Mn; % flexural strength required
182 g6 = Vu-0.9*Vn; % shear strength required
183 g7 = Vu/0.9-0.25*fc*x(2)*dv-Vp; % web requirment for shear
184 % limits of flexural reinf.
185 g8 = abs(Md)/(0.9*dv)+abs(Vu/0.9-Vp)-0.5*min([Vu/0.9,Vs])- ...
186     x(4)*fy-x(5)*fps; % longitudinal reinf.
187 g9 = Vu/0.9-0.5*Vs-Vp-x(4)*fy-x(5)*fps; % min. longitudinal reinf.
188 g10 = min([1.33*Md,1.2*Mcr])-0.9*Mn; % minimumu flexural reinf. reqd
189 g11 = 0.004*yb*x(2)-x(4)-x(5); % minimumu flexural reinf. reqd
190 g12 = Omp+Ompr-Omn-0.3; % maximumu limit of flexural reinf. reqd
191 g13 = c/de-0.42; % maximumu flexural reinf. reqd
192 % limits of traverse reinforcement
193 g14 = x(6)-fy*av/(0.083*x(2)*sqrt(fc)); %shear reinf.
194 if(abs(Vu-0.9*Vp)/(0.9*dv*x(2)) < 0.125*fc)
195     g15 = x(6)-min([0.8*dv,600]); % spacing of shear reinf.
196 else
197     g15 = x(6)-min([0.4*dv,300]); % spacing of shear reinf.
    <
    
```

Finally define  $c$  and  $c_{eq}$  and save it with its name 'Tpcintgirderconst' as follows.

```

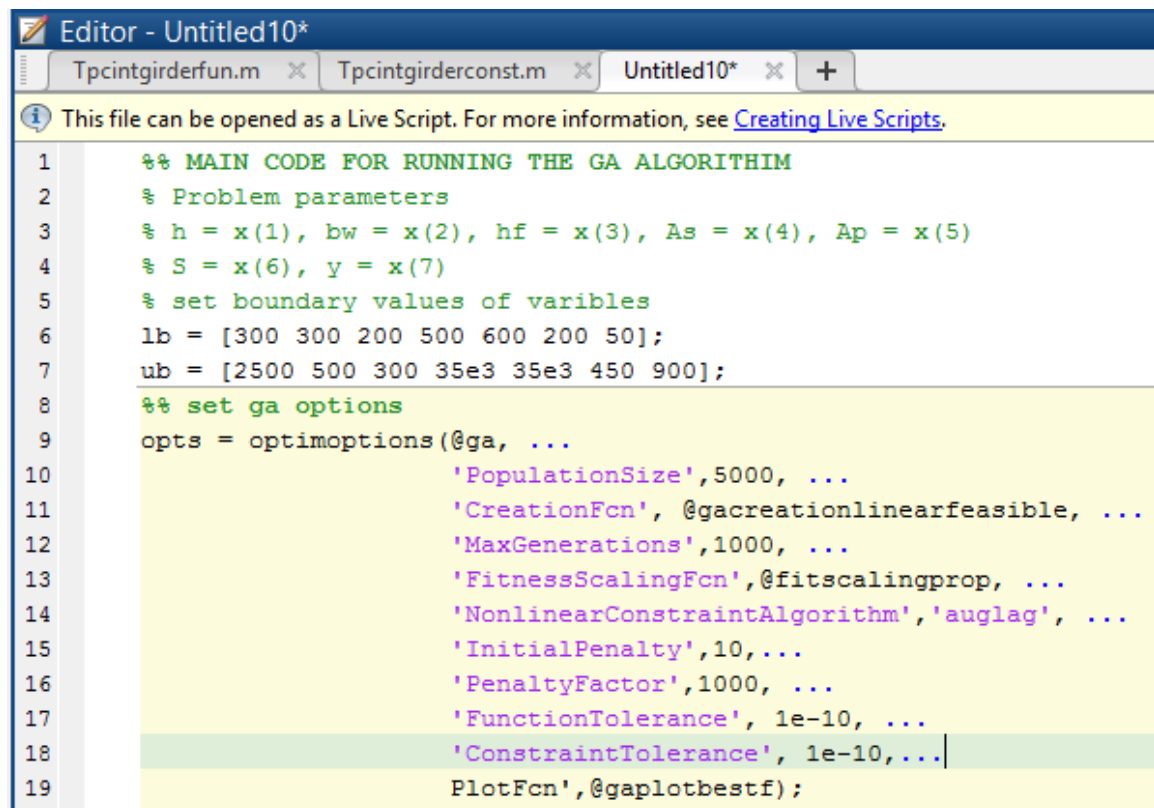
Editor - Untitled9*
Tpcintgirderfun.m x Untitled9* x +
234     g30 = -confcnvalm-tol;
235 end
236 if x(7) > x(3)
237     radf = 0;
238     tol = 1e-6;
239     confcnvalf = Ts+Tp+Cn-C-radf;
240     g31 = confcnvalf-tol; % sum of service load moments when NA depth y > hf
241     g32 = -confcnvalf-tol;
242 else
243     radf = 0;
244     tol = 1e-6;
245     confcnvalf = Ts+Tp-C-radf;
246     g31 = confcnvalf-tol; % sum of service load moments when NA depth y < hf
247     g32 = -confcnvalf-tol;
248 end
249 g33 = 0.20*x(1)-x(7);
250 g34 = x(7) - 0.75*x(1);
251 % non linear equality const. functions defn.
252 c = [g1;g2;g3;g4;g5;g6;g7;g8;g9;g10;g11;g12;g13;g14;g15;g16;g17;g18;g19;g20;...
253     g21;g22;g23;g24;g25;g26;g29;g30;g31;g32;g33;g34]; % non linear inequality const. functions defn.
254 ceq = [];
255
    
```

Step 3. Define the main function. Add new script and define boundaries of design variables in the main file as follows.



```
Editor - Untitled10*
Tpcintgirderfun.m x Tpcintgirderconst.m x Untitled10* x +
This file can be opened as a Live Script. For more information, see Creating Live Scripts.
1 %% MAIN CODE FOR RUNNING THE GA ALGORITHM
2 % Problem parameters
3 % h = x(1), bw = x(2), hf = x(3), As = x(4), Ap = x(5)
4 % S = x(6), y = x(7)
5 % set boundary values of variables
6 lb = [300 300 200 500 600 200 50];
7 ub = [2500 500 300 35e3 35e3 450 900];
```

Set the optimization options using the following syntaxes.

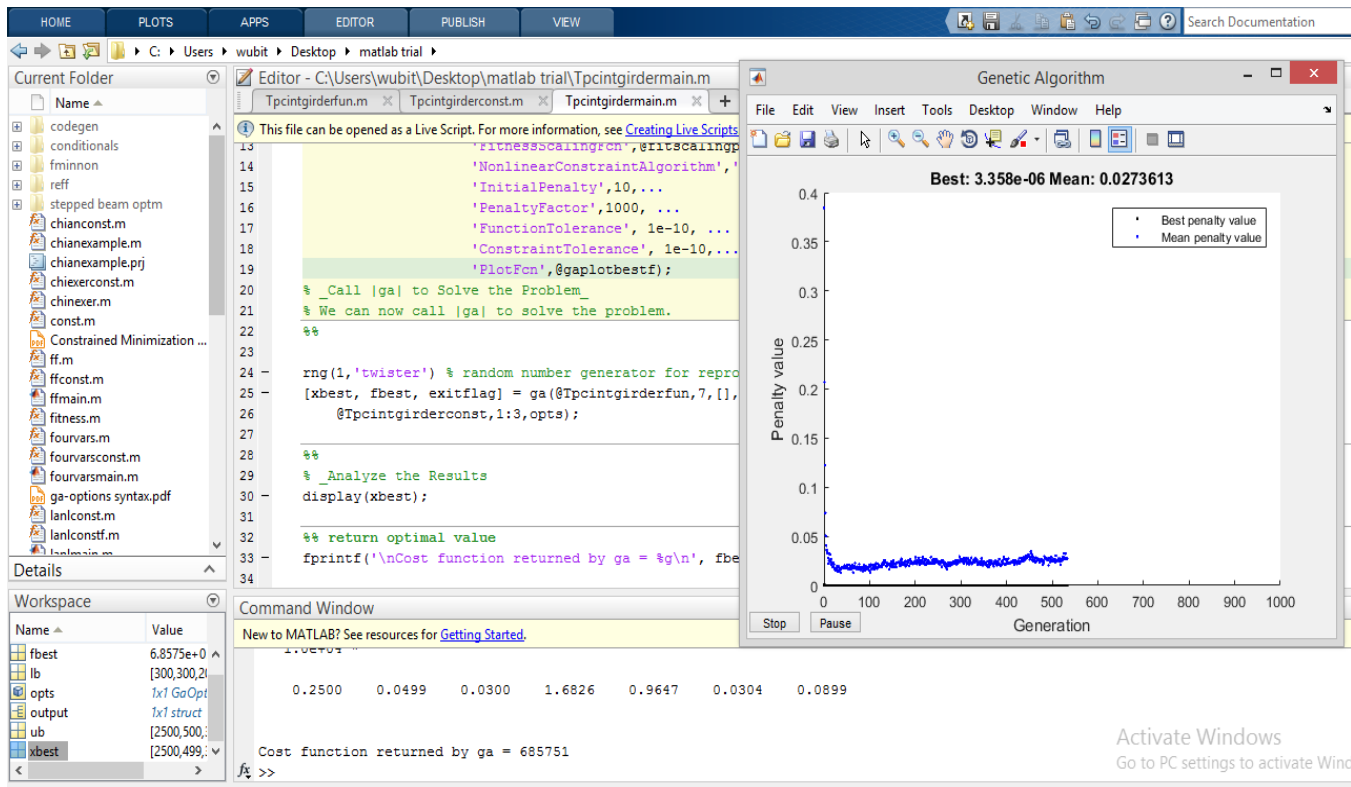


```
Editor - Untitled10*
Tpcintgirderfun.m x Tpcintgirderconst.m x Untitled10* x +
This file can be opened as a Live Script. For more information, see Creating Live Scripts.
1 %% MAIN CODE FOR RUNNING THE GA ALGORITHM
2 % Problem parameters
3 % h = x(1), bw = x(2), hf = x(3), As = x(4), Ap = x(5)
4 % S = x(6), y = x(7)
5 % set boundary values of variables
6 lb = [300 300 200 500 600 200 50];
7 ub = [2500 500 300 35e3 35e3 450 900];
8 %% set ga options
9 opts = optimoptions(@ga, ...
10     'PopulationSize',5000, ...
11     'CreationFcn', @gacreationlinearfeasible, ...
12     'MaxGenerations',1000, ...
13     'FitnessScalingFcn',@fitscalingprop, ...
14     'NonlinearConstraintAlgorithm','auglag', ...
15     'InitialPenalty',10,...
16     'PenaltyFactor',1000, ...
17     'FunctionTolerance', 1e-10, ...
18     'ConstraintTolerance', 1e-10,...
19     'PlotFcn',@gaplotbestf);
```

Note the three dots allows continuity of a sentence in a new line. Next call GA with the following syntax to solve the problem. Save this file with a name you want, let it be 'Tpcintgirdermain.m' for this case.

```
Editor - Untitled10*
Tpcintgirderfun.m x Tpcintgirderconst.m x Untitled10* x +
This file can be opened as a Live Script. For more information, see Creating Live Scripts.
13     'FitnessScalingFcn',@fitscalingprop, ...
14     'NonlinearConstraintAlgorithm','auglag', ...
15     'InitialPenalty',10,...
16     'PenaltyFactor',1000, ...
17     'FunctionTolerance', 1e-10, ...
18     'ConstraintTolerance', 1e-10,...
19     PlotFcn',@gaplotbestf);
20 % _Call |ga| to Solve the Problem_
21 % We can now call |ga| to solve the problem.
22 %%
23
24 rng(1,'twister') % random number generator for reproducibility
25 [xbest, fbest, exitflag] = ga(@Tpcintgirderfun,7, [], [], [], [], lb,ub,...
26     @Tpcintgirderconst,1:3,opts);
27
28 %%
29 % _Analyze the Results
30 display(xbest);
31
32 %% return optimal value
33 fprintf('\nCost function returned by ga = %g\n', fbest);
34
```

Step 4. Run the code. Press F5 to start running the code and generate optimization results. Note as soon you run it the program prompts you to change the folder so that click *change folder*. Unless the current folder is active or being opened by the program, Matlab couldn't understand your code and solve it. The following results obtained.



If all constraints are satisfied print the outputs and if not adjust lower and upper bounds and rerun it again. Note that Matlab is case sensitive due attention should be given to each characters you type in a code. If you miss even one character or mathematical symbol, the whole code could not run and correct it if such error warning message appeared following the error lines suggested in the message.

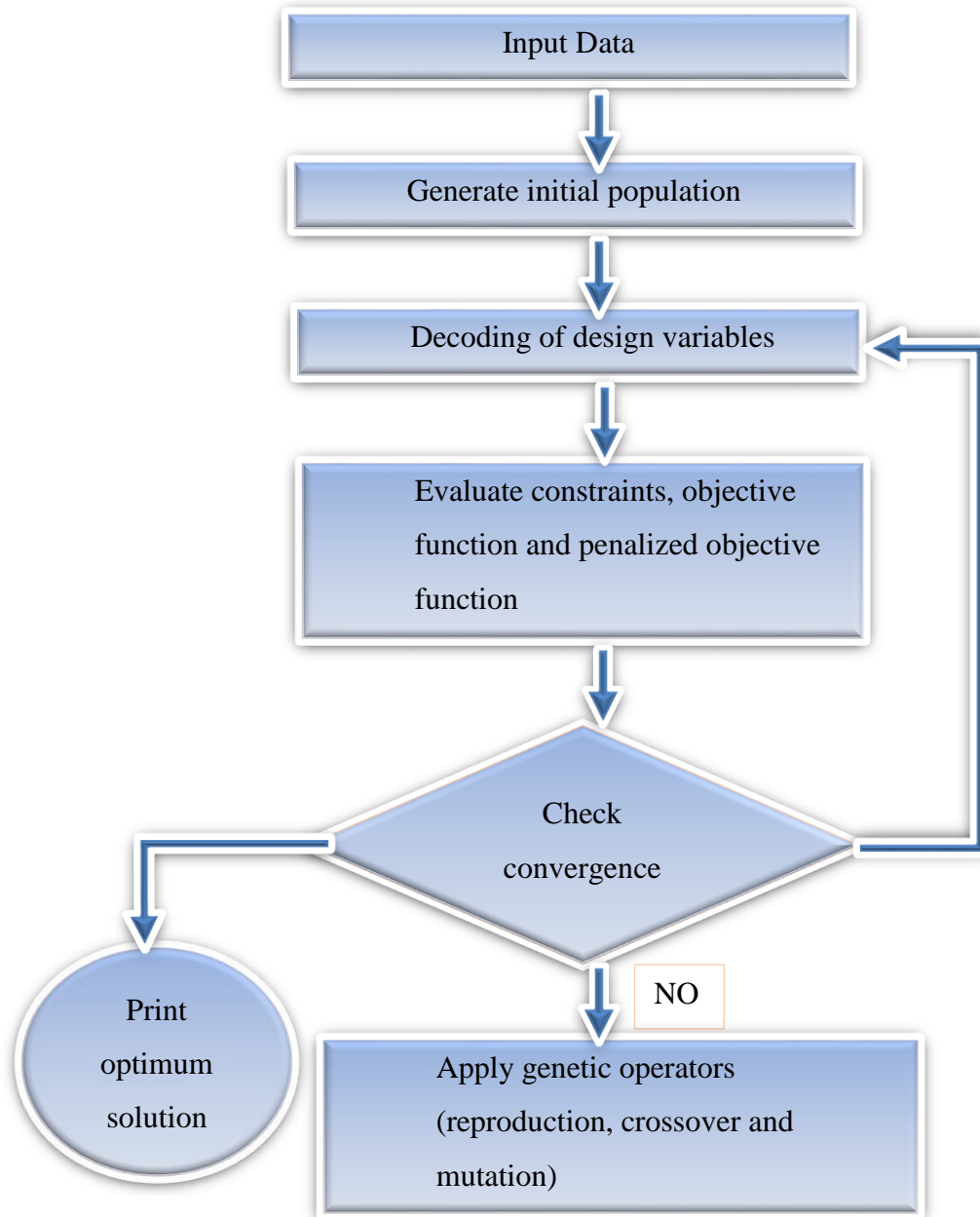


Figure 3.2 Flow Chart for GA Optimization

### **3.5 Study Variables**

#### **3.5.1 Independent variables**

- Girder Cross section types.
- Grades of Concrete
- Span of the bridge.
- Construction Materials.
- Girder spacing.

#### **3.5.2 Dependent variables**

- Cost optimization of prestressed concrete bridge girders.



## **CHAPTER FOUR**

### **OPTIMIZATION OF PRESTRESSED CONCRETE**

#### **GIRDERS**

Prestressed concrete bridge consists of a superstructure of either reinforced or prestressed concrete deck slab with prestressed concrete girders supported at the ends by abutments and at the intermediate there may or may not be pier supports at one or more points. The renewal of prestressed system in modern bridge engineering was due to the tendency of bridge engineers to obtain optimum structural performance through saving limited resources, construction materials. This fact is due to reduction of amount of steel and size of section required for relatively long span bridges by using prestressing system which reduces tensile stress, deflection and cracks in the section substantially and enhances bending, shear and torsional capacities of the member and hence its durability. Dimensioning of a particular bridge from economic consideration, meeting the safety and serviceability requirements is complicated due to wide possible range.

The studies of effects of individual parameter on relative optimum cost of bridge do not carry much significance. Thus in the present work the optimization is carried out by considering more design parameters as design variables simultaneously. In optimizing prestressed concrete girders, the cross sectional area of girders, amount of reinforcing steel and prestressing tendons, strength quality of concrete and steel are crucial and are decided based on the strength and stability criteria. In this research all the possible design parameters which affect the optimum cost of bridge significantly are considered as design

variables and all types of constraints, strength, stability, and serviceability are incorporated in the optimization routine.

## **4.2 Optimization Model of Simply Supported Prestressed Concrete Girders**

In this section, the model of PC girder of a bridge is described, showing the fixed parameters, the design variables' boundary, the design constraints and the objective function.

A simply supported Tee and box partially prestressed concrete and reinforced concrete girders with a variable span of  $L$  m and supporting a uniform superimposed gravity dead load of components  $W_{DC}$  kN/m, in addition to its own weight and design vehicular point live load of  $P_{LL}$  kN together with the design lane load,  $W_{LN}$  kN/m was applied. It is intended to optimize the design of bridge girders by keeping the provisions of the requirements of AASHTO LRFD Bridge Design Specifications are satisfied [28]. Figures (4.1), (4.2) and (4.3) below show the geometry of simply supported bridge girder.

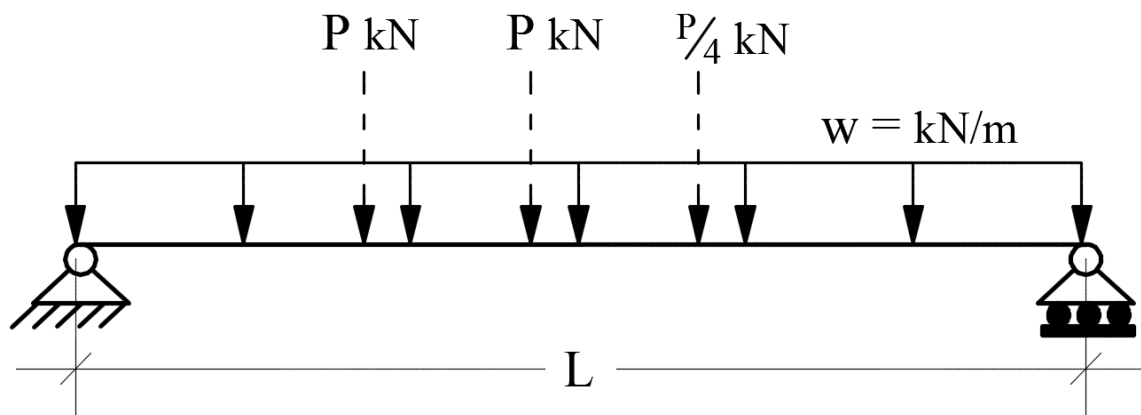


Figure 4.1 Longitudinal Model of the Bridge

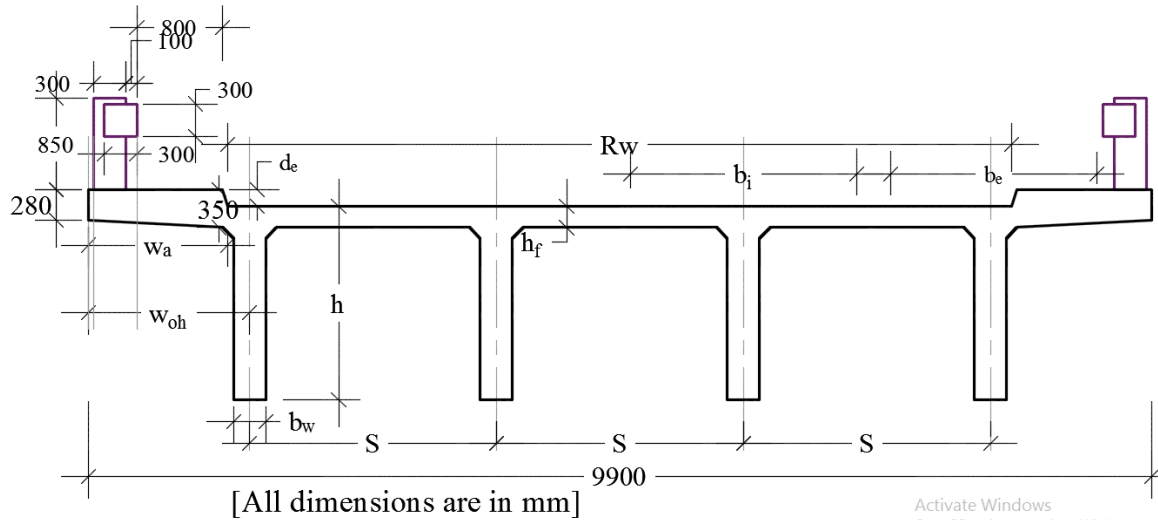


Figure 4.2 Cross Sectional Model of T-Girder Bridge

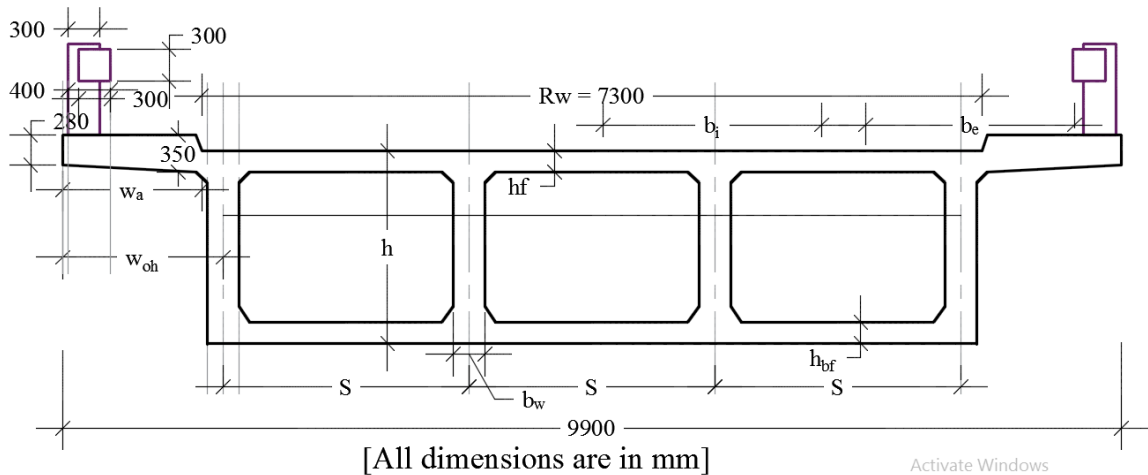


Figure 4.3 Cross Sectional Model of Box Girder Bridge

### 4.3 Load Analysis

Linear static structural analysis is made using distribution factors for shear and moment given under AASHTO Article 4.4.2.2. Girders are modelled as simply supported by abutments at their ends in which the support joints are assumed as roller and pin. In the analysis of loads dynamic load allowance or impact factor (IM) of 15% for fatigue and fracture limit state and 33% for all other limit states was considered as stated in AASTO Article 3.6.2.1. These factor accounts for hammering when riding surface discontinuities exist, and long undulations when settlement or resonant excitation occurs. Depending on

the number of lanes loaded multiple presence factor ( $m$ ) specified under AASHTO Article 3.6.1.1.2 was used to modify the vehicular live loads for the probability that vehicular live loads occur together in a fully loaded state.

### 4.3.1 Load Cases and Load Combinations

Three main load cases were considered for the structure to analyze and design, the action of dead, live and prestressing loads. During the optimization process routine structure analysis for maximum response under any live load pattern at any section was made with the use of influence lines. Dead load includes self-weight of bridge deck components, railings and girders which are accounted for as a uniform loads and self-weight of diaphragms applied as point loads. Live load is design vehicular live load of AASHTO LRFD designated as HL-93. It includes point loads of maximum effect of either design truck or design tandem combined with a uniform design lane load of  $9.3\text{kN/m}$  as shown in figure 4.4 and 4.5 below. Minimum prestressing force is obtained by selecting the maximum tensile prestress at the top fiber and a minimum compressive prestress at the bottom fiber. For post tensioning system a prestress loss factor of 0.85 is applied in this research.

Load combinations applicable to superstructure design that is strength limit state-I, strength limit state-IV, service limit state-I, service limit state-III, and fatigue limit state as specified in AASHTO LRFD Article 3.4 were used.

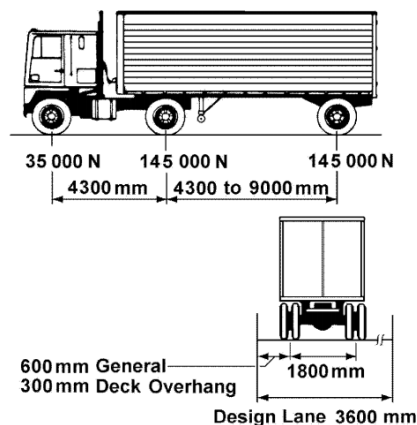


Figure 4.4 Characteristics of the Design Truck

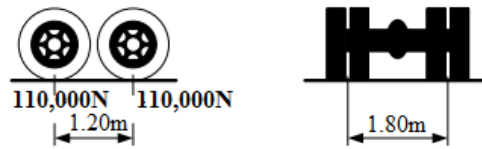


Figure 4.5 Characteristics of the Design Tandem

*(Adopted from AASHTO LRFD Code Book Article 3.6.1.2.2)*

#### **4.4 Design Philosophy**

Design philosophy used in this thesis is the AASHTO load and resistance factor design (LRFD) approach stated in Article 1.3.2 is given below.

$$\sum \eta_i \gamma_i Q_i \leq \phi R_n \quad (4.1)$$

Where,

$\eta$  – load modifier as per ODOT recommendation is a value of 1.05 used.

$\gamma$  – load factor, statically based multipliers applied to force effects.

$\Phi$  – resistance factor, statically based multipliers applied to nominal resistance; a value of 0.90 is used for both shear and flexure as given in AASHTO LRFD Article 5.5.4.

Q – force effects.

$R_n$  – nominal resistance.

#### **4.4 Optimization Problem Formulation**

In this thesis, problem formulation was based on linear elastic analysis and ultimate strength method of design with the consideration of serviceability constraints as per AASHTO LRFD 2005 Interim code is used. Two dimensional static linear analysis was adequate for all practical purposes in optimization of prestressed concrete bridges. While modeling the connection between girder and abutment, the connection node are separately considered and boundary conditions are applied independently.

Genetic algorithm (GA) deals with population that is collection of candidate solution and a population is a collection of N individuals. An important feature of a population, especially in the early generation of its evolution, is its genetic diversity. The too small population size may lead to scarcity of genetic diversity. It may result in a population dominated by almost equal chromosomes and then, after decoding the genes and evaluating the objective function it may converge quickly but may lead to local optimum. At the other extreme, in too large populations, the overabundance of genetic diversity can lead to clustering of individuals around different local optima. But the mating of individuals belonging to different clusters can produce children lacking the good genetic part of either of the parents. In addition, the manipulation of large populations may be excessively expensive in terms of computer time. Thus proper selection of population size is extremely important.

The formulation of optimization problem had been made by utilizing the interior penalty function method as an optimization method with the purpose of minimizing the objective function representing the cost of the girder. This cost includes cost of concrete, reinforcement, prestressing strands. Cost of form work was neglected as it takes only small portion of the material cost. Commonly used girder sections T and box sections made up of reinforced concrete and post tensioned partially prestressed concrete were intended to study.

#### **4.5 Fixed Design Variables**

The span of the girders, characteristic strength of concrete and reinforcement steel, modulus of elasticity, and unit weight of concrete and reinforcement, magnitude of dead and live loads were assumed to be fixed or pre-assigned parameters. It was also assumed that the total cost of concrete and reinforcement is proportional to volume and weight of each material, respectively. Consequently, the total cost of the structure was calculated using fixed parameters of the cost of unit volume of concrete and unit weight of reinforcement. Values of fixed parameters and the defined materials property are given in the following table.

Table 4.1. Fixed Values of Material Properties

<i>Items</i>	<i>Properties</i>	<i>Values</i>	<i>Remark</i>
Modulus of Elasticity of Concrete	$E_c$	27660 N/mm <sup>2</sup>	
Modulus of Elasticity of prestressing steel	$E_p$	195000 N/mm <sup>2</sup>	
Modulus of Elasticity of non prestressing steel	$E_s$	200000 N/mm <sup>2</sup>	
Specified compressive strength of concrete	$f_c'$	30 N/mm <sup>2</sup>	Grade C-35
Yield strength of reinforcing bars	$f_y$	420 N/mm <sup>2</sup>	Grade_420
Ultimate tensile strength of prestressing steel	$f_{pu}$	1860 N/mm <sup>2</sup>	Grade_270
Density of reinforcement steel	$\rho_s$	7.850x10 <sup>-9</sup>	ton/mm <sup>3</sup>

#### **4.6 Design Variables**

The formulations of an optimization problem begins with identifying the underlying geometric design variables. These variables should be independent of each other. If one of the design variables can be expressed in terms of the other then that variable can be eliminated from the model. Geometric design variables includes overall depth, web width, thickness of the flange, area of nonprestressed and prestressed reinforcement, spacing of traverse reinforcement and NA depth of the cracked transformed section. These variables are listed below.

- $d$  – effective depth of nonprestressed reinforcement
- $b_w$  – web width
- $h_f$  – thickness of the flange
- $A_s$  – Area of nonprestressed steel
- $A_p$  – Area of prestressing steel
- $S$  – spacing of traverse reinforcement
- $y$  – NA depth of the cracked transformed section

These variables can be assigned in terms of  $x_i$ 's as follows

Table 4.2 Designation of Design Variables

Variable designation	h	b <sub>w</sub>	h <sub>f</sub>	A <sub>s</sub>	A <sub>p</sub>	S	y
Matlab code designation	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>	x <sub>5</sub>	x <sub>6</sub>	x <sub>7</sub>

## 4.7 Objective Function

In structural design, the dominant objective was to minimizing structural cost. There can be multi objective functions such as minimizing cost, maximize performance, maximize reliability, and others in one problem, but generally it is avoided by choosing the most important objective as the objective function and the other objective functions were included as constraints by restricting their values within a certain range.

In this research minimization of the initial cost of bridge girders was carried out. The most important cost items in the initial cost of the girders are usually the cost of material. So the total cost is the cost of concrete and the cost of reinforcement and prestressing steel.

The function below defines the total cost of the PC simple girder model in terms of the cost of concrete and reinforcement used.

$$f(x) = C_c x \left\{ (A_c - A_s - A_p) x L - \frac{W_{str}}{\gamma} \right\} + C_s x \{ \gamma x A_s x L + W_{str} \} + C_p x \{ \gamma x A_p x L \} \quad (4.2)$$

Where:

$C_c$  – unit cost of concrete per cubic millimeter (ETB/mm<sup>3</sup>)

$C_s$  – unit cost of reinforcement steel per ton (ETB/ton)

$C_p$  – unit cost of pre-stressing tendons per ton (ETB/ton)

$A_c$  – Area of concrete cross section (mm<sup>2</sup>)

$A_s$  – Area of longitudinal reinforcement (mm<sup>2</sup>)

$A_p$  – Area of prestressing tendons (mm<sup>2</sup>)

$L$  – Span length of the girder (mm)

$W_{str}$  – Weight of stirrups (ton)

$\gamma$  – Unit weight of steel reinforcement bars and prestressing tendons (ton/mm<sup>3</sup>)

Where unit cost of materials assessed based on the current market trend and given in the following table.



Table 4.3 Unit Cost of Concrete

Grade of Concrete, Mpa	30	40	50	60	70
Unit Cost, (ETB/mm <sup>3</sup> x10 <sup>-9</sup> )	2,840	3,205	3,500	3,640	4,200

(Source: Own Survey)

Unit cost of reinforcing steel and prestressing steel were evaluated as 27,940ETB/ton and 46,450ETB/ton respectively. It may be noted that in evaluating the cost of prestressing strands, since it is an imported material its price is referred from market price of China Hong Kong [29] and all necessary custom taxes [30] and freight costs are also included.

#### 4.8 Constraint Function

The constraints reflect design requirements in the optimization problem. In other words they limit the range of acceptable designs in the problem. In this research, the constraints relevant to the design of PC girder are applied using a penalty function.

Generally, structural design is required to conform to number of inequality constraints related to stresses, deflection, dimensional relationships, and other code requirements.

*Referring to the optimization model Figure 4.2 and 4.3 above, width of the compression face, b is given by adopted from AASHTO Article 4.6.2.6*

$$i) \text{ for interior girder } b_i \leq \begin{cases} L/4 \\ 12h_f + b_w \\ \text{Avg. girder spacing, } g_s \end{cases} \Rightarrow b = \begin{cases} L/4 \\ 12h_f + b_w \leq 0 \\ g_s \end{cases}$$

$$ii) \text{ for exterior girder } b_e = \frac{b_i}{2} + \leq \begin{cases} L/8 \\ 6h_f + \frac{b_w}{2} \\ W_{oh} \end{cases} \Rightarrow b = \frac{b_{int}}{2} + \leq \begin{cases} L/8 \\ 6h_f + \frac{b_w}{2} \leq 0 \\ W_{oh} \end{cases}$$

Section properties of T-girder is given by;

$$\text{depth from centroid to extreme bottom fiber, } y_b = \frac{b_w \frac{h^2}{2} + (b-b_w)h_f \left( \frac{h-h_f}{2} \right)}{b_w h + (b-b_w)h_f}, \quad y_t = h - y_b$$

$$\text{Moment of inertia, } I = \frac{b_w h^3}{12} + b_w h \left( \frac{h}{2} - y_t \right)^2 + \frac{(b-b_w)h_f^3}{12} + (b-b_w)h_f \cdot \left( y_t - \frac{h_f}{2} \right)^2$$

$$\text{Section moduli, } Z_b = \frac{I}{y_b}, \quad Z_t = \frac{I}{y_t} \quad \text{and area of concrete section, } A = b_w h + (b - b_w)h_f$$

Section properties of box-girder is given by:

From symmetry of the section both extreme top and bottom fibers located at equidistance from centroid of the section that is  $y_b = y_t = \frac{h}{2}$ , moment of inertia of the section is given

$$\text{by: } I = \frac{b_w h^3}{12} + 2 \cdot \left( \frac{(b-b_w)h_f^3}{12} + (b-b_w)h_f \cdot \left( y_t - \frac{h_f}{2} \right)^2 \right),$$

Section moduli are:  $Z_b = \frac{I}{y_b}$  &  $Z_t = \frac{I}{y_t}$  and gross cross sectional area of concrete is

$$A_c = b_w h + 2 \cdot ((b - b_w)h_f)$$

Where,

$b_w$  – web width of the section (mm)

$b_i$  – width of compression face for interior girder (mm)

$b_e$  – width of compression face of the section of exterior girder (mm)

$h_f$  – thickness of the flange (mm)

$g_s$  – girder spacing (mm)

$w_{oh}$  – width of overhang (mm)

$h$  – over all depth of the section (mm)

$y_t$  – depth from extreme top fiber to centroid of the section (mm)

$y_b$  – depth from extreme bottom fiber to centroid of the section (mm)

$d_p$  – depth from extreme compression fiber to centroid of prestressing steel (mm)

$A$  – cross sectional area of concrete (mm<sup>2</sup>)

$I$  – second moment of area or moment of inertia of concrete cross section (mm<sup>4</sup>)

$Z_t$  – Section modulus of the extreme top fiber (mm<sup>3</sup>).

$Z_b$  – Section modulus of the extreme bottom fiber (mm<sup>3</sup>)

The constraint functions imposed in the design of a prestressed concrete flexural member are generally stated in the following articles:

**1. Allowable stresses in the concrete**

Stresses in the concrete at the two extreme outer fibers shall be less than the allowable values stated in code book of AASHTO Article 5.9.4. These stresses can be evaluated using the following equations adopted from the book of Krishna. R [27].

- i. Top fiber subjected to tension at stress transfer stage:

$$\frac{P}{A} - \frac{Pe}{Z_t} + \frac{M_g}{Z_t} \geq f_{tt} \quad \Rightarrow \quad g_1 = f_{tt} - \frac{P}{A} + \frac{Pe}{Z_t} - \frac{M_g}{Z_t} \leq 0 \quad (4.3a)$$

- i. Bottom fiber subjected to compression at stress transfer:

$$\frac{P}{A} + \frac{Pe}{Z_b} - \frac{M_g}{Z_b} \leq f_{ct} \quad \Rightarrow \quad g_2 = \frac{P}{A} + \frac{Pe}{Z_b} - \frac{M_g}{Z_b} - f_{ct} \leq 0 \quad (4.3b)$$

- ii. Top fiber subjected to compression at service loads:

$$\eta P \left( \frac{1}{A} - \frac{e}{Z_t} \right) + \frac{M_w}{Z_t} \leq f_{cw} \quad \Rightarrow \quad g_3 = \eta P \left( \frac{1}{A} - \frac{e}{Z_t} \right) + \frac{M_w}{Z_t} - f_{cw} \leq 0 \quad (4.3c)$$

- iii. Bottom fiber subjected to tension at service loads:

$$\eta P \left( \frac{1}{A} + \frac{e}{Z_b} \right) - \frac{M_w}{Z_b} \geq f_{tw} \Rightarrow g_4 = f_{tw} - \eta P \left( \frac{1}{A} + \frac{e}{Z_b} \right) + \frac{M_3}{Z_b} \leq 0 \quad (4.4d)$$

The extreme bottom fiber stress,  $f_{inf}$  developed at a given eccentricity  $e$  is given by;

$$f_{inf} = \frac{f_{tw}}{\eta} + \frac{M_w}{\eta \cdot Z_b}$$

and once knowing  $f_{inf}$  and using the section modulus  $Z_b$  of the provided

section, the minimum prestressing force required is given by:  $P = \frac{A \cdot f_{inf} \cdot Z_b}{Z_b + A \cdot e}$ . Depth from

extreme top fiber to centroid of prestressing steel is  $d_p = y_t + e$

Where:

P – Prestressing force (N)

e – eccentricity of prestressing force from the centroid of the section (mm)

$M_{min}$  – minimum moment due to self weight or during handling of the member (Nmm)

$M_w$  – working moment at service limit state I (Nmm)

$M_3$  – working moment at service limit state III (Nmm)

$f_{ct}$  – maximum allowable compressive stress in concrete at initial prestress (N/mm<sup>2</sup>)

$f_{it}$  – maximum allowable tensile stress in concrete at initial prestress (N/mm<sup>2</sup>)

$f_{cw}$  – maximum allowable compressive stress in concrete at service load (N/mm<sup>2</sup>)

$f_{tw}$  – maximum allowable tensile stress in concrete at service load (N/mm<sup>2</sup>)

$f_{tr}$  – stress range at the extreme top fiber (N/mm<sup>2</sup>)

$f_{br}$  – stress range at the extreme bottom fiber (N/mm<sup>2</sup>)

$f_{inf}$  – stress at the extreme bottom fiber for a given eccentricity  $e$  (N/mm<sup>2</sup>)

$\eta$  – Loss factor

## 2. **Strength requirement for flexure at the limit state of collapse**

NA axis depth for evaluate at the strength limit state is given by AASHTO LRFD Equation 5.7.3.1-3.

$$\text{if } c > h_f, \text{ then } c = \frac{A_p f_{pu} + A_s f_y - A'_s f'_y - 0.85 \beta_1 f_c (b - b_w) h_f}{0.85 f_c \beta_1 b_w + 0.28 A_p \frac{f_{pu}}{d_p}} \quad \dots \text{T section}$$

$$\text{else } c = \frac{A_p f_{pu} + A_s f_y - A'_s f'_y}{0.85 f_c \beta_1 b + 0.28 A_p \frac{f_{pu}}{d_p}} \quad \dots \text{rectangular section}$$

For rectangular or T section where  $f_{pe} \geq 0.5 f_{pu}$ , average stress in prestressing steel  $f_{ps}$  is given by AASHTO LRFD Equation 5.7.3.1.1-1 and 2

$$f_{ps} = f_{pu} \left( 1 - k \frac{c}{d_p} \right) = f_{pu} \left( 1 - \frac{0.28c}{d_p} \right) \text{ in which } k = 2 \left( 1.04 - \frac{f_{py}}{f_{pu}} \right) \text{ for } \frac{f_{py}}{f_{pu}} = 0.90, k = 0.28$$

Effective depth from extreme compression fiber to centroid of tensile force,  $d_e$  is given by AASHTO LRFD Equation 5.7.3.3.1-2.

$$d_e = \frac{A_p f_{ps} d_p + A_s f_y d}{A_p f_{ps} + A_s f_y}$$

Depth of equivalent rectangular stress block,  $a = \beta_1 \cdot c$  and the nominal moment of resistance  $M_n$  is given by AASHTO LRFD Equation 5.7.3.2.2-1 stated as follows.

$$\text{if } c > h_f, M_n = A_p f_{ps} \left( d_p - \frac{a}{2} \right) + A_s f_y \left( d - \frac{a}{2} \right) - A'_s f'_y \left( d'_s - \frac{a}{2} \right) + 0.85 f_c \beta_1 h_f (b - b_w) \left( \frac{a}{2} - \frac{h_f}{2} \right)$$

$$\text{else, } M_n = A_p f_{ps} \left( d_p - \frac{a}{2} \right) + A_s f_y \left( d - \frac{a}{2} \right) - A'_s f'_y \left( d'_s - \frac{a}{2} \right)$$

$$g_5 = M_u \leq \phi M_n \Rightarrow M_u - 0.90 M_n \leq 0 \quad (4.4)$$

Where,

$A_p$  – area of prestressing steel ( $\text{mm}^2$ )

$f_{pe}$  – effective stress in prestressing steel ( $\text{N/mm}^2$ )

$f_{pu}$  – ultimate tensile strength of prestressing steel ( $\text{N/mm}^2$ )

$f_{py}$  – yield strength of prestressing steel ( $\text{N/mm}^2$ )

$f_{ps}$  – average stress in prestressing steel (N/mm<sup>2</sup>)

$d_p$  – distance from extreme compression fiber to centroid of prestressing tendons (mm)

$d_e$  – depth from extreme compression fiber to centroid of tensile force (mm)

$A_s$  – area of nonprestressed steel tension reinforcement (mm<sup>2</sup>)

$f_y$  – yield strength of non prestressed steel tension reinforcement (N/mm<sup>2</sup>)

$d$  – distance from extreme compression fiber to centroid of nonprestressed tension reinforcement (mm)

$A_s'$  – area of nonprestressed steel compression zone reinforcement (mm<sup>2</sup>)

$f_y'$  – yield strength of non prestressed steel compression zone reinforcement (N/mm<sup>2</sup>)

$d_s'$  – distance from extreme compression fiber to centroid of nonprestressed compression zone reinforcement (mm)

$f_c'$  – specified cylindrical compressive strength of concrete (N/mm<sup>2</sup>)

$b$  – width of the cross section in compression zone (mm)

$b_w$  – web width of the cross section (mm)

$\beta_1$  – stress block factor,  $\beta_1 = 0.85$  for  $f_c' = 28\text{Mpa}$  and reduced by 0.05 for each 7Mpa increment of  $f_c'$  and  $\beta_1 \geq 0.65$

$h_s$  – depth of the deck slab or flange thickness (mm)

$c$  – depth of the neutral axis (mm)

$a$  – depth of equivalent rectangular stress block (mm)

$M_d$  – ultimate factored design moment due to all loads (Nmm)

$M_n$  – nominal moment of resistance (Nmm)

$\Phi$  – resistance factor

### **3. Strength requirements for shear design (AASHTO LRFD Article 5.8)**

Effective shear depth is given by AASHTO LRFD Article 5.8.2.9,  $d_v \geq \begin{cases} 0.9d_e \\ 0.72h \\ d_e - a/2 \end{cases}$

$$V_c = 0.083 \beta \sqrt{f'_c} b_w d_v = 0.166 \sqrt{f'_c} b_w d_v \quad \text{for } \beta = 2$$

$$V_s = \frac{A_v f_y d_v}{S}$$

$$V_p = \eta \cdot P \cdot \left( \frac{4e}{L} \right)$$

Thus nominal shear resistance,  $V_n \leq \begin{cases} V_c + V_s + V_p \\ 0.25 f'_c b_w d_v + V_p \end{cases}$

then  $V_d \leq \phi V_n$  from this we have the following

$$g_6 = V_u - 0.9V_n \leq 0 \quad (4.5)$$

$$g_7 = \frac{V_u}{0.9} - 0.25 f'_c b_w d_v - V_p \leq 0 \quad (4.6)$$

#### *Longitudinal reinforcement*

At each section the tensile capacity of the longitudinal reinforcement on the flexural tension side of the member shall satisfy the following requirement [28].

$$A_s f_y + A_p f_{ps} \geq \frac{|M_d|}{\phi d_v} + 0.5 \frac{N_d}{\phi} + \left( \left| \frac{V_u}{\phi} - V_p \right| - 0.5 V_s \right) \cot \theta, \quad V_s \leq \frac{V_u}{\phi} \quad \text{take } \theta = 45^\circ \rightarrow \cot \theta = 1$$

$$g_8 = \frac{|M_d|}{0.9 d_v} + \left| \frac{V_u}{0.9} - V_p \right| - 0.5 \min \left( \frac{V_u}{0.9}, \frac{A_v f_y d_v}{S} \right) - A_s f_y - A_p f_{ps} \leq 0 \quad (4.7)$$

Where,  $N_d$  – factored longitudinal tension force (N)

For simple end supports to the section of critical shear the longitudinal reinforcement on the flexural tension side should satisfy the following conditions, AASHTO LRFD article 5.8.3.5:

$$A_s f_y + A_p f_{ps} \geq \left( \frac{V_u}{\phi} - 0.5 V_s - V_p \right) \cot \theta \quad \text{assume } \theta = 45^\circ, \cot \theta = 1$$

$$g_9 = \frac{V_u}{0.9} - 0.5V_s - V_p - A_s f_y - A_p f_{ps} \leq 0 \quad (4.8)$$

*Minimum spacing of traverse reinforcement, S*

$$S \leq \frac{A_v f_y}{0.083 b_w \sqrt{f'_c}} \Rightarrow g_{10} = S - \frac{A_v f_y}{0.083 b_w \sqrt{f'_c}} \leq 0 \quad (4.9)$$

*Maximum spacing of traverse reinforcement, S*

if  $\frac{|V_u - \phi V_p|}{\phi b_w d_v} < 0.125 f'_c$  then,

$$g_{11} = S - \min \left\{ \begin{array}{l} 0.8d_v \\ 600 \end{array} \right\} \leq 0 \quad (4.10a)$$

else

$$g_{11} = S - \min \left\{ \begin{array}{l} 0.4d_v \\ 300 \end{array} \right\} \leq 0 \quad (4.10b)$$

Where;

$V_u$  – factored design shearing force d distance from face of support (N)

$V_n$  – nominal shear resistance (N)

$V_c$  - shear resisting force due to tensile stress in the concrete (N)

$V_s$  – shear resisting force due to tensile stress in traverse reinforcement (N)

$V_p$  – component of prestressing force in the direction of shearing force (N)

$S$  – spacing of stirrups (mm)

$A_v$ - cross sectional area of shear reinforcement within a distance  $S$  (mm<sup>2</sup>)

$d_v$  - effective depth of shearing force (N)

#### **4. Limits of reinforcement (AASHTO LRFD Article 5.7.3.3)**

*\_ Minimum amount of reinforcement*

Amount of prestressed and non prestressed tensile reinforcement shall be adequate to develop factored flexural resistance  $M_r$  which shall not be the lesser of 1.2 times cracking moment and 1.33 times factored design moment as equated below.



$$\text{Cracking moment, } M_{cr} = Z_c (f_r + f_{cpe}) - M_g \left( \frac{Z_c}{Z_{nc}} - 1 \right) \geq Z_c f_r$$

for monolithic sections substitute  $Z_{nc}$  for  $Z_c$  then,  $M_{cr} = (f_r + f_{cpe}) \frac{I}{y_b}$

$$f_r = 0.97 \sqrt{f'_c}$$

$$f_{cpe} = \eta P \left( \frac{1}{A} + \frac{e}{Z_b} \right)$$

$$\text{and } M_r = \phi M_n$$

$$M_r \geq 1.2M_{cr} \Rightarrow g_{12a} = 1.2M_{cr} - 0.9M_n \leq 0 \quad (4.11a)$$

$$M_r \geq 1.33M_d \Rightarrow g_{12b} = 1.33M_d - 0.9M_n \leq 0 \quad (4.11b)$$

As per ACI-318 1989 minimum area of flexural reinforcement shall not be less than 0.4% of the area of concrete found between centroid of the section and tension face [31].

$$(A_p + A_s) \geq 0.004 y_b b_w \Rightarrow g_{13} = 0.004 y_b b_w - A_p - A_s \leq 0 \quad (4.12)$$

Where,

$f_{cpe}$  – compressive stress in concrete due to effective prestress forces only (N/mm<sup>2</sup>)

$M_g$  – total unfactored dead load moment (Nmm)

$M_d$  – total factored design moment (Nmm)

$M_r$  – total factored moment of resistance of the section (Nmm)

$M_{cr}$  – cracking moment (Nmm)

$Z_c$  – section modulus for the extreme fiber of the composite section where tensile stress is caused by externally applied loads (mm<sup>3</sup>)

$Z_{nc}$  – section modulus for the extreme fiber of monolithic or non-composite section where tensile stress is caused by externally applied loads (mm<sup>3</sup>) that is  $Z_b$

$f_r$  – modulus of rupture (N/mm<sup>2</sup>)

– *Maximum amount of reinforcement*

For the section to develop enough ductility by yielding of steel before failure it should be designed as *under reinforced* section and the following equation shall meet. This will be done with the reinforcement index  $\omega$  as per the report addressed by ACI 423 [32].

$$\rho = A_s/bd \rightarrow \omega = \rho f_y/f_c', \quad \rho' = A_s'/bd \rightarrow \omega' = \rho' f_y/f_c', \quad \rho_p = A_p/bd_p \rightarrow \omega_p = \rho_p f_y/f_c'$$

$$\omega + \omega_p - \omega' \leq 0.3 \Rightarrow g_{14} = \omega + \omega_p - \omega' - 0.3 \leq 0 \quad (4.13a)$$

Where,

$\omega, \omega', \omega_p$  - reinforcement indices of tension, compression and prestressing steels respectively

$\rho, \rho', \rho_p$  - ratios of reinforcement of tension, compression and prestressing steels to area of concrete respectively

alternatively as per ASHTO equation 5.7.3.3.1-1, we have

$$\frac{c}{d_e} \leq 0.42 \Rightarrow g_{15} = \frac{c}{d_e} - 0.42 \leq 0 \quad (4.13b)$$

### **5. Permissible stresses in the reinforcement steels**

The cracked section analysis of partially prestressed flanged section with prestressed tendons and nonprestressed reinforcement was carried out under the assumptions: the strain distribution across the section is linear and the tensile strength of concrete below the NA is negligible. The stress and forces acting on a cracked partially prestressed concrete section subjected to a moment a working or service load moment of  $M_w$  in excess of the cracking moment  $M_{cr}$  is shown in figure 4.6 below. The analysis was made with use of equations given by N. Krishna R, see reference [27].

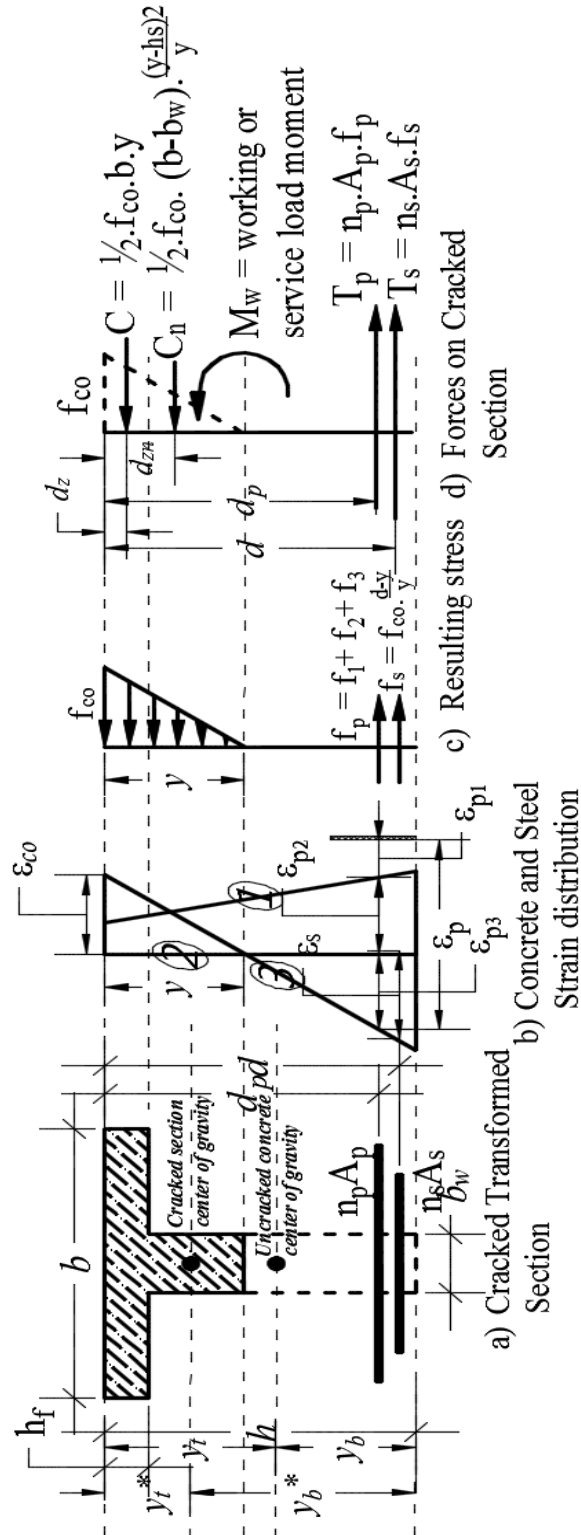


Figure 4.6 Cracked Transformed Section

From the cracked section analysis we have the following equations:

- a. Just prior to the application of  $M_w$  or at stage (1) stress in the prestressing tendons is

$$f_{p1} = f_{pe} = \frac{P_e}{A_p} \quad \Rightarrow \quad f_{p1} = \frac{\eta \cdot P}{A_p}$$

- b. Next, it is useful to consider a fictitious load stage (2) corresponding to complete decompression of the concrete, at which there is zero concrete strain throughout the entire depth as shown in Figure 4.6b. compatibility of deformation of concrete and steel requires that changes in strain in the tendon is the same as that in the concrete at that level and the stress in tendon due to this strain is given by:

$$\varepsilon_{ce} = \frac{P_e}{E_c} \left( \frac{e^2}{I} + \frac{1}{A} \right) \text{ from which } f_{p2} = E_p \varepsilon_{ce} \Rightarrow f_{p2} = n_p \eta P \left( \frac{e^2}{I} + \frac{1}{A} \right)$$

- c. During the application of  $M_w$ , the concrete compressive strain in the bottom fiber reduces to zero and then becomes tensile. With  $M_w$  acting the tensile strain in the reinforcing steel is  $\varepsilon_s$  and the strain in the concrete at the level of the tendon has changed from a compression  $\varepsilon_{oc}$  to a tension of  $\varepsilon_{cp}$ . From linearity of strain distribution these strains can be defined in terms of the neutral axis depth  $y$  and top fiber strain  $\varepsilon_o$ .

Let the extreme top fiber strain be  $\varepsilon_o$  and NA depth be  $y$ , then from strain compatibility one can drive the strains and stresses in reinforcing steel and prestressing tendons at their respective depths. Once the strains are evaluated the corresponding stresses can be obtained from the stress-strain relationship as shown in the following steps.

The strain in reinforcing steel at a depth  $d$  is  $\varepsilon_s = \varepsilon_o \frac{d-y}{y}$  and the corresponding stress is

$$f_s = \varepsilon_s E_s \quad \Rightarrow \quad f_s = E_s \varepsilon_o \frac{d-y}{y} . \text{ Similarly the tensile strain in the concrete at the level of}$$

prestressing steel or at a depth  $d_p$  is  $\varepsilon_{cp} = \varepsilon_o \frac{d_p - y}{y}$  . The prestressing tendons undergoes

this strain and thus the stress in the tendons is  $f_{p3} = E_p \varepsilon_{cp} \Rightarrow f_{p3} = E_p \varepsilon_{co} \frac{d_p - y}{y}$  and stress in the concrete at the extreme top fiber is  $f_{co} = \varepsilon_{co} E_c$ . The prestressing steel undergoes a stress of  $(f_{p2} + f_{p3})$  during the application of  $M_w$  so that the total tensile stress in the tendon is  $f_p = f_1 + f_2 + f_3$ . Tensile force in the prestressing and reinforcing steel respectively;  $T_p = A_p f_p$  and  $T_s = A_s f_s$ . In the concrete compressive zone, the resultant compressive force is;  $C = \frac{1}{2} f_{co} b y$  which is acting at a depth  $d_z = \frac{y}{3}$ . This equation is valid if the neutral axis lies in the flange that is  $y \leq h_f$  and if  $y > h_f$ , the force C shall be reduced by  $C_n$  given by  $C_n = \frac{1}{2} f_{co} (b - b_w) \frac{(y - h_f)^2}{y}$  which can be regarded as negative force and acting at a depth,  $d_{zn} = h_f + \frac{y - h_f}{3}$ .

The incremental strain,  $\varepsilon_{co}$  sought as loading passes from stage (2) to stage (3) can be defined in terms neutral axis depth,  $y$  as;

$$\varepsilon_{co} = \frac{A_p (f_{p1} + f_{p2})}{\frac{1}{2} E_c \left( b_w y + (b - b_w) h_f \left( 1 + \frac{y - h_f}{y} \right) \right) - \left( E_s A_s \left( \frac{d - y}{y} \right) + E_p A_p \left( \frac{d_p - y}{y} \right) \right)}$$

In this equation is for flanged section so that substitute  $b_w$  by  $b$  if  $y$  is less than or equal to  $h_f$ . From equilibrium of moments we have,

$$\text{if } y > h_f, \text{ then } \Rightarrow g_{16} = T_s d + T_p d_p + C_n d_{zn} - C d_z - M_w \leq 0 \quad (4.14a)$$

else

$$g_{17} = T_s d + T_p d_p - C d_z - M_w \leq 0 \quad (4.14b)$$

Since  $M_w$  is known solve for the strain  $\varepsilon_{co}$  and NA depth  $y$  and then equilibrium of x(horizontal of forces should be checked.

$$\sum F_x = 0, \quad C - C_n = T_s + T_p$$

$$\text{If } y > h_f, \text{ then } g_{18} = C - C_n - T_s - T_p \leq 0 \quad (4.15a)$$

else,

$$g_{18} = C - T_s - T_p \leq 0 \quad (4.15b)$$

Location of centroid of the cracked transformed section from extreme top fiber,  $y_{ct}$  is given

$$\text{if } y > h_f, y_{ct} = \frac{b_w \frac{y^2}{2} + (b - b_w) \frac{h_f^2}{2} + n_p A_p d_p + n_s A_s d}{b_w y + (b - b_w) h_f + n_p A_p + n_s A_s}$$

by

$$\text{else, } y_{ct} = \frac{b \frac{y^2}{2} + n_p A_p d_p + n_s A_s d}{b y + n_p A_p + n_s A_s}$$

The cross sectional area of the cracked transformed section,  $A_{ct}$  will be;

$$\text{if } y > h_f, A_{ct} = b_w y + (b - b_w) h_f + n_p A_p + n_s A_s$$

$$\text{else } A_{ct} = b y + n_p A_p + n_s A_s$$

Second moment of area or moment of inertia of the cracked transformed section,  $I_{ct}$  is;

$$\text{if } y > h_f, I_{ct} = b_w \frac{y^3}{12} + b_w y \left( y_{ct} - \frac{y}{2} \right)^2 + (b - b_w) \frac{h_f^3}{12} + (b - b_w) h_f \left( y_{ct} - \frac{h_f}{2} \right)^2 + n_p A_p (d_p - y_{ct})^2 + n_s A_s (d - y_{ct})^2$$

$$\text{else } I_{ct} = b \frac{y^3}{12} + b y \left( y_{ct} - \frac{y}{2} \right)^2 + n_p A_p (d_p - y_{ct})^2 + n_s A_s (d - y_{ct})^2$$

where,

$f_{p1}$  – incremental stress in prestressing tendons prior to the application of service loads or at stage (1) (N/mm<sup>2</sup>)

$f_{p2}$  – incremental stress in prestressing tendons as the section passes from prior to the application of service loads stage (1) to decompression stage (2) (N/mm<sup>2</sup>)

$f_{p3}$  – incremental stress in prestressing tendons due to change of stress from compression to tension in the concrete located at the level of the tendon (N/mm<sup>2</sup>)

$f_{co}$  – stress in the extreme top fiber during the application of service load moment (N/mm<sup>2</sup>)

$f_s$  – stress in nonprestressed steel reinforcement at the application of service loads (N/mm<sup>2</sup>)

$f_p$  – total stress in prestressing tendons at the application of service loads (N/mm<sup>2</sup>)

$\epsilon_{oc}$  – compressive strain in the concrete at the level of the tendon

$\epsilon_{cp}$  – tensile strain in the concrete at the level of the tendon

$\epsilon_o$  – compressive strain at the extreme top fiber

$\epsilon_s$  – tensile strain in the reinforcing steel at working loads

$n_p$  – modular ratio of prestressing steel

$n_s$  – modular ratio of reinforcing steel

$E_c$  – modulus of elastic of concrete (N/mm<sup>2</sup>)

$E_s$  – modulus of elastic of reinforcing steel (N/mm<sup>2</sup>)

$E_p$  – modulus of elastic of prestressing steel (N/mm<sup>2</sup>)

$F_x$  – forces acting in the horizontal direction (N)

$T_s$  – Tension force in the reinforcing steel at service limit state (N)

$T_p$  – Tension force in the prestressing steel at service limit state (N)

$C$  – resultant compressive force in compression zone of concrete (N)

$C_n$  – compressive force in compression zone of concrete used to reduce the resultant compressive force  $C$  when NA depth exceeds flange thickness (N)

$y$  – NA depth of the cracked section under service limit state (mm)

$d_z$  – depth from extreme compression fiber to centroid of resultant compression force  $C$  (mm)

$d_{zn}$  – depth from extreme compression fiber to centroid of compression force  $C_n$  (mm)

$y_{ct}$  – depth from extreme compression fiber to centroid of cracked section (mm)

$A_{ct}$  – area cracked transformed section under service limit state (mm<sup>2</sup>)

$I_{ct}$  – moment of inertia of cracked transformed section under service limit state ( $\text{mm}^4$ )

- *Permissible stresses in prestressing strands during stress transfer stage*

$$\frac{P}{A_p} \leq f_{pt} \quad \Rightarrow \quad g_{19} = \frac{P}{A_p} - f_{pt} \leq 0 \quad (4.16)$$

- *Permissible stresses in prestressing strands during service limit state*

$$f_p \leq f_{pe} \quad \Rightarrow \quad g_{20} = f_p - f_{pe} \leq 0 \quad (4.17)$$

- *Permissible stress in nonprestressed steel at service limit state*

$$f_s \leq \begin{cases} 0.5f_y = 0.5 \cdot 420 = 210 \\ 206 \end{cases} \quad \Rightarrow \quad g_{21} = f_s - 206 \leq 0 \quad (4.18)$$

### **6. Deflection control (ASHTO LRFD Article 5.7.3.6)**

Deflection and camber calculations shall consider dead load, live load, prestressing, erection loads, concrete creep and shrinkage, and steel relaxation. Immediate or instantaneous deflection is computed by taking the effective moment of inertia,  $I_e$ .

Effective moment of inertia used to calculate the instantaneous deflection is given by

$$I_e \leq \begin{cases} \left( \frac{M_{ck}}{M_w} \right)^3 I + \left( 1 - \left( \frac{M_{ck}}{M_w} \right)^3 \right) I_{ct} \\ I \end{cases}$$

$$M_{ck} = f_{rk} \frac{I}{y_b} \quad \text{and} \quad f_{rk} = 0.63\sqrt{f'_c}$$

Where,

$M_{ck}$  – Cracking moment (Nmm)

$f_{rk}$  – modulus of rupture of concrete (N/mm<sup>2</sup>)

*Deflection due to dead loads and prestressing force*

Instantaneous deflection for permanent loads calculation,  $\Delta_i$



Instantaneous deflection due to dead load,  $\Delta_{di} = \frac{\iint M_g(x) \cdot dx^2}{E \cdot I_e}$

Additional long term deflection  $\Delta_{d1} = \lambda \cdot \Delta_{di}$   $\lambda \geq \begin{cases} 3 - 1.2 \frac{A_s'}{A_s} = 3, \text{ setting } \frac{A_s'}{A_s} = 0.00 \\ 1.6 \end{cases}$

- Thus total deflection due to dead load,  $\Delta_d = 4 \Delta_{d1}$

Note in integrating the dead load moment equations integral constants should be evaluated based on boundary conditions.

For parabolic tendon profile with central anchor upward deflection due to prestress is,

- $\Delta_p = -\frac{5 \cdot \eta \cdot P \cdot e \cdot L^2}{48 \cdot E \cdot I_e}$

Hence, camber required is limited to,  $g_{22} = \Delta_d - \Delta_p \leq 0$  (4.19)

*Deflection due to live loads*

When investigating the maximum absolute deflection, all of the design lanes should be loaded and all of the girders may be assumed to deflect equally in supporting the loads. This statement is equivalent to a deflection distribution factor  $mg^d$  equal to the number of lanes divided by the number of girders [33]. Deflection of the bridge due to truck loads occurs at a wheel load position where maximum moment is occurring. For live load deflection evaluation, design vehicular live load of AASHTO HL-93 where the vehicle load includes the impact factor  $IM$  and the multiple presence factor  $m$ . In general, the deflection at the point of maximum moment,  $x$  due to each design truck load at a distance  $a$ , from the left support is given by:

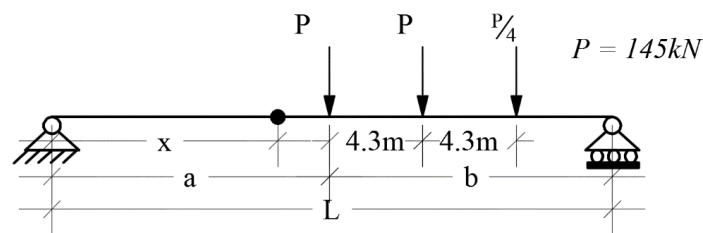


Figure 4.7 Design Truck Load Arrangement for Deflection Calculation

Live load deflection due to design truck load will be

- for  $x = a$ ,  $\Delta_{ki} = \frac{P \cdot a^2 \cdot b^2}{3E \cdot I_e \cdot L}$
- for  $x < a$ ,  $\Delta_{ki} = \frac{P \cdot b \cdot x}{6E \cdot I_e \cdot L} \cdot (L^2 - b^2 - x^2)$  individual truck load deflection

Thus total design truck deflection will be,  $\Delta_{kt} = \sum_{i=1}^3 \Delta_{ki}$

- Deflection due to each design lane load,  $\Delta_{Ln} = \frac{5 \cdot w_{Ln} \cdot L^4}{384 \cdot E \cdot I_e}$  where  $w_{Ln} = 9.3 \text{ kN/m}$

In computation of live load deflection design truck load alone or design lane load plus 25% of design truck load whichever is the greater as stated in AASHTO article 3.6.1.3.2.

- Thus live load deflection is,  $\Delta_{LL} \geq \begin{cases} \Delta_{kt} \\ \Delta_{Ln} + 0.25\Delta_{kt} \end{cases}$
- Allowable live load deflection is,  $\Delta_{all} = \frac{L}{1000}$

Thus limit of live load deflection is,  $\Delta_{LL} \leq \Delta_{all}$

$$\text{hence, } g_{23} = \Delta_{LL} - \frac{L}{1000} \leq 0 \quad (4.20)$$

Where:

$I_e$  – effective moment of inertia of the section ( $\text{mm}^4$ )

$\Delta_{di}$  – Immediate deflection due to dead load (mm)

$\Delta_d$  – total long term deflection due to dead load (mm)

$\Delta_p$  – upward deflection due to prestress force (mm)

$\Delta_{kl}$  – deflection due to truck load (mm)

$\Delta_{Ln}$  – deflection due to design lane load (mm)

$\Delta_{LL}$  – deflection due to live load (mm)

$\Delta_{all}$  – allowable deflection for live load (mm)

$x$  – distance from left support to a point at which maximum service load moment occurs.

$a$  – distance from the left support to the point of truck load for which deflection is to be computed.

$b$  – distance from the right support to the point of truck load for which deflection is to be computed.

## 7. *Limit of the crack width*

For dry air or protective membrane (class I) exposure condition an assumed allowable crack width is 0.41mm. The expressions that has figured prominently in the development of the crack control provisions in the ACI code is the one that developed by Gerley [34]. Crack width equation of the model code CEB-FIP-1970 is also used for determining the maximum crack width at the tension face of the girder. These equations are respectively.

$$w_1 = 0.076 \frac{h_2}{h_1} f_s \sqrt[3]{d_c A_{ct}} \times 0.1451$$

$$w_2 = (f_s - 40) \times 10^{-3}$$

$$g_{24} = \max \begin{cases} w_1 \\ w_2 \end{cases} - 0.41 \leq 0 \quad (4.21)$$

Where,

$f_s$  - service load stress in non prestressed steel (Mpa)

$h_1$  – distance from centroid of tensile steel to NA depth (mm)

$h_2$  – depth from extreme compression fiber to depth of NA (mm)

$d_c$  – thickness of concrete cover measured from extreme tension fiber to centroid of closest bar there to (mm)

$A_{ct}$  – effective tension area of concrete surrounding one bar (mm<sup>2</sup>)

### **8. Fatigue limit state (AASHTO LRFD Article 5.5.3)**

The stress range in reinforcing steel resulting from fatigue load is;  $f_{fs} = \frac{n_s \cdot M_f \cdot (d - y_{ct})}{I_{ct}}$  and

stress range in prestressing steel resulting from fatigue load is;  $f_{fp} = \frac{n_p \cdot M_f \cdot (d_p - y_{ct})}{I_{ct}}$ .

The allowable stress range in reinforcing steel is given by  $= 145 - 0.33 f_{min} + 55 \left( \frac{r}{h} \right) = 161.5$  Mpa by setting  $\frac{r}{h} = 0.3$ ,  $f_{min} = 0.00$  where,  $r/h$  – ratio of base radius-to-height of rolled-on transverse deformation (a value of 0.3 can be used in the absence of specific data). Also the allowable stress range in prestressing tendons with

radius of curvature larger than 9000mm shall be less than 125Mpa. Thus, the following constraints stated as;

$$f_{fs} \leq 161.5 \Rightarrow g_{25} = f_{fs} - 161.5 \leq 0 \quad (4.22)$$

$$f_{fpp} \leq 125 \Rightarrow g_{26} = f_{fpp} - 125 \leq 0 \quad (4.23)$$

Where,

$f_{fs}$  – stress range in reinforcing steel due to fatigue load (N/mm<sup>2</sup>)

$f_{fp}$  – stress range in prestressing steel due to fatigue load (N/mm<sup>2</sup>)

$M_f$  – maximum fatigue load moment (Nmm)

$f_{min}$  – minimum live load stress where there is stress reversal (N/mm<sup>2</sup>)

$r$  – base radius of the deformation (mm) and

$h$  – height of the deformation (mm)

#### 9. ***Partial Prestressing Ratio (AASHTO LRFD Article 5.5.4.2)***

For partial prestressed concrete (class-III) structures partial prestressing ratio shall be in between 0.50 and 1. If this ratio is less than 0.5, the structure will be reinforced concrete not prestressed concrete and if it is equal to 1 it under go fully prestressing system (class-D) structure. PPR is given in the following:

$$PPR = \frac{A_p f_{py}}{A_p f_{py} + A_s f_y} \quad \text{and } 0.50 \leq PPR < 1.00$$

From the above equation we the following constraints

$$g_{27} = 0.50 - PPR \leq 0 \quad (4.24)$$

$$g_{28} = PPR - 1 \leq 0 \quad (4.25)$$

The constraint functions  $g_1$  to  $g_{27}$  and fitness function  $f(x)$  formulated above were constrained nonlinear programming problem for numerical solutions of post tension T and box girders and reinforced concrete T and box girders. These formulations were coded in

the script for constraint function definition in GA packages of Matlab software. For RC girders, these constraint functions were developed in similar procedure.

#### ***10. Design Variables bounds***

A bound constraint for lower and the upper limits of design variables were derived from point of view of geometric requirements, minimum practical dimension, code restriction etc. It were defined in the main scripts field for the given optimization problem accordingly.

## CHAPTER FIVE

### RESULTS AND DISCUSSIONS

The four cases presented earlier in section four that was reinforced concrete T girder, partially prestressed concrete T girder, reinforced concrete box girder, and partially prestressed concrete box girder were solved using genetic algorithm. The formulated optimization problem was coded in to Mat lab software to run the optimization genetic algorithm. The various parameters such as effect of bridge construction materials and grades of concrete on optimum cost, optimal girder spacing, optimum girder cross sectional dimensions, and comparison of cost of concrete and steel and comparison of properties of optimization solvers, has been studied. A discussion and comparison among the results were presented here in.

#### 5.1 Effect of Construction Materials on Optimum Cost

Table 5.1 Summary of Cost Comparison of Girder Cross Sections

<i>Span, L (m)</i>	<i>Optimum Costs, ETB</i>			
	<i>RC T Girder</i>	<i>RC Box Girder</i>	<i>PC T Girder</i>	<i>PC Box Girder</i>
10	41,575.10	56,214.75	55,151.50	57,484.40
20	115,729.00	169,340.00	149,936.00	148,419.50
30	237,705.00	345,156.00	318,968.00	284,334.00
40	426,042.00	628,953.00	597,499.50	513,705.50
50	714,913.50	1,171,345.50	986,760.50	769,398.50
60	1,117,095.00	2,004,765.00	1,222,780.00	1,207,886.00
70	1,626,045.00	2,913,590.00	1,591,015.00	1,882,780.00
80	2,308,745.00	4,512,120.00	2,383,755.00	2,888,645.00
90	3,109,530.00	6,436,030.00	2,937,510.00	3,363,805.00
100	4,111,565.00	7,503,240.00	3,873,375.00	5,384,450.00

Graphical representation of effect construction materials was given below.

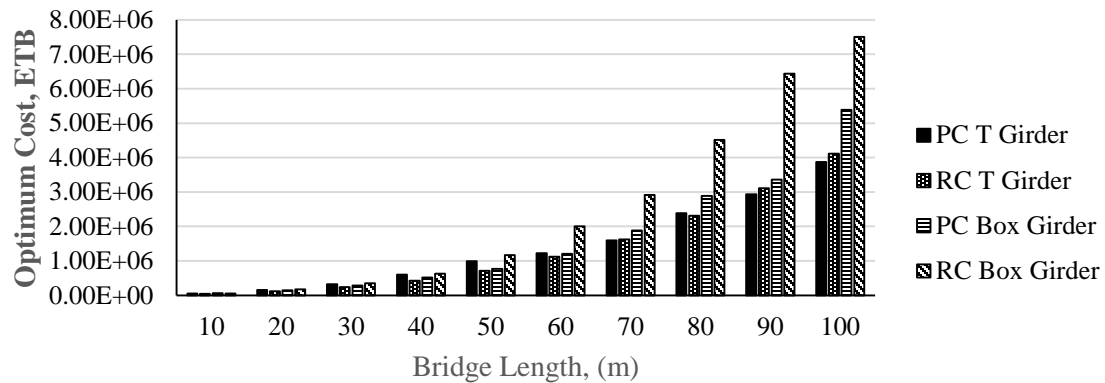


Figure 5.1 Effect of Construction Materials on Optimum Cost

Figure 5.1 shows cost comparison among commonly used girders in terms of cross sections and type of construction material. It can be noted from this figure that despite of the required stiffness, T section is economical for small to large spans preferably 20 to 40m [27]. However, partially prestressed box girder is stiffer than T girder and economical for spans larger than 40m.

## 5.2 Effect of Grades of Concrete on Optimum Cost

Summary of effect of grades of concrete on the optimum cost is given below.

Table 5.2 Effect of Grades of Concrete on Optimum Cost

<i>Grades of Concrete, Mpa</i>	<i>Optimum Costs, ETB</i>			
	<i>RC T Girder</i>	<i>RC Box Girder</i>	<i>PC T Girder</i>	<i>PC Box Girder</i>
30	710302	1471230	906221	1177935
40	765010	1175589.6	1120391	1344920
50	806293	1146074	1015500	1232800
60	817713	1195836.3	998500	1183500
70	922544	1375964.5	1023708	1232320

Graphical illustration of the effect of grades of concrete was shown below

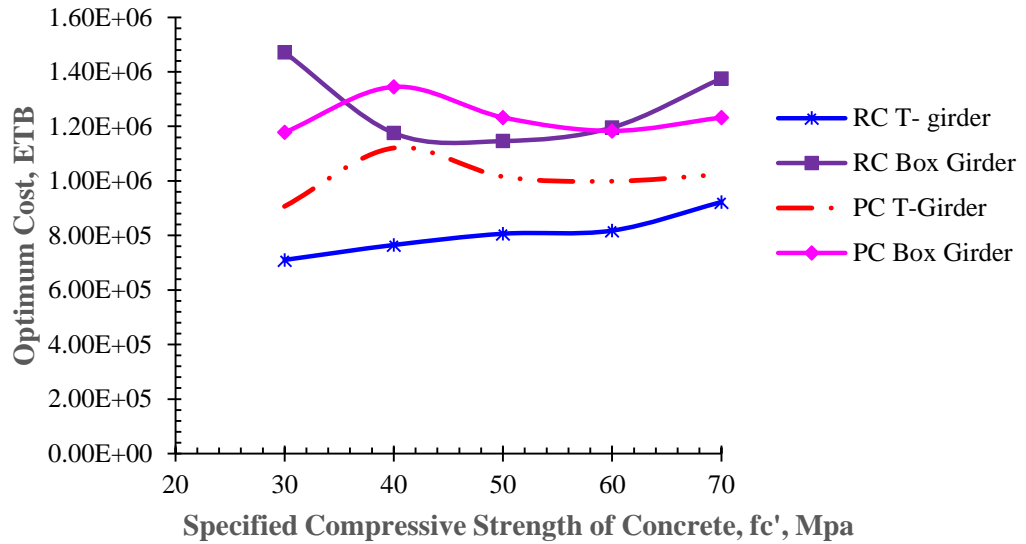


Figure 5.2 Effect of grades of Concrete on Optimum Cost

From the above graph 5.2 it may be considered that optimum cost will result for grades of concrete of values of specified cylindrical compressive strength of 30 to 50 Mpa.

### 5.3 Optimum Girder Spacing

Optimum cost for girder spacing of 1.5m to 4m was summarized in the following Table 5.3.

Table 5.3 Optimum Girder Spacing

Girder Spacing (m)	No. of girders	PC T Girders		PC Box Girders	
		Exterior Girder	Interior Girder	Exterior Girder	Interior Girder
1.50	7	1,002,376.09	444,694.31	908,045.40	1,105,531.62
2.00	5	680,826.32	436,000.00	987,791.12	1,130,507.70
2.50	4	362,089.48	427,408.79	1,390,056.63	1,147,931.00
3.00	4	301,546.37	369,756.89	1,410,000.00	1,169,016.59
3.50	3	347,500.00	388,528.33	1,405,463.04	1,196,196.31
4.00	3	391,533.86	439,414.84	1,425,282.03	1,218,812.15



Graphical representation of the optimal girder spacing was drawn in Figure 5.3 below.

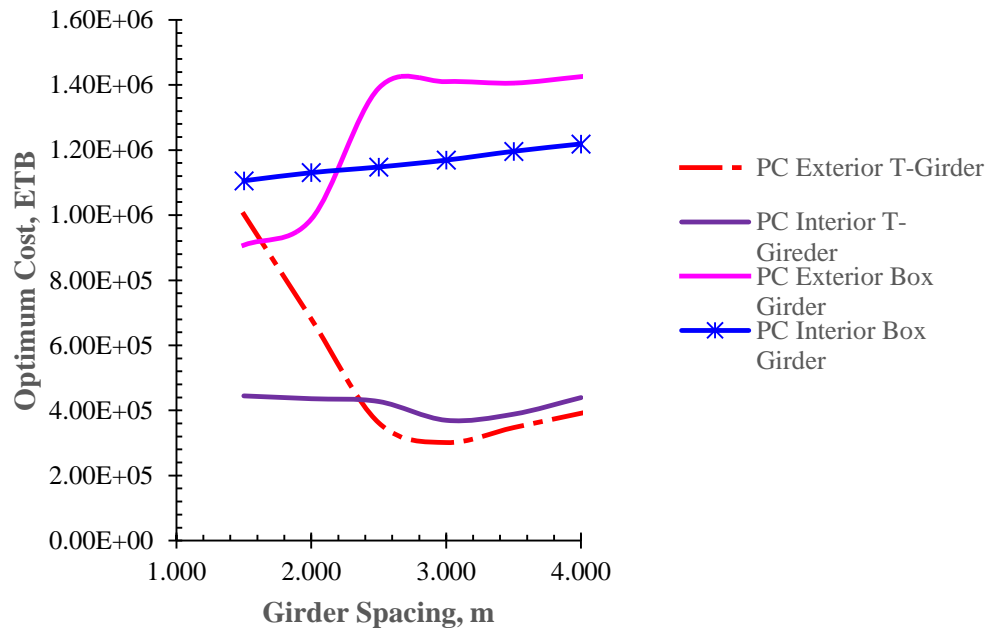


Figure 5.3 Optimum Girder Spacing

The plot above reveals an effort made in detrainning an optimal girder spacing as this is a parameter which determines load distribution factors between the girders. It is found that for road of two lanes of standard width of 3.65m each and an overhang of 1.3m wide both sides, the optimal girder spacing is 2.50m (point of intersection of exterior and interior girders).

### 5.4 Cost comparison of optimum design and conventional design approach

Summary of comparison of costs of optimum design and conventional design was tabulated below.

Table 5.4 Cost Comparison of Optimum and Conventional Design

Span, 50m	Optimum Design Cost (ETB)			Conventional Design Cost (ETB)			Cost Saving	
	Exterior Girder	Interior Girder	Average Cost	Exterior Girder	Interior Girder	Average Cost	in Amount	in %
T girder	835,552.50	1,028,626.50	<b>932,089.50</b>	1,505,008.48	1,519,951.96	<b>1,512,480.22</b>	580,390.72	38%
Box girder	1,122,090.00	1,498,140.00	<b>1,310,115.00</b>	1,605,376.43	1,905,971.53	<b>1,755,673.98</b>	445,558.98	25%

Cost comparison of the two design approaches was graphed below.

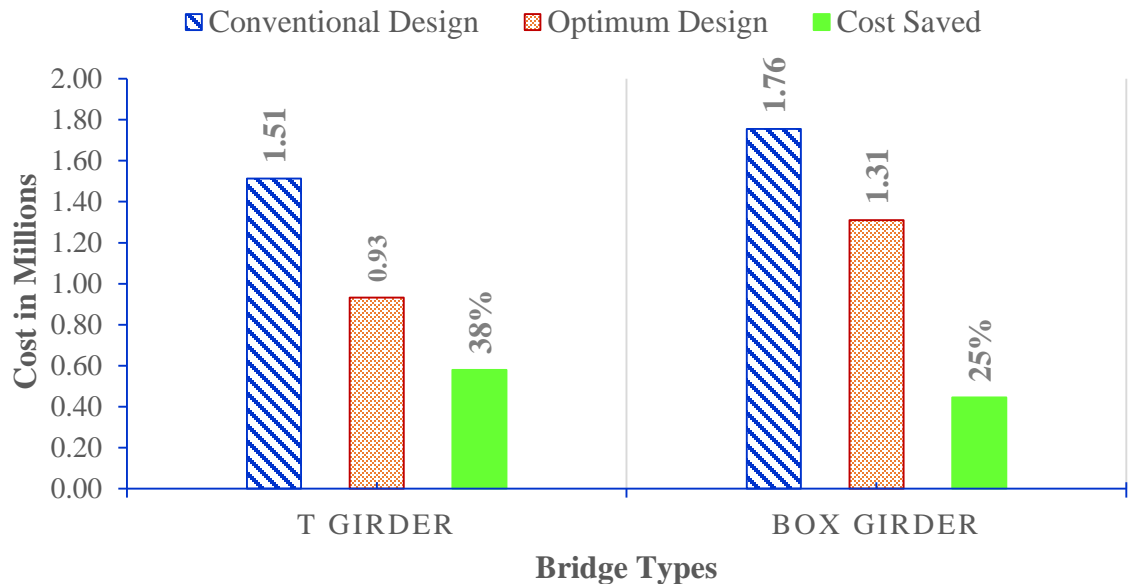


Figure 5.4 Cost comparison of Optimum Design and Conventional Design

From the Figure 5.4 it may be noted that optimum design of partially prestressed T and box girders could save a cost with an amount of 38% and 25% of the cost of conventional design approach respectively. This result was comparable to the one investigated by Bhawar, P.D see reference [17].

### **5.5 Effect of Grades of Concrete on Depth of the Girders**

The influence of grades of concrete on the optimum girder depth was given in Table 5.5 below.

Table 5.5 Effect of Grades of Concrete on the Optimum Girder Depth

<i>Grades of Concrete, Mpa</i>	<i>Optimum Girder Depth (mm)</i>			
	<i>RC T Girder</i>	<i>RC Box Girder</i>	<i>PC T Girder</i>	<i>PC Box Girder</i>
30	2981	2918	2796	2575
40	2808	2794	2800	2550
50	2726	2745	2792	2550
60	2693	2751	2780	2550
70	2689	2751	2766	2550

Graphical representation for elaborating the effects of grades of concrete on the girder depth was plotted in Figure 5.5.

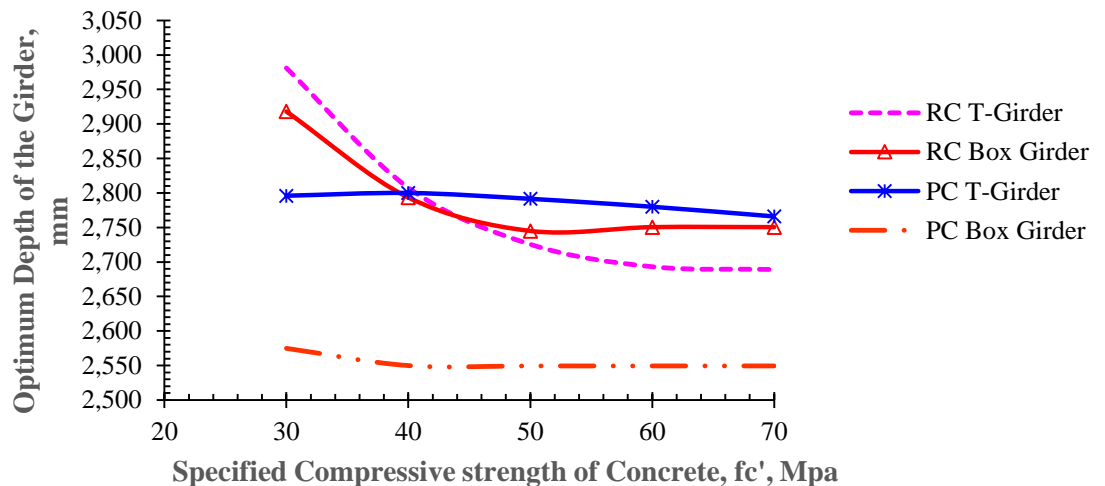


Figure 5.5 Effect of Compressive Strength of Concrete on Girder Depth

From the graph 5.5 it was obviously seen that depths of the girders reduces as grades of is increasing.

### 5.6 Comparison of Cost of Concrete and Steel

Ratio of cost of concrete to cost of reinforcement steel was given in Table 5.6 shown below.

Table 5.6 Cost Ratio of Concrete and Reinforcement Steel

Span (m)	Ratio of Cc/Cs			
	RC T	RC Box	PC T	PC Box
10	0.2907	0.1852	0.0929	0.2857
20	0.3461	0.20705	0.1075	0.5026
30	0.4023	0.22637	0.1221	0.614
40	0.3871	0.25106	0.1366	0.6361
50	0.3503	0.32104	0.1299	0.674
60	0.356	0.34352	0.1231	0.7118
70	0.3542	0.33842	0.1221	0.7195
80	0.3489	0.33194	0.121	0.7271
90	0.3672	0.32275	0.173	0.7294
100	0.3649	0.32326	0.1597	0.7317
110	0.3657	0.31512	0.1548	0.7432
120	0.3607	0.3095	0.15	0.7613
130	0.384	0.30388	0.1632	0.7623
140	0.3417	0.29826	0.1681	0.7564
150	0.3873	0.29264	0.173	0.7505

Comparison of construction materials cost ratio over a specific bridge length was plotted in Figure 5.6.

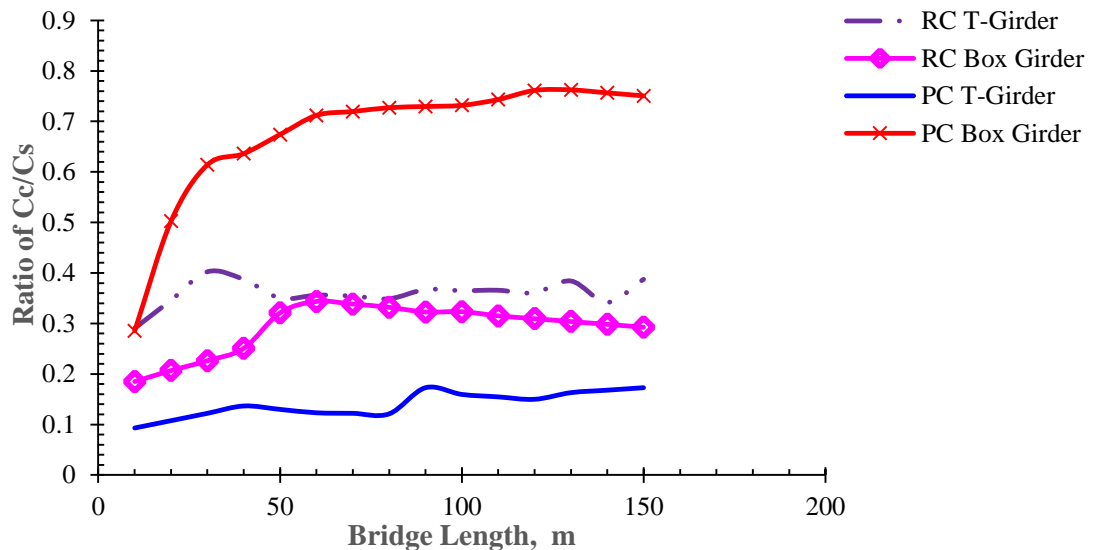


Figure 5.6 Cost Ratio of Concrete and Reinforcement Steel

In order to compare cost of concrete and steel reinforcement (prestressing steel and non prestressing steel reinforcement) Graph 5.6 was plotted as ratio cost of concrete to cost of steel versus bridge length. It was observed that cost of concrete is governing to a span of 40m and beyond this cost of steel reinforcement will dominate cost of bridge girders and due attention need to be considered in optimization process of these materials that is based on their nature cost dominance one over the other.

## 5.7 Optimum Girder Cross Sectional Dimensions

Table 5.7 Ratios of Optimum Girder Cross Sectional Dimensions

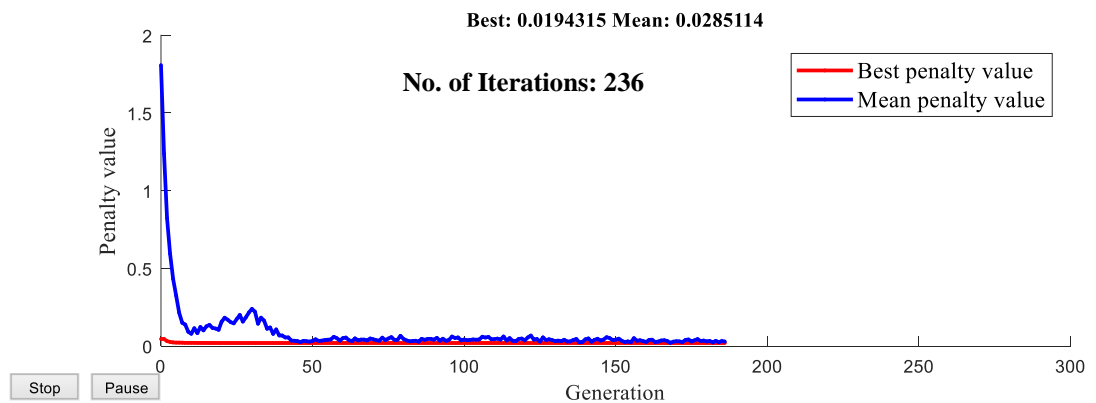
Bridge Length, m	RC T Girder			RC Box Girder			PC T Girder			PC Box Girder		
	$h/L$	$b_w/h$	$h_f/h$	$h/L$	$b_w/h$	$h_f/h$	$h/L$	$b_w/h$	$h_f/h$	$h/L$	$b_w/h$	$h_f/h$
10	0.080	0.043	0.020	0.085	0.040	0.020	0.109	0.042	0.023	0.099	0.035	0.028
20	0.070	0.018	0.010	0.060	0.020	0.011	0.070	0.024	0.013	0.058	0.021	0.013
30	0.070	0.011	0.007	0.060	0.013	0.007	0.065	0.018	0.010	0.051	0.017	0.010
40	0.071	0.008	0.006	0.062	0.013	0.007	0.059	0.014	0.008	0.050	0.013	0.008
50	0.070	0.008	0.005	0.068	0.012	0.006	0.055	0.016	0.006	0.048	0.012	0.006
60	0.070	0.007	0.005	0.070	0.012	0.005	0.051	0.014	0.005	0.048	0.010	0.005
70	0.070	0.007	0.004	0.074	0.011	0.004	0.049	0.012	0.004	0.049	0.010	0.004
80	0.070	0.007	0.004	0.080	0.011	0.004	0.049	0.013	0.004	0.050	0.010	0.004
90	0.070	0.006	0.003	0.083	0.011	0.003	0.047	0.012	0.003	0.049	0.009	0.003
100	0.070	0.006	0.003	0.082	0.010	0.003	0.047	0.012	0.003	0.050	0.009	0.003
<i>Minimum</i>	<i>0.070</i>	<i>0.006</i>	<i>0.003</i>	<i>0.060</i>	<i>0.010</i>	<i>0.003</i>	<i>0.047</i>	<i>0.012</i>	<i>0.003</i>	<i>0.048</i>	<i>0.009</i>	<i>0.003</i>
<i>Maximum</i>	<i>0.080</i>	<i>0.043</i>	<i>0.020</i>	<i>0.085</i>	<i>0.040</i>	<i>0.020</i>	<i>0.109</i>	<i>0.042</i>	<i>0.023</i>	<i>0.099</i>	<i>0.035</i>	<i>0.028</i>
<i>Mean values</i>	<i>0.071</i>	<i>0.012</i>	<i>0.007</i>	<i>0.072</i>	<i>0.015</i>	<i>0.007</i>	<i>0.060</i>	<i>0.018</i>	<i>0.008</i>	<i>0.055</i>	<i>0.015</i>	<i>0.008</i>

Table 5.7 ratios of depth to span ( $h/L$ ), web width to depth ( $b_w/h$ ), and flange thickness to depth ( $h_f/h$ ) which are outcomes of the design optimization process. From the table it was seen that  $h/L$  may be considered as 0.071, 0.072, 0.060, and 0.055 for RC T, RC box, PC T, and PC box girders respectively. In similar manner  $b_w/h$  can be taken as 0.012, 0.015, 0.018, and 0.015 for RC T, RC box, PC T, and PC box girders respectively. The ratio of  $h_f/h$  for reinforced concrete, and partially prestressed girders 0.007, and 0.008 respectively

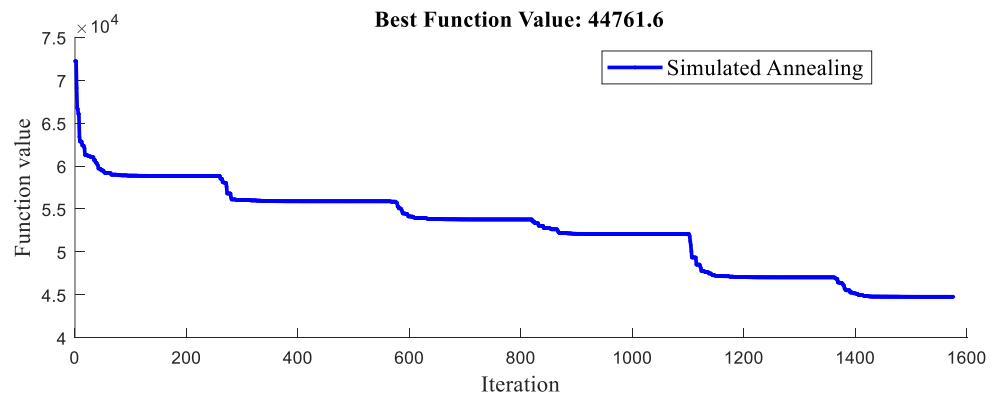
may be used. These are optimal girders section properties and may be used as a starting point for the design activities of bridge girders.

### 5.8 Comparison of Optimization Algorithms

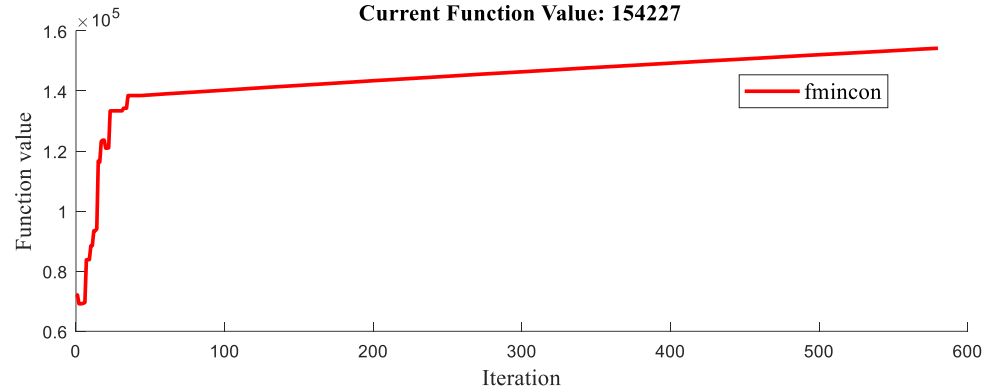
The graphs under Figure 5.7 below were the plots showing the comparison optimization operating programs GA (a), simulated annealing (b), and  $f_{mincon}$  (c). Display of the graphs indicated that genetic algorithm has a rapid convergence property and produced better results. It also capable in handling large scale multivariable fitness function with either linear or nonlinear constraints or both.



(a). Genetic Algorithm



(b). Simulated Annealing



(c). fmincon

Figure 5.7 Comparison of Efficiency of Optimization Solvers

Generally, it was observed that GA is the best algorithm specially in solving complex multi variable fitness either single or multi objective subjected linear and nonlinear constraints. It is only GA which is capable of optimizing integer constraint problems.

## **CHAPTER SIX**

### **CONCLUSIONS AND RECOMMENDATIONS**

#### **6.1 Conclusion**

The goal of this research was to optimize prestressed concrete bridge girders under the study variables construction materials, girder cross sections, span length, grades of concrete, and girder spacing. The following conclusions were drawn from the present work;

1. Effect of bridge materials reveals that reinforced concrete girders are economical for smaller bridge length up to 40m and for span larger than 40 the use of prestressed box girder was economical and stiffer type of structure which was the same findings from the reference wrote by N. Krishna.
2. Optimum cost of bridge girders may results for the specified compressive strength values of 30 to 50 Mpa.
3. For a bridge supporting dual traffic lanes with an extended overhang of 1.5m wide, it was obtained through a number of iterations that the economical girder spacing is 2.5m.
4. Optimum design of prestressed concrete girders could capable of reducing cost with 38% for partially prestressed concrete tee girder and 25% for partially prestressed concrete box girder as compared to the cost of conventional design approach.
5. This study shows that depth of bridge girders could be made shallower by increasing compressive strength of concrete.
6. Cost of concrete could govern the cost of the girders to a span of nearly 40m and beyond that cost dominance hierarchy is shifted to cost of steel reinforcement.
7. In this research it was obtained that the ratio of section depth to span  $h/L$  may be considered as 0.071, 0.072, 0.060, and 0.055 for RC T, RC box, PC T, and PC box girders respectively. In similar manner the ratio of web width to section depth  $b_w/h$  may be taken as 0.012, 0.015, 0.018, and 0.015 for RC T, RC box, PC T, and PC box girders respectively. The ratio of top flange thickness to section depth  $h_f/h$  for



reinforced concrete, and partially prestressed girders 0.007, and 0.008 respectively may be used.

8. GA is a robust tool for structural design optimization. The present study was carried out using genetic algorithm which could handle both single and multi-objective fitness functions constrained linearly or nonlinearly and having more number of design variables easily. It is also more general to accommodate discrete and continuous variables.

## **6.2 Recommendations for Future Studies**

In this study, the effect of bridge construction materials, grades of concrete, cross section properties, span length and girder spacing on the optimum cost have been investigated. The following ideas are recommended for further studies in the future.

- It is recommended to use T girder for a length less than 40m and box girder if the length is greater than 40m.
- Implement design optimization for continuously supported bridge system of variable depth of superstructure.
- Perform size optimization to available girder cross sections to obtain an effective and optimum cross section with the use of finite element analysis supplemented with simulation using programs like ANSYS.
- Apply design optimization to building structures also.
- Try out different optimization algorithms other than GA, it is of interest to find out which of the available algorithms that are most efficient for such optimization problems.

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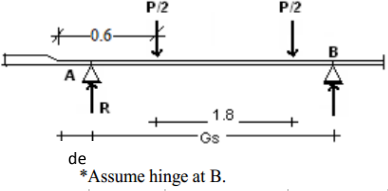
## Appendix A Bridge Girders Analysis Data

Fixed Geometric Parameters	
	Total road width = <b>9.9</b> m
	Clear roadway width, $R_w$ = <b>7.3</b> m
	Width of abutment, $W_A$ = 0.500 m
	Width of diaphragm, $w_{di}$ = 0.250 m
	Girder spacing, $G_s$ = <b>2.300</b> m, $G_s \leq 2h_{girder}$
	Number of girders $N_g = \text{INT}[R_w/G_s] + 1 = 5.000$
	Curb width, $d_c = 0.5[R_w - (N_g - 1)G_s] = 0.200$ m
	Additional curb width, $C_w$ = 1.300 m
	Width of the overhang, $W_{oh} = C_w + d_c = 1.500$ m
	Depth of overhang at the edge, $h_{oh} = 0.280$ m
	Curb depth, $C_d = 0.150$ m
	Width of post = 0.300 m
	Depth of post = 0.300 m
	Height of post = 0.850 m
	Average spacing between posts = 1.800 m
	Width of rail = 0.300 m
	Depth of rail = 0.300 m
	Number of design lanes, $N_L = \text{INT}[R_w/3600] = 2.000$
	Thickness of deck slab, $h_c = \text{MAX}[G_{seff}/10, 175\text{mm}] = 0.200$ m
	Size of fillet = 0.150 m
	Thickness of bottom flange of the girder, $t_b = 0.140$ m
	Web width of the girder, $b_w = 0.300$ m
	Bottom flange width of the girder, $b_b = 0.600$ m
	Thickness of asphalt layer, $h_s = 0.075$ m
	Skewness angle, $\Theta = 0.000$ deg
	Unit weight of concrete, $\rho_c = 24.000$ KN/m <sup>3</sup>
	Unit weight of bituminous asphalt, $\rho_b = 22.500$ KN/m <sup>4</sup>

for RC box  $t_b = \text{max}(140, (g_s - b_w)/16)$   
for PC box  $t_b = \text{max}(140, (g_s - b_w)/30)$

ASHTO art. 5.14.1.5, area of bottom slab reinf. And c/sectional dim. of T and Box girders

Distribution Factor for Moment and Shear [T-Girder]	
The following approximate distribution factor equations include multiple presence factor.	
<b>i. Distribution factor for moment</b>	
<b>- Interior Girder</b>	
One lane loaded: $m_g^{SL} = 0.06 + [G_s/4300]^{0.4} \cdot [G_s/L]^{0.3} \cdot [K_g/(L t_s^3)]^{0.1}$	
setting $[K_g/(L t_s^3)]^{0.1} = 1$ , $m_g^{SL} =$	<b>0.37</b>
Two or more lane loaded: $m_g^{ML} = 0.075 + [G_s/2900]^{0.6} \cdot [G_s/L]^{0.2} \cdot [K_g/(L t_s^3)]^{0.1}$	
setting $[K_g/(L t_s^3)]^{0.1} = 1$ , $m_g^{ML} =$	<b>0.55</b>
Skewness correction factor = $1.05 - 0.025 \tan \Theta = 1.05 \approx 1$ , $b/c \Theta = 0$	
<b>thus, <math>m_g^{ML} = 0.55</math></b>	
<b>- Exterior Girder</b>	
One lane loaded: $m_g^{SL} =$ lever rule (solving the reaction in the exterior girder as a function of truck load assuming hinge develops over each interior girders & multiply it with mpf, $m=1.2$ ) using the ff truck arrangement.	
	$m_g^{SL} =$
	<b>0.522</b>
Two or more lane loaded: $m_g^{ML} = m_g^{ML} (\text{INT}) \cdot [0.77 + d_c/2800]$	
	$m_g^{SL} =$
	<b>0.46</b>
<b>thus, <math>m_g^{ML} = 0.52</math></b>	
<b>ii. Distribution factor for shear</b>	
<b>- Interior Girder</b>	
One lane loaded: $m_g^{SL} = 0.36 + [G_s/7600]$	
	$m_g^{SL} =$
	<b>0.66</b>
Two or more lane loaded: $m_g^{ML} = 0.2 + [G_s/3600] - [G_s/10700]^2$	
	$m_g^{ML} =$
	<b>0.79</b>
<b>thus, <math>m_g^{ML} = 0.79</math></b>	

<b>- Exterior Girder</b>	
One lane loaded: $m_g^{SL} =$	$m_g^{SL} = 0.522 = 1.2/G_s$
Two or more lane loaded: $m_g^{ML} = m_g^{ML} (INT) \cdot [0.6 + d_e/3000]$	
$d_e$ - is from c/l of ext. girder to inner face of curb	$m_g^{SL} = 0.53$
	<b>thus, <math>m_g^{ML} = 0.53</math></b>
	
Fig.A1 Truck wheel load arrangement for the lever rule	
<b>Design Philosophy</b>	
Design Philosophy is based on ASHTO load and resistance factor design (LRFD) approach.	
<b>Dynamic load allowance (impact factor), IM</b>	
This factor accounts for hammering when riding surface discontinuities exist, and long undulations when settlement or resonant excitation occurs.	
	for fatigue and fracture limit state, IM = 15%
	for all other limit states, IM = 33%
<b>Multiple presence factor(m)</b>	
Multiple presence factors modify the vehicular live loads for the probability that vehicular live loads occur together in a fully loaded state.	
	Multiple presence factor for two design lanes, m = 1.00 <b>MPF is not applied to fatigue limit state!</b>
<b>Load Modifier</b>	
<i>Applicable only for strength limit state load combination. Using ODOT (Ohio Department of Transportation) recommendations,</i>	
	Ductility, $\eta_D = 1.00$ ..for all strength limit states
	Redundancy $\eta_R = 1.00$ ..redundant bridge if 4girders with $G_s < 3.66m$ used!
	Importance, $\eta_I = 1.05$ ..for important bridge
	<b>load modifier, <math>\eta_i \geq [\eta_D \cdot \eta_R \cdot \eta_I, 0.95]</math> 1.05</b>
<b>Applicable Load Combinations</b>	
<b>i. Strength limit state-I</b>	
<i>- used to ensure strength and stability</i>	
Ultimate factored shear force = $1.05[m_{gv} \cdot 1.75(1.33 \cdot \text{MAX}(V_{tr}, V_{tm}) + V_{in}) + 1.25DC + 1.5DW]$	
Ultimate factored bending moment = $1.05[m_{gm} \cdot 1.75(1.33 \cdot \text{MAX}(V_{tr}, V_{tm}) + V_{in}) + 1.25DC + 1.5DW]$	
<b>ii. Strength limit state-IV (for span &gt;300ft =91.5m)</b>	
Ultimate factored shear force = $1.05[1.5(DC+DW)]$	
Ultimate factored bending moment = $1.05[1.5(DC+DW)]$	
<b>iv. Service limit state-I</b>	
<i>- used to restrict stress, deflection, crack width and used to check COMPRESSIVE stress in prestressed concrete under normal service condition</i>	
Shear force = $[m_{gv} \cdot (1.33 \cdot \text{MAX}(V_{tr}, V_{tm}) + V_{in}) + DC + DW]$	
Bending moment = $[m_{gm} \cdot (1.33 \cdot \text{MAX}(V_{tr}, V_{tm}) + V_{in}) + DC + DW]$	
<b>v. Service limit state-III (for tension analysis of PC structure)</b>	
<i>- used to check TENSILE stress in prestress concere super structures with the objective of crack control.</i>	
Shear force = $[m_{gv} \cdot 0.8(1.33 \cdot \text{MAX}(V_{tr}, V_{tm}) + V_{in}) + DC + DW]$	
Bending moment = $[m_{gm} \cdot 0.8(1.33 \cdot \text{MAX}(V_{tr}, V_{tm}) + V_{in}) + DC + DW]$	
<b>vi. Fatigue and Fracture limit state - (based on single design truck in w/c rear axles spaced 9m apart)</b>	
Shear force = $0.75 \cdot [m_{gv}^{SL} \cdot 1.15 \cdot V_{tr}]$	
Bending moment = $0.75[m_{gm}^{SL} \cdot 1.15 \cdot V_{tr}]$	

**Calculation of loads**

**i. Weight of structural and non structural components**

Weight of top deck slab =	4.80	KN/m <sup>2</sup>
Weight of overhang at the its beginning =	8.40	KN/m <sup>2</sup>
Weight of overhang at its end =	6.72	KN/m <sup>2</sup>
Weight of Post and rail =	10.91	KN/m <sup>2</sup>
Weight of wearing surface =	1.69	KN/m <sup>2</sup>

Imposed dead load reaction transferred to girders

KN/m			
DC (ext.gird)	DW(ext.g)	DC (int.gird)	DW(int.grd)
24.880	1.920	5.740	4.250
24.88	1.92	5.74	4.25

....used as a constatnt

**Dead loads of DC & DW for variable girder spacing**

g.spac-gs	No. girde	DC(ext)	DW(ext)	DC(int)	DW(int)	W <sub>ho</sub> (m)	d <sub>c</sub> (m)
1.50	7	12.45	-0.03	7.292	2.34	0.45	-0.85
2.00	5	18.85	0.82	7.91	3.5666667	0.95	-0.35
2.50	4	21.65	1.7	8.795	4.64	1.20	-0.10
3.00	4	15.11	0.84	15.58	5.33	0.45	-0.85
3.50	3	25.31	2.5	10.75	7.35	1.45	0.15
4.00	3	20.85	2	19.67	8.35	0.95	-0.35

**ii. Influence lines for bending moment and shear force**

Design vehicular live load is ASHTO 2007, LRFD, HL-93 single vehicle is considered.

**i. influence line (IL)for live load moment**

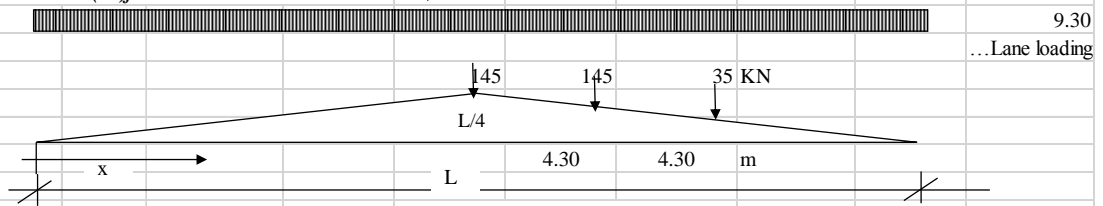
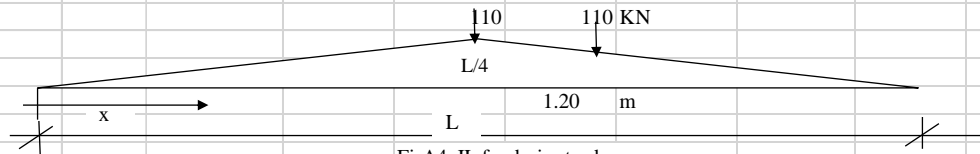


Fig. A3. IL for design truck



FigA4. IL for design tandem

**i. influence line for live load moment**

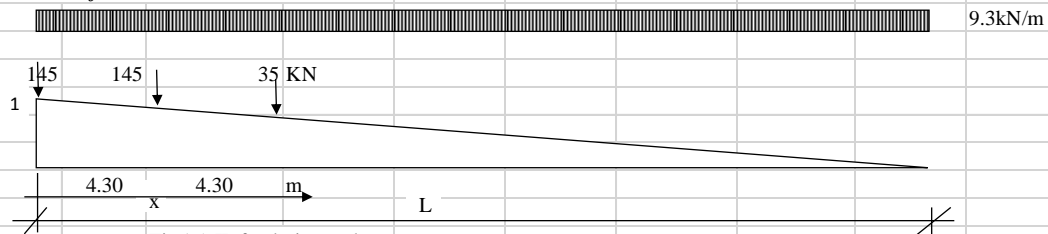


Fig A5. IL for design truck

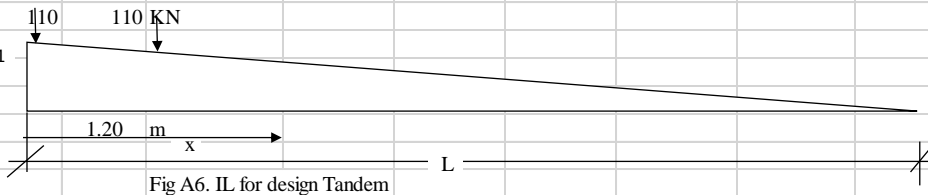
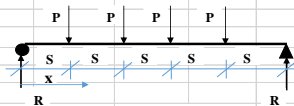


Fig A6. IL for design Tandem

*Optimization of Prestressed Concrete Girders for Bridge Design, Master's Thesis*  
By Wubishet Jemaneh June, 2018

1. Design Shear and Moment Computation for T-Girders																																	
		For bridge span, $L =$		50.00 m																													
		Estimated depth of the girder, $h =$		2.30 m																													
iii. Selfweight of girders and diaphragms																																	
		Diaphragm spacing, $S =$		5.000 m																													
		Number of diaphragms, $N_d =$		9.00																													
		Dead load of diaphragm on exterior girder, $P =$		11.40 KN		$R =$		51.30 KN																									
		Dead load of diaphragm on interior girder, $P =$		22.80 KN		$R =$		102.60 KN																									
		Self weight of the girders, $g = A_c \cdot \rho_c =$		17.100 KN/m																													
 Fig A7 Diaphragm Loadings																																	
Shear & Moment equations of DC and DW loads																																	
- for Exterior girder																																	
DC: $V(x) =$	1049.50	-41.98 x	+ if(x<S, R, if(x<2S, R-P, if(x<3S, R-2P,...)))																														
DW: $V(x) =$	48.00	-1.92 x																															
DC: $M(x) =$	1049.50 x	-20.99 x <sup>2</sup>	+ if(x<S, R.x, if(x<2S, R.x-P(x-S), if(x<3S, R.x-P(2x-3S), if(x<4S, R.x-p(3x-6S),...)))																														
DW: $M(x) =$	48.00 x	-0.96 x <sup>2</sup>																															
- for Interior girder																																	
DC: $V(x) =$	571.00	-22.84 x	+ if(x<S, R, if(x<2S, R-P, if(x<3S, R-2P,...)))																														
DW: $V(x) =$	106.25	-4.25 x																															
DC: $M(x) =$	571.00 x	-11.42 x <sup>2</sup>	+ if(x<S, R.x, if(x<2S, R.x-P(x-S), if(x<3S, R.x-P(2x-3S), if(x<4S, R.x-p(3x-6S),...)))																														
DW: $M(x) =$	106.25 x	-2.13 x <sup>2</sup>																															
<table border="1" style="float: right; margin-top: 10px;"> <thead> <tr> <th colspan="4" style="text-align: center;">Distribution Factors</th> </tr> <tr> <th colspan="2" style="text-align: center;">All other limit states</th> <th colspan="2" style="text-align: center;">Fatigue limit state</th> </tr> <tr> <th style="text-align: center;">Moment</th> <th style="text-align: center;">Shear</th> <th style="text-align: center;">Moment</th> <th style="text-align: center;">Shear</th> </tr> <tr> <th style="text-align: center;"><math>m_{gm}</math></th> <th style="text-align: center;"><math>m_{gv}</math></th> <th style="text-align: center;"><math>m_{gm}^{SL}</math></th> <th style="text-align: center;"><math>m_{gv}^{SL}</math></th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">Interior girder</td> <td style="text-align: center;">0.55</td> <td style="text-align: center;">0.79</td> <td style="text-align: center;">0.31</td> </tr> <tr> <td style="text-align: center;">Exterior girder</td> <td style="text-align: center;">0.52</td> <td style="text-align: center;">0.53</td> <td style="text-align: center;">0.43</td> </tr> </tbody> </table>										Distribution Factors				All other limit states		Fatigue limit state		Moment	Shear	Moment	Shear	$m_{gm}$	$m_{gv}$	$m_{gm}^{SL}$	$m_{gv}^{SL}$	Interior girder	0.55	0.79	0.31	Exterior girder	0.52	0.53	0.43
Distribution Factors																																	
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a) Interior girder												
i. Shear force												
677.25												
x	$V_{TR}$	$V_{TM}$	$V_{LN}$	$V_{DC}$	$V_{DW}$	Strength-I $V_{S-I}$	Strength-IV $V_{S-IV}$	Service-I $V_{SI}$	Service-III $V_{SIII}$	Fatigue $V_{FG}$		
0.00	306.51	217.36	232.50	673.60	106.25	1983.87	1228.26	1287.29	1185.80	130.48		
2.54	290.03	206.21	208.92	615.70	95.48	1824.65	1120.10	1182.56	1088.28	122.63		
5.07	273.56	195.05	185.35	535.00	84.70	1635.50	976.03	1055.03	967.96	114.78		
7.61	257.08	183.90	161.77	477.10	73.93	1476.28	867.87	950.29	870.44	106.93		
10.14	240.60	172.74	138.20	396.40	63.16	1287.14	723.80	822.76	750.12	99.09		
12.68	224.12	161.59	114.62	338.50	52.38	1127.91	615.64	718.03	652.60	91.24		
15.21	207.65	150.44	91.05	257.80	41.61	938.77	471.57	590.50	532.28	83.39		
17.75	191.17	139.28	67.47	199.90	30.83	779.55	363.41	485.76	434.76	75.54		
20.28	174.69	128.13	43.90	119.20	20.06	590.40	219.34	358.23	314.44	67.70		
22.82	158.21	116.97	20.32	61.31	9.29	431.18	111.18	253.50	216.92	59.85		
25.35	141.74	105.82	-3.26	-19.39	-1.49	242.03	-32.89	125.97	96.60	52.00		
27.89	125.26	94.67	-26.83	-77.29	-12.26	82.81	-141.05	21.23	-0.92	44.15		
30.42	108.78	83.51	-50.41	-157.99	-23.04	-106.33	-285.12	-106.30	-121.25	36.31		
32.96	92.30	72.36	-73.98	-215.89	-33.81	-265.56	-393.28	-211.03	-218.77	28.46		
35.49	75.83	61.20	-97.56	-296.59	-44.58	-454.70	-537.35	-338.57	-339.09	20.61		
$V_{max} =$						1983.87	1228.26	1287.29	1185.80	130.48		
ii. Bending Moment												
Service moment due to dead load selfweight												
x	$M_{TR}$	$M_{TM}$	$M_{LN}$	$M_{DC}$	$M_{DW}$	Strength-I $M_{S-I}$	Strength-IV $M_{S-IV}$	Service-I $M_{SI}$	Service-III $M_{SIII}$	Fatigue $M_{FG}$	$M_g$	$M_{min}$
0.000	462.25	66.00	0.00	0.00	0.00	615.74	0.00	335.10	268.08	338.53	0.00	0.00
2.535	874.19	344.85	559.51	1634.19	255.69	4272.41	2976.56	2828.56	2640.82	447.82	1889.88	1028.77
5.070	1286.13	623.70	1059.25	3120.01	484.06	7631.47	5676.41	5113.76	4811.82	557.11	3604.07	1947.65
7.605	1698.06	902.55	1499.22	4402.85	685.13	10621.25	8013.56	7136.10	6726.48	666.40	5087.98	2756.64
10.140	2110.00	1181.40	1879.44	5535.72	858.88	13311.33	10071.50	8948.59	8437.80	775.69	6394.60	3455.74
12.675	2521.94	1460.25	2199.89	6467.21	1005.33	15634.22	11769.25	10499.81	9894.36	884.98	7472.54	4044.96
15.210	2933.88	1739.10	2460.57	7247.14	1124.46	17655.32	13185.27	11839.59	11145.99	994.27	8371.60	4524.28
17.745	3345.81	2017.95	2661.50	7827.29	1216.28	19311.33	14243.61	12919.69	12144.46	1103.56	9043.56	4893.72
20.280	3757.75	2296.80	2802.66	8254.27	1280.78	20663.44	15017.71	13786.74	12936.41	1212.85	9535.06	5153.27
22.815	4169.69	2575.65	2884.05	8483.07	1317.98	21652.56	15436.65	14395.73	13476.79	1322.14	9801.05	5302.93
25.350	3543.38	2645.50	2905.68	8557.11	1327.86	20952.70	15568.84	14037.41	13206.93	1214.90	9884.98	5342.70
27.885	3131.44	2366.65	2867.55	8434.56	1310.44	20177.49	15348.37	13578.03	12811.42	1103.56	9745.00	5272.59
30.420	2719.50	2087.80	2769.65	8155.66	1265.70	19094.20	14838.64	12902.40	12206.19	994.27	9421.36	5092.58
32.955	2307.56	1808.95	2611.99	7681.76	1193.65	17652.10	13978.77	11971.90	11352.60	884.98	8875.41	4802.69
35.490	1895.63	1530.10	2394.56	7049.91	1094.29	15899.83	12827.12	10823.56	10287.69	775.69	8144.20	4402.91
$M_{max} =$						21652.56	15568.84	14395.73	13476.79	1322.14	9884.98	5342.70



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**b) Exterior girder**  
**i. Shear force**

x	V <sub>TR</sub>	V <sub>TM</sub>	V <sub>LN</sub>	V <sub>DC</sub>	V <sub>DW</sub>	Strength-I	Strength-IV	Service-I	Service-III	Fatigue
						V <sub>S-I</sub>	V <sub>S-IV</sub>	V <sub>SI</sub>	V <sub>SIII</sub>	V <sub>FG</sub>
0.00	306.51	217.36	232.50	1100.80	48.00	2142.02	1809.36	1487.10	1419.44	102.74
2.54	290.03	206.21	208.92	994.38	43.13	1950.50	1634.08	1351.77	1288.92	96.56
5.07	273.56	195.05	185.35	876.56	38.27	1744.03	1440.85	1205.04	1147.00	90.38
7.61	257.08	183.90	161.77	770.14	33.40	1552.51	1265.58	1069.72	1016.48	84.20
10.14	240.60	172.74	138.20	652.32	28.53	1346.04	1072.35	922.99	874.56	78.02
12.68	224.12	161.59	114.62	545.90	23.66	1154.52	897.07	787.66	744.04	71.84
15.21	207.65	150.44	91.05	428.08	18.80	948.05	703.84	640.94	602.13	65.66
17.75	191.17	139.28	67.47	321.66	13.93	756.53	528.56	505.61	471.61	59.48
20.28	174.69	128.13	43.90	203.85	9.06	550.05	335.33	358.89	329.69	53.30
22.82	158.21	116.97	20.32	97.43	4.20	358.54	160.05	223.56	199.17	47.12
25.35	141.74	105.82	-3.26	-20.39	-0.67	152.06	-33.18	76.83	57.25	40.94
27.89	125.26	94.67	-26.83	-126.81	-5.54	-39.45	-208.45	-58.49	-73.27	34.77
30.42	108.78	83.51	-50.41	-244.63	-10.41	-245.93	-401.68	-205.22	-215.18	28.59
32.96	92.30	72.36	-73.98	-351.05	-15.27	-437.44	-576.96	-340.55	-345.70	22.41
35.49	75.83	61.20	-97.56	-468.87	-20.14	-643.92	-770.19	-487.27	-487.62	16.23
<b>V<sub>max</sub> =</b>						<b>2142.02</b>	<b>1809.36</b>	<b>1487.10</b>	<b>1419.44</b>	<b>102.74</b>

**ii. Bending Moment**

x	M <sub>TR</sub>	M <sub>TM</sub>	M <sub>LN</sub>	M <sub>DC</sub>	M <sub>DW</sub>	Strength-I	Strength-IV	Service-I	Service-III	Fatigue	Service moment due to	
						M <sub>S-I</sub>	M <sub>S-IV</sub>	M <sub>SI</sub>	M <sub>SIII</sub>	M <sub>FG</sub>	M <sub>g</sub>	M <sub>min</sub>
0.00	462.25	66.00	0.00	0.00	0.00	589.40	0.00	320.76	256.61	478.50	0.00	0.00
2.54	874.19	344.85	559.51	2655.64	115.51	5318.50	4364.56	3669.68	3489.97	632.98	2771.15	1028.77
5.07	1286.13	623.70	1059.25	5040.71	218.68	9615.75	8283.55	6704.51	6415.48	787.45	5259.40	1947.65
7.61	1698.06	902.55	1499.22	7127.91	309.52	13445.31	11713.95	9397.94	9005.84	941.93	7437.43	2756.64
10.14	2110.00	1181.40	1879.44	8943.74	388.01	16841.97	14697.51	11776.48	11287.54	1096.41	9331.75	3455.74
12.68	2521.94	1460.25	2199.89	10462.49	454.17	19771.99	17193.74	13814.43	13234.88	1250.88	10916.66	4044.96
15.21	2933.88	1739.10	2460.57	11887.26	507.99	22501.93	19522.51	15714.88	15050.95	1405.36	12395.25	4524.28
17.75	3345.81	2017.95	2661.50	12924.26	549.47	24646.20	21221.12	17184.04	16441.98	1559.84	13473.73	4893.72
20.28	3757.75	2296.80	2802.66	13691.49	578.61	26359.67	22475.41	18339.91	17525.95	1714.31	14270.10	5153.27
22.82	4169.69	2575.65	2884.05	14188.95	595.42	27642.33	23285.38	19182.49	18302.86	1868.79	14784.37	5302.93
25.35	3543.38	2645.50	2905.68	14416.63	599.88	27170.34	23651.01	18991.32	18196.36	1023.61	15016.52	5342.70
27.89	3131.44	2366.65	2867.55	14374.55	592.01	26540.90	23572.33	18635.61	17901.80	869.13	14966.56	5272.59
30.42	2719.50	2087.80	2769.65	14062.69	571.80	25480.65	23049.31	17966.62	17300.19	714.66	14634.48	5092.58
32.96	2307.56	1808.95	2611.99	13481.05	539.25	23989.59	22081.97	16984.33	16391.52	560.18	14020.30	4802.69
35.49	1895.63	1530.10	2394.56	12629.65	494.36	22067.73	20670.31	15688.74	15175.80	405.70	13124.01	4402.91
<b>M<sub>max</sub> =</b>						<b>27642.33</b>	<b>23651.01</b>	<b>19182.49</b>	<b>18302.86</b>	<b>1868.79</b>	<b>15016.52</b>	<b>5342.70</b>

**2. Design Shear and Moment Computation for Box Girder**

For bridge span, L = 50.00 m  
Estimated depth of the girder, h = 2.300 m

**iii. Selfweight of girders and diaphragms**

Diaphragm spacing, S = 5.000 m  
Number of diaphragms, N<sub>d</sub> = 9.00  
Dead load of diaphragm on exterior girder, P = 11.40 KN R = 51.30 KN  
Dead load of diaphragm on interior girder, P = 22.80 KN R = 102.60 KN  
Self weight of the exterior girder, g = A<sub>e</sub>ρ<sub>c</sub> = 19.056 KN/m  
Self weight of the interior girder, g = A<sub>i</sub>ρ<sub>c</sub> = 22.920 KN/m

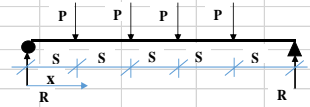


Fig A7 Diaphragm loading

**Shear & Moment equations of DC and DW loads**

**-for Exterior girder**

DC: V(x) =	1098.40	-47.80 x + if(x<S, R, if(x<2S, R-P, if(x<3S, R-2P,...)))
DW: V(x) =	48.00	-1.92 x
DC: M(x) =	1098.40	x - 21.97 x <sup>2</sup> + if(x<S, R.x, if(x<2S, R.x-P(x-S), if(x<3S, R.x-P(2x-3S), if(x<4S, R.x-p(3x-6S),...)))
DW: M(x) =	48.00	x - 0.96 x <sup>2</sup> + if(x<S, R, if(x<2S, R-P, if(x<3S, R-2P,...)))

**-for Interior girder**

DC: V(x) =	716.50	-28.66 x + if(x<S, R, if(x<2S, R-P, if(x<3S, R-2P,...)))
DW: V(x) =	106.25	-4.25 x
DC: M(x) =	716.50	x - 14.33 x <sup>2</sup> + if(x<S, R.x, if(x<2S, R.x-P(x-S), if(x<3S, R.x-P(2x-3S), if(x<4S, R.x-p(3x-6S),...)))
DW: M(x) =	106.25	x - 2.13 x <sup>2</sup>

Distribution Factors			
All other limit states		Fatigue limit state	
Moment	Shear	Moment	Shear
m <sub>gM</sub>	m <sub>gV</sub>	m <sub>gM<sup>SL</sup></sub>	m <sub>gV<sup>SL</sup></sub>
Interior girder	0.51	0.76	0.29
Exterior girder	0.62	0.52	0.51

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<b>a) Interior girder</b>											
<b>i. Shear force</b>											
x	V <sub>TR</sub>	V <sub>TM</sub>	V <sub>LN</sub>	V <sub>DC</sub>	V <sub>DW</sub>	Strength-I	Strength-IV	Service-I	Service-III	Fatigue	
						V <sub>S-I</sub>	V <sub>S-IV</sub>	V <sub>SI</sub>	V <sub>SIII</sub>	V <sub>FG</sub>	
0.00	306.51	217.36	232.50	819.10	106.25	2142.25	1457.43	1415.06	1317.12	125.93	
2.54	290.03	206.21	208.92	746.45	95.48	1965.98	1326.03	1296.83	1205.85	118.36	
5.07	273.56	195.05	185.35	650.99	84.70	1759.79	1158.72	1155.81	1071.78	110.78	
7.61	257.08	183.90	161.77	578.34	73.93	1583.52	1027.32	1037.58	960.52	103.21	
10.14	240.60	172.74	138.20	482.89	63.16	1377.32	860.02	896.55	826.45	95.64	
12.68	224.12	161.59	114.62	410.23	52.38	1201.05	728.62	778.33	715.19	88.06	
15.21	207.65	150.44	91.05	314.78	41.61	994.86	561.31	637.30	581.12	80.49	
17.75	191.17	139.28	67.47	242.13	30.83	818.59	429.92	519.07	469.85	72.91	
20.28	174.69	128.13	43.90	146.68	20.06	612.39	262.61	378.05	335.79	65.34	
22.82	158.21	116.97	20.32	74.02	9.29	436.12	131.21	259.82	224.52	57.76	
25.35	141.74	105.82	-3.26	-21.43	-1.49	229.93	-36.10	118.80	90.45	50.19	
27.89	125.26	94.67	-26.83	-94.08	-12.26	53.66	-167.49	0.57	-20.81	42.62	
30.42	108.78	83.51	-50.41	-189.54	-23.04	-152.53	-334.80	-140.46	-154.88	35.04	
32.96	92.30	72.36	-73.98	-262.19	-33.81	-328.80	-466.20	-258.68	-266.15	27.47	
35.49	75.83	61.20	-97.56	-357.64	-44.58	-535.00	-633.51	-399.71	-400.21	19.89	
<b>V<sub>max</sub> =</b>						<b>2142.25</b>	<b>1457.43</b>	<b>1415.06</b>	<b>1317.12</b>	<b>125.93</b>	

<b>ii. Bending Moment</b>												
x	M <sub>TR</sub>	M <sub>TM</sub>	M <sub>LN</sub>	M <sub>DC</sub>	M <sub>DW</sub>	Strength-I	Strength-IV	Service-I	Service-III	Fatigue	Service moment due to	
						M <sub>S-I</sub>	M <sub>S-IV</sub>	M <sub>SI</sub>	M <sub>SIII</sub>	M <sub>FG</sub>	M <sub>g</sub>	M <sub>min</sub>
0.00	462.25	66.00	0.00	0.00	0.00	575.49	0.00	313.19	250.55	314.98	0.00	0.00
2.54	874.19	344.85	559.51	1984.33	255.69	4619.22	3528.03	3117.34	2941.88	416.67	2240.02	1378.91
5.07	1286.13	623.70	1059.25	3786.08	484.06	8324.36	6725.48	5681.15	5398.95	518.36	4270.15	2610.53
7.61	1698.06	902.55	1499.22	5459.86	685.13	11762.57	9678.36	8059.24	7676.39	620.05	6144.99	3694.86
10.14	2110.00	1181.40	1879.44	7297.85	858.88	15317.35	12846.85	10543.77	10066.36	721.74	8156.73	4631.91
12.68	2521.94	1460.25	2199.89	8718.87	1005.33	18225.91	15315.60	12553.58	11987.71	823.42	9724.19	5421.66
15.21	2933.88	1739.10	2460.57	10882.08	1124.46	22009.62	18910.29	15247.83	14599.57	925.11	12006.53	6064.13
17.75	3345.81	2017.95	2661.50	12108.14	1216.28	24464.38	20985.96	16947.17	16222.62	1026.80	13324.42	6559.30
20.28	3757.75	2296.80	2802.66	14885.57	1280.78	28856.34	25462.00	20140.12	19345.36	1128.49	16166.35	6907.19
22.82	4169.69	2575.65	2884.05	15974.47	1317.98	30933.15	27235.62	21586.79	20727.92	1230.17	17292.45	7107.79
25.35	3543.38	2645.50	2905.68	19655.10	1327.86	35020.05	33048.18	24863.97	24087.77	673.82	20982.97	7161.10
27.89	3131.44	2366.65	2867.55	20664.65	1310.44	35769.09	34610.76	25557.56	24841.07	572.13	21975.09	7067.12
30.42	2719.50	2087.80	2769.65	25537.47	1265.70	41489.72	42215.00	30056.67	29405.97	470.44	26803.17	6825.85
32.96	2307.56	1808.95	2611.99	26525.46	1193.65	42012.53	43657.59	30613.18	30034.37	368.75	27719.11	6437.29
35.49	1895.63	1530.10	2394.56	32879.47	1094.29	49479.30	53508.66	36477.97	35977.13	267.06	33973.76	5901.44
<b>M<sub>max</sub> =</b>						<b>49479.30</b>	<b>53508.66</b>	<b>36477.97</b>	<b>35977.13</b>	<b>1230.17</b>	<b>33973.76</b>	<b>7161.10</b>

<b>b) Exterior girder</b>											
<b>i. Shear force</b>											
x	V <sub>TR</sub>	V <sub>TM</sub>	V <sub>LN</sub>	V <sub>DC</sub>	V <sub>DW</sub>	Strength-I	Strength-IV	Service-I	Service-III	Fatigue	
						V <sub>S-I</sub>	V <sub>S-IV</sub>	V <sub>SI</sub>	V <sub>SIII</sub>	V <sub>FG</sub>	
0.00	306.51	217.36	232.50	1149.70	48.00	2198.30	1886.38	1531.70	1464.90	102.74	
2.54	290.03	206.21	208.92	1028.53	43.13	1987.98	1687.86	1381.92	1319.87	96.56	
5.07	273.56	195.05	185.35	895.95	38.27	1762.70	1471.40	1220.75	1163.44	90.38	
7.61	257.08	183.90	161.77	774.78	33.40	1552.38	1272.88	1070.97	1018.41	84.20	
10.14	240.60	172.74	138.20	642.21	28.53	1327.11	1056.41	909.80	861.99	78.02	
12.68	224.12	161.59	114.62	521.04	23.66	1116.79	857.90	760.02	716.96	71.84	
15.21	207.65	150.44	91.05	388.46	18.80	891.51	641.43	598.85	560.53	65.66	
17.75	191.17	139.28	67.47	267.29	13.93	681.19	442.92	449.07	415.50	59.48	
20.28	174.69	128.13	43.90	134.72	9.06	455.91	226.45	287.90	259.08	53.30	
22.82	158.21	116.97	20.32	13.54	4.20	245.60	27.94	138.13	114.05	47.12	
25.35	141.74	105.82	-3.26	-119.03	-0.67	20.32	-188.53	-23.05	-42.38	40.94	
27.89	125.26	94.67	-26.83	-240.20	-5.54	-190.00	-387.04	-172.82	-187.41	34.77	
30.42	108.78	83.51	-50.41	-372.78	-10.41	-415.28	-603.51	-334.00	-343.83	28.59	
32.96	92.30	72.36	-73.98	-493.95	-15.27	-625.60	-802.03	-483.77	-488.86	22.41	
35.49	75.83	61.20	-97.56	-626.52	-20.14	-850.88	-1018.49	-644.95	-645.29	16.23	
<b>V<sub>max</sub> =</b>						<b>2198.30</b>	<b>1886.38</b>	<b>1531.70</b>	<b>1464.90</b>	<b>102.74</b>	

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ii. Bending Moment											Service moment due to	
	$M_{TR}$	$M_{TM}$	$M_{LN}$	$M_{DC}$	$M_{DW}$	Strength-I $M_{SI}$	Strength-IV $M_{SIV}$	Service-I $M_{SI}$	Service-III $M_{SIII}$	Fatigue $M_{FG}$	dead load $M_g$	selfweight $M_{min}$
0.00	462.25	66.00	0.00	0.00	0.00	696.20	0.00	378.88	303.11	565.20	0.00	0.00
2.54	874.19	344.85	559.51	2773.32	115.51	5772.12	4549.91	3950.17	3737.90	747.67	2888.83	1146.44
5.07	1286.13	623.70	1059.25	5263.50	218.68	10389.31	8634.43	7189.14	6847.75	930.14	5482.18	2170.43
7.61	1698.06	902.55	1499.22	7443.23	309.52	14511.94	12210.58	10068.51	9605.36	1112.61	7752.75	3071.96
10.14	2110.00	1181.40	1879.44	9339.03	388.01	18174.78	15320.09	12614.76	12037.22	1295.08	9727.04	3851.03
12.68	2521.94	1460.25	2199.89	10925.17	454.17	21344.11	17922.47	14802.20	14117.63	1477.54	11379.35	4507.64
15.21	2933.88	1739.10	2460.57	12226.59	507.99	24052.60	20056.96	16655.73	15871.50	1660.01	12734.58	5041.80
17.75	3345.81	2017.95	2661.50	13219.15	549.47	26268.63	21685.58	18151.25	17274.72	1842.48	13768.62	5453.49
20.28	3757.75	2296.80	2802.66	13926.18	578.61	28022.78	22845.06	19312.06	18350.61	2024.95	14504.80	5742.73
22.82	4169.69	2575.65	2884.05	14325.17	595.42	29285.50	23499.92	20115.65	19076.64	2207.42	14920.58	5909.51
25.35	3543.38	2645.50	2905.68	14437.81	599.88	28521.58	23684.37	19732.74	18793.73	1209.09	15037.70	5953.83
27.89	3131.44	2366.65	2867.55	14243.21	592.01	27590.16	23365.47	19169.12	18302.34	1026.62	14835.22	5875.70
30.42	2719.50	2087.80	2769.65	13761.48	571.80	26194.77	22574.91	18269.19	17482.01	844.15	14333.28	5675.10
32.96	2307.56	1808.95	2611.99	12973.29	539.25	24310.05	21282.25	17013.65	16313.43	661.69	13512.54	5352.05
35.49	1895.63	1530.10	2394.56	11897.17	494.36	21960.31	19516.67	15421.01	14815.11	479.22	12391.54	4906.54
$M_{max} =$						<b>29285.50</b>	<b>23684.37</b>	<b>20115.65</b>	<b>19076.64</b>	<b>2207.42</b>	<b>15037.70</b>	<b>5953.83</b>

**Deflection Computation**

**1. Dead Load Deflection T-Girder**

Exterior T- Girder dead load deflection calculation including long term effects							
Span, L (mm)	Uniform loads dead load on the girder		Defl. Due to $w_i, \Delta_w \cdot 1/EI_e$ (mm)	Diaph. point load on the girder, P (N)	End rxn due to diaph. Load, R (N)	Defl. Due to $P_i, \Delta_p \cdot 1/EI_e$ (mm)	Total dead load long term deflection, $\Delta_d = 4 \cdot [\Delta_w + \Delta_p]_{instant.} \cdot X 1/EI_e$ (mm)
	$w_1$ (coeff. of x) (N/mm)	$w_2$ (coeff. of $x^2$ ) (N/mm)					
10000	154.70	-15.47	5.63E+15	600	300	1.25E+13	2.26E+16
20000	338.2	-16.91	9.85E+16	3000	4500	7.51E+14	3.97E+17
30000	561.3	-18.71	5.52E+17	6000	15000	5.07E+15	2.23E+18
40000	806	-20.15	1.88E+18	8400	29400	1.69E+16	7.58E+18
<b>50000</b>	<b>1011.25</b>	<b>-20.225</b>	<b>4.60E+18</b>	<b>11400</b>	<b>51300</b>	<b>4.47E+16</b>	<b>1.86E+19</b>
60000	1403.4	-23.39	1.10E+19	13800	75900	9.27E+16	4.45E+19
70000	1763.3	-25.19	2.20E+19	16800	109200	1.81E+17	8.88E+19
80000	2130.4	-26.63	3.97E+19	19200	144000	3.09E+17	1.60E+20
90000	2558.7	-28.43	6.79E+19	22200	188700	5.10E+17	2.74E+20
100000	2987	-29.87	1.09E+20	24600	233700	7.76E+17	4.38E+20
110000	3483.7	-31.67	1.69E+20	27600	289800	1.16E+18	6.80E+20
120000	3973.2	-33.11	2.50E+20	30000	345000	1.64E+18	1.01E+21
130000	4538.3	-34.91	3.63E+20	33000	412500	2.29E+18	1.46E+21
140000	5089	-36.35	5.08E+20	35400	477900	3.08E+18	2.05E+21
150000	5722.5	-38.15	7.03E+20	38400	556800	4.11E+18	2.83E+21

Interior T- Girder dead load deflection calculation including long term effects							
Span, L (mm)	Uniform loads		Defl. Due to $w_i, \Delta_w \cdot 1/EI_e$ (mm)	diaphragm point load on the girder, P (N)	End rxn due to diaphragm Load, R (N)	Defl. Due to $P_i, \Delta_p \cdot 1/EI_e$ (mm)	Total dead load long term deflection, $\Delta_d = 4 \cdot [\Delta_w + \Delta_p]_{instant.} \cdot X 1/EI_e$ (mm)
	$w_1$ (coeff. of x) (N/mm)	$w_2$ (coeff. of $x^2$ ) (N/mm)					
10000	70.65	-7.07	2.571E+15	1200	600	3.750E+13	1.044E+16
20000	170.1	-8.505	4.953E+16	6000	9000	1.5015E+15	2.041E+17
30000	309.15	-10.305	3.038E+17	12000	30000	1.01453E+16	1.256E+18
40000	469.8	-11.745	1.094E+18	16800	58800	3.37008E+16	4.512E+18
<b>50000</b>	<b>763.375</b>	<b>-15.2675</b>	<b>3.473E+18</b>	<b>22800</b>	<b>102600</b>	<b>8.94188E+16</b>	<b>1.425E+19</b>
60000	899.1	-14.985	7.068E+18	27600	151800	1.85369E+17	2.901E+19
70000	1174.95	-16.785	1.467E+19	33600	218400	3.62311E+17	6.012E+19
80000	1458	-18.225	2.717E+19	38400	288000	6.18701E+17	1.112E+20
90000	1802.25	-20.025	4.782E+19	44400	377400	1.01958E+18	1.954E+20
100000	2146.5	-21.465	7.812E+19	49200	467400	1.55134E+18	3.187E+20
110000	2559.15	-23.265	1.240E+20	55200	579600	2.31893E+18	5.052E+20
120000	2964.6	-24.705	1.864E+20	60000	690000	3.27564E+18	7.589E+20
130000	3445.65	-26.505	2.755E+20	66000	825000	4.58569E+18	1.120E+21
140000	3912.3	-27.945	3.907E+20	70800	955800	6.15002E+18	1.587E+21
150000	4461.75	-29.745	5.481E+20	76800	1113600	8.2134E+18	2.225E+21

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**2. Live Load Deflection (for both T and Box Girders)**

- Dynamic load allowance and multiple presence factor should be applied to truck and tandem loads and all design loads should be fully loaded in calculating absolute deflection.  
- All components assumed to deflect equally.

Deflection distribution factor, $mg^d = N_L/N_g = 0.5$				IM = 33%			1.00, for two design lanes			
				Each truck wheel load deflection, $\Delta_{ik}$			Total truck deflection	Lane load deflection	Total LL def.	
Span, L (mm)	$x$ at max. $M_w$ (mm)	$a$ (mm)	$b$ (mm)	$\Delta_{ik1} \cdot 1/EI_e$ (mm)	$\Delta_{ik2} \cdot 1/EI_e$ (mm)	$\Delta_{ik3} \cdot 1/EI_e$ (mm)	$IM \cdot \Sigma \Delta_{ik} \cdot 1/EI_e$ (mm)	$\Delta_{Ln} \cdot 1/EI_e$ (mm)	$mg^d \cdot \Delta_{LL} \cdot 1/EI_e$ (mm)	
10000	4560	4560	5440	2.97E+15	9.79E+14	0.00E+00	5.26E+15	1.21E+15	2.63E+15	
20000	9126	9126	10874	2.38E+16	1.98E+16	1.89E+15	6.05E+16	1.94E+16	3.03E+16	
30000	13689	13689	16311	8.03E+16	7.53E+16	1.34E+16	2.25E+17	9.81E+16	1.12E+17	
40000	18252	18252	21748	1.90E+17	1.85E+17	3.83E+16	5.50E+17	3.10E+17	2.75E+17	
50000	22815	22815	27185	3.72E+17	3.67E+17	8.08E+16	1.09E+18	7.57E+17	5.45E+17	
60000	27378	27378	32622	6.43E+17	6.40E+17	1.45E+17	1.90E+18	1.57E+18	1.02E+18	
70000	31941	31941	38059	1.02E+18	1.02E+18	2.36E+17	3.03E+18	2.91E+18	1.83E+18	
80000	40560	40560	39440	1.55E+18	1.52E+18	3.47E+17	4.53E+18	4.96E+18	3.05E+18	
90000	45630	45630	44370	2.20E+18	2.17E+18	5.01E+17	6.48E+18	7.94E+18	4.78E+18	
100000	50700	50700	49300	3.02E+18	2.98E+18	6.95E+17	8.90E+18	1.21E+19	7.17E+18	
110000	55770	55770	54230	4.02E+18	3.97E+18	9.32E+17	1.19E+19	1.77E+19	1.03E+19	
120000	60840	60840	59160	5.22E+18	5.17E+18	1.22E+18	1.54E+19	2.51E+19	1.45E+19	
130000	59319	59319	70681	6.54E+18	6.57E+18	1.57E+18	1.95E+19	3.46E+19	1.97E+19	
140000	63882	63882	76118	8.16E+18	8.21E+18	1.97E+18	2.44E+19	4.65E+19	2.63E+19	
150000	60840	60840	89160	9.48E+18	9.65E+18	2.35E+18	2.86E+19	6.13E+19	3.42E+19	
$P = 145000$		$145000$	$35000$ N				$w_{Ln} = 9.3$ N/mm			

**3. Dead Load Deflection Box-Girder**

Exterior Box- Girder dead load deflection calculation including long term effects							
Span, L (mm)	Uniform loads dead load on the girder		Defl. Due to $w_i, \Delta_w \cdot 1/EI_e$ (mm)	Diaph. point load on the girder, P (N)	End rxn due to diaph. Load, R (N)	Defl. Due to $P_i, \Delta_p \cdot 1/EI_e$ (mm)	Total dead load long term deflection, $\Delta_d = 4 \cdot [\Delta_w + \Delta_p]_{instant.} \cdot 1/EI_e$ (mm)
	$w_1$ (coeff. of x) (N/mm)	$w_2$ (coeff. of $x^2$ ) (N/mm)					
10000	167.00	-16.70	6.08E+15	0	0	0.00E+00	2.43E+16
20000	362.8	-18.14	1.06E+17	2400	3600	6.01E+14	4.25E+17
30000	587.4	-19.58	5.77E+17	4800	12000	4.06E+15	2.33E+18
40000	840.8	-21.02	1.96E+18	7200	25200	1.44E+16	7.89E+18
50000	1064.95	-21.299	4.84E+18	9600	43200	3.77E+16	1.95E+19
60000	1434	-23.9	1.13E+19	12000	66000	8.06E+16	4.54E+19
70000	1773.8	-25.34	2.21E+19	14400	93600	1.55E+17	8.92E+19
80000	2142.4	-26.78	3.99E+19	16800	126000	2.71E+17	1.61E+20
90000	2539.8	-28.22	6.74E+19	19200	163200	4.41E+17	2.71E+20
100000	2966	-29.66	1.08E+20	21600	205200	6.81E+17	4.35E+20
110000	3421	-31.1	1.66E+20	24000	252000	1.01E+18	6.67E+20
120000	3904.8	-32.54	2.46E+20	26400	303600	1.44E+18	9.88E+20
130000	4417.4	-33.98	3.53E+20	28800	360000	2.00E+18	1.42E+21
140000	4958.8	-35.42	4.95E+20	31200	421200	2.71E+18	1.99E+21
150000	5529	-36.86	6.79E+20	33600	487200	3.59E+18	2.73E+21

Interior Box- Girder dead load deflection calculation including long term effects							
Span, L (mm)	Uniform loads		Defl. Due to $w_i, \Delta_w \cdot 1/EI_e$ (mm)	diaphragm point load on the girder, P (N)	End rxn due to diaphragm Load, R (N)	Defl. Due to $P_i, \Delta_p \cdot 1/EI_e$ (mm)	Total dead load long term deflection, $\Delta_d = 4 \cdot [\Delta_w + \Delta_p]_{inst.} \cdot 1/EI_e$ (mm)
	$w_1$ (coeff. of x) (N/mm)	$w_2$ (coeff. of $x^2$ ) (N/mm)					
10000	110.55	-11.06	4.024E+15	0	0	0.000E+00	1.609E+16
20000	249.9	-12.495	7.276E+16	4800	7200	1.2012E+15	2.959E+17
30000	418.05	-13.935	4.108E+17	9600	24000	8.1162E+15	1.676E+18
40000	615	-15.375	1.433E+18	14400	50400	2.88864E+16	5.846E+18
50000	922.075	-18.4415	4.195E+18	19200	86400	7.53E+16	1.708E+19
60000	1095.3	-18.255	8.611E+18	24000	132000	1.6119E+17	3.509E+19
70000	1378.65	-19.695	1.721E+19	28800	187200	3.10552E+17	7.009E+19
80000	1690.8	-21.135	3.151E+19	33600	252000	5.41363E+17	1.282E+20
90000	2031.75	-22.575	5.391E+19	38400	326400	8.81798E+17	2.192E+20
100000	2401.5	-24.015	8.740E+19	43200	410400	1.36215E+18	3.551E+20
110000	2800.05	-25.455	1.356E+20	48000	504000	2.01647E+18	5.506E+20
120000	3227.4	-26.895	2.030E+20	52800	607200	2.88256E+18	8.234E+20
130000	3683.55	-28.335	2.945E+20	57600	720000	4.00206E+18	1.194E+21
140000	4168.5	-29.775	4.163E+20	62400	842400	5.42036E+18	1.687E+21
150000	4682.25	-31.215	5.751E+20	67200	974400	7.18673E+18	2.329E+21

Similarly done for all other cases required

## Appendix B Unit Cost of Construction Materials

1. Unit Price of Concrete Material				
	Cement (ETB/ku)	sand (ETB/m <sup>3</sup> )	Aggrg. (ETB/m <sup>3</sup> )	Water (ETB/m <sup>3</sup> )
	285	531.25	593.75	17.5
	labour+equip cost =			25% of material cost
	over head + profit =			25% of total cost
	VAT =			15% of total cost
	total factor =			1.80 ... 80% of material cost
	mater=			56%
	labr+equip+overhd+prot+vnt =			44%
				100%
	assume shrinkage = 1.3 & wastage = 1.15			

comp. strength, f <sub>c</sub> (Mpa)	Quantity with in 1m <sup>3</sup> of concrete				Unit Rate				
	cement	sand	Aggr.	Water	cement (kg/m <sup>3</sup> )	sand (m <sup>3</sup> /m <sup>3</sup> )	Aggrg. (m <sup>3</sup> /m <sup>3</sup> )	water (m <sup>3</sup> /m <sup>3</sup> )	ETB/m <sup>3</sup>
15	1	2.02	2.72	0.5	306.25	0.48	0.65	0.12	2730
20	1	2.02	2.72	0.5	306.25	0.48	0.65	0.12	2730
25	1	2.02	2.72	0.5	306.25	0.48	0.65	0.12	2730
30	1	1.8	2.51	0.47	330.62	0.47	0.65	0.12	2840
35	1	1.51	2.24	0.42	369.63	0.44	0.65	0.12	3010
40	1	1.25	2	0.37	413.64	0.41	0.65	0.12	3205
45	1	1.07	1.82	0.34	451.77	0.38	0.64	0.12	3370
50	1	0.94	1.7	0.32	482.58	0.36	0.64	0.12	3500
55	1	0.87	1.63	0.3	502.90	0.34	0.64	0.12	3595
60	1	0.83	1.59	0.3	513.71	0.33	0.64	0.12	3640
65	1	0.8	1.57	0.29	522.13	0.33	0.64	0.12	3675
70	1	0.48	1.26	0.24	642.22	0.24	0.63	0.12	4200
75	1	0.33	1.12	0.21	718.82	0.19	0.63	0.12	4535
80	1	0.18	0.98	0.19	816.16	0.11	0.63	0.12	4960
85	1	0.03	0.84	0.16	944.00	0.02	0.62	0.12	5520
90	1	-0.12	0.69	0.13	1119.33	-0.11	0.61	0.12	6285
Summary of unit rate for concrete									
Grade of Concrete, Mpa					30	40	50	60	70
Unit Cost, ETB/mm <sup>3</sup> x10 <sup>9</sup>					2840	3205	3500	3640	4200

2. Unit Cost of reinforcing steel steel							
Reinforcing Steel unit cost							
Diam. (mm)	cost ETB/ton	kg/m	kg/12m	etb/berg	etb/kg	etb/ton	current
8	19010	0.395	4.736	90	19.01	19010	47460
10	16220	0.617	7.4	120	16.22	16220	44500
12	16900	0.888	10.656	180	16.9	16900	41250
14	24140	1.209	14.504	350	24.14	24140	42010
16	22700	1.579	18.944	430	22.7	22700	42180
20	19600	2.467	29.6	580	19.6	19600	42600
24	15250	3.552	42.624	650	15.25	15250	43500
30	11120	5.550	66.6	740	11.12	11120	43420
32	11220	6.315	75.776	850	11.22	11220	43650
avg =	17351.11				17.35	17351.11	43396.67

		<i>labr+prof+overhd (10%+15%+15% =40%) =</i>	<b>6940.44</b>	
<b>3. Unit Cost of Prestressing steel</b>			<i>Vat (15%)=</i>	<b>3643.73</b>
<i>7wire strand freight cost estimate (http/www.alibaba.com)</i>			<b>Total unit cost =</b>	<b>27,940.00 ETB/ton</b>
	weight of strand per container =		96000 kg/40ft container	
	weight of strand per container =		96 tone/40ft container	
	cost of freight per container =		208500 ETB/40ft container	
	cost of freight per ton =		<b>2171.875</b> ETB/ton	
	container size=2280x2591x12192mm=7.5x8.5x40ft			
	assume 20%+10%+15% = 45%			
<b>unit cost analysis for prestressing strands works</b>				
<b>Dia. (mm)</b>	<b>Cost (ETB/ton)</b>	<b>Calculation of unit cost including tax (ETB/ton)</b>		
9.53	19460	<i>Base value</i>	<i>Rate type</i>	<i>Rate</i>
11.11	20850	23583.05	Duty	5%
12.7	22240	24762.21	Excise	0%
15.24	22796	24762.21	Surtax	10%
<i>Arg selling price=</i>	<b>21336.5</b>	27238.43	VAT	15%
<i>Freight =</i>	2171.88	<b>23583.05</b>	Withhold	3%
<i>Insurance (0.3%) =</i>	64.01			<b>Total tax to be paid</b>
<i>Other cost (0.05%) =</i>	10.67			<b>8448.63</b>
<b>Total material cost =</b>	<b>23583.05</b>		<i>Equipment + labour + profit &amp; overhead</i>	<b>14414.26</b>
			<b>Total unit cost</b>	<b>46,450.00</b>
<small>(source: <a href="https://www.alibaba.com/showroom/prestressing-steel-strand-price.html">https://www.alibaba.com/showroom/prestressing-steel-strand-price.html</a>)</small>				
<small>(source for custom taxes: <a href="https://www.erca.gov.et/index.php/tax-calculator">https://www.erca.gov.et/index.php/tax-calculator</a>)</small>				
<b>Currency conversion factor as per the date 30/04/2018 GC is 1 US\$ = 27.80ETB (Source: EBC)</b>				

## Appendix C Design Optimization Code and Outputs using GA in Matlab

Case (1). PC T-girder (code for interior girder)

```
function z = Tpcintgirderfun(x)
% Cost parameters
Cc = 2840e-9; % unit rate of fc'= 30 concrete (ETB/m3)
Cs = 27940; % unit rate of reinforcing steel (ETB/ton)
Cp = 46450; % unit rate of prestressing 7-wire strands (ETB/ton)
L = 50000; % span length (mm)
NL = 4; % number of legs of vertical stirrups
dsh = 12; % diam. of shear rebar (mm)
av = NL*pi*dsh^2/4; % area of f12mm for shear reinforcement within a
distance S (mm2)
density = 7.850e-9; % density of steel_prestressing strands and
reinforcing bars (ton/mm3)
Ag = x(1)*x(2); % concrete cross sectional area of the girder (mm2)
Wstr = density*av*(L/x(6)+1)*2*(x(2)/2+2*(x(1)-280)); % weight of
stirrups (ton)
% Cost cost function prestressed exterior T-girder
% z = Cc*((Ag-As-Ap)*L-Wstr/density)+
Cs*(density*As*L+Wstr)+Cp*(density*Ap*L)
z = Cc*((Ag -x(4) - x(5))*L-Wstr/density)+...
Cs*(density*x(4)*L + Wstr)+ Cp*density*x(5)*L;
```

```
% NON LINEAR CONSTRAINT FUNCTIONS DEFINITION FOR PC EXTER. T-GIRDER
BRIDGE
function [c, ceq] = Tpcintgirderconst(x)
% Problem parameters
% h = x(1), bw = x(2), hf = x(3), As = x(4), Ap = x(5)
% S = x(6), y = x(7)
% Material properties
fc = 30; % cylindrical compr.strength (N/mm2)
fy = 420; % yield strength of reinforcing steel (N/mm2)
fpu = 1860; % Ultimate tensile strength of tendon (N/mm2)
Ec = 27660; % Young's modulus of concrete (N/mm2)
Es = 2e5; % Young's modulus of reinforcing steel (N/mm2)
Ep = 195e3; % Young's modulus of prestressing strands (N/mm2)
ns = Es/Ec; % Modular ratio os reinforcing steel
np = Ep/Ec; % Modular ratio os prestressing strands
% stress limits in concrete
fci = 0.8*fc; % Specified compressive strength of concrete at transfer
of prestress
fct = 0.6*fci; % Allowable compressive stress at transfer of prestress
ftt = 0.63*sqrt(fci); % Allowable tensile stress at transfer of
prestress
fcw = 0.45*fc; % Allowable compressive stress at working loads
ftw = 0.5*sqrt(fc); % Allowable tensile stress at working loads
% stress ranges at extreme fibers
% stress limits in prestressing tendons
fpy = 0.9*fpu; % Yield strength of tendon
fpt = 0.74*fpu; % Allowable stress in tendons at transfer of prestress
fpe = 0.8*fpy; % Allowable stress in tendons at working loads
% Loadings
Vd = 2166.86e3; % design shear force (Nmm)
Md = 23751.24e6; % design bending moment (Nmm)
Mw = 15861.19e6; % Service limit state-I bending moment (Nmm)
M3 = 14887.59e6; % Service limit state-III bending moment (Nmm) for
tension control of pc
Mf = 1388.69e6; % fatigue load design bending moment (Nmm)
Mg = 11086.97e6; % Permanent load (self weight+additional deadloads)
bending moment (Nmm)
% Geometric properties
L = 50000; % Span length of the girder (mm)
gs = 2500; % girder spacing (mm)
woh = 1200; % width of overhang (mm)
wsup = 500; % width of support (mm)
%be = 0.5*min([L/4, 12*x(3)+x(2), gs])+min([L/8, 6*x(3)+x(2)/2, woh]); %
effec.width for ext. girder
be = min([L/4, 12*x(3)+x(2), gs]); % effec.width for int. girder
% equations for effective depth of reinforcing steel
db = 32; % assumed diam. of bar assume it (mm).
Agg = 25; % maximum aggregate size (mm)
Sh = max([1.5*db, 1.5*Agg, 38]); % (mm) clear spacing of parallel bars
(horizontal)
Sv = max([25, db]); % (mm) clear spacing between layers of bars
(vertically)
as = pi*db^2/4; % area of a single reinf. bar (mm2)
nb = x(4)/as; % Number of bars
```

```

npr = min([(x(2)+Sh-124)/(Sh+db),nb]); % Number of bars per a row
nr = nb/npr; %Number of reinforcement rows
hr = nr*db+ Sv*(nr-1); % Height of reinforcement rows
dst = 62+hr/2; % depth from extreme tension fiber to centroid of reinf.
steel (mm)
d = x(1)-dst; % effective depth of reinf. steel(mm)
% effective depth of prestressing steel
dsrd = 15.24; % assumed diam. of prestressing low relaxation strand
(mm)
Nspt = 31;% number of strands per tendon
ap = 0.77*pi*dsrd^2/4; % steel area of a single strand (mm2) (using a
reduction factor of 77% of nominal area of the strand)
dduct = 125; % diameter of duct, (mm)
Sduct = 38; % clear vertical and horizontal spacing of ducts (mm)
nst = x(5)/ap ; % number of strands required
nt = nst/Nspt; % Number of tendons
ntr = min([(x(2)+Sduct-200)/(dduct+Sduct),nt]); %Number of tendons per
a row
nrt = nt/ntr; % Number of rows of prestressing tendons
hrt = dduct*nrt+Sduct*(nrt - 1); % height of rows of prestressing
tendons
dpt = 50+12+Sduct+25+hr+hrt/2; % Depth from extreme tension fiber to
centroid of prestressing tendons (mm)
dp = x(1)-dpt; % Depth from extreme top fiber to centroid of
prestressing steel (mm)
% shear reinforcement steel
NL = 4; % No. of legs of vertical stirrups
dsh = 12; % diam. of bar for shear reinforcement (mm)
av = NL*pi*dsh^2/4; % area of shear reinforcement within a distance S
(mm2)
% section properties
Ac = x(2)*x(1)+(be-x(2))*x(3); % cross sectional area of concrete (mm2)
yt = (x(2)*x(1)^2/2+(be-x(2))*x(3)^2/2)/(x(2)*x(1)+(be-x(2))*x(3)); %
depth from c.g of section to extreme top fiber (mm)
yb = x(1) - yt; % depth from c.g of section to extreme top fiber (mm)
I = x(2)*x(1)^3/12+x(2)*x(1)*(x(1)/2-yt)^2+(be-x(2))*x(3)^3/12+...
    (be-x(2))*x(3)*(yt-x(3)/2)^2; % Gross moment of inertia of concrete
mm4
Zb = I/yb; % section modulus of the extreme bootom fiber (mm3)
Zt = I/yt; % section modulus of the extreme top fiber (mm3)
% extreme fiber stresses for computing Prestressing force
%fsup = ftt-Mg/Zt; % extreme bottom fiber stress, finf developed at a
given eccentricity e (N/mm2)
finf = ftw/0.85+Mw/(0.85*Zb); % extreme bottom fiber stress, finf
developed at a given eccentricity e (N/mm2)
e = yb - dpt; % possible maximum eccentricity of prestressing force
from c.g.c (mm)
P = Ac*finf*Zb/(Zb+Ac*e); % x(5)*fpt; minimum prestressing force at a
kwnon eccentricity, e (N)
% NA depth c from equivalent stress block ananlysis
c0 = (x(5)*fpu+x(4)*fy-0.85^2*fc*(be-x(2))*x(3))/(0.85^2*fc*x(2)+...
    0.28*x(5)*fpu/dp);
if( c0 > x(3))
c = c0; % NA depth for T section (mm)
else

```



```

c = (x(5)*fpu+x(4)*fy)/(0.85^2*fc*be+0.28*x(5)*fpu/dp); % NA depth for
rectangular section (mm)
end
fps = fpu*(1-0.28*c/dp); % Average stress in prestressing steel (N/mm2)
de = (x(5)*fps*dp+x(4)*fy*d)/(x(5)*fps+x(4)*fy); % effective depth from
extreme compression fiber to centroid of tension force (mm)
a = 0.85*c; % depth of equivalent stress block (mm)
%Nominal flexural resistance, Mn
if(c>x(3))
Mn = x(5)*fps*(dp-a/2)+x(4)*fy*(d-a/2)+0.85^2*fc*x(3)*(be-...
x(2))*(a/2-x(3)/2); % Mn for T section (mm)
else
Mn = x(5)*fps*(dp-a/2)+x(4)*fy*(d-a/2); % Mn for rectangular section
(mm)
end
% shearing force parameters
dv = max([0.9*de,0.72*x(1),de-a/2]); % effective shear depth
Vu = Vd*(L/2-wsup/2-d)/(L/2); % ultimate design shear force at a
distance d from face of support (N)
Vc = 0.083*2*sqrt(fc)*x(2)*dv; %
Vs = av*fy*dv/(x(6)); %
Vp = 0.85*P*(4*e/L); %
Vn = min([(Vc+Vs+Vp),(0.25*fc*x(2)*dv+Vp)]; %
% limits of reinforcement
fcpe = 0.85*P*(1/Ac+e/Zb); % compressive stress in concrete due to
effective prestress forces only (N/mm2)
fr = 0.97*sqrt(fc); % modulus of rupture (N/mm2)
Mcr = (fcpe+fr)*I/yb; % cracking moment (Nmm)
% limits of max. reinf
% a). using reinf. index omega-om
Asn = 0;
rhp = x(4)/(be*d);
rhn = Asn/(be*d);
rhpr = x(5)/(be*dp);
Omp = rhp*fy/fc;
Omn = rhn*fy/fc;
Ompr = rhpr*fps/fc;
% b). using imperic.
% c/de <= 0.42
% cracked section analysis
fp1 = 0.85*P/Ac; % stress in the prestressing tendons prior to the
application of Mw (N/mm2)
fp2 = 0.85*np*P*(e^2/I+1/Ac); % stress in prestressing tendons due to
decompression (N/mm2)
% incremental strain during the appl. of Mw
% let NA depth of cracked section be y = x(7)
if(x(7)>x(3))
eo = (x(5)*(fp1+fp2))/(0.5*Ec*(x(2)*x(7)+(be-x(2))*x(3)*...
(1+(x(7)-x(3))/x(7)))-(Es*x(4)*(d-x(7))/x(7)+Ep*x(5)*(dp-
x(7))/x(7)));
else
eo = (x(5)*(fp1+fp2))/(0.5*Ec*be*x(7)-(Es*x(4)*(d-
x(7))/x(7)+Ep*x(5)*...
(dp-x(7))/x(7)));
end
fco = eo*Ec; % stress in concrete at service limit state (N/mm2)

```

```

fs = Es*eo*(d-x(7))/x(7); % tensile stress in reinforcing steel at
service stage (N/mm2)
fp3 = Ep*eo*(dp-x(7))/x(7); % tensile stress in prestressing steel at
service stage (N/mm2)
fp = fp1+fp2+fp3; % total tensil stress in prestressing steel at
service stage (N/mm2)
Ts = x(4)*fs; % tension force in reinforcing steel at service limit
state (N)
Tp = x(5)*fp; % tension force in prestressing steel at service limit
state (N)
C = 0.5*fco*be*x(7); % total compression force in concrete (N)
Cn = 0.5*fco*(be-x(2))*(x(7)-x(3))^2/x(7); % a force used to reduce c
if y>hf (N)
dz = x(7)/3; % location of centroid of comp. force C from top (mm)
dzn = x(3)+(x(7)-x(3))/3; % location of centroid of comp. force Cn from
top (mm)

% section properties of cracked transformed section
% -----moment of inertia of cracked section-----
---%
if(x(7)>x(3))
Ict = x(2)*x(7)^3/3+(be-x(2))*x(3)^3/12+(be-x(2))*x(3)*(x(7)-
x(3)/2)^2+...
    np*x(5)*(dp-x(7))^2+ns*x(4)*(d-x(7))^2; % 2nd moment of area of
cracked transformed section (mm4)
else
Ict = be*x(7)^3/3+np*x(5)*(dp-x(7))^2+ns*x(4)*(d-x(7))^2; % 2nd moment
of area of cracked transformed section (mm4)
end

% -----deflection parameters-----
----
frk = 0.63*sqrt(fc); % modulus of rupture for Ie computation (N/mm2)
Mck = frk*I/yb; % cracking moment for deflection computation(Nmm)
Ie = min([(Mck/Mw)^3*I+(1-(Mck/Mw)^3)*Ict, I]); %effective moment of
inertia for deflection calculation (mm4)
defD = 1.425E+19/(Ec*Ie); % total dead load deflection including long
term effcets (mm)
defLL = 5.45E+17/(Ec*Ie); % maximum live load deflection (mm)
defP = 0.85*5*P*e*L^2/(48*Ec*Ie); % % total effec. prestressing load
deflection (mm)
% maximum crack width
cw1 = (fs - 40)*1e-3; % CEB-FIP-1970, crack width eq. (mm)
h1 = d-x(7)-dst; % depth from steel centroid to NA (mm)
h2 = d-x(7); % depth from NA ~ tension face (mm)
dc = 62+db/2; % concrete cover to closest bar layer (mm)
Atc = x(2)*2*dst/nb; % effective tension area of concrete per bar (mm2)
cw2 = 0.076*(h2/h1)*fs*(dc*Atc)^(1/3)*1e-3*0.1451; % Gergely Lut2-1968
crack equation (mm)
cw = max([cw1, cw2]); % maximu of the crack width given by the above
eqns.
cwa = 0.41; % allowable crack width for moderate exposure condition
% fatigue stress ranges
ffs = ns* Mf*(d-x(7))/Ict; % fatigue stress range in reinforcing steel
(N/mm2)

```

```

ffp = np* Mf*(dp-x(7))/Ict; % fatigue stress range in prestressing
steel (N/mm2)
% partial prestressing ratio
PPR = x(5)*fpy/(x(5)*fpy+x(4)*fy); % partial prestressing ratio, 0.5 <
PPR < 1

%% Non linear inequality constraints [c] written of the form gi(xi)<= 0
g1 = ftt-P*(1/Ac+e/Zt)-Mg/Zt;
g2 = P*(1/Ac+e/Zb)-Mg/Zb-fct;
g3 = 0.85*P*(1/Ac-e/Zt)+Mw/Zt-fcw;
g4 = ftw-0.85*P*(1/Ac+e/Zb)+M3/Zb;
g5 = Md-0.9*Mn; % flexural strength required
g6 = Vu-0.9*Vn; % shear strength required
g7 = Vu/0.9-0.25*fc*x(2)*dv-Vp; % web requirement for shear
% limits of flexural reinf.
g8 = abs(Md)/(0.9*dv)+abs(Vu/0.9-Vp)-0.5*min([Vu/0.9,Vs])- ...
    x(4)*fy-x(5)*fps; % longitudinal reinf.
g9 = Vu/0.9-0.5*Vs-Vp-x(4)*fy-x(5)*fps; % min. longitudinal reinf.
g10 = min([1.33*Md,1.2*Mcr])-0.9*Mn; % minimumu flexural reinf. reqd
g11 = 0.004*yb*x(2)-x(4)-x(5); % minimumu flexural reinf. reqd
g12 = Omp+Ompr-Omn-0.3; % maximumu limit of flexural reinf. reqd
g13 = c/de-0.42; % maximumu flexural reinf. reqd
% limits of traverse reinforcement
g14 = x(6)-fy*av/(0.083*x(2)*sqrt(fc)); %shear reinf.
if(abs(Vu-0.9*Vp)/(0.9*dv*x(2)) < 0.125*fc)
g15 = x(6)-min([0.8*dv,600]); % spacing of shear reinf.
else
g15 = x(6)-min([0.4*dv,300]); % spacing of shear reinf.
end
% service load stress limit
g16 = P - x(5)*fpt; % stress limit in tendons at transfer
g17 = fp - fpe; % stress limit in tendons at service limit state
g18 = fs - min([206,0.6*fy]); % stress limit in reinforcing steel at
service limit state
% deflection limit
radd = 0;
tol = 1e-6;
confcnvald = defD-defP-radd;
g19 = confcnvald-tol; % camber due to prestressing shall counter
balanced by dead load deflection
g20 = -confcnvald-tol;
g21 = defLL-L/1000; % limit of vehicular live load deflection
% Crack width
g22 = cw-cwa; % spacing of longitudinal bars for crck control
% fatigue stress limit
g23 = ffs-161.5; % limit on fatigue stress limit in reinforcing steel
g24 = ffp-125; % limit on fatigue stress limit in prestressing steel
% PPR limit
g25 = 0.5-PPR; % limit on partial prestressing ratio PPR > 0.50
g26 = PPR-1; % limit on partial prestressing ratio PPR < 1.00
% service load degree of prestress
%g27 = 0.5-Mdec/Mw; % service load degree of prestress > 0.50
%g28 = Mdec/Mw-1; % service load degree of prestress < 1.00
% check equilibrium conditions
% summations of internal couple must equal to working moment

```

```
if x(7) > x(3)
rad = Mw;
tol = 1e-6;
confcnvalm = Ts*d+Tp*dp+Cn*dzn-C*dz-rad;
g29 = confcnvalm-tol; % sum of service load moments when NA depth y >
hf
g30 = -confcnvalm-tol;
else
rad = Mw;
tol = 1e-6;
confcnvalm = Ts*d+Tp*dp-C*dz-rad;
g29 = confcnvalm-tol; % sum of service load moments when NA depth y <
hf
g30 = -confcnvalm-tol;
end
if x(7) > x(3)
radf = 0;
tol = 1e-6;
confcnvalf = Ts+Tp+Cn-C-radf;
g31 = confcnvalf-tol; % sum of service load moments when NA depth y >
hf
g32 = -confcnvalf-tol;
else
radf = 0;
tol = 1e-6;
confcnvalf = Ts+Tp-C-radf;
g31 = confcnvalf-tol; % sum of service load moments when NA depth y <
hf
g32 = -confcnvalf-tol;
end
g33 = 0.20*x(1)-x(7);
g34 = x(7) - 0.75*x(1);
% non linear equality const. functions defn.
c =
[g1;g2;g3;g4;g5;g6;g7;g8;g9;g10;g11;g12;g13;g14;g15;g16;g17;g18;g19;g20
;...
g21;g22;g23;g24;g25;g26;g29;g30;g31;g32;g33;g34]; % non linear
inequality const. functions defn.
ceq = [];

%% MAIN CODE FOR RUNNING THE GA ALGORITHM
% Problem parameters
% h = x(1), bw = x(2), hf = x(3), As = x(4), Ap = x(5)
% S = x(6), y = x(7)
% set boundary values of variables
lb = [300 300 200 500 600 200 50];
ub = [2500 500 300 35e3 35e3 450 900];
%% set ga options
opts = optimoptions(@ga, ...
'PopulationSize',5000, ...
'CreationFcn', @gacreationlinearfeasible, ...
'MaxGenerations',1000, ...
'FitnessScalingFcn',@fitscalingprop, ...
'NonlinearConstraintAlgorithm','auglag', ...
'InitialPenalty',10,...
'PenaltyFactor',1000, ...
```

```

'FunctionTolerance', 1e-10, ...
'ConstraintTolerance', 1e-10);
%'PlotFcn',@gaplotbestf);
% _Call |ga| to Solve the Problem_
% We can now call |ga| to solve the problem.
%%

rng(1,'twister') % random number generator for reproducibility
[xbest, fbest, exitflag] = ga(@Tpcintgirderfun,7,[],[],[],[],lb,ub,...
    @Tpcintgirderconst,1:3,opts);

%%
% _Analyze the Results
display(xbest);

%% return optimal value
fprintf('\nCost function returned by ga = %g\n', fbest);

% Results
% xbest =
[2500,499,300,16825.9907946415,9647.44734325986,304.083639994683,899.44
6462904115]
% fbest = 685751

```

#### Case (2). PC box girder (code for interior girder)

```

function z = Bpcintgirderfun(x)
% Map the discrete variables
% Cost parameters
L = 50000; % span length (mm)
gs = 2500; % boottom flange width, mm
NG = 4; % number of girders
tb = max([140,(gs-x(2))/30]); % thickness of bottom flange (/16 for RC,
/30 for PC), mm
tbmin = min([140,(gs-x(2))/30]); % minimum of bottom slab thickness, mm
Asb = 0.004*tb*((NG-1)*gs+x(2))+0.005*tbmin*((NG-1)*gs+x(2)); % total
area of bottom slab reinf.
VAsb = 0.004*tb*((NG-1)*gs+x(2))*L+0.005*tbmin*((NG-1)*gs+x(2))^2; %
volume of bottom slab reinf.
Cc = 2840e-9; % unit rate of fc'= 30 concrete (ETB/m3)
Cs = 27940; % unit rate of reinforcing steel (ETB/ton)
Cp = 46450; % unit rate of prestressing 7-wire strands (ETB/ton)
NL = 4; % number of legs of vertical stirrups
dsh = 12; % diam. of shear rebar (mm)
av = NL*pi*dsh^2/4; % area of f12mm for shear reinforcement within a
distance S (mm2)
density = 7.850e-9; % density of steel_prestressing strands and
reinforcing bars (ton/mm3)
Ag = x(1)*x(2)+tb*gs; % concrete cross sectional area of the girder
(mm2)
Wstr = density*av*(L/x(6)+1)*2*(x(2)/2+2*(x(1)-280)); % weight of
stirrups (ton)
% Cost cost function prestressed exterior T-girder

```

```

% z = Cc*((Ag-As-Ap)*L-Wstr/density)+
Cs*(density*As*L+Wstr)+Cp*(density*Ap*L)
z = Cc*((Ag -x(4) - x(5))*L-Wstr/density)+...
    Cs*(density*x(4)*L + density*VAsb/NG + Wstr)+ Cp*density*x(5)*L;

% NON LINEAR CONSTRAINT FUNCTIONS DEFINITION FOR PC EXTER. T-GIRDER
BRIDGE
function [c, ceq] = Bpcintgirderconst(x)
% Problem parameters
% h = x(1), bw = x(2), hf = x(3), As = x(4), Ap = x(5)
% S = x(6), y = x(7)
% Material properties
fc = 30; % cylindrical compr.strength (N/mm2)
fy = 420; % yield strength of reinforcing steel (N/mm2)
fpu = 1860; % Ultimate tensile strength of tendon (N/mm2)
Ec = 27660; % Young's modulus of concrete (N/mm2)
Es = 2e5; % Young's modulus of reinforcing steel (N/mm2)
Ep = 195e3; % Young's modulus of prestressing strands (N/mm2)
ns = Es/Ec; % Modular ratio os reinforcing steel
np =Ep/Ec; % Modular ratio os prestressing strands
% stress limits in concrete
fci = 0.8*fc; % Specified compressive strength of concrete at transfer
of prestress
fct = 0.6*fci; % Allowable compressive stress at transfer of prestress
ftt = 0.63*sqrt(fci); % Allowable tensile stress at transfer of
prestress
fcw = 0.45*fc; % Allowable compressive stress at working loads
ftw = 0.5*sqrt(fc); % Allowableee tensile stress at working loads
% stress limits in prestressing tendons
fpy = 0.9*fpu;% Yield strength of tendon
fpt = 0.74*fpu; %Allowable stress in tendons at transfer of prestress
fpe = 0.8*fpy; % Allowable stress in tendons at working loads
% Loadings
Vd = 2357.21e3; % design shear force (Nmm)
Md = 58671.06e6; % design bending moment (Nmm)
Mw = 40218.79e6; % Service limit state-I bending moment (Nmm)
M3 = 39625.33e6; % Service limit state-III bending moment (Nmm) for
tension control of pc
Mf = 1466.48e6; % fatigue load design bending moment (Nmm)
Mg = 37251.47e6; % Permanent load (self weight+additional deadloads)
bending moment (Nmm)

% Geometric properties
L = 50000; % Span length of the girder (mm)
gs = 2500; % gider spacing (mm)
woh = 1200; % width of overhang (mm)
wsup = 500; % width of support 9mm)
%be = 0.5*min([L/4,12*x(3)+x(2),gs])+min([L/8,6*x(3)+x(2)/2,woh]);%
effec.width for ext. girder
be = min([L/4,12*x(3)+x(2),gs]);% effec.width for int. girder
bb = be; % boottom flange width, mm
tb = max([140,(gs-x(2))/30]); % thickness of bottom flange (/16 for RC,
/30 for PC), mm
% equations for effective depth of reinforcing steel

```

```

db = 32; % assumed diam. of bar assume it (mm).
Agg = 25; % maximum aggregate size (mm)
Sh = max([1.5*db,1.5*Agg,38]); % (mm) clear spacing of parallel bars
(horizontal)
Sv = max([25,db]); % (mm) clear spacing between layers of bars
(vertically)
as = pi*db^2/4; % area of a single reinf. bar (mm2)
nb = x(4)/as; % Number of bars
npr = min([(x(2)+Sh-124)/(Sh+db),nb]); % Number of bars per a row
nr = nb/npr; %Number of reinforcement rows
hr = nr*db+ Sv*(nr-1); % Height of reinforcement rows
dst = 62+hr/2; % depth from extreme tension fiber to centroid of reinf.
steel (mm)
d = x(1)-dst; % effective depth of reinf. steel(mm)
% effective depth of prestressing steel
dsrd = 15.24; % assumed diam. of prestressing low relaxation strand
(mm)
Nspt = 31;% number of strands per tendon
ap = 0.77*pi*dsrd^2/4; % steel area of a single strand (mm2) (using a
reduction factor of 77% of nominal area of the strand)
dduct = 125; % diameter of duct, (mm)
Sduct = 38; % clear vertical and horizontal spacing of ducts (mm)
nst = x(5)/ap ; % number of strands required
nt = nst/Nspt; % Number of tendons
ntr = min([(x(2)+Sduct-200)/(dduct+Sduct),nt]); %Number of tendons per
a row
nrt = nt/ntr; % Number of rows of prestressing tendons
hrt = dduct*nrt+Sduct*(nrt - 1); % height of rows of prestressing
tendons
dpt = 50+12+Sduct+25+hr+hrt/2; % Depth from extreme tension fiber to
centroid of prestressing tendons (mm)
dp = x(1)-dpt; % Depth from extreme top fiber to centroid of
prestressing steel (mm)
% shear reinforcement steel
NL = 4; % No. of legs of vertical stirrups
dsh = 12; % diam. of bar for shear reinforcement (mm)
av = NL*pi*dsh^2/4; % area of shear reinforcement within a distance S
(mm2)
% section properties
Ac = x(2)*x(1)+(be-x(2))*x(3)+(bb-x(2))*tb; % cross sectional area of
concrete (mm2)
yt = (x(2)*x(1)^2/2+(be-x(2))*x(3)^2/2+(bb-x(2))*tb*(x(1)-
tb/2))/(x(2)*x(1)+(be-x(2))*x(3)+(bb-x(2))*tb); % depth from c.g of
section to extreme bottom fiber (mm)
yb = x(1) - yt; % depth from c.g of section to extreme top fiber (mm)
I = x(2)*x(1)^3/12+x(2)*x(1)*(x(1)/2-yt)^2+(be-x(2))*x(3)^3/12+...
    (be-x(2))*x(3)*(yt-x(3)/2)^2+(bb-x(2))*tb^3/12+(bb-x(2))*tb*...
    (yt-(x(1)-tb/2))^2; % Moment of inertia mm4
Zb = I/yb; % section modulus of the extreme bottom fiber (mm3)
Zt = I/yt; % section modulus of the extreme top fiber (mm3)
% extreme fiber stresses for computing Prestressing force
%fsup = ftt-Mg/Zt; % extreme bottom fiber stress, finf developed at a
given eccentricity e (N/mm2)
finf = ftw/0.85+Mw/(0.85*Zb); % extreme bottom fiber stress, finf
developed at a given eccentricity e (N/mm2)

```

```

e = yb - dpt; % possible maximum eccentricity of prestressing force
from c.g.c (mm)
P = Ac*finf*Zb/(Zb+Ac*e); % x(5)*fpt; minimum prestressing force at a
known eccentricity, e (N)
% NA depth c from equivalent stress block analysis
c0 = (x(5)*fpu+x(4)*fy-0.85^2*fc*(be-x(2))*x(3))/(0.85^2*fc*x(2)+...
0.28*x(5)*fpu/dp);
if( c0 > x(3))
c = c0; % NA depth for T section (mm)
else
c = (x(5)*fpu+x(4)*fy)/(0.85^2*fc*be+0.28*x(5)*fpu/dp); % NA depth for
rectangular section (mm)
end
fps = fpu*(1-0.28*c/dp); % Average stress in prestressing steel (N/mm2)
de = (x(5)*fps*dp+x(4)*fy*d)/(x(5)*fps+x(4)*fy); % effective depth from
extreme compression fiber to centroid of tension force (mm)
a = 0.85*c; % depth of equivalent stress block (mm)
%Nominal flexural resistance, Mn
if(c>x(3))
Mn = x(5)*fps*(dp-a/2)+x(4)*fy*(d-a/2)+0.85^2*fc*x(3)*(be-...
x(2))*(a/2-x(3)/2); % Mn for T section (mm)
else
Mn = x(5)*fps*(dp-a/2)+x(4)*fy*(d-a/2); % Mn for rectangular section
(mm)
end
% shearing force parameters
dv = max([0.9*de,0.72*x(1),de-a/2]); % effective shear depth
Vu = Vd*(L/2-wsup/2-d)/(L/2); % ultimate design shear force at a
distance d from face of support (N)
Vc = 0.083*2*sqrt(fc)*x(2)*dv; %
Vs = av*fy*dv/(x(6)); %
Vp = 0.85*P*(4*e/L); %
Vn = min([(Vc+Vs+Vp),(0.25*fc*x(2)*dv+Vp)]; %
% limits of reinforcement
fcpe = 0.85*P*(1/Ac+e/Zb); % compressive stress in concrete due to
effective prestress forces only (N/mm2)
fr = 0.97*sqrt(fc); % modulus of rupture (N/mm2)
Mcr = (fcpe+fr)*I/yb; % cracking moment (Nmm)
% limits of max. reinf
% a). using reinf. index omega-om
Asn = 0;
rhp = x(4)/(be*d);
rhn = Asn/(be*d);
rhpr = x(5)/(be*dp);
Omp = rhp*fy/fc;
Omn = rhn*fy/fc;
Ompr = rhpr*fps/fc;
% b). using imperic.
% c/de <= 0.42
% cracked section analysis
fp1 = 0.85*P/Ac; % stress in the prestressing tendons prior to the
application of Mw (N/mm2)
fp2 = 0.85*np*P*(e^2/I+1/Ac); % stress in prestressing tendons due to
decompression (N/mm2)
% incremental strain during the appl. of Mw
% let NA depth of cracked section be y = x(7)

```



```

if(x(7)>x(3))
eo = (x(5)*(fp1+fp2))/(0.5*Ec*(x(2)*x(7)+(be-x(2))*x(3)*...
      (1+(x(7)-x(3))/x(7)))-(Es*x(4)*(d-x(7))/x(7)+Ep*x(5)*(dp-
x(7))/x(7)));
else
eo = (x(5)*(fp1+fp2))/(0.5*Ec*be*x(7)-(Es*x(4)*(d-
x(7))/x(7)+Ep*x(5)*...
      (dp-x(7))/x(7)));
end
fco = eo*Ec; % stress in concrete at service limit state (N/mm2)
fs = Es*eo*(d-x(7))/x(7); % tensile stress in reinforcing steel at
service stage (N/mm2)
fp3 = Ep*eo*(dp-x(7))/x(7); % tensile stress in prestressing steel at
service stage (N/mm2)
fp = fp1+fp2+fp3; % total tensil stress in prestressing steel at
service stage (N/mm2)
Ts = x(4)*fs; % tension force in reinforcing steel at service limit
state (N)
Tp = x(5)*fp; % tension force in prestressing steel at service limit
state (N)
C = 0.5*fco*be*x(7); % total compression force in concrete (N)
Cn = 0.5*fco*(be-x(2))*(x(7)-x(3))^2/x(7); % a force used to reduce c
if y>hf (N)
dz = x(7)/3; % location of centroid of comp. force C from top (mm)
dzn = x(3)+(x(7)-x(3))/3; % location of centroid of comp. force Cn from
top (mm)

% section properties of cracked transformed section
% -----moment of inertia of cracked section-----
---%
if(x(7)>x(3))
Ict = x(2)*x(7)^3/3+(be-x(2))*x(3)^3/12+(be-x(2))*x(3)*(x(7)-
x(3)/2)^2+...
      np*x(5)*(dp-x(7))^2+ns*x(4)*(d-x(7))^2; % 2nd moment of area of
cracked transformed section (mm4)
else
Ict = be*x(7)^3/3+np*x(5)*(dp-x(7))^2+ns*x(4)*(d-x(7))^2; % 2nd moment
of area of cracked transformed section (mm4)
end

% -----deflection parameters-----
----
frk = 0.63*sqrt(fc); % modulus of rupture for Ie computation (N/mm2)
Mck = frk*I/yb; % cracking moment for deflection computation(Nmm)
Ie = min([(Mck/Mw)^3*I+(1-(Mck/Mw)^3)*Ict, I]); %effective moment of
inertia for deflection calculation (mm4)
defD = 1.708E+19/(Ec*Ie); % total dead load deflection including long
term effcets (mm)
defLL = 5.45E+17/(Ec*Ie); % maximum live load deflection (mm)
defP = 0.85*5*P*e*L^2/(48*Ec*Ie); % % total effec. prestressing load
deflection (mm)
% maximum crack width
cw1 = (fs - 40)*1e-3; % CEB-FIP-1970, crack width eq. (mm)
h1 = d-x(7)-dst; % depth from steel centroid to NA (mm)
h2 = d-x(7); % depth from NA ~ tension face (mm)

```

```

dc = 62+db/2; % concrete cover to closest bar layer (mm)
Atc = x(2)*2*dst/nb; % effective tension area of concrete per bar (mm2)
cw2 = 0.076*(h2/h1)*fs*(dc*Atc)^(1/3)*1e-3*0.1451; % Gergely Lut2-1968
crack equation (mm)
cw = max([cw1, cw2]); % maximum of the crack width given by the above
eqns.
cwa = 0.41; % allowable crack width for moderate exposure condition
% fatigue stress ranges
ffs = ns* Mf*(d-x(7))/Ict; % fatigue stress range in reinforcing steel
(N/mm2)
ffp = np* Mf*(dp-x(7))/Ict; % fatigue stress range in prestressing
steel (N/mm2)
% partial prestressing ratio
PPR = x(5)*fpy/(x(5)*fpy+x(4)*fy); % partial prestressing ratio, 0.5 <
PPR < 1

%% Non linear inequality constraints [c] written of the form gi(xi)<= 0
g1 = ftt-P*(1/Ac+e/Zt)-Mg/Zt;
g2 = P*(1/Ac+e/Zb)-Mg/Zb-fct;
g3 = 0.85*P*(1/Ac-e/Zt)+Mw/Zt-fcw;
g4 = ftw-0.85*P*(1/Ac+e/Zb)+M3/Zb;
g5 = Md-0.9*Mn; % flexural strength required
g6 = Vu-0.9*Vn; % shear strength required
g7 = Vu/0.9-0.25*fc*x(2)*dv-Vp; % web requirement for shear
% limits of flexural reinf.
g8 = abs(Md)/(0.9*dv)+abs(Vu/0.9-Vp)-0.5*min([Vu/0.9,Vs])- ...
x(4)*fy-x(5)*fps; % longitudinal reinf.
g9 = Vu/0.9-0.5*Vs-Vp-x(4)*fy-x(5)*fps; % min. longitudinal reinf.
g10 = min([1.33*Md,1.2*Mcr])-0.9*Mn; % minimum flexural reinf. reqd
g11 = 0.004*yb*x(2)-x(4)-x(5); % minimum flexural reinf. reqd
g12 = Omp+Ompr-Omn-0.3; % maximum limit of flexural reinf. reqd
g13 = c/de-0.42; % maximum flexural reinf. reqd
% limits of traverse reinforcement
g14 = x(6)-fy*av/(0.083*x(2)*sqrt(fc)); %shear reinf.
if(abs(Vu-0.9*Vp)/(0.9*dv*x(2)) < 0.125*fc)
g15 = x(6)-min([0.8*dv,600]); % spacing of shear reinf.
else
g15 = x(6)-min([0.4*dv,300]); % spacing of shear reinf.
end
% service load stress limit
g16 = P - x(5)*fpt; % stress limit in tendons at transfer
g17 = fp - fpe; % stress limit in tendons at service limit state
g18 = fs - min([206,0.6*fy]); % stress limit in reinforcing steel at
service limit state
% deflection limit
radd = 0;
tol = 1e-6;
confcnvald = defD-defP-radd;
g19 = confcnvald-tol; % camber due to prestressing shall counter
balanced by dead load deflection
g20 = -confcnvald-tol;
g21 = defLL-L/1000; % limit of vehicular live load deflection
% Crack width
g22 = cw-cwa; % spacing of longitudinal bars for crack control
% fatigue stress limit

```

```
g23 = ffs-161.5; % limit on fatigue stress limit in reinforcing steel
g24 = ffp-125; % limit on fatigue stress limit in prestressing steel
% PPR limit
g25 = 0.5-PPR; % limit on partial prestressing ratio PPR > 0.50
g26 = PPR-1; % limit on partial prestressing ratio PPR < 1.00
% service load degree of prestress
% check equilibrium conditions
% summations of internal couple must equal to working moment
if x(7) > x(3)
rad = Mw;
tol = 1e-6;
confcnvalm = Ts*d+Tp*dp+Cn*dzn-C*dz-rad;
g27 = confcnvalm-tol; % sum of service load moments when NA depth y >
hf
g28 = -confcnvalm-tol;
else
rad = Mw;
tol = 1e-6;
confcnvalm = Ts*d+Tp*dp-C*dz-rad;
g27 = confcnvalm-tol; % sum of service load moments when NA depth y <
hf
g28 = -confcnvalm-tol;
end
if x(7) > x(3)
radf = 0;
tol = 1e-6;
confcnvalf = Ts+Tp+Cn-C-radf;
g29 = confcnvalf-tol; % sum of service load moments when NA depth y >
hf
g30 = -confcnvalf-tol;
else
radf = 0;
tol = 1e-6;
confcnvalf = Ts+Tp-C-radf;
g29 = confcnvalf-tol; % sum of service load moments when NA depth y <
hf
g30 = -confcnvalf-tol;
end
g31 = 0.20*x(1)-x(7);
g32 = x(7) - 0.75*x(1);
% non linear equality const. functions defn.
c =
[g1;g2;g3;g4;g5;g6;g7;g8;g9;g10;g11;g12;g13;g14;g15;g16;g17;g18;g19;g20
;...
g21;g22;g23;g24;g25;g26;g27;g28;g29;g30;g31;g32]; % non linear
inequality const. functions defn.
ceq = [];
```

%% MAIN CODE FOR RUNNING THE GA ALGORITHM  
% Problem parameters  
% h = x(1), bw = x(2), hf = x(3), As = x(4), Ap = x(5)  
% S = x(6), y = x(7)  
% set boundary values of variables  
lb = [300 300 200 500 600 200 50];  
ub = [2500 700 300 10e3 21e3 450 700];

```
%% set ga options
opts = optimoptions(@ga, ...
    'PopulationSize',500, ...
    'CreationFcn', @gacreationlinearfeasible, ...
    'MaxGenerations',1000, ...
    'FitnessScalingFcn',@fitscalingprop, ...
    'NonlinearConstraintAlgorithm','auglag', ...
    'InitialPenalty',1,...
    'PenaltyFactor',1, ...
    'FunctionTolerance', 1e-10, ...
    'ConstraintTolerance', 1e-10);
    %'PlotFcn',@gaplotbestf);
% _Call |ga| to Solve the Problem_
% We can now call |ga| to solve the problem.
%%

rng(1,'twister') % random number generator for reproducibility
[xbest, fbest, exitflag] = ga(@Bpcintgirderfun,7,[],[],[],[],lb,ub,...
    @Bpcintgirderconst,1:3,opts);

%%
% _Analyze the Results
display(xbest);

%% return optimal value
fprintf('\nCost function returned by ga = %g\n', fbest);
% Press F5 to run the code and get the ff Results:
% xbest =
[2500,700,300,9994.54948775170,20995.0724864966,334.602112918657,681.61
7397769954]
%zbest = 941015
```

For all other case it was done in the same way.

## Appendix D Design Optimization Validation in Excel spreadsheet

(a). Optimization results validation for PC T interior girder

<u>Check up of Validity of Optim. Outputs for TPC Girders</u>								
Note: All dimensions are in {mm, mm <sup>2</sup> , mm <sup>3</sup> , mm <sup>4</sup> , N, Nmm N/mm <sup>2</sup> }								
Input fixed variab.	Optim. Output variab.							
$E_c = 27660$	h	bw	hf	As	Ap	S	y	
$E_s = 200000$	x1	x2	x3	x4	x5	x6	x7	
$E_p = 197000$	2500	499	300	16826	9647.45	304.08	899.4465	
$f_c' = 30$	LB =	300	300	180	1000	1000	100	875
$f_y = 420$	UB =	700	500	250	5.00E+04	6.00E+04	450	350
$f_{pu} = 1860$	<b>Paste the optim output values below!!</b>							
$n_s = 7.23$	2500	499	300	16825.9908	9647.44734	304.0836	899.4464629	
$n_p = 7.12$	No of legs of vert. stirrups =				4.0			
$L = 50000$	Diam.stirrup, $d_{sh} =$		12	..... $a_v =$		452.4	b =	2500
$G_s = 2500$							bex =	2450
$woh = 1200$							bb =	140
$wsup = 500$							tb =	66.7
$V_d = 2.17E+06$				<i>Effect. depth of prestr. Steel</i>				
$M_d = 2.38E+10$	<i>Effec. depth of reinf. Steel</i>			diam. Strand, dsrd = 15.24				
$M_w = 1.59E+10$	Diam. Of bar, db = 32			area of single strand, ap = 140				
$M_3 = 1.49E+10$	area of single bar, as = 804			duct diam, DD = 125.0				
$M_f = 1.39E+09$	max. aggr. Size, Agg. = 25			clr vert&hri. Duct spcg, SD = 38				
$M_g = 1.11E+10$	clr spacing of // bars, Sh = 48			No. strands, nsrd = 68.68				
$E_c I_e \cdot defDL = 1.42E+19$	clr spac. of bar layers, Sv = 32			No strand per tendn = 35				
$E_c I_e \cdot defLL = 5.45E+17$	No. of bars, nb = 20.92			<b>No.of Tdn, nT = 1.96</b>				
<b>Concrete stress limits</b>	$f_{ci} = 0.8f_c' = 24.000$			No, Tdn/row, ntr = 2.07				
$f_{ct} = 0.6f_{ci} = 14.400$	No. bars per row, npr = 5.29			No. of rows, nrt = 0.95				
$ftt = 0.63\sqrt{f_{ci}} = 3.086$	No. of rows of bars, nr = 3.96			ht of rows of Tdn, hrt = 116.72				
$fcw = 0.45f_c' = 13.500$	ht of rows of bars, hr = 221.23			d' = dst = 172.62				
$ftw = 0.5\sqrt{f_c'} = 2.739$				dpt = 404.59				
				<b>d = 2327.38</b>		<b>dp = 2095.41</b>		
<i>prstressing steel stress limits</i>								
$f_{py} = 0.9f_{pu} = 1674.000$								
$f_{pt} = 0.74f_{pu} = 1376.400$								
$f_{pe} = 0.8f_{py} = 1339.200$								

<b>1) Constraints Functions definition</b>	
<b>a). Section properties</b>	
$A_c = x(2)*x(1)+(be-x(2))*x(3) =$	1847800
$yt = (x(2)*x(1)^2/2+(be-x(2))*x(3)^2/2)/(x(2)*x(1)+(be-x(2))*x(3)) =$	892.6398961
$y_b = x(1) - yt =$	1607.360104
$I = x(2)*x(1)^3/12+x(2)*x(1)*(x(1)/2-yt)^2+(be-x(2))*x(3)^3/12+... (be-x(2))*x(3)*(yt-x(3)/2)^2 =$	1.14463E+12
$Z_t = I/yt =$	1282296748
$Z_b = I/y_b =$	712117485.8
<b>b). Prestressing force, P</b>	
$f_{inf} = ftw/0.85+Mw/(0.85Z_b) =$	2.94E+01
$e = y_b - d_{pt} =$	1202.77
$P = A_c*f_{inf}*Z_b/(Z_b+A_c*e) =$	13194299.55
<b>c). Flexural resistance</b>	
$c_0 = (x(5)*f_{pu}+x(4)*f_y-0.85^2*f_c*(be-x(2))*x(3))/(0.85^2*f_c*x(2)+0.28*x(5)*f_{pu}/d_p) =$	908.1275369
$c = \text{if}(c_0 > hf, c_0, (x(5)*f_{pu}+x(4)*f_y)/(0.85^2*f_c*be+0.28*x(5)*f_{pu}/d_p)) =$	908.1275369
$f_{ps} = f_{pu}*(1-0.28*c/d_p) =$	1634.290846
$d_e = (x(5)*f_{ps}*d_p+x(4)*f_y*d)/(x(5)*f_{ps}+x(4)*f_y) =$	2167.203572
$a = 0.85*c =$	771.9084064
$M_n = \text{if}(c > hf, x(5)*f_{ps}*(d_p-a/2)+x(4)*f_y*(d-a/2)+0.85^2*f_c*x(3)*(be-x(2))*(a/2-x(3)/2), \text{else}, M_n = x(5)*f_{ps}*(d_p-a/2)+x(4)*f_y*(d-a/2)) =$	43742545163
<b>d). Shear force resistance</b>	
$d_v = \max([0.9*d_e, 0.72*x(1), d_e-a/2]) =$	1950.483215
$V_u = V_d*(L/2-w_{sup}/2-d)/(L/2) =$	1.98E+06
$V_c = 0.083^2*\sqrt{f_c}*x(2)*d_v =$	884935.2161
$V_s = f_y*d_v*av/(x(6)) =$	1218739.304
$V_p = 0.85*P*(4*e/L) =$	1.08E+06
$V_n = \min([(V_c+V_s+V_p), (0.25*f_c*x(2)*d_v+V_p)]) =$	3182813.158
<b>e). Minimum flexural reinf.</b>	
$f_{cpe} = 0.85*P*(1/A_c+e/Z_b) =$	25.01188976
$f_r = 0.97*\sqrt{f_c} =$	5.312908808
$M_{cr} = (f_{cpe}+f_r)*I/y_b =$	21594819315
<b>f). Maximum limits of reinf.</b>	
$a. c/d_e =$	0.419031949
$A_{sn} =$	0
$\rho_{hp} = x(4)/(be*d) =$	0.00289183
$\rho_{hn} = A_{sn}/(be*d) =$	0.00E+00
$\rho_{hpr} = A_p/(be*d_p) =$	1.84E-03
$O_{mp} = \rho_{hp}*f_y/f_c =$	0.040485614
$O_{mn} = \rho_{hn}*f_y/f_c =$	0.00E+00
$O_{mpr} = \rho_{hpr}*f_{ps}/f_c =$	2.58E-02
<b>Omp+Ompr - Omn =</b>	<b>6.63E-02</b>
<b>g). Cracked section analysis</b>	
$f_{p1} = 0.85*P/A_c =$	6.07E+00
$f_{p2} = 0.85*n_p*P*(e^2/l+1/A_c) =$	144.1807431
$e_o = \text{if}(y > hf) = x(5)*(f_{p1}+f_{p2})/(0.5*E_c*(x(2)*x(7)+(be-x(2))*x(3)*(1+(x(7)-x(3))/x(7))))-... (E_s*x(4)*(d-x(7))/x(7)+E_p*x(5)*(d_p-x(7))/x(7)), \text{else}, x(5)*(f_{p1}+f_{p2})/(0.5*E_c*be*x(7)-(E_s*x(4)*(d-x(7))/x(7)+E_p*x(5)*(d_p-x(7))/x(7))) =$	1.19E-04

$f_s = E_s \cdot e_o \cdot (d - x(7)) / x(7) =$	3.78E+01
$f_{p3} = E_p \cdot e_o \cdot (d_p - x(7)) / x(7) =$	3.12E+01
$f_p = f_{p1} + f_{p2} + f_{p3} =$	1.81E+02
$f_{co} = e_o \cdot E_c =$	3.29E+00
$T_s = x(4) \cdot f_s =$	636179.014
$T_p = x(5) \cdot f_p =$	1750454.100
$C = 0.5 \cdot f_{co} \cdot b_e \cdot x(7) =$	3.70E+06
$C_n = 0.5 \cdot f_{co} \cdot (b_e - x(2)) \cdot (x(7) - x(3))^2 / x(7) =$	1316526.845
$d_z = x(7) / 3 =$	299.8154876
$d_{zn} = x(3) + (x(7) - x(3)) / 3 =$	499.8154876
$I_{ct} = \begin{cases} \text{if } (y > h_f) = x(2) \cdot y^3 / 3 + (b_e - x(2)) \cdot x(3)^3 / 12 + \dots \\ (b_e - x(2)) \cdot x(3) \cdot (y - x(3)) / 2 + n_p \cdot x(5) \cdot (d_p - y)^2 + \dots \\ \text{else, } I_{ct} = b_e \cdot y^3 / 3 + n_p \cdot x(5) \cdot (d_p - y)^2 + \dots \end{cases}$	8.09057E+11
<b>h). Deflection limit</b>	
$f_{rk} = 0.63 \cdot \sqrt{f_c} =$	3.450652112
$M_{ck} = f_{rk} \cdot I / y_b =$	2457269706
$I_e = \min\left(\left[\frac{M_{ck}}{M_w}\right]^3 I + \left(1 - \left[\frac{M_{ck}}{M_w}\right]^3\right) I_{ct}, I\right] =$	8.10304E+11
$def_D = 2.26e16 / (E_c \cdot I_e) =$	<b>635.769341</b>
$def_P = 0.85 \cdot 5 \cdot P \cdot e \cdot L^2 / (48 \cdot E_c \cdot I_e) =$	<b>156.7312845</b>
$def_{LL} = 2.63e15 / (E_c \cdot I_e) =$	<b>24.33152184</b>
<b>i). Crack width limit</b>	
$cw_1 = (f_s - 40) \cdot 1e-3 =$	<b>-0.002190695</b>
$h_1 = d - x(7) - d_{st} =$	1255.32
$h_2 = d - x(7) =$	1427.94
$d_c = 62 + d_b / 2 =$	78
$A_{tc} = x(2) \cdot 2 \cdot d_{st} / n_b =$	8234.213324
$cw_2 = 0.076 \cdot (h_2 / h_1) \cdot f_s \cdot (d_c \cdot A_{tc})^{1/3} \cdot 1e-3 \cdot 0.1451 =$	<b>0.040920384</b>
<b>j). Fatigue stress limit</b>	
$f_{fs} = n_s \cdot M_f \cdot (d - y) / I_{ct} =$	<b>17.72202299</b>
$f_{fp} = n_p \cdot M_f \cdot (d_p - y) / I_{ct} =$	<b>14.62035029</b>
<b>k). Prestressing indices</b>	
$M_{dec} = x(5) \cdot (f_{p1} + f_{p2}) \cdot e =$	1.74E+09
$PPR = x(5) \cdot f_{py} / (x(5) \cdot f_{py} + x(4) \cdot f_y) =$	0.695611217

<b>CONSTRAINT FUNCTIONS</b>	<b>≤ 0</b>	<b>status</b>
$g1 = f_{tt} - P \cdot (1/A_c + e/Z_t) - M_g/Z_t =$	<b>-2.51E+01</b>	<b>OK!</b>
$g2 = P \cdot (1/A_c + e/Z_b) - M_g/Z_b - f_{ct} =$	<b>-5.43E-01</b>	<b>OK!</b>
$g3 = 0.85 \cdot P \cdot (1/A_c - e/Z_t) + M_w/Z_t - f_{cw} =$	<b>-5.58E+00</b>	<b>OK!</b>
$g4 = f_{tw} - 0.85 \cdot P \cdot (1/A_c + e/Z_b) + M_3/Z_b =$	<b>-1.37E+00</b>	<b>OK!</b>
$g5 = M_d - 0.9 \cdot M_n =$	<b>-1.56E+10</b>	<b>OK!</b>
$g6 = V_u - 0.9 \cdot V_n =$	<b>-8.88E+05</b>	<b>OK!</b>
web requirement for shear, $g6' = V_u/0.9 - 0.25 \cdot f_c' \cdot b_w \cdot d_v - V_p =$	<b>-6.18E+06</b>	<b>OK!</b>
$g7 = \text{abs}(M_d)/(0.9 \cdot d_v) + \text{abs}(V_u/0.9 - V_p) - 0.5 \cdot \min(V_u/0.9, V_s) -$ $x(4) \cdot f_y - x(5) \cdot f_{ps} =$	<b>-8.80E+06</b>	<b>OK!</b>
$g8 = V_u/0.9 - 0.5 \cdot V_s - V_p - x(4) \cdot f_y - x(5) \cdot f_{ps} =$	<b>-2.23E+07</b>	<b>OK!</b>
$g9 = x(6) \cdot f_y \cdot a_v / (0.083 \cdot x(2) \cdot \text{sqrt}(f_c)) =$	<b>-533.4888216</b>	<b>OK!</b>
$b_s(V_u - 0.9 \cdot V_p) / (0.9 \cdot d_v \cdot x(2)) < 0.125 \cdot f_c, x(6) - \min([0.8 \cdot d_v, 600]), x(6) - \min([0.4 \cdot d_v, 300]) =$	<b>-295.91636</b>	<b>OK!</b>
$g11 = \min([1.33 \cdot M_d, 1.2 \cdot M_{cr}]) / (0.9 \cdot M_n) - 1 =$	<b>-1.35E+10</b>	<b>OK!</b>
$g12 = 0.004 \cdot y_b \cdot x(2) - (x(4) + x(5)) =$	<b>-23265.14737</b>	<b>OK!</b>
$g13a = O_{mp} + O_{mpr} - O_{mn} - 0.3 =$	<b>-2.34E-01</b>	<b>OK!</b>
$g13b = c/d_e - 0.42 =$	<b>-0.000968051</b>	<b>OK!</b>
$g16 = P/x(5) - f_{pt} =$	<b>-8.753297588</b>	<b>OK!</b>
$g17 = f_p - f_{pe} =$	<b>-1.16E+03</b>	<b>OK!</b>
$g18 = f_s - 206 =$	<b>-1.68E+02</b>	<b>OK!</b>
$g20 = \text{def}_{LL} - L/1000 =$	<b>-2.57E+01</b>	<b>OK!</b>
$g21 = c_w - 0.41 =$	<b>-0.369079616</b>	<b>OK!</b>
$g22 = f_{fs} - 161.5 =$	<b>-143.777977</b>	<b>OK!</b>
$g23 = f_{fp} - 125 =$	<b>-110.3796497</b>	<b>OK!</b>
$g26 = 0.5 - \text{PPR} =$	<b>-0.195611217</b>	<b>OK!</b>
$g27 = \text{PPR} - 1 =$	<b>-0.304388783</b>	<b>OK!</b>

Optimization results of all other cases were verified in similar way.



## Appendix E conventional design of post tensioned girders

Conventional Design of Partially Prestressed post tensioned Girders[Class-III type struc.]			
[All dimensions Are mm, mm2, mm3, mm4, N, Nmm, N/mm2]			
i. Concrete	Design is for: Ext. T girder	i. Section Property	
Specified compressive strength of concrete, $f'_c =$		Depth, h =	3700
$f'_c = 0.8f'_c =$		Flange thickness, $h_f =$	250
Short term modulus elasticity of concrete, $E_c =$		web width, $b_w =$	1332
Long term modulus elasticity of concrete, $E_{cr} =$		Width of comp. face, $b_1 =$	2500
Elastic modulus of rupture of concrete, $f_{cr} =$		Width of comp. face, $b_c =$	2450
<b>°. Permissible stresses in Concrete</b>			
allowable compressive stress at transfer of prestress, $f_{ci} = 0.60f'_{ci} =$	14.40	Factored shear, $V_d =$	1.98E+06 $\Phi M_n \dots$
Allowable tensile stress at transfer of prestress, $f_{ti} = 0.63\sqrt{f'_{ci}} =$	2.84	Factored B/Moment, $M_d =$	2.57E+10 1.20E+11
Allowable compressive stress at working loads, $f_{cw} = 0.45f'_c =$	13.50	Service Ls. I moment, $M_1 =$	1.79E+10
Allowable tensile stress at working loads, $f_{tw} = 0.50\sqrt{f'_c} =$	2.74	Service Ls. III moment, $M_3 =$	1.71E+10
Stress range at top fiber, $f_{tr} = f_{ct} - \eta f_{ci} =$	11.08	Fatigue load Moment, $M_f =$	1.72E+09
Stress range at bottom fiber, $f_{br} = \eta f_{ci} - f_{cw} =$	9.50	Dead load moment, $M_g =$	1.41E+10
<b>ii. Reinforcing steel (Grade 420 steel)</b>			
Characteristic yield strength of reinforcing bars, $f_y =$	420	<b>Deflection</b>	
Allowable service stress, $f_{sa} = 0.6f_y =$	252	$\Delta_{pt,x} 1/EI_{e[ext.girder]} =$	1.86E+19
Modulus elasticity of reinforcing steel, $E_s =$	200000	$\Delta_{pt,x} 1/EI_{e[int.girder]} =$	1.42E+19
Modular ratio of reinforcing steel, $n_s =$	7	$\Delta_{LL,x} 1/EI_{e[ext.girder]} =$	5.45E+17
<b>***. Reinforcement bars</b>			
Assume $c/d_p =$ 0.40 0.40			
Ultimate tensile strength of tendon, $f_{pu} =$	1860	$f_{ps} =$	1652
Yield strength of tendon, $f_{py} = 0.9f_{pu} =$	1674	Assume PPR =	0.92
Modulus elasticity of prestressing steel, $E_p =$	197000	Assume area of prest.reinf. $A_p =$	26000 Enter(Trial)ly
Modular ratio of prestressing strands, $n_p =$	7.00	Area of non prest.reinf. $A_s =$	8891
Density of reinforcing steel and prestressing tendons, $\rho_s =$	0.00	Concrete Cover =	50
Loss factor, $\eta =$	0.85	Max. aggregate size =	25
<b>***. Permissible stresses in the tendons</b>			
allowable stress in tendons at transfer of prestress, $f_{pi} = 0.74f_{pu} =$	1376.40	Horiz. Spacing of bars, $S_b =$	48
Allowable stress in tendons at working loads, $f_{pe} = 0.8f_{py} =$	1339.20	Vert. Spacing of bars, $S_v =$	32
<b>Resistance factors, <math>\Phi</math> (ASHTO art.5.5.4.2)</b>			
for flexure and tension of RC member, $\Phi =$	0.90	No. of legs of vert.stirr. $N_L =$	4
for flexure and tension of PPC member, $\Phi = 0.9+0.1xPPR =$	0.90	Diam. stirrup., $\Phi_{str} =$	12
for shear and torsion of normal weight concrete, $\Phi =$	0.90	Area of stirrups, $A_v =$	452
<b>Design for Flexure</b>			
		assume $c_0 > h_1$ , then $c_0 =$	1406.38
		<b>Effective depth of reinforcement</b>	
		Use diam.bars, $\Phi_b =$	32
		No. bars =	15.00
		No. of bars per row =	15
		No. of reinf. Rows =	1
		Height of reinf. Rows =	32
		$d'_p =$	78
		Effec-Depth-reinf. $Stl. d = h-d'_p =$	3622
		Use diam.strands, $\Phi_{st} =$	15.24
		No. of strands =	185.11
		Diam. of duct =	125
		Clear hor. & vert. duct spac. =	38
		No. strands per tendon =	31
		No. of tendons =	6
		No. of tendons per row =	6
		No. of rows of prestr. Tendons =	1
		Height of tendon Rows =	125
		$d_p =$	194.5
		$dp = h - d'_p =$	3506

*Optimization of Prestressed Concrete Girders for Bridge Design, Master's Thesis  
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<b>Design is for: Int. T girder</b>		<b>i. Section Property</b>	
<b>i. Geometric data</b>		Depth, h = 3700	
Bridge span, L =	50000	Flange thickness, h <sub>f</sub> =	250
Girder spacing, G <sub>s</sub> =	2500	web width, b <sub>w</sub> =	1332
width of overhang, w <sub>oh</sub> =	1200	Width of comp. face, b <sub>c</sub> =	-
		Width of comp. face, b <sub>1</sub> =	2500
<b>Load Data</b>			
Factored shear, V <sub>d</sub> =	2.17E+06	Area of Conc., A <sub>c</sub> =	5.E+06
Factored B/Moment, M <sub>d</sub> =	2.38E+10	Depth top fiber~c.g. y <sub>t</sub> =	1754
Service I <sub>s</sub> I moment, M <sub>w</sub> =	1.59E+10	Depth bott. fiber~c.g. y <sub>b</sub> =	1946
Service I <sub>s</sub> III moment, M <sub>s</sub> =	1.49E+10	Moment of inertia, I =	6.E+12
Fatigue load Moment, M <sub>f</sub> =	1.39E+09	Sec. mod. of top fiber, Z <sub>t</sub> =	4.E+09
Dead load moment, M <sub>g</sub> =	1.11E+10	Sec. mod. of bott. fiber, Z <sub>b</sub> =	3.E+09
<b>Deflection</b>		<b>Effective depth of reinforcement</b>	
Δ <sub>DLX</sub> 1/EI <sub>e[ext.girder]</sub> =	1.86E+19	Use diam.bars, Φ <sub>b</sub> =	32
Δ <sub>DLX</sub> 1/EI <sub>e[int.girder]</sub> =	1.42E+19	No. bars =	18.00
Δ <sub>LLX</sub> 1/EI <sub>e[ext.girder]</sub> =	5.45E+17	No. of bars per row =	15
<b>** Reinforcement bars</b>	...c/dp...	No. of reinf. Rows =	2
Assume c/d <sub>p</sub> =	0.41	Height of reif. Rows =	96
f <sub>ps</sub> =	1646	d <sub>s</sub> ' =	110
Assume PPR =	0.90	Effec-Depth-reinf. Stl, d = h-d' =	3590
Assume area of prest.reinf.Ap =	25500 [EnterTrial1y]	Use diam.strands, Φ <sub>st</sub> =	15.24
Area of non prest.reinf.As =	11107	No. of strands =	181.55
Concrete Cover =	50	Diam. of duct =	125
Max. aggregate size =	25	Clear hor. & vert. duct spac. =	38
Horiz. Spacing of bars, S <sub>h</sub> =	48	No. strands per tendon =	31
Vert. Spacing of bars, S <sub>v</sub> =	32	No. of tendons =	6
No. of legs of vert.stirr. NL =	4	No. of tendons per row =	6
Diam. stirrup., Φ <sub>stirr</sub> =	12	No. of rows of prestr. Tendons =	1
Area of stirrups, A <sub>v</sub> =	452	Height of tendon Rows =	125
<b>Design for Flexure</b>		d <sub>p</sub> ' = 258.5	
assume c <sub>0</sub> > h <sub>f</sub> , then c <sub>0</sub> =	1398.29	dp = h- d <sub>p</sub> ' =	3442
NA depth, c =	1398.29	Eff. Dept. of tens. force, de =	3456
Rect. Stres. Block depth, a =	1188.54	f <sub>inf</sub> =	8.86E+00
Nomin. Flexural resis., Mn =	1.31E+11	eccentricityof prestr. force, e =	1688
Check flex. Capacity, M <sub>d</sub> ≤ ΦM <sub>n</sub> =	OK!	<b>Prestr. Force required, P =</b>	<b>1.E+07</b>
<b>Design for shear</b>		<b>Check Stresses of extr. Fibers</b>	
width of support =	500	a. f <sub>ti</sub> -P*(1/A <sub>c</sub> +e/Z <sub>t</sub> )-M <sub>g</sub> /Z <sub>t</sub> ≤ 0	-8.E+00 OK!
Spacing of stirrups, s =	310	b. P*(1/A <sub>c</sub> +e/Z <sub>b</sub> )-M <sub>g</sub> /Z <sub>b</sub> -f <sub>ct</sub> ≤ 0	-9.E+00 OK!
shear depth, d <sub>v</sub> =	3110.715	ΦV <sub>n</sub> ... c. 0.85*P*(1/A <sub>c</sub> -e/Z <sub>t</sub> )+M <sub>w</sub> /Z <sub>t</sub> -f <sub>tw</sub> ≤ 0	-1.E+01 OK!
Design shear dv dist. From face of supp., V <sub>U</sub> =	1.88E+06	d. f <sub>tw</sub> -0.85*P*(1/A <sub>c</sub> +e/Z <sub>b</sub> )+M <sub>s</sub> /Z <sub>b</sub> ≤ 0	-3.E-01 OK!
Concrete shear resis. V <sub>c</sub> =	3.77E+06	<b>Service load stresses, fs &amp; fp</b>	
Shear reinf. resis. V <sub>s</sub> =	1.91E+06	fp1 = η)P/A <sub>c</sub> =	2.06
Shear reis. of Prestressing, V <sub>p</sub> =	1.45E+06	fp2 = η) .n <sub>p</sub> .P(e <sup>2</sup> /I+1/A <sub>c</sub> ) =	47.62
Nominal shear resi. V <sub>n</sub> =	7.12E+06	Cracked NA depth, y =	976.7556835
Check shear. Capacity, V <sub>u</sub> ≤ ΦV <sub>n</sub> =	OK!	Concrete strain, ε <sub>0</sub> =	0.000197402
<b>Minimum area of reif. Area</b>		f <sub>co</sub> = ε <sub>0</sub> .Ec =	5.460126918
f <sub>cpe</sub> = 0.85*P*(1/A <sub>c</sub> +e/Z <sub>b</sub> ) =	8.E+00	fs = Es*eo*(d-y)/y =	105.6269636
frp =	5.312908808	fp3 = Ep*eo*(dp-y)/y =	98.1302455
M <sub>cr</sub> =	4.E+10	chk! fp =fp1+fp2+fp3 =	147.8048156
Min[1.2M <sub>cr</sub> ,1.33Md]-ΦM <sub>n</sub> ≤ 0 =	-9.E+10	Ts = fs.As =	1173214.78
Max. area of steel, c/d <sub>e</sub> ≤ 0.42 =	0.404555717	Tp = fp.Ap =	3769022.798
0.004*γ <sub>b</sub> *b <sub>w</sub> *A <sub>p</sub> -A <sub>s</sub> ≤ 0 =	-26236.2704	C = 0.5*f <sub>co</sub> *be*y =	6666512.499
...Check deflection & camber...		Cn = 0.5*f <sub>co</sub> *(be-bw)*(y-hf) <sup>2</sup> /y =	1724274.921

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NA depth, c =	1406.38		Eff. Dept. of tens. force, de =	3515	
Rect. Stres. Block depth, a =	1195.42		$f_{inf} =$	9.59E+00	
Nomin. Flexural resis., Mn =	1.33E+11		eccentricity of prestr. force, e =	1748	
Check flex. Capacity, $M_d \leq \Phi M_n$ =	OK!		<b>Prestr. Force required, P =</b>	<b>1.E+07</b>	
<b>Design for shear</b>			<b>Check Stresses of extr. Fibers</b>		Check!
width of support =	500		a. $f_{it} - P*(1/A_c + e/Z_t) - M_g/Z_t \leq 0$	-1.E+01	OK!
Spacing of stirrups, s =	310		b. $P*(1/A_c + e/Z_b) - M_g/Z_b - f_{ct} \leq 0$	-9.E+00	OK!
shear depth, d <sub>v</sub> =	3163.338	$\Phi V_n \dots$	c. $0.85*P*(1/A_c - e/Z_t) + M_w/Z_t - f_{cw} \leq 0$	-1.E+01	OK!
Design shear dv dist. From face of supp., $V_U =$	1.71E+06	6.61E+06	d. $f_{tw} - 0.85*P*(1/A_c + e/Z_b) + M_3/Z_b \leq 0$	-2.E-01	OK!
Concrete shear resis. $V_c =$	3.83E+06		<b>Service load stresses, fs &amp; fp</b>		
Shear reinf. resis. $V_s =$	1.94E+06		$fp1 = \eta P/A_c =$	2.17	
Shear res. of Prestressing, $V_p =$	1.58E+06		$fp2 = \eta \cdot n_p \cdot P(e^2/l + 1/A_c) =$	52.87	
Nominal shear resi. $V_n =$	7.35E+06		Cracked NA depth, y =	971.7999877	
Check shear. Capacity, $V_u \leq \Phi V_n$ =	OK!		Concrete strain, $\epsilon_0 =$	0.000222369	
<b>Minimum area of reif. Area</b>			$f_{co} = \epsilon_0 \cdot E_c =$	6.150730952	
$f_{cpe} = 0.85*P*(1/A_c + e/Z_b) =$	8.E+00		$f_s = E_s \cdot \epsilon_0 \cdot (d-y)/y =$	121.2848298	
$f_{rp} =$	5.312908808		$fp3 = E_p \cdot \epsilon_0 \cdot (dp-y)/y =$	114.2139773	
$M_{cr} =$	4.E+10	chk!	$fp = fp1 + fp2 + fp3 =$	169.2509163	
$\text{Min}[1.2M_{cr}, 1.33M_d] - \Phi M_n \leq 0 =$	-9.E+10	OK!	$T_s = f_s \cdot A_s =$	1078347.188	
Max. area of steel, $c/d_e \leq 0.42 =$	0.40012744	OK!	$T_p = fp \cdot A_p =$	4400523.823	
$0.004 \cdot y_b \cdot b_w \cdot A_p \cdot A_s \leq 0 =$	-24540.9749	OK!	$C = 0.5 \cdot f_{co} \cdot b_e \cdot y =$	7322168.323	
<b>...Check deflection &amp; camber...</b>			$C_n = 0.5 \cdot f_{co} \cdot (b_e - b_w) \cdot (y - hf)^2 / y =$	1843297.311	
$\Delta_{DL} =$	203.4539444		$dz = y/3 =$	323.9333292	
$\Delta_p = 0.85 \cdot 5 \cdot P \cdot e \cdot L^2 / (48 \cdot E_c \cdot I_e) =$	56.29944061		$dz_n = hf + (y - hf)/3 =$	490.5999959	
Camber, $a = \Delta_{DL} - \Delta_p =$	147.1545038	NOT OK!	<b>...Verify y using equilibrium...</b>		
$\Delta_{UL} =$	5.971114598		$\Sigma M - M_w = 0$	-3.8147E-06	OK!
$\Delta_{LL(allow.)} = L/1000 =$	50	OK!	$\Sigma F_x = 0 =$	0	OK!
<b>...Crack Control...</b>			<b>...Cracked Moment of inertia...</b>		
$h1 = d - y - d' =$	893.7999877		$I_{ct} =$	2.21486E+12	
$h2 = d - y =$	2650.200012		$f_{rup} = 0.63 \cdot \text{sqrt}(f_c') =$	3.450652112	
$dc = 62 + db/2 =$	78		$M_{crk} = f_{rup} \cdot Z_b =$	11388009587	
$Atc = bw \cdot 2 \cdot ds' / nb =$	13852.8		<b>...Effective Moment of inertia...</b>		
$cw1 = 0.076 \cdot (h2/h1) \cdot f_s \cdot (dc \cdot Atc)^{1/3} \cdot 1e-3 \cdot 0.1451 =$	0.40694575		$I_e =$	3.30189E+12	
$cw2 = (f_s - 40) \cdot 1e-3 =$	0.08128483		<b>...Check service stresses...</b>		
$cw = \max([cw1, cw2]) =$	0.40694575		$f_{sa} = \text{min}(0.5f_y, 206) =$	206	
$cwa =$	0.41	OK!	$f_s - f_{sa} \leq 0 =$	-84.71517023	OK!
			$fp - f_{pe} \leq 0 =$	-1169.949084	OK!
			<b>...Check fatigue stresses...</b>		
			$ffa = 145 - 0.33f_{min} + 55(r/h) =$	161.5	
			$ffpa =$	125	
			$ffs = ns \cdot M_f \cdot (d-y) / I_{ct} =$	14.40050367	OK!
			$ffp = np \cdot M_f \cdot (dp-y) / I_{ct} =$	13.7674727	OK!
===== The Design is completed!! =====					

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$\Delta_{DL} =$	135.1266802		$dz = y/3 =$	325.5852278	
$\Delta_p = 0.85*5*P*e*L^2/(48*E_c*I_e) =$	44.74772582		$dz_n = hf + (y-hf)/3 =$	492.2518945	
Camber, $a = \Delta_{DL} - \Delta_p =$	90.37895442		...Verify y using equilibrium...		
$\Delta_{LL} =$	5.171431775		$\Sigma M - M_w = 0$	1.33514E-05	OK!
$\Delta_{LL(allow.)} = L/1000 =$	50	OK!	$\Sigma F_x = 0 =$	-9.31323E-10	OK!
...Crack Control...			...Cracked Moment of inertia...		
$h_1 = d - y - d' =$	866.7556835		$I_{ct} =$	2.24246E+12	
$h_2 = d - y =$	2613.244317		$f_{rup} = 0.63\sqrt{f_c'} =$	3.450652112	
$dc = 62 + db/2 =$	78		$M_{crk} = f_{rup} \cdot Z_b =$	11424165879	
$Atc = bw*2*ds'/nb =$	16280		...Effective Moment of inertia...		
$cw_1 = 0.076*(h_2/h_1)*f_s*(dc*Atc)^{(1/3)}*1e-3*0.1451 =$	0.380296316		$I_e =$	3.81247E+12	
$cw_2 = (f_s - 40)*1e-3 =$	0.065626964		...Check service stresses...		
$cw = \max\{cw_1, cw_2\} =$	0.38029632		$f_{sa} = \min(0.5f_y, 206) =$	206	
$cwa =$	0.41	OK!	$f_s - f_{sa} \leq 0 =$	-100.3730364	OK!
			$f_p - f_{pe} \leq 0 =$	-1191.395184	OK!
			...Check fatigue stresses...		
			$ffa = 145 - 0.33f_{min} + 55(r/h) =$	161.5	
			$ffpa =$	125	
			$ffs = ns * M_f(d-y)/I_{ct} =$	11.32816362	OK!
			$ffp = np * M_f(dp-y)/I_{ct} =$	10.68443035	OK!
===== The Design is completed!! =====					

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<b>Design is for: Ext. Box girder</b>			<b>i. Section Property</b>		
<b>i. Geometric data</b>			Depth, h = 3750		
Bridge span, L =	50000		Flange thickness, h <sub>f</sub> =	250	
Girder spacing, G <sub>s</sub> =	2500		web width, b <sub>w</sub> =	1350	
width of overhang, w <sub>oh</sub> =	1200		Width of comp. face, b <sub>i</sub> =	2500	
			Width of comp. face, b <sub>e</sub> =	2450	
			effec. Width of bottom slab, b <sub>eb</sub> =	1225	
<b>Load Data</b>			thickness of bott. Slab, t <sub>b</sub> = 140		
Factored shear, V <sub>d</sub> =	2.05E+06	ΦM <sub>n</sub> ...	Area of Conc., A <sub>c</sub> =	5.E+06	
Factored B/Moment, M <sub>d</sub> =	2.73E+10	1.21E+11	Depth top fiber~c.g. y <sub>t</sub> =	1779	
Service I <sub>s</sub> I moment, M <sub>w</sub> =	1.88E+10		Depth bott. fiber~c.g. y <sub>b</sub> =	1971	
Service I <sub>s</sub> III moment, M <sub>3</sub> =	1.78E+10		Moment of inertia, I =	7.E+12	
Fatigue load Moment, M <sub>f</sub> =	2.04E+09		Sec. mod. of top fiber, Z <sub>t</sub> =	4.E+09	
Dead load moment, M <sub>g</sub> =	1.41E+10		Sec. mod. of bott. fiber, Z <sub>b</sub> =	3.E+09	
<b>Deflection</b>			<b>Effective depth of reinforcement</b>		
Δ <sub>DLX</sub> 1/EI <sub>e[ext.girder]</sub> =	1.95E+19		Use diam.bars, Φ <sub>b</sub> =	32	
Δ <sub>DLX</sub> 1/EI <sub>e[int.girder]</sub> =	1.71E+19		No. bars =	24.00	
Δ <sub>LLX</sub> 1/EI <sub>e[ext.girder]</sub> =	5.45E+17		No. of bars per row =	15	
<b>** Reinforcement bars</b>			No. of reinf. Rows = 2		
Assume c/d <sub>p</sub> =	0.41	0.41	Height of reif. Rows =	96	
f <sub>ps</sub> =	1646		d <sub>p</sub> ' =	110	
Assume PPR =	0.87		Effec-Depth-reinf. Stl, d = h-d' =	3640	
Assume area of prest.reinf. A <sub>p</sub> =	25000	[EnterTrial1y]	Use diam.strands, Φ <sub>st</sub> =	15.24	
Area of non prest.reinf. A <sub>s</sub> =	14644		No. of strands =	177.99	
Concrete Cover =	50		Diam. of duct =	125	
Max. aggregate size =	25		Clear hor. & vert. duct spac. =	38	
Horiz. Spacing of bars, S <sub>h</sub> =	48		No. strands per tendon =	31	
Vert. Spacing of bars, S <sub>v</sub> =	32		No. of tendons =	6	
No. of legs of vert.stirr. N <sub>L</sub> =	4		No. of tendons per row =	6	
Diam. stirrup., Φ <sub>stirr</sub> =	12		No. of rows of prestr. Tendons =	1	
Area of stirrups, A <sub>v</sub> =	452		Height of tendon Rows =	125	
<b>Design for Flexure</b>			d <sub>p</sub> ' = 258.5		
assume c <sub>0</sub> > h <sub>f</sub> , then c <sub>0</sub> =	1415.26		dp = h- d <sub>p</sub> ' =	3492	
NA depth, c =	1415.26		Eff. Dept. of tens. force, de =	3511	
Rect. Stres. Block depth, a =	1202.97		f <sub>inf</sub> =	9.75E+00	
Nomin. Flexural resis., Mn =	1.35E+11		eccentricityof prestr. force, e =	1713	
Check flex. Capacity, M <sub>d</sub> ≤ ΦM <sub>n</sub> =	OK!		<b>Prestr. Force required, P =</b>	1.E+07	
<b>Design for shear</b>			<b>Check Stresses of extr. Fibers</b>		
width of support =	500		a. f <sub>tt</sub> -P*(1/A <sub>c</sub> +e/Z <sub>t</sub> )-M <sub>g</sub> /Z <sub>t</sub> ≤ 0	-1.E+01	OK!
Spacing of stirrups, s =	300		b. P*(1/A <sub>c</sub> +e/Z <sub>b</sub> )-M <sub>g</sub> /Z <sub>b</sub> -f <sub>ct</sub> ≤ 0	-9.E+00	OK!
shear depth, d <sub>v</sub> =	3159.7245	ΦV <sub>n</sub> ...	c. 0.85*P*(1/A <sub>c</sub> -e/Z <sub>t</sub> )+M <sub>w</sub> /Z <sub>t</sub> -f <sub>ctw</sub> ≤ 0	-1.E+01	OK!
Design shear dv dist. From face of supp., V <sub>U</sub> =	1.77E+06	6.76E+06	d. f <sub>tw</sub> -0.85*P*(1/A <sub>c</sub> +e/Z <sub>b</sub> )+M <sub>3</sub> /Z <sub>b</sub> ≤ 0	-3.E-01	OK!
Concrete shear resis. V <sub>c</sub> =	3.88E+06		<b>Service load stresses, fs &amp; fp</b>		
Shear reinf. resis. V <sub>s</sub> =	2.00E+06		fp1 = η P/A <sub>c</sub> =	2.24	
Shear reis. of Prestressing, V <sub>p</sub> =	1.64E+06		fp2 = η .n <sub>p</sub> .P(e <sup>2</sup> /I+1/A <sub>c</sub> ) =	52.47	
Nominal shear resi. V <sub>n</sub> =	7.52E+06		Cracked NA depth, y =	1016.0479	
Check shear. Capacity, V <sub>u</sub> ≤ ΦV <sub>n</sub> =	OK!				
<b>Minimum area of reif. Area</b>					

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$f_{cpe} = 0.85 \cdot P \cdot (1/A_c + e/Z_b) =$	8.E+00		Concrete strain, $\epsilon_0 =$	<b>0.000225045</b>
$f_{rp} =$	5.312908808		$f_{co} = \epsilon_0 \cdot E_c =$	6.224730832
$M_{cr} =$	5.E+10	chk!	$f_s = E_s \cdot \epsilon_0 \cdot (d-y)/y =$	116.2359012
$\text{Min}[1.2M_{cr}, 1.33Md] - \Phi M_n \leq 0 =$	-9.E+10	<b>OK!</b>	$f_{p3} = E_p \cdot \epsilon_0 \cdot (d_p-y)/y =$	108.0127795
Max. area of steel, $c/d_e \leq 0.42 =$	0.403116629	<b>OK!</b>	$f_p = f_{p1} + f_{p2} + f_{p3} =$	162.7252629
$0.004 \cdot \gamma_b \cdot b_w \cdot A_p \cdot A_s \leq 0 =$	-28998.7692	<b>OK!</b>	$T_s = f_s \cdot A_s =$	1702195.565

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<b>...Check deflection &amp; camber...</b>			$T_p = f_p \cdot A_p =$	4068131.572	
$\Delta_{DL} =$	202.9198442		$C = 0.5 \cdot f_{co} \cdot b_e \cdot y =$	7747665.243	
$\Delta_p = 0.85 \cdot 5 \cdot P \cdot e \cdot L^2 / (48 \cdot E_c \cdot I_e) =$	55.32783022		$C_n = 0.5 \cdot f_{co} \cdot (b_e - b_w) \cdot (y - hf)^2 / y =$	1977338.106	
Camber, $a = \Delta_{DL} - \Delta_p =$	147.592014	NOT OK!	$dz = y/3 =$	338.6826332	
$\Delta_{LL} =$	5.666063104		$dzn = hf + (y - hf)/3 =$	505.3492999	
$\Delta_{LL(allow.)} = L/1000 =$	50	OK!	<b>...Verify y using equilibrium...</b>		
			$\Sigma M - M_w = 0$	-3.8147E-06	OK!
<b>...Crack Control...</b>			$\Sigma F_x = 0 =$	0	OK!
$h1 = d - y - d' =$	906.0478997		<b>...Cracked Moment of inertia...</b>		
$h2 = d - y =$	2623.9521		$I_{ct} =$	2.46996E+12	
$dc = 62 + db/2 =$	78		$f_{rup} = 0.63 \cdot \sqrt{f_c'} =$	3.450652112	
$Atc = b_w \cdot 2 \cdot ds' / n_b =$	12375		$M_{crk} = f_{rup} \cdot Z_b =$	11674460878	
$cw1 = 0.076 \cdot (h2/h1) \cdot f_s \cdot (dc \cdot Atc)^{(1/3)} \cdot 1e-3 \cdot 0.1451 =$	0.36686474		<b>...Effective Moment of inertia...</b>		
$cw2 = (f_s - 40) \cdot 1e-3 =$	0.076235901		$I_e =$	3.47966E+12	
$cw = \max\{cw1, cw2\} =$	0.36686474		<b>...Check service stresses...</b>		
$cwa =$	0.41	OK!	$f_{sa} = \min(0.5f_y, 206) =$	206	
			$f_s - f_{sa} \leq 0 =$	-89.7640988	OK!
			$f_p - f_{pe} \leq 0 =$	-1176.474737	OK!
			<b>...Check fatigue stresses...</b>		
			$ffa = 145 - 0.33f_{min} + 55(r/h) =$	161.5	
			$ffpa =$	125	
			$ffs = n_s \cdot M_f \cdot (d - y) / I_{ct} =$	15.17640481	OK!
			$ffp = n_p \cdot M_f \cdot (d_p - y) / I_{ct} =$	14.31751104	OK!
===== The Design is completed! =====					
<b>Design is for: Int. Box girder</b>			<b>i. Section Property</b>		
<b>i. Geometric data</b>			<b>i. Section Property</b>		
Bridge span, L =	50000		Depth, h =	3750	
Girder spacing, G <sub>s</sub> =	2500		Flange thickness, h <sub>f</sub> =	250	
width of overhang, w <sub>oh</sub> =	1200		web width, b <sub>w</sub> =	1350	
			Width of comp. face, b <sub>i</sub> =	2500	
			Width of comp. face, b <sub>e</sub> =	2500	
			effec. Width of bottom slab, b <sub>eb</sub> =	2500	
			thickness of bott. Slab, t <sub>b</sub> =	140	
<b>Load Data</b>					
Factored shear, V <sub>d</sub> =	2.36E+06	ΦM <sub>n</sub> ...	Area of Conc., A <sub>c</sub> =	6.E+06	
Factored B/Moment, M <sub>d</sub> =	5.87E+10	1.47E+11	Depth top fiber~c.g. y <sub>t</sub> =	1836	
Service I <sub>s</sub> _I moment, M <sub>w</sub> =	4.02E+10		Depth bott. fiber~c.g. y <sub>b</sub> =	1914	
Service I <sub>s</sub> _III moment, M <sub>3</sub> =	3.96E+10		Moment of inertia, I =	7.E+12	
Fatigue load Moment, M <sub>f</sub> =	1.47E+09		Sec. mod. of top fiber, Z <sub>t</sub> =	4.E+09	
Dead load moment, M <sub>g</sub> =	3.73E+10		Sec. mod. of bott. fiber, Z <sub>b</sub> =	4.E+09	
<b>Deflection</b>			<b>Effective depth of reinforcement</b>		
$\Delta_{DL} \times I / E I_{e[ext.girder]} =$	1.95E+19		Use diam.bars, Φ <sub>b</sub> =	32	
$\Delta_{DL} \times I / E I_{e[int.girder]} =$	1.71E+19		No. bars =	40.00	
$\Delta_{LL} \times I / E I_{e[ext.girder]} =$	5.45E+17		No. of bars per row =	15	
<b>** Reinforcement bars</b>			<b>Effective depth of reinforcement</b>		
Assume c/d <sub>p</sub> =	0.59	0.59	No. of reinf. Rows =	3	
f <sub>ps</sub> =	1553		Height of reif. Rows =	160	
Assume PPR =	0.84		d <sub>s</sub> ' =	142	
Assume area of prest.reinf.A <sub>p</sub> =	35000	[EnterTrial1y]			

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Area of non prest.reinf. $A_s =$	24646		Effec-Depth-reinf. Stl. $d = h-d' =$	3608	
Concrete Cover =	50		Use diam.strands, $\Phi_{st} =$	15.24	
Max. aggregate size =	25		No. of strands =	249.18	
Horiz. Spacing of bars, $S_n =$	48		Diam. of duct =	125	
Vert. Spacing of bars, $S_v =$	32		Clear hor. & vert. duct spac. =	38	
No. of legs of vert.stirr. $N_L =$	4		No. strands per tendon =	31	
Diam. stirrup., $\Phi_{stirr} =$	12		No. of tendons =	9	
Area of stirrups, $A_v =$	452		No. of tendons per row =	7.288343558	
<b>Design for Flexure</b>			No. of rows of prestr. Tendons =	1.234848485	
assume $c_0 > h_i$ , then $c_0 =$	2000.04		Height of tendon Rows =	163.280303	
NA depth, $c =$	2000.04		$d_p' =$	341.6401515	
Rect. Stres. Block depth, $a =$	1700.03		$dp = h - d_p' =$	3408	
Nomin. Flexural resis., $M_n =$	1.63E+11		Eff. Dept. of tens. force, $d_e =$	3440	
Check flex. Capacity, $M_d \leq \Phi M_n =$	OK!		$f_{int} =$	1.56E+01	
<b>Design for shear</b>			eccentricity of prestr. force, $e =$	1572	
width of support =	500		<b>Prestr. Force required, <math>P =</math></b>	3.E+07	
Spacing of stirrups, $s =$	300		<b>Check Stresses of extr. Fibers</b>		Check!
shear depth, $d_v =$	3096.272045	$\Phi V_n \dots$	a. $f_{ti} - P^*(1/A_c + e/Z_t) - M_g/Z_t \leq 0$	-2.E+01	OK!
Design shear $d_v$ dist. From face of supp., $V_U =$	2.04E+06	7.72E+06	b. $P^*(1/A_c + e/Z_b) - M_g/Z_b - f_{ct} \leq 0$	-9.E+00	OK!
Concrete shear resis. $V_c =$	3.80E+06		c. $0.85 * P^*(1/A_c - e/Z_t) + M_w/Z_t - f_{cw} \leq 0$	-8.E+00	OK!
Shear reinf. resis. $V_s =$	1.96E+06		d. $f_{iw} - 0.85 * P^*(1/A_c + e/Z_b) + M_3/Z_b \leq 0$	-2.E-01	OK!
Shear reis. of Prestressing, $V_p =$	2.81E+06		<b>Service load stresses, <math>f_s</math> &amp; <math>f_p</math></b>		
Nominal shear resis. $V_n =$	8.57E+06		$fp1 = \eta P/A_c =$	4.06	
Check shear. Capacity, $V_u \leq \Phi V_n =$	OK!		$fp2 = \eta \cdot \eta_p \cdot P(e^2/(1+1/A_c)) =$	81.18	
<b>Minimum area of reif. Area</b>			Cracked NA depth, $y =$	1198.561439	
$f_{cpe} = 0.85 * P^*(1/A_c + e/Z_b) =$	1.E+01		Concrete strain, $\epsilon_0 =$	0.000433715	
$f_{rp} =$	5.312908808		$f_{co} = \epsilon_0 \cdot E_c =$	11.99655911	
$M_{cr} =$	7.E+10	chk!	$f_s = E_s \cdot \epsilon_0 \cdot (d-y)/y =$	174.3774185	
$\text{Min}[1.2M_{cr}, 1.33M_d] - \Phi M_n \leq 0 =$	-7.E+10	OK!	$fp3 = E_p \cdot \epsilon_0 \cdot (dp-y)/y =$	157.5300006	
Max. area of steel, $c/d_c \leq 0.42 =$	0.41	OK!	$fp = fp1 + fp2 + fp3 =$	242.7737874	
$0.004 * y_b * b_w - A_p - A_s \leq 0 =$	-49313.2367	OK!	$T_s = f_s \cdot A_s =$	4297788.894	
<b>...Check deflection &amp; camber...</b>			$T_p = f_p \cdot A_p =$	8497082.558	
$\Delta_{DL} =$	179.0582418		$C = 0.5 * f_{co} * b_e * y =$	17973266.43	
$\Delta_p = 0.85 * 5 * P * e * L^2 / (48 * E_c * I_e) =$	95.99203058		$C_n = 0.5 * f_{co} * (b_e - b_w) * (y - hf)^2 / y =$	5178394.979	
Camber, $a = \Delta_{DL} - \Delta_p =$	83.0662112	NOT OK!	$dz = y/3 =$	399.5204795	
$\Delta_{UL} =$	5.717103584		$dz_n = hf + (y - hf)/3 =$	566.1871462	
$\Delta_{LL(allow.)} = L/1000 =$	50	OK!	<b>...Verify y using equilibrium...</b>		
			$\Sigma M - M_w = 0$	0	OK!
<b>...Crack Control...</b>			$\Sigma F_x = 0 =$	0	OK!
$h1 = d - y - d' =$	1056.561439		<b>...Cracked Moment of inertia...</b>		
$h2 = d - y =$	2409.438561		$I_{ct} =$	3.30562E+12	
$dc = 62 + db/2 =$	78		$f_{rup} = 0.63 \cdot \text{qrt}(f_c') =$	3.450652112	
$Atc = bw * 2 * ds' / nb =$	9585		$M_{crk} = f_{rup} \cdot Z_b =$	13220049759	
$cw1 = 0.076 * (h2/h1) * f_s * (dc * Atc) * (1/3)^3 * 1e-3 * 0.1451 =$	0.398004445		<b>...Effective Moment of inertia...</b>		
$cw2 = (f_s - 40) * 1e-3 =$	0.134377419		$I_e =$	3.44859E+12	
$cw = \text{max}([cw1, cw2]) =$	0.39800445		<b>...Check service strsses...</b>		
$cwa =$	0.41	OK!	$f_{sa} = \text{min}(0.5f_y, 206) =$	206	
			$f_s - f_{sa} \leq 0 =$	-31.62258148	OK!
			$fp - f_{pe} \leq 0 =$	-1096.426213	OK!
			<b>...Check fatigue strsses...</b>		
			$ffa = 145 - 0.33f_{min} + 55(r/h) =$	161.5	
			$ffpa =$	125	
			$ffs = n_s * M_f * (d-y) / I_{ct} =$	7.482330935	OK!
			$ffp = n_p * M_f * (dp-y) / I_{ct} =$	6.862363402	OK!
===== The Design is completed!! =====					



## Appendix F Design Optimization Results

Optimization output for PC Girders with variable span																	
Optim. OValues for PC T-Girders																	
Span, L (mm)	Exterior Girder								Interior Girder								
	h	bw	hf	As	Ap	S	y	Opt. Cost	h	bw	hf	As	Ap	S	y	Opt. Cost	
	x1	x2	x3	x4	x5	x6	x7	Z(x)	x1	x2	x3	x4	x5	x6	x7	Z(x)	
10000	997	396	200	2618.357	4748.863	359.9743	193.1033	4.33E+04	1189	440	253	5392.146	7826.691	338.4656	285.3397	6.70E+04	
20000	1300	447	200	3997.218	6999.014	375.6838	276.589	1.25E+05	1500	500	300	7982.893	8895.015	321.1482	360.235	1.75E+05	
30000	1900	543	300	7476.768	9936.481	387.4355	416.7794	2.98E+05	2000	550	300	12848.22	9731.936	388.7149	799.7575	3.40E+05	
40000	2250	550	300	10957.91	11572.26	233.7503	50.85817	5.45E+05	2500	600	300	27988.9	7559.128	292.7523	799.7329	6.50E+05	
50000	2250	700	300	9983.929	13032.69	276.6278	51	7.20E+05	3200	850	300	26064	12448	409	873	1.25E+06	
60000	2800	800	300	9832.46	16271.83	314.1705	50.21755	1.07E+06	3357	846	297	28311.41	13418.64	330.1903	946.129	1.38E+06	
70000	3150	800	300	522.5449	20135.89	334.7989	50.15386	1.27E+06	3700	900	300	39952.5	15573.1	404.2692	741.0425	1.91E+06	
80000	3600	950	300	11888.24	21495.15	246.5598	50.71023	2.06E+06	4200	1100	300	40228.27	18803.18	315.5659	1257.925	2.71E+06	
90000	4100	1000	300	9821.459	25552.41	348.9301	54.16227	2.48E+06	4400	1200	300	35355.67	23455.4	265.516	1294.822	3.39E+06	
100000	4600	1100	300	9991.4	30293.68	265.0967	50.00993	3.43E+06	4800	1300	300	38239.49	27855.66	268.5104	1427.909	4.32E+06	
Average optimum values for exterior & interior T-girders									Cost of concrete & steel								
h	bw	hf	As	Ap	S	Z(x)	Wstr	ost of Con	Cost of Steel								
1.09E+03	4.18E+02	2.27E+02	4.01E+03	6.29E+03	3.49E+02	5.52E+04	4.30E-01	1.14E+04	4.37E+04								
1.40E+03	4.74E+02	2.50E+02	5.99E+03	7.95E+03	3.48E+02	1.50E+05	1.13E+00	3.43E+04	1.16E+05								
1.95E+03	5.47E+02	3.00E+02	1.02E+04	9.83E+03	3.88E+02	3.19E+05	2.16E+00	8.41E+04	2.35E+05								
2.38E+03	5.75E+02	3.00E+02	1.95E+04	9.57E+03	2.63E+02	5.97E+05	5.18E+00	1.43E+05	4.55E+05								
2.73E+03	7.75E+02	3.00E+02	1.80E+04	1.27E+04	3.43E+02	9.87E+05	5.91E+00	3.92E+05	5.95E+05								
3.08E+03	8.23E+02	2.99E+02	1.91E+04	1.48E+04	3.22E+02	1.22E+06	8.54E+00	4.08E+05	8.14E+05								
3.43E+03	8.50E+02	3.00E+02	2.02E+04	1.79E+04	3.70E+02	1.59E+06	9.66E+00	5.55E+05	1.04E+06								
3.90E+03	1.03E+03	3.00E+02	2.61E+04	2.01E+04	2.81E+02	2.38E+06	1.68E+01	8.70E+05	1.51E+06								
4.25E+03	1.10E+03	3.00E+02	2.26E+04	2.45E+04	3.07E+02	2.94E+06	1.89E+01	1.16E+06	1.78E+06								
4.70E+03	1.20E+03	3.00E+02	2.41E+04	2.91E+04	2.67E+02	3.87E+06	2.68E+01	1.54E+06	2.34E+06								
Optim. OValues for PC Box-Girders																	
Span, L (mm)	Exterior Girder								Interior Girder								
	h	bw	hf	As	Ap	S	y	Opt. Cost	h	bw	hf	As	Ap	S	y	Opt. Cost	
	x1	x2	x3	x4	x5	x6	x7	Z(x)	x1	x2	x3	x4	x5	x6	x7	Z(x)	
10000	900	312	289	2372.525	3999.996	324.6204	449.9999	4.49E+04	1070	384	263	5111.904	6795.234	391.3076	226.2217	7.01E+04	
20000	1100	350	203	3295.317	5000	351.6707	450	1.09E+05	1200	500	300	11661.86	7311.886	339.3878	293.3983	1.88E+05	
30000	1450	400	276	5993.789	8581.932	448.978	449.3894	2.39E+05	1600	600	300	5591.196	10667.94	302.2286	348.8622	3.30E+05	
40000	1900	450	300	9276.953	9596.698	317.3982	799.9137	4.33E+05	2100	600	300	14655.74	12725.18	349.9817	539.053	5.94E+05	
50000	2300	500	300	5201.908	12109.47	330.2099	999	6.05E+05	2500	700	300	9922.609	17490.64	237.9945	631.2583	9.34E+05	
60000	2800	550	300	6146.748	13757.59	333.9222	996.2223	8.75E+05	3000	700	300	6406.02	32100.43	208.6663	993.3222	1.54E+06	
70000	3300	600	300	3727.073	16552.65	315.9206	1497.169	1.20E+06	3500	800	300	7339.057	53227.64	203.524	700.0185	2.57E+06	
80000	4000	650	300	8404.882	18467.32	394.7657	1041.412	1.64E+06	4000	950	300	8810.585	83674.39	208.3659	800.0036	4.14E+06	
90000	4100	700	300	29343.93	26875.95	306.203	1350.133	2.71E+06	4800	900	300	8262.683	55779.97	203.9949	960.8919	4.02E+06	
100000	4800	700	300	45816.96	44951.89	257.3342	1924.718	4.37E+06	5200	1100	300	10295.32	94953.75	205.0751	1475.613	6.40E+06	
Average optimum Values forexterior & interior box girders									Cost of concrete & steel								
h	bw	hf	As	Ap	S	Z(x)	Wstr	ost of Con	Cost of Steel								
9.85E+02	3.48E+02	2.76E+02	3.74E+03	5.40E+03	3.58E+02	5.75E+04	3.61E-01	9.34E+03	4.81E+04								
1.15E+03	4.25E+02	2.52E+02	7.48E+03	6.16E+03	3.46E+02	1.48E+05	9.05E-01	2.67E+04	1.22E+05								
1.53E+03	5.00E+02	2.88E+02	5.79E+03	9.62E+03	3.76E+02	2.84E+05	1.72E+00	6.30E+04	2.21E+05								
2.00E+03	5.25E+02	3.00E+02	1.20E+04	1.12E+04	3.34E+02	5.14E+05	3.40E+00	1.15E+05	3.98E+05								
2.40E+03	6.00E+02	3.00E+02	7.56E+03	1.48E+04	2.84E+02	7.69E+05	6.08E+00	1.99E+05	5.70E+05								
2.90E+03	6.25E+02	3.00E+02	6.28E+03	2.29E+04	2.71E+02	1.21E+06	9.25E+00	3.01E+05	9.07E+05								
3.40E+03	7.00E+02	3.00E+02	5.53E+03	3.49E+04	2.60E+02	1.88E+06	1.33E+01	4.60E+05	1.42E+06								
4.00E+03	8.00E+02	3.00E+02	8.61E+03	5.11E+04	3.02E+02	2.89E+06	1.56E+01	7.08E+05	2.18E+06								
4.45E+03	8.00E+02	3.00E+02	1.88E+04	4.13E+04	2.55E+02	3.36E+06	2.30E+01	8.86E+05	2.48E+06								
5.00E+03	9.00E+02	3.00E+02	2.81E+04	7.00E+04	2.31E+02	5.38E+06	3.18E+01	1.24E+06	4.15E+06								

**Optimization output for RC Girders with variable span**

**Optim. Values for RC T-Girders**

Span, L (mm)	Exterior Girder						Interior Girder					
	h	bw	hf	As	S	Opt. Cost	h	bw	hf	As	S	Opt. Cost
	x1	x2	x3	x4	x5	Z(x)	x1	x2	x3	x4	x5	Z(x)
10000	800	369	200	10075.41	449.0458	3.58E+04	800	498	200	13055.24773	334.7253	4.74E+04
20000	1400	339	208	14197.92	449.6289	1.10E+05	1400	393	200	14684.94072	363.6127	1.22E+05
30000	2100	315	203	19548.65	448.4084	2.34E+05	2100	361	221	18606.92714	405.2733	2.42E+05
40000	2798	327	224	27312.25	439.6861	4.34E+05	2900	338	223	23640.61908	420.2813	4.18E+05
50000	3500	417	235	34993.71	449.9574	7.32E+05	3499	408	284	31869.95181	437.2142	6.98E+05
60000	4200	487	283	44998.65	449.9982	1.15E+06	4199	379	300	41767.01116	345.3561	1.09E+06
70000	4896	466	300	57816.71	408.6033	1.65E+06	4877	483	300	51855.84762	372.9205	1.60E+06
80000	5600	604	300	69999.82	449.9959	2.36E+06	5599	484	280	64578.83753	329.6219	2.25E+06
90000	6293	595	294	86060.03	423.8833	3.15E+06	6297	479	297	79544.82431	289.6212	3.07E+06
100000	6993	603	297	103728.3	374.2931	4.16E+06	6991	587	296	94211.93219	315.6275	4.06E+06

Average Values for exterior & interior T-girders						Cost of conc.& steel		
h	bw	hf	As	S	Z(x)	Wstr	Cost of Conc	Cost of Steel
8.00E+02	4.34E+02	2.00E+02	1.16E+04	3.92E+02	4.16E+04	2.71E-01	9.42E+03	3.22E+04
1.40E+03	3.66E+02	2.04E+02	1.44E+04	4.07E+02	1.16E+05	1.30E+00	2.78E+04	8.79E+04
2.10E+03	3.38E+02	2.12E+02	1.91E+04	4.27E+02	2.38E+05	2.91E+00	5.78E+04	1.80E+05
2.85E+03	3.33E+02	2.24E+02	2.55E+04	4.30E+02	4.26E+05	5.33E+00	1.03E+05	3.23E+05
3.50E+03	4.13E+02	2.60E+02	3.34E+04	4.44E+02	7.15E+05	8.03E+00	1.97E+05	5.18E+05
4.20E+03	4.33E+02	2.92E+02	4.34E+04	3.98E+02	1.12E+06	1.30E+01	2.98E+05	8.19E+05
4.89E+03	4.75E+02	3.00E+02	5.48E+04	3.91E+02	1.63E+06	1.80E+01	4.44E+05	1.18E+06
5.60E+03	5.44E+02	2.90E+02	6.73E+04	3.90E+02	2.31E+06	2.38E+01	6.68E+05	1.64E+06
6.30E+03	5.37E+02	2.96E+02	8.28E+04	3.57E+02	3.11E+06	3.30E+01	8.31E+05	2.28E+06
6.99E+03	5.95E+02	2.97E+02	9.90E+04	3.45E+02	4.11E+06	4.22E+01	1.14E+06	2.97E+06

**Optim. Values for RC Box-Girders**

Span, L (mm)	Exterior Girder						Interior Girder					
	h	bw	hf	As	S	Opt. Cost	h	bw	hf	As	S	Opt. Cost
	x1	x2	x3	x4	x5	Z(x)	x1	x2	x3	x4	x5	Z(x)
10000	700	397	200	15095	309	5.59E+04	1000	393	207	12038	432	5.65E+04
20000	1200	398	223	19494	319	1.52E+05	1200	398	200	22708	204	1.86E+05
30000	1799	398	203	24852	370	2.97E+05	1799	400	219	33549	230	3.94E+05
40000	2398	397	254	32692	365	5.16E+05	2600	613	282	50323	450	7.42E+05
50000	3000	400	300	42215	337	8.28E+05	3800	779	300	81986	450	1.51E+06
60000	3.60E+03	400	254	53303	288	1.26E+06	4800	1000	300	116584	305	2.75E+06
70000	4198	486	288	63598	318	1.77E+06	6196	1049	299	150700	397	4.06E+06
80000	4800	596	296	75274	362	2.42E+06	7983	1198	297	210801	346	6.61E+06
90000	5399	656	298	89764	365	3.23E+06	9500	1250	300	264163	241	9.64E+06
100000	5999	753	277	105188	366	4.27E+06	10432	1205	296	273163	346	1.07E+07

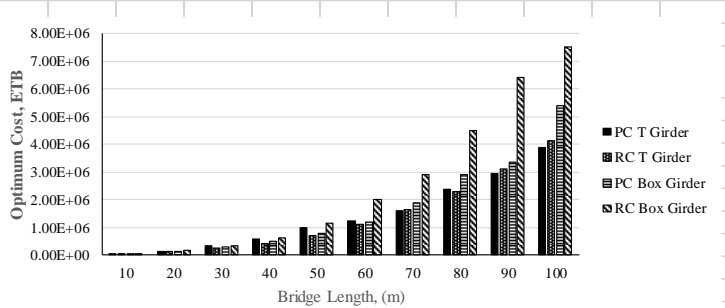
Average Values for exterior & interior box girders						Cost of conc.& steel		
h	bw	hf	As	S	Z(x)	Wstr	Cost of Conc	Cost of Steel
8.50E+02	3.95E+02	2.04E+02	1.36E+04	3.71E+02	5.62E+04	3.11E-01	9.04E+03	4.72E+04
1.20E+03	3.98E+02	2.12E+02	2.11E+04	2.62E+02	1.69E+05	1.34E+00	2.54E+04	1.44E+05
1.80E+03	3.99E+02	2.11E+02	2.92E+04	3.00E+02	3.45E+05	2.82E+00	5.76E+04	2.88E+05
2.50E+03	5.05E+02	2.68E+02	4.15E+04	4.08E+02	6.29E+05	4.01E+00	1.37E+05	4.92E+05
3.40E+03	5.90E+02	3.00E+02	6.21E+04	3.94E+02	1.17E+06	7.22E+00	2.73E+05	8.98E+05
4.20E+03	7.00E+02	2.77E+02	8.49E+04	2.97E+02	2.00E+06	1.44E+01	4.81E+05	1.52E+06
5.20E+03	7.68E+02	2.94E+02	1.07E+05	3.58E+02	2.91E+06	1.74E+01	7.65E+05	2.15E+06
6.39E+03	8.97E+02	2.97E+02	1.43E+05	3.54E+02	4.51E+06	2.49E+01	1.26E+06	3.25E+06
7.45E+03	9.53E+02	2.99E+02	1.77E+05	3.03E+02	6.44E+06	3.82E+01	1.76E+06	4.68E+06
8.22E+03	9.79E+02	2.87E+02	1.89E+05	3.56E+02	7.50E+06	4.00E+01	2.22E+06	5.29E+06

# Optimization of Prestressed Concrete Girders for Bridge Design, Master's Thesis

By Wubishet Jemaneh June, 2018

## 1. Comparison of Girder Cross Sections

Span, L (m)	Optimum Costs, ETB			
	RC T Girder	RC Box Girder	PC T Girder	PC Box Girder
10	41575.1	56214.75	55151.5	57484.4
20	115729	169340	149936	148420
30	237705	345156	318968	284334
40	426042	628953	597500	513706
50	714913.5	1171345.5	986761	769399
60	1117095	2004765	1222780	1207886
70	1626045	2913590	1591015	1882780
80	2308745	4512120	2383755	2888645
90	3109530	6436030	2937510	3363805
100	4111565	7503240	3873375	5384450



## 2. Effect of grades of concrete on optimum cost

### Optim. Values for RC T-Girders and Concrete Comp. Strength

fc' (Mpa)	Exterior Girder						Interior Girder						Z(x)avg.	havg
	h	bw	hf	As	S	Opt. Cost	h	bw	hf	As	S	Opt. Cost		
	x1	x2	x3	x4	x5	Z(x)	x1	x2	x3	x4	x5	Z(x)		
30	3051.00	3.00E+02	2.98E+02	48086.9	239.969	1.92E+05	2911	300	284	46199	200.831	1.14E+14	7.10E+05	2.98E+03
40	2782.00	3.00E+02	3.00E+02	6.04E+04	214.807	1.93E+05	2833	300	292	59944.1	200.068	1.22E+14	7.65E+05	2.81E+03
50	2675.00	3.00E+02	2.86E+02	7.00E+04	201.843	1.99E+05	2776	300	288	69999.3	200	1.29E+14	8.06E+05	2.73E+03
60	2673.00	3.00E+02	2.72E+02	6.99E+04	200.443	2.07E+05	2713	300	283	69993.4	200	1.31E+14	8.18E+05	2.69E+03
70	2690.00	3.00E+02	2.64E+02	6.99E+04	200.156	2.38E+05	2688	300	225	69978.3	200.004	1.48E+14	9.23E+05	2.69E+03

### Optim. Values for RC Box-Girders and Concrete Comp. Strength

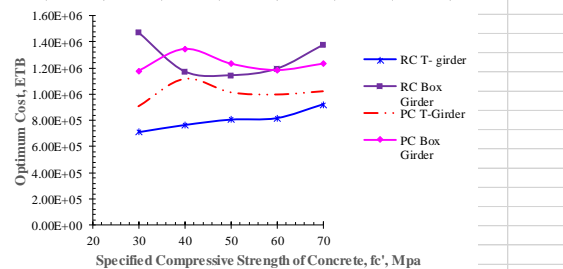
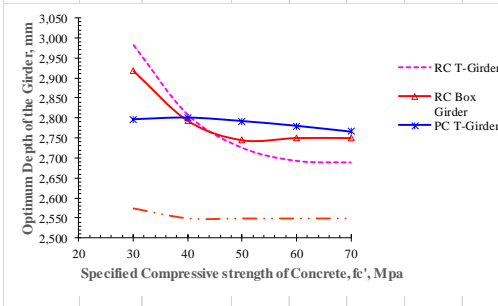
fc' (Mpa)	Exterior Girder						Interior Girder						Z(x)avg.	havg
	h	bw	hf	As	S	Opt. Cost	h	bw	hf	As	S	Opt. Cost		
	x1	x2	x3	x4	x5	Z(x)	x1	x2	x3	x4	x5	Z(x)		
30	2.84E+03	3.00E+02	2.93E+02	46020	213	7.10E+05	3000	974	300	81248	321	4.52E+14	1.47E+06	2.92E+03
40	2.59E+03	3.00E+02	2.93E+02	5.82E+04	235	7.32E+05	2996	668	300	86884	252	3.61E+14	1.18E+06	2.79E+03
50	2.49E+03	3.02E+02	2.79E+02	6.79E+04	221	7.67E+05	3000	589	287	89998	204	3.52E+14	1.15E+06	2.75E+03
60	2.50E+03	3.00E+02	2.84E+02	6.77E+04	208	7.95E+05	3000	589	289	89987	306	3.68E+14	1.20E+06	2.75E+03
70	2.50E+03	3.00E+02	2.69E+02	6.74E+04	202	9.15E+05	3000	589	300	89978	233	4.23E+14	1.38E+06	2.75E+03

### Optim. Values for PC T-Girders and Concrete Comp. Strength

fc' (Mpa)	Exterior Girder							Interior Girder							Z(x)avg.	havg		
	h	bw	hf	As	Ap	S	y	Opt. Cost	h	bw	hf	As	Ap	S			y	Opt. Cost
	x1	x2	x3	x4	x5	x6	x7	Z(x)	x1	x2	x3	x4	x5	x6			x7	Z(x)
30	2.60E+03	7.14E+02	3.00E+02	582.348	69893.6	307.405	1552.12	6.69E+05	2992	1194	297	13578.7	20122.1	271.791	783.662	1.14E+06	9.06E+05	2.80E+03
40	2.60E+03	8.90E+02	3.00E+02	5.18E+02	69889.8	328.636	1508.27	9.50E+05	3000	1199	300	761.727	56133.8	296.748	1200.35	1.29E+06	1.12E+06	2.80E+03
50	2.60E+03	8.53E+02	3.00E+02	5.04E+02	69882.2	297.892	1550.3	9.92E+05	2983	343	300	819.708	22028.8	261.471	995.622	3.95E+05	1.02E+06	2.79E+03
60	3.00E+03	7.95E+02	3.00E+02	5.00E+02	69998.5	200.567	1799.96	1.47E+06	2231	390	300	504.584	24467.1	264.562	965.99	3.48E+05	9.99E+05	2.78E+03
70	2.60E+03	8.75E+02	3.00E+02	5.05E+02	69924.1	240.634	1583.92	1.22E+06	2932	615	300	513.372	69605.1	293.825	1676.94	8.26E+05	1.02E+06	2.77E+03

### Optim. Values for PC Box-Girders and Concrete Comp. Strength

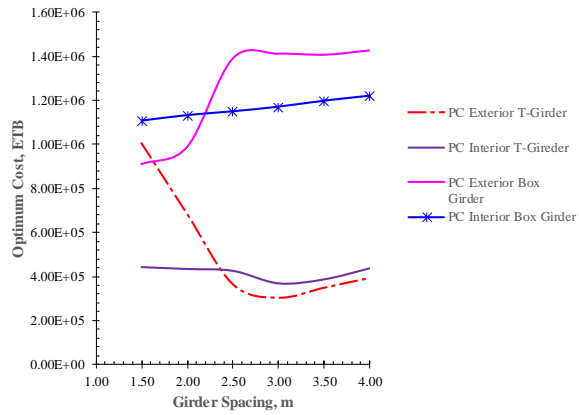
fc' (Mpa)	Exterior Girder							Interior Girder							Z(x)avg.	havg		
	h	bw	hf	As	Ap	S	y	Opt. Cost	h	bw	hf	As	Ap	S			y	Opt. Cost
	x1	x2	x3	x4	x5	x6	x7	Z(x)	x1	x2	x3	x4	x5	x6			x7	Z(x)
30	2.52E+03	1.70E+03	3.00E+02	507.228	34995.5	223.121	1467.29	1.42E+06	2630	1000	300	508.999	21804.1	296.624	769.854	9.31E+05	1.18E+06	2.58E+03
40	2.50E+03	1.76E+03	2.93E+02	2.57E+04	34967.3	205.434	918.899	1.65E+06	2600	1000	300	7277.71	34993.7	271.57	851.576	1.04E+06	1.34E+06	2.55E+03
50	2.50E+03	1.27E+03	3.00E+02	6.93E+02	34980.2	229.54	814.13	1.33E+06	2600	998	300	862.374	31884.6	259.513	884.305	1.14E+06	1.23E+06	2.55E+03
60	2.50E+03	8.17E+02	3.00E+02	5.20E+02	34966.6	250.34	886.657	9.05E+05	2599	845	300	1007.87	34982.1	247.968	925.342	1.02E+06	1.18E+06	2.55E+03
70	2.50E+03	5.67E+02	3.00E+02	5.05E+02	34999.6	269.588	905.651	7.34E+05	2599	882	298	10330.6	34984	272.435	890.678	1.22E+06	1.23E+06	2.55E+03



**3. optimum Girder Spacing**

**Optim. Values for PC T-Girders and Girder Spaci**

Girder Spacing, S (mm)	No. of girdres	Opt. Cost of ext.G, Z(x)	Opt. Cost of int.G, Z(x)
1.500	7	1.00E+06	4.45E+05
2.000	5	6.81E+05	4.36E+05
2.500	4	3.62E+05	4.27E+05
3.000	4	3.02E+05	3.70E+05
3.500	3	3.48E+05	3.89E+05
4.000	3	3.92E+05	4.39E+05



**Optim. Values for PC Box-Girders and Girder Spa**

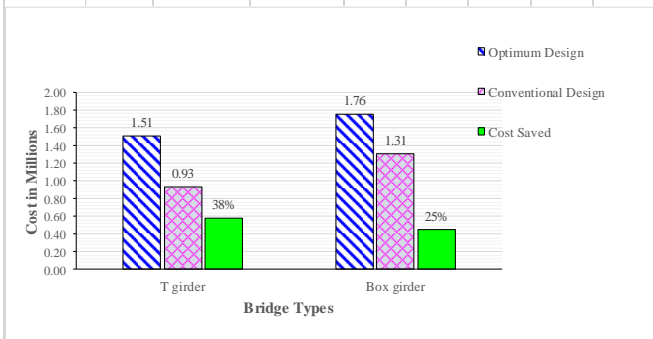
Girder Spacing, S (mm)	No. of girdres	Opt. Cost of ext.G, Z(x)	Opt. Cost of int.G, Z(x)
1.500	7	9.08E+05	1.11E+06
2.000	5	9.88E+05	1.13E+06
2.500	4	1.39E+06	1.15E+06
3.000	4	1.41E+06	1.17E+06
3.500	3	1.41E+06	1.20E+06
4.000	3	1.43E+06	1.22E+06

**4. Comparison of Conventional and Optimum Design**

span, 50m	optimum design cost			conventional design cost			Cost Saving	
	ext. girder	int. girder	avg	ext. girde	int. girde	avg	in Amount	in % tage
	8.36E+05	1.03E+06	9.32E+05	1.51E+06	1.52E+06	1.51E+06	5.80E+05	38%
	1.12E+06	1.50E+06	1.31E+06	1.61E+06	1.91E+06	1.76E+06	4.46E+05	25%
							Avg =	32%

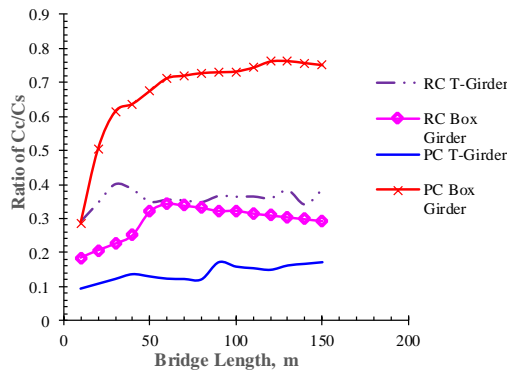
  

Box		T-girder	
ex	int	ext	int
L = 50000	50000	L = 50000	50000
Gs = 2500	2500	Gs = 2500	2500
Ng = 4	4	Ng = 4	4
tb = 140	140	tb = 140	140
tbmin = 38.33	38.33	tbmin = 38.33	38.33
h = 3750	3750	h = 3700	3700
bw = 1350	1350	bw = 1332	1332
Ap = 25000	35000	Ap = 26000	25500
As = 14644	24646	As = 8891	11107
V <sub>Asb</sub> = 6.6E+07	6.6E+07	S = 310	310
S = 300	300	Wstr = 8.65	8.65
Wstr = 9.07	9.07	Vc = 2.44E+11	2.43E+11
Vc = 2.54E+11	2.57E+11	Wsteel = 12.14	13.01
Wsteel = 15.33	19.26	Wprestre = 10.21	10.01
Wprestre = 9.81	13.74		



**6. Materials Cost Ratio**

Bridge Length, m	RC T		RC Box		PC T		PC Box		Ratio of Cc/Cs			
	coc.	steel	Conc.	steel	Conc	steel	Conc	steel	RC T	RC Box	PC T	PC Box
10	10927.12	37582.94514	13732.6808	74150.8	16124	173517	11684.2	40901.7	0.29075	0.1852	0.09292	0.28566
20	29389.99	84907.64748	36506.368	176321	37246	476867	47308.4	94133.2	0.34614	0.20705	0.1075	0.50257
30	66313.88	164850.1729	70319.42864	310636	86508	251650	170436	277584	0.40227	0.22637	0.12207	0.614
40	111071.2	286928.878	134952.613	537534	144820	723317	311565	489782	0.3871	0.25106	0.13665	0.63613
50	163183.7	465827.0993	402380.6149	1253354	202416	735990	484746	768743	0.35031	0.32104	0.12989	0.67398
60	276255.9	776104.8143	763473.9406	2222488	246699	2003526	857097	1204067	0.35595	0.34352	0.12313	0.71184
70	441449.7	1246297.269	1111983.627	3285766	357901	6782194	973223	1824468	0.35421	0.33842	0.12207	0.71945
80	653950.9	1874268.318	2521872.735	7597261	462072	3818298	1726532	2374660	0.34891	0.33194	0.12102	0.72707
90	681580.1	1856260.747	3521852.204	1.1E+07	827040	4780358	2009937	3814306	0.36718	0.32275	0.17301	0.72939
100	975359.1	2672875.043	4273959.565	1.3E+07	1001857	6275311	3134432	4283642	0.36491	0.32326	0.15965	0.73172
110	1348897	3688587.336	5630553.565	5.6E+07	1076789	9810217	4048931	5448118	0.36569	0.31512	0.15481	0.74318
120	1798822	4986434.634	6911708.17	6.9E+07	1454474	9698991	4786667	6287806	0.36074	0.3095	0.14996	0.76126
130	2506380	6526935.514	8581830.425	8.7E+07	1859968	1.1E+07	5701354	7479073	0.38401	0.30388	0.16318	0.76231
140	2881607	8432029.937	10040992.05	1.1E+08	2175177	1.5E+07	7011218	9269258	0.34175	0.29826	0.16812	0.75639
150	3889730	10042013.56	11549468.15	1.2E+08	2434377	1.4E+07	7916309	1.1E+07	0.38735	0.29264	0.17305	0.75048



**7. Optimum Girder Cross sectional dimensions**

Bridge Length, m	RC T Girder			RC Box Girder			PC T Girder			PC Box Girder		
	h/L	b <sub>w</sub> /h	h <sub>f</sub> /h	h/L	b <sub>w</sub> /h	h <sub>f</sub> /h	h/L	b <sub>w</sub> /h	h <sub>f</sub> /h	h/L	b <sub>w</sub> /h	h <sub>f</sub> /h
10	0.080	0.043	0.020	0.085	0.040	0.020	0.109	0.042	0.023	0.099	0.035	0.028
20	0.070	0.018	0.010	0.060	0.020	0.011	0.070	0.024	0.013	0.058	0.021	0.013
30	0.070	0.011	0.007	0.060	0.013	0.007	0.065	0.018	0.010	0.051	0.017	0.010
40	0.071	0.008	0.006	0.062	0.013	0.007	0.059	0.014	0.008	0.050	0.013	0.008
50	0.070	0.008	0.005	0.068	0.012	0.006	0.055	0.016	0.006	0.048	0.012	0.006
60	0.070	0.007	0.005	0.070	0.012	0.005	0.051	0.014	0.005	0.048	0.010	0.005
70	0.070	0.007	0.004	0.074	0.011	0.004	0.049	0.012	0.004	0.049	0.010	0.004
80	0.070	0.007	0.004	0.080	0.011	0.004	0.049	0.013	0.004	0.050	0.010	0.004
90	0.070	0.006	0.003	0.083	0.011	0.003	0.047	0.012	0.003	0.049	0.009	0.003
100	0.070	0.006	0.003	0.082	0.010	0.003	0.047	0.012	0.003	0.050	0.009	0.003
Minimum	0.070	0.006	0.003	0.060	0.010	0.003	0.047	0.012	0.003	0.048	0.009	0.003
Maximum	0.080	0.043	0.020	0.085	0.040	0.020	0.109	0.042	0.023	0.099	0.035	0.028
Mean value	0.071	0.012	0.007	0.072	0.015	0.007	0.060	0.018	0.008	0.055	0.015	0.008

**Summary of Optimum Girder Cross sectional dimension**

Sections	RC T Girder			RC Box Girder			PC T Girder			PC Box Girder		
Ratios	h/L	bw/h	hf/h	h/L	bw/h	hf/h	h/L	bw/h	hf/h	h/L	bw/h	hf/h
Values	0.071	0.012	0.007	0.072	0.015	0.007	0.060	0.018	0.008	0.055	0.015	0.008