

JIMMA UNIVERSITY
JIMMA INSTITUTE OF TECHNOLOGY
FACULTY OF CIVIL AND ENVIRONMENTAL ENGINEERING STRUCTURAL ENGINEERING STREAM

## Weight Optimization of Reinforced Concrete Mat foundation A Case study on Addis Ababa Housing Condominium

A Research Thesis Submitted to School of Graduate Studies of Jimma University in Partial Fulfilment of the Requirement for the Degree of Masters of Science in Structural Engineering

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## DECLARATION

I declare that this research titled "Weight Optimization of Reinforced Concrete Mat foundation a case study on Addis Ababa Housing Condominium" is an authentic record of my work and has not been submitted before for degree to any other university.

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Signature


#### Abstract

With the advancement of construction technology and increase in the land scarcity in the cities due to urbanization, the demand of high-rise buildings is at highest priority in real estate projects. The construction of this buildings takes highest investment economies and the designs must concern not only for the serviceability and durability but also the economic advantages. The design of these reinforced concrete structures is performed by conventional methods following the paradigm "estimate-analysis-check" making the design process time consuming and having a larger design margin making it safe but uneconomical. This leads to the optimization concept for having the best performance within the available resources.

In this research, a case study of uniform flat plate mat foundation of $3 B+G+21$ building of the Addis Ababa Housing Condominium project was carried out mainly because the project is currently under review and will be implemented in large scale soon. The research mainly focused on the minimization of weight of the reinforced concrete Mat foundation while satisfying the limitations and specifications described by ES EN 1992-1-1-2015 design code. Structural optimization problems were formulated with inclusion of weight minimization as objective function, design constraints and design variables. The design variables were taken as the area of the reinforcement and the cross-sections of the structural members. The design constraints were formulated based on the design book of ES EN. For the study, MATLAB optimization toolbox software was used for the optimization process. The case study mat has been designed in SAFE Foundation software which were validated in ABAQUS software for accepting the loads for the optimization process.

The optimization process was carried out and was able to reduce the total structural weight of the mat foundation by $18.7 \%$ while having structural members of $22.24 \%$ and $16.58 \%$ reduction for beam and slab respectively as compared to the original design. In conclusion, the optimized design is more economical than the conventional design due to the weight reduction the consumption of materials has been reduced and it's better to consider a more optimized design by satisfying the safety constraints for future use.


Keywords: Structural optimization, Structural weight, Mat foundation

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## ACRONYMS

| ACI | American Concrete Institute |
| :---: | :---: |
| Asc | Area of Longitudinal compression reinforcement |
| Ast | Area of Longitudinal tensile reinforcement |
| Av | Area of Shear reinforcement |
| Bb | Width of beam |
| CDPM | Concrete Damage Plasticity Model |
| CEA | Complete ABAQUS Environment |
| Ceq | Equality Constraint |
| Db | Effective Depth of beam |
| Ds | Effective Depth of slab |
| ES EN | Ethiopian Standards in Euro Norms |
| FEA | Finite element analysis |
| Lb | Centre to centre length of beam |
| RC | Reinforced concrete |
| S | Spacing of shear reinforcement |
| SLS | Serviceability limit state |
| SSI | Soil-structure interaction |
| Vbc | Volume of concrete in a beam |
| Vbs | Volume of reinforcing steel in a beam |
| Vv | Volume of stirrups in a beam |
| Vsc | Volume of concrete in a slab |
| Vss | Volume of main and secondary reinforcing steel in both direction of slab |
| ULS | Ultimate limit state |

## CHAPTER ONE

## INTRODUCTION

### 1.1 Background of the study

Construction of buildings dates back to the old ancestors of mankind built for different purposes, in recent years the lack of space in major cities tall buildings are in demand for this reason development of new technologies and methodologies must be developed and implemented. The main requirement of present-day engineering designs are safety and economy. Safety is guaranteed by design parameter constraints and the economy can be achieved by minimization of cost functions. The design parameter constraints are fulfilled by following the design codes and standards in this case Ethiopian building codes of standards in Euro-norms and for the economy we will be using optimization methodologies. The most frequently used geotechnical structure in low bearing capacity soils and high-rise building is Mat foundation. According to the standard ES EN 2: Design of Concrete Structure - Part 1 : general rules for building states that structures must satisfy ultimate limit state (ULS) and serviceability limit state (SLS) conditions. Mat foundation must be designed to withstand structural failure (bending failure, shear failure and design constraints of a concrete design code) and geotechnical failure (bearing failure, sliding failure and settlement) and also equilibrium caused by combined failure in ground and structure by overturning due to eccentricity. In this study, structural failure and geotechnical failure by bearing failure has been consider and no equilibrium case since the mat foundation has symmetrical shape which omitted the situation.

Mat foundations are parts of shallow foundation type which are thick concrete slab placed on the ground as the foundation of the structure, used for their best ability to transferring of loads simultaneously to the ground which has a week bearing capacity to support the coming load. Mat foundation will transfer the load uniformly and prevents sudden settlements of the structure.

Recently, various methods and algorithms are being developed for the optimization of structures. Optimization is getting best result under given circumstances to achieve the goal of minimizing effort and maximize desired benefit in this case minimize the over cost and maximize the performance of the mat foundation. For nonlinear problem there are different
solving techniques and for this case optimization of nonlinear programming was done using MATLAB toolbox software.

### 1.2 Statement of the problem

The increasing demand of high rising building due to land scarcity and designing of the structures with the conventional methodology based on the design books and codes that follows the "estimate - analysis - check" view which leads to larger designs only to satisfy the strength and durability rather makes it uneconomical. This design approach increases the focus to stability and being in safe zones rather choosing a better economical sections and approaches. Due to the increase cost of construction materials and economical views, the increase demand for optimized projects of reinforced concrete structures is at highest level.

In this research, the design process is formulated as the minimization of the total weight of the whole structure under consideration under a series of design constraints, which are designed to consider different design criteria. The considered design criteria are from the requirements of newly revised Ethiopian Building Codes of Standard including section capacity (Moment and shear capacity), Minimum and Maximum requirements of reinforcement bar. There are some reasons for choosing weight function optimization rather than cost functions: since the study is planning to reduce the cost of the structure by reducing the dimensions not on material and labour cost since those are varied based on location and time due to inflation is hard to estimate and validate.

### 1.3 Objective of the study

### 1.3.1 General objective

The general objective of this thesis was to optimize and compare design of mat foundation for safe and economical design using MATLAB Toolbox.

### 1.3.2 Specific objective

- To model and study the performance of already designed mat foundation using ABAQUS software
- To optimize weight of the design by reducing the cross-sectional area of mat beam and slab and reinforcement using MATLAB optimization tool box using fmincon
- To compare the optimized design with original weight of concrete and reinforcement


### 1.4 Research questions

The research addressed in this study are following questions:

1. How to optimize the weight of Mat Foundation (concrete and rebar) for satisfaction requirements stated under the design code?
2. What are the percentage difference of the reduced or optimized structure versus the originally designed structure?

### 1.5 Significance of the study

Now days, the rapid growth building construction and need of structural stable and economical designs, Mat foundation are used for high rising buildings and also small buildings with low bearing capacity soils for transferring loads from above uniformly which makes it a core area of study since it is the fundamental part of structural engineering and the size and scale of the projects within the country is boosting with scarce resources and capital optimized structures come in handy. This will make the designed structure economical by minimizing the total material usage as a structure is built under constraints. Simple mathematical expression for objective function and constraints have been developed and solved using MATLAB optimization tool box. Through that process best optimization tool was proposed for design engineers based on the optimum solution obtained from software solutions. Therefore, the study of optimized design of Mat foundation is critical and lay a methodology of optimization techniques for sub-structures and also this material can be used for future students and researchers conducting studies on optimization of Mat foundations and helps the government saving additional costs required on this and similar projects proposed and to be launched soon.

### 1.6 Scope and limitation of the study

The study is an attempt to optimize the uniform flat plate mat Foundation of 3B+G+21 Addis Ababa Housing Condominium Apartment designed in the city where soil test conducted on site having bearing capacity of 550 KPa and the design have been done on conventional methodology for adequate transfer of the lateral load. Also, the research deals with the optimization of mat foundation and limited to studying the lateral loadings and their effect without soil structure interaction and also limited to the weight reduction optimization process by MATLAB Toolbox optimization and the 3D analysis has been carried out by ABAQUS software. The material nonlinearity for concrete and reinforcement grades for yield strength capacity are not design constraints.

## CHAPTER TWO

## RELATED LITERATURE REVIEW

### 2.1 General

In this chapter, development of commonly used optimization problem formulas, previous studies on the concepts of optimization of reinforced concrete structures and review on optimization techniques using MATLAB tool box and also reviews on the concept of Mat foundation design and methodologies. Optimization is a process of finding a minimum or maximum value of a function subjected to constraints helping to find the maximum desirable benefit from the available resources. The basic requirements for an effective structural design are that the response of the structure should be acceptable as per various specifications from design books and limitations. There can be variety of feasible designs but it's essential to choose the best from the candidates. The best designs could be in terms of minimum cost, minimum weight or maximum performance or a combination of those (S. M. Thomas and P. A. G, 2017).

### 2.2 Design of Mat foundation

Mat foundation is a large rigid slab covering the entire area under the structure supporting several columns and walls to transfer the load to the soil strata uniformly (Venkatramaiah, C., 1993). It's preferred for soils with low bearing capacity. Under some conditions, if a spread footing would cover more than half the building area Mat foundation is more economical. There are several types of mat foundations used currently. Some of the common are listed below:
a. Flat plate, uniform mat thickness
b. Flat plate thickened under column
c. Beam and slab, columns are located at the intersection of the beams
d. Flat plates with pedestals
e. Slab with walls as a part of the mat


Figure 2.1 Types of Mat foundation
Now days due to advanced aesthetic design in construction it difficult to provide a regular spacing of columns and isolated footings or combined footings hence the overlapping and failure of soil, its common to design mat foundations which reduces the cost, effort of excavation and prevent sliding of adjacent footing. In the paper analysis and design of raft foundation, superstructural analysis was done in ETABS 13 and foundation was done in SAFE foundation V12. The area of steel obtained for moment obtained from the envelop combination was the same as the strip method and the punching shear ratio is less than 1 also for the deflection there is allowable deflection within the limit making the design safe. (Zia-abe D. S. Punekar et at.,2017)

There are several important structural design parameters to address while modelling and designing of mat foundations those includes dimensions of the mat, soil bearing capacity, and structural loads. By using those parameters feed into the Finite element software analysing all
the elements giving solutions and make sure none of the critical cases have not been missed (David B. Lorenz, 1989).

Mat foundation is a large rigid concrete slab rests directly on soil or rock which covers the entire area under the structure and supports all the columns and the walls. This type of foundation is best suitable when the allowable bearing capacity of the soil pressure is low or the load is very heavy in a case when spread footings cover more than half of the plan area. And it's also a better choice for soil strata which are likely to have considerable differential settlements (Venkatramaiah C., 1993).

Design of Mat foundations are as inverted continuous flat slab floor supported without any upward deflection at the columns and walls. In the analysis of Mat foundations, it is assumed that the soil pressure acting against the slab is uniformly distributed and equal to the total load from all columns. Mat slabs have uniform thickness throughout the entire area for relatively small loads but for larger column loads, the slab under the column is thickened to provide sufficient strength for negative moment and shear (Peck, R.B. et al., 1974).

Mat foundations are reinforced structures used to transfer structural loads to the underlying soil and are well suited in reduction of differential settlement. In the study on analysis of mat foundation finite element has been used for the analysis. The mat foundation was analysed as an inverted flat slab by flat plate analysis and FEA. The result from plate analysis were similar to the FEA. Hence the direct design method analysis overestimates the moment making the sections larger certainly making the design safe when compared to the FEA method but making it an economical. (Rashed C. 2012)

A very commonly used type of foundation this time is Mat foundation having many advantages in adopting this type of foundation which includes the compensation of loadings, utilization of underground space, lowering foundation settlements and increase in safety factor of the bearing capacity for the foundation. In foundation design differential settlement affects the whole structure in worst case it will lead to structural failure threatening the inhibitors within the building. Therefore, designing of foundation needs to follow strict attention and evaluation (CM Ma and Y-Y Chen, 2019).

### 2.3 Nonlinear analysis

Study on the Analysis of Mat Foundation using finite element and direct design method shows that the FE analysis gives a more economical sections rather than the direct method which is based on higher safety standards making it safe but not economical (Rashedul H. Chowdhury et al. 2013).

Finite element analysis using ABAQUS software was performed to investigate the behaviour of post tensioned concrete beams attempting to examine the concrete damage plasticity model in ABAQUS as well as the effect of an external post tensioning steel rod system. In the paper, the finite element analysis has been done with some assumptions including perfect bond of steel and concrete. The simulated tensile deformation result from FE analysis and experimental results are similar to actual crack patterns in tests and analytical responses such as strength and deflections are in good agreement with the measured responses from the experimental testes (Swoo-Heon Lee, et al., 2020).

A novel biaxial constitutive model and a non-linear model using updated algorithm have been implemented in ABAQUS software for simulating the reinforced concrete shear walls. An optimization algorithm based on the discrete Fréchet distance is proposed to quantify and minimize the difference between test results and FE simulation results of the loaddisplacement curves. The comparison shows the proposed model with optimization method predicts high accuracy simulation results (Jia-Ji Wang, et al., 2021).

Research on shear behaviour of reinforced concrete beams done to investigate the effect of the orientation of stirrups on the shear capacity of RC structure as a shear span to depth ratio varies and for the analysis using general purpose finite element package, ABAQUS software. the shear capacity of the RC beam obtained from the numerical analysis gives a good agreement with the experimental result. The results have been compared to analytically calculated values by using shear provisions of ACI 318-14 and EN-2 (Chalachew B. Hunegnawand Temesgen W. Aure, 2021).

### 2.4 Structural optimization

The use modern numerical techniques of optimized design of structures dated back to 1960 . Schmit is the most widely referenced beginning of method in this field. During the decade of 1960s, study of the application of optimization technique has been used for Varity of structures
(Schmit, 1960). At the time the problems where relatively simple but through time the problems become sophisticated.

A review on design optimization of reinforced concrete beams was presented and have been discussed that the optimal design of concrete beam either individually or as part of frame have been addressed using various optimization techniques depending on the formulation of the problems. It was explained that the objective of optimization (minimization of weight, cost) the design variables and constraints vary depending on the problems arise therefore methodologies differ for each problems providing the optimal design (I. Rahmanian, 2014).

Optimization of T-beam have been studied for both minimization of cost and weight under geometrical constraints, shear and moment capacity for deflection have been considered. Optimal solutions for minimum cost and weight were compared showing results were affected significantly by the optimal size. Cost is significantly affected since the size reduction leads to lesser material uses making is economical too (Fedghouche Ferhat, 2018).

Optimization is a methodology of getting the best result under given circumstances to achieve the goals of minimizing effort and maximize desired benefit in this case minimize the over cost and maximize the performance of the mat foundation. For nonlinear problem there are different solving techniques, since there is no define way to solve all problems, in this problem from mathematical programming or optimization technique, nonlinear programming will be used using MATLAB software using optimization tool box for solving.

Optimization can include a wide range of problem with the aim of searching for certain optimality. Therefore, there are many ways of classification of optimization problems and techniques differ from problem to problem. A unified approach is not possible and complexity depends on the function forms of its objective functions and constraints.

Mathematically it's possible to write most optimization problems in the generic forms as:
Minimize

$$
\begin{equation*}
\mathrm{f}_{\mathrm{i}}(\mathrm{x}), \quad(\mathrm{i}=1,2, \ldots, \mathrm{M}), \tag{2.1}
\end{equation*}
$$

Subjected to $\quad \operatorname{jj}(\mathrm{x})=0 \quad(\mathrm{j}=1,2, \ldots, \mathrm{~J})$,

$$
\begin{aligned}
& \Psi \mathrm{k}(\mathrm{x}) \leq 0, \quad(\mathrm{k}=1,2, \ldots, \mathrm{~K}) \\
& \mathrm{X}=\left(\mathrm{X}_{1}, \mathrm{X}_{2}, \ldots \mathrm{X}_{\mathrm{n}}\right)^{\mathrm{T}}
\end{aligned}
$$

Where: - $\mathrm{fi}(\mathrm{x}), \phi \mathrm{j}(\mathrm{x})$ and $\Psi \mathrm{k}(\mathrm{x})$ are functions of the design vector.

- Xi of X are called design or decision variables, and they can be real continuous, discrete or the mixed of these two.
- The functions fi(x) where $\mathrm{i}=1,2, \ldots, \mathrm{M}$ are called the objective functions (cost function/ energy function), in the case of $\mathrm{M}=1$, there is only a single objective and in the case of $\mathrm{M}>1$, there are multi objectives.
- The equalities for $\phi \mathrm{j}$ and inequalities of $\Psi \mathrm{k}$ are called constraints. It is worth pointing out that we can also write the inequalities in the other way $\geq 0$ for $\Psi \mathrm{k}$, and we can also formulate the objectives as maximization problem.


### 2.4.1 Design variables

Design variable also called control variable is under control of the designer and can impact the solution of the optimization problem. Combination of different variables represents different designs. The main aim of the design optimization is to find the best combinations of design variables that can give best solution for the intended objective and also maintains the required constraints. Design variables have lower limit and upper limit specifying the minimum value and maximum values.

### 2.4.2 Objective function

Objective functions define the objective of the optimization process. It's a numeric value that is formulated from sets of design variables whose values is either be minimized or maximized over the set of alternatives/ constraints. In our case minimization of weight or size of the mat foundation.

### 2.4.3 Design constraints

Design constraints are values limiting the function which is either equality or inequality mathematical forms. This can be the resources available, time available or permeable limits of such design within design codes.

### 2.5 Classification of Optimization

The classification of optimization is not well established and there are some confusions in literature especially about the use of terminologies. Here we can use the most widely used terminologies. If we try to classify optimization problems according to the number of objectives, then there are two categories: single objective $M=1$ and multi-objective $M>1$. Multi-objective optimization is also referred to as multicriteria or even multi-attributes
optimization in literature. Most real-world optimization problems are multi-objective. Similarly, we can classify optimization as constrained and unconstrained. If there are no constraints at all $\mathrm{J}=\mathrm{K}=0$ it called an unconstraint optimization problem. And if there is inequality in either or both $\mathrm{J} \& \mathrm{~K}$ the optimization problem is called inequality constraint and if the if $\mathrm{J}=0 \& \mathrm{~K} \geq 1$ the problem is called equality constraint. Here is a diagram of the classification of optimization problem (Xin-She Yang, 2010).


Figure 2.2 Classification of Optimization problems
Generally, the optimization problem focusses on weight reduction it's a single objective problem with different inequality constraints depends on the values for different variable constraints bound under the code of standards. Therefore, for this research study the optimization problem is single objective inequality constrained nonlinear optimization problem.

### 2.6 Optimization Techniques

Optimization problems are defined by its objective functions and their constraints, a suitable method is chosen to find the best solution for the problems arise and wide ranges of method are available as per the design functional forms. If the functional forms are all linear it can be
taken as a linear optimization problem where as if any of the constraints or objectives are nonlinear the optimization technique will be nonlinear optimization problem. In Reinforced concrete structure design nonlinearity is common due to the objective functions and constraints are nonlinear. Therefore, nonlinear optimization method is taken for solving the RC problems.

### 2.6.1 Optimization Tool Box in MATLAB

MATLAB's Optimization Toolbox is an integrated part in MATLAB software which provides functions for solving linear and nonlinear programming, mixed-integer linear programming, quadratic programming, and nonlinear least square problems (Optimization toolbox user guide, 2010).

Based on types of optimization problems, there are several functions of MATLAB Optimization Toolbox. Those are:

Linear and Quadratic Minimization problems.

- linprog - Linear programming
- quadprog - Quadratic programming.

Nonlinear zero finding (equation solving).

- fzero - Scalar nonlinear zero finding
- fsolve - Nonlinear system of equations solve (function solve)

Linear least squares (of matrix problems).

- lsqlin - Linear least squares with linear constraints.
- lsqnonneg - Linear least squares with nonnegativity.

Nonlinear minimization of functions

- fminbnd - Scalar bounded nonlinear function minimization.
- fmincon - Multidimensional constrained nonlinear minimization.
- fminsearch - Multidimensional unconstrained nonlinear minimization, by Nelder-Mead direct search method.
- fminunc - Multidimensional unconstrained nonlinear minimization
- fseminf - Multidimensional constrained minimization, semi-infinite constraints

Nonlinear minimization of multi-objective functions

- fgoalattain - Multidimensional goal attainment optimization
- fminimax - Multidimensional minimax optimization

For the purpose of this research, the nonlinear minimization of function, fmincon was used to solve the formulated optimization problems.

## CHAPTER THREE

## RESEARCH METHODOLOGY

### 3.1 General

Structural optimization is a procedure of improving a preliminary design done by a designer according to the design codes and design criteria stated (Structural Strength, stability durability and comfort). While improving the design, limitations are being considered for each elements cross section and capacity for performing the intended purpose while reducing the unnecessary parts for better economical usages. There are plenty numbers of optimization techniques and software for performing optimization and are different for different types of problems faced. Those includes Optimization Toolbox in MATLAB, Genetic algorithms, Simulated annealing, Particle swarm optimization, Ant colony optimization, Excel Solver Fuzzy optimization, and Neural-network-based methods. The optimization techniques in general enable designer to find the best design for the structures under consideration.

To achieve the objective of the study, the methodology describes in brief on how to execute the works, what tools are being used and the methods to execute the task for the mat foundation which is a reinforced concrete structure. The use of optimization technique was based on the nature of optimization problems and depends on the mathematical structure of the problem. Optimization techniques play an important role in structural design, the very purpose of which is to find the best ways so that a designer or a decision maker can derive a maximum benefit from the available resources. The code of practice applied was ES EN 1992-1-1:2015, Structural use of concrete in buildings.


Figure 3.1 Optimization methodology flow chart

### 3.2 Formulation of the optimization problem

Identification of design variables for structural systems, an objective function that must be minimized, and design restrictions that must be placed on the systems are all required for the formulation of the optimum design issue (A. A. A. Aga and F. M. Adam, 2015). The general form of optimization problem is as follows:

- Constant parameters
- Objective function
- Design variables
- Design constraints


### 3.2.1 Constant parameters

In this study, mat foundation system is used for this specific project. The mat foundation provides adequate bearing capacity and to transfer the loads over large foot print of the building area. The mat foundation reduces differential settlement and is preferable for compressible soils. The foundation was dimensioned in such a way that the allowable soil bearing capacity is not exceeded. The allowable bearing capacity of the soil for economical shallow foundation system is 550 kpa with allowable settlement of 50 mm . This bearing capacity of soil should be achieved by replacing and properly compacting suitable selected soil to appropriate depth soil under the foundation in case of a soil with a lower bearing capacity than 550kpa. The concrete grade of the concrete is C25/30 type of concrete of cylindrical strength of 25 MPa or a cubic strength of 30 MPa and the steel yield strength of 466 MPa which satisfies the requirement in the Ethiopian Building Code of Standards ES EN-1-1:2015 valid yield strength range from 400 to 600 MPa .

### 3.2.2 Design variables

An important first step in the formulation of an optimization problem is to identify the design variables. For the present formulation, cross sectional dimensions and reinforcement areas (tensile, compressive and shear reinforcement) for beams and slabs are taken as design variables. Specifically, for beams there exist six design variables: the width, Bb , the effective depth, db, the longitudinal tensile reinforcing steel area, Ast, the longitudinal compression reinforcing steel area, Asc, shear reinforcement area, Av. and spacing of shear reinforcements, Sv. For slab, there exist three design variables: effective depth of slab, ds, area of reinforcement
(longitudinal and lateral reinforcements in both directions) and spacing of reinforcement in both directions.

### 3.2.3 Design variable boundaries

In designing an optimization methodology, it must be clearly stated for the minimum and maximum boundaries of each design variables to satisfy their limits under the design codes and considerations. The upper and lower bounds state the feasible solution ranges to consider for the optimum design needed. The following bounds are considered in the modelling of the equation for the optimization problem in this research.

Depth - $\mathrm{D}_{\min } \leq \mathrm{D} \leq \mathrm{D}_{\max }$ for overall depth of the Beams and the Mat slab
Width $-\mathrm{B}_{\min } \leq \mathrm{B} \leq \mathrm{B}_{\max }$ for overall depth of the Beams
Reinforcement $-\mathrm{As}_{\min } \leq \mathrm{As} \leq \mathrm{As}_{\max }$ for main tensile reinforcement for the Beam and Mat slab
Shear Rebar- $\quad \mathrm{As} \mathrm{min}_{\min } \leq \mathrm{As} \leq \mathrm{As}_{\max }$ for main tensile \& compressive reinforcement for the Beam

Spacing of shear reinforcement $-S_{\min } \leq S \leq S_{\max }$ for spacing of shear reinforcement

### 3.2.3.1 Minimum and Maximum Cross- sectional Dimensions of Structural Elements

Minimum width of beams should not be greater than minimum width of column for the safe transfer of the load. According to ES EN 1992-1-1:2015, section 5.3.1, the width of primary seismic beams shall be not less than 200 mm and the Maximum allowable width of the beam is limited to the thickness of the beam section $b_{\max }=h$. In general, in rectangular reinforced concrete structure beams the ratio of overall depth ' D ' to width ' b ' ranges from 1.5 to 2 and for heavy loads it can range up to 3. In most cases beam sections are restricted by the Architects for aesthetics purpose for super structure, the substructures are not restricted by the architect since they don't disturb the aesthetics and the structural engineer is the one who is responsible for the cross-sectional area needed to provide adequate strength and stability to transfer and support the upcoming loads from superstructure and shall design it to withstand the moments and deflections.
$D_{\text {min }}=\mathrm{d}+\phi_{\text {min }} / 2+\phi_{\mathrm{t}}+$ concrete cover
d = effective span/20 for continuous beams of highly stressed concrete (ES EN 1992-1-1:2013, section 7.4).

The maximum depth and width of the beam dependent on the designer whether it supports the upcoming load.

### 3.2.4 Objective Function

The main objective of this optimization process is to design a structure capable of supporting the applied loads while having a minimum possible size to save economy within the design constraints stated under the design codes. In this paper, the objective function of the optimization is structural weight function and it is expressed in terms of beam and mat slab weight. The total weight of reinforced concrete building can be expressed as:

Minimize $\mathrm{W}=\mathrm{W}_{\text {beam }}+\mathrm{W}_{\text {mat slab }}$
Where $\mathrm{W}_{\text {beam }}=$ Weight of beam
$\mathrm{W}_{\text {mat slab }}=$ Weight of mat slab
These weights can be calculated in the following formulations as:
$\mathrm{W}_{\text {beam }}=\sum_{i=1}^{N s}[(\gamma c(\mathrm{Vbc}-\mathrm{Vbs}-\mathrm{Vv})+\gamma s(V b s+V v))]$
$\mathrm{W}_{\text {mat slab }}=[\gamma c(V s c-V s s)+\gamma s V s s]$
Where
$\gamma \mathrm{c}, \gamma \mathrm{s}$ : are unit weight of concrete and steel respectively
Vbc: volume of concrete in a beam

Vbs: volume of reinforcing steel in a beam
Vv: volume of stirrups in a beam
Vsc: volume of concrete in a slab

Vss: volume of main and secondary reinforcing steel in both direction of slab

$$
\mathrm{V}_{\mathrm{bc}}=\mathrm{A}_{\mathrm{gb}} \mathrm{~L}_{\mathrm{b}}
$$

Where $\mathrm{Agb}=$ gross cross-sectional area of beam \&

$$
\begin{array}{r}
\mathrm{Lb}=\text { Centre to centre length of beam } \\
\mathrm{V}_{\mathrm{bc}}=\mathrm{B}_{\mathrm{b}} \mathrm{D}_{\mathrm{b}} \mathrm{~L}_{\mathrm{b}}=\mathrm{B}_{\mathrm{b}}\left(\mathrm{~d}_{\mathrm{b}}+\mathrm{d}^{\prime}\right) \mathrm{L}_{\mathrm{b}} \ldots \ldots \ldots \ldots \ldots \ldots . . \tag{3.4}
\end{array}
$$

Where $\mathrm{B}_{\mathrm{b}}$ and $\mathrm{d}_{\mathrm{b}}$ are width and effective depth of beam and $\mathrm{d}^{\prime}$ is concrete cover (to centre of reinforcing steel bars)

$$
\mathrm{V}_{\mathrm{bs}}=\mathrm{As} \mathrm{~L}_{\mathrm{bbars}}
$$

Where As = cross-sectional area of longitudinal bars include tension and compression steel.
Lbbars $=$ length of beam longitudinal reinforcing steel bars.
Since As= Ast +Asc where Ast and Asc areas of tensile and compressive steel respectively, Vbs can be rewritten as:

$$
\begin{equation*}
\text { Vbs }=(\text { Ast }+ \text { Asc }) \text { Lbbars } \tag{3.5}
\end{equation*}
$$

$\mathrm{Vv}=\mathrm{Av} \operatorname{Lv} \mathrm{ns}$

Where $\quad \mathrm{Av}=$ cross-sectional area of bars used for stirrups.
$\mathrm{Lv}=$ length of one stirrup.
$\mathrm{ns}=$ number of stirrups in one beam.
$\mathrm{Lv}=2\left(\mathrm{Bb}+\mathrm{db}+\mathrm{d}^{\prime}\right)-8\left(\mathrm{~d}^{\prime}-\Phi / 2-\phi \mathrm{t}\right)$,
$\Phi=$ diameter of longitudinal bar and $\phi t=$ diameter of stirrups and $\Phi=(4 / \pi)^{1 / 2} \mathrm{As}^{1 / 2}=1.128 \mathrm{As}^{1 / 2}$, $\phi t=1.128 \mathrm{Av}^{1 / 2}$

$$
\mathrm{Ns}=\frac{\text { length of beam }}{\text { spacing }}+1
$$

$\mathrm{Vv}=\mathrm{Av}\left[2\left(\mathrm{Bb}+\mathrm{db}+\mathrm{d}^{\prime}\right)-8\left(\mathrm{~d}^{\prime}-\Phi / 2-\phi \mathrm{t}\right)\right]\left(\frac{\text { length of beam }}{\text { spacing }}+1\right)$
$\mathrm{Vv}=\mathrm{Av}\left[2\left(\mathrm{Bb}+\mathrm{db}+\mathrm{d}^{\prime}\right)-8\left(\mathrm{~d}^{\prime}-0.564 \mathrm{As} 1 / 2-1.128 \mathrm{Av} 1 / 2\right)\right]\left(\frac{\text { length of beam }}{\text { spacing }}+1\right)$
$\mathrm{Vsc}=\mathrm{LWh}=\mathrm{LW}\left(\mathrm{ds}+\mathrm{d}^{\prime}\right)$
Where L and W are clear span length and clear span width of slab respectively
$\mathrm{h}=$ overall thickness of slab=effective depth and $\mathrm{d}^{\prime}=$ concrete cover

$$
\begin{equation*}
\mathrm{Vss}=\operatorname{AsL}\left(\frac{\mathrm{L} 2}{s}+\mathrm{L}\right)+\mathrm{A}_{\mathrm{ss}} \mathrm{~W}\left(\frac{\mathrm{~W} 2}{\mathrm{~S}}+\mathrm{W}\right) . \tag{3.8}
\end{equation*}
$$

Where ASL = cross section area of reinforcement bars parallel to span length of slab
ASW $=$ cross section area of reinforcement bars parallel to span width of slab

### 3.2.5 Design Constraints

Constraints are the functional and structural requirements of the structural optimization expressed as equality or inequality equations. For the optimization problem the design constraints are of two types: Structural optimization such as code requirements and serviceability criteria and size limitations. In our case, the structural constraints are in accordance with ES EN 1992-1-1:2015 code for Mat slab constraints and Beam constraints categorized as geometric constraints. The mathematical expression contains only inequality constraints as in the design book stated the lower limit and upper limits ranges where equality constraints are not usually found in the case of structural optimizations.

### 3.2.5.1 Beam Constraints

For the optimal design problem of beams, the dimensions taken as design variables are width of beam $\left(B_{b}\right)$, effective depth of beam $\left(d_{b}\right)$ and area of tensile and compressive reinforcement (Ast) \& (Asc) respectively and area of shear reinforcement and spacing of reinforcements (Av) \& (Sv).

## i. Flexural Capacity

The applied bending moment shall be lower than the moment resistance capacity of the cross section. The applied bending moment includes the self-weight of the section which imposed a nonlinear constraint to the optimization. In this study the Beams are all double reinforced rectangular sections.

## Double reinforced rectangular sections

Double reinforced sections are those having reinforcement in the tensile and compression zones. If the required area of tension steel exceeds the maximum area of recommended by the code the compression reinforcement is provided. The compression reinforcement also reduces the long-term deflection and beam ductility. The ultimate limit state is shown for the rectangular section with compression reinforcement below.


Figure 3.2 Double reinforced concrete cross section and resistive forces
From the section property by taking moments about centre of tensile reinforcement:
Muc $=\mathrm{Fc}(\mathrm{db}-\mathrm{a} / 2)+\mathrm{Fsc}\left(\mathrm{db}-\mathrm{d}^{\prime}\right), \quad \mathrm{Fc}=0.567 \mathrm{fck} \mathrm{aBb}, \mathrm{Fsc}=0.87 \mathrm{fykAsc}$
$\mathrm{a}=0.8 \mathrm{c}$, Asc is area of compression reinforcements
a is the depth of stress block and c is the depth of neutral axis from top outer most surface of beam. To ensure that all beams have the desirable characteristics of visible warning, if failure is sudden as well as reasonable ductility at failure, it is recommended that depth of neutral axis should be limited.

In ES EN 1992-1-1:2015 $\frac{c}{d} \leq 0.8(\delta-0.44)$, where $\delta=\%$ moment redistribution,
$=\frac{\text { Moment after redistribution }}{\text { Original moment }}$, when no moment is redistributed, $\delta=1$
In such case, $\frac{c}{d}=0.45$, or $\mathrm{c}=0.45 \mathrm{~d}, \mathrm{a}=0.8 * 0.45 \mathrm{~d}=0.36 \mathrm{~d}$
$\mathrm{Mu}=0.1674 \mathrm{fckBbdb} 2+0.87 \mathrm{fyk}$ Asc (db- d')
All beams are designed to ensure that the moment produced by factored loads $\mathrm{M}_{\mathrm{d}}$ does not exceed the available flexural design strength $M u$ of the cross section at any point along the length of the beam.
$\mathrm{Md} \leq \mathrm{Mu}=0.1674 \mathrm{fckBbdb} 2+0.87 \mathrm{fyk}$ Asc (db- d')
Md - 0.1674fckBbdb2-0.87fykAsc (db- d') $\leq 0$

## ii. Shear Strength Requirement

For reinforced concrete members with vertical shear reinforcement, the shear resistance, VRd,s should be taken to be the lesser, either (ES EN 1992-1-1-2015 section 6.2.3).

$$
\mathrm{VRd} ; \mathrm{s}=\frac{A v}{s} \mathrm{zfyd} \cot \theta \quad \text { or } \quad \text { VRd } ; \max =\alpha c b z v f c d ~ /(\cot \theta+\tan \theta)
$$

where VRd,s is the design value of the shear force that can be sustained by the yielding shear reinforcement; VRd, max is the design value of the maximum shear force that can be sustained by the member, limited by compression strut crushing; VRd, max is the design value of the maximum shear force that can be sustained by the member, limited by crushing of the compression struts; VRd, max is the design value of the maximum shear force that can be sustained by the member, limited by fyd is the yield strength of the shear reinforcement; is the angle of the inclined struts; $b$ is the width of the member; fcd is the design compressive cylinder strength of concrete after 28 days; and c is a coefficient that accounts for the effect of normal stresses on the shear strength. The recommended value of $\alpha \mathrm{c}$ is as follows:

Table 3.1 Recommended value of $\alpha c$

| 1 | for non-pre stressed structures |
| :---: | :---: |
| $(1+\sigma \mathrm{cp} / \mathrm{fcd})$ | for $0<\sigma c \mathrm{c} \leq 0.25 \mathrm{fcd}$ |
| 1.25 | for $0.25 \mathrm{fcd}<\sigma \mathrm{cp} \leq 0.5 \mathrm{fcd}$ |
| $2.5(1-\sigma c p / \mathrm{fcd})$ | for $0.5 \mathrm{fcd}<\sigma c \mathrm{c}<1.0 \mathrm{fcd}$ |

$\sigma c p$ is the mean compressive stress in the concrete caused by the design axial force. v is a coefficient that accounts for the rise in fragility and decrease in shear transmission by aggregate interlock as compressive concrete strength increases. It is taken to be 0.6 for fck $\leq 60 \mathrm{MPa}$. The recommended limiting values for $\cot \theta$ are given by $1 \leq \cot \theta \leq 2.5$ and $\tan \theta$ is zero for vertical shear reinforcement. For the purpose of this study, $\alpha \mathrm{c}=1$ as the building under consideration is non-pre-stressed, $\mathrm{v}=0.6$ since fck considered is less than 60 Mpa and $\cot \theta=1$ which is the initial value. Therefore the maximum shear resistance of beam member, VRd;max is given by:

VRd;max $=\alpha c b z v f c d=B b * 0.9 \mathrm{db} * 0.6 * f c d=0.54 \mathrm{Bbdbfcd}$
VRd; $\mathrm{s}=\frac{A v}{s} \mathrm{zfyd}=\frac{A v}{s} 0.9 \mathrm{dfyd}$

Factored design shear force of member must be lesser of shear resistance of the sections
$\mathrm{Vd} \leq \mathrm{VRd} ; \max =0.54 \mathrm{~B}_{\mathrm{b}} \mathrm{d}_{\mathrm{b}} \mathrm{fcd}$
Vd - $0.54 \mathrm{Bbdbfcd} \leq 0$
$\mathrm{Vd}-\frac{A v}{s} 0.9 \mathrm{dfyd} \leq 0$

## iii. Minimum and Maximum limits of Area of reinforcement

According to ES EN 1992-1-1:2015, section 9.2 for longitudinal reinforcement minimum reinforcement area for tension reinforcement should not be taken as less than $A_{s, \text { min }}$
$A s, \min =0.26 \frac{f c t m}{f y k} \mathrm{~B}_{\mathrm{b}} \mathrm{d}_{\mathrm{d}} \quad$ But not less than $0.0013 \mathrm{~B}_{\mathrm{b}} \mathrm{d}_{\mathrm{b}}$
The cross-sectional area of tension or compression reinforcement should not exceed 0.04 Ac outside lap locations.

Therefore, for the minimum tensile reinforcement:
$0.26 \frac{f c t m}{f y k} \mathrm{~B}_{\mathrm{b}} \mathrm{d}_{\mathrm{d}}-$ Ast $\leq 0$ or $0.0013 \mathrm{~B}_{\mathrm{b}} \mathrm{d}_{\mathrm{b}}-\mathrm{Ast} \leq 0$
For minimum compression reinforcement:
$0.26 \frac{f c t m}{f y k} \mathrm{~B}_{\mathrm{b}} \mathrm{d}_{\mathrm{d}}-\mathrm{Act} \leq 0$ or $0.0013 \mathrm{~B}_{\mathrm{b}} \mathrm{d}_{\mathrm{b}}-\mathrm{Act} \leq 0$.
For maximum reinforcing steel area:
As, $\max =0.04 \mathrm{Ac}=0.04 \mathrm{Bb}\left(\mathrm{db}+\mathrm{d}^{\prime}\right)$
For tension reinforcement,
Ast $-0.04 \mathrm{Bb}\left(\mathrm{db}+\mathrm{d}^{\prime}\right) \leq 0$
For compression steel,
Asc -0.04Bb $\left(\mathrm{db}^{+}+\mathrm{d}^{\prime}\right) \leq 0$

## iv. Shear Reinforcement Spacing Constraints

The transverse spacing of the legs in a series of shear links should not exceed St,max
$\mathrm{St}, \max =0.75 \mathrm{~d} \leq 600 \mathrm{~mm}$
$S v-0.75 \mathrm{~d} \leq 0$.

$$
\begin{equation*}
\mathrm{Sv}-600 \mathrm{~mm} \leq 0 \tag{3.16}
\end{equation*}
$$

### 3.2.5.2 Mat Slab Constraints

## i. Flexural Resistance

The slab of the case study building in this research is a two-way slab since the ratio of longer side to shorter side is less than two. Bending will occur in two directions in a dish-like shape in the case of two-way slabs. As a result, for two-way slabs, the reinforcement against bending is assessed in both short and long directions. A rectangular stress distribution is shown in figure below:


Figure 3.3 Rectangular Stress Distribution
The factor $\lambda$ defining the effective height of the compression zone and the factor $\eta$ defining the effective strength and the values:
$\lambda=0.8, \eta=1.0 \quad$ for fck $\leq 50 \mathrm{MPa} \&$
$\lambda=0.8-($ fck -50$) / 400, \eta=1.0-($ fck -50$) / 200 \quad$ for $50<\mathrm{fck} \leq 90 \mathrm{MPa}$.
Therefore, for this study since the fck $\leq 50 \mathrm{MPa}$ we take $\lambda=0.8$ and $\eta=1.0$

## ii. Bending Moment Resistance Capacity Along Longer Direction

From the section properties shown above taking moments about Centre of compressive steel, moment resistance of the section is given by:

$$
\mathrm{Mul}=\mathrm{Fs}(\mathrm{~d}-\lambda \mathrm{x} / 2)=0.87 \mathrm{fykAsl}\left(\mathrm{~d}-\frac{0.87 f y k A s l}{1.6 f c d b}\right)
$$

Where $b$ is per meter length and Asl is area of reinforcement along longer direction.
Moment due to external actions, Medl which is along the longer side should not be greater than the section capacity, Mul.

$$
\begin{array}{r}
\mathrm{M}_{\mathrm{edl}} \leq \mathrm{Mul}=\mathrm{Fs}(\mathrm{~d}-\lambda \mathrm{x} / 2)=0.87 \mathrm{fykAsl}\left(\mathrm{~d}-\frac{0.87 f y k A s l}{1.6 f c d b}\right) \\
\mathrm{M}_{\mathrm{edl}}-0.87 \mathrm{fykAsl}\left(\mathrm{~d}-\frac{0.87 f y k A s l}{1.6 f c d b}\right) \leq 0 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \tag{3.17}
\end{array}
$$

## iii. Bending Moment Resistance Capacity Along Shorter Direction

Again, from the section properties shown above taking moments about Centre of compressive steel, moment resistance of the section is given by:

$$
\text { Mus }=\text { Fs }(\mathrm{d}-\lambda \mathrm{x} / 2)=0.87 \mathrm{fykAss}\left(\mathrm{~d}-\frac{0.87 f y \mathrm{fAss}}{1.6 f c d b}\right)
$$

Where $b$ is per meter length and Ass is area of reinforcement along shorter direction.
Moment due to external actions, Meds which is along the longer side should not be greater than the section capacity, Mus.

$$
\begin{array}{r}
\mathrm{M}_{\mathrm{eds}} \leq \mathrm{Mus}=\mathrm{Fs}(\mathrm{~d}-\lambda \mathrm{x} / 2)=0.87 \mathrm{fykAss}\left(\mathrm{~d}-\frac{0.87 f y k A s s}{1.6 f c d b}\right) \\
\mathrm{M}_{\mathrm{eds}}-0.87 \mathrm{fykAss}\left(\mathrm{~d}-\frac{0.87 f y k A s s}{1.6 f c d b}\right) \leq 0 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \tag{3.18}
\end{array}
$$

## iv. Minimum And Maximum Area of Reinforcing Steel Constraint

Minimum areas of steel reinforcement must be provided to control crack. The provision of minimum area ensures that the steel reinforcement does not yield when the concrete in the tension zone cracks with a sudden transfer of stress to the reinforcement. The area of reinforcement in primary direction should not be less than $0.26(f \mathrm{ftm} / \mathrm{fyk})$ bds or 0.0013 bds where $b$ is width per meter or length per meter and ds is effective depth of slab.
$0.26 \frac{f c t m}{f y k}$ bd - Ass $\leq 0$ or $0.0013 \mathrm{bd}-$ Ass $\leq 0$.
For the maximum areas of reinforcing steel constraint:
As -0.04B $\left(\mathrm{ds}^{2}+\mathrm{d}^{\prime}\right) \leq 0$.

## v. Reinforcement spacing

The spacing of bars should not exceed Smax,slabs.
The recommended value of Smax,slabs. is:
For the principal reinforcement, $3 h \leq 400 \mathrm{~mm}$, where $h$ is the total depth of the slab;

Based on the mentioned specification for the maximum spacing of bars, constraints on spacing can be set out as shown below in equations 3.21 and 3.22.
$S-3 h \leq 0, S-3\left(d+d^{\prime}\right) \leq 0$
$S-400 \leq 0$

### 3.3 Description of the case study

In this case study a Mat foundation have been selected for optimization which is taken from a structural design for Addis Ababa Housing of 40/60 condominium which is a 3B+G+21 building designed for residential purpose only to be built in Addis Ababa. The super structural design has been made using ETABS and the sub-structure have been designed using SAFE Foundation software's. From the ETABS analysis the final factored loads from the base of the column have been taken to analysis on the SAFE Foundation software. The characteristic strength of the concrete is taken as $\mathrm{C} 25 / 30$ with a cubic strength of 30MPa and yield strength of main bars are taken to be 466 MPa for all sections of the slab and the beams.

Table 3.2 Case study Mat slab dimension and details

| Parameters | Values |
| :---: | :---: |
| Plan dimension | $47.6 * 30.1 \mathrm{~m}$ |
| Beam size (width*depth) | $1 \mathrm{~m}^{*} 2.4 \mathrm{~m}$ |
| Mat slab thickness | 1.2 m |



Figure 3.4 Plan view of the case study mat slab

### 3.3.1 Design strength of concrete and reinforced steel

## Concrete

Design of reinforced concrete in Eurocode2 is based on the characteristic cylindrical strength rather than cubic strength, the value of the design compressive strength of concrete is defined as:
$\mathrm{fcd}=\alpha \mathrm{ccfck} / \gamma \mathrm{c} \quad \gamma \mathrm{c}=1.5$
$\alpha c c$ is the coefficient taking account of long-term effects on the compressive strength and of unfavourable effects resulting from the way the load is applied and equal to 0.85

The value of the design tensile strength, fctd is defined as fctd $=\alpha c t f c t k, 0.05 / \gamma \mathrm{c}$ $\alpha c t$ is coefficient taking account of long-term effects on the tensile strength and of unfavourable effects, resulting from the way the load is applied and equal to 0.85 .

## Reinforcement steel

In ES-EN 1992-1-1-2015 section 3.2.2, characteristic strength of reinforcing steel ranges from $400-600 \mathrm{MPa}$. In this study, 466 MPa yield strength is taken for both the longitudinal reinforcement and shear reinforcement.
fyd $=\mathrm{fyk} / \gamma \mathrm{s}, \gamma \mathrm{s}=1.15$

## Concrete cover

The concrete cover is the distance between the surfaces of the reinforcement close to the nearest concrete surface which is exposed to external atmosphere. The minimum nominal cover for foundation for this case study has been taken 50 mm .

The loads from the original design have been taken from the SAFE Foundation software and the load display are shown in the figure below and the moment and shear forces are tabulated in the Appendix A.2.


Figure 3.5 Load display form SAFE Foundation software

### 3.4 Nonlinear Finite Element software

In this study, Nonlinear finite element software called ABAQUS 6.14-5 is used for the analysis of the Mat foundation. The software ABAQUS includes Abaqus CEA, Abaqus viewer, Abaqus standard and Abaqus explicit.

Abaqus CEA is a Complete Abaqus Environment providing an interface for creating, modelling, analysis, and evaluation of result.

Abaqus Viewer provides a graphical display of FE models and results.
Abaqus standard is a solver of a wide range of linear and nonlinear problems including static, dynamic, thermal, electrical, and electromagnetic responses of models.

Abaqus Explicit solve using explicit time integration for nonlinear transient dynamic analysis. In this study, Abaqus standard has been used for the nonlinear analysis of mat foundation

### 3.4.1 Modelling of Mat foundation

Modelling and analysis of the mat foundation are described below:

### 3.4.1.1 Parts (Geometry)

## I. Mat Slab and Beam

Mat foundation which has been already designed for $3 \mathrm{~B}+\mathrm{G}+21$ building contains a beam and slab part of which the slab thickness is 1.2 m and the beams are all 2.4 m in depth and has been modelled in 3D modelling space as deformable type with solid and extrusion base feature. The mat foundation is shown in the fig below:


Figure 3.6 Mat foundation model in ABAQUS

## II. Longitudinal Reinforcement Bars

Longitudinal reinforcement bars were modelled on 3D modelling space as deformable type with wire and planar base feature. The dimensions vary as the length varies. The reinforcement bar is shown in the fig below:

Figure 3.7 Longitudinal Reinforcement Bar

## III. Stirrups

Stirrups were modelled on 3D modelling space as deformable type with wire and planar base feature. The dimensions vary as the length varies and it's shown in figure below:


Figure 3.8 Stirrups

## IV. Loading plate and support plate

For loading plate steel plates were used at the top of the beams where the columns are in contact with the beam and support plates were used under the mat slab for supporting the plate. Since the plates are not study variables, they have same width and length of the column resting at the points and have thickness of 25 mm . the parts where face partitioned for the purpose of support, loading and boundary condition.


Figure 3.9 Loading plate and Support plate

### 3.4.1.2 Material property

## Concrete

(B. Alfarah a,2017) states that due to micro cracking concrete shows nonlinear stress-strain response which generates failure modes. Stress strain behaviour in tension is characterized by sudden softening go along with reduction in unloading stiffness and in compression failure begins mostly from outside and is more complex involving in crushing and inclined slipping. Stress-strain behaviour involves ductile hardening followed by softening and reduction in the unloading stiffness. Concrete damage plasticity model has shown good performance in capturing concrete behaviour in which it has been tested on practical structures (Houls and Giang D Nguyen, 2007). The unloading response of the concrete specimen is found to be diminished when it is unloaded from any position on the strain-softening branch of the stressstrain curves: the material's elastic stiffness seems to be compromised (or degraded). Between tension and compression testing, the deterioration of elastic stiffness differs substantially. In any instance, the impact becomes more severe as the temperature rises. The degraded response of concrete is characterized by two independent uniaxial damage variables dt and dc which are assumed to be functions of the plastic strain, temperature, and field variables. The uniaxial degradation variables are increasing functions of the equivalent plastic strains. They can take values ranging from zero, for the undamaged material to one for the fully damaged material. If Eo is the initial (undamaged) elastic stiffness of the material, the stress-strain relations under uniaxial tension and compression loading are, respectively: (Abaqus Theory, 2013).

```
\(\sigma_{\mathrm{t}}=\left(1-\mathrm{d}_{\mathrm{t}}\right) \operatorname{Eo}\left(\varepsilon_{\mathrm{t}}-\varepsilon_{\mathrm{t}}{ }^{\mathrm{pl}}\right)\)
\(\sigma c=(1-\mathrm{dc}) \operatorname{Eo}\left(\varepsilon_{\mathrm{t}}-\varepsilon_{\mathrm{t}}{ }^{\mathrm{pl}}\right)\)
```

The parameters, which were used in the ABAQUS was discussed in the table 3.3.
Table 3.3 Concrete parameter

| Characteristic and cubic strength of concrete <br> $\mathrm{f}_{\mathrm{ck}} / \mathrm{f}_{\mathrm{ck}, \text { cube }}(\mathrm{MPa})$ | $25 / 30$ |
| :---: | :---: |
| Modulus of Elasticity (MPa) | 31000 |
| Dilation Angle | 31 (default) |
| Eccentricity | 0.1 (default) |
| fb $_{0} / \mathrm{fc}_{0}$ | 1.16 (default) |
| Constant Kc | 0.6667 (default) |
| Viscosity | 0 (default) |
| Poisson ratio | 0.2 (ES EN 1992-1-1:2013) |

## Uniaxial compressive strength of concrete

Characteristic and cubic strength of concrete used in this study was 25/30 MPa. In ES EN 1992-1-1: 2015 provides uniaxial compressive stress-strain diagram for nonlinear analysis was used.
$\frac{\sigma c}{f c m}=\frac{k \eta-\eta^{2}}{1+(k-2) \eta}$
Where $\eta=\frac{\varepsilon c}{\varepsilon c 1} \quad, \quad k=\frac{1.05 E c m|\varepsilon c 1|}{f c m}$
where $\varepsilon c 1$-peak strain at peak stress
$\varepsilon_{\mathrm{cu1}}$ - nominal ultimate strain

Ecm -secant modulus of elasticity of concrete
$\mathrm{f}_{\mathrm{cm}}-$ mean compressive strength at $28^{\text {th }}$ day

Equation has been provided in ES EN 1992-1-1:2015, Table 3.1 for $\varepsilon_{\mathrm{cl}}$, Ecm, $\mathrm{f}_{\mathrm{cm}}$ and $\varepsilon_{\mathrm{cu}}$ which is listed below.
$\mathrm{f}_{\mathrm{cm}}=\mathrm{fck}+8(\mathrm{MPa})$
Eqn 3-4
$\mathrm{Ecm}=22[(\mathrm{fcm}) / 10]^{0.3}$
Eqn 3-5
$\mathrm{E}_{\mathrm{cl}}(\% 0)=0.7 \mathrm{fcm}^{0.31} \leq 2.8$
Eqn 3-6
$\varepsilon_{\mathrm{cu1}}(\% 0)=3.5$
$0<\left|\varepsilon_{\mathrm{cu}}\right|<3.5$ has been considered. Using the above noted compressive strength each point of the curve for C25/30MPa is presented figure 3.10.


Figure 3.10 Compressive stress-strain diagram for C25/30 Concrete
However, the input data which was used for ABAQUS is compressive stress- crushing strain which was calculated by deducting elastic strain from total strain.

Compressive damage variables (dc)
Based on (B. Alfarah a et al., 2017) proposed methodology and equation the damage variable has been prepared.
$\mathrm{dc}=1-\frac{1}{2+a c}\left[2(1+\mathrm{ac}) \exp \left(-\mathrm{bc} \varepsilon_{\mathrm{c}}{ }^{\mathrm{ch}}\right)-\mathrm{ac} \exp \left(-2 \mathrm{bc} \varepsilon_{c}{ }^{\mathrm{ch}}\right)\right]$
where $\varepsilon_{\mathrm{c}}{ }^{\mathrm{ch}}$ is compressive crushing strain (inelastic strain).
$\mathrm{Ac}=7.873, \mathrm{bc}=\frac{1.97(f c k+8) l e q}{G c h}$
where fck is the cylindrical compressive strength of concrete
leq is the characteristic length of the element
Gch is crushing energies

$$
\mathrm{Gch}=\left(\frac{f t m}{f t m}\right)^{2} \mathrm{Gf}
$$

$\mathrm{fcm}(\mathrm{MPa})=0.073 \mathrm{fcm}^{0.18}$
The compressive damage variable crushing strain is shown below in figure calculated using the above equations.


Figure 3.11 compressive damage - crushing strain diagram of concrete

## Concrete Tensile strength

For tensile behaviour equation given on Hordijk (Lantsoght et al., 2016), the ratio of tensile stress (w) for crack width ' w ', and maximum tensile strength ftm is given below:
$\frac{\sigma t(w)}{f t m}\left[1+\left(c 1\left(\frac{w}{w c}\right)^{3}\right] e^{-c 1\left(\frac{w}{w c}\right)}-\frac{w}{w c}\left(1+c 1^{3}\right) e^{-c 1}\right.$

In equation 3-8, $\mathrm{c} 1=3, \mathrm{c} 2=6.93$ (Lantsoght, et al., 2016) and wc is the critical crack opening. The equation 3-12 shows that $\sigma t(0)=\mathrm{ftm}$ and $\sigma \mathrm{t}(\mathrm{wc})=0$. Therefore, wc can be considered as the fracture crack opening (B. Alfarah a et al., 2017).
$\mathrm{Wc}=\frac{5.14 G f}{f t m}$

$$
\mathrm{ftm}=0.3016 \mathrm{fck}^{2 / 3}
$$

The exact crack spacing was not examined in this suggested approach, but a single crack per element was assumed. The assumption is acceptable for global-purpose simulation, according to Alfarah et al. Following this assumption, the strain in terms of crack opening may be calculated using the kinematic relation below in the descending portion of the tensile stressstrain curve. (B. Alfarah a el al., 2017).
$\varepsilon t=\varepsilon \mathrm{tm}+\frac{W}{l e q}$

The tensile stress-strain of concrete was calculated, tabulated, and displayed in appendix A using the method described above. Figure below, on the other hand, depicts the stress-cracking strain that was utilized as input data for the program. By subtracting elastic strain from total strain, cracking strain was determined.

## Tensile stress-cracking strain



Figure 3.12 Concrete tensile stress- cracking strain diagram

## Tensile damage variables

As discussed in compressive damage variable the method used is applied here for tensile damage variables described below:
$\mathrm{dc}=1-\frac{1}{2+a t}\left[2(1+\mathrm{ac}) \exp \left(-\mathrm{bc} \varepsilon_{c}{ }^{\mathrm{ck}}\right)-\mathrm{ac} \exp \left(-2 \mathrm{bc} \varepsilon_{\mathrm{c}}{ }^{\mathrm{ck}}\right)\right]$
Where: $\varepsilon \mathrm{t}^{\mathrm{ck}}$ is tensile cracking strain (inelastic strain).
$\mathrm{a}_{\mathrm{t}}=1, \mathrm{bt}=\frac{0.453 f c k^{0.667}}{G f} \mathrm{leq}$
Using the above equations, the tensile damage variables were calculated and tabulated. The table is presented in appendix A but the tensile damage variables- cracking strain is shown in figure below.


Figure 3.13 Tensile damage- cracking strain diagram of concrete

## Reinforcing steel

Youngu's modulus Es=221Gpa and a Poisson ratio of 0.3 were used for all the following steels. The 3.8.2 data for steel was derived from a study by Christina Claeson1 and Kent Gylltoff (Christina Claeson1 and Kent Gylltoff, 1998), as stated at the beginning of this section. Steel reinforcement was provided using deformed Swedish type Ks40S bars. Figure and table show the mechanical characteristics of the steel reinforcement.


Figure 3.14 Mechanical property of reinforcement bars.

Table 3.4 Mechanical property of reinforcement bars

| Type | As (mm)2 | fsy (MPa) | fsu (MPa) | $\varepsilon_{\text {sh }} \%$ | $\varepsilon_{\mathrm{u}} \%$ | Es (GPa) |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Ks40S | 50 | 466 | 620 | 4.0 | 12 | 221 |

Input data used for the software was true stress-logarithmic plastic strain. The following equation was used to calculate real stress and logarithmic plastic strain from nominal stress and nominal strain:
$\sigma_{\text {true }}=\sigma$ nom $(1+\varepsilon n o m)$
$\varepsilon_{\text {ln }}{ }^{\mathrm{pl}} \ln (1+\varepsilon$ nom $)-\frac{\sigma \text { true }}{E}$
Using nominal stress and strain, true stress and logarithmic plastic strain was computed and tabulated in appendix A but the diagram is shown in figure 3.15.


Figure 3.15 Stress-strain and true stress-logarithmic plastic strain of reinforcing steel

### 3.4.1.3 Part Assembly and their Interaction

After defining the material properties, profiles and sections were created and designed for the individual parts; for all parts, instances were created and assembled to their relative position. Dependent instance was used for all parts.


Figure 3.16 Assembled Mat Foundation with its beam using ABAQUS


Figure 3.17 Assembled longitudinal bars, stirrups and loading plates
In the model reinforcement bars are embedded region constraints was used by considering the reinforcement bars as embedded parts and the concrete as a host. However, for the loading plates the connection was surface to surface contact interaction has been used in which the steel plate as a master and the concrete beams as slave. This constraint makes the steel plate act as discrete rigid object and prevents local deformation due to applied loading and supporting reaction on its surface.

### 3.4.1.4 Meshing

Meshing is the critical part of finite element analysis which helps reduce the number of degrees of freedom from infinite to finite. Meshing is dividing the parts into small but finite parts where each piece represents an element. The accuracy of the finite element analysis is increases as
the elements gets finer (smaller) in which the analysis is interpolated to give the most accurate solution of the main structure.


Figure 3.18 Meshing of parts

### 3.5 Validation

This validation was done to check the workability of the software ABAQUS with reinforced concrete design by remodelling the existing mat foundation, which have been done originally in a commercially used Safe Foundation software, for the loading purposes later to be used in the formulation of objective function and design constraints in MATLAB optimization toolbox.

### 3.5.1 Reference experiment

Validation of the software was done by comparing the result of previously done experimental research on beams by T. Tejaswini on the paper Analysis of RCC beams using Abaqus.
T. Tejaswini, 2015 conducted a study on RCC Beam dimension of $1200 \times 200 \times 100$ of M30 grade concrete with under, balanced and over reinforced sections and Finite Element Analysis (FEA) have also been performed using ABAQUS. In the validation experiment it has been used the balanced reinforcement which has been casted with 2- $\varnothing 12,2-\varnothing 8$ bars at bottom and 2- $\varnothing 8$ hanger bars and $\varnothing 8$ @ 135c/c stirrups are provided.

In the experimental study of the RCC beam it has been calculated the deflection vs load applied and it has been shown in table 3.5.

Table 3.5 Load and corresponding deflection for a balanced section

| Load (KN) | Deflection |
| :---: | :---: |
| 20 | 1.25 |
| 40 | 2.35 |
| 60 | 3.56 |
| 80 | 4.65 |
| 88.8 (Ultimate) | 5.2 |

In $T$, Tejaswini research the experimental study Vs ABAQUS result were 88.8 KN and 78.4KN respectively.

### 3.6 Optimization

### 3.6.1 Load on the mat foundation slab and beam sections

In designing reinforced concrete structure, the most effected panels are the longest span to span length of slabs and beams in our case study the longest beam length is 6.5 m located from 9.8 16.3 m and $28.3-34.8 \mathrm{~m}$ in the longer span direction and the largest spans of slabs are 5.5 x 4.8 m length and width respectively. Loads on beams and slab panels are listed below:


Figure 3.19 Slab panels and beam of the longest spans layout

Table 3.6 Maximum Moment and Shear forces on beams

| Beam <br> $(\mathbf{m})$ | Location <br> $(\mathbf{m})$ | Max Moment <br> $(\mathbf{K N}-\mathbf{m})$ | Max Shear <br> $(\mathbf{K N})$ |
| :---: | :---: | :---: | :---: |
| BM1- 1x2.4 | 11.3 | 4814.8794 | 2404.137 |
| BM1 -1x2.4 | 29.8 | 2784.8389 | 2063.477 |
| BM2 -1x2.4 | 11.3 | 6579.4204 | 3365.518 |
| BM2 -1x2.4 | 29.8 | 6394.4141 | 3287.823 |
| BM3 -1x2.4 | 11.3 | 4879.9702 | 3179.692 |
| BM3 -1x2.4 | 29.8 | 4358.5957 | 1756.984 |
| BM4 -1x2.4 | 14.9 | 4898.1938 | 3045.455 |
| BM4 -1x2.4 | 29.8 | 6420.3164 | 3168.539 |
| BM5- 1x2.4 | 11.3 | 4828.21 | 3090.264 |
| BM5 -1x2.4 | 36.3 | 6149.9312 | 3329.831 |
| BM6 -1x2.4 | 11.3 | 2688.9763 | 1971.231 |
| BM6 - 1x2.4 | 36.3 | 4692.1733 | 2403.912 |

Table 3.7 Maximum top and bottom moment on slab

| Panel | Direction | Max Top Moment <br> $(\mathbf{K N}-m)$ | Max Bottom Moment <br> $(\mathbf{K N}-m)$ |
| :---: | :---: | :---: | :---: |
| P1 | Short | -745.2195 | 605.0936 |
| P1 | Long | -310.6944 | 1394.8662 |
| P2 | Short | -716.6424 | 451.8846 |
| P2 | Long | -554.434 | 1166.4778 |
| P3 | Short | -709.3483 | 1228.3608 |
| P3 | Long | -535.1248 | 1780.8226 |
| P4 | Short | -766.4835 | 1385.411 |
| P4 | Long | -567.7619 | 1518.9607 |

### 3.6.2 Initialization values for design variables

To start the algorithm, it needs a feasible starting point. If the point provided is not feasible, then using linear programming solution can be found. This feasible point is the initial value for
design variables. For the purpose of this study, the upper values for decision variables (crosssectional dimensions of structural elements) are taken from the case study under consideration and the initial values have been used as per ES EN 1992-1-1-2015. Accordingly, the initial lower and upper bound values of the cross-sectional dimensions of the structure is given bellow in the table. The quantities initial value (x0), lower bound (LB) and upper bound (UB) are arguments in fmincon in MATLAB solver and the quantities are given in the following table.

Table 3.8 Initial, Lower and Upper bound values for the design variable

| Structural <br> Element | Design variable | Initial Value <br> $(\mathrm{mm})$ | Lower bound <br> $(\mathrm{mm})$ | Upper bound <br> $(\mathrm{mm})$ |
| :---: | :---: | :---: | :---: | :---: |
| Beam | Width | 800 | 800 | 1000 |
|  | Effective Depth | 1000 | 500 | 2400 |
|  | Stirrup Spacing | 100 | 100 | 600 |
|  | Ast $\left(\mathrm{mm}^{2}\right)$ | 0 | 0 | 6328 |
|  | Asc $\left(\mathrm{mm}^{2}\right)$ | 0 | 0 | 6328 |
|  | Av $\left(\mathrm{mm}^{2}\right)$ | 0 | 0 | 3660 |
| Slab | Effective depth | 500 | 500 | 1200 |
|  | Spacing | 100 | 100 | 400 |
|  | As (top) $\left(\mathrm{mm}^{2}\right)$ | 0 | 0 | 1667 |
|  | As (bottom) $\left(\mathrm{mm}^{2}\right)$ | 0 | 0 | 1667 |

### 3.6.3 Formulation of Optimization Problems Using Optimization Tool Box In MATLAB

In order to solve optimization problems using optimization tool box in MATLAB the formulated nonlinear programming problems should be converted from nonlinear programming mathematical form into MATLAB's Optimization Toolbox solver syntax through the following steps:

1. Define the objective function in the MATLAB language, as a function file or anonymous function.
2. Define the constraints as a separate file or anonymous function

The nonlinear programming problems were separately formulated for beam and slab.

### 3.6.3.1 Solution for formulated optimization problems using MATLAB Optimization toolbox

The optimization problems formulated for beams and slab were solved by using optimization tool box in MATLAB. Beams where targeted for the longest spans having higher bending moments and shear forces and for the slab was targeted for the longest span to span length having the maximum moment. The formulated optimization problems were solved through the following seps.

1. Creating M-file for objective and constraint functions
2. Running optimization for obtaining optimum solution for design variables

For this case study objective functions and constraints for the beams and for the slab functions having same objective with different constraints are stated below.

## M file for objective function for beams

An m-file is a text file with the extension ".m" containing MATLAB commands which can be created in any text editor, or using the built in MATLAB editor. For the objective function of beams is written in M -file as shown below.

## function $\mathrm{f}=\mathrm{beam}(\mathbf{x})$

$\mathrm{f}=8.25 * \mathrm{Bb} * \mathrm{db}+0.495 * \mathrm{Bb}-1.925^{*} \mathrm{Ast}-1.925^{*} \mathrm{Asc}-3.85 * \mathrm{Av} * \mathrm{Bb} / \mathrm{S}-3.85^{*} \mathrm{Av} * \mathrm{db} / \mathrm{S}$
$+\left(0.639^{*} \mathrm{Av}\right) / \mathrm{S}-\left(29.057 \mathrm{Av} v^{\wedge} 1.5 / \mathrm{S}-0.7^{*} \mathrm{Av} * \mathrm{Bb}-0.7^{*} \mathrm{Av}{ }^{*} \mathrm{db}-27.97^{*} \mathrm{Av}-3.158^{*} \mathrm{Av}^{\wedge} 1.5\right.$;

```
f=8.25*x(1)*x(2)+0.495*x(1)-1.925*x(3)-1.925*x(4)-3.85*x(5)*x(1)/x(6)-3.85*x(5)*x(1))/
x(6)+(0.639*x(5))/x(6)-(29.057x(5)^1.5)/x(6)-0.7*x(5)*x(1)-0.7*x(5)*x(2)-27.97*x(5)-
3.158*x(5)^1.5;
```

M-file for constraint functions for beams

Constraint functions must be formulated so that they are in the form $c(x) \leq 0$ (for inequality constraint) or $\operatorname{ceq}(x)=0$ (for equality constraint). In this study, there is only an inequality constraint, so the equality constraint was passed by an empty array [ ] as the equality constraint function ceq. The constraint function for beam is written in M-file as shown below.

## Function [c, ceq]=constraint(x)

$\mathrm{c}(1)=\mathrm{Md}^{*} 10^{\wedge} 3-4.175^{*} \mathrm{Bb}^{*} \mathrm{db}^{\wedge} 2-466^{*} \mathrm{db} * \mathrm{Asc}+27.96^{*} \mathrm{Asc}$;

```
c(2)= Vd*10^3-7.646*Bb*db;
c(3)= Vd*10^3-(364.5Av*db/S+(18.78*Av/S);
c(4)= 0.00145*Bb*db-Asc;
c(5)= 0.00145*Bb*db-Ast;
c(6)= 0.0013*Bb*db-Asc;
c(7)= 0.0013*Bb*db-Ast;
c(8)= Asc-0.04*Bb*db-0.0024*Bb;
c(9)= Ast-0.04*Bb*db-0.0024*Bb;
c(10)= S-0.75*x(2);
c(11)= S -600;
ceq = [];
```

Therefore, the constraint function for beam is:

```
c(1)=6579.42*10^3-4.175* x(1)*x(2)^2-466*x(2)*x(4)+27.96*x(4);
c(2)=3365.518*10^3-7.646*x(1)*x(2);
c(3)=3365.518*10^3-(364.5x(5)*x(2))/x(6)+(18.78 *x(5))/x(6);
c(4)= 0.00145*x(1)*x(2)-x(3);
c(5)= 0.00145*x(1)*x(2)-x(4);
c(6)= 0.0013*x(1)*x(2)-x(3);
c(7)=0.0013*x(1)*x(2)-x(4);
c(8)= x(3)-0.04*x(1)*x(2)-0.0024*x(1);
c(9)= x(4)-0.04*x(1)*x(2)-0.0024*x(1);
c(10)= x(6)-0.75*x(2);
c(11)=x(6)-600;
ceq = [];
```

The constraint function for the beam were written by selecting the longest beam section ranging from $9.8-16.3 \mathrm{~m}$ and $28.3-34.8 \mathrm{~m}$ which is 6.5 m long beam having the maximum shear force and bending moment.

## Running the optimization in Optimization toolbox

By crating m. file for objective function and for the constraints as a script file separate files then by opening the optimization tool box app from the selectable apps available we can insert the values.


Figure 3.20 MATLAB optimization tool box window

Figure 3.21 m . files for objective function and constraints for beam


Figure 3.22 Graphical user interface of optimization toolbox for beam

## M file for objective function for Slab

The objective function of slab is written in M-file as shown below.

## function $\mathrm{f}=\operatorname{slab}(\mathbf{x})$

$\mathrm{f}=39.6^{*} \mathrm{ds}+1.99+(10.587 * \mathrm{Asl} / \mathrm{S})-1.925 \mathrm{Asl}-\left(38.705^{*}\right.$ Ass/S)-8.064*Ass;
$\mathrm{f}=39.6 * x(1)+1.99+(10.587 * x(2) / \mathrm{x}(4))-1.925 \mathrm{x}(2)-(38.705 * \mathrm{x}(3) / \mathrm{x}(4))-8.064 * \mathrm{x}(3)$;
M-file for constraint functions for Maximum Top slab

## Function [ $\mathbf{c}, c e q]=$ constraint( $\mathbf{x}$ )

$\mathrm{c}(1)=$ Medl-405.42*Asl*ds-7254.828*Asl;
$c(2)=$ Meds-405.42*Ass*ds-7254.828*Ass;
$\mathrm{c}(3)=0.00145^{*} \mathrm{ds}-\mathrm{Ass} ;$

```
c(4)= 0.0013*ds-Ass;
c(5)= Asl-0.04*ds-0.002;
c(6)= Ass-0.04*ds-0.002;
c(7)= S-3*ds-0.15;
c(8)=S-400;
ceq = [];
```

Therefore, constraint functions are:

```
c(1)= -567.762*10^3-405.42*x(2)*x(1)-7254.828*x(2);
c(2)= -745.22*10^3-405.42*x(3)x(1)-7254.828*x(3);
c(3)=0.00145*x(1)-x(3);
c(4)= 0.0013*x(1)-x(3);
c(5)=x(2)-0.04*x(1)-0.002;
c(6)=x(3)-0.04*x(1)-0.002;
c(7)= x(4)-3*x(1)-0.15;
c(8)= x(4)-400;
ceq = [];
```



Figure 3.23 m . files for objective function and constraints for top slab


Figure 3.24 Graphical user interface of optimization toolbox for top slab M-file for constraint functions for Maximum Bottom slab

## Function [ $\mathbf{c}, c e q]=$ constraint( $\mathbf{x}$ )

$c(1)=1780.823^{*} 10 \wedge 3-405.42 * x(2) x(1)-7254.828^{*} x(2)$;
$c(2)=1394.8662 * 10 \wedge 3-405.42 * x(3) * x(1)-7254.828 * x(3)$;
$c(3)=0.00145 * x(1)-x(3)$;
$c(4)=0.0013 * x(1)-x(3)$;
$c(5)=x(2)-0.04 * x(1)-0.002$;
$c(6)=x(3)-0.04 * x(1)-0.002 ;$
$c(7)=x(4)-3 * x(1)-0.15$;
$c(8)=x(4)-400 ;$
ceq $=[] ;$

## Running the optimization

The optimization may be conducted in two ways: using the Optimization Tool or using command line methods. The optimization tool was utilized in this study to find the best value for design variables and goal functions. As a result, the M-files for the objective function and constraint functions for all grouped beams were constructed and run for the best solution, as shown below.


Figure 3.25 m . files for objective function and constraints for bottom slab


Figure 3.26 Graphical user interface of optimization toolbox for bottom slab
By changing the ultimate moment and shear force in the design constraint functions based on different types of soil bearing capacities it's possible to generate an optimized mat foundation. To illustrate this concept, bearing capacity of 450 KPa was taken and the foundation was analysed. The design made in the conventional method were tabulated below:

Table 3.9 Design result of conventional design based on 450 KPa soil bearing capacity

| Conventional design of mat foundation with bearing capacity of 450 KPa |  |
| :---: | :---: |
| Beam section | 1.2 m X 2.5 m |
| Mat slab section | 1.8 m |
| Maximum moment in beam | 4924.6914 KNm |
| Maximum shear in beam | 2989.884 KN |
| Maximum moment in slab top short | -1533.7656 KNm |
| Maximum moment in slab top long | -1410.5824 KNm |
| Maximum moment in slab bottom short | 2760.2886 KNm |
| Maximum moment in slab bottom long | 3746.6743 KNm |

## CHAPTER FOUR

## RESULT AND DISCUSSION

### 4.1 General

In this chapter, result from the original Mat Foundation design and optimized design from MATLAB optimization tool box are discussed in terms of total weight and compare these results.

### 4.2 Validation result of ABAQUS Vs experiment

The balanced reinforced beam has been analysed in ABAQUS and has been found that the ultimate load carrying capacity of the modelled beam was found to be 91.5 KN which is $97.05 \%$ of the result Vs experimental result. Moreover, the load-displacement curve of the FEAc result matched with the load-displacement curve of the experimental result. This indicates that FEA result well conformed to the experimental result.



Figure 4.1 Graphical representation of Validation beam load Vs Displacement in ABAQUS
Table 4.1 Validation ABAQUS VS Experimental

| Abaqus |  | Experiment |  |
| :---: | :---: | :---: | :---: |
| Disp. | Load | Disp. | Load |
| 0 | 0 | 0 | 0 |
| 0.042259 | 4.47168 | 0.118065 | 3.60375 |
| 0.135163 | 7.039 | 0.248393 | 5.1123 |
| 0.143073 | 9.452 | 0.378722 | 6.87228 |
| 0.159313 | 10.3288 | 0.50905 | 8.38082 |
| 0.162433 | 14.8749 | 0.639378 | 11.1465 |
| 0.175895 | 17.9387 | 0.769706 | 14.1636 |
| 0.188132 | 20.8145 | 0.900034 | 16.175 |
| 0.19967 | 25.5856 | 1.46479 | 23.2149 |
| 0.206682 | 27.6528 | 1.59512 | 25.4777 |
| 0.2239 | 40.3305 | 2.11643 | 37.2947 |
| 0.240138 | 46.7455 | 2.24676 | 39.5575 |
| 0.248901 | 47.9899 | 2.37709 | 41.066 |
| 0.249125 | 48.0217 | 2.50741 | 43.3289 |
| 0.249576 | 48.0853 | 2.63774 | 44.8374 |
| 0.249917 | 48.133 | 2.76807 | 47.3517 |
| 0.25121 | 48.3118 | 2.8984 | 50.3688 |
| 0.252371 | 48.4727 | 3.02873 | 52.883 |
| 0.256681 | 49.0757 | 3.15906 | 54.3916 |
| 0.260512 | 49.6179 | 3.28938 | 55.6487 |
| 0.274658 | 51.6447 | 3.41971 | 56.403 |
| 0.287023 | 53.4575 | 3.55004 | 57.4086 |
| 0.330944 | 60.1038 | 3.68037 | 58.6658 |
| 0.349848 | 62.9851 | 3.8107 | 60.1743 |


| 0.368532 | 65.6062 | 3.94102 | 61.18 |
| :---: | :---: | :---: | :---: |
| 0.384743 | 67.8893 | 4.07135 | 62.6886 |
| 0.46518 | 75.6804 | 4.20168 | 64.9514 |
| 0.506423 | 79.6712 | 4.33201 | 66.7114 |
| 0.552787 | 84.2048 | 4.46234 | 68.4713 |
| 0.558472 | 84.6033 | 4.59267 | 70.2313 |
| 0.586919 | 85.9887 | 4.72299 | 71.4884 |
| 0.612981 | 87.0881 | 4.85332 | 72.7456 |
| 0.648696 | 89.5701 | 4.98365 | 74.2541 |
| 0.65932 | 90.3803 | 5.11398 | 75.0084 |
| 0.674938 | 91.4907 | 5.24431 | 76.0141 |
| 0.675071 | 91.5 | 5.37463 | 76.5169 |
| 0.675071 | 91.5 | 5.50496 | 77.0198 |
| 0.675071 | 91.5 | 5.63529 | 78.2769 |
| 0.675071 | 91.5 | 5.93939 | 79.534 |
| 0.675071 | 91.5 | 6.02627 | 78.7798 |



Figure 4.2 Load Vs Displacement curve of FEA and Experimental

### 4.3 Result of the original Mat Foundation design

The original mat foundation design has been modelled initially in SAFE Foundation, a commercial software, it has been remodelled in ABAQUS to study how the design performs by defining sections and material properties getting the moments and shear forces for further optimization process. From the original design the weight of the Mat slab and beam are taken. Having Mat surface area of $1465.54 \mathrm{~m}^{2}$ and section of 1 m beam width, 2.4 m depth of beam and
slab depth of 1.2 m and reinforcement of Mat slab with diameter 20 mm bar spaced centre to centre of 150 mm top and bottom. The results are tabulated below in the table 4.2:

Table 4.2 Weight of the original Mat foundation design

| Structural Element |  | Weight (Kg) |
| :---: | :---: | :---: |
| Beam | Concrete | $4,147,200$ |
|  | Reinforcement | 222,328 |
| Slab | Concrete | $4,396,620$ |
|  | Reinforcement | 122,442 |
| Total concrete weight |  | $8,543,820$ |
| Total Rebar weight |  | 344,770 |
| Total Structural weight |  | $8,815,739$ |

### 4.4 Result of the Optimized Mat Foundation design

Here the optimized Mat foundation weight is given from the result obtained from the MATLAB optimization toolbox. Having the Mat surface area of 1465.54 m 2 and section of 0.8 m beam width, 2.3 m beam depth and 1 m Mat slab depth and rounding the numbers for the purpose of workability area of reinforcement of Mat slab having diameter 16 mm bar centre to centre 120 mm on the top and diameter 20 mm bar centre to centre 170 mm on the bottom area. The results are tabulated in the table 4.3:

Table 4.3 Weight of the optimized Mat foundation design

| Structural Element |  | Weight (Kg) |
| :---: | :---: | :---: |
| Beam | Concrete | $3,180,903$ |
|  | Reinforcement | 216,864 |
| Slab | Concrete | $3,666,781$ |
|  | Reinforcement | 102,913 |
| Total concrete weight |  | $6,847,684$ |
| Total Rebar weight |  | 319,777 |
| Total Structural weight |  | $7,167,461$ |

### 4.5 Comparison of the results

Here the parameter for comparison is based on weight. The result original design done by conventional methods and the optimized design done by optimization toolbox in MATLAB will be compared.

### 4.5.1 Comparison of Total weight

As shown above, the cross-sectional dimensions of all structural elements, Mat slab depth, beam depth and width and also area of reinforcement bars have been reduced compared to the conventional design done for the original Mat foundation. Hence showing a reduction of weight of the project making it an economical design satisfying the necessary conditions.

Table 4.4 Weight and percentage variation of original design and optimized design

| Design | Element | Weight | Percentage difference <br> w.r.t original design |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Original design | $4,147,200$ | - |
|  |  | Optimized design | $3,180,903$ | $23.3 \%$ |
|  | Rebar | Original design | 222,328 | - |
|  |  | Optimized design | 216,864 | $2.46 \%$ |
| Slab | Concrete | Original design | $4,396,620$ | - |
|  |  | Optimized design | $3,666,781$ | $16.6 \%$ |
|  | Rebar | Original design | 122,442 | - |
|  |  | Optimized design | 102,913 | $15.95 \%$ |
| Total weight |  | Original design | $8,815,739$ | - |
|  |  | Optimized design | $7,167,461$ | $18.7 \%$ |



Figure 4.3 Graphical representation of total weight of case study design and optimized design


Figure 4.4 Graphical representation of Beam weight of case study design Vs optimized design


Figure 4.5 Graphical representation of Beam weight of case study design Vs optimized design

As shown above figure 4.1, the case study mat foundation done in conventional methods has higher weight value than the optimized design using MATLAB optimization toolbox making the design uneconomical. While structural elements, beam weight decrease by $22.24 \%$ and Mat slab weight decreased by $16.58 \%$, reducing the total weight of the optimized design up to $18.7 \%$ making the design more economical and preferable the projects constructed ahead.

### 4.6 Result of the Optimized Mat Foundation design based on 450KPa bearing capacity

Having the Mat surface area of 1465.54 m 2 and section of 1.2 m beam width, 2.5 m beam depth and 1.8 m Mat slab depth and rounding the numbers for the purpose of workability area of reinforcement of Mat, the weight of original design by conventional method and optimized sectional design using MATLAB optimization Toolbox result is shown in table 4.5:

Table 4.5 Weight and percentage variation of original design and optimized design

| Structural Element |  | Weight (Kg) <br> Conventional | Weight (Kg) <br> Optimized | \% Difference w.r.t <br> conventional design |
| :---: | :---: | :---: | :---: | :---: |
| Beam | Concrete | $4,320,000$ | $3,337,239$ | $22.75 \%$ |
|  | Reinforcement | 231,595 | 225,574 | $2.6 \%$ |
| Slab | Concrete | $6,594,930$ | $5,488,504$ | $16.78 \%$ |
|  | Reinforcement | 183,669 | 154,466 | $15.9 \%$ |
| Total concrete weight |  | $10,914,930$ | $8,825,743$ | $19.14 \%$ |


| Total Rebar weight | 415,264 | 380,040 | $8.48 \%$ |
| :---: | :---: | :---: | :---: |
| Total Structural weight | $11,330,194$ | $9,205,783$ | $18.75 \%$ |

### 4.7 Verification of result using SAFE Foundation

Using the commercially used software SAFE Foundation V12 the optimized result has been designed to verify if the solution gives the best optimized result and compare the originally designed foundation to the optimized solution. The model and analysis are done following the methods as follow:

- Define material properties
- Define section dimensions
- Define loads onto the Mat
- Defining stripes (1m) horizontally and vertically.
- Analyse


### 4.7.1 Result from original design using SAFE Foundation

The original Mat foundation design having beam width of 1 m , beam depth of 2.4 m and slab thickness of 1.2 m and soil bearing capacity of 550KPA. The analysis shown in the figure below shows the soil pressure diagram showing the maximum pressure of 542.75 KPA .


Figure 4.6 Soil pressure diagram of original design on SAFE Foundation


Figure 4.7 Structural design drawing of the original Mat foundation design

### 4.7.2 Result from optimized design using SAFE Foundation

In the optimized Mat foundation having beam width of 0.8 m , beam depth of 2.3 m and slab thickness of 1 m and soil bearing capacity of 550KPA. The analysis shown in the figure below shows the soil pressure diagram showing the maximum pressure of 550.84KPA and also the punching shear ratio calculated in the SAFE was 0.79 .


Figure 4.8 Soil pressure diagram of optimized design on SAFE Foundation


Figure 4.9 Structural design drawing of the optimized Mat foundation design

In the figure 4.8 the structural members adequately transfer the load coming from the superstructure without any failures pushing the limits to the ultimate capacity of 550 KPA of soil pressure which is the ultimate bearing capacity of the given soil bearing capacity done in laboratory soil test and also the punching shear ratio calculated in the SAFE was 0.87 which is less than 1 which makes it safe and performs well.

## CHAPTER FIVE

## CONCLUSION AND RECOMMENDATION

This chapter summarizes the major conclusions from the previous chapters and makes recommendations based on the obtained results and recommendations for further studies.

### 5.1 Conclusion

This study was conducted by considering multiple constraints to optimize the total weight of Mat foundation, a case study on 3B+G+21 Addis Ababa condominium housing project which is one of the large-scale projects planned to be executed in the near future. Mainly, the total weight of structure is the key parameter focused on as it results from the values of design variables predominantly the size of cross-sectional dimensions of structural elements. MATLAB Optimization toolbox were used for the optimization analysis for structural elements (Mat beam and slab) using the ultimate moment and shear capacities from the case study done by a commercial analysis software SAFE analysis verified by ABAQUS.

The following conclusions were drawn:

- When optimization toolbox in MATLAB is used to optimize the total weight of the case study, the weight has decreased by 18.7 \% while having structural members of $22.24 \%$ and $16.58 \%$ reduction for beam and slab respectively with respect to the weight obtained by conventional design method.
- Significant weight reduction on concrete of beam about $23.3 \%$ and the slab concrete about $16.6 \%$ and reinforcement bar of $15.95 \%$ but for the beam reinforcement was about $2.46 \%$ showing minimum reduction showing the design was at its capacity.
- The soil pressure on the original design was 542.75 KPA where the Optimized design reached the ultimate capacity of 550 KPa showing the optimized design have been pushed to the maximum limit.
- The punching shear ratio of the original design was 0.79 while the optimized design was 0.87 showing that the optimized structural member adequately transfers the loads to the ground without structural failure due to punching in which both designs where within the safe limit.
- By changing the ultimate moment and shear values in the constraint functions based on bearing capacity $450 \mathrm{KPa}, 18.75 \%$ weight reduction obtained w.r.t optimized design done for bearing capacity of 550 KPa .


### 5.2 Recommendation

The following recommendations are suggested for future researches, which are not covered in the present study.

- For better optimized Mat Foundation design the soil-structure interaction shall be studied since design of foundation is based on the type of soil and it is essential to determine the best suited type of foundation design.
- Design and study the performance of mat foundation by considering different grade of concrete strength and steel capacity and propose an economical section with better capacity.
- There are different methodologies available to optimize reinforced concrete designs, select methods that suits the optimization problems and compare for the better solutions having best economical section and capacity.
- Verify the performance of optimized design by performing experimental investigation in laboratories by studying the tensile and compressive strength of members.
- In practical implementation, pushing the soil to its ultimate bearing capacity is risky due to different factors therefore other measurements have to be considered like using subsurface drainage perforated pipes.


## References

A. A. A. Aga and F. M. Adam, (2015). Design Optimization of Reinforced Concrete Frames. Open J. Civ. Eng., vol. 05, No. 01, Pages. 74-83,

ABAQUS/CAE User's Guide. Abaqus/CAE, Abaqus 6.14 (User's Guide).
ACI, C. R. b., 2014. An ACI Standard and Report, Building Code Requirements for Structural Concrete (ACI 318-14), Commentary on Building Code Requirements for Structural Concrete (ACI 318R-14). American/Texas: American Concrete Institute.
B. Alfarah a, F. L.-A. b. S. O. a., 2017. "New methodology for calculating damage variables evolution in Plastic Damage Model for RC structures". Engineering Structures, Volume 132, pp. 70-86

Chalachew B. Hunegnaw, Temesgen W. Aure (2021). Effect of orientation of stirrups in combination with shear span to depth ratio on shear capacity of RC beams. Heliyon, vol. 07.

Christina Claeson1 and Kent Gylltoff, 1998. "Slender high-strength concrete columns subjected to eccentric loading". J. Struct. Eng, Volume 124, pp. 233-240.

C-M Ma and Y-Y Chen (2019). Advances in Civil and Ecological Engineering Research. IOP Conf. Series, Vol 351.

David B. Lorenz (1989). Some aspects of the structural design of a mat foundation for various soil conditions", Marin Structures, Vol. 02, pp. 385-401.

ES EN:1992-1-1., 2015. Ethiopian Standard based on European Norm, Design of Concrete Structures. Ministry of Urban Development \& Construction.

Houls and Giang D Nguyen, G. T., 2007. "A coupled damage-plasticity model for concrete based on thermodynamic principles: Part I: model formulation and parameter identification" International journal for numerical and analytical methods in geomechanics, Volume 32, p. 353-389.

Jia-Ji Wang, Cheng Liu, Xin Nie, Jian- Sheng Fan, Ying- Jie Zhu (2021). Nonlinear model updating algorithm for biaxial reinforced concrete constitutive model of shear walls. Journal of Building Engineering 44.

Lantsoght, E., Yuguang Yang, Cor van der Veen, Ane de Boer and Dick A Hordijk (2016). Engineering Structures, Volume 128, pp. 111-123.

Rashed H. Chowdhury, Mavinakere E. Raghunandan, A.M ASCE and Abdul Muqtadir (2012). Study on the analysis of mat foundation using different approach. $2^{\text {nd }}$ International conference for sustainable design, Engineering and construction, ASCE, pp 465-472.

Schmit, L.A (1960). Structural design by systematic synthesis. Proc. 2nd Conference on Electronic Computation, ASCE, New York, Pages. 105-122.
S. M. Thomas and P. A. G (2017). Optimization of singly reinforced RC beams. vol. 5, Pages. 199-207.
T. Tejaswini, "Analysis of RCC Beams using ABAQUS", International Journal of Innovations in Engineering and Technology, volume 5, 2015.

The Mathworks (2010). Optimization Toolbox ${ }^{\text {TM }} 4$ User's Guide. Order A J. Theory Ordered Sets Its Appl., vol. 200, p. 20.
I. Rahmanian, Y. Lucet, and S. Tesfamariam (2014). Optimal design of reinforced concrete beams: A review. Comput. Concr. vol. 13, no. 4, pp. 457-482.

Fedghouche Ferhat (2018). Design optimization of reinforced ordinary and high strength concrete beams with Eurocode2", IntechOpen.

Peck, R.B., Hanson, W.E., and Thornburn, T.H (1974). Foundation Engineering. 2nd Edt., John Wiley and Sons, New York.

Rashedul H. Chowdhury, Mavinakere E. Raghunandan, A.M. ASCE and Abdul Muqtadir (2013). Study on the analysis of Mat foundation using a different approach. International Conference on Sustainable Design, Engineering, and Construction, ASCE.

Swoo-Heon Lee, Ali Abolmaali, Kyung-Jae Shin, Hee-Du-Lee (2020). ABAQUS modelling for post tensioned reinforced concrete beams. Journal of Building Engineering 30.

Venkatramaiah, C (1993). Geotechnical Engineering", Wiley Eastern Limited, New Delhi,
Xin-She Yang (2010). An introduction with Metaheuristic Application. Engineering optimization, page 16.

Zia-abe Deen. S Punekar, M H Kolhar, Anjum Algur and Kushappa M K (2017). Analysis and design of raft foundation. International journal of research in engineering and technology vol. 06 pp.14-19

## APPENDIX A <br> LOADS FROM SAFE MODEL OF THE ORIGINAL DESIGN

TABLE A. 1 Concrete Slab Design Summary 01 - Flexural and Shear Data

| TABLE: Concrete Slab Design Summary 01 - Flexural and Shear Data |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Strip | SpanID | Location | FTopMoment | FBotMoment | VForce |
| SA6 | Span 1 | Start | -0.1343 | 547.045 | 330.691 |
| SA6 | Span 1 | Middle | -13.8402 | 604.1533 | 39.85 |
| SA6 | Span 1 | End | -0.1832 | 910.1267 | 476.537 |
| SA6 | Span 2 | Start | -188.4598 | 910.1267 | 476.537 |
| SA6 | Span 2 | Middle | -188.4598 | 352.2479 | 735.942 |
| SA6 | Span 2 | End | -4.4238 | 1228.3608 | 355.18 |
| SA6 | Span 3 | Start | -174.1984 | 1228.3608 | 814.549 |
| SA6 | Span 3 | Middle | -709.3483 | 0.5917 | 914.111 |
| SA6 | Span 3 | End | -240.6817 | 1016.1155 | 661.382 |
| SA6 | Span 4 | Start | -238.5712 | 1016.1155 | 1074.428 |
| SA6 | Span 4 | Middle | -715.2907 | 13.4885 | 867.631 |
| SA6 | Span 4 | End | -310.3186 | 902.987 | 1207.63 |
| SA6 | Span 5 | Start | -280.4253 | 902.987 | 1207.63 |
| SA6 | Span 5 | Middle | -766.4835 | 0.299 | 665.447 |
| SA6 | Span 5 | End | -355.4912 | 1385.411 | 918.048 |
| SA6 | Span 6 | Start | -115.3407 | 1385.411 | 1410.872 |
| SA6 | Span 6 | Middle | -508.6405 | 673.6854 | 746.733 |
| SA6 | Span 6 | End | -508.6405 | 1221.1801 | 260.476 |
| SA6 | Span 7 | Start | -1.1653 | 1221.1801 | 797.027 |
| SA6 | Span 7 | Middle | -221.8417 | 873.8387 | 347.199 |
| SA6 | Span 7 | End | -124.4645 | 829.3794 | 76.625 |
| SB3 | Span 1 | Start | -0.3631 | 507.183 | 659.301 |
| SB3 | Span 1 | Middle | -258.5328 | 439.1104 | 98.296 |
| SB3 | Span 1 | End | -218.6671 | 1338.8312 | 664.32 |
| SB3 | Span 2 | Start | -0.4534 | 1338.8312 | 1208.046 |
| SB3 | Span 2 | Middle | 0 | 635.2172 | 53.159 |
| SB3 | Span 2 | End | 0 | 1294.8662 | 569.904 |


| SB3 | Span 3 | Start | -210.1536 | 1394.8662 | 930.877 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| SB3 | Span 3 | Middle | -310.6944 | 133.4392 | 834.607 |
| SB3 | Span 3 | End | -300.1927 | 1032.88 | 1178.012 |
| SB3 | Span 4 | Start | -99.8307 | 1032.88 | 1178.012 |
| SB3 | Span 4 | Middle | -269.5794 | 13.7323 | 333.58 |
| SB3 | Span 4 | End | -1.8479 | 861.4501 | 507.334 |
| SB3 | Span 5 | Start | -525.1332 | 861.4501 | 843.248 |
| SB3 | Span 5 | Middle | -541.3324 | 13.8889 | 499.688 |
| SB3 | Span 5 | End | -337.2499 | 1166.4778 | 964.793 |
| SB3 | Span 6 | Start | -399.6544 | 1166.4778 | 1305.704 |
| SB3 | Span 6 | Middle | -554.434 | 29.6286 | 815.878 |
| SB3 | Span 6 | End | -362.2061 | 1099.205 | 1303.019 |
| SB3 | Span 7 | Start | -96.047 | 1099.205 | 1303.019 |
| SB3 | Span 7 | Middle | -282.3283 | 869.7185 | 258.184 |
| SB3 | Span 7 | End | -282.3283 | 1049.4982 | 164.727 |
| SB3 | Span 8 | Start | -554.2616 | 1049.4982 | 658.519 |
| SB3 | Span 8 | Middle | -533.8922 | 525.7196 | 426.255 |
| SB3 | Span 8 | End | -0.1509 | 857.2291 | 218.864 |

TABLE A. 2 Concrete Beam Design - Flexural and Shear Data

| Concrete Beam Design - Flexural and Shear Data |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| Line | Location | Moment | VForce | Line | Location | Moment | VForce |  |
| BM1 | 0 | 72.6156 | 154.738 | BM1 | 23.2 | 0 | 830.852 |  |
| BM1 | 0.75 | 369.8372 | 422.704 | BM1 | 23.8 | 442.2766 | 1084.39 |  |
| BM1 | 1.5 | 686.8651 | 720.852 | BM1 | 24.4 | 0 | 1084.39 |  |
| BM1 | 2.25 | 63.4171 | 720.852 | BM1 | 25 | 0 | 852.813 |  |
| BM1 | 2.96667 | 0 | 453.858 | BM1 | 25.85 | 0 | 527.027 |  |
| BM1 | 3.68333 | 0 | 173.725 | BM1 | 26.7 | 0 | 379.739 |  |
| BM1 | 4.4 | 0 | 587.472 | BM1 | 27.475 | 0 | 841.056 |  |
| BM1 | 5.36667 | 313.1012 | 1173.557 | BM1 | 28.25 | 0 | 1366.39 |  |
| BM1 | 6.33333 | 2038.2112 | 1990.088 | BM1 | 29.025 | 1185.6444 | 2063.477 |  |


| BM1 | 7.3 | 4128.2373 | 1990.088 | BM1 | 29.8 | 2784.8389 | 2063.477 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| BM1 | 8.3 | 2934.28 | 1193.957 | BM1 | 30.76667 | 702.4129 | 2045.356 |
| BM1 | 9.3 | 2415.4041 | 458.085 | BM1 | 31.73333 | 0 | 1202.671 |
| BM1 | 10.1 | 2925.458 | 1072.361 | BM1 | 32.7 | 0 | 543.874 |
| BM1 | 10.7 | 3762.5693 | 1753.85 | BM1 | 33.3 | 0 | 467.794 |
| BM1 | 11.3 | 4814.8794 | 2404.137 | BM1 | 34.3 | 0 | 1153.729 |
| BM1 | 12.3 | 2121.793 | 2404.137 | BM1 | 35.3 | 400.1925 | 1998.106 |
| BM1 | 13.3 | 3.0204 | 1394.035 | BM1 | 36.3 | 2724.9836 | 1998.106 |
| BM1 | 14.3 | 0 | 643.795 | BM1 | 36.9 | 1985.345 | 1232.731 |
| BM1 | 14.9 | 0 | 392.746 | BM1 | 37.5 | 1354.1744 | 687.061 |
| BM1 | 15.86667 | 0 | 1088.573 | BM1 | 38.3 | 1020.3499 | 432.684 |
| BM1 | 16.83333 | 889.394 | 1971.798 | BM1 | 39.3 | 1645.4213 | 1219.206 |
| BM1 | 17.8 | 2877.7959 | 1980.798 | BM1 | 40.3 | 2864.6272 | 1611.13 |
| BM1 | 18.575 | 1342.677 | 1980.798 | BM1 | 41.26667 | 1145.9169 | 1611.13 |
| BM1 | 19.35 | 0 | 1279.16 | BM1 | 42.23333 | 0 | 855.309 |
| BM1 | 20.125 | 0 | 776.857 | BM1 | 43.2 | 0 | 251.598 |
| BM1 | 20.9 | 0 | 387.898 | BM1 | 46.1 | 965.0286 | 625.427 |
| BM1 | 21.75 | 0 | 322.331 | BM1 | 46.85 | 495.9582 | 625.427 |
| BM1 | 22.6 | 0 | 612.158 | BM1 | 47.6 | 90.3204 | 225.591 |


| Concrete Beam Design - Flexural and Shear Data |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Line | Location | Moment | VForce | Line | Location | Moment | VForce |
| BM2 | 0 | 73.9745 | 167.215 | BM2 | 23.2 | 0 | 799.128 |
| BM2 | 0.75 | 317.4003 | 498.818 | BM2 | 23.8 | 0 | 799.128 |
| BM2 | 1.5 | 691.5139 | 1041.837 | BM2 | 24.4 | 0 | 741.746 |
| BM2 | 2.25 | 0 | 1041.837 | BM2 | 25 | 0 | 482.958 |
| BM2 | 2.96667 | 0 | 668.54 | BM2 | 25.85 | 0 | 364.821 |
| BM2 | 3.68333 | 0 | 255.434 | BM2 | 26.7 | 0 | 898.424 |
| BM2 | 4.4 | 0 | 847.565 | BM2 | 27.475 | 0 | 1493.513 |
| BM2 | 5.36667 | 0 | 1729.636 | BM2 | 28.25 | 1094.8657 | 2233.919 |
| BM2 | 6.33333 | 1955.1146 | 2950.385 | BM2 | 29.025 | 3846.3511 | 3287.823 |
| BM2 | 7.3 | 5261.1094 | 2950.385 | BM2 | 29.8 | 3394.4141 | 3287.823 |


| BM2 | 8.3 | 3394.2986 | 1866.811 | BM2 | 30.76667 | 3235.0232 | 2972.933 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| BM2 | 9.3 | 2932.228 | 636.292 | BM2 | 31.73333 | 157.2197 | 1691.613 |
| BM2 | 10.1 | 3819.6294 | 1539.997 | BM2 | 32.7 | 0 | 733.971 |
| BM2 | 10.7 | 5065.6929 | 2522.88 | BM2 | 33.3 | 0 | 707.57 |
| BM2 | 11.3 | 5379.4204 | 3365.518 | BM2 | 34.3 | 0 | 1725.545 |
| BM2 | 12.3 | 2736.7808 | 3365.518 | BM2 | 35.3 | 1643.6257 | 3080.382 |
| BM2 | 13.3 | 0 | 1957.028 | BM2 | 36.3 | 5030.8989 | 3080.382 |
| BM2 | 14.3 | 0 | 906.721 | BM2 | 36.9 | 3612.4075 | 2364.152 |
| BM2 | 14.9 | 0 | 556.729 | BM2 | 37.5 | 2414.9114 | 1422.64 |
| BM2 | 15.86667 | 0 | 1501.107 | BM2 | 38.3 | 1572.1991 | 552.126 |
| BM2 | 16.83333 | 1762.4927 | 2735.123 | BM2 | 39.3 | 2026.1526 | 1869.368 |
| BM2 | 17.8 | 4593.4707 | 3093.605 | BM2 | 40.3 | 3895.5208 | 2825.452 |
| BM2 | 18.575 | 2195.927 | 3093.605 | BM2 | 41.26667 | 787.5793 | 2825.452 |
| BM2 | 19.35 | 0 | 2090.969 | BM2 | 42.23333 | 0 | 1639.875 |
| BM2 | 20.125 | 0 | 1382.414 | BM2 | 43.2 | 0 | 784.7 |
| BM2 | 20.9 | 0 | 808.769 | BM2 | 46.1 | 781.186 | 290.759 |
| BM2 | 21.75 | 0 | 289.667 | BM2 | 46.85 | 380.5763 | 700.34 |
| BM2 | 22.6 | 0 | 542.233 | BM2 | 47.6 | 95.0382 | 1074.097 |


| Concrete Beam Design - Flexural and Shear Data |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Line | Location | Moment | VForce | Line | Location | Moment | VForce |
| BM3 | 0 | 95.0382 | 248.819 | BM3 | 23.2 | 0 | 2828.97 |
| BM3 | 0.75 | 63.1156 | 775.72 | BM3 | 23.8 | 0 | 2828.97 |
| BM3 | 1.5 | 353.1603 | 1628.385 | BM3 | 24.4 | 0 | 1145.121 |
| BM3 | 2.25 | 934.9502 | 1628.385 | BM3 | 25 | 0 | 1069.867 |
| BM3 | 2.96667 | 0 | 1094.172 | BM3 | 25.85 | 1325.9716 | 2697.758 |
| BM3 | 3.68333 | 0 | 600.52 | BM3 | 26.7 | 3619.0662 | 2697.758 |
| BM3 | 4.4 | 0 | 568.805 | BM3 | 27.475 | 1892.75 | 1173.289 |
| BM3 | 5.36667 | 0 | 1470.663 | BM3 | 28.25 | 2108.5132 | 834.252 |
| BM3 | 6.33333 | 0 | 2701.164 | BM3 | 29.025 | 2900.9812 | 1756.984 |
| BM3 | 7.3 | 0 | 2701.164 | BM3 | 29.8 | 4358.5957 | 1756.984 |
| BM3 | 8.3 | 2770.5422 | 1825.292 | BM3 | 30.76667 | 2857.0759 | 1553.296 |


| BM3 | 9.3 | 1052.8959 | 740.562 | BM3 | 31.73333 | 2478.3311 | 909.692 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| BM3 | 10.1 | 876.2693 | 1693.465 | BM3 | 32.7 | 4074.9922 | 2999.288 |
| BM3 | 10.7 | 1892.634 | 2732.307 | BM3 | 33.3 | 3275.4197 | 2999.288 |
| BM3 | 11.3 | 3240.5862 | 3179.692 | BM3 | 34.3 | 702.8326 | 1513.352 |
| BM3 | 12.3 | 4279.9702 | 3179.692 | BM3 | 35.3 | 282.9864 | 1661.232 |
| BM3 | 13.3 | 0 | 1695.262 | BM3 | 36.3 | 1898.311 | 2528.377 |
| BM3 | 14.3 | 0 | 617.797 | BM3 | 36.9 | 348.4527 | 2528.377 |
| BM3 | 14.9 | 0 | 818.881 | BM3 | 37.5 | 0 | 1569.581 |
| BM3 | 15.86667 | 517.0086 | 1757.025 | BM3 | 38.3 | 0 | 662.925 |
| BM3 | 16.83333 | 3470.7236 | 3007.762 | BM3 | 39.3 | 0 | 1828.388 |
| BM3 | 17.8 | 3378.2275 | 3157.737 | BM3 | 40.3 | 1484.5371 | 2620.528 |
| BM3 | 18.575 | 3888.4209 | 3157.737 | BM3 | 41.26667 | 0 | 2620.528 |
| BM3 | 19.35 | 701.968 | 2090.335 | BM3 | 42.23333 | 0 | 1419.119 |
| BM3 | 20.125 | 0 | 1282.018 | BM3 | 43.2 | 0 | 537.223 |
| BM3 | 20.9 | 0 | 567.684 | BM3 | 46.1 | 960.0372 | 613.659 |
| BM3 | 21.75 | 0 | 1046.606 | BM3 | 46.85 | 370.5528 | 1104.23 |
| BM3 | 22.6 | 0 | 1603.691 | BM3 | 47.6 | 73.971 | 1638.074 |


| Concrete Beam Design - Flexural and Shear Data |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Line | Location | Moment | VForce | Line | Location | Moment | VForce |
| BM4 | 0 | 73.1277 | 259.028 | BM4 | 23.2 | 0 | 1154.16 |
| BM4 | 0.75 | 363.2524 | 808.128 | BM4 | 23.8 | 0 | 2879.362 |
| BM4 | 1.5 | 969.3486 | 1656.438 | BM4 | 24.4 | 0 | 2879.362 |
| BM4 | 2.25 | 0 | 1656.438 | BM4 | 25 | 0 | 1947.027 |
| BM4 | 2.96667 | 0 | 1108.301 | BM4 | 25.85 | 0 | 1080.722 |
| BM4 | 3.68333 | 0 | 610.61 | BM4 | 26.7 | 0 | 557.604 |
| BM4 | 4.4 | 0 | 547.043 | BM4 | 27.475 | 0 | 1281.429 |
| BM4 | 5.36667 | 0 | 1432.309 | BM4 | 28.25 | 771.1515 | 2097.15 |
| BM4 | 6.33333 | 0 | 2638.878 | BM4 | 29.025 | 3950.8416 | 3168.539 |
| BM4 | 7.3 | 1539.7615 | 2638.878 | BM4 | 29.8 | 6420.3164 | 3168.539 |
| BM4 | 8.3 | 0 | 1840.988 | BM4 | 30.76667 | 3506.1323 | 3014.673 |
| BM4 | 9.3 | 0 | 665.754 | BM4 | 31.73333 | 490.7888 | 1767.675 |


| BM4 | 10.1 | 0 | 1579.308 | BM4 | 32.7 | 0 | 838.248 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| BM4 | 10.7 | 289.7137 | 2545.826 | BM4 | 33.3 | 0 | 569.226 |
| BM4 | 11.3 | 1806.6143 | 2545.826 | BM4 | 34.3 | 0 | 1615.058 |
| BM4 | 12.3 | 125.2189 | 1687.252 | BM4 | 35.3 | 1108.6503 | 3046.458 |
| BM4 | 13.3 | 501.5182 | 1529.855 | BM4 | 36.3 | 4351.1816 | 3046.458 |
| BM4 | 14.3 | 3070.9209 | 3045.455 | BM4 | 36.9 | 2778.3748 | 2621.345 |
| BM4 | 14.9 | 4808.1938 | 3045.455 | BM4 | 37.5 | 1486.0627 | 1623.287 |
| BM4 | 15.86667 | 2372.175 | 921.418 | BM4 | 38.3 | 316.6129 | 705.89 |
| BM4 | 16.83333 | 2777.2473 | 1605.936 | BM4 | 39.3 | 704.9335 | 1767.183 |
| BM4 | 17.8 | 4329.6519 | 1790.369 | BM4 | 40.3 | 2380.8416 | 2599.291 |
| BM4 | 18.575 | 2872.6162 | 1790.369 | BM4 | 41.26667 | 0 | 2599.291 |
| BM4 | 19.35 | 2128.1301 | 838.485 | BM4 | 42.23333 | 0 | 1408.417 |
| BM4 | 20.125 | 1979.9324 | 1231.686 | BM4 | 43.2 | 0 | 1684.528 |
| BM4 | 20.9 | 4047.5427 | 2763.631 | BM4 | 46.1 | 989.0264 | 1684.528 |
| BM4 | 21.75 | 1698.4565 | 2763.631 | BM4 | 46.85 | 375.9979 | 817.371 |
| BM4 | 22.6 | 0 | 1093.621 | BM4 | 47.6 | 69.0685 | 261.821 |


| Concrete Beam Design - Flexural and Shear Data |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Line | Location | Moment | VForce | Line | Location | Moment | VForce |
| BM5 | 0 | 105.0317 | 185.545 | BM5 | 23.2 | 649.9258 | 1392.115 |
| BM5 | 0.75 | 408.5561 | 541.297 | BM5 | 23.8 | 1485.1948 | 1452.564 |
| BM5 | 1.5 | 814.5287 | 1085.923 | BM5 | 24.4 | 515.4133 | 1452.564 |
| BM5 | 2.25 | 0 | 1085.923 | BM5 | 25 | 0 | 1097.802 |
| BM5 | 2.96667 | 0 | 709.383 | BM5 | 25.85 | 0 | 662.114 |
| BM5 | 3.68333 | 0 | 295.731 | BM5 | 26.7 | 0 | 540.283 |
| BM5 | 4.4 | 0 | 789.677 | BM5 | 27.475 | 0 | 1182.859 |
| BM5 | 5.36667 | 0 | 1651.377 | BM5 | 28.25 | 0 | 1945.486 |
| BM5 | 6.33333 | 697.3499 | 2844.461 | BM5 | 29.025 | 1485.609 | 2990.113 |
| BM5 | 7.3 | 3801.3679 | 2844.461 | BM5 | 29.8 | 3802.9465 | 2990.113 |
| BM5 | 8.3 | 1924.9812 | 1876.387 | BM5 | 30.76667 | 957.0699 | 2783.013 |
| BM5 | 9.3 | 1402.7032 | 557.599 | BM5 | 31.73333 | 0 | 1525.554 |
| BM5 | 10.1 | 2222.98 | 1432.299 | BM5 | 32.7 | 0 | 570.645 |


| BM5 | 10.7 | 3401.252 | 2378.263 | BM5 | 33.3 | 0 | 892.065 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| BM5 | 11.3 | 4828.21 | 3090.264 | BM5 | 34.3 | 0 | 1934.332 |
| BM5 | 12.3 | 1392.0735 | 3090.264 | BM5 | 35.3 | 2308.3628 | 3329.831 |
| BM5 | 13.3 | 0 | 1727.467 | BM5 | 36.3 | 6149.9312 | 3329.831 |
| BM5 | 14.3 | 0 | 700.904 | BM5 | 36.9 | 4652.5171 | 2495.69 |
| BM5 | 14.9 | 0 | 761.077 | BM5 | 37.5 | 3457.2366 | 1521.457 |
| BM5 | 15.86667 | 0 | 1738.537 | BM5 | 38.3 | 2621.0415 | 625.025 |
| BM5 | 16.83333 | 2688.3987 | 3047.1 | BM5 | 39.3 | 3170.0017 | 1869.192 |
| BM5 | 17.8 | 5888.8506 | 3178.93 | BM5 | 40.3 | 5039.1934 | 2969.159 |
| BM5 | 18.575 | 3425.1794 | 3178.93 | BM5 | 41.26667 | 1740.4032 | 2969.159 |
| BM5 | 19.35 | 839.3829 | 2084.724 | BM5 | 42.23333 | 0 | 1743.397 |
| BM5 | 20.125 | 0 | 1291.027 | BM5 | 43.2 | 0 | 854.038 |
| BM5 | 20.9 | 0 | 628.149 | BM5 | 46.1 | 755.8152 | 1076.964 |
| BM5 | 21.75 | 0 | 595.315 | BM5 | 46.85 | 360.9893 | 526.434 |
| BM5 | 22.6 | 0 | 1035.898 | BM5 | 47.6 | 87.4316 | 177.777 |


| Concrete Beam Design - Flexural and Shear Data |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| Line | Location | Moment | VForce | Line | Location | Moment | VForce |  |
| BM6 | 0 | 100.6418 | 240.5 | BM6 | 23.2 | 299.6552 | 1425.055 |  |
| BM6 | 0.75 | 543.9094 | 665.131 | BM6 | 23.8 | 1188.2219 | 1425.055 |  |
| BM6 | 1.5 | 1042.7578 | 665.131 | BM6 | 24.4 | 394.1645 | 1323.429 |  |
| BM6 | 2.25 | 383.4478 | 321.308 | BM6 | 25 | 0 | 1000.168 |  |
| BM6 | 2.96667 | 117.4351 | 232.015 | BM6 | 25.85 | 0 | 639.139 |  |
| BM6 | 3.68333 | 0 | 813.355 | BM6 | 26.7 | 0 | 249.688 |  |
| BM6 | 4.4 | 0 | 813.355 | BM6 | 27.475 | 0 | 699.907 |  |
| BM6 | 5.36667 | 0 | 285.987 | BM6 | 28.25 | 0 | 1230.007 |  |
| BM6 | 6.33333 | 0 | 841.938 | BM6 | 29.025 | 568.035 | 1942.767 |  |
| BM6 | 7.3 | 1089.9308 | 1605.928 | BM6 | 29.8 | 2073.6794 | 1942.767 |  |
| BM6 | 8.3 | 2805.5825 | 1605.928 | BM6 | 30.76667 | 232.5797 | 1893.897 |  |
| BM6 | 9.3 | 1591.6952 | 1213.887 | BM6 | 31.73333 | 0 | 1021.925 |  |
| BM6 | 10.1 | 969.1935 | 423.814 | BM6 | 32.7 | 0 | 346.51 |  |
| BM6 | 10.7 | 1304.5452 | 697.845 | BM6 | 33.3 | 0 | 669.987 |  |


| BM6 | 11.3 | 1943.8497 | 1241.878 | BM6 | 34.3 | 0 | 1408.088 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| BM6 | 12.3 | 2688.9763 | 1971.231 | BM6 | 35.3 | 1982.2833 | 2403.912 |
| BM6 | 13.3 | 386.0539 | 1971.231 | BM6 | 36.3 | 4692.1733 | 2403.912 |
| BM6 | 14.3 | 0 | 1130.903 | BM6 | 36.9 | 3653.2021 | 1731.619 |
| BM6 | 14.9 | 0 | 447.96 | BM6 | 37.5 | 2843.6355 | 1055.802 |
| BM6 | 15.86667 | 0 | 560.038 | BM6 | 38.3 | 2376.0083 | 446.832 |
| BM6 | 16.83333 | 0 | 1219.102 | BM6 | 39.3 | 2916.9509 | 1199.374 |
| BM6 | 17.8 | 522.2141 | 2068.166 | BM6 | 40.3 | 4116.3247 | 2000.871 |
| BM6 | 18.575 | 2598.4333 | 2068.166 | BM6 | 41.26667 | 0 | 1177.176 |
| BM6 | 19.35 | 1046.4784 | 2002.522 | BM6 | 42.23333 | 0 | 497 |
| BM6 | 20.125 | 0 | 1290.426 | BM6 | 43.2 | 119.6548 | 784.451 |
| BM6 | 20.9 | 0 | 746.665 | BM6 | 46.1 | 800.4507 | 784.451 |
| BM6 | 21.75 | 0 | 261.246 | BM6 | 46.85 | 442.0038 | 477.929 |
| BM6 | 22.6 | 0 | 719.667 | BM6 | 47.6 | 88.957 | 176.427 |

## APPENDIX B

## MATERIAL PROPERTIES

## B. 1 Concrete Properties

Table B. 1 Concrete properties

| C-30 |  |  |  |
| :---: | :---: | :---: | :---: |
| Compression |  |  |  |
| бc | ein | dc | epl |
| 15.19998722 | 0 |  |  |
| 21.45086476 | $9.14195 \mathrm{E}-05$ |  |  |
| 26.72372722 | 0.000180841 |  |  |
| 31.00522546 | 0.000300452 |  |  |
| 34.2817663 | 0.000450669 |  |  |
| 36.53950688 | 0.000631912 |  |  |
| 37.76434896 | 0.000844611 |  |  |
| 38 | 0.00100463 | 0 | 0 |
| 37.46904943 | 0.0012708 | 0.02631 | 0.00124 |
| 35.86627915 | 0.00156961 | 0.03405 | 0.00153 |
| 33.1766019 | 0.001901522 | 0.04322 | 0.00186 |
| 29.38464592 | 0.002267001 | 0.05397 | 0.00222 |
| 22.58865097 | 0.002807225 | 0.07092 | 0.00275 |
| 22.4735262 | 0.00281564 | 0.0712 | 0.00276 |


| C-30 |  |  |  |
| :---: | :---: | :---: | :---: |
| Tension |  |  |  |
| бt | $\varepsilon c r$ | dt | $\varepsilon \mathrm{cr}$ |
| 2.896468154 | 0 | 0 | 0 |
| 2.620832766 | 0.000108394 | 0.05611099 | 0.000108394 |
| 2.371427553 | 0.00021599 | 0.111125953 | 0.00021599 |
| 2.145756384 | 0.000322862 | 0.164572899 | 0.000322862 |
| 1.941560666 | 0.000429081 | 0.216122779 | 0.000429081 |
| 1.75679674 | 0.000534707 | 0.265556052 | 0.000534707 |
| 1.589615426 | 0.000639799 | 0.31273694 | 0.000639799 |
| 1.438343518 | 0.000744406 | 0.357593517 | 0.000744406 |
| 1.301467035 | 0.000848574 | 0.40010228 | 0.000848574 |
| 1.177616072 | 0.000952346 | 0.440276166 | 0.000952346 |
| 1.065551086 | 0.001055758 | 0.478155215 | 0.001055758 |
| 0.964150493 | 0.001158847 | 0.513799306 | 0.001158847 |
| 0.872399443 | 0.001261641 | 0.547282494 | 0.001261641 |
| 0.789379659 | 0.001364169 | 0.578688596 | 0.001364169 |
| 0.714260253 | 0.001466457 | 0.608107764 | 0.001466457 |
| 0.646289403 | 0.001568527 | 0.635633829 | 0.001568527 |
| 0.584786835 | 0.0016704 | 0.661362248 | 0.0016704 |
| 0.52913701 | 0.001772094 | 0.685388538 | 0.001772094 |
| 0.478782966 | 0.001873628 | 0.707807082 | 0.001873628 |
| 0.456896099 | 0.001921086 | 0.71777107 | 0.001921086 |
|  |  |  |  |

## B. 2 Steel Properties

Table B. 2 Steel properties

| fy | $\varepsilon \mathrm{t}$ | бtrue | $\varepsilon \mathrm{pl}$ |
| :---: | :---: | :---: | :---: |
| 0 | 0 |  |  |
| 466 | 0.00211 | 466.983 | 0 |
| 466 | 0.005 | 468.33 | 0.00287 |
| 466 | 0.04 | 484.64 | 0.03703 |
| 484.64844 | 0.045 | 506.458 | 0.04173 |
| 502.09375 | 0.05 | 527.198 | 0.0464 |
| 518.33594 | 0.055 | 546.844 | 0.05107 |
| 533.375 | 0.06 | 565.378 | 0.05571 |
| 547.21094 | 0.065 | 582.78 | 0.06034 |
| 559.84375 | 0.07 | 599.033 | 0.06495 |
| 571.27344 | 0.075 | 614.119 | 0.06954 |
| 581.5 | 0.08 | 628.02 | 0.07412 |
| 590.52344 | 0.085 | 640.718 | 0.07868 |
| 598.34375 | 0.09 | 652.195 | 0.08323 |
| 604.96094 | 0.095 | 662.432 | 0.08776 |
| 610.375 | 0.1 | 671.413 | 0.09227 |
| 614.58594 | 0.105 | 679.117 | 0.09677 |
| 617.59375 | 0.11 | 685.529 | 0.10126 |
| 619.39844 | 0.115 | 690.629 | 0.10573 |
| 620 | 0.12 | 694.4 | 0.11019 |

## B. 3 Validation ABAQUS VS Experimental

Table B. 3 Validation ABAQUS VS Experimental

| Abaqus |  | Experiment |  |
| :---: | :---: | :---: | :---: |
| Disp. | Load | Disp. | Load |
| 0 | 0 | 0 | 0 |
| 0.042259 | 4.47168 | 0.118065 | 3.60375 |
| 0.135163 | 7.039 | 0.248393 | 5.1123 |
| 0.143073 | 9.452 | 0.378722 | 6.87228 |
| 0.159313 | 10.3288 | 0.50905 | 8.38082 |
| 0.162433 | 14.8749 | 0.639378 | 11.1465 |
| 0.175895 | 17.9387 | 0.769706 | 14.1636 |
| 0.188132 | 20.8145 | 0.900034 | 16.175 |
| 0.19967 | 25.5856 | 1.46479 | 23.2149 |
| 0.206682 | 27.6528 | 1.59512 | 25.4777 |
| 0.2239 | 40.3305 | 2.11643 | 37.2947 |
| 0.240138 | 46.7455 | 2.24676 | 39.5575 |
| 0.248901 | 47.9899 | 2.37709 | 41.066 |


| 0.249125 | 48.0217 | 2.50741 | 43.3289 |
| :---: | :---: | :---: | :---: |
| 0.249576 | 48.0853 | 2.63774 | 44.8374 |
| 0.249917 | 48.133 | 2.76807 | 47.3517 |
| 0.25121 | 48.3118 | 2.8984 | 50.3688 |
| 0.252371 | 48.4727 | 3.02873 | 52.883 |
| 0.256681 | 49.0757 | 3.15906 | 54.3916 |
| 0.260512 | 49.6179 | 3.28938 | 55.6487 |
| 0.274658 | 51.6447 | 3.41971 | 56.403 |
| 0.287023 | 53.4575 | 3.55004 | 57.4086 |
| 0.330944 | 60.1038 | 3.68037 | 58.6658 |
| 0.349848 | 62.9851 | 3.8107 | 60.1743 |
| 0.368532 | 65.6062 | 3.94102 | 61.18 |
| 0.384743 | 67.8893 | 4.07135 | 62.6886 |
| 0.46518 | 75.6804 | 4.20168 | 64.9514 |
| 0.506423 | 79.6712 | 4.33201 | 66.7114 |
| 0.552787 | 84.2048 | 4.46234 | 68.4713 |
| 0.558472 | 84.6033 | 4.59267 | 70.2313 |
| 0.586919 | 85.9887 | 4.72299 | 71.4884 |
| 0.612981 | 87.0881 | 4.85332 | 72.7456 |
| 0.648696 | 89.5701 | 4.98365 | 74.2541 |
| 0.65932 | 90.3803 | 5.11398 | 75.0084 |
| 0.674938 | 91.4907 | 5.24431 | 76.0141 |
| 0.675071 | 91.5 | 5.37463 | 76.5169 |
| 0.675071 | 91.5 | 5.50496 | 77.0198 |
| 0.675071 | 91.5 | 5.63529 | 78.2769 |
| 0.675071 | 91.5 | 5.93939 | 79.534 |
| 0.675071 | 91.5 | 6.02627 | 78.7798 |

## APPENDIX C

## LOADS FROM SAFE MODEL OF THE OPTIMIZED DESIGN

Table Concrete beam design flexural and shear data of optimized model

| TABLE: Concrete Beam Design 01 - Flexural And Shear Data |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Line | Location | FTopMoment | FTopArea | FBotMoment | FBotArea | VForce | VArea |
| Text | m | kN-m | mm2 | kN-m | mm 2 | kN | $\mathrm{mm} 2 / \mathrm{m}$ |
| BM1 | 0 | 0 | 0 | 67.5605 | 0 | 150.732 | 640 |
| BM1 | 0.75 | 0 | 0 | 349.7585 | 347.615 | 413.099 | 640 |
| BM1 | 1.5 | 0 | 0 | 659.583 | 670.885 | 709.118 | 814.468 |
| BM1 | 2.25 | -312.3839 | 215.135 | 42.9362 | 0 | 709.118 | 814.468 |
| BM1 | 2.96667 | -845.2275 | 770.531 | 0 | 0 | 448.049 | 640 |
| BM1 | 3.68333 | -997.3287 | 930.673 | 0 | 0 | 160.959 | 640 |
| BM1 | 4.4 | -881.9749 | 809.156 | 0 | 0 | 562.372 | 645.921 |
| BM1 | 5.36667 | -195.6352 | 0 | 194.1591 | 0 | 1135.238 | 1303.894 |
| BM1 | 6.33333 | 0 | 0 | 1834.1584 | 1784.884 | 1948.521 | 2238.001 |
| BM1 | 7.3 | 0 | 0 | 3862.4956 | 3996.577 | 1948.521 | 2238.001 |
| BM1 | 8.3 | 0 | 0 | 2674.6926 | 2686.468 | 1187.803 | 1364.268 |
| BM1 | 9.3 | 0 | 0 | 2163.6609 | 2133.394 | 450.125 | 640 |
| BM1 | 10.1 | 0 | 0 | 2665.4846 | 2679.568 | 1059.408 | 1216.799 |
| BM1 | 10.7 | 0 | 0 | 3495.0784 | 3591.466 | 1746.14 | 2005.554 |
| BM1 | 11.3 | 0 | 0 | 4542.7622 | 4764.933 | 2357.108 | 2707.29 |
| BM1 | 12.3 | 0 | 0 | 1921.1703 | 1889.289 | 2357.108 | 2707.29 |
| BM1 | 13.3 | -585.7817 | 505.897 | 0 | 0 | 1350.333 | 1550.944 |
| BM1 | 14.3 | -1384.3018 | 1362.855 | 0 | 0 | 616.655 | 708.268 |
| BM1 | 14.9 | -1445.9135 | 1428.312 | 0 | 0 | 391.144 | 640 |
| BM1 | 15.86667 | -1007.2254 | 972.633 | 0 | 0 | 1071.105 | 1230.233 |
| BM1 | 16.83333 | 0 | 0 | 843.9944 | 842.204 | 1950.702 | 2240.506 |
| BM1 | 17.8 | 0 | 0 | 2810.8223 | 2978.309 | 1950.702 | 2240.506 |
| BM1 | 18.575 | 0 | 0 | 1301.7448 | 1357.557 | 1947.197 | 2236.481 |
| BM1 | 19.35 | -355.9315 | 409.139 | 0 | 0 | 1243.831 | 1428.619 |
| BM1 | 20.125 | -1115.4055 | 1217.406 | 0 | 0 | 749.489 | 860.837 |
| BM1 | 20.9 | -1427.3716 | 1552.253 | 0 | 0 | 370.24 | 640 |
| BM1 | 21.75 | -1455.8373 | 1582.435 | 0 | 0 | 318.655 | 640 |
| BM1 | 22.6 | -1039.1367 | 1142.266 | 0 | 0 | 601.001 | 690.289 |
| BM1 | 23.2 | -360.7548 | 442.052 | 0 | 0 | 816.934 | 938.301 |
| BM1 | 23.8 | 0 | 0 | 483.537 | 581.667 | 1060.944 | 1218.563 |
| BM1 | 24.4 | -669.1497 | 801.705 | 0 | 0 | 1060.944 | 1218.563 |
| BM1 | 25 | -1630.1725 | 1810.455 | 0 | 0 | 832.519 | 956.202 |
| BM1 | 25.85 | -2330.2983 | 2544.598 | 0 | 0 | 515.073 | 640 |
| BM1 | 26.7 | -2394.2869 | 2613.612 | 0 | 0 | 366.294 | 640 |
| BM1 | 27.475 | -2042.8064 | 2216.222 | 0 | 0 | 816.099 | 937.342 |
| BM1 | 28.25 | -1053.2578 | 1144.188 | 0 | 0 | 1332.73 | 1530.727 |
| BM1 | 29.025 | 0 | 0 | 1222.8474 | 1283.324 | 2032.051 | 2333.941 |
| BM1 | 29.8 | 0 | 0 | 2797.6868 | 2972.948 | 2032.051 | 2333.941 |
|  |  |  |  |  |  |  |  |


| BM1 | 30.76667 | -56.2235 | 0 | 743.2501 | 757.6 | 2017.925 | 2317.717 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BM1 | 31.73333 | -1812.9678 | 1848.81 | 0 | 0 | 1177.288 | 1352.191 |
| BM1 | 32.7 | -2561.8745 | 2639.717 | 0 | 0 | 533.224 | 640 |
| BM1 | 33.3 | -2589.4653 | 2669.685 | 0 | 0 | 450.047 | 640 |
| BM1 | 34.3 | -2021.1992 | 2040.473 | 0 | 0 | 1119.304 | 1285.592 |
| BM1 | 35.3 | -385.6685 | 293.888 | 381.5413 | 281.489 | 1958.182 | 2249.098 |
| BM1 | 36.3 | 0 | 0 | 2639.4363 | 2667.96 | 1958.182 | 2249.098 |
| BM1 | 36.9 | 0 | 0 | 1900.1379 | 1868.908 | 1232.164 | 1415.219 |
| BM1 | 37.5 | 0 | 0 | 1263.1769 | 1193.38 | 683.339 | 784.859 |
| BM1 | 38.3 | 0 | 0 | 912.4658 | 825.497 | 421.023 | 640 |
| BM1 | 39.3 | 0 | 0 | 1505.8408 | 1453.929 | 1203.927 | 1382.788 |
| BM1 | 40.3 | 0 | 0 | 2709.7681 | 2750.978 | 1583.52 | 1818.774 |
| BM1 | 41.26667 | 0 | 0 | 1026.553 | 954.835 | 1583.52 | 1818.774 |
| BM1 | 42.23333 | -637.0843 | 573.027 | 0 | 0 | 831.997 | 955.603 |
| BM1 | 43.2 | -1005.8351 | 971.138 | 0 | 0 | 246.441 | 640 |
| BM1 | 43.91667 | -829.219 | 785.261 | 0 | 0 | 745.022 | 855.706 |
| BM1 | 44.6 | -933.5771 | 934.01 | 0 | 0 | 745.022 | 855.706 |
| BM1 | 44.63333 | -933.5607 | 933.993 | 62.5388 | 0 | 213.47 | 640 |
| BM1 | 45.35 | 0 | 0 | 299.6003 | 275.048 | 301.468 | 640 |
| BM1 | 46.1 | 0 | 0 | 932.505 | 948.135 | 614.432 | 705.715 |
| BM1 | 46.85 | 0 | 0 | 471.6809 | 465.504 | 614.432 | 705.715 |
| BM1 | 47.6 | 0 | 0 | 83.2851 | 0 | 220.925 | 640 |
| BM2 | 0 | 0 | 0 | 71.355 | 0 | 163.849 | 640 |
| BM2 | 0.75 | 0 | 0 | 308.1717 | 323.161 | 491.281 | 640 |
| BM2 | 1.5 | 0 | 0 | 676.6321 | 707.408 | 1023.821 | 1175.924 |
| BM2 | 2.25 | -1105.0945 | 1056.551 | 0 | 0 | 1023.821 | 1175.924 |
| BM2 | 2.96667 | -2140.252 | 2165.8 | 0 | 0 | 656.984 | 754.589 |
| BM2 | 3.68333 | -2593.4226 | 2659.608 | 0 | 0 | 255.268 | 640 |
| BM2 | 4.4 | -2453.1868 | 2507.405 | 0 | 0 | 816.2 | 937.458 |
| BM2 | 5.36667 | -1502.5072 | 1489.792 | 0 | 0 | 1680.448 | 1930.102 |
| BM2 | 6.33333 | 0 | 0 | 1780.8652 | 1794.189 | 2899.323 | 3330.059 |
| BM2 | 7.3 | 0 | 0 | 4992.646 | 5331.795 | 2899.323 | 3330.059 |
| BM2 | 8.3 | 0 | 0 | 3131.0588 | 3246.969 | 1861.587 | 2138.152 |
| BM2 | 9.3 | 0 | 0 | 2638.0979 | 2713.735 | 627.616 | 720.858 |
| BM2 | 10.1 | 0 | 0 | 3505.1365 | 3665.142 | 1526.422 | 1753.194 |
| BM2 | 10.7 | 0 | 0 | 4741.9941 | 5050.815 | 2517.848 | 2891.91 |
| BM2 | 11.3 | 0 | 0 | 6252.7026 | 5495.622 | 3306.077 | 3797.242 |
| BM2 | 12.3 | 0 | 0 | 2503.1807 | 2597.007 | 3306.077 | 3797.242 |
| BM2 | 13.3 | -1406.3018 | 1423.731 | 0 | 0 | 1899.769 | 2182.007 |
| BM2 | 14.3 | -2522.1221 | 2623.194 | 0 | 0 | 870.942 | 1000.333 |
| BM2 | 14.9 | -2605.2966 | 2713.468 | 0 | 0 | 553.36 | 640 |
| BM2 | 15.86667 | -1746.7091 | 1790.486 | 0 | 0 | 1478.12 | 1697.716 |
| BM2 | 16.83333 | 0 | 0 | 1680.0142 | 1722.372 | 2710.84 | 3113.573 |
| BM2 | 17.8 | 0 | 0 | 4493.939 | 4801.112 | 3042.647 | 3494.675 |
| BM2 | 18.575 | 0 | 0 | 2135.8877 | 2202.217 | 3042.647 | 3494.675 |
| BM2 | 19.35 | -1004.9515 | 992.853 | 0 | 0 | 2035.264 | 2337.632 |


| BM2 | 20.125 | -2521.1528 | 2609.035 | 0 | 0 | 1337.104 | 1535.749 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BM2 | 20.9 | -3100.0339 | 3144.541 | 0 | 0 | 778.167 | 893.775 |
| BM2 | 21.75 | -3334.1199 | 3152.551 | 0 | 0 | 275.395 | 640 |
| BM2 | 22.6 | -2940.1023 | 3052.632 | 0 | 0 | 529.997 | 640 |
| BM2 | 23.2 | -2065.1667 | 2096.293 | 0 | 0 | 783.035 | 899.366 |
| BM2 | 23.8 | -1077.8965 | 1039.372 | 0 | 0 | 783.035 | 899.366 |
| BM2 | 24.4 | -1833.7498 | 1853.785 | 0 | 0 | 731.825 | 840.548 |
| BM2 | 25 | -2502.5828 | 2585.948 | 0 | 0 | 476.833 | 640 |
| BM2 | 25.85 | -2787.3101 | 2905.727 | 0 | 0 | 344.947 | 640 |
| BM2 | 26.7 | -2494.1055 | 2586.975 | 0 | 0 | 862.839 | 991.026 |
| BM2 | 27.475 | -1602.2167 | 1635.959 | 0 | 0 | 1443.942 | 1658.46 |
| BM2 | 28.25 | 0 | 0 | 1016.7756 | 1022.899 | 2174.943 | 2498.062 |
| BM2 | 29.025 | 0 | 0 | 3652.6577 | 3871.882 | 3235.369 | 3716.029 |
| BM2 | 29.8 | 0 | 0 | 5160.0684 | 5228.496 | 3235.369 | 3716.029 |
| BM2 | 30.76667 | 0 | 0 | 3033.6292 | 3197.51 | 2940.286 | 3377.107 |
| BM2 | 31.73333 | -1105.4896 | 1119.127 | 64.157 | 0 | 1658.315 | 1904.681 |
| BM2 | 32.7 | -2328.0776 | 2421.96 | 0 | 0 | 719.288 | 826.149 |
| BM2 | 33.3 | -2344.4434 | 2439.641 | 0 | 0 | 684.266 | 785.923 |
| BM2 | 34.3 | -1634.824 | 1676.069 | 0 | 0 | 1681.028 | 1930.769 |
| BM2 | 35.3 | 0 | 0 | 1501.729 | 1533.89 | 3032.621 | 3483.161 |
| BM2 | 36.3 | 0 | 0 | 4821.541 | 5159.957 | 3032.621 | 3483.161 |
| BM2 | 36.9 | 0 | 0 | 3407.009 | 3574.954 | 2357.553 | 2707.802 |
| BM2 | 37.5 | 0 | 0 | 2221.7012 | 2284.519 | 1408.79 | 1618.086 |
| BM2 | 38.3 | 0 | 0 | 1402.5234 | 1410.493 | 543.818 | 640 |
| BM2 | 39.3 | 0 | 0 | 1897.1588 | 1938.963 | 1862.679 | 2139.407 |
| BM2 | 40.3 | 0 | 0 | 3759.8379 | 3971.992 | 2779.454 | 3192.381 |
| BM2 | 41.26667 | -60.8966 | 0 | 735.1032 | 724.995 | 2779.454 | 3192.381 |
| BM2 | 42.23333 | -2143.6428 | 2217.711 | 0 | 0 | 1596.116 | 1833.242 |
| BM2 | 43.2 | -2903.4932 | 3039.875 | 0 | 0 | 758.574 | 871.271 |
| BM2 | 43.91667 | -3015.2708 | 3162.145 | 0 | 0 | 285.97 | 640 |
| BM2 | 44.63333 | -2426.1133 | 2518.328 | 0 | 0 | 684.385 | 786.06 |
| BM2 | 45.35 | -1249.9349 | 1255.644 | 0 | 0 | 1051.964 | 1208.248 |
| BM2 | 46.1 | 0 | 0 | 766.7386 | 795.706 | 1051.964 | 1208.248 |
| BM2 | 46.85 | 0 | 0 | 371.5627 | 382.982 | 526.901 | 640 |
| BM2 | 47.6 | 0 | 0 | 92.2861 | 0 | 179.155 | 640 |
| BM3 | 0 | 0 | 0 | 60.4744 | 0 | 246.347 | 640 |
| BM3 | 0.75 | 0 | 0 | 349.9428 | 380.171 | 771.64 | 886.279 |
| BM3 | 1.5 | 0 | 0 | 928.6732 | 985.18 | 1597.136 | 1834.414 |
| BM3 | 2.25 | -1287.1567 | 1249.91 | 0 | 0 | 1597.136 | 1834.414 |
| BM3 | 2.96667 | -2684.6001 | 2757.992 | 0 | 0 | 1067.669 | 1226.286 |
| BM3 | 3.68333 | -3269.2761 | 3122.564 | 0 | 0 | 585.385 | 672.353 |
| BM3 | 4.4 | -3335.0046 | 3165.31 | 0 | 0 | 550.929 | 640 |
| BM3 | 5.36667 | -2962.4888 | 3068.968 | 0 | 0 | 1433.594 | 1646.574 |
| BM3 | 6.33333 | -1039.3751 | 1008.219 | 0 | 0 | 2660.464 | 3055.714 |
| BM3 | 7.3 | 0 | 0 | 2697.5723 | 2776.514 | 2660.464 | 3055.714 |
| BM3 | 8.3 | 0 | 0 | 941.3153 | 900.363 | 1822.777 | 2093.577 |


| BM3 | 9.3 | 0 | 0 | 736.0895 | 686.316 | 725.769 | 833.593 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BM3 | 10.1 | 0 | 0 | 1717.9908 | 1725.427 | 1673.014 | 1921.564 |
| BM3 | 10.7 | 0 | 0 | 3047.0566 | 3162.189 | 2719.68 | 3123.728 |
| BM3 | 11.3 | 0 | 0 | 4678.8647 | 4981.774 | 3137.026 | 3603.076 |
| BM3 | 12.3 | 0 | 0 | 1353.4393 | 1362.419 | 3137.026 | 3603.076 |
| BM3 | 13.3 | -1412.5477 | 1417.665 | 0 | 0 | 1655.611 | 1901.576 |
| BM3 | 14.3 | -2012.6492 | 2058.546 | 0 | 0 | 600.101 | 689.255 |
| BM3 | 14.9 | -1924.0261 | 1960.688 | 0 | 0 | 797.505 | 915.986 |
| BM3 | 15.86667 | -643.309 | 600.54 | 398.5929 | 340.972 | 1716.565 | 1971.585 |
| BM3 | 16.83333 | 0 | 0 | 3230.7507 | 3369.478 | 2966.903 | 3407.678 |
| BM3 | 17.8 | 0 | 0 | 5598.7563 | 5422.746 | 3109.183 | 3571.096 |
| BM3 | 18.575 | 0 | 0 | 3660.6309 | 3830.157 | 3109.183 | 3571.096 |
| BM3 | 19.35 | -688.9246 | 614.918 | 621.2039 | 549.349 | 2037.455 | 2340.148 |
| BM3 | 20.125 | -2748.4866 | 2811.068 | 0 | 0 | 1241.284 | 1425.695 |
| BM3 | 20.9 | -3552.2029 | 3135.781 | 0 | 0 | 544.783 | 640 |
| BM3 | 21.75 | -3562.6216 | 3169.158 | 0 | 0 | 1027.577 | 1180.238 |
| BM3 | 22.6 | -3549.564 | 3183.259 | 0 | 0 | 1872.341 | 2150.504 |
| BM3 | 23.2 | -2493.5842 | 2526.09 | 0 | 0 | 2797.559 | 3213.177 |
| BM3 | 23.8 | -815.0486 | 733.617 | 0 | 0 | 2797.559 | 3213.177 |
| BM3 | 24.4 | -1422.7858 | 1352.992 | 0 | 0 | 1183.312 | 1359.11 |
| BM3 | 25 | -1266.7281 | 1192.537 | 0 | 0 | 1015.441 | 1166.3 |
| BM3 | 25.85 | 0 | 0 | 1228.8381 | 1162.248 | 2625.08 | 3015.074 |
| BM3 | 26.7 | 0 | 0 | 3460.1565 | 3577.544 | 2625.08 | 3015.074 |
| BM3 | 27.475 | 0 | 0 | 1695.3376 | 1699.482 | 1157.125 | 1329.032 |
| BM3 | 28.25 | 0 | 0 | 1877.1823 | 1896.029 | 812.797 | 933.55 |
| BM3 | 29.025 | 0 | 0 | 2605.0007 | 2683.862 | 1723.894 | 1980.003 |
| BM3 | 29.8 | 0 | 0 | 4027.72 | 4264.545 | 1723.894 | 1980.003 |
| BM3 | 30.76667 | 0 | 0 | 2569.9673 | 2659.487 | 1530.134 | 1757.458 |
| BM3 | 31.73333 | 0 | 0 | 2203.1606 | 2262.63 | 890.793 | 1023.134 |
| BM3 | 32.7 | 0 | 0 | 4785.7534 | 5134.445 | 2937.597 | 3374.019 |
| BM3 | 33.3 | 0 | 0 | 3023.1948 | 3168.596 | 2937.597 | 3374.019 |
| BM3 | 34.3 | 0 | 0 | 525.824 | 498.375 | 1465.893 | 1683.672 |
| BM3 | 35.3 | 0 | 0 | 176.9735 | 0 | 1672.475 | 1920.945 |
| BM3 | 36.3 | 0 | 0 | 1849.4484 | 1898.596 | 2503.131 | 2875.007 |
| BM3 | 36.9 | 0 | 0 | 342.9839 | 301.944 | 2503.131 | 2875.007 |
| BM3 | 37.5 | -670.1829 | 644.41 | 0 | 0 | 1541.165 | 1770.127 |
| BM3 | 38.3 | -1184.4878 | 1185.548 | 0 | 0 | 642.881 | 738.39 |
| BM3 | 39.3 | -573.5004 | 542.549 | 0 | 0 | 1827.538 | 2099.044 |
| BM3 | 40.3 | 0 | 0 | 1534.8949 | 1554.098 | 2581.204 | 2964.679 |
| BM3 | 41.26667 | -1702.4564 | 1745.162 | 0 | 0 | 2581.204 | 2964.679 |
| BM3 | 42.23333 | -3430.646 | 3118.921 | 0 | 0 | 1384.05 | 1589.671 |
| BM3 | 43.2 | -3434.7393 | 3148.477 | 0 | 0 | 521.476 | 640 |
| BM3 | 43.91667 | -3385.4087 | 3110.94 | 0 | 0 | 596.584 | 685.216 |
| BM3 | 44.63333 | -2909.7903 | 3044.969 | 0 | 0 | 1075.909 | 1235.75 |
| BM3 | 45.35 | -1427.1752 | 1445.95 | 0 | 0 | 1605.165 | 1843.635 |
| BM3 | 46.1 | 0 | 0 | 954.5664 | 998.776 | 1605.165 | 1843.635 |


| BM3 | 46.85 | 0 | 0 | 367.9367 | 385.124 | 782.173 | 898.376 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BM3 | 47.6 | 0 | 0 | 71.3033 | 0 | 250.66 | 640 |
| BM4 | 0 | 0 | 0 | 70.4861 | 0 | 256.584 | 640 |
| BM4 | 0.75 | 0 | 0 | 360.5829 | 377.06 | 804.26 | 923.744 |
| BM4 | 1.5 | 0 | 0 | 963.7776 | 1008.056 | 1623.9 | 1865.154 |
| BM4 | 2.25 | -1423.4536 | 1437.202 | 0 | 0 | 1623.9 | 1865.154 |
| BM4 | 2.96667 | -2902.6936 | 3031.605 | 0 | 0 | 1080.343 | 1240.843 |
| BM4 | 3.68333 | -3571.1428 | 3160.249 | 0 | 0 | 593.984 | 682.229 |
| BM4 | 4.4 | -3512.1226 | 3143.89 | 0 | 0 | 530.623 | 640 |
| BM4 | 5.36667 | -3099.1865 | 3074.728 | 0 | 0 | 1396.475 | 1603.941 |
| BM4 | 6.33333 | -1656.3647 | 1682.833 | 0 | 0 | 2598.801 | 2984.89 |
| BM4 | 7.3 | 0 | 0 | 1588.4489 | 1586.694 | 2598.801 | 2984.89 |
| BM4 | 8.3 | -563.2073 | 503.331 | 0 | 0 | 1840.142 | 2113.521 |
| BM4 | 9.3 | -1206.9825 | 1177.725 | 0 | 0 | 645.524 | 741.426 |
| BM4 | 10.1 | -690.5633 | 633.854 | 0 | 0 | 1550.613 | 1780.979 |
| BM4 | 10.7 | 0 | 0 | 285.9036 | 208.265 | 2520.302 | 2894.729 |
| BM4 | 11.3 | 0 | 0 | 1797.2145 | 1800.044 | 2520.302 | 2894.729 |
| BM4 | 12.3 | 0 | 0 | 70.6885 | 0 | 1698.487 | 1950.822 |
| BM4 | 13.3 | -50.3114 | 0 | 337.3241 | 254.207 | 1481.418 | 1701.504 |
| BM4 | 14.3 | 0 | 0 | 2829.3743 | 2908.815 | 2982.315 | 3425.381 |
| BM4 | 14.9 | 0 | 0 | 4618.7637 | 4899.043 | 2982.315 | 3425.381 |
| BM4 | 15.86667 | 0 | 0 | 2105.3325 | 2117.807 | 903.131 | 1037.304 |
| BM4 | 16.83333 | 0 | 0 | 2490.1521 | 2533.732 | 1581.06 | 1815.949 |
| BM4 | 17.8 | 0 | 0 | 4002.5271 | 4198.207 | 1758.755 | 2020.044 |
| BM4 | 18.575 | 0 | 0 | 2576.3416 | 2611.136 | 1758.755 | 2020.044 |
| BM4 | 19.35 | 0 | 0 | 1890.67 | 1870.658 | 818.797 | 940.44 |
| BM4 | 20.125 | 0 | 0 | 1769.6069 | 1741.646 | 1213.046 | 1393.261 |
| BM4 | 20.9 | 0 | 0 | 3855.4285 | 3976.491 | 2691.895 | 3091.814 |
| BM4 | 21.75 | 0 | 0 | 1567.3182 | 1479.78 | 2691.895 | 3091.814 |
| BM4 | 22.6 | -767.745 | 634.074 | 0 | 0 | 1040.08 | 1194.599 |
| BM4 | 23.2 | -855.5438 | 723.792 | 0 | 0 | 1191.664 | 1368.703 |
| BM4 | 23.8 | -238.2715 | 0 | 0 | 0 | 2845.46 | 3268.194 |
| BM4 | 24.4 | -1949.6183 | 1912.72 | 0 | 0 | 2845.46 | 3268.194 |
| BM4 | 25 | -3097.4041 | 3159.716 | 0 | 0 | 1912.976 | 2197.176 |
| BM4 | 25.85 | -3019.6479 | 3071.867 | 0 | 0 | 1058.919 | 1216.236 |
| BM4 | 26.7 | -3134.5881 | 3161.562 | 0 | 0 | 537.091 | 640 |
| BM4 | 27.475 | -2613.8525 | 2638.578 | 0 | 0 | 1242.845 | 1427.487 |
| BM4 | 28.25 | -603.9449 | 498.133 | 678.3784 | 580.752 | 2046.202 | 2350.194 |
| BM4 | 29.025 | 0 | 0 | 3713.9885 | 3861.952 | 3121.764 | 3585.547 |
| BM4 | 29.8 | 0 | 0 | 5536.1553 | 5411.239 | 3121.764 | 3585.547 |
| BM4 | 30.76667 | 0 | 0 | 3263.8457 | 3378.779 | 2971.355 | 3412.792 |
| BM4 | 31.73333 | -701.6324 | 635.137 | 377.8464 | 291.466 | 1724.769 | 1981.008 |
| BM4 | 32.7 | -2026.115 | 2045.817 | 0 | 0 | 814.354 | 935.338 |
| BM4 | 33.3 | -2160.6455 | 2195.112 | 0 | 0 | 554.389 | 640 |
| BM4 | 34.3 | -1606.2567 | 1600.876 | 0 | 0 | 1578.444 | 1812.944 |
| BM4 | 35.3 | 0 | 0 | 1005.0604 | 976.984 | 3006.371 | 3453.01 |


| BM4 | 36.3 | 0 | 0 | 4193.3018 | 4420.182 | 3006.371 | 3453.01 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BM4 | 36.9 | 0 | 0 | 2628.2642 | 2690.739 | 2608.396 | 2995.911 |
| BM4 | 37.5 | 0 | 0 | 1354.1908 | 1326.313 | 1603.078 | 1841.238 |
| BM4 | 38.3 | 0 | 0 | 417.3971 | 344.574 | 691.445 | 794.169 |
| BM4 | 39.3 | 0 | 0 | 631.4235 | 571.349 | 1764.186 | 2026.282 |
| BM4 | 40.3 | 0 | 0 | 2341.5188 | 2391.739 | 2560.351 | 2940.728 |
| BM4 | 41.26667 | -1261.2473 | 1258.889 | 0 | 0 | 2560.351 | 2940.728 |
| BM4 | 42.23333 | -3106.1868 | 3248.085 | 0 | 0 | 1373.39 | 1577.427 |
| BM4 | 43.2 | -3530.5952 | 3127.787 | 0 | 0 | 512.492 | 640 |
| BM4 | 43.91667 | -3544.3713 | 3132.241 | 0 | 0 | 613.203 | 704.304 |
| BM4 | 44.63333 | -2733.9465 | 2843.142 | 0 | 0 | 1102.751 | 1266.58 |
| BM4 | 45.35 | -1308.0535 | 1310.509 | 0 | 0 | 1651.896 | 1897.309 |
| BM4 | 46.1 | 0 | 0 | 982.5906 | 1027.474 | 1651.896 | 1897.309 |
| BM4 | 46.85 | 0 | 0 | 372.5328 | 389.115 | 813.41 | 934.254 |
| BM4 | 47.6 | 0 | 0 | 66.281 | 0 | 259.328 | 640 |
| BM5 | 0 | 0 | 0 | 102.141 | 0 | 182.15 | 640 |
| BM5 | 0.75 | 0 | 0 | 398.8369 | 406.299 | 533.83 | 640 |
| BM5 | 1.5 | 0 | 0 | 799.2093 | 824.712 | 1063.155 | 1221.102 |
| BM5 | 2.25 | -1259.9935 | 1255.61 | 0 | 0 | 1063.155 | 1221.102 |
| BM5 | 2.96667 | -2465.5149 | 2546.347 | 0 | 0 | 692.901 | 795.841 |
| BM5 | 3.68333 | -3075.6638 | 3080.639 | 0 | 0 | 290.575 | 640 |
| BM5 | 4.4 | -3064.1187 | 3086.43 | 0 | 0 | 763.535 | 876.969 |
| BM5 | 5.36667 | -2214.0422 | 2266.892 | 0 | 0 | 1607.394 | 1846.196 |
| BM5 | 6.33333 | -132.0375 | 0 | 654.5009 | 596.417 | 2798.147 | 3213.852 |
| BM5 | 7.3 | 0 | 0 | 3677.134 | 3827.864 | 2798.147 | 3213.852 |
| BM5 | 8.3 | 0 | 0 | 1807.509 | 1788.686 | 1869.625 | 2147.385 |
| BM5 | 9.3 | 0 | 0 | 1251.5511 | 1176.116 | 549.088 | 640 |
| BM5 | 10.1 | 0 | 0 | 2050.7776 | 2019.922 | 1418.119 | 1628.801 |
| BM5 | 10.7 | 0 | 0 | 3219.0403 | 3283.186 | 2371.269 | 2723.554 |
| BM5 | 11.3 | 0 | 0 | 4641.8013 | 4874.004 | 3043.158 | 3495.262 |
| BM5 | 12.3 | 0 | 0 | 1275.7208 | 1195.251 | 3043.158 | 3495.262 |
| BM5 | 13.3 | -1969.6335 | 1915.492 | 0 | 0 | 1683.933 | 1934.106 |
| BM5 | 14.3 | -2722.7471 | 2725.027 | 0 | 0 | 679.112 | 780.004 |
| BM5 | 14.9 | -2698.3042 | 2698.362 | 0 | 0 | 743.76 | 854.256 |
| BM5 | 15.86667 | -1504.1215 | 1405.988 | 0 | 0 | 1701.793 | 1954.618 |
| BM5 | 16.83333 | 0 | 0 | 2524.7722 | 2495.481 | 3010.264 | 3457.482 |
| BM5 | 17.8 | 0 | 0 | 5683.6245 | 6032.981 | 3134.67 | 3600.37 |
| BM5 | 18.575 | 0 | 0 | 3254.2551 | 3282.791 | 3134.67 | 3600.37 |
| BM5 | 19.35 | -71.904 | 0 | 762.9756 | 597.99 | 2034.655 | 2336.932 |
| BM5 | 20.125 | -1568.7178 | 1446.962 | 0 | 0 | 1251.272 | 1437.166 |
| BM5 | 20.9 | -2118.6428 | 2034.974 | 0 | 0 | 603.33 | 692.963 |
| BM5 | 21.75 | -2116.0796 | 2032.208 | 0 | 0 | 587.54 | 674.828 |
| BM5 | 22.6 | -1220.838 | 1072.355 | 0 | 0 | 1018.683 | 1170.023 |
| BM5 | 23.2 | 0 | 0 | 659.8406 | 476.483 | 1373.388 | 1577.425 |
| BM5 | 23.8 | 0 | 0 | 1483.8735 | 1348.18 | 1426.552 | 1638.487 |
| BM5 | 24.4 | -238.9432 | 0 | 535.1392 | 348.767 | 1426.552 | 1638.487 |


| BM5 | 25 | -1722.3018 | 1612.563 | 0 | 0 | 1073.362 | 1232.825 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BM5 | 25.85 | -2860.5105 | 2848.087 | 0 | 0 | 647.189 | 743.338 |
| BM5 | 26.7 | -3022.9431 | 3028.619 | 0 | 0 | 521.559 | 640 |
| BM5 | 27.475 | -2618.7349 | 2586.487 | 0 | 0 | 1148.475 | 1319.097 |
| BM5 | 28.25 | -1375.9891 | 1250.615 | 0 | 0 | 1899.788 | 2182.029 |
| BM5 | 29.025 | 0 | 0 | 1476.8959 | 1360.262 | 2948.387 | 3386.412 |
| BM5 | 29.8 | 0 | 0 | 3761.896 | 3850.094 | 2948.387 | 3386.412 |
| BM5 | 30.76667 | -33.678 | 0 | 941.8339 | 803.636 | 2755.319 | 3164.661 |
| BM5 | 31.73333 | -2271.5254 | 2232.509 | 0 | 0 | 1499.801 | 1722.618 |
| BM5 | 32.7 | -3071.1384 | 3108.87 | 0 | 0 | 565.036 | 648.981 |
| BM5 | 33.3 | -2994.1138 | 3024.305 | 0 | 0 | 858.042 | 985.517 |
| BM5 | 34.3 | -1838.905 | 1776.956 | 0 | 0 | 1878.683 | 2157.788 |
| BM5 | 35.3 | 0 | 0 | 2121.5015 | 2098.604 | 3271.756 | 3757.822 |
| BM5 | 36.3 | 0 | 0 | 5468.2554 | 5278.252 | 3271.756 | 3757.822 |
| BM5 | 36.9 | 0 | 0 | 4373.6064 | 4563.669 | 2491.081 | 2861.167 |
| BM5 | 37.5 | 0 | 0 | 3183.7434 | 3244.643 | 1508.49 | 1732.598 |
| BM5 | 38.3 | 0 | 0 | 2363.5339 | 2356.773 | 617.014 | 708.68 |
| BM5 | 39.3 | 0 | 0 | 2934.1812 | 2994.137 | 1863.382 | 2140.214 |
| BM5 | 40.3 | 0 | 0 | 4797.563 | 5073.995 | 2918.561 | 3352.154 |
| BM5 | 41.26667 | 0 | 0 | 1589.5603 | 1569.152 | 2918.561 | 3352.154 |
| BM5 | 42.23333 | -1668.1377 | 1667.716 | 0 | 0 | 1694.749 | 1946.528 |
| BM5 | 43.2 | -2601.8584 | 2679.763 | 0 | 0 | 823.415 | 945.746 |
| BM5 | 43.91667 | -2740.7671 | 2830.934 | 0 | 0 | 266.604 | 640 |
| BM5 | 44.63333 | -2250.7786 | 2304.201 | 0 | 0 | 679.201 | 780.106 |
| BM5 | 45.35 | -1159.8925 | 1142.025 | 0 | 0 | 1057.414 | 1214.508 |
| BM5 | 46.1 | 0 | 0 | 739.8184 | 759.283 | 1057.414 | 1214.508 |
| BM5 | 46.85 | 0 | 0 | 350.8811 | 353.196 | 518.583 | 640 |
| BM5 | 47.6 | 0 | 0 | 84.5747 | 0 | 174.256 | 640 |
| BM6 | 0 | 0 | 0 | 93.2587 | 0 | 235.634 | 640 |
| BM6 | 0.75 | 0 | 0 | 518.5558 | 508.649 | 653.698 | 750.814 |
| BM6 | 1.5 | 0 | 0 | 1008.829 | 1022.752 | 653.698 | 750.814 |
| BM6 | 2.25 | 0 | 0 | 338.8325 | 306.869 | 313.76 | 640 |
| BM6 | 2.96667 | -965.2118 | 954.373 | 92.2146 | 0 | 226.613 | 640 |
| BM6 | 3 | -965.2292 | 954.392 | 0 | 0 | 800.497 | 919.422 |
| BM6 | 3.68333 | -830.9333 | 754.812 | 0 | 0 | 800.497 | 919.422 |
| BM6 | 4.4 | -1032.9093 | 967.552 | 0 | 0 | 281.827 | 640 |
| BM6 | 5.36667 | -685.9186 | 584.007 | 0 | 0 | 819.605 | 941.37 |
| BM6 | 6.33333 | 0 | 0 | 976.5811 | 843.417 | 1579.034 | 1813.622 |
| BM6 | 7.3 | 0 | 0 | 2657.1453 | 2624.806 | 1579.034 | 1813.622 |
| BM6 | 8.3 | 0 | 0 | 1458.9396 | 1333.527 | 1198.206 | 1376.216 |
| BM6 | 9.3 | 0 | 0 | 864.1209 | 693.403 | 411.961 | 640 |
| BM6 | 10.1 | 0 | 0 | 1217.8177 | 1045.847 | 694.009 | 797.114 |
| BM6 | 10.7 | 0 | 0 | 1861.7178 | 1722.133 | 1241.075 | 1425.455 |
| BM6 | 11.3 | 0 | 0 | 2606.363 | 2527.959 | 1931.904 | 2218.916 |
| BM6 | 12.3 | -418.4703 | 190.679 | 368.6044 | 0 | 1931.904 | 2218.916 |
| BM6 | 13.3 | -2061.6677 | 1936.179 | 0 | 0 | 1097.214 | 1260.22 |


| BM6 | 14.3 | -2638.6035 | 2565.196 | 0 | 0 | 431.03 | 640 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BM6 | 14.9 | -2601.1895 | 2524.402 | 0 | 0 | 548.479 | 640 |
| BM6 | 15.86667 | -1872.5129 | 1738.905 | 0 | 0 | 1192.761 | 1369.962 |
| BM6 | 16.83333 | -157.4581 | 0 | 565.4637 | 355.908 | 2039.701 | 2342.728 |
| BM6 | 17.8 | 0 | 0 | 2613.1653 | 2537.877 | 2039.701 | 2342.728 |
| BM6 | 18.575 | 0 | 0 | 1082.8265 | 894.093 | 1974.631 | 2267.99 |
| BM6 | 19.35 | -1078.0643 | 882.18 | 0 | 0 | 1260.399 | 1447.649 |
| BM6 | 20.125 | -1981.3264 | 1841.865 | 0 | 0 | 725.416 | 833.186 |
| BM6 | 20.9 | -2240.9678 | 2117.447 | 0 | 0 | 251.551 | 640 |
| BM6 | 21.75 | -2051.9255 | 1913.176 | 0 | 0 | 704.365 | 809.008 |
| BM6 | 22.6 | -1202.3192 | 1001.149 | 0 | 0 | 1073.562 | 1233.055 |
| BM6 | 23.2 | -104.5599 | 0 | 368.6835 | 0 | 1401.667 | 1609.905 |
| BM6 | 23.8 | 0 | 0 | 1238.8752 | 1042.659 | 1401.667 | 1609.905 |
| BM6 | 24.4 | 0 | 0 | 456.6032 | 217.944 | 1303.787 | 1497.483 |
| BM6 | 25 | -941.1787 | 729.648 | 0 | 0 | 980.969 | 1126.706 |
| BM6 | 25.85 | -1723.23 | 1562.656 | 0 | 0 | 626.873 | 720.004 |
| BM6 | 26.7 | -1906.5023 | 1754.986 | 0 | 0 | 238.255 | 640 |
| BM6 | 27.475 | -1721.8546 | 1556.736 | 0 | 0 | 677.177 | 777.782 |
| BM6 | 28.25 | -1033.5171 | 817.454 | 0 | 0 | 1198.235 | 1376.25 |
| BM6 | 29.025 | 0 | 0 | 583.3074 | 338.014 | 1912.249 | 2196.341 |
| BM6 | 29.8 | 0 | 0 | 2065.3005 | 1914.862 | 1912.249 | 2196.341 |
| BM6 | 30.76667 | -133.9723 | 0 | 239.1125 | 0 | 1874.204 | 2152.644 |
| BM6 | 31.73333 | -1351.8251 | 1153.179 | 0 | 0 | 1005.788 | 1155.212 |
| BM6 | 32.7 | -1703.8634 | 1535.412 | 0 | 0 | 346.051 | 640 |
| BM6 | 33.3 | -1622.7726 | 1448.596 | 0 | 0 | 641.746 | 737.087 |
| BM6 | 34.3 | -767.0548 | 542.397 | 0 | 0 | 1363.32 | 1565.861 |
| BM6 | 35.3 | 0 | 0 | 1799.391 | 1645.937 | 2355.85 | 2705.845 |
| BM6 | 36.3 | 0 | 0 | 4435.4326 | 4551.968 | 2355.85 | 2705.845 |
| BM6 | 36.9 | 0 | 0 | 3400.2029 | 3392.702 | 1725.383 | 1981.713 |
| BM6 | 37.5 | 0 | 0 | 2596.4126 | 2515.619 | 1044.354 | 1199.508 |
| BM6 | 38.3 | 0 | 0 | 2133.6968 | 2031.947 | 440.369 | 640 |
| BM6 | 39.3 | 0 | 0 | 2663.8652 | 2620.026 | 1191.738 | 1368.787 |
| BM6 | 40.3 | 0 | 0 | 3855.6028 | 3935.63 | 1960.292 | 2251.521 |
| BM6 | 41.26667 | 0 | 0 | 1818.7299 | 1731.47 | 1960.292 | 2251.521 |
| BM6 | 42.23333 | -236.8635 | 0 | 162.0737 | 0 | 1139.973 | 1309.332 |
| BM6 | 43.2 | -925.0295 | 843.551 | 0 | 0 | 559.326 | 642.422 |
| BM6 | 43.91667 | -1032.5118 | 956.879 | 0 | 0 | 170.879 | 640 |
| BM6 | 44.63333 | -862.578 | 788.717 | 0 | 0 | 489.423 | 640 |
| BM6 | 45.35 | -283.6637 | 196.284 | 97.5241 | 0 | 770.796 | 885.308 |
| BM6 | 46.1 | 0 | 0 | 770.4312 | 780.151 | 770.796 | 885.308 |
| BM6 | 46.85 | 0 | 0 | 419.6728 | 413.526 | 467.678 | 640 |
| BM6 | 47.6 | 0 | 0 | 83.3495 | 0 | 172.124 | 640 |
|  |  |  |  |  |  |  |  |

## APPENDIX D

## LOADS FROM SAFE MODEL DESIGN BASED ON BEARING CAPACITY 450KPa

Table Maximum moment and shear force on beam

| Beam <br> $(\mathbf{m})$ | Location <br> $(\mathbf{m})$ | Max Moment <br> $(\mathbf{K N} \mathbf{- m})$ | Max Shear <br> $(\mathbf{K N})$ |
| :---: | :---: | :---: | :---: |
| BM1- 1x2.4 | 11.3 | 4139.6797 | 2090.569 |
| BM1 -1x2.4 | 29.8 | 1763.6329 | 1963.097 |
| BM2 -1x2.4 | 11.3 | 4924.6914 | 2920.726 |
| BM2 -1x2.4 | 29.8 | 4130.21 | 2989.884 |
| BM3 -1x2.4 | 11.3 | 3880.7688 | 2701.812 |
| BM3 -1x2.4 | 29.8 | 3569.4629 | 1821.19 |
| BM4 -1x2.4 | 14.9 | 3344.4429 | 2729.206 |
| BM4 -1x2.4 | 29.8 | 4351.7407 | 2750.167 |
| BM5- 1x2.4 | 11.3 | 3497.0583 | 2645.761 |
| BM5 -1x2.4 | 36.3 | 4547.8242 | 2881.505 |
| BM6 -1x2.4 | 11.3 | 2049.5161 | 1743.577 |
| BM6 - 1x2.4 | 36.3 | 3924.1221 | 2104.402 |

Table Maximum top and bottom moment of mat slab

| Panel | Direction | Max Top Moment <br> $(\mathbf{K N}-m)$ | Max Bottom Moment <br> $(\mathbf{K N}-\mathbf{m})$ |
| :---: | :---: | :---: | :---: |
| P1 | Short | -1029.491 | 270.9785 |
| P1 | Long | -539.6213 | 3445.5005 |
| P2 | Short | -1174.3625 | 724.4664 |
| P2 | Long | -1410.5824 | 1564.056 |
| P3 | Short | -1205.8719 | 1700.2184 |
| P3 | Long | -895.3745 | 3742.6743 |
| P4 | Short | -1533.7656 | 2760.2886 |
| P4 | Long | -1392.7419 | 3174.835 |

