JIMMA UNIVERSITY

JIMMA INSTITUTE OF TECHNOLOGY

SCHOOL OF GRADUATE STUDIES

FACULTY OF ELECTRICAL AND COMPUTER ENGINEERING

MODELING OF FRACTIONAL ORDER SLIDING MODE CONTROL FOR FRACTIONAL ORDER SEPIC CONVERTER

By

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This thesis is submitted to School of Graduate Studies of Jimma University in partial fulfilment of the requirements for the degree of

Master of Science

in

Control and Instrumentation Engineering

January 2022

Jimma, Ethiopia

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Submission Date: January 2022 ,

Declaration

I,Sena Alemu declare that this thesis entitled "Modeling of fractional order sliding mode controller for fractional order SEPIC converter" and work presented in my own, except I refer different works that are previously done. I also declare that this thesis has not been previously submitted by any researcher in this and other institutions

RESEARCH THESIS SUBMITTED BY

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Acknowledgement

First and above all, I express gratitude toward God, the almighty for all his blessing throughout my life and for granting me the capacity to continue effectively in my investigations. I am heartily appreciative to every one of the individuals who supported me in any regard towards the completion of my Master thesis. I owe my deepest appreciation to my advisors Dr. Prashanth Alluvada and Mr. Tadele Tegegne, who supported me with there well-explained illustrations enabled me to build up a solid understanding of the subject in both practical works as well as the documentation. It is an honor for me to recognize the Jimma University which offered me a scholarship during the entire time of my master studies. Last, however, by no means least, I would like to express my deep gratitude to my lovely parents and friends for their personal support. Thanks to them for always being my main source of motivations for success.

Abstract

This thesis presents fractional order sliding mode control (FOSMC) for fractional order single-ended primary-inductor converters (FOSEPIC). Integer order single ended primary-inductor converter is not accurate, has low flexibility, defined uniformly and has low degrees of freedom. Like the conventional SMC methods, the proposed FOSMC method employs a sliding surface function based upon the input current error only. We achieve output voltage control indirectly by controlling the input inductor current. This investigation of a fractional order SMC on a fractional order SEPIC highlights the merits of the fractional order systems and fractional order controllers. The input current reference is generated by a proportional-integral (PI) regulator. The performance of the proposed FOSMC approach is investigated by using a MATLAB/SIMULINK program. The buck/boost modes for voltage regulation are studied by varying the following parameters: the input voltage, orders of the system, order of the controller and load resistance. Simulation results are presented for fractional order SEPIC converter of fractional orders (0.25, 0.35, 0.52, and 0.65) and FOSMC of fractional order 0.35.

Keyword: ***Fractional calculus ,Fractional order sliding mode control, Fractional order SEPIC converter***

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Abbreviations and Acronyms

C	**C**aputo
CRONE	**C**ommande **R**obusted **O**rdre **N**on **E**ntier
D	**D**erivative **or** **D**iode
DC	**D**irect **D**urrent
FO	**F**ractional **O**rder
FOMCON	**F**ractional **O**rder **M**odeling **CON**trol
FOM	**F**ractional **O**rder **M**odel
FOFT	**F**ractional **O**rder **F**unction **T**ransfer
FOPID	**F**ractional **O**rder **P**roportional **I**ntegral **D**erivative
FOSEPIC	**F**ractional **O**rder **S**ingle **E**nded **P**rimary **I**nductor **C**onverter
FOSMC	**F**ractional **O**rder **S**liding **M**odel **C**ontrol
GL	**G**rünwald **L**etnikov
GUI	**G**raphical **U**ser **I**nterface
IOM	**I**nteger **O**rder **M**odel
IOSMC	**I**nteger **O**rder **S**liding **M**odel **C**ontrol
MATLAB	**MAT**trix **LAB**ratory
MIMO	**M**ultiple **I**nput **M**ultiple **O**utput
ML	**M**ittag **L**effler
PID	**P**roportional **I**ntegral **D**erivative
RL	**R**iemann **L**iouville
SMC	**S**liding **M**odel **C**ontrol
SEPIC	**S**ingle **E**nded **P**rimary **I**nductor **C**onverter
TF	**T**ransfer **F**unction

Chapter 1

Introduction

1.1 Background

The term "non-integer calculus" is sometimes used instead of "fractional calculus" in this study. However, the fractional calculus covers integer orders as well as generalized functional orders such as fractional, irrational and complex[1]. For this reason, the fractional calculus is often referred as the generalized calculus. These names are used interchangeably in the current literature. The concept of fractional calculus as an extension of ordinary calculus goes back to 1695 AD. In the letter to L'Hospital, Leibniz proposed the possibility of generalizing the differentiation to half order derivative. Though fractional calculus has a long history, only in the recent years have the applications of fractional calculus to physics and engineering become an important aspect of modern technology[2, 3]. Recently, it has been widely used in modeling the dynamics of many natural phenomena, which is attributed to its higher capability of providing accurate description than integer order dynamic systems [3–6]. Applications of Fractional Calculus have also been reported in areas such as control Engineering, Biology, Biomedical Engineering, Financial Market and Signal Processing. In Electrical Engineering, use of Fractional Calculus has also been growing[5, 6]. Modeling of electrical equipment and wireless power transmission systems, design as well as Study of chaos in fractional order dynamic systems and related phenomena is receiving growing attention [6].Practical fractional designs of electrode-electrolyte polarization, viscoelastic fluids, chaotic systems, and power converters[2, 6] have also been made. Concepts from fractional-order circuits and systems have recently attracted much attention from the electrical engineering community. Many novel ideas have been generated by exploiting concepts of fractional circuits. For example, fractional order models of capacitors, inductors, memristors, and CMOS metamaterial transmission lines [7] are built. Practical fractional-order elements are fabricated [7, 8]. In addition, new topologies of circuits based on fractional-order elements are constructed, though the underlying characteristics of these circuits continue being studied[7]. In comparison with the classical calculus, the main advantage of fractional calculus is that it can provide an elegant description for the memory and hereditary properties of various real objects [9–11]. The kernel dynamics of most real systems are actually fractional. IOM can describe the features of many systems which have less fractionalities but it will not be highly accurate. The main reason for using the IOM was the absence of solution methods for fractional differential equations. However, the recent improvements in hardware implementation has renewed the interest in the modeling and analysis of new class of fractional-order systems[12]. Many systems can be described more accurately and more conveniently by fractional differential equations (FDEs)[4, 9, 12, 13]. Fractional order control is ubiquitous when the dynamic system has distributed parameter nature [14].

1.1.1 Advantages of Fractional Calculus

Compared to the classical theory, fractional differential equations can more accurately describe many systems in interdisciplinary fields and has higher capability of providing accurate description than integer order dynamic systems. This has been true with the control theory of robotic systems. The integer-order models currently in use to describe the characteristics of inductors and capacitors are not accurate enough, even incorrect [8, 11, 15–20].Further, fractional calculus is also a powerful method to describe data memory and heredity [17, 21, 22]. Fractional order sliding mode has also been used to eliminate the chattering effect caused by the switching control action and realize high-precision performance and without deteriorating the robust tracking performance [20]. Unlike integer-order systems, fractional-order systems do not permit fractional derivatives to be defined uniformly [7]. In recent years, fractional-order capacitors or fractional order inductors have been incorporated into DC-DC converters. Their results show that the output voltage gain can be not only controlled by duty cycle, but also the orders of the fractional-order components [8]. Additional attractive features of fractional-orders over integer-order system occur in stability and differentiability some functions which are not differentiable in classical sense are found to be differentiable in fractional (RL) sense[1]. These properties of fractional calculus motivate researchers to seek applications to other physical and natural phenomena.

1.1.2 Sliding Mode Control

Sliding Mode Control (SMC) is one of the most efficient control strategies to deal with uncertainties. It is a widely used method with fast dynamic and good transient response in linear and nonlinear systems, is robust against external disturbances and parameter variations [18]. The main objective of SMC class of controllers is to force the system states to stay in a predefined manifold (sliding surface) and maintain it there in spite of the presence of uncertainties in the system. Therefore, the sliding mode based design consists of two phases (i) Reaching Phase and (ii) the sliding phase. In reaching phase, the system states are driven from the initial state to reach the sliding manifold in finite time. The trajectories are sensitive to disturbances and parameter variations in the reaching phase. For this reason, various methods have been suggested to eliminate or lessen the system sensitivity by minimizing or even removing the reaching phase. In Sliding Phase, the closed-loop system is induced into sliding motion. The considerations of robustness and order reduction, which are the most important aspects of the sliding mode based design, come into picture. In addition, in the sliding phase, the trajectories are insensitive to disturbances and parameter variations. This feature makes SMC a robust control method. It is worth noting that during the reaching phase, there is no guarantee of robustness [19]. When integer order SMC methods is used to deal with fractional order system, they always reject the disturbances in a robust way, but chattering is a serious problem that needs to be solved[23]. The most important property of SMC is that the sliding motion of the state on the sliding surface is ensured. Conventional SMC usually chooses a predefined, constant sliding surface. When the initial value of a system is far from the sliding surface, a long reaching time occurs. Hence, the control performance reduces and robustness cannot be ensured in the reaching phase. In conventional SMC, while increasing discontinuous control gain there is possibility of shorten reaching phase, however still it has problem of chattering [24].Fractional calculus has been shown to be effective in eliminating chattering, realizing high precision performance [20]. Considering the advantages of fractional order calculus, the fractional order is incorporated into the design of sliding mode control, which reduce the chattering problem and speed up the response of the closed-loop system[6, 19, 23]. Fractional-order controllers may further improve the closed loop system performance [17]. A fractional-order SMC is advantageous due to its additional design parameters (i.e., adjustable non-integer differentiator/integrator order). Thus, the fractional order can be tuned for the best dynamic response, with reference to the orders of the differentiation, while the main benefits of the conventional SMC remain intact [17, 25].

1.1.3 SEPIC Converter

DC-DC converters is a power electronic circuit, which is used for DC voltage step-up (boost) and step-down (buck). This system has a wide variety of applications, including DC motor drives, active filters, computers, power supplies, and medical instrumentation[2]. Other related DC/DC converters, include the Cuk, Zeta, and SEPIC [23]. The Fig.1.1 shows the SEPIC DC-to-DC converter circuit with switches realized by means of semiconductor devices (Q, D). These operate in a complementary fashion i.e., when the transistor Q is in the conducting mode then the diode D is inversely polarized and vice versa. DC– DC SEPIC converters are widely used in applications where low ripple current is desired at the input and output terminals of the converter. These converters with both step-down and step-up capability are suitable in the off-grid photovoltaic (PV) applications due to their interconnection ability with different batteries and PV modules. The inverse polarity at the output terminals of the Cuk converter is its major disadvantage. Furthermore, the low-power conversion efficiency resulting from the hard switching condition can be considered as another drawback for the Cuk converter. The SEPIC converter offers the similar features as Cuk converter without inverting the polarity of the output voltage. Moreover, the SEPIC converter provides great benefit for power conversion since it can generate a wide range of output non-inverted voltage [26].

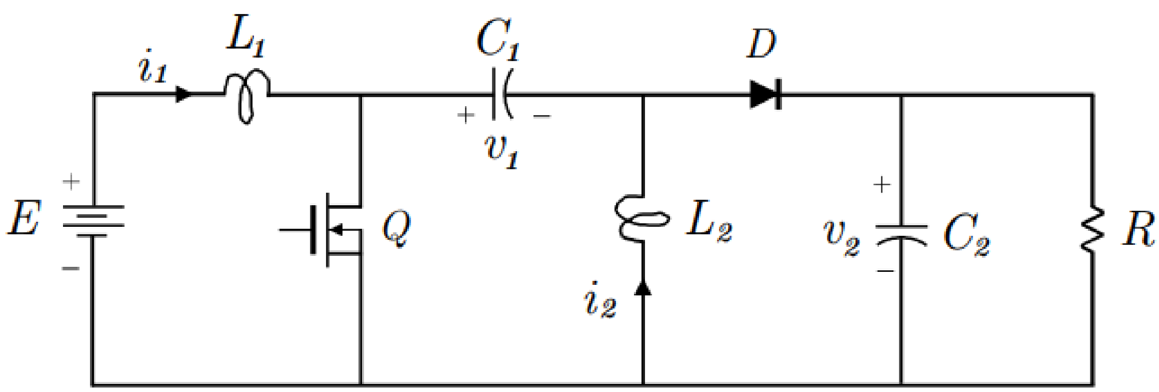

FIGURE 1.1: The SEPIC DC-to-DC Power Converter

1.1.3.1 Variables in the Steady State

The steady-state variables of the system corresponding to a constant value of the average control input u can be obtained. Assuming that the output voltage and inductor current equal to their references ($v_{\text{out}} = v_{\text{ref}}$ and $i_{L_1} = i^*_{L_1}$) and this means that the steady state values are obtained by equating the integer order derivatives to zero. The differential equations can be written as

$$\begin{aligned} 0 &= v_{in} - (1 - u_{ss}) \left(v^s_{C_1} + v^*_{\text{out}} \right) \\ 0 &= u_{ss} v^{ss}_{C_1} - (1 - u_{ss})\, v^*_{\text{out}} \\ 0 &= (1 - u_{ss})\, i^*_{L_1} - i^{ss}_{L_1} u_{ss} \\ 0 &= (1 - u_{ss}) \left(i^*_{L_1} + i^{ss}_{L_2} \right) - \tfrac{v^*_{\text{out}}}{R_L} \end{aligned} \tag{1.1}$$

Solving for $u_{ss}, i^{ss}_{L_2}, i^*_{L_1}$ and $v^{ss}_{c_1}$ in terms of v_{ref} and v_{in} yields

$$u_{ss} = \frac{v^*_{out}}{v_{in} + v^*_{out}} \tag{1.2}$$

$$\begin{aligned} i^{ss}_{L_2} &= \tfrac{v^*_{\text{out}}}{R_L} \\ i^*_{L_1} &= \tfrac{\left(v^*_{\text{out}}\right)^2}{v_{\text{in}}\, R_L} \\ v^{ss}_{C_1} &= v_{\text{in}} \end{aligned} \tag{1.3}$$

where $u_{ss}, i^{ss}_{L_2}$, and $v^{ss}_{C_1}$ denote the steady-state values of u, i_{L_2}, and v_{C_1}, respectively. It is important to note that u_{ss} is the duty cycle of the converter which should satisfy $0 < u_{ss} < 1$. From equation (1.2) the voltage transfer ratio can be deduced as

$$\frac{v^*_{\text{out}}}{v_{\text{in}}} = \frac{u_{ss}}{1 - u_{ss}} \tag{1.4}$$

Converters consists of the passive power switch, the active power switch, and the storage elements. All key components of power electronic converters are considered as integer-order components in the traditional models of power electronic converters, which cannot actually reflect their operating characteristics. In this study we incorporate a fractional inductor and a fractional capacitor in the SEPIC, making it a FOSEPIC. The fractional-order modeling of power electronic converters considering the fractional-orders of inductors and capacitors is considered in[8, 27]. The main objective of most closed-loop feedback controlled DC/DC converters is to ensure that the converter operates with fast dynamic response, small steady-state output error, and low overshoot, while maintaining high efficiency and low noise emission in terms of rejection of input voltage changes, parameter uncertainties, and load variations[2]. The modeling of fractional-order converters has important significance in practice[15]. In fact, in addition to integer-order components, there are also fractional-order components that are described by fractional-order calculus, such as fractional- order capacitors .The fractional-order components are not yet standard market-oriented components, but various fractional-order capacitors and fractional-order inductors have been manufactured in the laboratory, leading to a practical application of fractional-order components. Some recent results suggest that the output voltage gain can be not only controlled by duty cycle, but also by the orders of the fractional-order components[8].The above studies demonstrate that fractional-order components could bring in more flexibility and higher performance than the integer-order components in circuit design and applications. The non-uniformity of the fractional derivative renders ineffective, existing numerical methods for computing fractional derivatives [5, 7, 28]. Therefore, taking the fractional-order DC/DC SEPIC converter as an example, this thesis presents a time-domain modeling and analysis scheme. In case of buck and buck-boost converters significant harmonics are present at the input. This is due to the absence of Inductor at the input side. In this configuration, very less number of capacitors (C) and inductors (L) have been taken into consideration. However, the converter experiences more ripple in current at input due to minimal number of L and C. One way to minimize the ripple is to use a filter. But, filter requires large values of L and C, this makes filter bulky and also increases the cost. The most important advantage of using SEPIC over conventional converter is, non-inverting output and it uses state space analysis [29]. The proposed study demonstrates that fractional order components could bring in more flexibility and higher performance than the integer-order components in circuit design and applications [8] and the characteristics of FOC(fractional order capacitor) of different orders [30].Therefore, by interrelating the above all theoretical aspect of fractional calculus, sliding mode control and SEPIC converter as a plant we are going to model fractional order SEPIC converter with controller of FOSMC. The introduction part of this thesis give the highlight of fractional calculus and its applications.

1.2 Statement of the Problem

Many natural phenomena may not be better described by a classical calculus formulation, because they do not take into account the past behavior and are not compact when expressing high-order dynamics. IOM can describe the features of many systems but may not have good accuracy [12]. They may not represent an accurate model or the characteristic response of systems[7]. Integer order systems permit the derivative and integration to be defined uniformly. The main reason for using the IOM was the absence of solution methods for fractional differential equations [12]. In general, systems including energy storage components such as the capacitor and the inductor could be described by integral or differential equations. On the other hand, fractional order systems come with further parameters, such as differentiation orders and degrees of freedom which may be set as desired. These further parameters are unavailable in the IOM. Fractional order systems though, do not have simple analytical solutions because of their long memory characteristic [12]. Hence, numerical algorithms are widely applied to the analysis of fractional-order systems, which may cause an exponential increasing of computational efforts. In this thesis, we apply the principles of fractional order calculus to the study and analysis of the SEPIC. The SEPIC itself is described through fractional order and coupled differential equations (the FOSEPIC). The control designed is a fractional order SMC, where a fractional version of a PI controller is used to compute a fractional order SMC, leading to a FOSMC.

1.3 Objectives of the Research

1.3.1 General Objective

The main objective of this thesis is to model and give comparative analysis of fractional order and integer order systems with sliding mode controller for SEPIC converter.

1.3.2 Specific Objectives

- To model FOSEPIC(Fractional Order SEPIC) converter using state variables
- To investigate fractional order model of SEPIC converter in terms of different performances(steady state performance, performance under input voltage variations ,performance underload variation)
- To compare and evaluate the performance of fractional SEPIC converter under two different sliding controllers: fractional order sliding mode control and integer order sliding mode control.
- To provide other modulating/regulating variable for DC/DC SEPIC converter(duty cycle and order of the system)

1.4 Significance of the Study

The kernel dynamics of most real systems are actually fractional. IOM can describe the features of many systems which have less fractionalities but it will not be highly accurate. The main reason for using the IOM was the absence of solution methods for fractional differential equations. However, the recent evolution in hardware implementation has brought a renewed wave in the modeling and analysis of new class of fractional-order systems [12]. Many systems can be described more accurately and more conveniently by fractional differential equation [4, 9, 12, 13]and fractional order control is ubiquitous when the dynamic system has distributed parameter nature[14].

1.5 Scope of the Thesis

The scope of this thesis is comparing the performance of SMC, FOSMC, FOSEPIC and IOSEPIC. The thesis conducts the comparisons of these dynamic systems through simulations performed in a MATLAB/SIMULINK environment. No practical (hardware) implementation is done.

1.6 Methodology

Methodology used to solve the problem is as follows. The study begins with gathering and studying literatures that related to this thesis.

- Modeling of SEPIC converter using state variables
- Investigate fractional order mathematical model of SEPIC converter.
- Design fractional order sliding mode controller and integer order sliding mode controller (conventional sliding mode controller) for fractional order SEPIC.
- Performance evaluation and analysis.

Simulation of the control of fractional order SEPIC using fractional order and integer order sliding mode methods is performed using a MATLAB/SIMULINK program. Performance of the fractional control of the fractional system is described through the following four variables: steady state performance, performance under input voltage variation, performance under load variation and performance under order of controller variation. The overall schematic diagram of fractional order sliding mode controller and fractional order system (SEPIC) is shown in Fig.1.2 [26].

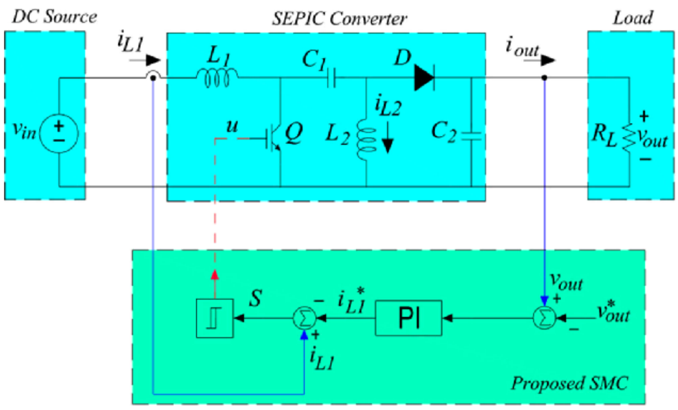

FIGURE 1.2: Overall Schematic Diagram of SEPIC Converter with SMC

where x_{4d}=v^*_{out} denotes the reference of x_4=v_{out} state variable and x_{1d}=i^*_{L1} denotes the reference of x_1=i_{L1}.

1.7 Organization of the thesis

The thesis has organized accordingly.The second chapter describes a literature review and the current state of the art of the subject, chapter three describes the Control design and Stability analysis Chapter four presents simulation results obtained using MATLAB/SIMULINK and Chapter five talks about conclusion and recommendations for future works.

Chapter 2

Literature Review

2.1 Fractional Calculus

Several research articles and monographs exist in the area of fractional calculus. Articles of fractional order systems, relevant to the current study are referenced here. Fractional calculus allows alleviating the limitations of conventional differential equations where only integer operator powers are used. This gives rise to system models that take into account dynamics such as self-similarity (Self similarity means that it is invariant under linear scale change in time [1] and system state history dependence. Contemporary, industrial control systems are of considerable complexity, therefore such systems are likely to exhibit such dynamism. Hence, interest for fractional system has been witnessed in the area of identification, control among others [31]. Many authors pointed out that, fractional-order calculus is most suitable for the description of memory and generic properties of various materials and processes [4, 9, 10], which are neglected in the classical integer-order models . The order of differentiation is a key difference between IOM and FOM. While the orders are integers in the IOM systems, they are fractions in the FOM systems. The values of fractional orders are often used as tuning parameters [8]. This gives FOM more flexibility and higher degrees of freedom and hence possesses richer dynamics than IOM [1, 10, 12, 32–34]. The results show that the fractional-order model has less root-mean-square error than the integer order models[34]. Moreover, FOM is a good candidate to, accurately, explain system with memory (The past is considered to explain the present) [23], since it has a memory in the model [35]. Recently, fractional-order control, which is the generalization of integer-order control, has emerged as an attractive control strategy. The existence of adjustable fractional order enables it to achieve the optimum dynamic responses over the variable fractional order. As mentioned in [36, 37], SMC possesses robustness property and powerful ability to reject the plant uncertainties and disturbances. Further, nonlinear dynamics can also be incorporated. Combining the merits of fractional-order control and SMC, fractional-order sliding mode control (FOSMC) is proposed and investigated for various dynamic systems in recent years [33, 37]. In fact, FOSMC is an improvement of traditional integer-order SMC. By tuning the order of fractional order system appropriately, designers have the opportunity to obtain satisfactory transient and steady state responses. In the meantime, the advantages of traditional integer-order SMC can still be retained. Hence, the merits of FOSMC are that it can achieve better control performances (such as faster and smoother dynamic responses) than corresponding integer-order SMC [37, 38]. The behavioral attractive relation of plant with fractional order controllers would be an advantage, because the responses are not restricted to a sum of exponential functions. Therefore a wide range of responses not occurring in integer order calculus would be visible. Some recent research shows that the fractional sliding surfaces have better control effects than integer ones. The fractional sliding mode control has faster response and convergence speed in the initial stage due to the fractional operator [38]. The input impedance of a FOSEPIC converter can be changed by regulating the duty cycle[2, 8, 26] as well as by regulating the order of differentiation of the system as well [8]. However, the controller design for the SEPIC converter is very complicated due to its inherent fourth-order and non-linear nature. Furthermore, its behavior depends on operating conditions and load variations [26]. The SMC is a model-based variable structure control system that was proposed and which has a number of advantages, including an inherent robustness to external disturbances and inherent insensitivity to system uncertainties. The SMC is a powerful technique that can control both linear and nonlinear systems [2]. Moreover, SMC is a popular control approach and some scholars presented fractional-order sliding mode control (FOSMC) that combined fractional calculus theory and SMC to design for DC-DC buck converter. However, it is necessary to design controllers based on fractional order model because capacitors and inductors have been identified as having the nature of fractional-order differential electronic components [8]. The thesis will contribute the following:

- To our knowledge, it is the first time to develop the fractional-order models of a SEPIC converter. The fractional order models are the general expression of a SEPIC converter, which can describe the operating characteristics more accurately.
- The influence of the orders of the inductors and the capacitor on the operating characteristics of a SEPIC converter has been analyzed. The study found that the order of inductors affects both the dynamic characteristics and static stability of a SEPIC converter significantly, while the capacitor order mainly affects the dynamic characteristics.
- We discovered that the SEPIC converter which contain the fractional-order inductors and the fractional-order capacitor can present better static and dynamic performance indexes (such as smaller overshoot and shorter regulation time) than the traditional integer-order SEPIC converters, by appropriately selecting the orders of the inductors and the capacitor

2.2 Review of Software Reference for Fractional Order Systems

In recent years, as fractional calculus becomes more and more broadly used across different academic disciplines, there are increasing demands for the numerical tools for the computation of fractional integration/differentiation, or the simulation of fractional order systems. Time to time, being asked about which tool is suitable for a specific application.

2.2.1 @FOTF

@fotf (fractional order transfer function) is a control toolbox for fractional order systems developed. Most of the functions inside are extended from the Matlab built-in functions. The code and usage of the @fotf toolbox are described in very detail. In order to describe fo models, this toolbox adds further ovedrload to certain built-in functions of matlab. The transfer function objects generated from it can be interactive with those generated from the Matlab transfer function class. Yet, the overloading of associated functions such as impulse (), step (), etc. lost the plotting functionality. As a work around, users can simply define a time vector as the second input to these functions. Fotf toolbox supports time delay in the TF, e.g. fotf (a, na, b, nb, delay). It does not directly support transfer function matrix, hence, MIMO systems cannot be simulated directly. However, since it provides Simulink block encapsulation of the involved function fotf (), multiple input/output relationship can be established by manually adding loop interactions in Simulink block diagrams. Therefore, the remark “could” is put in the “MIMO” column in [39, 40] , (where the ‘Delay’ column denotes if the script/toolbox is able to handle time delay in the FO model; and the ‘MIMO’ column denotes if the script/toolbox is able to handle MIMO FO models). A small drawback with @fotf is that the sampling time has relatively big impact on the accuracy, which has been remarked in the validation comments. Encouragingly, an update is upcoming according to the author.

2.2.2 Ninteger

Ninteger, non-integer control toolbox for Matlab, is a toolbox intended to help with developing fractional order controllers and assessing their performance[39, 40]. It uses integer order transfer functions to approximate the fractional order integrator or differentiator,$C(s) = ks^{\nu}, \nu \in \mathbb{R}$. It offers three frequency domain approximation methods. The CRONE(Commande Robuste d'Ordre Non Entier) methods, which uses a recursive distribution,

$$C(s) = k' \prod_{n=1}^{N} \frac{1 + s/\omega_{zn}}{1 + s/\omega_{pn}} \tag{2.1}$$

The Carlson's method that solves $C^{\alpha}(s)$ using Newton's iterative method,

$$C_n(s) = C_{n-1}(s) \frac{(\alpha - 1)C_{n-1}^{\alpha}(s) + (\alpha + 1)g(s)}{(\alpha + 1)C_{n-1}^{\alpha}(s) + (\alpha - 1)} g(s) \tag{2.2}$$

The Matsuda's methods, that approximates $C(s)$ with a gain known at several frequencies.

$$\begin{aligned} C(s) &= [d_0(\omega_0); \quad (s - \omega_k - 1)/d_k(\omega_k)]_{k-1}^{+\infty} \\ d_0(\omega) &= |C(j\omega)|, \quad d_{k+1}(\omega) = \tfrac{\omega - \omega_k}{d_k} \tfrac{\omega}{(\omega) - d_k(\omega_k)} \end{aligned} \tag{2.3}$$

It offers three frequency domain approximation methods, it also provides Simulink block encapsulation of the involved functions, such as 'nid' and 'nipid' blocks. Moreover, it offers a user-friendly GUI for fractional order PID controller design. There is a problem with ninteger toolbox in Matlab version 2013a or later[39]. Without additional editing, it has conflicts with some built-in functions due to the overload editing of the Matlab built-in function isinteger (). For example, calling the mean () function will prompt an error.

2.2.3 OoCrone Toolbox

The CRONE(Commande Robuste d'Ordre Non Entier) Toolbox, developed since the nineties by the CRONE team, is a Matlab and Simulink toolbox dedicated to applications of non-integer derivatives in engineering and science. It evolved from the original script version to the current object-oriented version[41]. A good feature of the Crone toolbox is that some of the methods are implemented for MIMO fractional transfer functions. For example, executing sys MIMO= [sys, sys; sys2, sys2] generates a two-input-two-output TF matrix. Many simulation results in the literature are obtained using the CRONE toolbox such as the design of centralized CRONE controller with the combination of the MIMO-QFT approach. Several other toolboxes are inspired by CRONE, e.g. ninteger and FOMCON. A drawback of the CRONE toolbox is that time delay cannot be incorporated into the generated FOTF. Manually multiplying the delay to the frac tf object does not work either because the exp () operation is not overloaded by frac tf class. CRONE is a toolbox much more powerful than merely simulating fractional order systems[39, 40]. In spite of this basic functionality, it is also capable of fractional order system identification and robust control analysis and design[42].

2.2.4 FOMCON

The FOMCON toolbox for MATLAB is a fractional-order calculus based toolbox for system modeling and control design. The core of the toolbox is derived from an existing toolbox FOTF ("Fractional-order Transfer Functions"), the source code for which is provided in literature[31, 39, 40]. Consequently, the main object of analysis in FOMCON is a fractional-order transfer function of the form

$$G(s) = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \cdots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \cdots + a_0 s^{\alpha_0}} \tag{2.4}$$

FOMCON is related to other existing fractional-order calculus oriented MATLAB toolboxes, such as CRONE and Ninteger through either system model conversion features or shared code, and this relation is depicted in Fig.2.1initial motivation for developing FOMCON was the desire to obtain a set of useful and convenient tools to facilitate the research of fractional-order systems. This involved writing convenience functions, e.g., the polynomial string parser, building graphical user interfaces to improve the general work flow. However, a full suite of tools was also desired due to certain limitations in existing toolboxes. The basic functionality

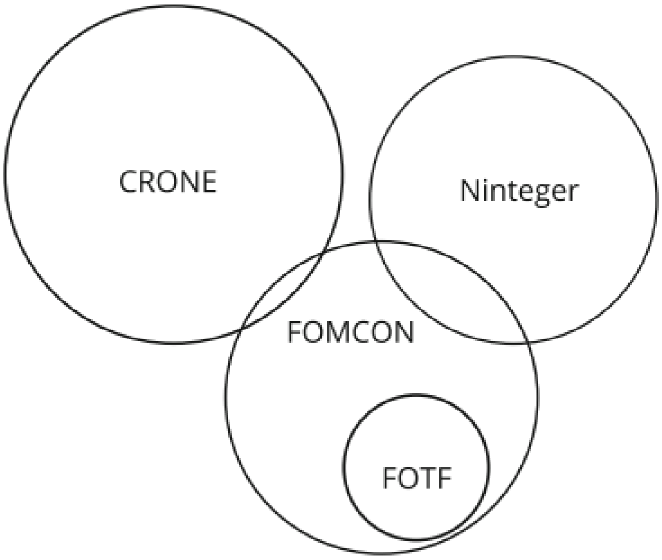

FIGURE 2.1: Relation of FOMCON Toolbox for MATLAB/Simulink to Similar Packages [31, 39, 40]

of the toolbox was then extended with advanced features, such as fractional-order system identification and FOPID controller design[31]. With all previous considerations, the motivations for developing the toolbox can now be established.

- It is a product suitable for both beginners and more demanding users to availability of graphical user interfaces and advanced functionality.
- It focuses on extending conventional control schemes (PID and lead-lag compensator loops) with concepts of fractional calculus;
- Tools for implementing fractional-order systems and controllers are available;
- With the Simulink block set the toolbox aims at a more sophisticated modeling approach. Real-time control application support is provided through, e.g., Real-time Windows Target toolbox for MATLAB/Simulink.
- It can be viewed as a “missing link” between CRONE and Ninteger;

- Due to availability of the source code the toolbox can be ported to other computational platforms such as Scilab or Octave (some limitations and/or restrictions may apply).

Most of the research results discussed in this thesis are implemented in FOMCON toolbox. Toolbox documentation is available on the official website.Structure of the Toolbox has a modular structure depicted in Fig. 2.2 and currently consists of the following modules:

- Main module (core-fractional system analysis);
- Identification module (system identification in both time and frequency domains);
- Control module (FOPID controller design, tuning and optimization tools, as well as some additional features);
- Implementation module (continuous and discrete time approximations, implementation of corresponding analog and digital filters)

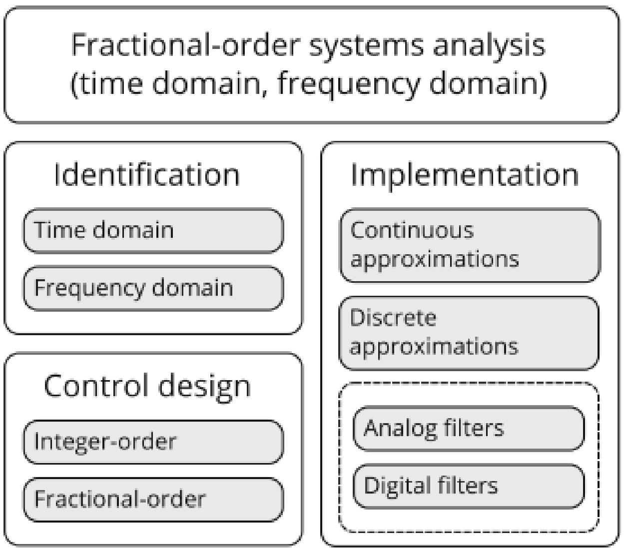

FIGURE 2.2: Modular Structure of the FOMCON Toolbox [31]

All the modules are interconnected. Most features are supported by graphical user interfaces. A Simulink block set is also provided in the toolbox allowing more complex modeling tasks to be carried out. General approach to block construction was used where applicable. The following blocks are currently realized:

- General fractional-order operators: fractional integrator and differentiator;
- Continuous and discrete time fractional transfer function;
- Continuous and discrete time FOPID controller. Several variants of these blocks are provided for convenience.

Dependencies: The toolbox relies on the following MATLAB products:

- Control System toolbox—required for most features;
- Optimization toolbox—required for time domain identification and conventional PID tuning, and also partially for fractional-order PID tuning.

Several other tools are used directly (without or with minor changes) per the BSD(Berkeley Software Distribution)or(Berkeley Standard Distribution) license:

- Nelder-Mead algorithm based function for nonlinear optimization subject to bounds and constraints;
- Ninteger toolbox frequency domain identification functions.

Identification Module: It is also possible to export fractional-order systems to the CRONE toolbox format. This feature requires the object-oriented CRONE toolbox to be installed.The module provides the following main features:

Time domain identification:

- Commensurate and noncommensurate order system identification;

- Parametric identification, which is applicable to closed-loop identification problems;
- Approximation of fractional systems by conventional process models.

Frequency domain identification:

- Commensurate transfer function identification based on algorithms by Hartley, Levy and Vinagre;
- Best fit algorithm for choosing an optimal commensurate order and pseudo orders of the fractional transfer function.

In addition, functions for manipulating the obtained model are provided, including truncation, rounding and normalization of coefficients and orders, as well as functions for validating the models and carry out residual analysis. In general FOMCON toolbox for MATLAB/Simulink was presented. The main focus was on MATLAB based features. The application of the tools available in the toolbox to solving identification, control, and analog and digital implementation problems for fractional systems[39, 40].

Chapter 3

Fractional Order Control Design and Stability Analysis

3.1 GL Fractional-Order Derivative

Extrapolating the applicability of classical backward difference formula to derivative of non-integer order gives rise to the formation of the Grunwald-Letnikov (GL) fractional-order derivative.GL is the first definition proposed for differentiation of noninteger order[43, 44]. Let us consider a real function $f(t)(t \in [0, b])$. The function is said to be in the space $C_\mu, \mu \in R$ if there exists a real number $p(> \mu)$, such that $f(t) = t^p f_1(t)$, where $f_1(t) \in C[a, \infty)$, and it is said to be in the space C_μ^n if and only if $f^{(n)} \in C_\mu, n \in \mathrm{N}$ We now express the n^{th} order derivative (n is an integer) of the casual function $f(t)$ (i.e., $f(t) = 0, t < 0$) in terms of backward difference formula: GL fractional-order derivative equations:

$$\frac{d^n f(t)}{dt^n} \cong \frac{\nabla^n f(t)}{h^n} = h^{-n} \sum_{i=0}^{N} \left((-1)^i \begin{pmatrix} n \\ i \end{pmatrix} f(t - ih) \right) \tag{3.1}$$

where $\begin{pmatrix} n \\ i \end{pmatrix} = \frac{n!}{i!(n-i)!}$, $h = (b-a)/N$, N is the total number of equidistant nodes in the interval $[0, b]$. Equation (3.1) is the discretized form of n^{th} order derivative of the function, $f(t)$. Because $\begin{pmatrix} n \\ i \end{pmatrix}$ becomes zero for all values of i greater than n, the upper limit of summation in this definition can be increased to infinity. Rewrite Equation the above equation as follows

$$ {}_0^{GL}D_t^n f(t) \cong h^{-n} \sum_{i=0}^{\infty} \left((-1)^i \begin{pmatrix} n \\ i \end{pmatrix} f(t-ih) \right), D = \frac{d}{dt}. \tag{3.2} $$

The following definition for the Grunwald-Letnikov fractional-order derivative can be obtained by putting α in place of n in Equation (3.2).

$$ {}_0^{GL}D_t^\alpha f(t) \cong h^{-\alpha} \sum_{i=0}^{\infty} \left((-1)^i \begin{pmatrix} \alpha \\ i \end{pmatrix} f(t-ih) \right) \tag{3.3} $$

When the sign of α is negative, Equation above) turns out to be a fractional order integral. The integral transform definition of GL fractional-order derivative is

$$ {}_0^{GL}D_t^a f(t) = \sum_{k=0}^{n-1} \frac{f^{(k)}(0)t^{-\alpha+k}}{\Gamma(-\alpha+k+1)} + \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau) d\tau \tag{3.4} $$

where $n - 1 \leq \alpha < n, n \in Z^+, t > 0$

The reason why the fractional-order derivative possesses nonlocal property is that the term $\begin{pmatrix} \alpha \\ i \end{pmatrix}$ in Equation (3.3) will never become zero; that is, determining the fractional-order derivative of any function requires its entire history. Therefore, it needs infinite memory and thus is more suitable to explain long memory processes mathematically. It is worth mentioning here that classical calculus is a particular case of the fractional calculus. The GL fractional-order derivative in Equation (3.4) is the left fractional-order derivative, because the lower terminal of the fractional integral is fixed at the left end of the interval $[0, b]$ and the upper terminal moves in the interval. If the upper terminal of the fractional integral in Equation (3.4) is fixed at the right end of the interval $[0, b]$ and the lower terminal is moving, then the GL fractional derivative is called the right fractional derivative. For the current purpose, we may assume that, the independent variable t is time and the function $f(t)$ describes the dynamic behavior of a process. If $\tau < t(t$ is the current instant), then the past of this process can be described by the state $f(\tau)$. If $\tau > t$, then the state $f(\tau)$belongs to the future of the process[43, 44].

3.2 Riemann-Liouville (RL) Fractional-Order Integral

Cauchy's formula for repeated integration, which reduces n-fold integration of function $f(t)$ to single integral, is:

$$f^{-n}(t) = {}_0J_t^n f(t) = \frac{1}{(n-i)!}\int_0^t (t-x)^{n-1} f(x)dx \tag{3.5}$$

where n is a positive integer.

$${}_0J_t^n f(t) = \frac{1}{\Gamma(n)}\int_0^t (t-x)^{n-1} f(x)dx \tag{3.6}$$

where $\Gamma(n)$ is a well-known Euler's Gamma function: $\Gamma(n) = \int_0^\infty e^{-x}x^{n-1}dx$.
the above Equation permits us to replace n with α to obtain a fractional-order integral:

$${}_0J_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-x)^{\alpha-1} f(x)dx \tag{3.7}$$

3.3 Riemann-Liouville (RL) Fractional-Order Derivative

The left Riemann-Liouville (RL) fractional-order derivative of function $f(t)$ is defined as

$$ {}_{0}^{RL}D_{t}^{\alpha} f(t) = D^{n} J^{n-\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \frac{d^{n}}{dt^{n}} \int_{0}^{t} (t-\tau)^{n-\alpha-1} f(\tau) d\tau, t > 0 \tag{3.8} $$

where α is a noninteger that satisfies the relation $n-1 < \alpha \leq n, n \in Z^{+}$ The right Riemann-Liouville fractional-order derivative is

$$ {}_{0}^{RL}D_{t}^{\alpha} f(t) = D^{n} J^{n-\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^{n}}{dt^{n}} \int_{t}^{b} (t-\tau)^{n-\alpha-1} f(\tau) d\tau, t < b \tag{3.9} $$

3.4 Caputo Fractional Derivative

Riemann-Liouville fractional differential equations lack widespread physical applications because of the need for fractional-order initial conditions. To enable fractional calculus concepts to be applied in different applied branches of science and technology [43, 44], Caputo modified Equation (3.9) as shown in the following definition. The left Caputo fractional-order derivative is

$$ {}_{0}^{c}D_{t}^{a} f(t) = J^{n-\alpha} f^{n}(t) = \frac{1}{\Gamma(n-\alpha)} \int_{0}^{t} (t-\tau)^{n-\alpha-1} f^{n}(\tau) d\tau, t > 0 \tag{3.10} $$

and the right Caputo fractional derivative is

$$ {}_{0}^{c}D_{t}^{a} f(t) = J^{n-\alpha} f^{n}(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t}^{b} (t-\tau)^{n-\alpha-1} f^{n}(\tau) d\tau, t < b \tag{3.11} $$

3.5 Properties of GL,RL and Caputo Fractional Order Derivative

Some useful properties of fractional-order operators that we shall use in the following chapters are provided here[43, 44].For

$$f(t) \in C_\mu, \mu > -1 and n - 1 \leq \alpha < n, p - 1 \leq \beta < p, p, n, q \in Z^+, \alpha, \beta \in R^+ :$$

property 1 semi group and commutative property.

$$_0J_{t0}^{\alpha} J_t^{\beta} f(t) = {}_0J_{t0}^{\beta} J_t^{a} f(t) = {}_0J_t^{\alpha+\beta} f(t) \tag{3.12}$$

property 2 Consistency property with the integer order integral.

$$\lim_{a \to n} \left({}_0J_t^a f(t) \right) = {}_0J_t^n f(t) \tag{3.13}$$

property 3 C is a constant.

$$\begin{aligned} &{}_0^{GL} D_t^\alpha c = \left(ct^{-\alpha} \right) / \Gamma(1-\alpha) \\ &{}_0^{RL} D_t^\alpha c = \left(ct^{-\alpha} \right) / \Gamma(1-\alpha) \\ &{}_0^{C} D_t^\alpha c = 0. \end{aligned} \tag{3.14}$$

property 4

$$_0^c D_t^a f(t) = {}_0^{RL} D_t^a \left(f(t) - \sum_{k=0}^{n-1} \frac{t^k}{k!} f^{(k)}(0) \right) \tag{3.15}$$

property 5

$$\begin{aligned} &{}_0^c D_{t0}^\alpha J_t^\alpha f(t) = {}_0^{RL} D_{t0}^a J_t^\alpha f(t) = f(t) \\ &{}_0J_{t0}^{\alpha C} D_t^a f(t) = f(t) - \textstyle\sum_{k=0}^{n-1} \frac{t^k}{k!} f^{(k)}(0) \end{aligned} \tag{3.16}$$

property 6

$$\begin{aligned} &{}_0^c D_t^\alpha \left({}_0D_t^q f(t) \right) = {}_0D_t^q \left({}_0^c D_t^\alpha f(t) \right) = {}_0^c D_t^{a+q} f(t) \\ &f^{(s)}(0) = 0, s = n, n+1, \ldots, q \end{aligned} \tag{3.17}$$

3.6 Mittag-Leffler Function

The following one-parameter Mittag-Leffler function, introduced by Mittag Leffler is an essential function used in modelling physical processes with the help of the fractional calculus concepts:

$$E_\alpha(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + 1)}. \tag{3.18}$$

The classical exponential function can be acquired from Equation (3.18) if $\alpha = 1$. The two-parameter Mittag-Leffler function, which is equally important as Equation (3.18) in fractional calculus, is given in (3.19) equation[43, 44].

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}, \alpha, \beta > 0 \tag{3.19}$$

3.7 Fractional Order Controller Design

Fractional order systems have no state variables, but it is possible to obtain for them representations similar to those that use the state variables of integer systems. This section addresses first the general case of multiple input, multiple-output (MIMO) systems , and then the particular case of single-input, single-output (SISO) systems. This represent the fractional-order SEPIC converter model by fractional calculus.Where α is the fractional order with $0 < \alpha < 1$,$x_1(t)$), $x_2(t)$),$x_3(t)$) and $x_4(t)$) are the state variables of the fractional-order system (3.21), a, b, d, f and g are system parameters[44]. The conventional SEPIC converter describe by

$$\begin{aligned}
{}_0D_t x_1(t) &= av_{in} - a(1-u)(x_3 + x_4) \\
{}_0D_t x_2(t) &= bx_3 u - b(1-u)(x_4) \\
{}_0D_t x_3(t) &= f(1-u)x_1 - fu(x_2) \\
{}_0D_t x_4(t) &= d(1-u)(x_1 + x_2) - g(x_4)
\end{aligned} \tag{3.20}$$

TABLE 3.1: Description for Variables

No.	Variables	Definitions
1	$x_1(t)$	input current
2	$x_2(t)$	second inductor current
3	$x_3(t)$	first capacitor voltage
4	$x_4(t)$	output voltage
5	x_{1r}	difference of x_4 and x_{4d}
6	x_r	input current reference
7	x_{4d}	output voltage reference
8	x_{rd}	derivative of input current reference
9	$u(t)$	Controller

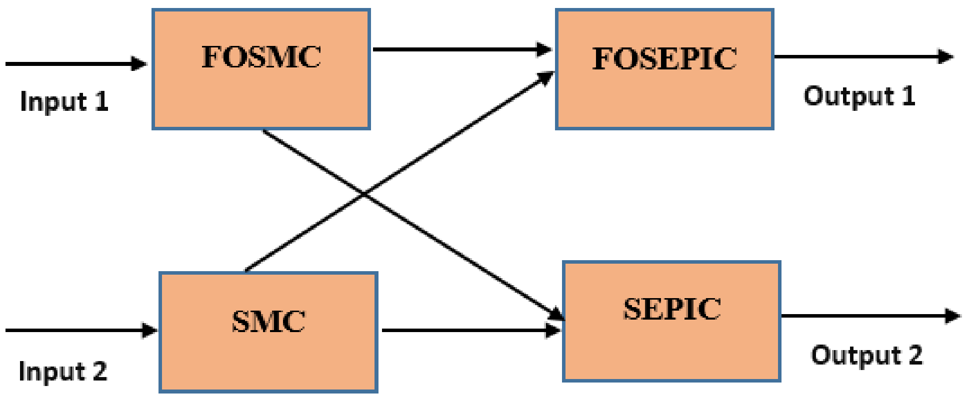

FIGURE 3.1: General Bock diagram for Proposed System

The SEPIC converter fractional-order system is defined as (3.21) with the simulink model in (3.2)

$$\begin{aligned}
{}_0D_t^\alpha x_1(t) &= av_{in} - a(1-u)(x_3+x_4) \\
{}_0D_t^\alpha x_2(t) &= bx_3u - b(1-u)(x_4) \\
{}_0D_t^\alpha x_3(t) &= f(1-u)x_1 - fu(x_2) \\
{}_0D_t^\alpha x_4(t) &= d(1-u)(x_1+x_2) - g(x_4)
\end{aligned} \tag{3.21}$$

The pseudo state space equation in (3.21) is modeled after the different aspects

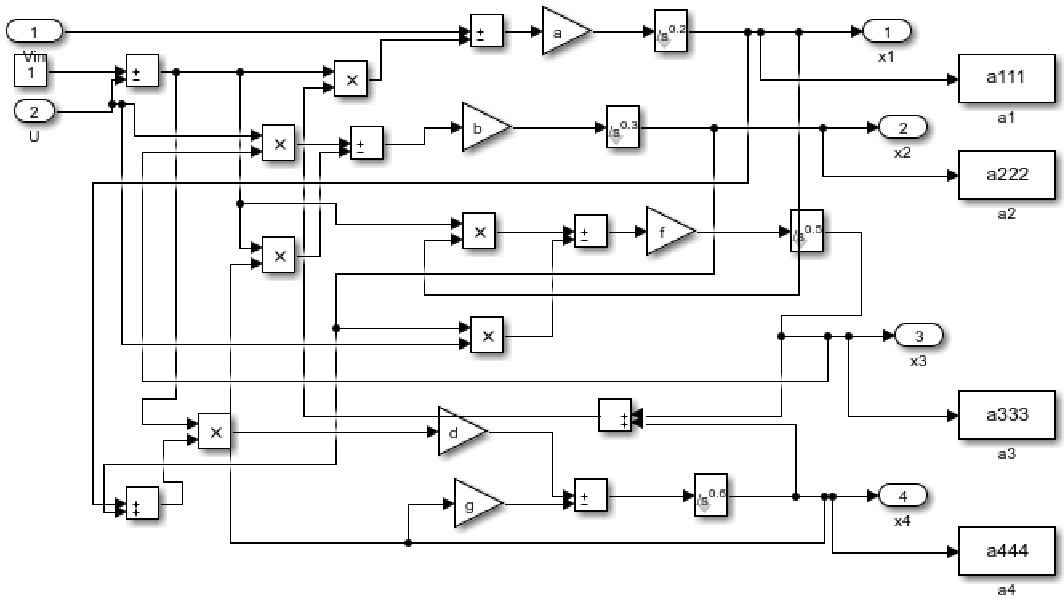

FIGURE 3.2: The Model FOSEPIC

mentioned in [44, 45]. The output voltage is controlled indirectly by controlling the input inductor current through x_1 state variable. In order to achieve such control, the sliding surface function given in (3.22)

$$s(t) = x_1 - x_r \tag{3.22}$$

where x_r denotes the reference of x_1 state variable and use the method presented in [26], the inductor current reference can be generated by using a proportional-integral (PI) controller without employing compensation term (3.23)

$$x_r = -\lambda x_{1r} - \varepsilon \int_0^t x_{1r} dt \tag{3.23}$$

where λ and ϵ are the proportional and integral gains, respectively. The derivative of equation (3.23)can be written as

$$Dx_r = -\lambda D x_{1r} - \varepsilon x_{1r} \tag{3.24}$$

Change equation (3.24) into equation (3.25) by using properties of fractional calculus in (3.16)

$$D^{\alpha} D^{-\alpha} D x_r = -\lambda D x_{1r} - \varepsilon x_{1r} \tag{3.25}$$

where $D = D^1$

$$D^{\alpha} D^{1-\alpha} x_r = -\lambda D x_{1r} - \varepsilon x_{1r} \tag{3.26}$$

Apply the $D^{\alpha-1}$ operator on both sides to obtain equation (3.27)

$$D^{\alpha} x_r = \lambda D^{\alpha-1} D x_{1r} - \varepsilon D^{\alpha-1} x_{1r} \tag{3.27}$$

Now using properties defined in equation(3.15) of fractional calculus we obtain the following result

$$D^{\alpha} x_r = \lambda D^{\alpha} x_{1r} - \varepsilon D^{\alpha-1} x_{1r} \tag{3.28}$$

Substitute equation (3.29) into the equation (3.28) and to obtain equation (3.30)

$$x_{1r} = x_4 - x_{4d} \tag{3.29}$$

where x_{4d} denotes the reference of x_4 state variable and x_{1r} denotes the difference between x_4 state variable and x_{4d} denotes the reference [26].

$$D^{\alpha} x_r = -\lambda D^{\alpha} x_4 + \lambda D^{\alpha} x_{4d} - \varepsilon D^{\alpha-1} x_4 + \varepsilon D^{\alpha-1} x_{4d} \tag{3.30}$$

Apply the D^{α} operator to both sides of equation (3.22)

$$D^{\alpha} s = D^{\alpha} x_1 - D^{\alpha} x_r \tag{3.31}$$

Substitute equation (3.30) into the equation (3.31)

$$D^{\alpha} s = D^{\alpha} x_1 + \lambda D^{\alpha} x_4 - \lambda D^{\alpha} x_{4d} + \varepsilon D^{\alpha-1} x_4 - \varepsilon D^{\alpha-1} x_{4d} \tag{3.32}$$

Equation (3.32) represent the sliding surface on which the controller slide according to predefined definition of sliding surface. In the following section we will check the stability of this sliding surface.

3.7.1 Check Stability of the Sliding Dynamics

Theorem 3.1 [1] If $x = 0$ is the equilibrium point of the system ${}_{t_0}{}^{C}D_t^{\alpha}x(t) = f(t,x)$, f is Lipschitz by a constant L and is piecewise continuous with respect to t, then the solution of the system satisfies $\|x(t)\| \leq \|x(t_0)\| E_{\alpha}\left(L(t-t_0)^{\alpha}\right)$,(3.18) where $\alpha \in (0,1)$.

Definition 3.1 [1] The solution of ${}_{t_0}D_t^{\alpha}x(t) = f(t,x)$ is said to be Mittag-Leffler stable if

$$\|x(t)\| \leq \left\{m\left[x(t_0)\right] E_{\alpha}\left(-\lambda(t-t_0)^{\alpha}\right)\right\}^b$$

where $E_{\alpha}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k\alpha+1)}, t_0$ is the initial time $\alpha \in (0,1), \lambda > 0, b > 0, m(0) = 0, m(x) \geq 0$, and $m(x)$ is locally Lipschitz on $x \in \mathbb{B} \in \mathbb{R}^n$ with Lipschitz constant m_0. The system output is forced to track this surface with the help of a reaching law. The reaching law designed in such a way that it guarantees the stability of the closed loop system. Generally, four reaching laws are widely used and have been reported in the literature namely exponential, constant rate,general and power rate laws[46]. In the present work, exponential law as given in equation (3.33) is utilized. According to this law,

$$ {}_0D_t^{\alpha}s = -\gamma\,\mathrm{sign}(s) - \rho s \tag{3.33}$$

where $s \in R, \rho > 0, \gamma > 0$. For proving the stability of (3.33), choose a Lyapunov candidate function $V = s^2$. According to the Leibniz rule of fractional differentiation in equation (3.34), the α th-order time derivative of V can be given in equation (3.35).

$$ {}_aD_t^{\alpha}(\phi(t)f(t)) = \sum_{r=0}^{\infty} \begin{pmatrix} \alpha \\ r \end{pmatrix} \phi^{(r)}(t)\,{}_aD_t^{\alpha-r}f(t) \tag{3.34}$$

if $\phi(t)$ and $f(t)$ and all their derivatives are continuous in the interval [a, t]

$$\begin{aligned} V^{\alpha} &= s(-\gamma \operatorname{sgn}(s) - \rho s) + \Delta \\ &= -\gamma s(\operatorname{sgn}(s)) - \rho s^2 + \beta |s|^2 \\ &= -\gamma s(\operatorname{sgn}(s)) - (\rho - \beta)|s|^2 \end{aligned} \tag{3.35}$$

where Δ [1] is given by

$$\Delta := \sum_{r=1}^{\infty} \frac{\Gamma(1+\alpha)}{\Gamma(1+r)\Gamma(1-r+\alpha)} {}_0D_t^{\alpha} s_0 D_t^{\alpha-r} s \tag{3.36}$$

$$|\Delta| := \left| \sum_{r=1}^{\infty} \frac{\Gamma(1+\alpha)}{\Gamma(1+r)\Gamma(1-r+\alpha)} {}_0D_t^{\alpha} s_0 D_t^{\alpha-r} s \right| \leq \beta |s|^2 \tag{3.37}$$

$${}_0D_t^{\alpha} V = s(-\gamma \operatorname{sign}(s) - \rho s) + \Delta = -\gamma |s| - \rho s + \Delta. \tag{3.38}$$

According to equation (3.35),${}_0D_t^{\alpha} V \leq 0$,Where $\rho \geq \beta$ and this proves that the system given by (3.33) is Mittag-Leffler stable, which implies that the trajectories in the phase space are attracted by the subspace (manifold) described by $s = 0$. Depends on all the concept we dealt at the top now ,in the following we are going to find controller for fractional order SEPIC converter in equation(3.21). Equate equation (3.21) with the equation (3.33) we obtain equation (3.39)

$$D^{\alpha} x_1 + \lambda D^{\alpha} x_4 - \lambda D^{\alpha} x_{4d} + \varepsilon D^{\alpha-1} x_4 - \varepsilon D^{\alpha-1} x_{4d} = -\gamma sign(s(t)) - \rho s(t) \tag{3.39}$$

From equation (3.21) the fractional order state space the ${}_0D_t^{\alpha} x_1(t)$

$$D_t^{\alpha} x_1(t) = a v_{in} - a(1 - u(t))(x_3 + x_4) \tag{3.40}$$

By substituting equation (3.40) into (3.39) and solve for u(t) which means the control law of fractional order sliding mode control obtained (3.41) [46].

$$u(t) = 1 - [\frac{a v_{in} + \lambda D_t^{\alpha} x_4(t) + \epsilon D_t^{\alpha-1} x_4(t) + \gamma sign(s(t)) + \rho s(t) - \lambda_0 D_t^{\alpha} x_{4d}(t) - \epsilon_0 D_t^{\alpha-1} x_{4d}(t)}{a(x_3(t) + x_4(t))}] \tag{3.41}$$

The controller work for both fractional order SEPIC coverter and conventional

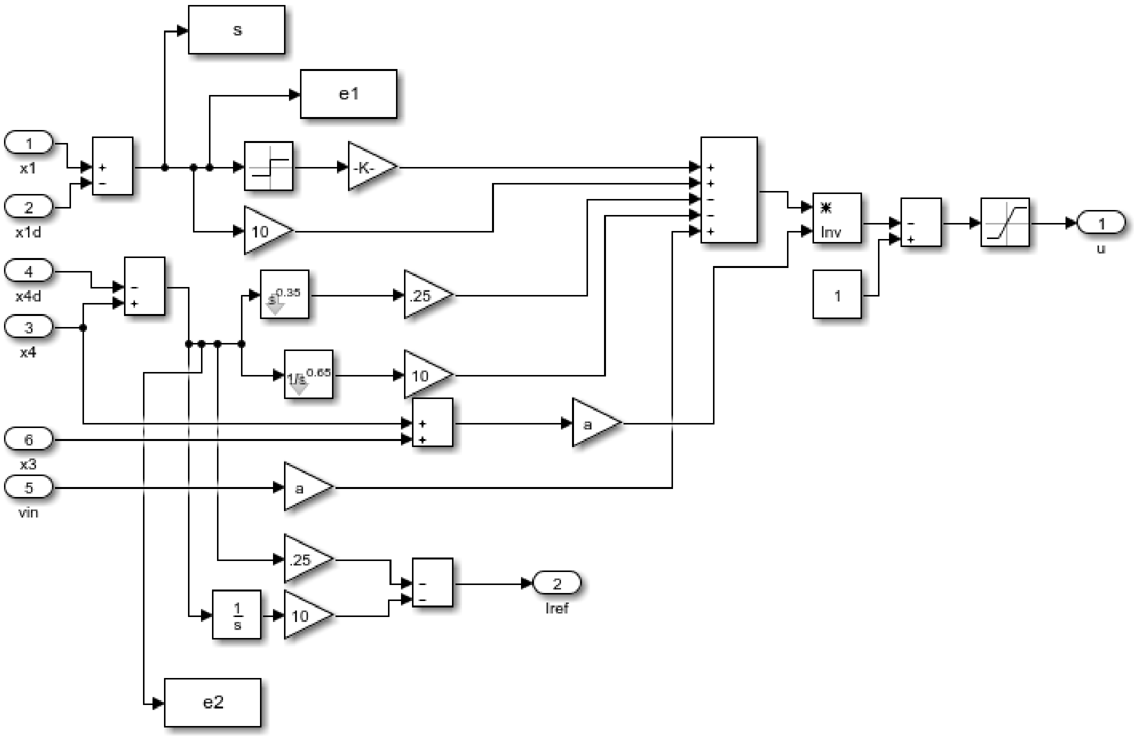

FIGURE 3.3: Controller Model.

SEPIC , depends on the value of α. This means , when $0 < \alpha < 1$ use equation (3.41) as fractional order slide mode and if $\alpha = 1$, it works as conventional slide mode controller and the SIMULINK mode of this controller in (3.3)

Chapter 4

Result and Discussion

4.1 Simulation Results and Discussion

In this section, simulation results of modeled system is presented to illustrate the effectiveness of the proposed fractional-order SMC-based sliding-mode control scheme for the fractional-order nonlinear system. The theoretical considerations are verified by simulations. The proposed fractional order sliding mode control strategy simulated using MATLAB/SIMULINK program in fig.4.1. The perfor-

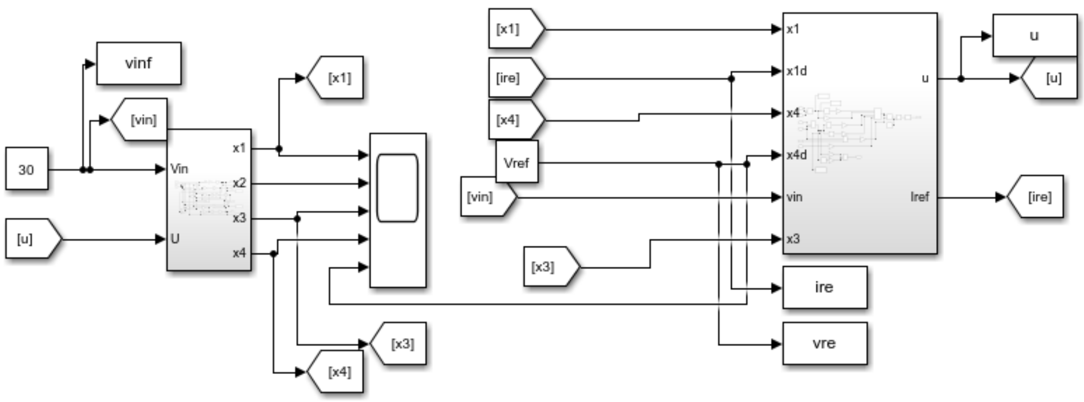

Figure 4.1: Overall System Model

mance of the proposed control method is tested in terms of voltage regulation ability for four parameters. These are: variable input voltage, variable controller order, variable system orders and different load conditions. The system and control parameters used in [26] are used here, the simulation parameters are given in Table 4.1

TABLE 4.1: Parameters Used to Evaluate the Model

No.	Parameters	values used
1	input voltage(v_{in})	30V and 60V
2	Capacitors(C_1, C_2)	330μF
3	Inductors(L_1, L_2)	800μH
4	(v_{ref})	48V
5	Proportional and integral gains(λ, ϵ)	.25 and 10
6	load1 (R_{L1})	50Ω
7	load2 (R_{L2})	33.33Ω
8	input voltages	30V,60V

4.1.1 Steady-State Performance

Fig.4.2 show the steady-state results of input voltage (v_{in}), output voltage (v_{out}), and inductor currents (i_{L1} and i_{L2}) under $R_L = 50\Omega$ in the buck and boost modes. It is clear from Fig.4.2 (a) and (c) that the output voltage is 48V which means that the controller regulates the output voltage at its reference and the inductors current in Fig.4.2 (b) and (d), for boost and buck mode of operation respectively. In addition, the converter with the proposed control method successfully operates both in buck and in boost modes as shown in Fig.4.2. Fractional order sliding mode controller for FOSEPIC converter at steady-state responses of the input voltage, output voltage, and inductor currents under $R_L = 50\Omega$.

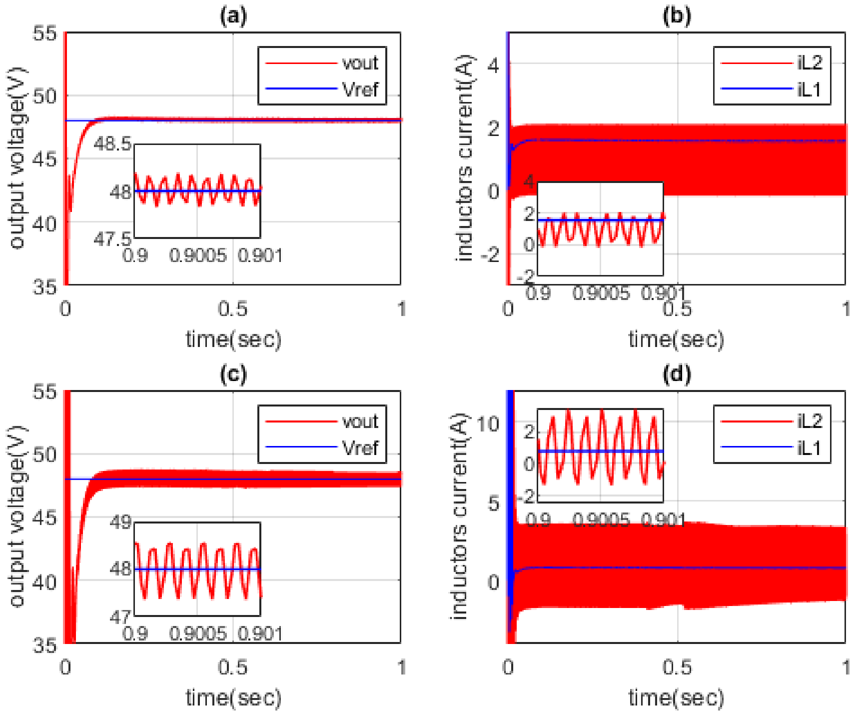

FIGURE 4.2: Steady-State Performance for FOSEPIC Converter
(a) Output voltage for boost mode (c) Output voltage for buck mode, (b)Inductors current for boost mode (d) Inductors current buck mode

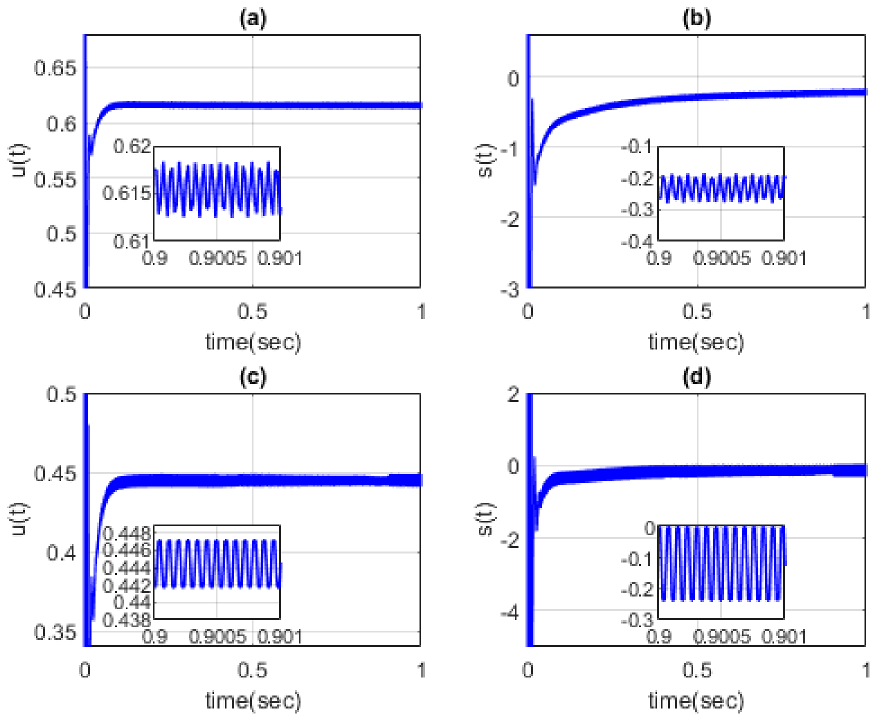

FIGURE 4.3: FOSEPIC Converter Sliding Surface and FOSMC
(a)FOSMC for boost mode (b) Sliding surface for boost mode, (c)FOSMC for buck mode (d) sliding surface buck mode

4.1.2 Performance Under Input Voltage Variations

Fig.4.4 show the dynamic responses of i_{L1} and i_{out} currents for a change in (v_{in}) under $V_{ref} = 48V$ and $R_L = 50\Omega$. The results presented correspond to the input voltage variation from 60V to 30V and from 30V to 60V. Initially, the converter operates in the buck mode for (v_{in})=60V. However, when the input voltage is changed from 60V to 30V, the operation mode of the converter is changed from buck mode to boost mode. Similarly, when the input voltage is changed from 30V to 60V, the converter's operation is changed from boost mode to buck mode. In these operating mode changes, the input current is also changed accordingly so that the power delivered to the load is unchanged as seen on Fig.4.5 for boost and buck mode operation. It can be noticed that the output voltage regulate its reference successfully at 48V in both operating modes. Fig. 4.4 there exist small

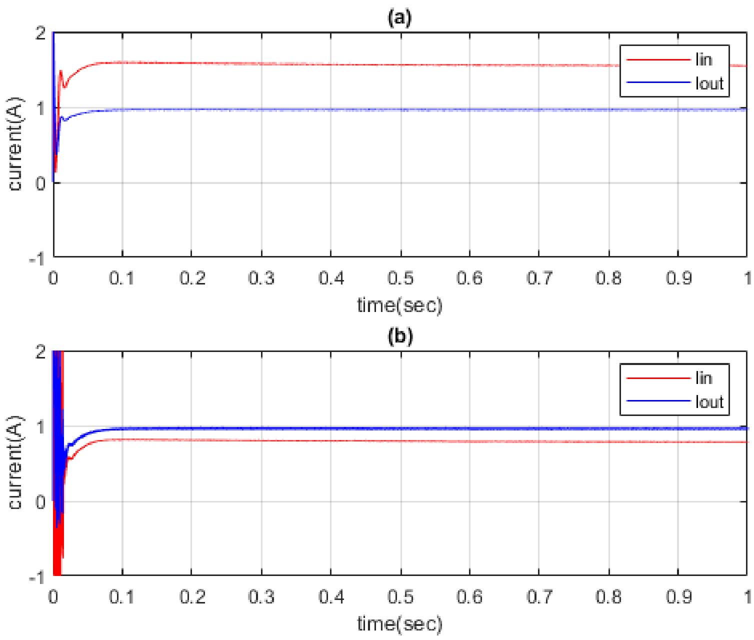

FIGURE 4.4: Input and Output Current for FOSEPIC Converter (a)Input and Output Current for Boost Mode of Operation (b) Input and Output Current for Buck Mode of Operation

undesired ripples on the output current which occur due to the noise disturbance in the system. Clearly, the output voltage is regulated at 48V in both operating modes.Again, in order to maintain the load power against this input voltage

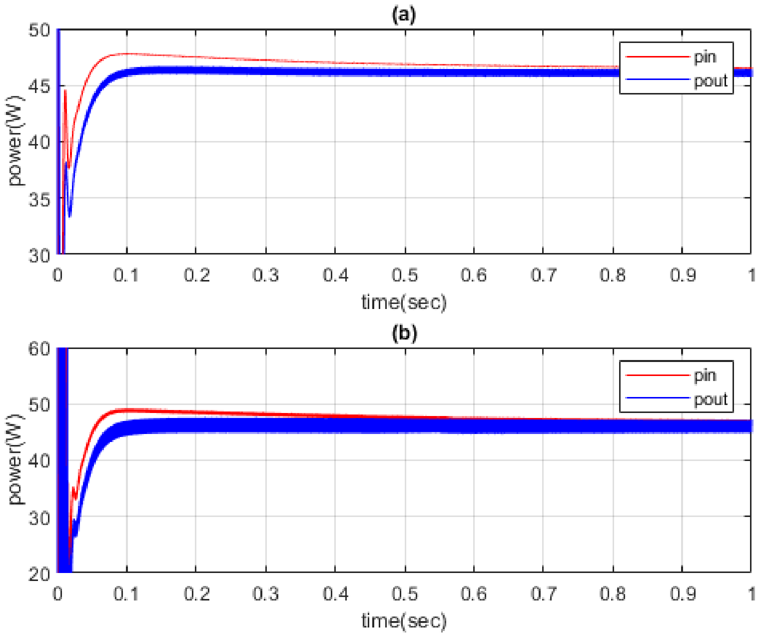

FIGURE 4.5: Input and Output Current for FOSEPIC Converter (a)Output and Input Power for Boost Mode of Operation (b)Output and Input Power for Buck Mode of Operation

variations, the input power should also be changed which is possible if the input current is changed.

4.1.3 Performance under Load Variations

The performance of the proposed control strategy is also tested under 50 to 70 percent load variations. Fig.4.6 show the dynamic responses of output voltage for an abrupt change in the load resistance when V_{ref}= 48V. The load change was from 50Ω to 33.33Ω and from 33.33Ω to 50Ω . Fig.4.6 and Fig.4.7 show the dynamic responses due these load changes when the converter operates in both boost mode and buck mode respectively. As can be seen clearly, the output voltage is almost not affected from these load changes. This means that the proposed controller is able to regulate the output voltage under load variations. Again, it can be seen that, except for the small overshoot and undershoot occurring during the transient period, the output voltage is not affected from the load changes. Under load variation the two operating modes act different with reference to peak to

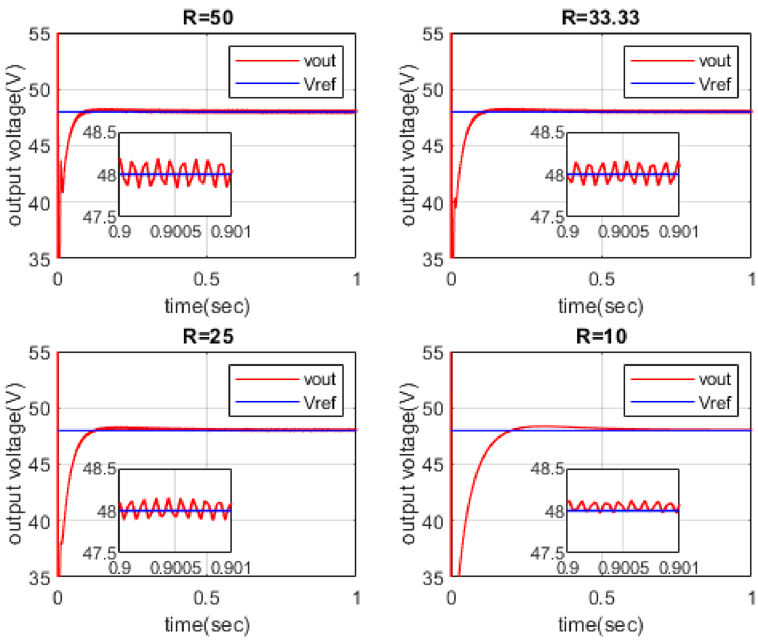

FIGURE 4.6: FOSEPIC Converter Under Load Variations for Boost Mode of Operation

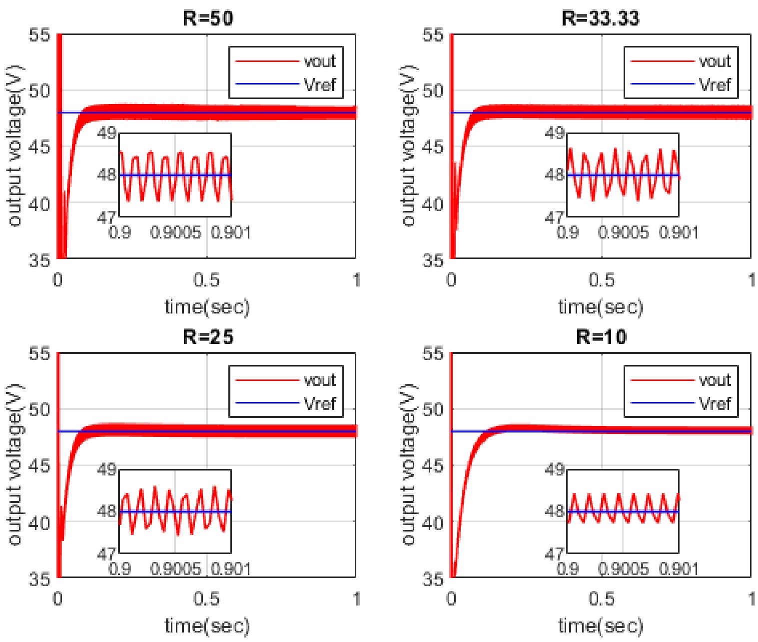

FIGURE 4.7: FOSEPIC Converter Under Load Variations for Buck Mode of Operation

peak voltage output. The boost mode has lower peak to peak values and the buck mode has higher peak to peak values. Further, the response has high peak to peak values for the output voltage as well as the output current. This is evident from tables 4.2 and 4.3 for boost and buck mode of operation respectively . Load variation for the boost mode for load resistance in the range 1Ω to $3k\Omega$ and for buck mode in the range 1Ω to 58Ω showed consistent result for the FOSEPIC of orders $(0.25, 0.35, 0.52, 0.65)$ for regulation around a V_{ref}= 48V. variation of load

TABLE 4.2: Relationship between load variation and Peak to Peak Output Voltage for FOSEPIC Converter (Boost Mode Operation)

Load(R)	10	16	25	50	100	120	140	160
$V_{\max}$	48.09	48.11	48.16	48.45	48.2	48.23	48.23	48.24
$V_{\min}$	47.93	47.71	47.88	47.83	47.79	47.78	47.77	46.97
V_{pp}	0.1595	0.2053	0.3042	0.3524	0.4313	0.4485	0.46	0.4677

Where $V^{*}_{\mathbf{max}}$ =maximum output voltage $V^{**}_{\mathbf{min}}$=minimum output voltage $V^{***}_{\mathbf{pp}}$=peak to peak voltage

TABLE 4.3: Relationship between load variation and Peak to Peak Output Voltage for FOSEPIC Converter (Buck Mode Operation)

Load(R)	1	10	16	25	50	55	58
$V_{\max}$	48.11	48.41	48.5	48.59	48.58	48.6	48.67
$V_{\min}$	47.96	47.71	47.52	47.42	47.25	47.24	47.16
V_{pp}	0.1561	0.7002	0.9776	1.169	1.333	1.36	1.512

Where $V^{*}_{\mathbf{max}}$ =maximum output voltage $V^{**}_{\mathbf{min}}$=minimum output voltage $V^{***}_{\mathbf{pp}}$=peak to peak voltage

by using order of the Fractional SEPIC converter $(0.25, 0.35, 0.52, 0.65)$ on voltage regulation and tracking the reference voltage $V_{ref} = 48V$

4.1.4 Output Voltage Variation under Orders Variation of the plant and the Controller

4.1.4.1 Under Orders Variation of the Plant

Under variation of plant orders, for FOSMC order given by (0.35), with the reference voltage $V_{ref} = 48V$ and it regulates the output voltage successfully as shown in fig.4.8

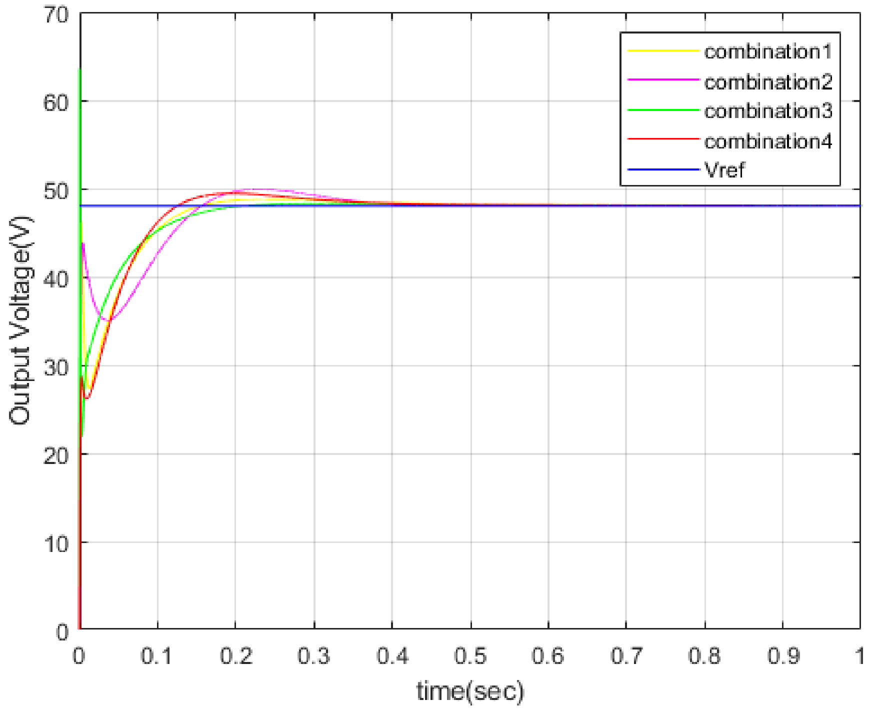

FIGURE 4.8: Output Voltage under Variation of FOSEPIC Orders (a)[Yellow] $(0.6, 0.28, 0.4, 0.9)$ Combination1 (b)[Magenta] $(0.9, 0.5, 0.4, 0.9)$ Combination2 (c)[Green] $(0.17, 0.25, 0.35, 0.69)$ Combination3 (d)[Red] $(0.75, 0.5, 0.2, 0.85)$ Combination4

4.1.4.2 Under Variation of Controller Order

Under variation of controller order, for FOSEPIC converter orders given by $(0.25, 0.35, 0.52, 0.65)$, with the reference voltage $V_{ref} = 48V$ the peak to peak values increase as the order of controller increases. This is shown in tables 4.4 and 4.5, respectively for boost and buck mode of operation.

TABLE 4.4: Relationship between Order of the Controller and Peak to Peak Output Voltage for FOSEPIC Converter (Boost Mode Operation)

Order (α)	0.07	0.08	0.1	0.2	0.3	0.4	0.5	0.52
$V_{\max}$	48.08	48.08	48.08	48.08	48.09	48.09	48.09	48.12
$V_{\min}$	47.91	47.92	47.92	47.93	47.93	47.93	47.93	47.95
V_{pp}	0.1547	0.1582	0.1585	0.1585	0.1591	0.159	0.1642	0.1691

Where $V^{*}_{\mathbf{max}}$ =maximum output voltage $V^{**}_{\mathbf{min}}$=minimum output voltage $V^{***}_{\mathbf{pp}}$=peak to peak voltage

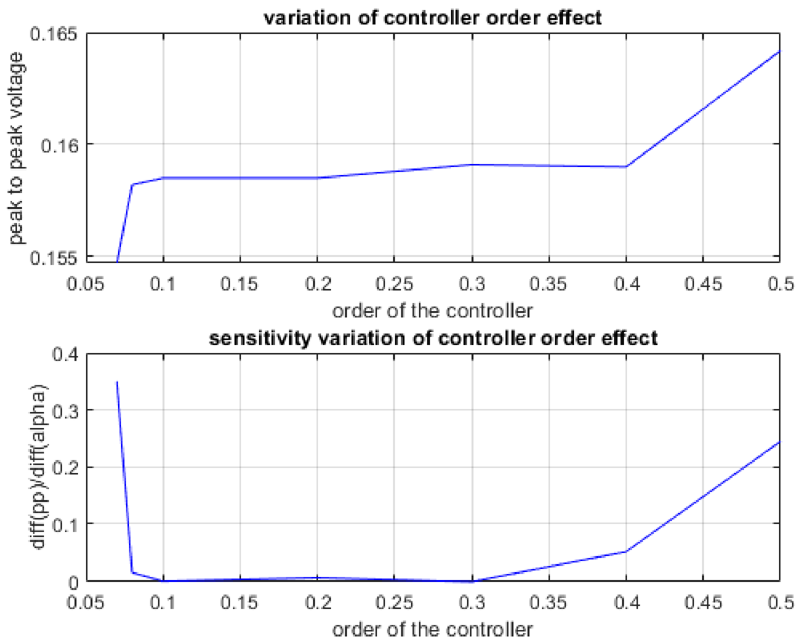

FIGURE 4.9: Relationship between Order of the Controller and Peak to Peak Output Voltage for FOSEPIC Converter (Boost Mode Operation)
(a) Variation of Order Effect on Peak to Peak output Voltage (b)Sensitivity of the Variation of Order Effect on Peak to Peak output Voltage

Table 4.5: Relationship between Order of the Controller and Peak to Peak Output Voltage for FOSEPIC Converter (Buck Mode Operation)

Order(α)	0.07	0.08	0.1	0.2	0.3	0.4	0.5
$V_{\max}$	48.4	48.41	48.4	48.41	48.41	48.41	48.41
$V_{\min}$	47.71	47.71	47.71	47.71	47.71	47.71	47.7
V_{pp}	0.6956	0.695	0.6958	0.696	0.699	0.7042	0.7092

Where $V^{*}_{\mathbf{max}}$ =maximum output voltage $V^{**}_{\mathbf{min}}$=minimum output voltage $V^{***}_{\mathbf{pp}}$=peak to peak voltage

4.1.5 Comparison of FOSMC and SMC for FOSEPIC converter

Under FOSEPIC orders of $(0.25, 0.35, 0.52, 0.65)$ with two different controllers, as show in fig 4.12 (boost mode), the oscillation effect is removed with FOSMC for FOSEPIC and SMC for SEPIC. The same is true for buck mode operation as shown in fig.4.13. fig 4.12(a) and 4.13(a) show that FOSMC and SMC can track the reference voltage well for boost and buck modes respectively.

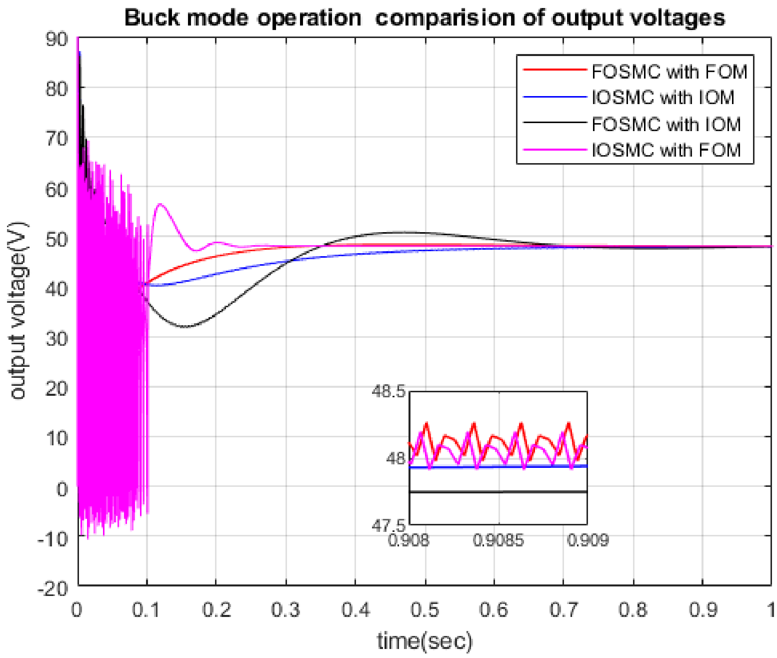

FIGURE 4.10: Comparison Output Voltage of SEPIC and FOSEPIC converter with FOMSC and IOSMC for Buck Mode Operation
(a)[Red] FOSMC with FOM (b)[Blue] IOSMC with IOM(c)[Black] FOSMC with IOM (d)[Magenta] IOSMC with FOM

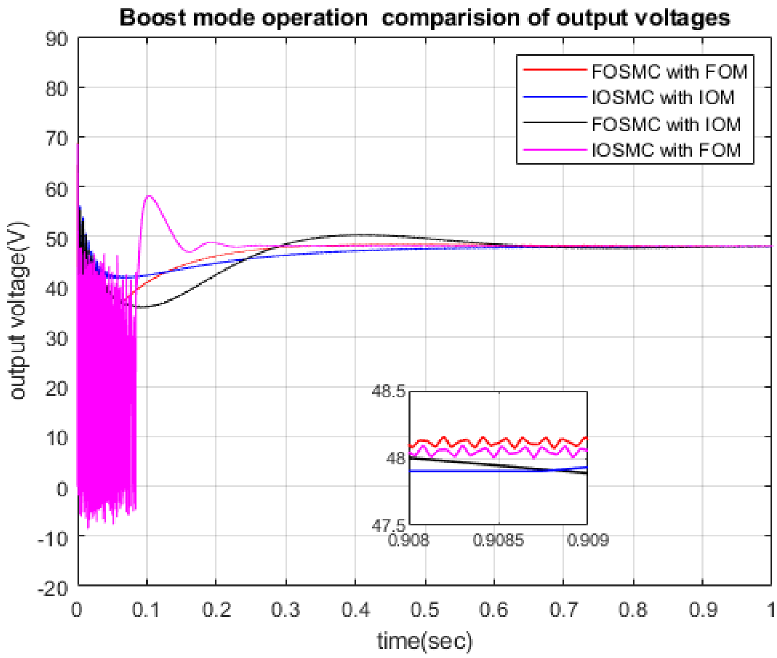

FIGURE 4.11: Comparison Output Voltage of SEPIC and FOSEPIC converter with FOMSC and IOSMC for Boost Mode Operation

(a)[Red] FOSMC with FOM (b)[Blue] IOSMC with IOM(c)[Black] FOSMC with

IOM (d)[Magenta] IOSMC with FOM

In Fig 4.12 , one can see that there are some oscillations in the output voltage which occur during comparison between FOSMC and SMC.

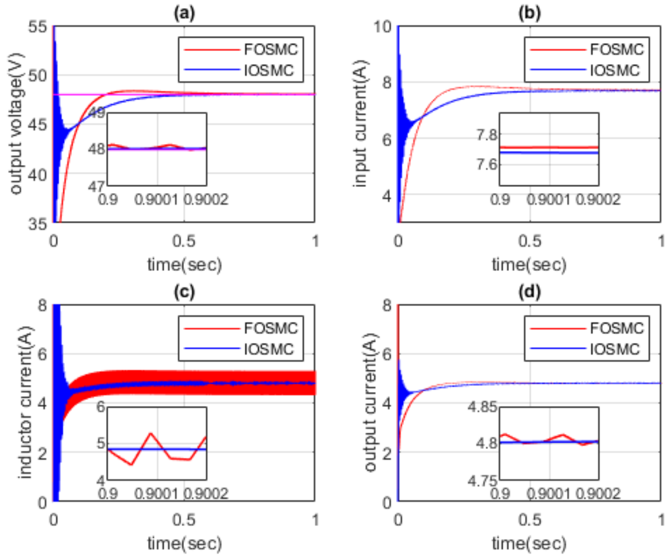

FIGURE 4.12: Comparison of Fractional and Integer SMC's for FOSEPIC Converter Under Boost Mode
(a) Output Voltage FOSMC and SMC.(b) Input current for FOSMC and SMC.(c) Output inductor current for FOSMC and SMC.(d) Output Current for FOSMC and SMC

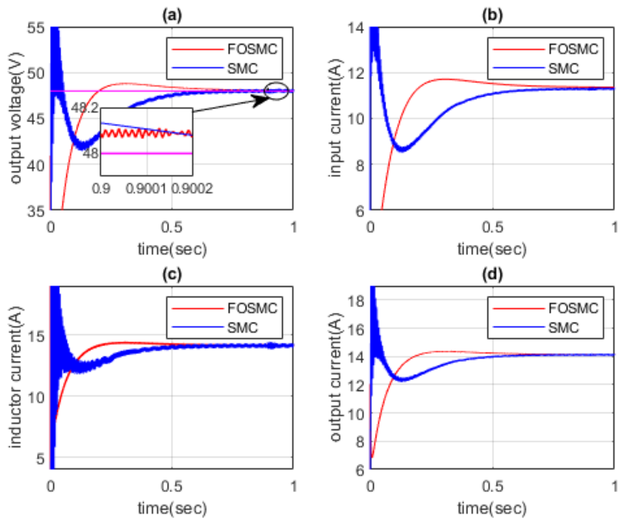

FIGURE 4.13: Comparison of Fractional and Integer SMC's for FOSEPIC Converter Under Buck Mode

(a) Output Voltage FOSMC and SMC.(b) Input current for FOSMC and SMC.(c) Output inductor current for FOSMC and SMC.(d) Output Current for FOSMC and SMC

Such undesired oscillations do not occur in the output voltage obtained by the proposed control method as shown Fig 4.12. Fig 4.12 shows the dynamic responses of output voltage, input and output currents obtained with the SMC method.

Chapter 5

Conclusion and Recommendation

5.1 Conclusions

Fractional order sliding mode control with simplified (single state variable) sliding surface function is proposed for DC-DC FOSEPIC converters. It is shown that the output voltage control can be achieved indirectly by using the sliding surface function based on the input inductor current error. The use of such sliding surface function not only simplifies the simulation of the system, but also the mathematical modeling of fractional order sliding mode control. The performance of the proposed strategy is cross checked under different mechanisms. The performance of the proposed FOSMC method is tested using MATLAB/SIMULINK for two different operations, buck and boost modes, in terms of the voltage regulation ability under two values of input voltage and different load resistances. The theoretical considerations are validated by the simulation results. These results show that the proposed FOSMC method offers advantages in terms of flexibility of design and degrees of freedom.

5.2 Recommendations

As recommendations for the future the following points:

- The sliding surface could be time varying
- Change sliding surface regulator from PI to PD to PID
- Build adaptive fractional order slide mode controller for more accuracy
- Conduct an experiment on the modeled system for validation and to implement it
- Automate the orders tuning process to identify the best combination

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