DESIGN AND FINITE ELEMENT ANALYSIS OF SINGLE-CYLINDER CRANKSHAFTSUBJECTED TO DYNAMIC LOAD

BY

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THESIS SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES OF JIMMA UNIVERSITY IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN MECHANICAL ENGINEERING (DESIGN OF MECHANICAL SYSTEM)

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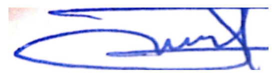
"Thesis submitted to the School of Graduate Studies of Jimma University in Partial Fulfillment of the Requirements for the Degree of Master of Science in Mechanical Engineering (Design of Mechanical System)"

Jimma, Ethiopia
January, 2022

Declaration

By my signature below, I declare and affirm that this thesis is my own work. I have followed all ethical and technical principles in the literature review. Any scholarly matter that is included in the thesis has been given recognition through citation.

Tilahun Mohammed		**10/3/2022**
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As a member of the Examination Board of the final Master of Science online defense, we certify that we have read and evaluated the thesis prepared by Tilahun Mohammed Muktar entitled "***Design and Finite Element Analysis of Single Cylinder Crankshaft Subjected to Dynamic Load ".*** We recommend that it could be accepted as a fulfilling the thesis requirement for the Degree of Master of Science in Mechanical Engineering (Design of Mechanical System).

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ABSTRACT

The static structural and dynamic loads were analyzed analytically which, results in the load applied to crankpin bearings. The study selection of best material by comparing the Static structural analysis on a crankshaft from a single cylinder crankshaft. The need of load history in the FEM analysis necessitates performing a detailed static and dynamic load analysis. Therefore, this study consists of two major sections: (1) static and load analysis, (2) FEM and stress analysis. In this study static and dynamic simulation was conducted on three crankshafts Forged steel (AISI 1045 steel), Ductile cast iron and Medium carbon steel.. Finite element analysis was performed to obtain the variation of stress magnitude at critical locations. Finally select suitable material.The stress and dynamic analysis was done analytically and was verified by simulations in SOLIDWORK which resulted in the load spectrum applied to crankpin bearing. This load was then applied to the FE model in ANSYS, and boundary conditions were applied according to the engine mounting conditions. The analysis was done for different engine speeds and as a result, critical engine speed and critical region on the crankshafts were obtained.

The value of von-misses stresses of analysis is less than the material yield stress so this design is safe. By comparing von-mises stress and deformation values, from three different materials Therefore the finite element results show that the forged steel (AISI 1045 steel material) is the best material of all. The maximum load occurs at the crank angle of 355 degrees for this specific engine. At this angle only bending load is applied to the crankshaft.

Keywords: *Crankshaft, Static Analysis, Finite Element Analysis (FEA), MATLAB.*

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NOMENCLATURE

α_p, α_{px}	Linear acceleration of piston, linear acceleration of piston in X direction
α_{rx}, α_{ry}	X, Y components of linear acceleration of C.G. of connecting rod
D, F_p	Bore diameter, force applied on the crankpin
F_Q	Force of connecting rod
F_x, Fy	X, Y components of the global load applied on the crankpin
Fr, Ft	r, t components of the global load applied on the crankpin
I_{xx}, I_{yy}, I_{zz}	Moment of Inertia about X, Y, and Z axis located at the C.G. of connecting rod.
I_{xy}, I_{yz}, I_{xz}	Moment of Inertia about XY, YZ, and XZ planes located at the C.G. of connecting rod.
L /r, r	Ratio of stroke length to crank radius, crank radius
L_c	Connecting rod length
L_g	Distance of the C.G. of connecting rod from crankpin end center
m_p, m_r	Mass of piston assembly, connecting rod
r_c	Crank radius
r_g	Location of C.G. of connecting rod
t	Crankpin bearing length

V_g	Linear velocity of C.G. of connecting rod
V_{gx}, V_{gy}	X, Y components of linear velocity of C.G. of connecting rod
V_{px}	Linear velocity of piston in the X direction
α_1, α_2	Angular acceleration of crankshaft, connecting rod

CHAPTER ONE
INTRODUCTION

1.1 Background of research

Crankshaft is one of the most important moving parts in internal combustion engine and it is a large component with a complex geometry in the engine [1]. The most common application of a crankshaft is in an automobile engine. However, there are many other applications of a crankshaft which range from small one-cylinder lawnmower engines to very large multi cylinder marine crankshafts and everything in between. Crankshaft is a part of an engine that is used to convert reciprocating motion into rotary motion, in order to convert this, it uses crank throws and additional bearing surfaces to which each cylinder is attached. We know that it takes a huge number of loads during the working of an engine, the performance of the fatigue and components durability must be taken care during the design consideration. The crankshaft consists of main journals, crank pins, crank webs, counter weights and oil holes.

Crankshaft is a large component with a complex geometry in the engine, which converts the reciprocating displacement of the piston to a rotary motion with a four-link mechanism. Design developments have always been an important issue in the crankshaft production industry, in order to manufacture a less expensive component with the minimum weight possible and proper fatigue strength and other functional requirements. These improvements result in lighter and smaller engines with better fuel efficiency and higher power output. The crankshaft consists of the shaft parts which revolve in the main bearings, the crankpins to which the big ends of the connecting rod are connected, the crank arms or webs (also called cheeks) which connect the crankpins and the shaft parts. The crankshaft main journals rotate in a set of supporting bearings causing the offset rod journals to rotate in a circular path around the main journal centers, the diameter of that path is the engine "stroke": the distance the piston moves up and down in its cylinder. The big ends of the connecting rods contain bearings ("rod bearings") which ride on the offset rod journal [2].

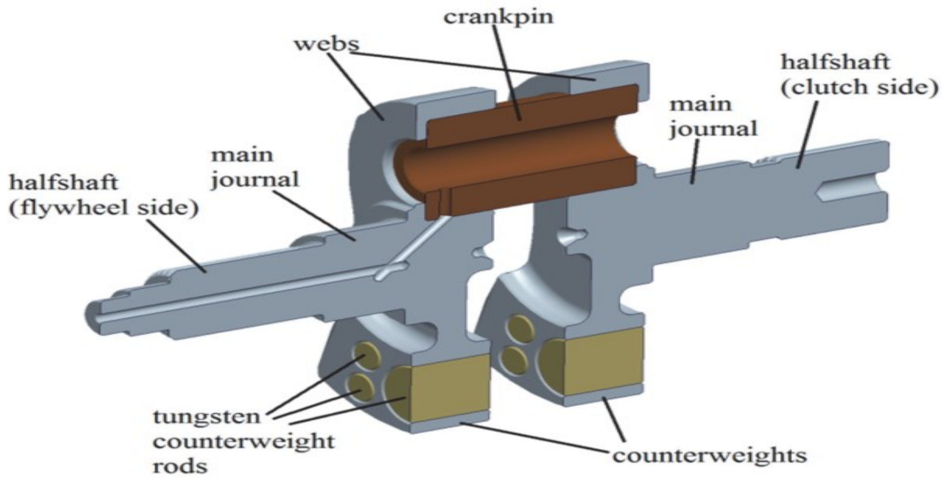


Figure 0.1 Typical Crankshaft with main journals that support the crankshaft in the engine block [2]

Crankshaft experiences large forces from gas combustion. This force is applied to the top of the piston and since the connecting rod connects the piston to the crank shaft, the force will be transmitted to the crankshaft. It must be strong enough to take the downward force of the power stroke without excessive bending so the reliability and life of the internal combustion engine depend on the strength of the crankshaft largely.

Section geometry changes in the crankshaft cause stress concentration at fillet areas where bearings are connected to the webs of the crank. In addition, these component experiences both bending and torsional load during its life service. Therefore, areas of filleted portion are locations that experience the most critical stresses during the service life of the crankshaft [3]. The type of crankshaft depends on number of crankpins. The number of crank-pins depends on the type of engine and number of cylinders. A single cylinder engine will have only one crank-pin and two webs. Multi-cylinder engines will have one crank-pin per piston if the engine is a straight engine, meaning that all cylinders are in a line. If the engine is a V-engine, one bank of cylinders on each side of the crankshaft, two pistons will attach to the same crank-pin. Commonly a crankshaft will be classified by the number of “crank throws” or simply “throws”, which simply refers to the combination of the two webs and crank-pin. There are several different material options available for manufacturing crankshafts, with the two most popular being steel and iron. Crankshafts can be machined by forging, casting and billet. Machining a crankshaft from a billet is not typically done due to the prohibitively long machining times; however, for low production custom pieces it is still done. The steel crankshaft is usually forged to near net shape and then finished by machining processes. Generally, a crankshaft can be classified as forged steel or cast iron, however, within these two categories there are many options.

Single-cylinder engines are simple, compact and economical in construction. The vibration they generate is acceptable in many applications. As a result, these engines are important auxiliary agricultural tool in rural areas for water pumping and lawnmower [4]

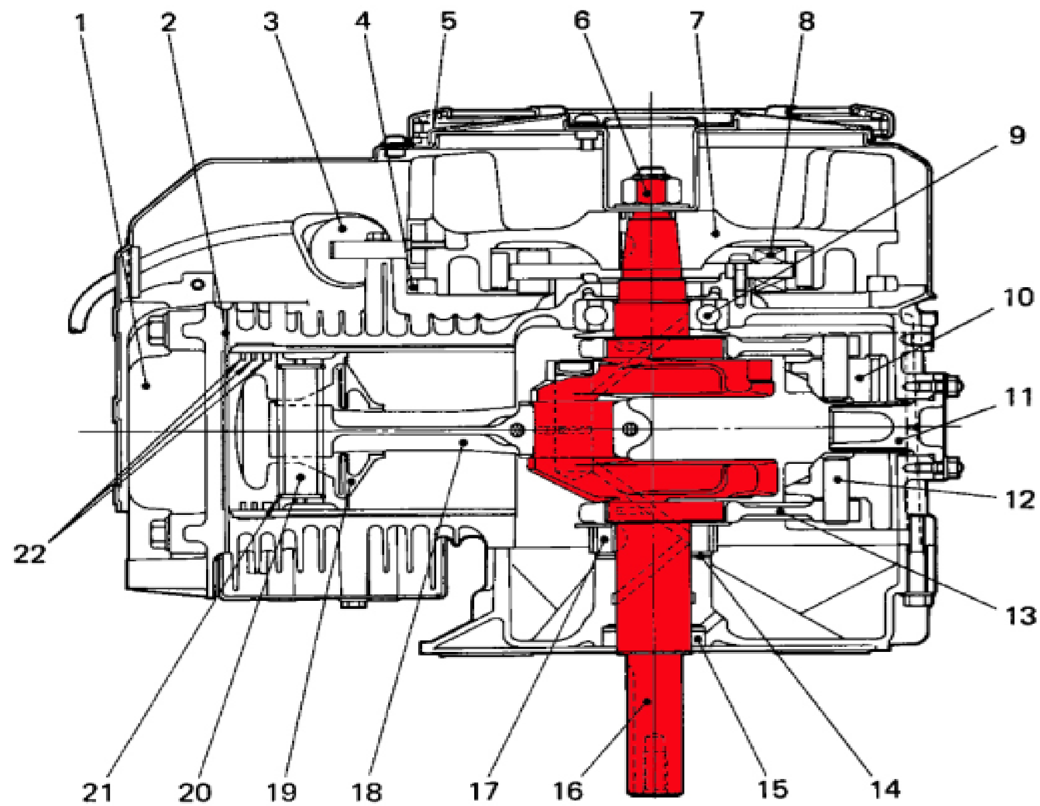


Figure 0.2 Single Cylinder Crankshaft in Engine Block (taken from the engine manual)

The objective of this study was to analyzing stress in critical location for improving fatigue life with geometry optimization of a single cylinder engine typical to that used in a riding lawnmower. Rate failure of crankshaft is not limited to selecting a material, such as steel or iron, a process, such as forging or casting, and surface treatment. Farzin H. Montazersadgh et al [5] suggested the modifications for improvement in fatigue life such as changing the main bearing radius, crank pin radius, fillet radius of crankpin pin and changing the type of material in crankshaft, which are very common modifications usually done in crankshaft geometry.

1.2 Failure of a crankshaft

Failure of the crankshaft makes an engine unworkable, resulting into costly procurement and replacement. There are many sources of failure in the engine one of the most common crankshaft failure is fatigue at the fillet areas due to the bending and torsional load caused by the combustion. The moment of combustion the load from the piston is transmitted to the crankpin, causing a large bending moment on the entire geometry of the crankshaft. At the root of the fillet areas stress concentrations exist and these high stress range locations are the points where cyclic loads could cause fatigue crack initiation leading to fracture.

The failure of crankshafts classified into three categories [6]

Operating source

- Oil absence
- Defective lubrication on journals
- High operating oil temperature
- Improper use of the engine

Mechanical sources

- Misalignments of the crankshaft on assembly
- Improper journal bearings (wrong size)
- Crankshaft vibrations (like misalignment of the crankshaft, incorrect fillet size)

Repairing sources

- Misalignments of the journals (due to improper grinding)
- Misalignments of the crankshaft (due to improper alignment of the crankshaft)
- High stress concentrations (due to improper grinding at the fillet radius areas)
- Improper welding

1.3 Statement of the problem

Due to dynamic loading systems, fatigue failure is a common problem in cyclic load of the crankshaft. In fillet areas due to flexural loads due to combustion. In the moment of fire, the load of the piston is transmitted to the crank pin, causes large bending moments over the entire crankshaft geometry. Fillet areas where these high concentrations of stresses exist and locations are points where cyclic loading can cause crank fatigue start Crankshaft fillet areas experiencing possible stress concentration lead to cracking start and eventually crankshaft failure.

In other cases, the engine is running, the power pulse hits the crankshaft in one place and then another. Torsion vibration occurs when an electrical pulse touches crankpin forward the engine and drive stroke ends. If not checked, it may have broken crankshaft. Furthermore, the linear displacement of the engine is not smooth, because the displacement is due to the combustion of the gas in combustion chamber. Therefore, sudden shocks and uses input from another device may damage it. To avoid problems with the crankshaft must be strong enough to withstand the downward force during power stroke without excessive bending. Therefore, the reliability and life of the internal combustion largely depends on the strength of the crankshaft materials.

1.4 Objective of the study

1.4.1 General objective

The main objective of this study to Design and Stress Analysis of Single Cylinder Crankshaft for a forged steel crankshaft.

1.4.2 Specific Objective

The specific objectives of the project include:

- To determine the magnitude and direction of the static and dynamic loads that act on the bearing between connecting rod and crankshaft torque.
- To analyzed the stresses acting on the crankpin due to gas force.

- To evaluate the von-misses stress and factor of safety and life at different material properties using analytical and ANSYS solution.
- To select suitable material.

1.4.3 Research Methodology

The methods employed to achieve the specific objectives are

- **Literature survey: -**There is a vast amount of literature related to Finite Element Analysis and optimization of Crankshaft. Many research publications, journals, reference manuals, newspaper articles, handbooks; books are available of national and international editions dealing with basic concepts of finite element analysis.
- **Modeling and Finite Element Analysis:** 3D model will be constructed using Solid work and Finite Element Analysis will be conducted by ANSYS or other dynamic software tool.
- **Theoretical Analysis:** The analytical approach discusses detail in this section
- **Generation of the Geometry of Crankshafts:** Finite element analysis of crankshaft geometry generation by using SOLIDWORK software and analysis is using through the ANSYS 18 WORKBENCH software. Using proper type of loading and boundary condition.
- **Mesh Generation:** The Discretization (Mesh generation) is the first step of Finite Element Method. In this step the component or part is divided into number of small parts. FE models were done for each optimized to analyze stress and improve life of crankshaft with different mesh size or by using mesh optimization to obtain good result which is close to analytical result. Import IGES format model in ANSYS WORKBENCH simulation module.
- **Loading And Boundary Conditions** Boundary conditions are playing the important role in finite element analysis. Load applied on the component when the crank is at the position of maximum bending moment or is at the dead Centre and also load data were taken from the calculated result. Boundary condition is based on under supporting condition of crankshaft then obtaining results

- **Result and Data Analysis:** the result from simulation will be analyzed and interpreted finally.
- **Conclusion and Recommendation.**

1.5 Scope of the Study

- ☞ Dynamic simulation should selected crankshaft materials
- ☞ FEA were performed to obtain the variation of stress magnitude at critical locations.
- ☞ Prepared model of crankshaft of single cylinder engine using SOLIDWRK
- ☞ FEA of crankshaft using ANSYS software.

1.6 Organization of the Paper

This thesis is organized into six main chapters. In the first chapter introduction, background, statement of the problem and objective to be achieved are discussed. In chapter two a review of literature appropriate to this thesis is stated. In chapter three Methodology and condition, chapter four material, dimension and static and dynamic analysis of crankshaft by analytical method used were discussed. In chapter five stress analysis from FEM and analytical method at different material and different Diameter are analyzed and discussed. Finally, in chapter six conclusions and recommendation works are presented.

CHAPTER TWO

LITERATURE REVIEW

2.1 Finite Element Modeling

The crankshaft is a huge component with a complicated shape, finite element models have been proposed to provide an accurate and appropriate solution when testing is not possible in a specific laboratory.

Bhumesh J. Bagde [7] to model the crankshaft with dimensions, then simulate the crankshaft for structural and static fatigue analysis. The theme was chosen because of the growing concern for higher loads, lower weight, higher efficiency and shorter load cycles in the crankshaft. The modeling software used was PROE wildfire 4.0 to model the crankshaft. ANSYS analysis software was used for crankshaft fatigue and structural analysis. Analysis was performed on the existing crankshaft material and four alternative materials were also considered for the crankshaft. The analysis shows the important part where stress action is maximum and possibility of crack formation is maximum. Induced stress is minimal for crankshaft material SAE 1137 compared to other materials.

R. Metkar et al [8] described finite element method as the most favorite method to solve stress and fatigue analysis and it is commonly used for analyzing engineering problems. He also studied stress life, strain life and LEFM methods to solve fatigue analysis. There are lots of softwares available for use in finite element analysis applications, such as, ANSYS, Abaqus, Nastran, and MSC.

Rinkle garg and Sunil Baghl [9] has been analyzed crankshaft model and crank throw were created by Pro/E Software and then imported to ANSYS software. The result shows that the improvement in the strength of the crankshaft as the maximum limits of stress, total deformation, and the strain is reduced. The weight of the crankshaft is reduced. There by, reduces the inertia force. As the weight of the crankshaft is decreased this will decrease the cost of the crankshaft and increase the I.C engine performance.

Zhenpeng et al. (2013)[10] has presented complicated work related to the analysis of dynamic and lubricating properties Dynamic properties which are affecting lubrication system are studied with the help of tribology, Finite Element Method (FEM), Finite Difference Method (FDM), multi-body dynamics method, and Component Mode Synthesis method (CMS)

Bhumesh J. Bagde[11] *described* the crankshaft model was created by Pro-E Wildfire 4.0 software. Then, the model created by Pro-E Wildfire 4.0 was imported to ANSYS software. The analysis of the crank is done using five different materials. Static Structural Analysis and fatigue analysis of crank shaft was performed on ANSYS software and the deformation and stresses were compared. Analysis has been performed on existing material of Crank shaft and four alternate materials also considered for crank shaft. Analysis shows the critical portion where stress acting is maximum and the chances of crack formation are maximum. The stresses induced is minimum.

Xiaorong Zhou et al. [12] described the stress concentration in static analysis of the crankshaft model. The stress concentration is mainly occurred in the fillet of spindle neck and the stress of the crank pin fillet is also relatively large. Based on the stress analysis, calculating the fatigue strength of the crankshaft will be able to achieve the design requirements.

Farzin[13] conducted finite element analysis on crankshaft to obtain the variation of stress magnitude at critical locations. The dynamic analysis was done analytically and was verified by simulations in ADAMS which resulted in the load spectrum applied to crankpin bearing. This load was then applied to the FE model in ABAQUS, and boundary conditions were applied according to the engine mounting conditions. Results from FE analysis were verified by strain gages attached to several locations on the forged steel crankshaft. Optimization of Crankshafts with Geometry, Material

Crankshaft is among large volume production components in the internal combustion engine industry. Optimizations like geometry, material and manufacturing of this component will result in high cost saving increase the fuel efficiency of the engine.

2.2 Geometry optimization

The modifications for improvement in fatigue life such as changing the main bearing radius, crank pin radius, fillet radius of crankpin pin and changing the type of material in crankshaft, which are very common modifications usually done in crankshaft geometry [14].

Farzin H.Montazersadgh and Ali Fatemi (2007) [15] conducted research on stress analysis and optimization of the crankshaft when subjected to dynamic loads. In this study, a kinematic simulation was performed on the crankshaft of a four-stroke single cylinder engine. A finite element analysis was performed to obtain the variation of stress amplitudes at critical locations. The optimization of the shape resulted in an 18% reduction in the weight of the forged steel crankshaft, which was achieved by modifying the dimensions.

Momin Muhammad Zia Muhammad Idris, etal [16] investigated Optimization of Crankshaft using Strength Analysis. This paper presents results of strength analysis done on crankshaft of a single cylinder two stroke petrol engine, to optimize its design, using PRO/E and ANSYS software. The paper also proposes a design modification in the crankshaft to reduce its mass. Therefore, web thickness was reduced from 13 mm to 10 mm. The reduction in mass obtained by this design modification is 16.98 %.

Amitpal Singh Punewale et al [17] studied torsional vibration with modified crankshaft geometry by adding some material at face inclination on crankshaft and found out improvement in results as compared to original geometry of crankshaft. Thus, Addition of material in Faceinclination area results in an increased torsional stiffness. Also, the fatigue life of crankshaft increases by this change.

Park et al. (2001) [18] showed that without any dimensional modification, fatigue life of crankshaft could be improved significantly by applying various surface treatments. Fillet rolling and nitriding were the surface treatment processes that were studied in their research. As mentioned above by many researcher's geometry optimizations like addition of material on face inclination, fillet rolling, reducing web thickness and so on are done for weight reduction and to reduce rate of failure. Generally, when geometry optimization is concluded Crankshaft is among large volume production components in the internal combustion engine industry and it is necessary to improve its reliability. The modifications for improvement in fatigue life such as changing the main bearing radius, crank pin radius, fillet radius of crankpin pin and changing the type of material in crankshaft, which are very common modifications usually done in crankshaft geometry.

2.3 Dynamic Load Determination and Analysis

An analytical investigation on bending vibrations was done for a V6 engine by Mourelatos (1995). He used a crankshaft system model (CRANKSYM) to analytically verify a vibration problem related to the flywheel for the mentioned crankshaft. As described in their study, CRANKSYM could perform an analysis considering the crankshaft structural dynamics, main bearing hydrodynamics and engine block flexibility.

Ampital Singhn [19] used the advantages of both the classical method and finite element technique in their studies to design crankshafts. They used the classical method in order to calculate the initial and the approximate results. In order to obtain more accurate results and to evaluate the results from the program, they created the finite element model of the crankshaft. Time varying radial and tangential forces acting on the crankpin were derived from the cylinder pressures. The displacements and stresses were calculated using the superposition method. They used an engine load duty cycle corresponding to automotive applications for generating load histories for accelerated fatigue testing.

BushmanJentel [20] studied the dynamic and inertial loading characteristics of the slider crank mechanism are studied and the necessary equations for the same are deduced. The torque and the loads acting on the crankpin are analytically determined. The numerical values required are determined using MATLAB.

C.M.Ramesha et al. (2015) studied the dynamic and inertial loading characteristics of the slider crank mechanism are studied and the necessary equations for the same are deduced. The torque and the loads acting on the crankpin are analytically determined. The numerical values required are determined using MATLAB.

2.4 Inference from the Literature

From the above literatures, to observe that different analysis were explained about the crank shaft, but the followings are not included:

- Detail analysis of static and dynamic load in simultaneously
- Crankshaft with different material not included in all literature
- Weight can be reduced by changing the material of the current materials not included.

CHAPTER THREE

METHODS AND MATERIALS

A one cylinder Honda engine model with an existing crankshaft was utilized in this thesis. Initially, the static and dynamic load and stress on the existing crankshaft are analyzed. The crankshaft of an existing model is evaluated first with forged steel (AISI 1045), medium carbon steel, and Ductile Cast Iron. The values of the stresses were determined by structural analysis. Figure 3.1 shows the flow charts for the thesis techniques.

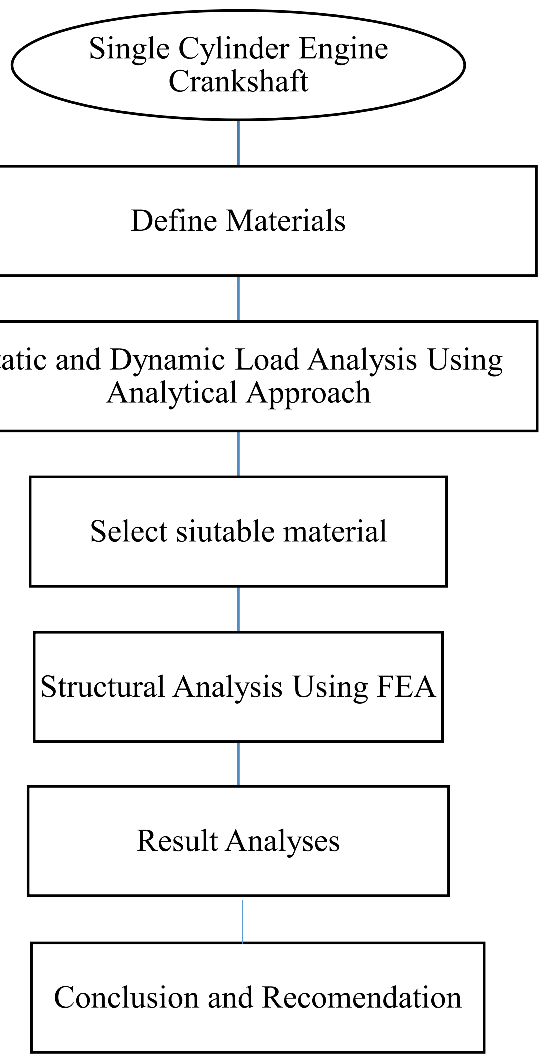


Figure 3.1 Methodology flow Chart

3.1 Crankshaft Materials

Materials for crankshafts should be easily formed, machined, and heat-treated, with sufficient strength, toughness, hardness, and fatigue strength. The crankshafts are made of steel and are either forged or cast. Cast crankshafts are weaker than forged crankshafts. Crankshafts that have been forged are constructed of structural steel. Forging produces a thick, strong shaft with a grain that runs along to the main stress direction. The performance of a forged crankshaft is more stable than that of a cast crankshaft in terms of internal organization. Internal grain of forged crankshaft is homogeneous; raw material flaw, tissue defect, and other defects are eliminated. The most significant difference between cast and forged crankshafts is that we can obtain superior internal structure and minimize crankshaft fracture with forged crankshafts (Ali Fatemi, 2007).

3.2 Physical Dimensions of Crankshaft Assembly

All necessary parameters of the crankshaft components are calculated as next section using the required formula. The calculations are based on engine specification as shown Table 3.1 and depend on the cylinder bore diameters.

Table 3-1 Specifications Engine:

Parameter	Specification
Capacity	390cc
Number of cylinders	1
Bore x stroke	86x68mm
Compression ratio	18.1
Maximum power	8.1 hp@3600rpm
Maximum torque	16.7Nm@2200rpm
Maximum gas pressure	25 bar

Piston

Within a cylinder, the piston is a disc that reciprocates. The piston's principal purpose in an internal combustion engine is to accept the energy from the expanding gas and deliver it to the crankshaft through the connecting rod. A substantial quantity of heat must be dispersed from the combustion chamber to the cylinder walls by the piston. The required physical dimensions of the piston can be found from Table 3.2.[11]

Table 3-2 Physical Parameters of Piston

S.No.	Parameters	Values (mm)
1	Cylinder Bore Diameter, D	60
2	Thickness of the Piston Head	25
3	The axial Thickness of Ring	1.5
4	Outer Diameter	58
5	Inner Diameter	48
6	Inner Pin Diameter	18
7	Total Piston Length	52

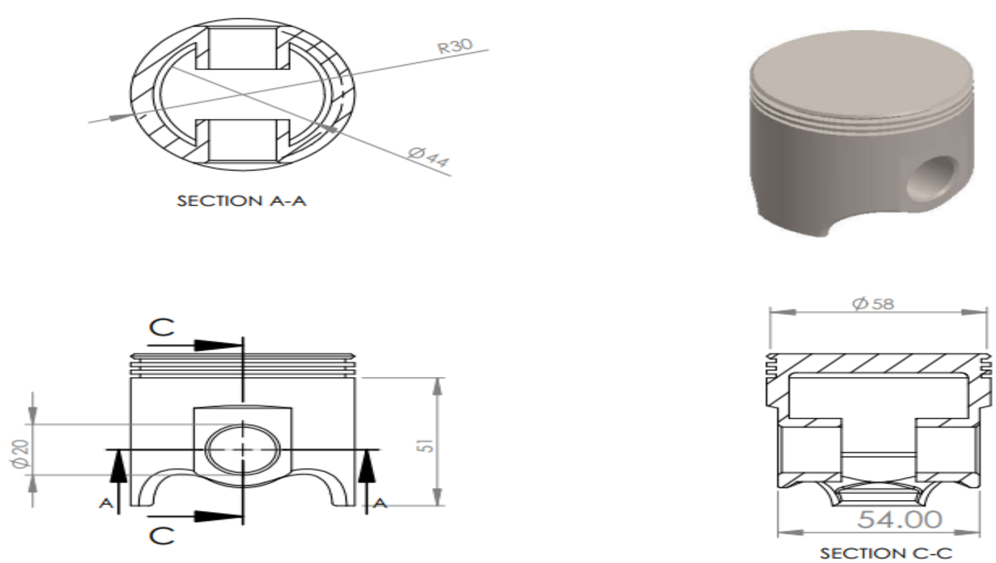


Figure 3.2 Dimension of Piston Head

Gudgeon pin

The piston pin (also known as a gudgeon pin or a wrist pin) connects the piston to the connecting rod. It's normally hollow on the inside and tapering, with the smallest internal diameter near the pin's center. The piston pin connects through bosses on the inside of the piston skirt and the bush on the connecting rod's small end. "(J.K. GUPTA, 2005)"all required diminutions are calculated in Table 3.3.

Table 3-3 Physical Parameter of piston pin

Dimension of the gudgeon pin	**Value in(mm)**
Outer diameter	18
Inner diameter	14
length	48

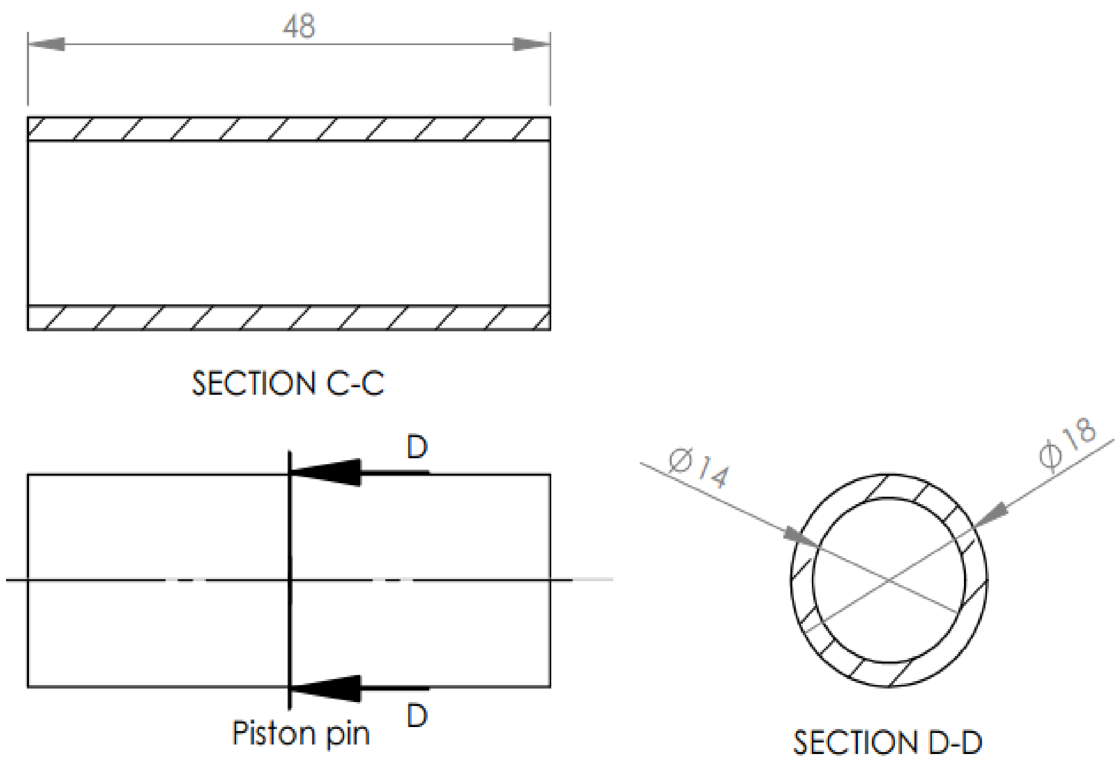


Figure 3.3 Dimension of Piston Pin

3.2.1 Connecting Rod

Between the piston and the crankshaft, the connecting rod serves as a link. Its major duty is to transmit push and pull from the piston pin to the crankpin, converting the piston's reciprocating action into the crank's rotating motion. Figure 3.4 shows the dimensions of the connecting rod.

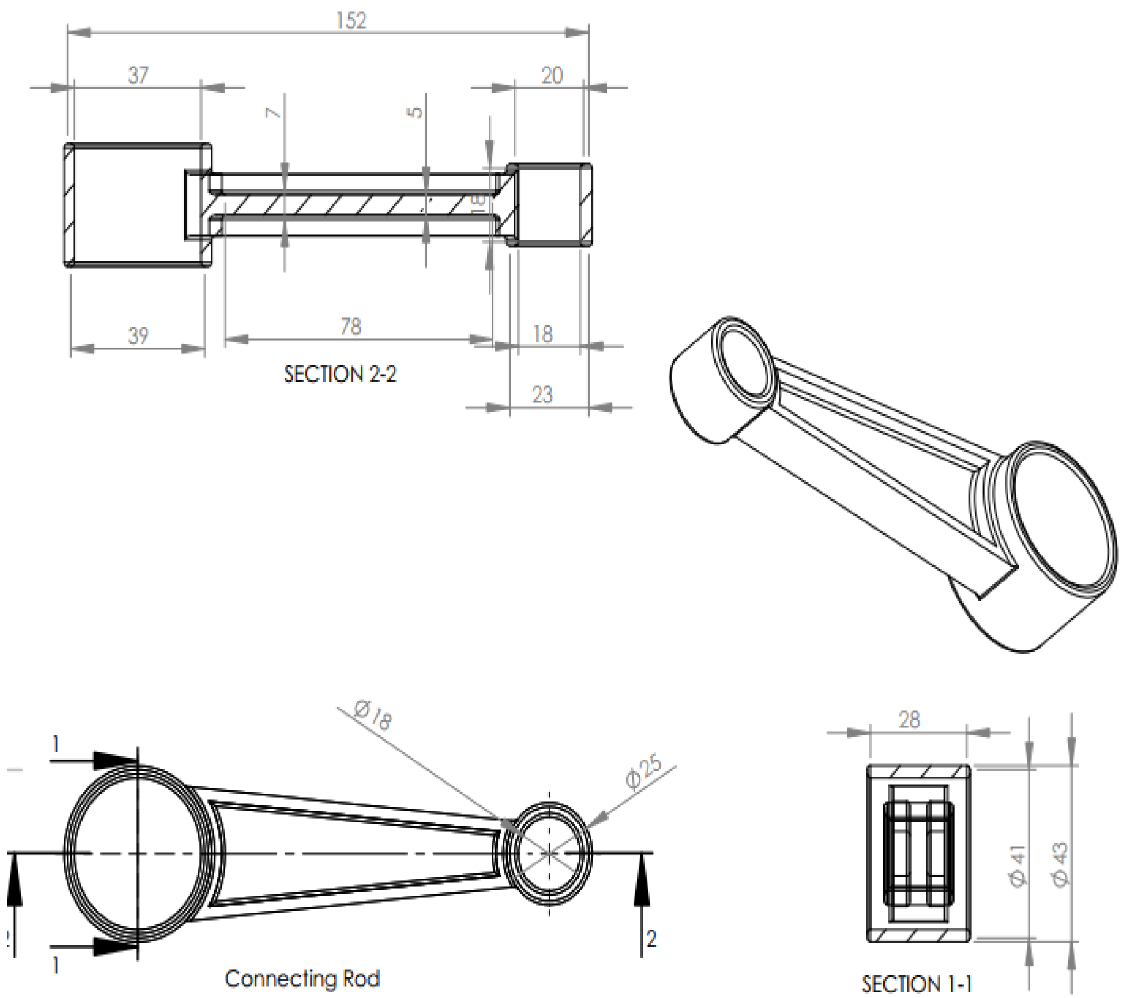


Figure 3.4 Dimension of connecting Rod

3.2.2 Crankshaft

The engine's crankshaft is subjected to gas forces, inertial forces, and moments, which result in a periodic functional angle of the knee. Torsion stress, bending, tension, and compression were all caused by these forces and moments. Deformation and bending vibrations are also caused by periodically changing moments.

The crankshaft is supposed to be a beam with two or more supports in the design of the crankshafts. The crankpin is developed by taking into account two crank positions: dead center (or when the crankpin is subjected to maximal bending moment) and an angle at which the twisting moment is greatest. In designing crankshaft we can use Dimension of crankshaft as shown Table 3.4

Table 3.4 Dimension of single cylinder engine crankshaft

N0	Parameter	Symbol	Value
1	Diameter of the Crank Pin	$\mathrm{d_c}$	38mm
2	Length of the Crank Pin	L_{c}	30mm
3	Crankpin oil hole diameter	C_{oh}	16mm
4	Diameter of the shaft/journal	d_s	34mm
5	Web Thickness (Both Left and Right Hand)	w_t	20mm
6	Web Width (Both Left and Right Hand)	w_w	64.5mm
7	Length of the crank shaft	L	355mm

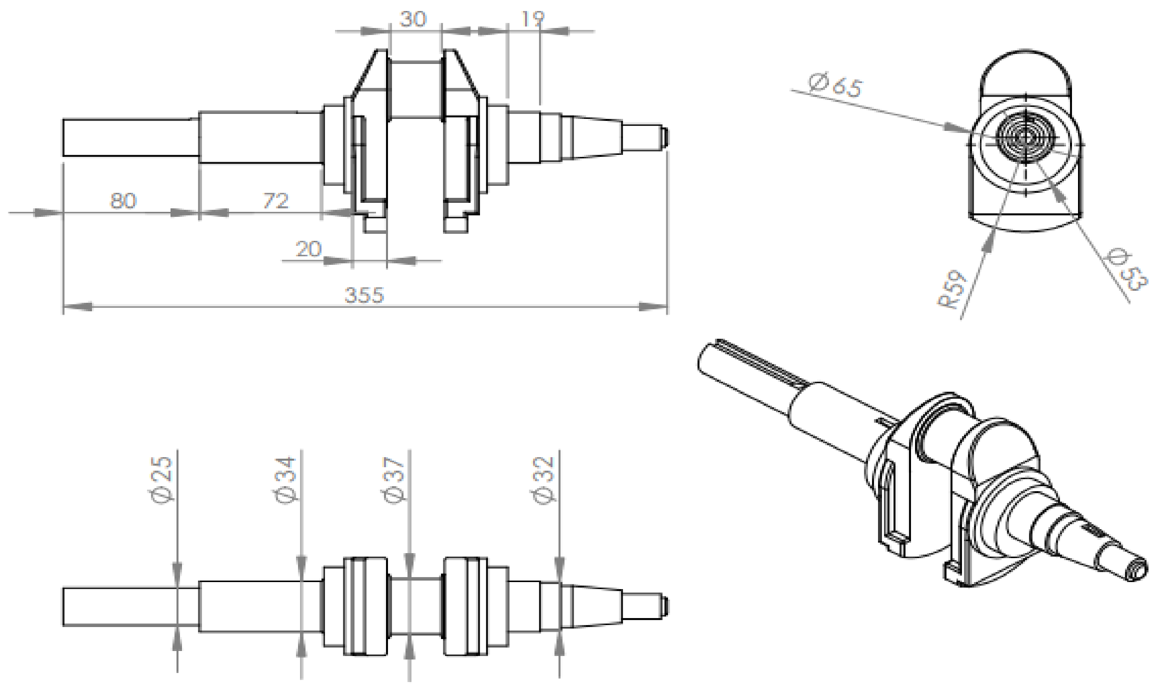
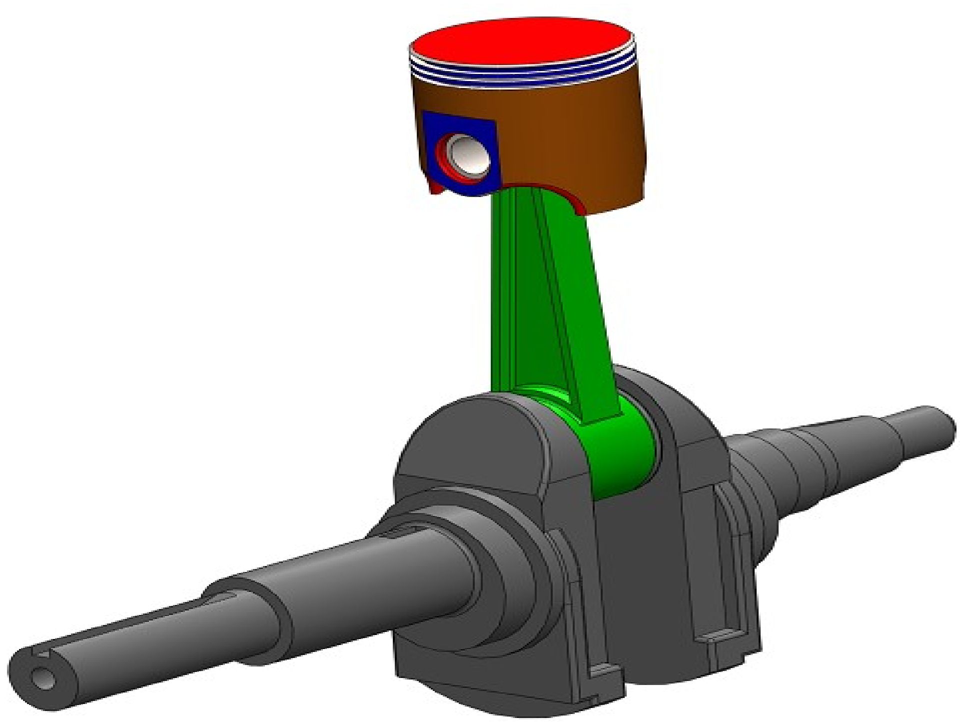


Figure 3.5 Dimension of crankshaft



3.3 Assumption

- Assume the distance (b) between bearings 1 and 2 is two times the piston diameter (D)
- The Piston force will act in the middle of the crankpin, and the responses will balance it out.
- The force acts on the crankpin's center or dead center and The crankpin is exposed to various forces and examined in two positions: maximum bending (when the crank is at dead center and twisting [21]

  The bending moment on the shaft is greatest when the crank is at dead center.Piston force is calculated using maximum cylinder pressure, and bore diameter of engine cylinder [22].
- Considering tangential and radial forces during crankshaft is an angle of twisting
- In terms of fillet radius calculation shoulder fillet and Gerber Criteria is used.

3.4 Conditions

- ☞ When the crank is at an angle of 25 to 30 degrees from the dead center for petrol engines and 30 to 40 degrees for diesel engines, the tangential force reaches its greatest magnitude. [23]. in this study since it is diesel engine 30^0 to 40^0 so $35^0$is taken for this analytical calculation.
- ☞ To find the thrust in the connecting rod (FQ), the angle of inclination of the connecting rod with the stroke line (i.e. angle ϕ),Take L/ r =4 [24]
- ☞ K_b = Combine shock, fatigue factor for bending = 1 and Kt = Combine shock, fatigue factor for torsion = 1 [25]

CHAPTER FOUR

STATIC AND DYNAMIC ANALYSES OF CRANKSHAFT

The crankshaft in the design of crankshafts must be a beam with two or more supports. Each crankshaft must be designed or tested for at least two crankshaft positions, one with the highest bending moment and the other with the highest torque. Additional points must also be considered due to flywheel weight, belt tension and other forces. To simplify calculations without sacrificing accuracy, it is assumed that bending forces do not affect the two bearings between which the force is applied.

The major goal of this chapter is to calculate the amount and direction of the static and dynamic loads acting on the bearing between the connecting rod and the crankshaft, which will be utilized in the FEA during the cycle. On the basis of a single degree of freedom slider crank mechanism, an analytical technique is applied. The dynamic load equations will be solved using Matlab. The analytical method solves for a general slider crank mechanism, yielding equations that may be applied to any crank radius, connecting rod geometry, connecting rod mass, connecting rod inertia, engine speed, engine acceleration, piston diameter, piston and pin mass, and cylinder pressure.

4.1 Design and Static Analyses of Crankshaft

Consider the two crank locations while designing the center crankshaft: dead center (the crankshaft is subjected to the maximum bending moment) and angle (the crankshaft is exposed to the highest twisting moment). The following are the details of these two cases: (J.K. GUPTA, 2005).

4.1.1 Crank is at Dead Centre

The maximum gas pressure on the piston transfers maximum force on the crankpin in the plane of the crank, creating only bending at this region of the crank. Only the bending moment is applied to the crankpin. The bending force on the shaft is highest and the twisting moment is zero when the crank is at dead center. When the crank is dead center, the crankshaft experiences the greatest bending moment. The connecting rod thrust will be equivalent to the piston gas load (Fp).

Force Due to Gas Pressure

The thrust of the connecting rod is equal to the piston gas load (Fp).at dead center as shown below Figure 4.1. So that gas load on the piston is,

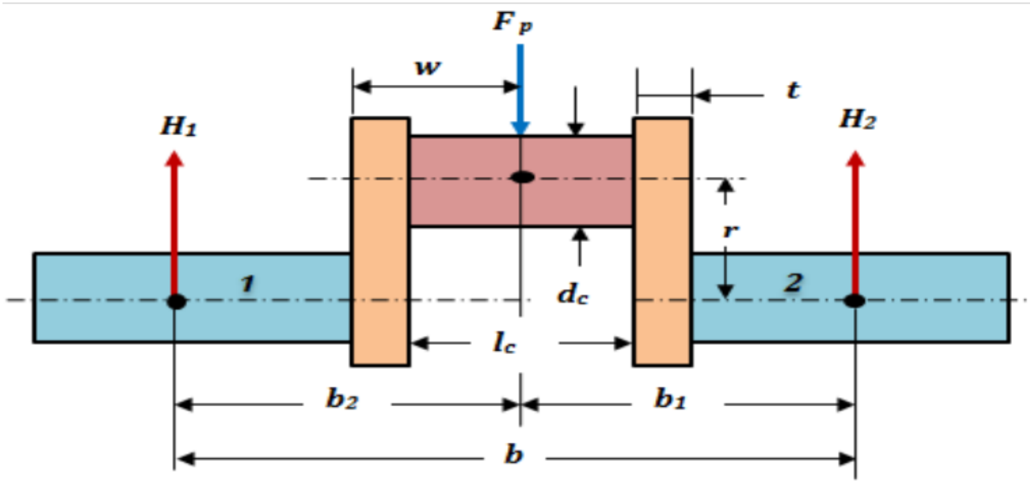


Figure 4.1 Force Analysis of Crank at Dead Center

$$F_p = \frac{\pi}{4} D^2 P_{max} \qquad 4.1$$

Consider a single throw three bearing crankshaft figure4.1

D = Piston diameter or cylinder bore in mm,

P_{max} = Maximum intensity of pressure on the piston in N/mm^2

D = Bore Diameter = 86mm

Maximum gas pressure = 25bar = 2.5N/mm^2

Gas load on the piston = area of bore x maximum gas pressure.

$$F_p = \frac{\pi}{4} 86^2 * 2.5N/mm^2 (mm^2)$$

$$F_p = 14{,}522N = 14.522kN$$

Due to this piston gas load (FP) acting horizontally, there will be two horizontal reactions H1 and H2 at bearings 1 and 2 respectively as shown Figure 4.1 such that, Taken as clockwise direction taken as a positive bending moment and counter clockwise as negative bending moment

$$\sum \mathrm{M}_1 = 0$$

$$F_p(b_2) - H_2(b_1 + b_2) = 0$$

$$H_2(b_1 + b_2) = F_p(b_2)$$

$$H_2 = \frac{F_p(b_2)}{(b_1 + b_2)} = \frac{F_p(b_2)}{b} \qquad 4.2$$

Where $b = 2D = 2 \times 86 = 172mm$

Summation of Y direction equal to zero

Taken as upward force taken as a positive and downward force as negative.

$$\sum F_y = 0$$

$$H_1 + H_2 - F_p = 0$$

$$H_1 = F_p - H_2$$

$$H_1 = \frac{F_p b_1}{(b_1 + b_2)} = \frac{F_p(b_1)}{b} \qquad 4.3$$

Where $b_1 = b_2$

$$H_1 = H_2 = \frac{F_p}{2}\ N$$

$$H_1 = H_2 = \frac{14{,}520}{2}\ N = 7{,}260N$$

We know that the bending moment at the Centre of the crankpin,

$$M_c = H_1 * b_2 \qquad 4.4$$

$$M_c = 7.26kN * 86mm.$$

$$M_c = 624{,}360N.mm.$$

We know that,

$$\sigma_b = \frac{32Mc}{(d_c)^3 x\pi} \qquad 4.5$$

$$\sigma_b = \frac{32 \times 624.36kN.mm.}{(d_c)^3 x\pi}$$

$$\sigma_b = \frac{32 \times 624{,}360 N.mm.}{(37\text{mm})^3 x\pi} = 125.5 MPa.$$

The maximum shear stress of crankpin at dead center is:

$$\tau = \frac{16xMc}{(dc)^3\pi} \qquad 4.6$$

$$\tau = \frac{16x\ 624{,}360 N.mm}{(37mm)^3\pi} = 62.75 MPa.$$

i. Design of Right and Left Hand Crank Web

The crank web is designed for eccentric loading. There are two stresses acting on the crank web, one is direct compressive stress and the other is bending stress due to piston gas load (F_P).

The maximum bending moment on the crank web figure 4.1

$$M_{bmax} = H1\left(b_2 - \frac{lc}{2} - \frac{t}{2}\right) \qquad 4.7$$

$$M_{bmax} = 7{,}260N\left(86 - \frac{32}{2} - \frac{20}{2}\right) = 4{,}356 N.m$$

And section modules,

$$Z = \frac{1}{6}(\text{W}.t^2) \qquad 4.8$$

Where W= Width of Crank Web

t =thickness of Crank Web

$$Z = \frac{1}{6}(65\text{mm} \times 20mm^2) = 2.16 \times 10^{-7} m^3$$

Now the bending stress,

$$\sigma_{wb} = M_{\frac{bmax}{Z}} \qquad 4.9$$

$$\sigma_{wb} = {}^{4{,}356 N.m}\!\big/_{2.16 \times 10^{-7} m^3}$$

The direct compressive stress on the crank web,

$$\sigma_{wc} = \frac{H1}{Wt}$$

Total stress on the crank web,

$$\sigma_w = \sigma_{wb} + \sigma_{wc}$$

4.1.2 Crank at an angle of maximum twisting moment

The twisting moment on the crankshaft is maximum when the tangential force on the crank (F_T) is maximum. The maximum value of the tangential force lies when the crank is at an angle of 25^0 to 30^0 from the dead center for petrol engines and $30^0$to 40^0 for diesel engines. [5]

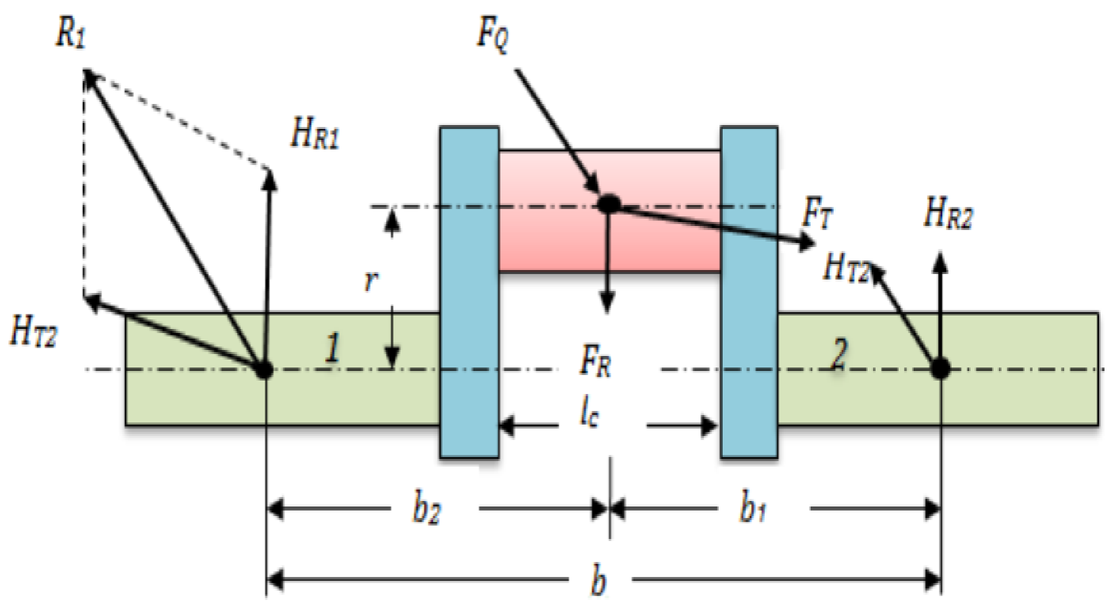


Figure 4.2 Force Analysis of Crank at angle of maximum twisting Moment

The tangential force two reactionsH$_{T1}$ and H$_{T2}$at bearing 1 and 2 respectively and while the Radial force reactions H$_{R1}$ and H$_{R1}$bearing 1 and 2 respectively. The different sections will be subjected to both bending and torsional moments and these must be checked for combined stress.

In order to find the thrust in the connecting rod (F_Q), angle of inclination of the connecting rod with the line of stroke (i.e. angle Ø), Take = $\frac{L}{R} = 4$ the maximum value of tangential force lies when the crank is at an angle of $\theta = 35^0$(from 30^0 to $40^0$for diesel engines.

$$\sin\phi = \frac{\sin\theta}{L/_R} \qquad 4.12$$

$$\sin\phi = \frac{\sin 35^0}{4} = 0.143$$

$$\phi = 8.244^0$$

We know that thrust in the connecting rod or the force exerted on the crankpin by the piston:

$$F_Q = \frac{F_P}{\cos\phi} \qquad 4.13$$

$$\mathbf{F_Q} = \frac{\mathbf{14{,}522N}}{\mathbf{COS\,8.244^0}} = 14{,}670\text{N}.$$

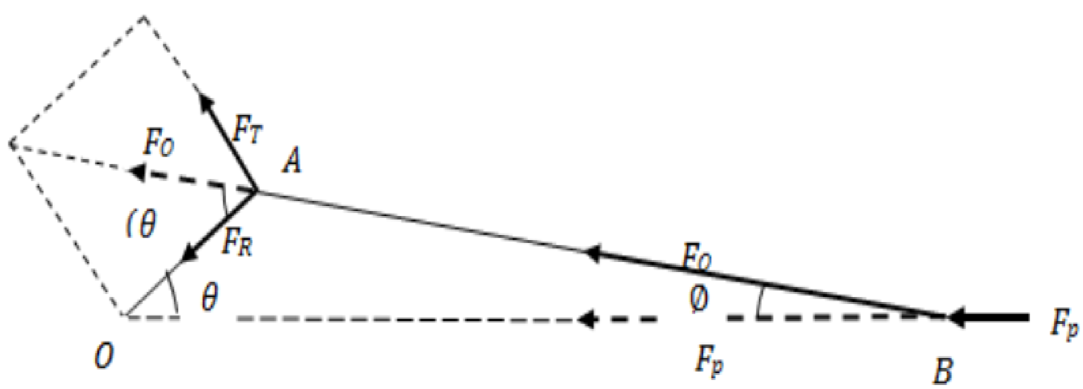


Figure 4.3 Forces Acting on the Crank.

The thrust in the connecting rod (FQ) may be divided into two components, one perpendicular to the crank and the other along the crank. The component of FQ perpendicular to the crank is the tangential force (FT) and the component of FT along the crank is the radial force (FT) as shown Figure 4.3, which produces thrust on the crankshaft bearings.

$$F_T = F_Q \sin(\theta + \phi) \quad 4.14$$

$$\theta = 35^0 \ and \ \phi = 8.244^0$$

$$F_T = 14.67kN \sin(43.244^0) = 10{,}050N.$$

The radial force along the crank from the Forces on crank Arm geometry.

$$F_R = F_Q \cos(\theta + \phi) \quad 4.15$$

$$F_R = 14.67kN \cos(43.244^0) = 10{,}680N$$

It may be noted that the tangential force were caused twisting of the crankpin and shaft while the radial force will cause bending of the shaft. Due to the tangential force (FT), there will be two reactions at bearings 1 and 2, such that,

$$H_{T1} = \frac{F_T\, b_1}{b}\ ,\ and \ \ H_{T2} = \frac{F_T\, b_2}{b} \quad 4.16$$

$$H_{T1} = H_{T2} = \frac{10{,}050N \times \ 86mm}{172mm} = 5{,}025N$$

Due to the radial force (F_R), there will be two reactions at the bearings 1 and 2, such that

$$H_{R1} = \frac{F_R\, b_1}{b} \ \ and \ \ H_{R2} = \frac{F_R\, b_2}{b} \quad 4.17$$

$$H_{T1} = H_{T2} = \frac{10{,}680N \times \ 86mm}{172mm} = 5{,}340N$$

ii. Design of Crankpin

The value of crankpin diameter at maximum twisting angle is should be equal or less than the value of crankpin diameter at dead center. The bending moment at the center of the crankpin.

The bending moment at the Centre of the crankpin is equal to

$$M_c = H_{R1}b_2 \tag{4.18}$$

The Twisting moment is

$$T_c = H_{T1}r \tag{4.19}$$

Therefore, Equivalent twisting moment on the crankpin,

$$T_e = \sqrt{{M_c}^2 + {T_c}^2} \tag{4.20}$$

$$T_e = \sqrt{(H_{R1}b_2\,)^2 + (H_{T1}r)^2}$$

We also know that twisting moment.

$$\frac{T_e}{J} = \frac{\tau}{r}$$

Where

$\tau = allowable\ shear\ stress\ in\ Mpa = 40\ Mpa = 40N/mm^2$

$J = polar\ moment\ of\ inertia\ in\ mm^4 = \frac{\pi}{32}{d_c}^4$

$T_e = Torsional\ moment\ in\ N.mm$

$r =$ Distance from the Neutral axis to the top most fibre, mm $= {}^{d_c}/_2$

$$T_e = \frac{\pi}{16}\tau {d_c}^3 \tag{4.21}$$

$${d_c}^3 = \frac{16T_e}{\pi\tau}$$

$$d_c = \sqrt[3]{{}^{16T_e}/_{\pi\tau}}$$

$$d_c = \sqrt[3]{{}^{16T_e}/_{\pi\tau}}$$

Let dc = Diameter of crankpin in mm.

We know that the bending moment at the Centre of the crankpin

$$M_c = H_{R1}b_2 \tag{4.22}$$

$$M_c = 5.34kN * 86 = 459.24kN.mm$$

$$T_c = 5.026kN * 34mm = 170.88kN.mm$$

$$T_e = \sqrt{{M_c}^2 + {T_c}^2} \qquad 4.22$$

$$T_e = \sqrt{(459.24)^2 + (170.88)^2}$$

$$T_e = 490kN.mm$$

According to distortion energy theory, the Von-Misses stress induced in the crank-pin **is** [4.22],

$$M_{ev} = \sqrt{(K_b M_c)^2 + \frac{3}{4}(K_t T_t)^2} \qquad 4.23$$

Where,

K_b = combined shock and fatigue factor for bending (Take K_b=1)

K_t= combined shock and fatigue factor for torsion (Take K_t =1)

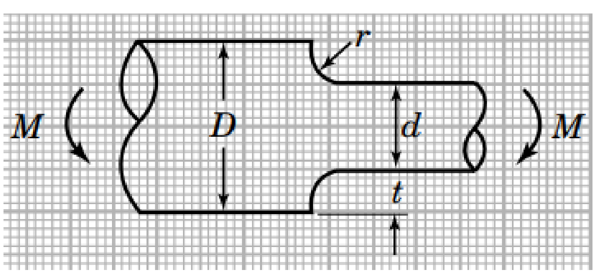
$$M_{ev} = \sqrt{(1(459.24))^2 + \frac{3}{4}(1(170.88))^2}$$

$$M_{ev} = 482.5kN.mm$$

4.2 Von misses Stress related to shoulder fillet

Shoulder fillets are the most common type of concentrated stress encountered in mechanical design practice. Shaft, shaft, spindle, rotor, etc. usually of different diameters joined by shoulder to round fillets.

Stress concentration factors K_t for **Bending** of a stepped bar of circular cross section with a shoulder fillet.



Where D = Journal Diameter

d = Crankpin Diameter

$D = 53\text{mm}$

$d = 37mm$

$r = 3mm$

$t = 6.5mm$

$^{t}/_{r} = 2.16$

$^{2t}/_{D} = 0.26$

Stress concentration factors $K_t = \frac{\boldsymbol{\sigma}_{\mathbf{max}}}{\boldsymbol{\sigma}_{\mathbf{nom}}}$

$$\sigma_{\text{max}} = K_t \sigma_{\text{nom}} \qquad 4.24$$

$$\sigma_{\text{nom}} = \frac{32M}{\pi d^3} \qquad 4.25$$

$$\sigma_{\text{nom}} = \frac{32\times 459.24kN\_mm}{\pi(37^3-18^3)} = \quad 104.36N/mm^2$$

To obtain the stress concentration factor (K_t)

$$\text{k}_\text{t} = C_1 + C_2\left(\frac{2t}{D}\right) + C_3\left(\frac{2t}{D}\right)^2 + C_4\left(\frac{2t}{D}\right)^3 \qquad 4.26$$

$2.0 \leq {}^{t}/_{r} =\leq 20.0 \qquad where \frac{t}{r} = 2.16$

$$C_1 = 1.232 + 0.832\sqrt{t/r} - 0.008\,t/r \qquad 4.27$$

$C_1 = 1.232 + 0.832\sqrt{2.16} - 0.008(2.16) \quad = 2.43$

$$C_2 = -3.813 + 0.968\sqrt{t/r} - 0.260\,t/r \qquad 4.28$$

$C_2 = -3.813 + 0.968\sqrt{2.16} - 0.260(2.16) = -2.95$

$$C_3 = 7.423 - 4.868\sqrt{t/r} + 0.869\,t/r \qquad 4.29$$

$C_3 = 7.423 - 4.868\sqrt{2.16} + 0.869(2.16) \quad = \ 2.14$

$$C_4 = -3.839 + 3.070\sqrt{t/r} - 0.600\,t/r \qquad 4.30$$

$C_4 = -3.839 + 3.070\sqrt{2.16} - 0.600(2.16) \ = -0.65$

$\text{K}_\text{t} = C_1 + C_2\left(\frac{2t}{D}\right) + C_3\left(\frac{2t}{D}\right)^2 + C_4\left(\frac{2t}{D}\right)^3$

$\text{K}_\text{t} = 2.43 \ - 2.95\,(0.26) + 2.14\,(0.26)^2 - 0.65(0.26)^3$

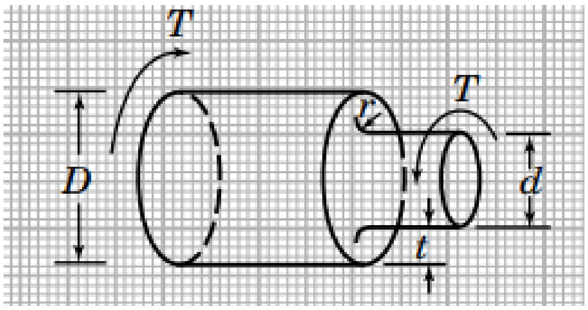
$\text{K}_\text{t} = 1.79$ (Stress concentration factor for bending)

$$\sigma_{\max} = K_t \sigma_{\text{nom}}$$

$$\sigma_{\max} = 1.79 \times 104.36 N/mm^2$$

$$\sigma_{\max} = 186.8 N/mm^2$$

Stress concentration factors K_t for **Torsion** of a stepped bar of circular cross section with a shoulder fillet



Stress concentration factors for Torsion $K_t = \frac{\tau_{\max}}{\tau_{\text{nom}}}$

$$\tau_{\max} = K_t \tau_{\text{nom}} \qquad 4.31$$

$$\tau_{\text{nom}} = \frac{16T}{\pi d^3} \qquad 4.32$$

$Where\ T\ is\ the\ twisting\ moment\ =\ 170.68 kN - mm$

$$\tau_{\text{nom}} = \frac{16 \times 170680 N\_mm}{\pi(37^3 - 18^3)}$$

$$\tau_{\text{nom}} = 19.4 N/mm^2$$

To obtain the stress concentration factor (K_t) for torsion

$$\mathrm{K_t} = C_1 + C_2\left(\frac{2t}{D}\right) + C_3\left(\frac{2t}{D}\right)^2 + C_4\left(\frac{2t}{D}\right)^3 \qquad 4.33$$

$$0.25 \le {}^{t}/_{r} = \le 4.0 \qquad where \frac{t}{r} = 2.16$$

$$C_1 = 0.905 + 0.783\sqrt{t/r} - 0.075\, t/r \qquad 4.34$$

$$C_1 = 0.905 + 0.783\sqrt{2.16} - 0.075(2.16) \quad = 1.89$$

$$C_2 = -0.437 - 1.969\sqrt{t/r} + 0.553\, t/r \qquad 4.35$$

$$C_2 = -0.437 - 1.969\sqrt{2.16} + 0.553(2.16) = -2.13$$

$$C_3 = 1.557 + 1.073\sqrt{t/r} - 0.578\, t/r \qquad 4.36$$

$$C_3 = 1.557 + 1.073\sqrt{2.16} - 0.578(2.16) \quad = 1.88$$

$$C_4 = -1.061 + 0.171\sqrt{t/r} + 0.086\, t/r \qquad 4.37$$

$$C_4 = -1.061 + 0.171\sqrt{2.16} + 0.086(2.16) = -0.62$$

$$\mathrm{K_t} = C_1 + C_2\left(\frac{2t}{D}\right) + C_3\left(\frac{2t}{D}\right)^2 + C_4\left(\frac{2t}{D}\right)^3$$

$$\mathrm{K_t} = 1.89 - 2.13\left(\frac{2 \times 6.5}{50}\right) + 1.88\left(\frac{2 \times 6.5}{50}\right)^2 - 0.62\left(\frac{2 \times 6.5}{50}\right)^3$$

$$\mathrm{K_t} = 1.44$$

$$\tau_{\max} = K_t \tau_{\mathrm{nom}}$$

$$\tau_{\max} = 1.45 \times 17.16 N/mm^2$$

$$\tau_{\max} = 27.92 N/mm^2$$

The maximum principal stresses are

$$\sigma_1 = \frac{\sigma_{\max}}{2} + \sqrt{\left(\frac{\sigma_{\max}}{2}\right)^2 + \tau_{\max}{}^2} \qquad 4.38$$

$$\sigma_1 = \frac{186.8}{2} + \sqrt{\left(\frac{186.8}{2}\right)^2 + (27.92)^2}$$

$$\sigma_1 = 190.8 MPa$$

$$\sigma_2 = \frac{\sigma_{\max}}{2} - \sqrt{\left(\frac{\sigma_{\max}}{2}\right)^2 + \tau_{\max}{}^2} \qquad 4.39$$

$$\sigma_2 = \frac{186.8}{2} - \sqrt{\left(\frac{186.8}{2}\right)^2 + (27.92)^2}$$

$$\sigma_2 = -4.1 MPa$$

Hence $\sigma_1 = \sigma_{\max}$ and $\sigma_2 = \sigma_{\min}$

To obtain the equivalent alternating stress

$$\sigma_{\mathrm{equ}} = \frac{\sigma_a}{1 - \left(\frac{\sigma_m}{\sigma_{ut}}\right)^n} \qquad 4.40$$

Where $\sigma_a = alternative\ stress$

$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2} \qquad 4.41$$

$$\sigma_a = \frac{186.8 - (-4.1)}{2} = 95.44 MPa.$$

$$\sigma_m = mean\ stress$$

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2} \qquad 4.42$$

$$\sigma_m = \frac{186.8-4.1}{2} = 91.38MPa$$

Alternating stress

$\sigma_{equ} = \frac{\sigma_a}{1-\left(\frac{\sigma_m}{\sigma_{ut}}\right)^n}$ For Gerber criteria n=2

Table 4.1 Equivalent alternating stress in terms of material properties

Material type	Ultimate tensile strength (MPa)	Equivalent alternating stress (MPa)
Forged steel (AISI 1045steel)	827	96.74
For medium carbon Steel	620	97.62
For Ductile cast Iron	770	96.89

4.3 Finite Element Analysis

The basis of FEA relies on the decomposition of the domain into a finite number of subdomains (elements) for which the systematic approximate solution is constructed by applying the vibrational or weighted residual methods. It is not always possible to obtain the exact analytical solution at any location in the body, especially for those elements having complex shapes or geometries. Always the most important are the boundary conditions and material properties. In such cases, the analytical solution that satisfies the governing equation or gives extreme values for the governing functional is difficult to obtain. Hence for most of the practical problems, the engineers' resort to numerical methods like the finite element method to obtain approximate but most probable solutions.

Five steps are used to solve any problem using finite elements. The six steps of finite element analysis are summarized as follows:

a. Discretizing the domain: This step involves subdividing the domain into elements and nodes. For continuous system this step is very important and the answers obtained are only approximate. In this case, the accuracy of the solution depends on the discretization used.

b. Writing the element stiffness matrices**:** The element stiffness equations need to be written for each element in the domain.

c. Assembling the global stiffness matrix: This is done using the direct stiffness approach to obtain the stiffness matrix for the entire system.

d. Applying the boundary conditions**:** This involves specifying the load conditions and restraints like supports and applied loads and displacements. In this study this step is performed manually.

e. Solving the equation: This is done by partitioning the global stiffness matrix and then solving the resulting equation using Gaussian elimination and the reactions and element stresses and visualization of the resulting solution.

Types of elements used

1. **Linear bar element:** the linear bar element is a one-dimensional finite element where the local and global coordinates coincide. It is characterized by linear shape functions. the linear bar element has modulus of elasticity E, cross-sectional area A, and length L, each linear bar element has two nodes as shown in Figure 1

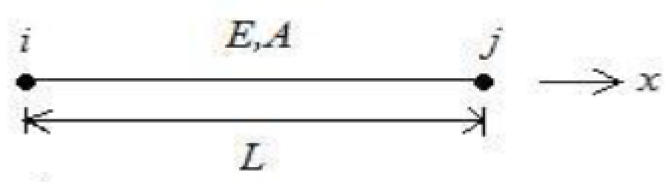


Figure 4.4 Linear Bar Element

In this case the element stiffness matrix is given by,

$$K = \begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix}$$

The linear bar element has only two degrees of freedom – one at each node. Consequently, for a structure with n nodes, the global stiffness matrix is of size n × n (since there is one degree of freedom at each node). Once the global stiffness matrix K is obtained, the structure equation is as follows:

$$\{K\}\{U\} = \{F\} \qquad 4.43$$

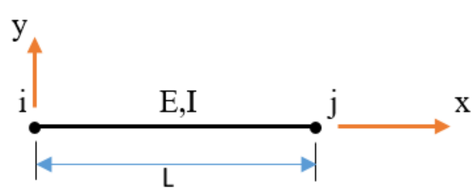
Where $\{U\}$ is the global nodal displacement is vector.

$\{F\}$ Is the global nodal force vector.

2. Beam element:

In this study beam element is used since the crankshaft is assumed to abeam element with two or more supports. The beam element is a two-dimensional finite element where the local and global coordinates coincide. It is characterized by linear shape functions.

In the case of transverse loading, the beam element has modulus of elasticity E, moment of inertia I, and the length L. Each beam element has two nodes and is assumed to be horizontal as shown in Figure below.



In this case the element stiffness matrix is given by the following matrix,

$$K = {EI}/{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} V_1 \\ \theta_1 \\ V_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix} \qquad 4.44$$

It is clear that the beam element has four degrees of freedom -two at each node (a transverse displacement and a rotation. The global stiffness matrix K is obtained, the structure equation is as follows:

$$\{F\} = \{K\}\{U\}$$

4.3.1 Finite element analysis of crankpin

The crank pin is like a built-in beam with a distributed load along its length that varies with crank positions. Each web is like a cantilever beam subjected to bending and twisting.

A. **Load analysis:** When the crank is at dead center, the maximum gas load on the piston is transmitted to the crankpin and is assumed to act as uniformly distributed load acting along the length of the crankpin.
   The gas load on the piston obtained during conventional design is$14.522\ KN$. When the crank is at the angle of maximum twisting moment, the equivalent twisting moment due to the bending and twisting loads is assumed to act at the center of the crankpin. The equivalent twisting moment acting at the center of crankpin obtained during conventional design is $480KN.mm$.
B. **Boundary conditions:** The crankpin is assumed to be acting as a fixed-fixed beam under all conditions. When at the dead center, it is assumed that a uniformly distributed load is acting on all the nodes except the end nodes which are fixed. When at the position of maximum twisting moment, it is assumed that the equivalent twisting moment is acting on the central node and the end nodes are fixed.
C. **Finite element analysis:** The conventional deign method gives the geometry of the crankpin, and the load conditions are obtained afterload analysis. These results are finally used to carry out the finite element analysis of the crankpin on the basis of the assumed boundary conditions.

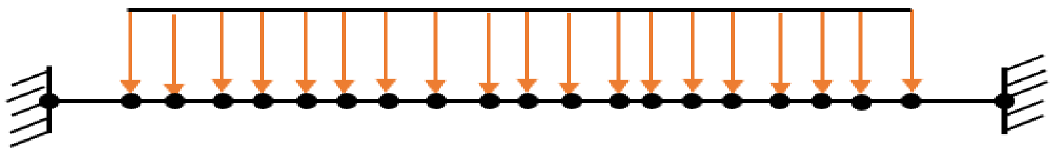


Figure 4.5 Distribute load at dead center

First, the analysis is carried out at the dead center. The crankpin is discretized using beam elements. Beam members can support distributed loading as well as concentrated nodal loading. Therefore, we must be able to account for distributed loading. Consider the fixed-fixed beam subjected to a uniformly distributed. These reactions are called fixed-end reactions. In general, Fixed-end reactions are those reactions at the ends of an element if the ends of the element are assumed to be fixed that is, if displacements and rotations are prevented. The force exerted on the crankpin by the piston:

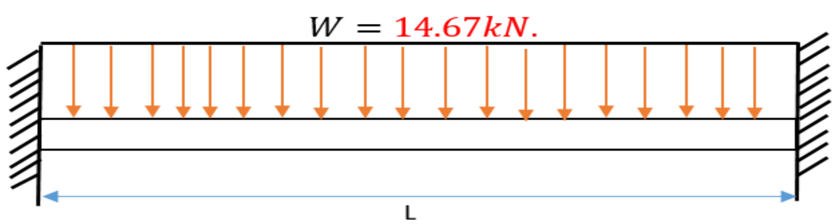
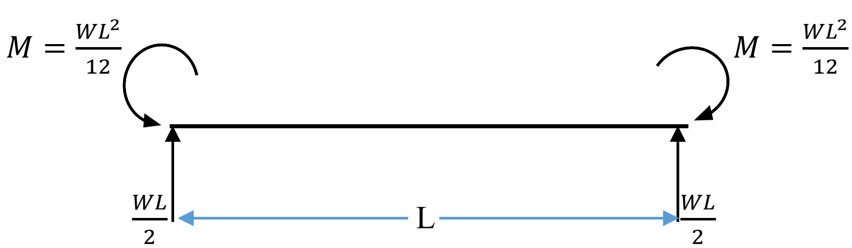


Figure 4-6 Fixed-fixed beam subjected to a uniformly distributed

$F_Q = W = 14.67kN.$



Where $I = \frac{\pi (d_c)^4}{64}$

$I = \frac{\pi ({d_o}^4 - {d_i}^4)}{64} = 86800.64 \text{ mm}^4$

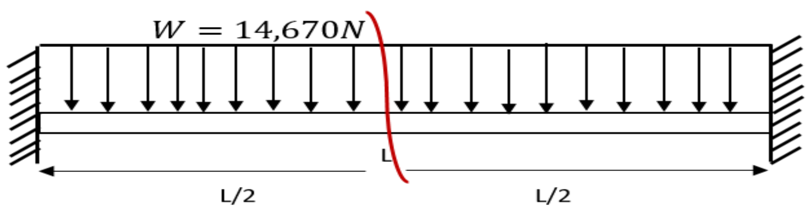
L=30mm

To find the unite load

Unit load W is defined as total load per total length of beam

$Unit\ load\ W\ =\ total\ load/total\ length$

$Unit\ load\ W = {}^{14670N}\!/_{32mm} = 458.43N/mm$



From the elementary of beam theory to compute each column of the stiffness matrix

$$N_1(x) = 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3} \qquad 4.45$$

$$N_2(x) = x - \frac{2x^2}{l} + \frac{x^3}{l^2} \qquad 4.46$$

$$N_3(x) = \frac{3x^2}{l^2} - \frac{2x^3}{l^3} \qquad 4.47$$

$$N_4(x) = -\frac{x^2}{l} + \frac{x^3}{l^2} \qquad 4.48$$

Define the linear shape function $N_1(s), N_2(s), N_3(s)\ and\ N_4(s)$

Where $s = \frac{x}{l} \quad 0 \le s \le 1$

$$N_1(x) = 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3} \qquad \text{Hence } \frac{3x^2}{l^2} = 3s^2\ and\ \frac{2x^3}{l^3} = 2s^3$$

$$N_1(s) = 1 - 3s^2 + 2s^3 \qquad 4.49$$

$$N_2(s) = l(s - 2s^2 + s^3) \qquad 4.50$$

$$N_3(s) = 3s^2 - 2s^3 \qquad 4.51$$

$$N_4(s) = l(-s^2 + s^3) \qquad 4.52$$

Uniformly distributed load

$$F_1 = WL\int_0^1 N_1(s)ds = WL\int_0^1 (1-3s^2+2s^3)ds = \frac{WL}{2} \qquad 4.53$$

$$M_1 = WL\int_0^1 N_2(s)ds = WL\int_0^1 l(s-2s^2+s^3)\,ds = \frac{WL^2}{12} \qquad 4.54$$

$$F_2 = WL\int_0^1 N_3(s)ds = WL\int_0^1 (3s^2-2s^3\,)ds = \frac{WL}{2} \qquad 4.55$$

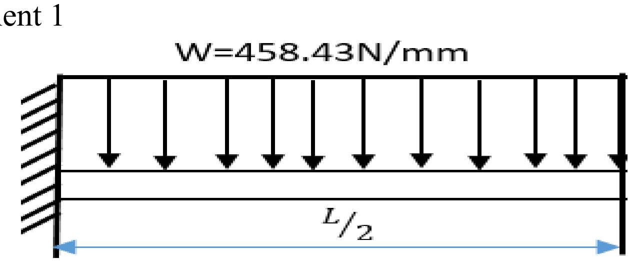
$$M_2 = WL\int_0^1 N_4(s)ds = WL\int_0^1 l(-s^2+s^3)\,ds = -\frac{WL^2}{12} \qquad 4.56$$

Therefore, the Finite Element equation of fixed-fixed beam

$$K = {}^{EI}/_{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} V_1 \\ \theta_1 \\ V_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix}$$

$$K = {}^{EI}/_{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} V_1 \\ \theta_1 \\ V_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} {}^{WL}/_{2} \\ {}^{WL^2}/_{12} \\ {}^{WL}/_{L} \\ -{}^{WL^2}/_{12} \end{Bmatrix}$$

For element 1



L=16mm

Unit load W = 458.43N/mm^2

Total load for element 1-unit load multiply by length

Total load W= 7335N

$$\begin{array}{cc} & \begin{array}{cccc} v_1 & \theta_1 & v_2 & \theta_2 \end{array} \\ K = {}^{EI}/_{L^3} & \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{array}{c} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{array} \end{array} \qquad 4.57$$

$$\begin{array}{cc} & \begin{array}{cccc} v_1 & \theta_1 & v_2 & \theta_2 \end{array} \\ K = {}^{EI}/_{L^3} & \begin{bmatrix} 12 & 96 & -12 & 96 \\ 96 & 1024 & -96 & 512 \\ -12 & -96 & 12 & -96 \\ 6L & 512 & -6L & 1024 \end{bmatrix} \begin{array}{c} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{array} \end{array}$$

Distributed loads must be converted to equivalent nodal loads

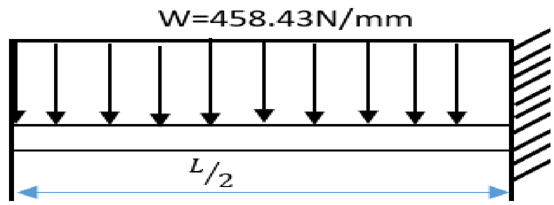
$$\begin{Bmatrix} F_{1e} \\ M_{1e} \\ F_{2e} \\ M_{2e} \end{Bmatrix} = WL \int_0^1 \begin{Bmatrix} N_1(s) \\ N_2(s) \\ N_3(s) \\ N_4(s) \end{Bmatrix} ds \qquad 4.58$$

$$\begin{Bmatrix} F_{1e} \\ M_{1e} \\ F_{2e} \\ M_{2e} \end{Bmatrix} = WL \int_0^1 \begin{Bmatrix} (1-3s^2+2s^3) \\ l(s-2s^2+s^3) \\ 3s^2-2s^3 \\ l(-s^2+s^3) \end{Bmatrix} ds = WL \begin{Bmatrix} 1/2 \\ L/12 \\ 1/2 \\ -L/12 \end{Bmatrix} = \begin{Bmatrix} 3667.5 \\ 9779.84 \\ 3667.5 \\ -9779.84 \end{Bmatrix}$$

Finite Element Matrix Equation for Element-1

$$\begin{array}{cc} & \begin{array}{cccc} v_1 & \theta_1 & v_2 & \theta_2 \end{array} \\ K = {}^{EI}/_{L^3} & \begin{bmatrix} 12 & 96 & -12 & 96 \\ 96 & 1024 & -96 & 512 \\ -12 & -96 & 12 & -96 \\ 6L & 512 & -6L & 1024 \end{bmatrix} \begin{array}{c} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{array} = \begin{Bmatrix} 3667.5 \\ 9779.84 \\ 3667.5 \\ -9779.84 \end{Bmatrix} \end{array}$$

For element -2



$$\begin{array}{c} \quad\quad\quad v_2 \quad\quad \theta_2 \quad\quad v_3 \quad\quad \theta_3 \end{array}$$

$$K = {}^{EI}/_{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{matrix} v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{matrix} \qquad 4.59$$

$$\begin{array}{c} \quad\quad\quad v_2 \quad\quad \theta_2 \quad\quad v_3 \quad\quad \theta_3 \end{array}$$

$$K = {}^{EI}/_{L^3}\begin{bmatrix} 12 & 96 & -12 & 96 \\ 96 & 1024 & -96 & 512 \\ -12 & -96 & 12 & -96 \\ 6L & 512 & -6L & 1024 \end{bmatrix}\begin{matrix} v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{matrix}$$

Work-equivalent nodal forces

$$\begin{Bmatrix} F_{1e} \\ M_{1e} \\ F_{2e} \\ M_{2e} \end{Bmatrix} = WL\int_0^1 \begin{Bmatrix} N_1(s) \\ N_2(s) \\ N_3(s) \\ N_4(s) \end{Bmatrix} ds \qquad 4.60$$

$$\begin{Bmatrix} F_{2e} \\ M_{2e} \\ F_{3e} \\ M_{3e} \end{Bmatrix} = WL\int_0^1 \begin{Bmatrix} (1-3s^2+2s^3) \\ l(s-2s^2+s^3) \\ 3s^2-2s^3 \\ l(-s^2+s^3) \end{Bmatrix} ds = WL\begin{Bmatrix} 1/2 \\ L/12 \\ 1/2 \\ -L/12 \end{Bmatrix} = \begin{Bmatrix} 3667.5 \\ 9779.84 \\ 3667.5 \\ -9779.84 \end{Bmatrix}$$

Finite Element Matrix Equation for Element-2

$$\begin{array}{c} \quad\quad\quad v_1 \quad\quad \theta_1 \quad\quad v_2 \quad\quad \theta_2 \end{array}$$

$$K = {}^{EI}/_{L^3}\begin{bmatrix} 12 & 96 & -12 & 96 \\ 96 & 1024 & -96 & 512 \\ -12 & -96 & 12 & -96 \\ 6L & 512 & -6L & 1024 \end{bmatrix}\begin{matrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{matrix} = \begin{Bmatrix} 3667.5 \\ 9779.84 \\ 3667.5 \\ -9779.84 \end{Bmatrix}$$

The governing equation for element -1

$$
\begin{array}{c}
\begin{array}{cccccc} v_1 & \theta_1 & v_2 & \theta_2 & v_2 & \theta_3 \end{array} \\
{EI}/{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} F1 \\ M1 \\ F2 \\ M2 \\ F3 \\ M3 \end{Bmatrix}
\end{array}
\qquad 4.61
$$

$$
\begin{array}{c}
\begin{array}{cccccc} v_1 & \theta_1 & v_2 & \theta_2 & v_2 & \theta_3 \end{array} \\
{EI}/{L^3}\begin{bmatrix} 12 & 96 & -12 & 96 & 0 & 0 \\ 96 & 1024 & -96 & 512 & 0 & 0 \\ -12 & -96 & 24 & 0 & -12 & 96 \\ 96 & 512 & 0 & 2048 & -96 & 512 \\ 0 & 0 & -12 & -96 & 12 & -96 \\ 0 & 0 & 96 & 512 & -96 & 1024 \end{bmatrix}\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} F1 \\ M1 \\ F2 \\ M2 \\ F3 \\ M3 \end{Bmatrix}
\end{array}
$$

By applying boundary equation

$$
{EI}/{L^3}\begin{bmatrix} 12 & 96 & -12 & 96 & 0 & 0 \\ 96 & 1024 & -96 & 512 & 0 & 0 \\ -12 & -96 & 24 & 0 & -12 & 96 \\ 96 & 512 & 0 & 2048 & -96 & 512 \\ 0 & 0 & -12 & -96 & 12 & -96 \\ 0 & 0 & 96 & 512 & -96 & 1024 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ v_2 \\ \theta_2 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} F1 \\ M1 \\ F2 \\ M2 \\ F3 \\ M3 \end{Bmatrix}
$$

$$K = {EI}/{L^3}\begin{bmatrix} 24 & 0 \\ 0 & 2048 \end{bmatrix}\begin{Bmatrix} v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 3667.5 \\ -9779.84 \end{Bmatrix}$$

$${EI}/{L^3} = 468 \times 10^4$$

$$K = 468 \times 10^4 \begin{bmatrix} 24 & 0 \\ 0 & 1024 \end{bmatrix}\begin{Bmatrix} v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 3667.5 \\ -9779.84 \end{Bmatrix}$$

$$468 \times 10^4 \times 24 v_2 = 36667.5$$

$$v_2 = 3.26 \times 10^{-4} mm$$

$$468 \times 10^4 \times 10 \times 2048 \theta_2 = -9779.84$$

$$\theta_2 = -1.02 \times 10^{-6} rad$$

Therefore, the effective global nodal forces $F^{(e)}$

{F}=K {d}

$$\begin{Bmatrix} F_{1y}{}^{(e)} \\ M_1{}^{(e)} \\ F_{2y}{}^{(e)} \\ M_2{}^{(e)} \\ F_{3y}{}^{(e)} \\ M_3{}^{(e)} \end{Bmatrix} = 468 \times 10^4 \begin{bmatrix} 12 & 96 & -12 & 96 & 0 & 0 \\ 96 & 1024 & -96 & 512 & 0 & 0 \\ -12 & -96 & 24 & 0 & -12 & 96 \\ 96 & 512 & 0 & 2048 & -96 & 512 \\ 0 & 0 & -12 & -96 & 12 & -96 \\ 0 & 0 & 96 & 512 & -96 & 1024 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 3.26 \times 10^{-4} \\ -1.02 \times 10^{-6} \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} F1 \\ M1 \\ F2 \\ M2 \\ F3 \\ M3 \end{Bmatrix}$$

$$F_{1y}{}^{(e)} = 468 \times 10^4[-12 \times 3.26 \times 10^{-4} + 96(-1.02 \times 10^{-6})]$$

$$\begin{Bmatrix} F_{1y}{}^{(e)} \\ M_1{}^{(e)} \\ F_{2y}{}^{(e)} \\ M_2{}^{(e)} \\ F_{3y}{}^{(e)} \\ M_3{}^{(e)} \end{Bmatrix} = \begin{Bmatrix} A \\ B \\ C \\ D \\ E \\ F \end{Bmatrix}$$

$$\{F\} = [K\{d\} - \{F_o\} \qquad 4.62$$

Where,

$\{F\}$ is concentrated nodal forces

$\{F_o\}$ are equivalent nodal forces

$$\{F_o\} = WL \begin{Bmatrix} 1/2 \\ L/12 \\ 1/2 \\ -L/12 \end{Bmatrix} = \begin{Bmatrix} 3667.5 \\ 9779.84 \\ 3667.5 \\ -9779.84 \end{Bmatrix}$$

$$\{F\} = [K\{d\} - \{F_o\}$$

$$\begin{Bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{Bmatrix} = \begin{Bmatrix} -1080.67 \\ -14809.7 \\ 3664.24 \\ -610.87 \\ -1083.39 \\ 14504.76 \end{Bmatrix} - \begin{Bmatrix} 3667.5 \\ 9779.84 \\ 3667.5 \\ -9779.84 \end{Bmatrix}$$

$$\begin{Bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{Bmatrix} = \begin{Bmatrix} -5528.76 \\ -15920.45 \\ -36.76 \\ 128.9 \\ 5470.45 \\ 13892.86 \end{Bmatrix}$$

Bending stress (σ_b)

$$\sigma_b = \frac{m_c}{I}$$

$$c = \frac{d_c}{2}$$

$$I = \frac{\pi(d_c)^2}{64}$$

$$\sigma_b = \frac{32M}{\pi({d_o}^3 - {d_i}^3)} \qquad 4.63$$

$$\sigma_b = 104.4 N/mm^2$$

4.3.2 Calculation for Factor of Safety, Weight, and Stiffness for each materials.

As yield stress is considered as a criterion of design, calculations are done based on **Superbug's** equation.

$$\frac{1}{F.S} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

Where

$F.s$ =Factor pf safety

σ_e =Endurance Stress

σ_y =Yield Stress

σ_m =Mean Stress

σ_v =Variable Stress

a) Safety of factor for Forged Steel (AISI 1045)

$\sigma_m = \frac{\sigma_{max}+\sigma_{min}}{2} = \frac{194+(-4)}{2} = 95MPa$

$\sigma_v = \frac{\sigma_{max}-\sigma_{min}}{2} = \frac{194-(-4)}{2} = 99MPa$

$\sigma_y = 625MPa$

$\sigma_e = 0.6\sigma_y = 0.6 \times 625 = 375MPa$

$\frac{1}{F.S} = \frac{95}{625} + \frac{99}{375} = 0.416$

Factor of Safety [F.S]= 2.4

b) Calculation for Weight and Stiffness for Forged Steel (AISI 1045)

Density of Forged Steel (AISI 1045) = $7833kg/m$^3 = 7.833 × (10^(−6) kg)/(mm^3)

Volume= $46447mm^3$

Weight of forged steel=volume ×density

Weight of forged steel= $46447mm^3 \times 7.833 \times 10^{-6}kg/mm^3$

Weight of forged steel= $3.65k.g$

Stiffness = weight/deformation

Stiffness= $^{3.53N}/_{0.009481mm} = 372.35N/mm$

c) Safety of factor for Forged medium carbon steel

$\sigma_m = \frac{\sigma_{max}+\sigma_{min}}{2} = \frac{190+(-4)}{2} = 93MPa$

$\sigma_v = \frac{\sigma_{max}-\sigma_{min}}{2} = \frac{190-(-4)}{2} = 97MPa$

$\sigma_y = 415MPa$

$\sigma_e = 0.6\sigma_y = 0.6 \times 415 = 249MPa$

$\frac{1}{F.S} = \frac{93}{415} + \frac{97}{249} = 0.613$

Factor of Safety [F.S]= 1.63

d) Weight and Stiffness for Medium carbon steel

Density $= 7845kg/m\text{^}3 = 7.845 \times (10\text{^}(-6)\ kg)/(mm\text{^}3\ )$

Volume= $46447mm^3$

Deformation= 0.005243

Weight of forged steel=volume ×density

Weight of forged steel= $46447mm^3 \quad \times 7.845 \times 10^{-6}kg/mm^3$

Weight of forged steel= $3.7kg$

Weight of forged steel= $0.37 \times 9.81 = 3.65N$

Stiffness = weight/deformation

Stiffness= ${}^{3.53N}/_{0.005243mm} = 992.3N/mm$

e) Safety of factor for Forged Ductile cast iron.

$\sigma_m = \frac{\sigma_{max}+\sigma_{min}}{2} = \frac{190+(-4)}{2} = 93MPa$

$\sigma_v = \frac{\sigma_{max}-\sigma_{min}}{2} = \frac{190-(-4)}{2} = 97MPa$

$\sigma_y = 515MPa$

$\sigma_e = 0.6\sigma_y = 0.6 \times 515 = 309MPa$

$\frac{1}{F.S} = \frac{93}{515} + \frac{97}{309} = 0.494$

$Factor\ of\ Safety[F.S] = 2.0$

f) Weight and Stiffness for Ductile cast iron steel

Density $= 7200kg/m\text{^}3 = 7.2 \times (10\text{^}(-6)\ kg)/(mm\text{^}3\ )$

$Volume$= $46447mm^3$

Deformation= 0.007642

Weight of forged steel=volume ×density

Weight of forged steel= $46447mm^3 \quad \times 7.2 \times 10^{-6}kg/mm^3$

Weight of forged steel= $3.8kg$

Stiffness = weight/deformation

Stiffness= ${3.53N}/{0.007642\ mm} = 429.25N/mm$

4.4 Dynamic Analysis of Crankshaft

The crankshaft experiences a complex dynamic loading due to the motion of the connecting rod, which transforms two sources of loading to the crankshaft. The significance of torsion during a cycle and its maximum compared to the total magnitude of loading should be investigated. In addition, there is a need for obtaining the stress variation during a loading cycle and this requires FEA over the entire engine cycle.

In general, this section explains the analytical approach steps and the equations that could be used in MATLAB to obtain angular velocity and acceleration of connecting rod, linear velocity and acceleration of piston assembly, and the most important forces between different joints in the mechanism. It is shown that the results from the analytical approach are verified by a simple model in Solidwork18 and the output of analytical approach is discuses using FEA.

4.4.1 Analytical Vector Approach

The analytical approach discusses detail in this section. The slider-crank mechanism with a single degree of freedom consider for solving the equations of motion as shown in Figure 4.4. The angle θ shown in figure represents the crankshaft angle, which is used as the generalized degree of freedom in the mechanism; therefore every other dynamic property in this mechanism would be a function of this angle.

The angle θ shown in Figure 4.4 represents the crankshaft angle, which is used as the generalized degree of freedom in the mechanism; therefore every other dynamic property in this mechanism would be a function of this angle. Calculation of other dynamic properties of the mechanism such as angular velocity, angular acceleration, and forces at pin joints.

i. ***Equivalent Masses of the Crank Mechanism***

In analyzing the inertia forces due to the connecting rod of an engine, it is often convenient to concentrate a portion of the mass at the crankpin and the remaining portion at the piston pin as in below Figure 4.4.

Mass of connecting rod at crankpin B,

$$m_{PB} = \frac{M_P L}{L_r} \qquad 4.64$$

ii. ***Displacement, Velocity and Accelerations of the Crank Mechanism***

The following analysis is required in order to obtain the load history applied to the crankshaft bearings. These calculations are based on the single degree of freedom mechanism shown in Figure 4.7.

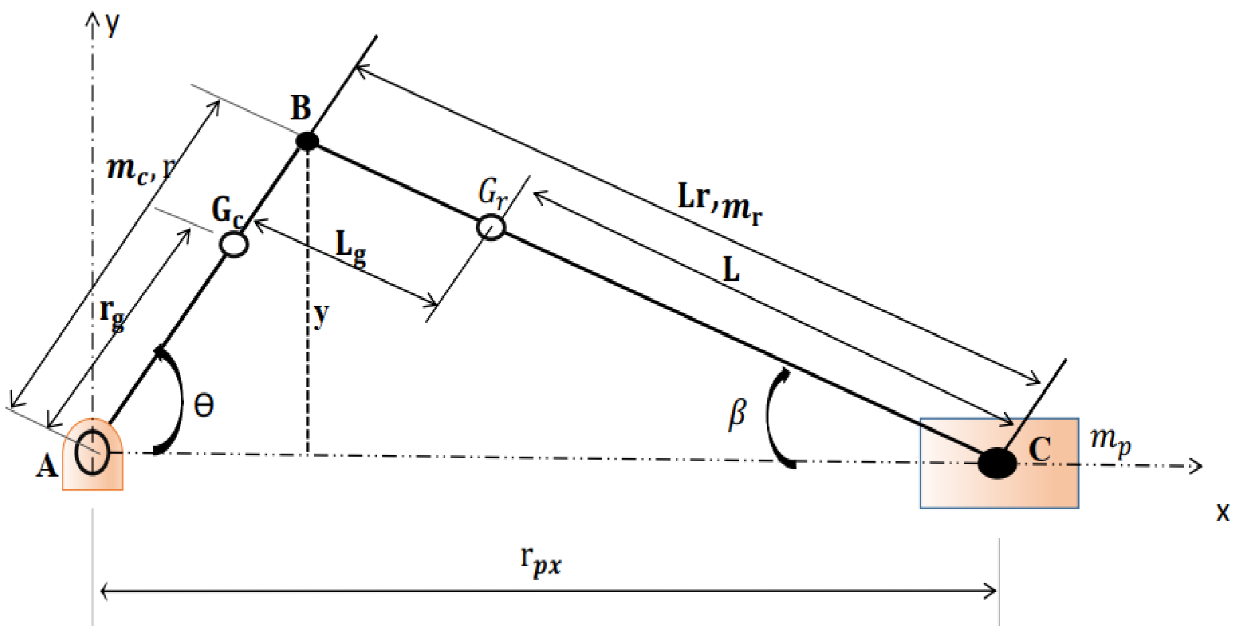


Figure 4.7 Parametric Representation of the Crankshaft, Connecting Rod, and Piston Assembly

The angular velocity and acceleration of the crankshaft are given by:

$$\omega_1 = \frac{d\theta}{dt} \qquad 4.65$$

$$\alpha_1 = \frac{d\omega_1}{dt} \qquad 4.66$$

Angular velocity and acceleration of the connecting rod are obtained by differentiating the angle β and ω2 with respect to time, respectively:

$$\omega_2 = \frac{d\beta}{\mathrm{dt}} \qquad 4.67$$

$$\alpha_2 = \frac{d\omega_2}{\mathrm{dt}} \qquad 4.68$$

From figure 4.4 geometry triangulation, the angle of connecting rod β related to angle θ.

$$y = l_r sin\beta = rsin\theta$$

$$sin\beta = \frac{rsin\theta}{l_r} \quad let\ n = \frac{l_r}{r} \ , \quad sin\beta = \frac{sin\theta}{n} \qquad 4.69$$

$$\sin^2\beta + \cos^2\beta = 1$$

$$\cos\beta = \sqrt{1 - \sin^2\beta}$$

$$\cos\beta = \sqrt{1 - \left(\frac{y}{l_r}\right)^2} \ , \ but\ y = rsin\theta$$

$$\cos\beta = \sqrt{1 - \frac{r^2\sin^2\theta}{(l_r)^2}} = \frac{1}{n}\sqrt{n^2 - \sin^2\theta} \qquad 4.70$$

The angular velocity of connecting rod ω_2,

Differentiating both side Equation (3.33) with respect to time.

$$\frac{d}{dt}(sin\beta) = \frac{d}{dt}\left(\frac{sin\theta}{n}\right)$$

$$\left(\frac{d}{d\beta}sin\beta\right)\frac{d\beta}{dt} = \frac{1}{n}\frac{d}{d\theta}(sin\theta)\frac{d\theta}{dt}$$

$$\omega_2\cos\beta = \frac{1}{\mathrm{n}}\omega_1\cos\theta$$

$$\omega_2 = \frac{\omega_1\cos\theta}{\mathrm{n}\cos\beta} = \frac{\omega_1\cos\theta}{\mathrm{n}\cos\beta} = \frac{\omega_1\cos\theta}{\sqrt{\mathrm{n}^2 - \sin(\theta)^2}} \qquad 4.71$$

The angular acceleration of the connecting rod, α2, is calculated by differentiating Equation (4.35) with respect to time.

$$\alpha_2 = \frac{\mathrm{d}\omega_2}{\mathrm{dt}} = \frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\omega_1\cos\theta}{\sqrt{\mathrm{n}^2 - \sin(\theta)^2}}\right)$$

$$\alpha_2 = \frac{d}{d\theta}\left(\frac{\omega_1 \cos\theta}{\sqrt{n^2 - \sin^2\theta}}\right)\frac{d\theta}{dt}, \text{since } \omega_1 = \frac{d\theta}{dt}$$

$$\alpha_2 = \omega_1{}^2 \frac{d}{d\theta}\left(\frac{\cos\theta}{\sqrt{n^2 - \sin(\theta)^2}}\right) \quad \text{where } n = \frac{l_r}{r}$$

$$\alpha_2 = -\omega_1{}^2 sin\theta \left[\frac{n^2 - 1}{(n^2 - sin(\theta)^2)^{\frac{3}{2}}}\right] \qquad 4.72$$

Location of center of mass of the connecting rod, r_g from the origin, A, is given in X and Y directions by the following equations:

$$r_{gx} = r\cos\theta + l_g \cos\beta$$

$$r_{gx} = r\cos\theta + l_g \frac{1}{n}\sqrt{n^2 - \sin(\theta)^2} \qquad 4.73$$

$$r_{gy} = r\sin\theta + l_g \sin\beta$$

$$r_{gy} = r\sin\theta + \frac{l_g \sin\theta}{n} \qquad 4.74$$

Linear velocity of the center of mass of the connecting rod, Vg, where ($r_g = r$) in X and Y, directions is obtained by differentiating Equations (4.37) and Equations (4.38) with respect to time:

$$v_{gx} = -r\omega_1 sin\theta - \frac{l_g \omega_1 sin2\theta}{2n^3\sqrt{n^2 - sin(\theta)^2}} \qquad 4.75$$

$$v_{gy} = r\omega_1 cos\theta - \frac{l_g\ \omega_1 cos\theta}{n} \qquad 4.76$$

Differentiating Equation (4.39) with respect to time will result in the linear acceleration of the center of mass of the connecting rod in the X direction, a_{rx}:

$$a_{rx} = -r\alpha_1 sin\theta - r{\omega_1}^2 cos\theta - \frac{l_g \alpha_1 sin2\theta}{2n^3\sqrt{n^2 - sin(\theta)^2}}$$

$$- \frac{1}{2{l_r}^2 - 2r^2 sin(\theta)^2}\left({\omega_1}^2{l_g}^2 r^2\left(2\,cos(2\theta)\frac{1}{n}\sqrt{n^2 - sin(\theta)^2}\right.\right.$$

$$\left.\left.+ \frac{sin(2\theta)^2}{2n^3\sqrt{n^2 - sin(\theta)^2}}\right)\right) \qquad 4.77$$

For the linear acceleration of the center of mass of the connecting rod in the Y direction:

$$a_{ry} = r\alpha_1 cos\theta - r{\omega_1}^2 sin\theta - \frac{\alpha_1 l_g cos\theta}{n} + \frac{\omega_1 l_g sin\theta}{n} \qquad 4.78$$

Considering the location of the piston as rp with respect to the origin, the following equation could be written:

$$r_{px} = rcos\theta + l_r cos\beta \qquad 4.79$$

Now, $r_{py} = 0$, since the piston does not have displacement in the Y direction.

$$r_{px} = rcos\theta + l_r \frac{1}{n}\sqrt{n^2 - \sin^2\theta} \qquad 4.80$$

Linear velocity of the piston is obtained by differentiating Equation (4.44) with respect to time:

$$v_{px} = -r\omega_1 sin\theta - \frac{r\omega_1 sin2\theta}{2n^3\sqrt{n^2 - sin(\theta)^2}} \qquad 4.81$$

Differentiating Equation (4.45) with respect to time will give the linear acceleration of the piston, a_{px}

$$a_{px} = -r\alpha_1 sin\theta - r{\omega_1}^2 cos\theta - \frac{r^2\alpha_1 sin2\theta}{2n^3\sqrt{n^2 - sin(\theta)^2}}$$

$$-\frac{1}{2{l_r}^2 - \frac{2r^2 sin(\theta)^2}{l_r}}\left({\omega_1}^2 r^2\left(2\cos(2\theta)\frac{1}{n}\sqrt{n^2 - sin(\theta)^2}\right.\right.$$

$$\left.\left.+\frac{r^2 sin(2\theta)^2}{2n^3\sqrt{n^2 - sin(\theta)^2}}\right)\right) \quad 4.82$$

Summing all forces acting on the piston in the X direction will result in:

$$F_{Px} = m_p a_{px} + \pi {R_p}^2 P_c \quad 4.83$$

Where m_p =is the mass of the piston assembly consisting of the piston, piston pin, and piston rings?

P_c =The pressure in the cylinder applied on the top of the piston.

R_p =The radius of the piston.

Substituting for a_{px} from Equation (4.46) into Equation (4.47), the following equation will be obtained for the force on the piston pin in the X direction F_{Px} :

$$F_{Px} = -m_p r\alpha_1 sin\theta - m_p r{\omega_1}^2 cos\theta - \frac{m_p r^2\alpha_1 sin2\theta}{2n^3\sqrt{n^2 - sin(\theta)^2}} + \pi {R_p}^2 P_c$$

$$-\frac{m_p}{2{l_r}^2 - \frac{2r^2 sin(\theta)^2}{l_r}}\left({\omega_1}^2 r^2\left(2\cos(2\theta)\frac{1}{n}\sqrt{n^2 - sin(\theta)^2}\right.\right.$$

$$\left.\left.+\frac{r^2 sin(2\theta)^2}{2n^3\sqrt{n^2 - sin(\theta)^2}}\right)\right) \quad 4.84$$

Solving the equations of forces in X and Y direction and moment about the center of mass of the connecting rod will result in forces on the joint between the connecting rod and the crankshaft, C. These forces are given by:

$$F_{ax} = m_r a_{rx} + F_{px} \quad 4.85$$

$$F_{ay} = \frac{1}{l_r}\left(\left(\frac{I_{zz}\alpha_2 - F_{ax}L_g - F_{px}Lsin\beta}{cos\beta}\right) + m_r a_{ry} L\right) \quad 4.86$$

Where I_{zz} and m_r are the moment of inertia and the mass of the connecting rod, respectively.

F_{ax} And F_{ay} are expressed in the global coordinate system, which is not rotating with the crankshaft. Forces expressed in a coordinate system attached to the crankshaft, better explain the loading history applied to the crankshaft.

These forces are given by:

$$F_x = F_{ax}\cos\theta + F_{ay}\sin\theta \quad 4.87$$

$$F_y = F_{ay}\cos\theta - F_{ax}\sin\theta \quad 4.88$$

The analytical approach was solved for a general slider crank mechanism which results in equations that could be used for any crank radius, connecting rod geometry, and connecting rod mass, connecting rod inertia, engine speed, engine acceleration, piston diameter, piston and pin mass, pressure inside cylinder diagram, and any other variables of the engine. The results of the MATLAB code include linear velocity and acceleration of piston assembly, various forces between different joints in the mechanism and the crankshaft torque. In this analysis it was assumed that the crankshaft rotates at a constant angular velocity, which means the angular acceleration was not included in the analysis. These equations have been used in a MATLAB program, in order to solve the mechanism for different angles of the crankshaft (Appendix-A).

4.4.2 Verification of Analytical Approach with MATLAB

The analytical approach used in this study verified by 3D dynamic simulation of the crankshaft, connecting rod, and piston assembly. The analysis is based on simulation of the simple slider-crank mechanism. For the purpose of this simulation crankshaft and connecting rod were digitized and the generated geometries were used to obtain the accurate location of the center of gravity of the connecting rod and the magnitude of its inertia.

Since the only concerning factor in the piston assembly that would affect the dynamic of the mechanism is the mass, there was no need to generate the piston assembly geometry. Material density of 7800 kg/m3was used for the connecting rod, which is the density of the Forged steel used in the component. The crankshaft AB rotational speed was taken to be the maximum operating speed of the engine, which is 3600 rev/min, which specify from engine specifications. The engine speed effect within the operating engine speed 2000 rpm, 2800 rpm, and 3600 rpm. The 3D model of slider crank mechanism used in SOLIWORK.

The details and any other information required are tabulated in below Table 4.2, input in the MATLAB program. The simulation is performs in a 3D space, the mechanism is a simple 2D linkage; therefore forces are expected to be in the plane of crankshaft motion. Therefore, forces in the longitudinal direction of the crankshaft would be zero in this case. Since the joints at different locations of this mechanism are pin joints, there would be no moment resistance.

Table4.1: Input Values in Mat Lab.

Parameter	Crank AB	Connecting Rod BC	Slider C
Calculated mass(kg)	3.72	283.35E-3	417.63E-3
Length (mm)	40	121	-
$\mathrm{I_{xx}}(\mathrm{kg}-\mathrm{mm}^2)$	-	608.58	-
$\mathrm{I_{yy}}(\mathrm{kg}-\mathrm{mm}^2)$	-	80.32	-
$\mathrm{I_{zz}}(\mathrm{kg}-\mathrm{mm}^2)$	-	662.52	-
$\mathrm{I_{xy}}(\mathrm{kg}-\mathrm{mm}^2)$	-	8.04	-

Table4.2: Configuration of the engine to which the crankshaft belongs [11]

Crankshaft radius	37 mm
Piston diameter	89 mm
Mass of the connecting rod	0.283 kg
Mass of the piston assembly	0.417 kg
Connecting rod length	152 mm
I_{zz} of connecting rod about the center of gravity	$662.52\ \mathrm{kg}-\mathrm{mm}^2$
Distance of C.G. of connecting rod from crank end center	28.6 mm
Maximum gas pressure	25 bar

The engine configuration from which the crankshaft was taken is shown in Table 4.2. The pressure versus crank angle of this specific engine was not available, so the pressure versus volume (thermodynamic engine cycle) diagram of a similar engine was considered. This diagram was scaled between the minimum and maximum of pressure and volume of the engine. The four link mechanism was then solved by MATLAB programming to obtain the volume of the cylinder as a function of the crank angle.

Figure 4.8 shows pressure versus crankshaft angle, which was used as the applied force on the piston during the dynamic analysis. It should be noted that the pressure versus volume of the cylinder graph changes as a function of engine speed

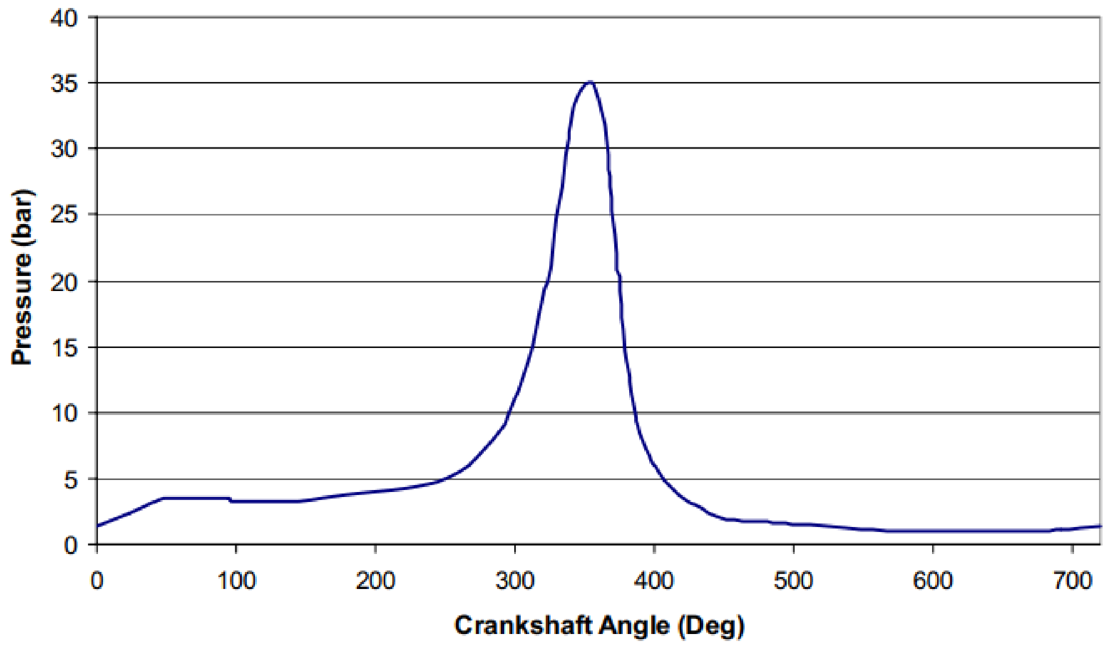


Figure 4.8 Pressure versus crankshaft angle used to calculate the forces at the surface of the piston.

The results of the MATLAB programming are linear velocity and acceleration of the piston assembly, angular velocity and angular acceleration of the connecting rod, linear acceleration of center of gravity of the connecting rod, and forces that are being applied to the bearing between the crankshaft and the connecting rod. The program was run for different engine speeds in the operating engine speed range.

The results of the analytical approach for the slider crank mechanism using the MATLAB program at the engine speed of 3600 rpm are plotted in Figures 4.9 through 4. 14. Figures 4.9 show the variation of linear velocity of the piston assembly and Figures 4.10 show the variation of linear acceleration of the piston assembly over $2\times 360\ degree$.

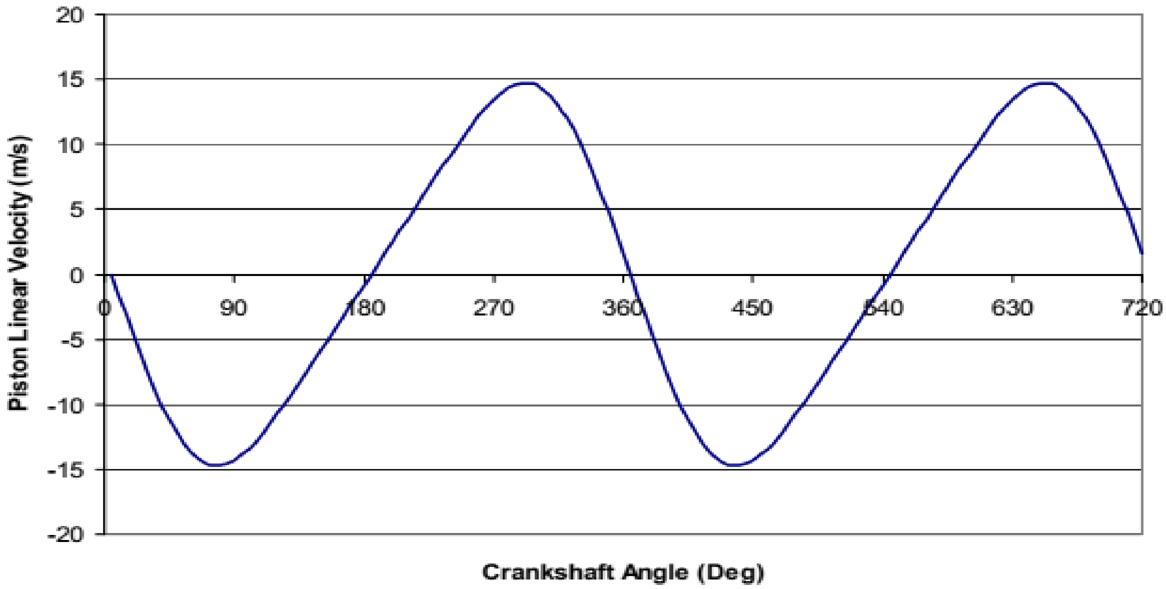


Figure 4.9 Variation of linear velocity of the piston assembly over one complete engine cycle at crankshaft speed of 3600 rpm.

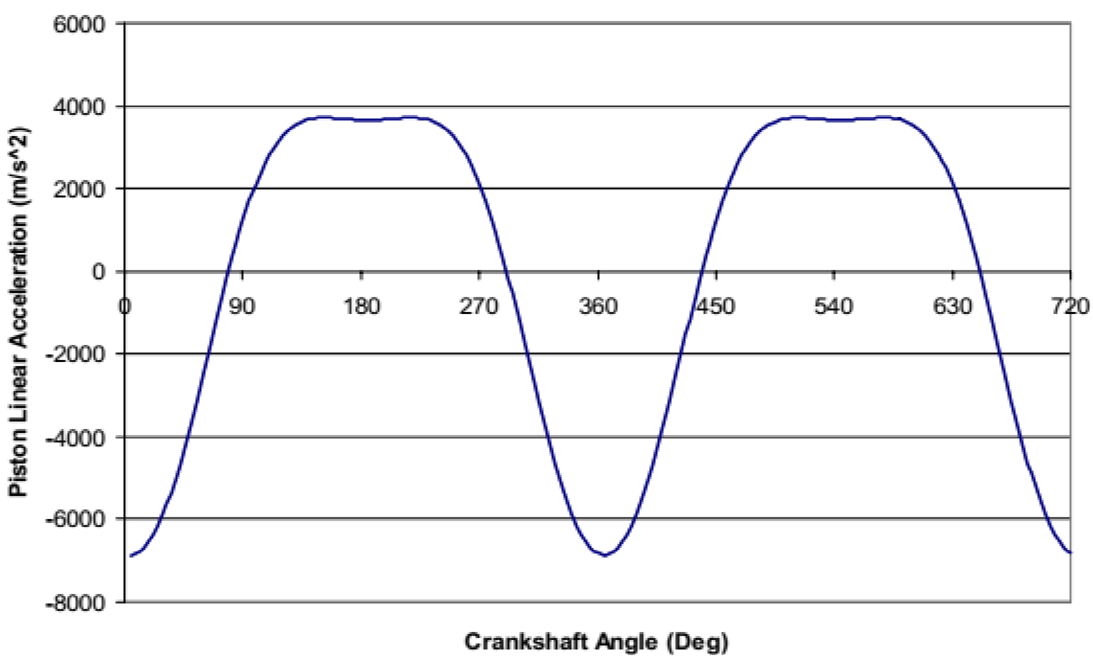


Figure 4.10 Variation of linear acceleration of the piston assembly over one complete engine cycle at crankshaft speed of 3600 rpm.

Figure 4.11 shows the variation of the force at the journal bearing between crankshaft and connecting rod defined in the global/non-rotating coordinate system. Figure 4.15 shows the variation of the same force defined in the local/rotating coordinate system. Fx in Figure 4.13 is the force that causes bending during service life and Fy is the force that causes torsion on the crankshaft. As can be seen in this figure, the maximum loading happens at the angle of 355° where the combustion takes place. The only difference between these figures is their reference coordinate system, therefore the magnitude, which is not dependent on the coordinate system chosen, is the same in both plots.

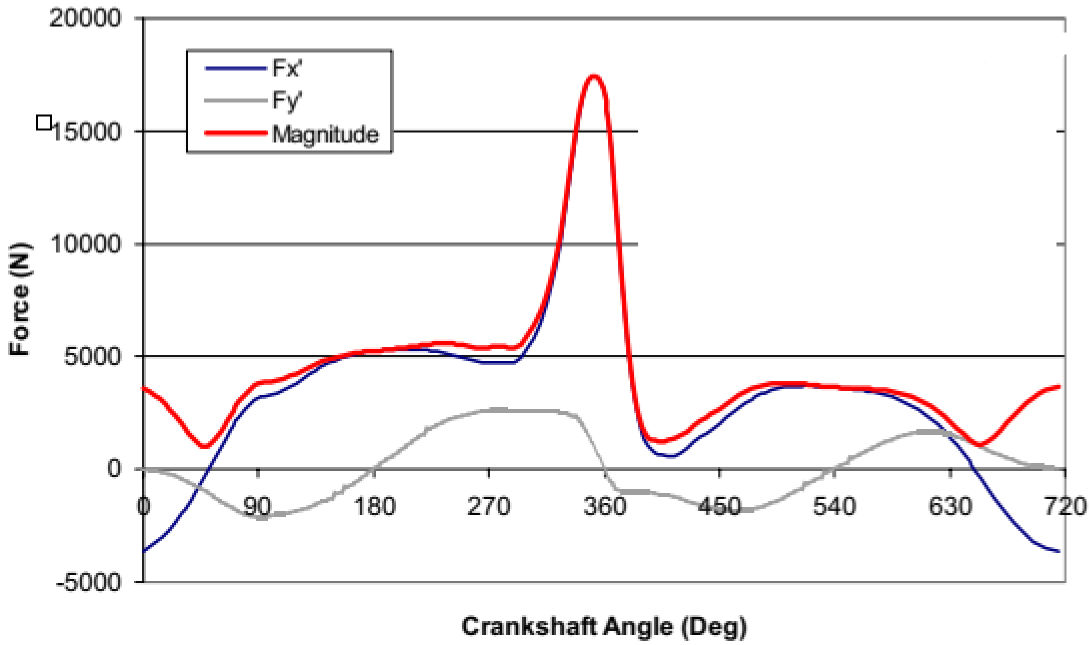


Figure 4.11 Variation of the force components over one complete cycle at the crank end of the connecting rod at crankshaft speed of 3600 rpm.

The variation of forces defined in the local coordinate system at 2000 rpm and 2800 rpm engine speeds are shown in Figures 4.12 and 4.13, respectively. Figure 4.14 compares the magnitude of maximum torsional load and bending load at different engine speeds.

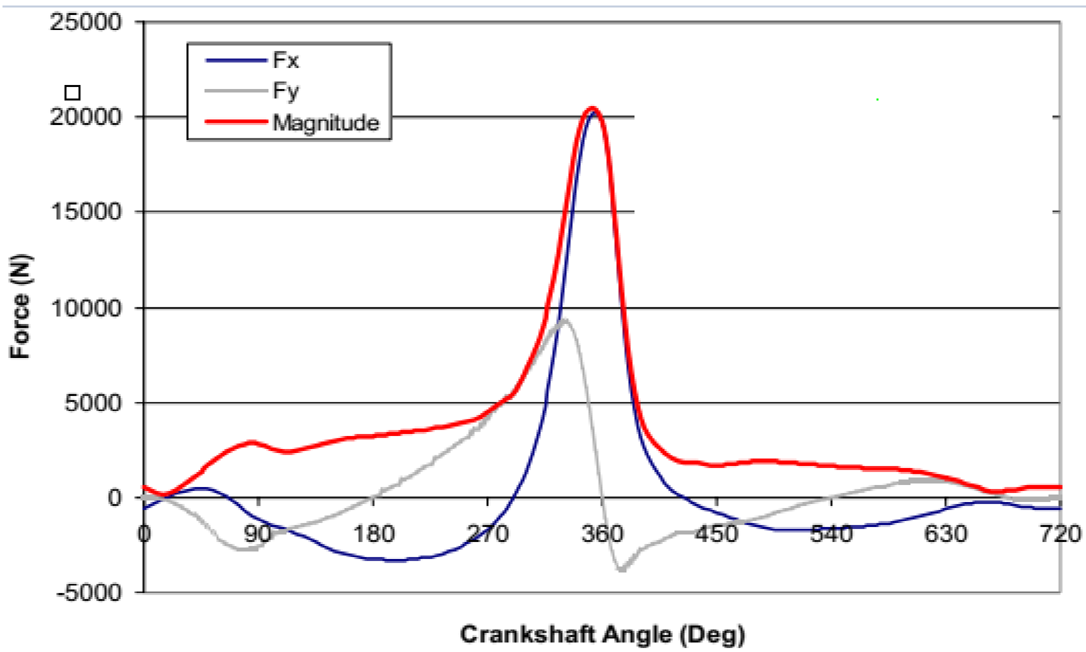


Figure 4.12 Variation of the force components over one complete cycle at the crank end of the connecting rod defined in the local/rotating coordinate system at crankshaft speed of 2000 rpm

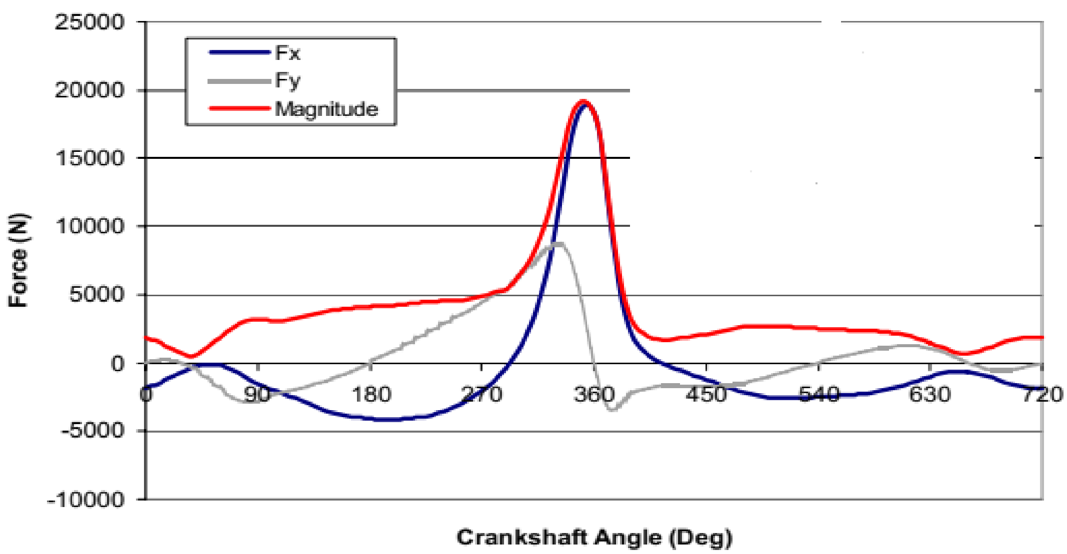


Figure 4.13 Variation of the force components over one complete cycle at the end of the connecting rod defined in the local/rotating coordinate system at crankshaft speed of 2800 rpm.

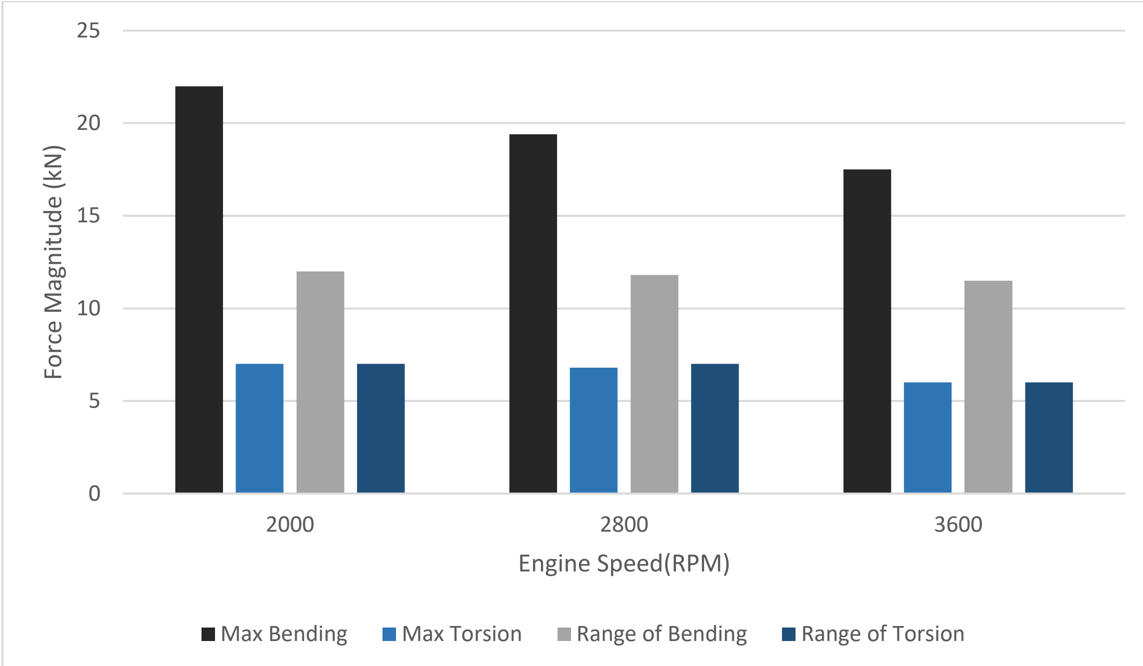
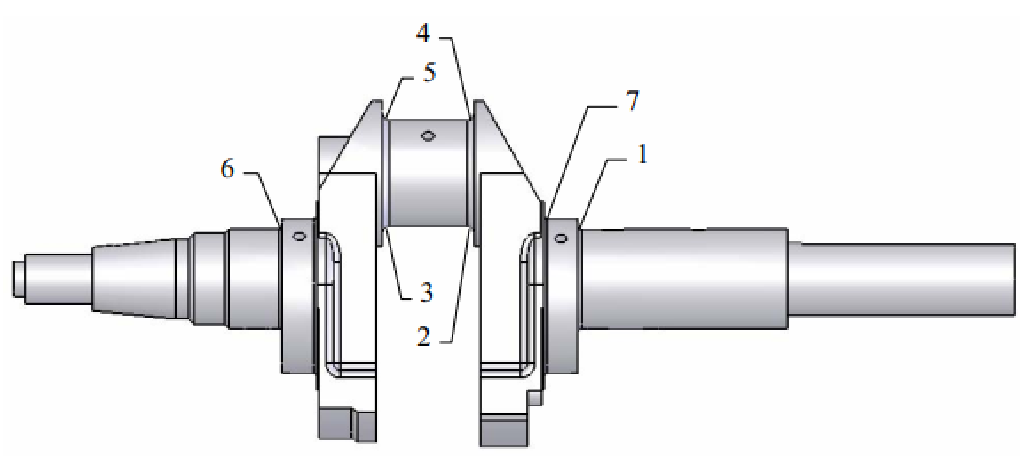


Figure 4.14 Comparison of maximum, minimum, mean, and range of load between bending and torsional load at different engine speeds.

4.4.2.1 Finding the Critical Location

Finite Element analyses were performed on the forged steel crankshaft as well as the cast iron crankshaft at different engine speeds. Investigation of the FE models shows that the fillet areas experience the highest stresses during service life of the crankshaft. Therefore, six points on the fillets were selected and labeled in for crankshaft.



Locations 1 and 6 are located on the boundary conditions of the loading conditions. For the loading condition, where the load direction is toward the center of the crankshaft, locations 1 and 6 are located far from boundary conditions. This loading condition is the only loading condition used at the time of maximum bending load, because at this time the torsional load is zero. Therefore, using the stress results and scaling them according to the maximum dynamic load at this moment will give the maximum stress at these locations.

Figures 5.6 show the von Mises stress with sign at these six locations at the engine speed of 2000 rpm crankshafts. The sign of von Mises stress is determined by the sign of the principal stress that has the maximum absolute value. As can be seen from both figures, the maximum von Mises stress occurs at location 2, while other locations experience stresses lower than location 2. Therefore, other five locations were not considered to be critical in the rest of the analysis. Since locations 1 and 6 have lower stresses than location 2 at the critical loading condition, finding stresses at these locations for the other three boundary conditions was unnecessary. According to the obtained results, the maximum von Mises stress value at location 2 for the forged steel at the engine speed of 2000 rpm.

Since stress range and mean stress are the main controlling parameters for calculating fatigue life of the component, these parameters have to be calculated. Figures5.7 show minimum, maximum, mean, and range of stress at selected locations on the crankshafts at the engine speed of 2000 rpm. As can be seen from these figures, location 2 has the highest maximum stress as well as the maximum value of stress range in crankshafts. This location also has a positive mean stress, which has a detrimental effect on the fatigue life of the component. Therefore, location 2 is the critical location on crankshaft. Therefore, this loading condition is the most saver case of loading resulting in the maximum magnitude of von Mises stress at location 2.

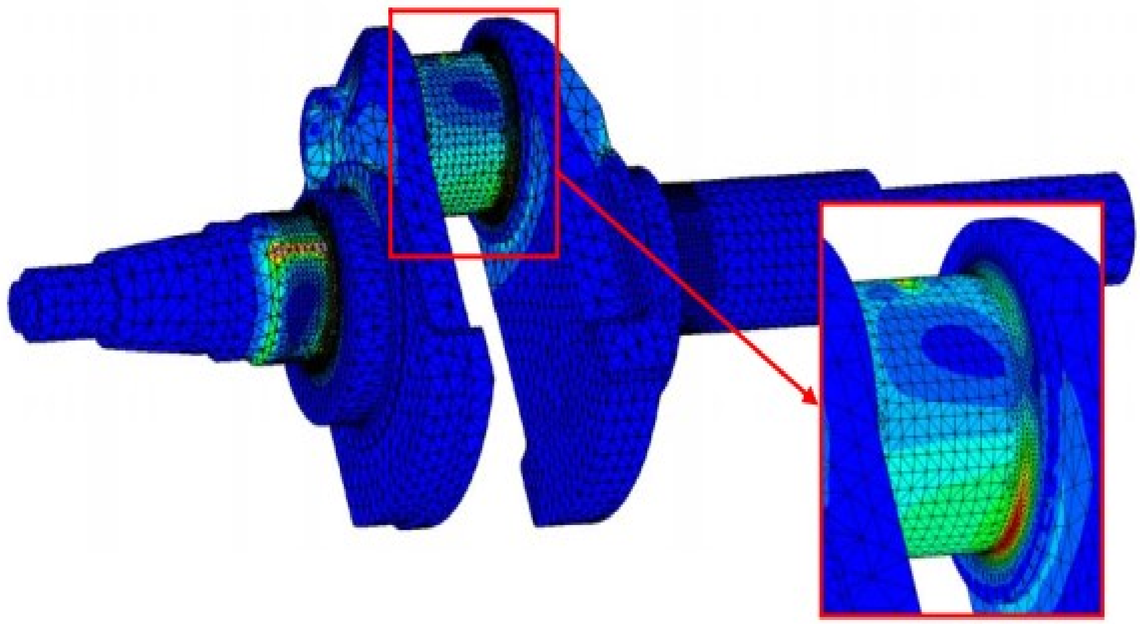


Figure 4.15 Stress distribution under critical loading condition at the crank angle of 355 degree

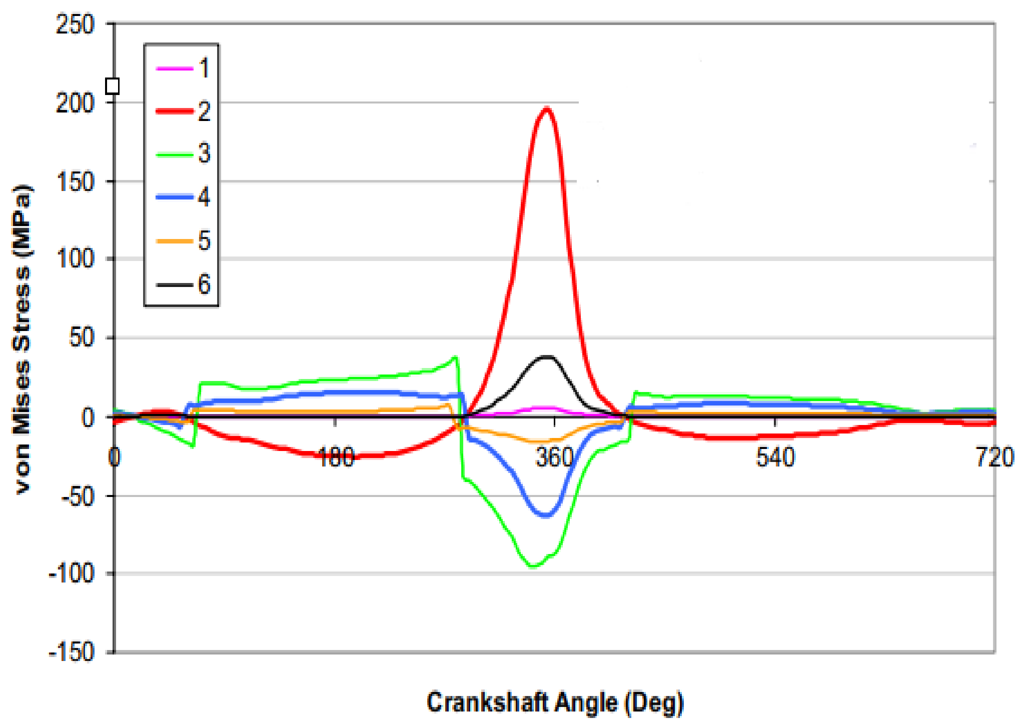


Figure 4.16 von Mises stress history at different locations on the crankshaft at the engine speed of 2000 rpm.

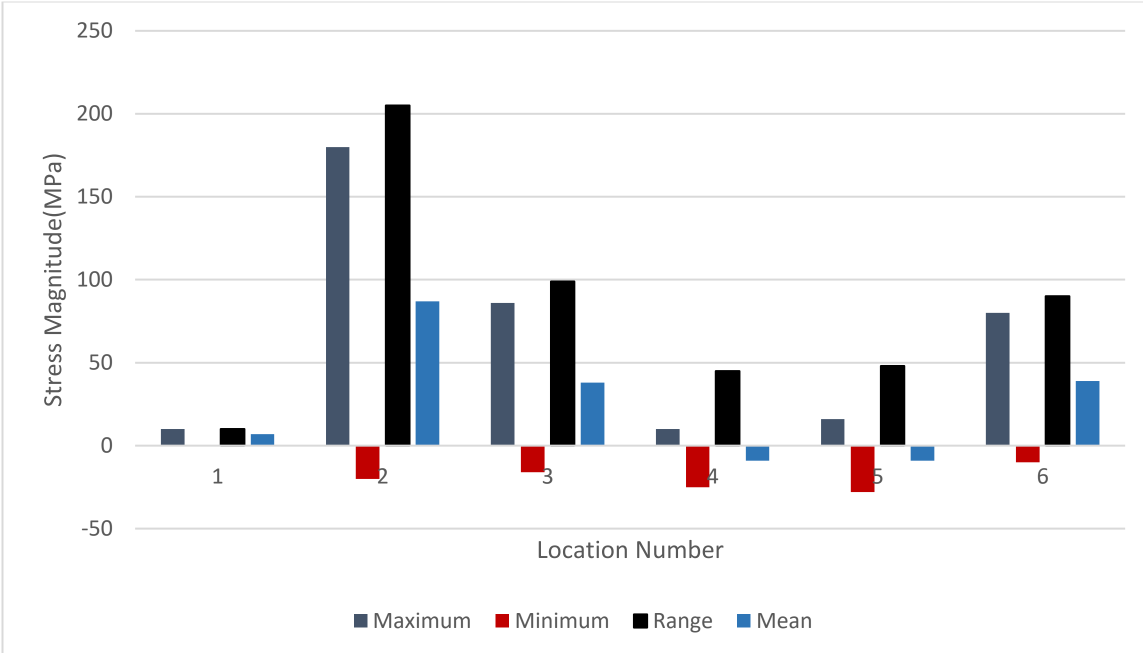


Figure 4.17 Comparison of maximum, minimum, mean, and range of stress at the engine speed of 2000 rpm at different locations on the crankshaft.

4.4.2.2 Finding Critical Engine Speed

Dynamic load applied on the crankshaft is a function of engine speed. Figures 5.8 show comparison plots of von Mises stress at location 2 at different engine speeds for crankshafts. As can be seen in these figures, with the increase of engine speed the maximum stress and, therefore, the stress range decreases, although not very significantly. Therefore, the critical engine speed will be the lowest operating engine speed, which is 2000 rpm according to the engine manual. This issue should not be misunderstood as the higher the engine runs the longer the service life, since there are many other factors to consider in an engine.

The most important issue when the engine speed increases is wear and lubrication. As these issues were not of concern in this study, further discussion is avoided. Engine speeds lower than 2000 rpm are transient, which means the engine speeds up in few seconds to its operating speed. Since an electric rotor starts the engine, combustion does not occur during the transient speed up and torque output is not taken from the engine. Therefore, speed engines lower than 2000 rpm are not considered.

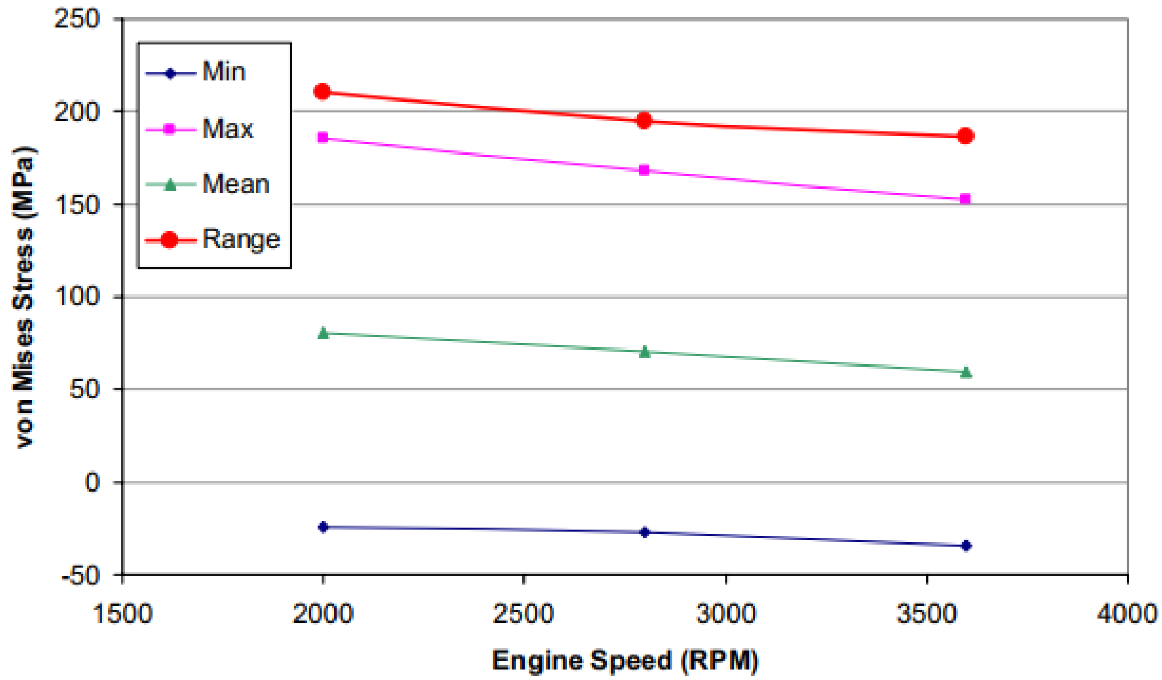


Figure 4.18 Variations of minimum stress, maximum stress, mean stress, and stress range at location 2 on the crankshaft as a function of engine speed.

CHAPTER FIVE

FINITE ELEMENT ANALYSIS OF CRANKSHAFT

The finite element method is numerical analysis technique for obtaining approximate solution to a wide variety of engineering problems. It is not possible to obtain analytical mathematical solutions for many engineering problems. An analytical solution is a mathematical expression that gives the values of the desired unknown quality at any location in a body. For problems involving complex material properties and boundary conditions, the engineer resorts to numerical methods that provide approximate but acceptable solutions.

This chapter discusses geometry generation used for finite element analysis, describes the accuracy of the model and explains the simplifications that are made to obtain an efficient FE model. Mesh generation and its convergence are discussed. Using proper boundary conditions and type of loading are necessary since they strongly affect the results of the finite element analysis. Identifying appropriate boundary conditions and loading situation are also discussed. Finite element models analyzed the crankshaft. Above mentioned FE models are used for static and dynamic analysis considering the boundary conditions according to the mounting of the crankshafts in the engine.

5.1 3D Modeling

Finite element modeling of any solid component consists of geometry generation, applying material properties, meshing the component, defining the boundary constraints, and applying the proper load type. These steps will lead to the stresses and displacements in the component. In this study, similar FE analysis procedures are performed for both static stress and dynamic stress of crankshafts. Figure 5.1, shows solid model of original crankshaft, which is directly imported to the ANSYS software to evaluate the analysis.

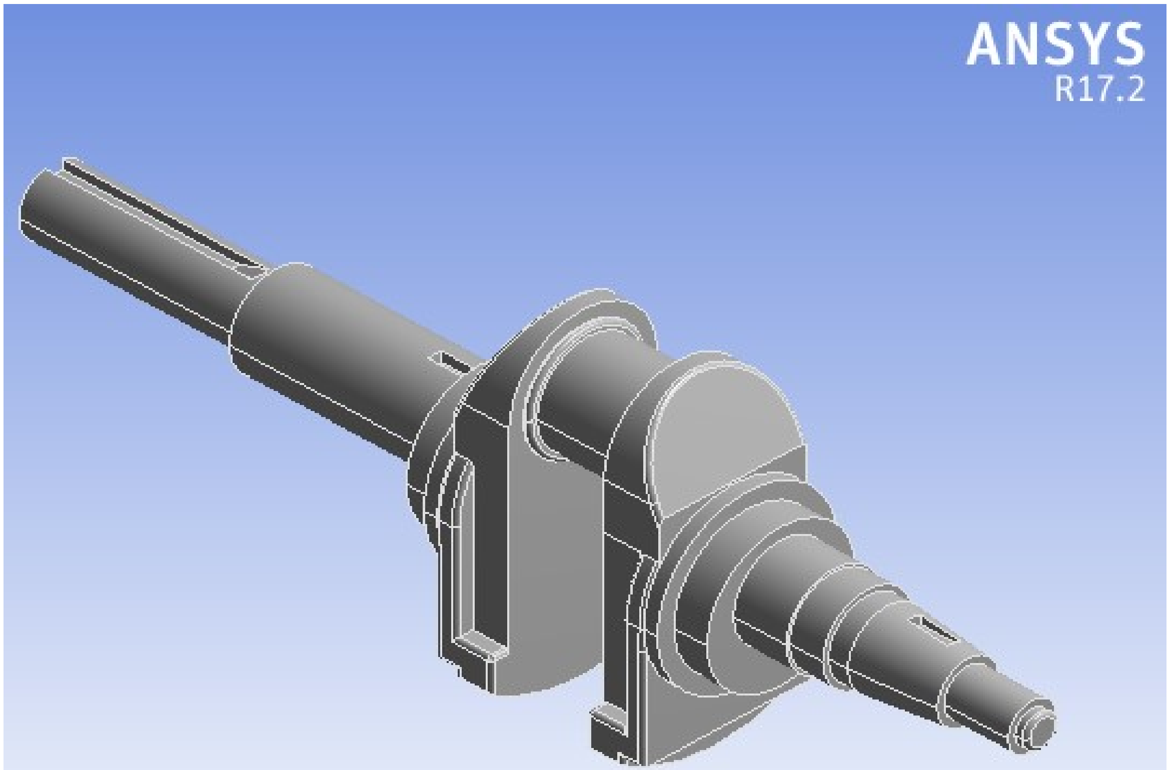


Figure 5-1 Solid work Model of Crankshaft

5.2 Material Properties

There are total three materials are used for this model Forged steel (AISI 1045 steel), Medium carbon steel and Ductile cast iron. The material properties of the crankshaft are given below in table.

Table 5-1 Forged steel (AISI 1045 steel properties.

Forged steel (AISI 1045 steel*)*	
Density	$7833 kg/m^3$
Yield Tensile Strength	$625 MPa$
Ultimate Tensile Strength	$827 MPa$
Poisson's Ratio	0.3
Young's Modulus	$221 GPa$

Table 5-2 Ductile Cast Iron properties

Ductile cast iron properties

Density	$7200kg/m^3$
Yield Tensile Strength	$515MPa$
Ultimate Tensile Strength	$770MPa$
Poisson's Ratio	0.290
Young's Modulus	$180GPa$

Table 5-3 Table 5.3 Medium Carbon Steel

Medium carbon steel properties	
Density	$7845kg/m^3$
Yield Tensile Strength	$415MPa$
Ultimate Tensile Strength	$620MPa$
Poisson's Ratio	0.285
Young's Modulus	$200GPa$

5.3 Mesh Generation

FEA analysis was performed on crankshafts for both loads, the static and dynamic load analysis. Since boundary conditions of dynamic FEA and static FEA are different. In this section, meshing of both dynamic FEA and static FEA are presented for the Forged Steel, Ductile Cast Iron and Medium Carbon Steel crankshafts.

Tetrahedral shape of element is used for meshing the imported complex geometries to the ANSYSWORKBENCH software. Three dimensional model (3D) of crankshafts is performed in Solid work. After that, model is exported by *ANSYS* and profile is subdivided into nodes and elements. Collection of elements is called mesh and it is necessary to make mesh optimization to get more accuracy results. Mesh optimization is carried out until the *FEA* results and analytical solutions are close to each other. Meshing of Model: We discretized the solid model into small elements.

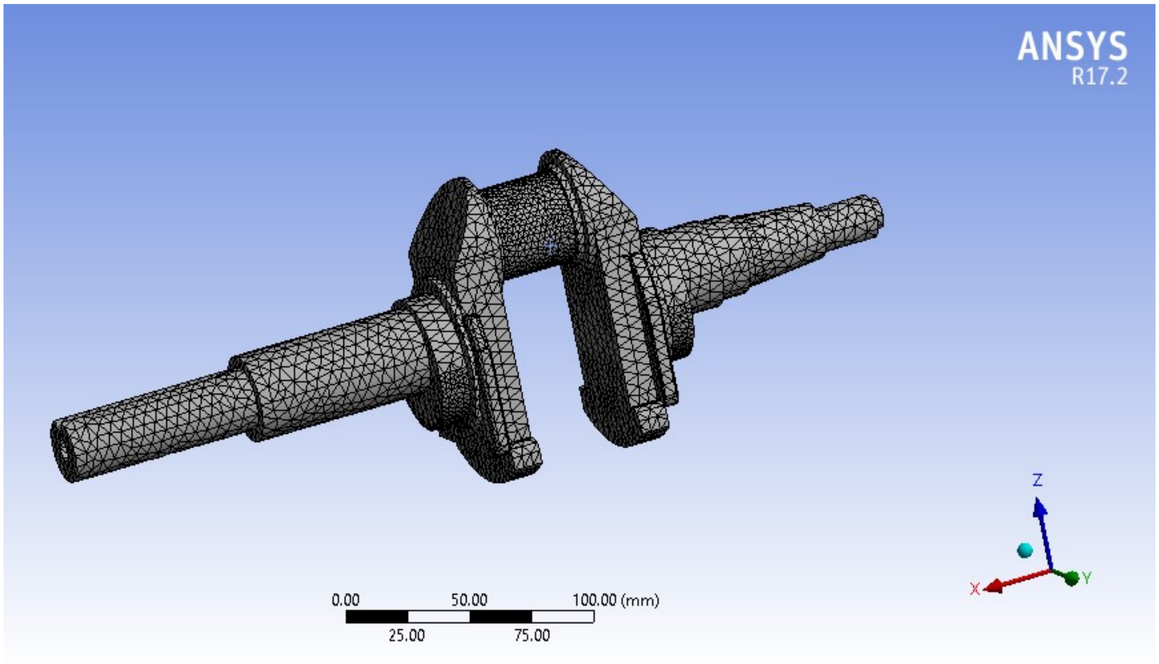


Figure 5-2 Mesh Generate Model

Figure 5.2 shows the meshed model of crankshaft. The Discretization (Mesh generation) is the first step of Finite Element Method. In this step the component or part is divided into number of small parts. In discretization the Number of nodes is 247,006 and number of elements is 147,139. The effect of force on each portion of the component is not same. The purpose of discretization is to perform the analysis on each small division separately.

In Dynamic FEA applied the same mesh that was used for static FEA, (Number of nodes is 247,006 and number of elements is 147,139 with element size of 0.005 m.) as presented in the above section. Convergence was checked at locations where high bending stresses are expected.

5.4 Boundary and loading conditions

The crankshaft is fixed at both side with cylindrical support using baring which show in Red color the load of 14670 N generated due to maximum gas pressure is applied at crankpin in vertical downward direction.

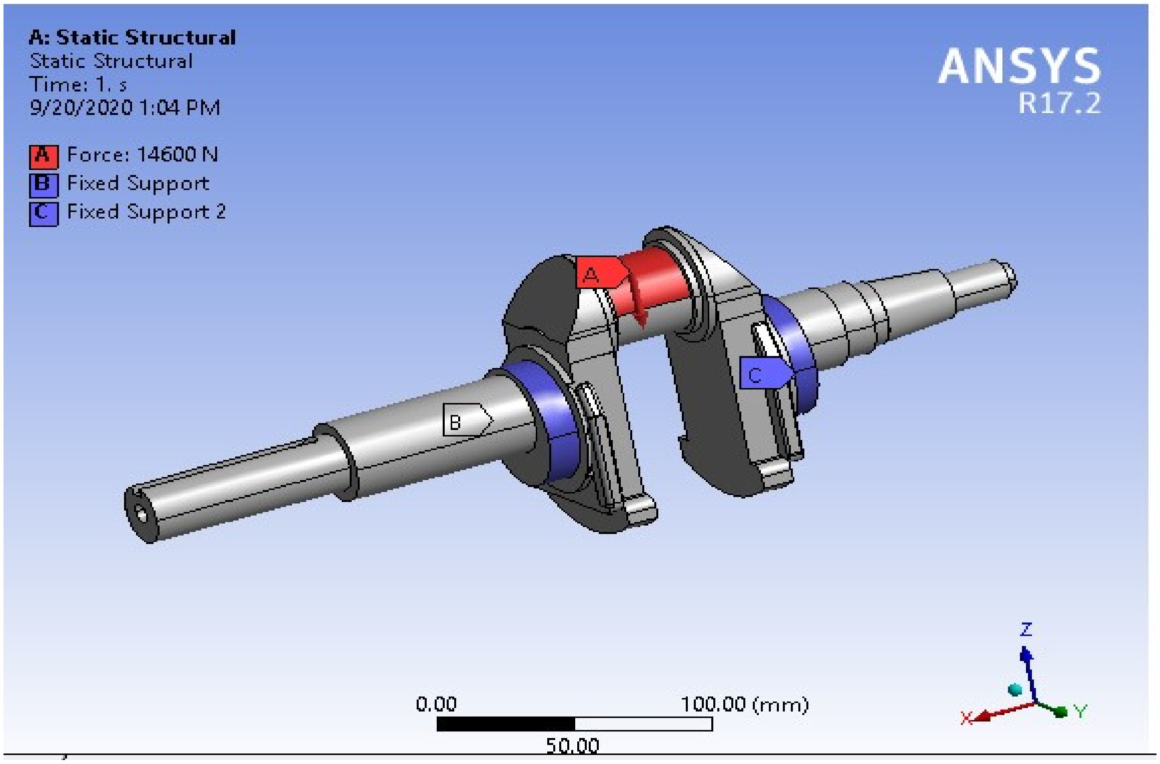


Figure 5-3 Static structure model

5.4.1 Static Structural Finite Element Analysis

The crankshaft bears the constraints of main journals and longitudinal thrust bearing. The bearing is press fits to the crankshaft and do not allow the crankshaft to have any motion other than rotation about its main axis. Crank at dead center the load 14, 600 N in downward and upward, z – direction constricts the motion of the crankshaft as shown Figure (5.3).

The crankshaft bears the constraints of main journals and longitudinal thrust bearing. The bearing is press fits to the crankshaft and do not allow the crankshaft to have any motion other than rotation about its main axis. Because of the effect of load, crankshaft main journals appear bend deformation between the lower main-bearing half and upper main-bearing half. Longitudinal thrust bearing can prevent effectively the crankshaft axial movement and ensure the piston-and connecting-rod assembly normally works. Five surfaces radial symmetry constrains were exerted on the five main journals surfaces.

5.5 Finite Element Analysis Results and Discussions

The crankshaft is checked for von-misses stress, shear stress, deformation and analytical calculation with different three materials for the validation of work. After the application of forces, the next step is to perform the structural analysis of crankshaft. In this structural analysis, mainly concern with the deformation and stresses acting on the crankshaft (von-misses stresses). When the forces are applied, the slight deformation and also the stresses take place in the crankshaft. Now, when applied the gas force due to maximum gas pressure to crankshaft, it may be deform to check the total deformation of crankshaft due to maximum gas load on crankshaft. Normal deformation may be occur due to maximum load of crankshaft that should take for validation of work with von-mises stress as below the figure of total deformation of crankshaft with deferent three material and compare it in result.

The deformation of original crankshaft is shown in Figure. 5.6, the deformation are 0.0076703mm, 0.0094048mm and 0.0084584mm in the Forged steel (AISI 1045 steel), Ductile cast iron and Medium carbon steel crankshaft respectively is not same throughout. The portion in red color shows that the deformation at that region is maximum and the portion in blue color shows that the deformation is minimum in that. In static finite element analysis, the maximum equivalent stress (von-Mises stress) 89.65MPa, 90.091MPa and 90.308MPa in the Forged steel (AISI 1045 steel), Ductile cast iron and Medium carbon steel respectively and the maximum shear stress are 46.956 MPa Forged steel (AISI 1045 steel), 47.003MPa for Ductile cast iron and 42.025MPa Medium carbon steel for crank position .

i. Von mises stress

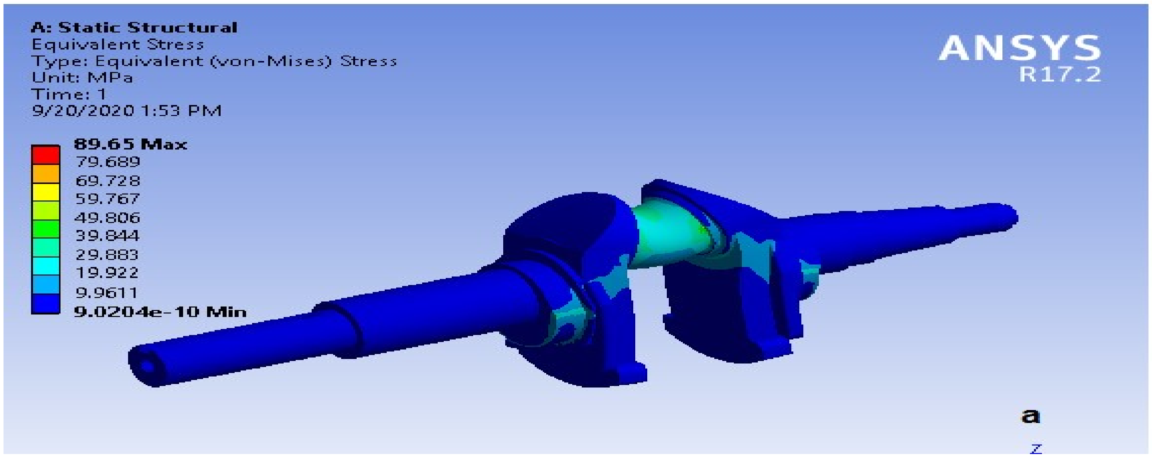


Figure 5.4(a) Von misses stress for Forged steel

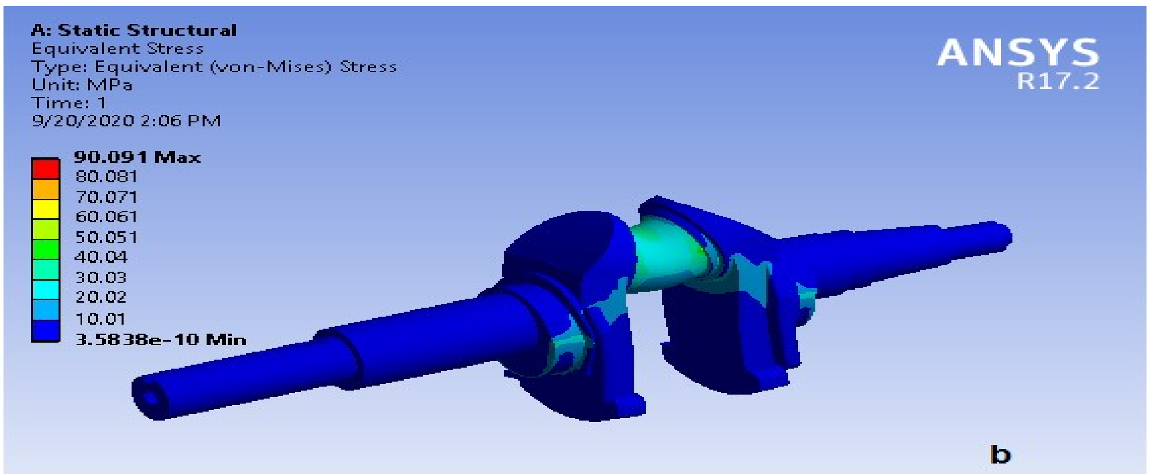


Figure5.4 (b) Von misses stress for ductile cast iron

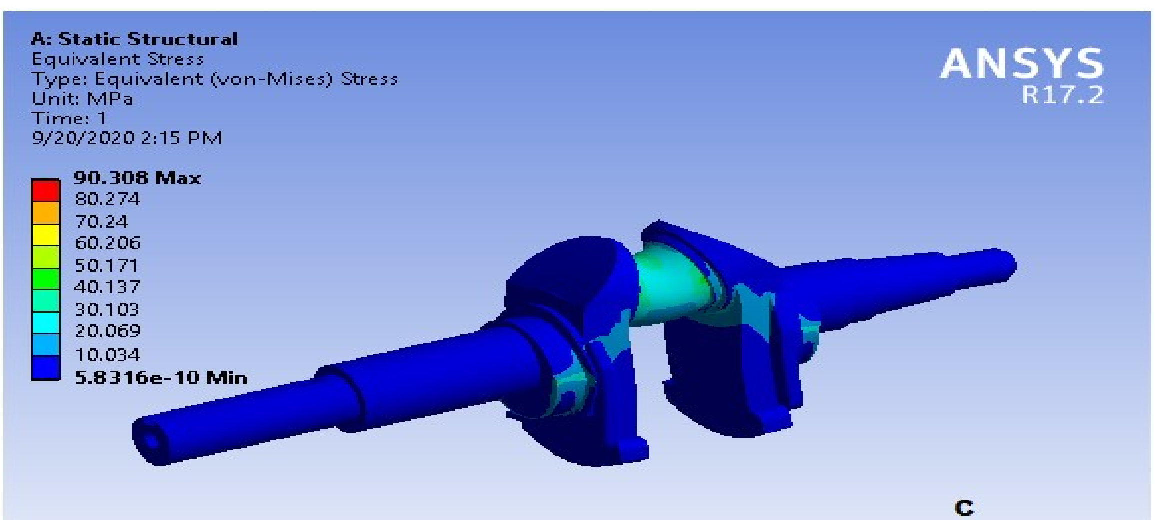


Figure 5.4(c) Von misses stress for Medium carbon steel.

ii. Shear stress at different material

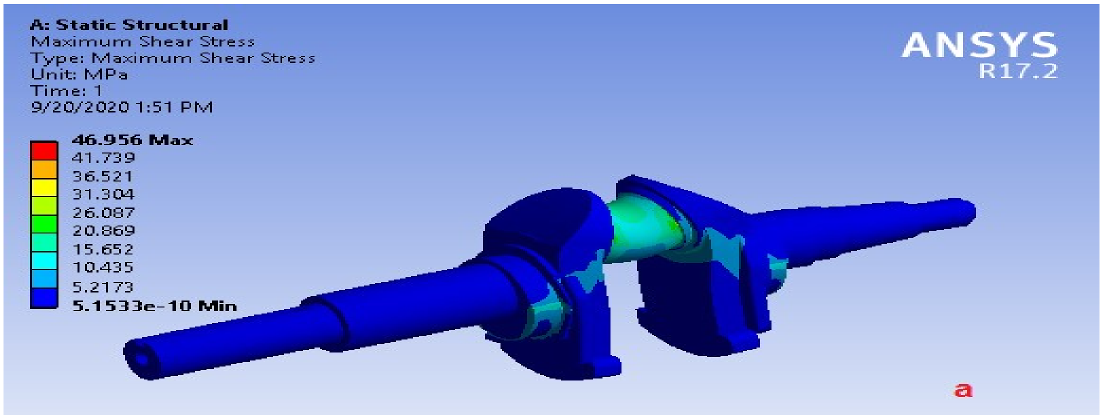


Figure 5.5(a) Shear stress for Forged steel

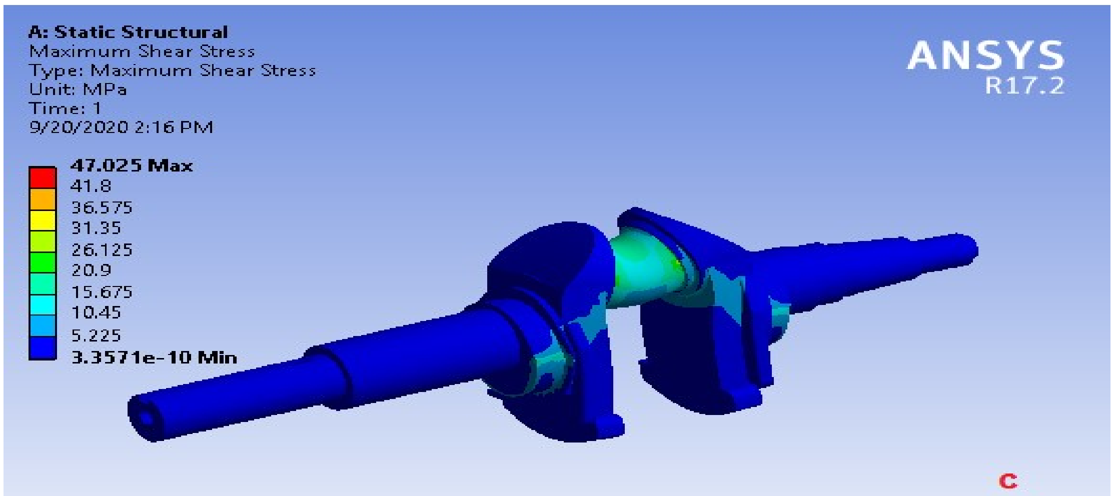


Figure 5.5(b) Shear stress for ductile cast iron

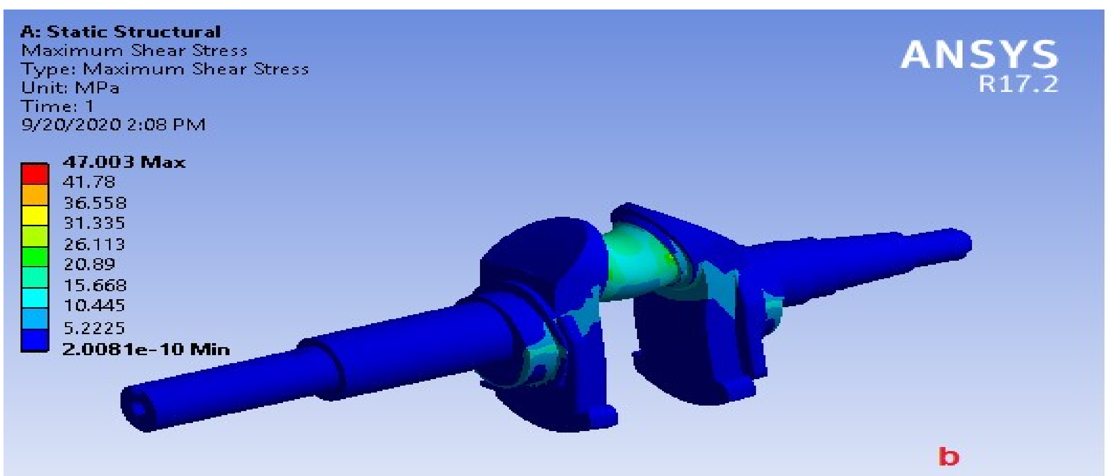


Figure 5.5(c) Shear stress for medium carbon steel

iii. Deformation at different material

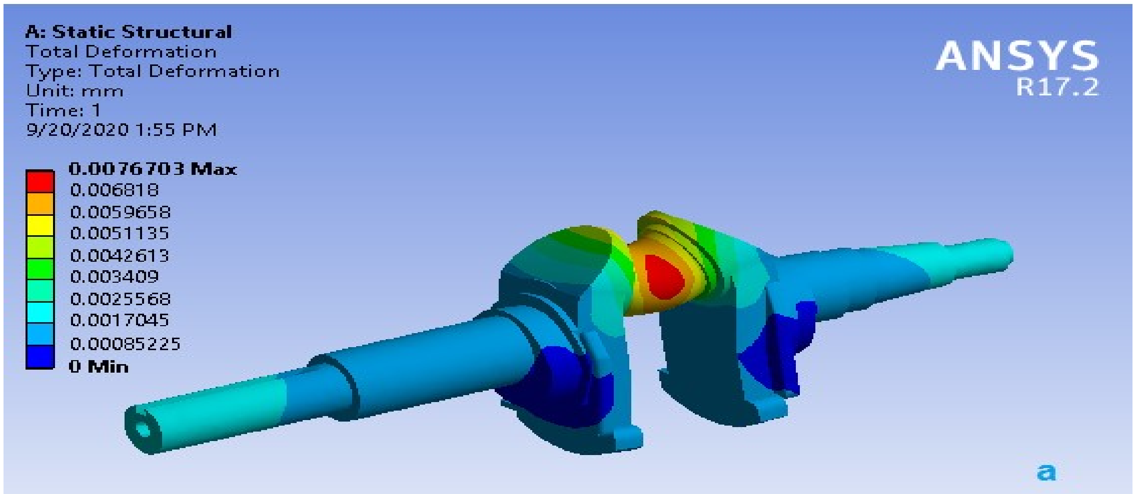


Figure 5.6(a) Deformation for Forged steel

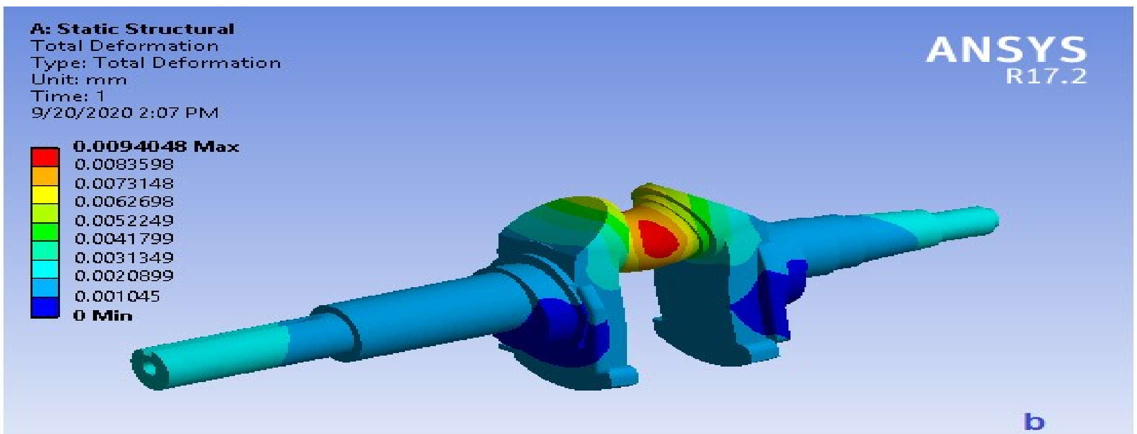


Figure 5.6(b) Deformation for ductile cast iron

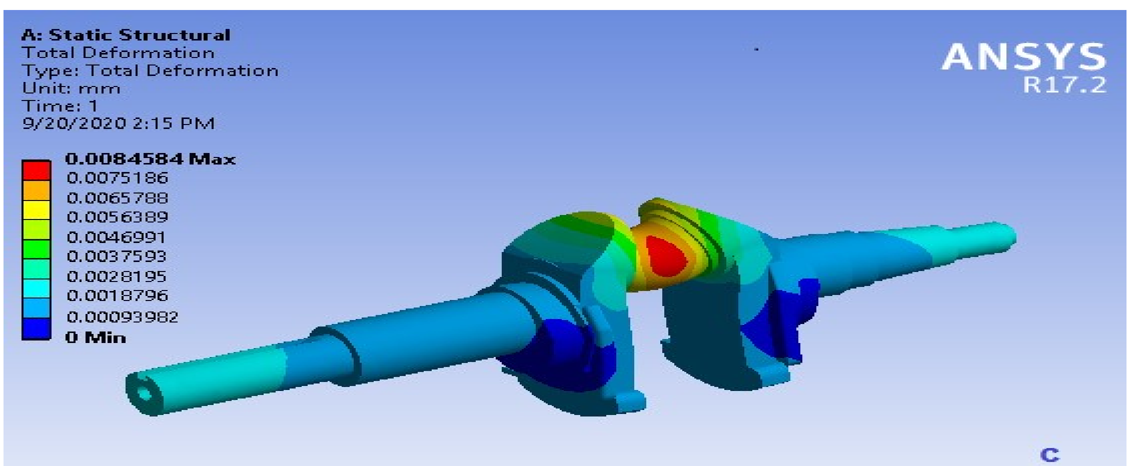


Figure 5.6(c) Deformation medium carbon steel

Analytical calculation and software calculation of von-mises stress are compare of three material. A difference in the von-mises stress is observed between the analytical and ANSYS software result which is satisfactorily as 10% variation to tolerance is set due to meshing technique and solution algorithm. The result stress is below the yield stress of the material, hence the design is safe.

Table 5-4 Results of Von misses, Deformation

No	Materials	Analytical Result	Software Result	Deformation (mm)
		Von-misses stress (MPa)	Von-misses stress (MPa)	
1	Forged steel (AISI 1045steel)	96.74	89.65	0.0076703
2	Ductile cast Iron	97.62	90.091	0.0094048
3	Medium carbon Steel	96.89	90.308	0.0084584

From above analysis we can see that there is total three material uses for analysis and got the different result with parameter, from that the Forged steel (AISI 1045steel) is best of them. Usually crankshaft is made from steel by using casting or forging but we can use Forged steel (AISI 1045steel) as a material for crankshaft made. Vonmises stress of Forged steel (AISI 1045steel) is got 89.65 MPa and deformation is 0.0076703 mm. The crankshaft chosen for this project Comparing chat of three materials is as below.

Table 5-5 Compare Analytical and software result

No	Materials	Analytical Result stress (MPa)	Software Result Von-misses stress (MPa)
1	Forged steel (AISI 1045steel)	96.74	**89.65**
2	Ductile cast Iron	97.62	90.091
3	Medium carbon Steel	96.89	90.308

Figure 5.7 Comparison of Maximum Equivalent (Von-Misses) Stress

SUMMARY OF COMPUTATIONAL ANALYSIS:

- Material Forged steel (AISI 1045steel) is meeting the maximum no of requirements.
- The minimum available deformation and the difference between the minimum deformation available and the deformation when Forged steel (AISI 1045steel) is used, is 0. .0076703 mm which is too less & can be ignored.
- Forged steel (AISI 1045steel) will give optimum results as compared to the other materials given in the case study

5.5.1 Dynamic Finite Element Analysis

Quadratic tetrahedral elements were used to mesh the crankshaft finite element geometry. Tetrahedral elements are the only option for meshing the imported complex geometries to the ANSYS software. Using linear tetrahedral elements will result in a rigid model with less accuracy, whereas using quadratic tetrahedral elements will increase the accuracy and lessen the rigidity of the geometry. In order to mesh the geometry with this element type, the free meshing feature of ANSYS software was used. In this feature, the global mesh size could be defined, while for critical locations free local meshing could be used to increase the number of elements for accurate stresses at locations with high stress gradients.

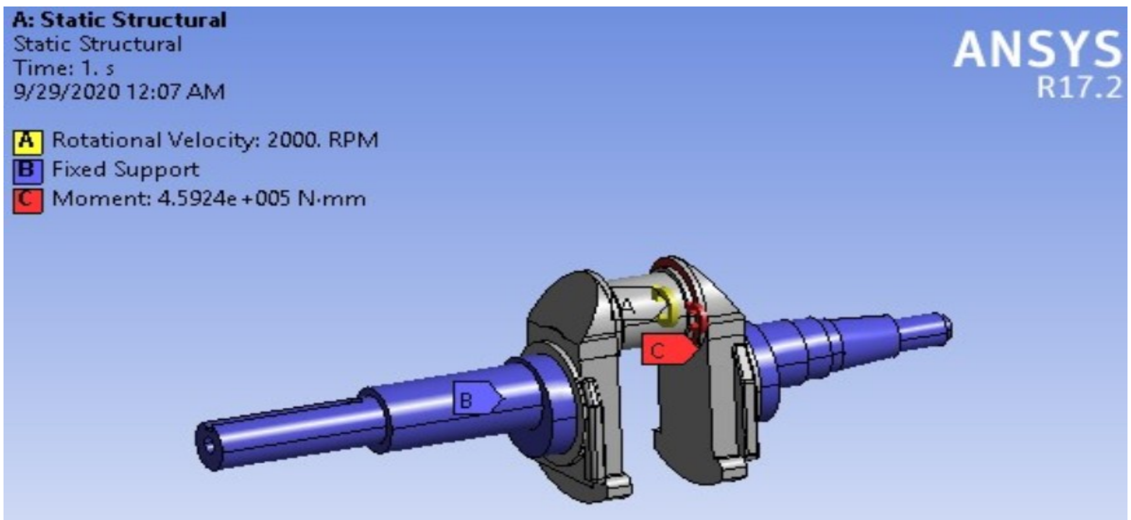
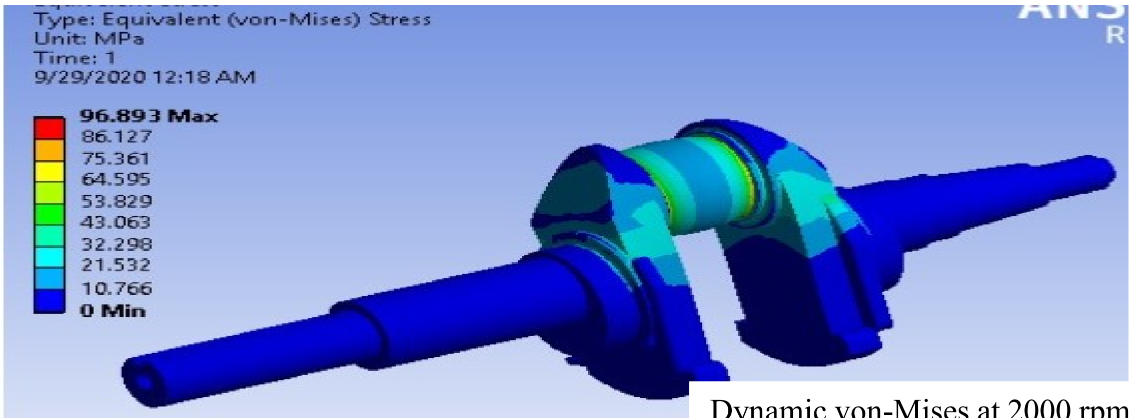


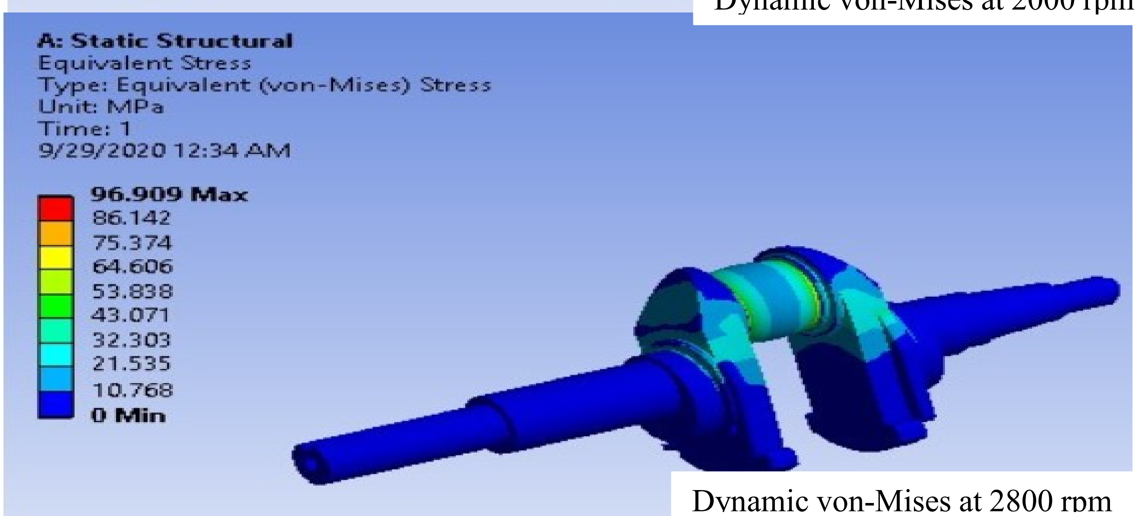
Figure5.8 boundary conditions of dynamic model of crankshaft

FEA model in which load is applied at the crank journal end in both sides is restrained. The crankshaft bears the constraints of main journals and longitudinal thrust bearing. Boundary conditions rotate with the direction of the load applied such that the inner face of the fixed semicircular surface and sliding ring face the direction of the load. Moment 459.24kN.mm and the radial force 10680N in at crank speed of 200, 2800 and 3600 rpm are applied on the crankpin surfaces.

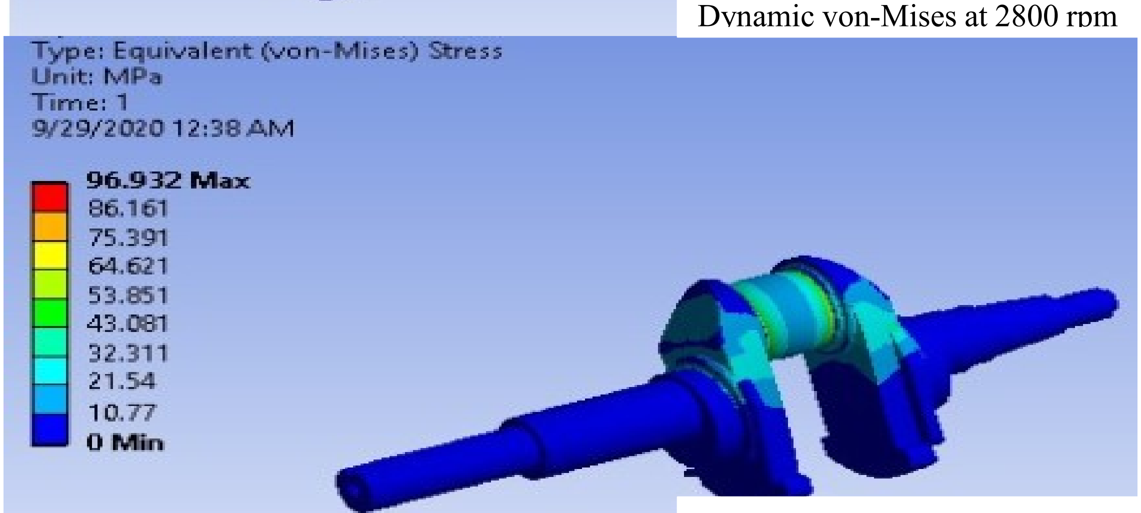
In dynamic finite element analysis, the maximum equivalent stress (von-Mises) and the maximum shear stress are 96.932 MPa and 47.079 MPa crank speed of 3600 rpm respectively, the maximum equivalent stress (von-Mises) and the maximum shear stress are 96.909 MPa and 45.079 MPa crank speed of 2800 rpm respectively. And maximum equivalent stress 96.893 MPa at the crank speed 2000 rpm. Generally the maximum concentrated stress are developed at the some portion of crankshaft, fillet area of crankpin and main journal, crankpin center and main journal surfaces.



Dynamic von-Mises at 2000 rpm



Dynamic von-Mises at 2800 rpm



Dynamic von-Mises at 3600 rpm

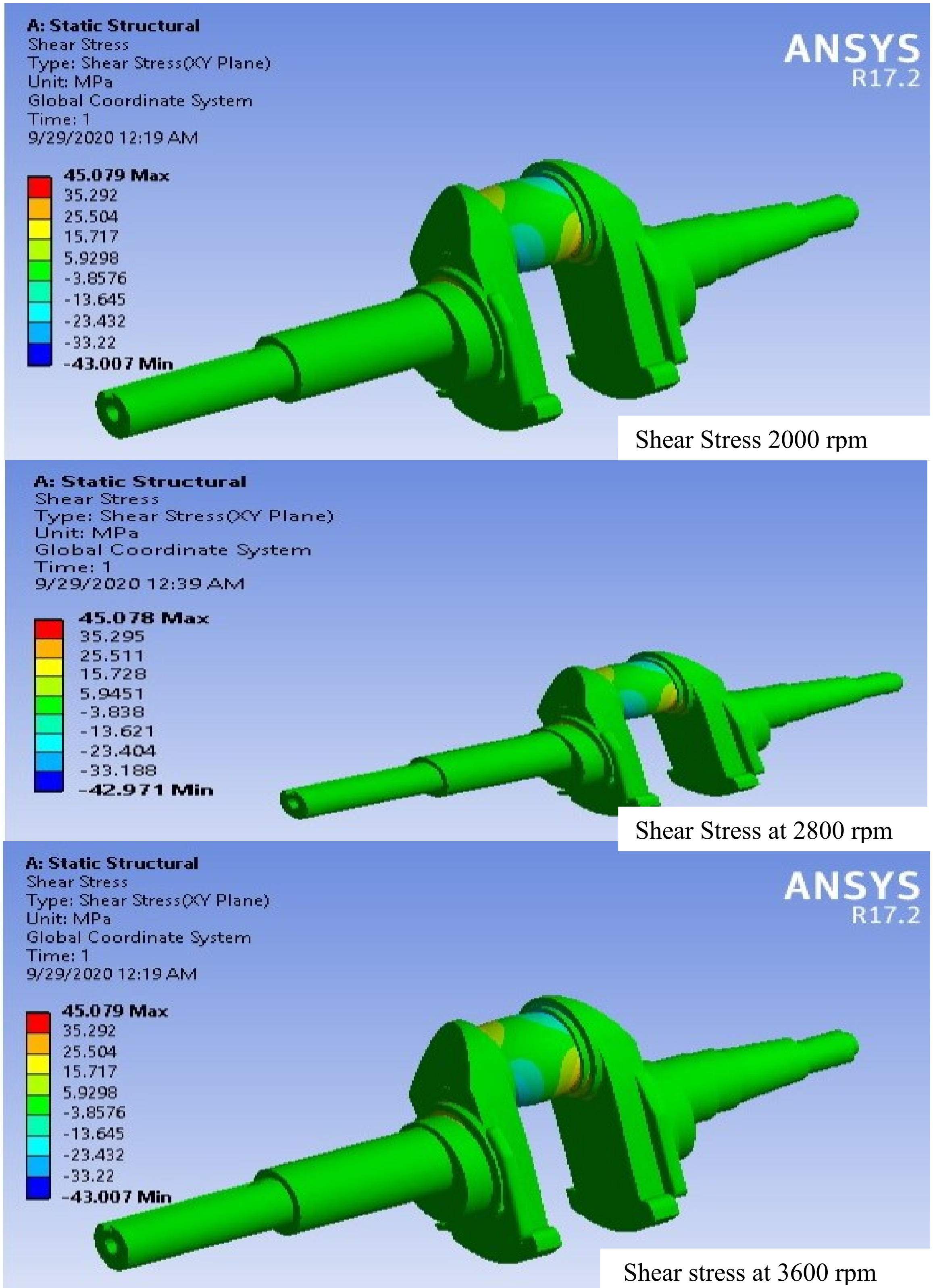
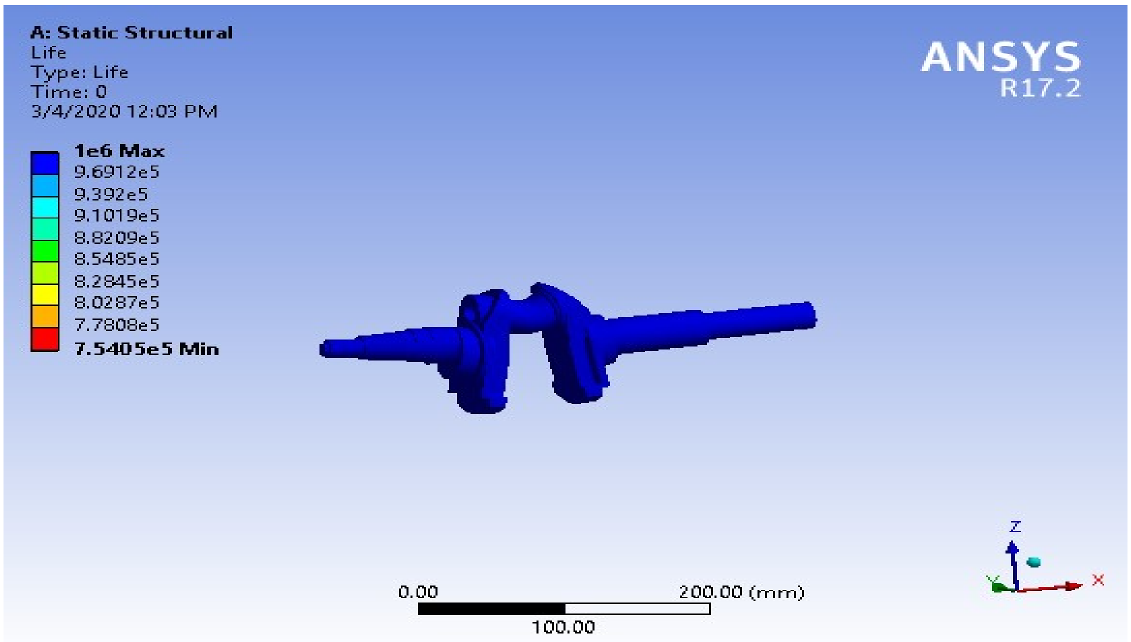


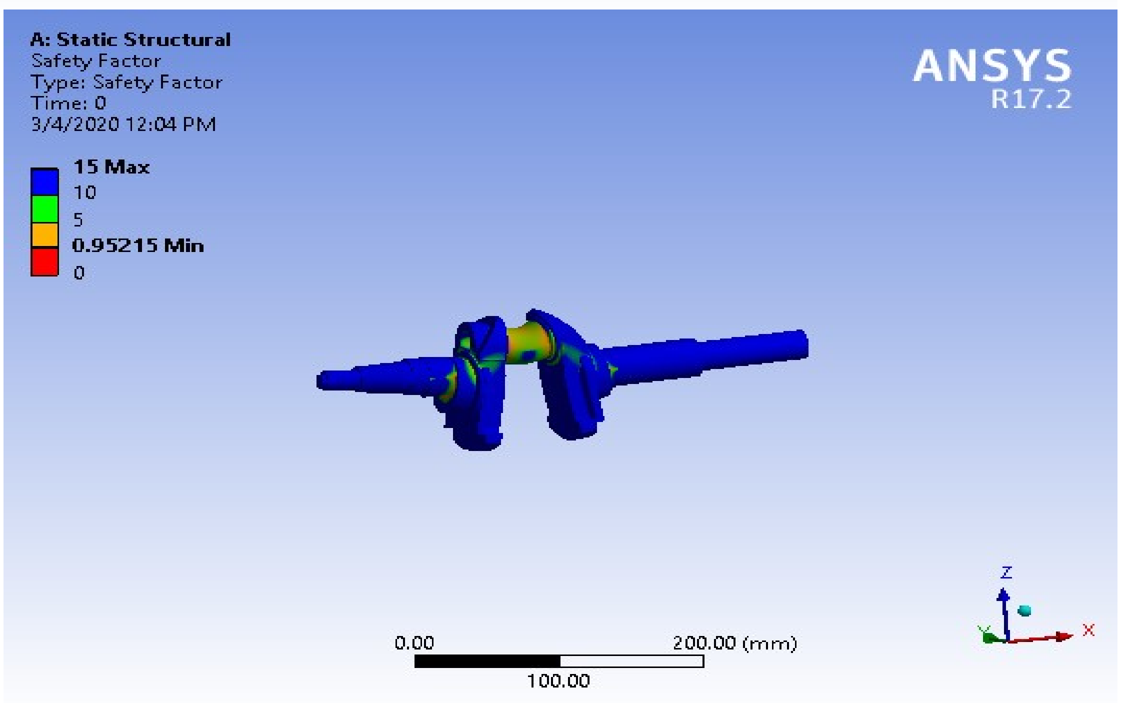
Figure 5.8 Dynamic shear Stress at different Crank Speed.

5.6 Fatigue life prediction

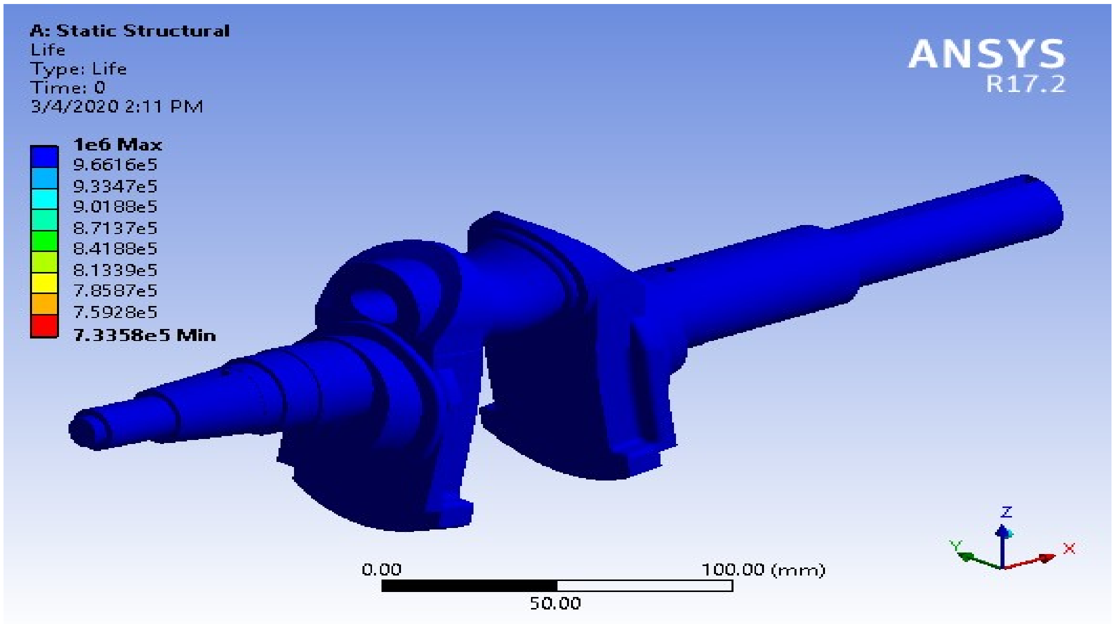
Compare the computational analysis result of three materials Forged steel (AISI 1045steel), Ductile cast Iron and Medium carbon Steel we define the following parameter

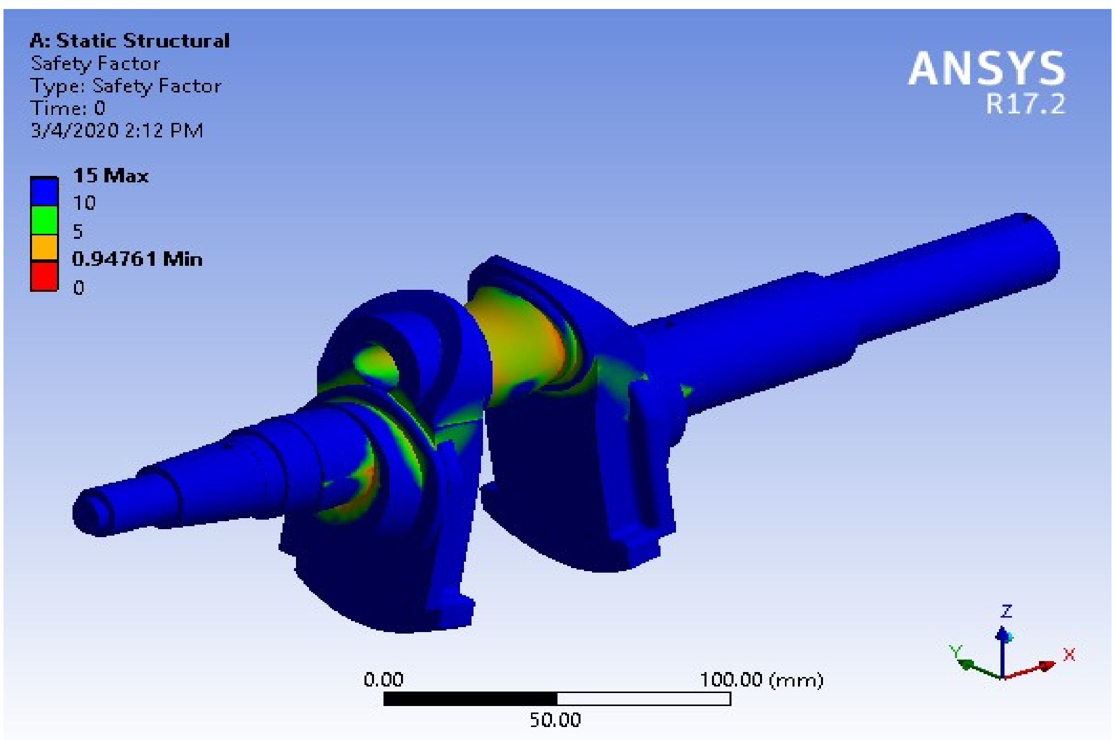
5.6.1 Life and safety factor in terms of Forged steel (AISI 1045 steel



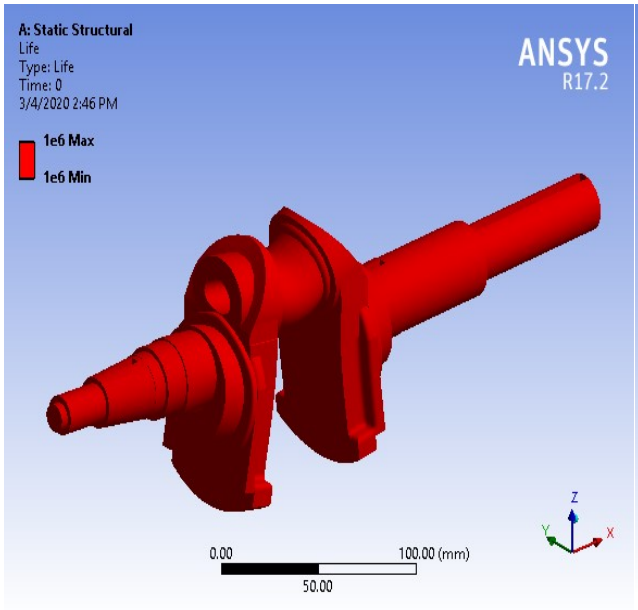


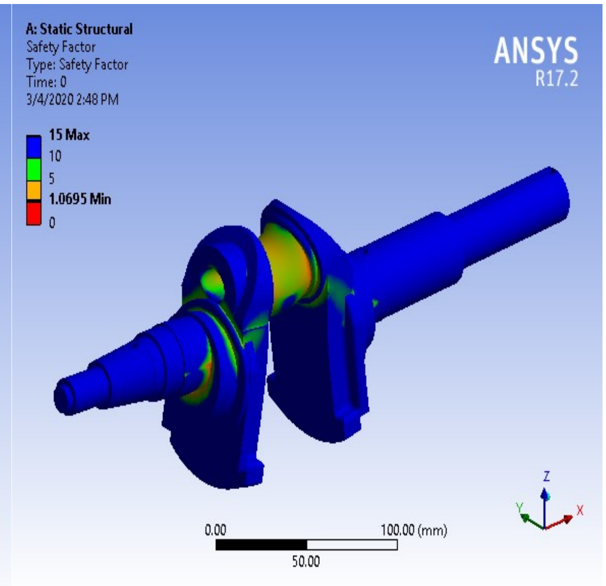
5.6.2 Life and safety factor in terms of carbon steel





5.6.3 Life and safety factor in terms of Cast Iron





According to Finite Element analysis, shown in Figure above in both parameters red location shows maximum stress and blue locations have low stresses during service life.

Single cylinder engine crankshaft model in ANSYS predicts that the maximum value of the equivalent alternating stress decreases, and fatigue life increases.

Structural Steel > Alternating Stress Mean Stress

Alternating Stress MPa	Cycles	Mean Stress MPa
3999	10	0
2827	20	0
1896	50	0
1413	100	0
1069	200	0
441	2000	0
262	10000	0
214	20000	0
138	1.e+005	0
114	2.e+005	0
86.2	1.e+006	0

Fatigue Sensitivity Figure shows fatigue sensitivity curve and how the fatigue results change as a function of the loading at the critical location on the model.

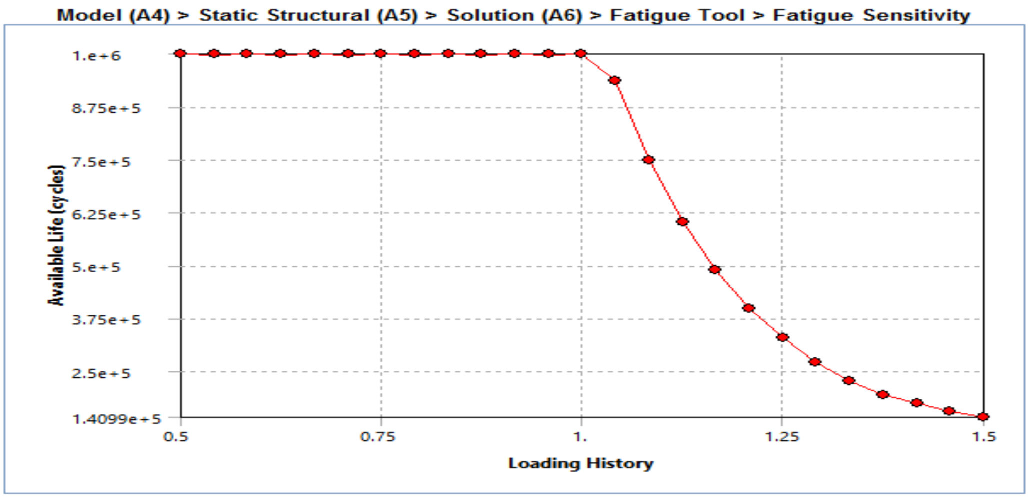


Figure 5-13 fatigue sensitivity

Figure shows fatigue sensitivity curve and between 0.5 and 1.25 is very safe region as load going to increase materials.

CHAPTER SIX

CONCLUS ION AND RECOMMENDATION

6.1 CONCLUSION

The crankshaft model was created by SOLIDWORK software. Then, the model created by SOLIDWORK was imported to ANSYS software. The examination of the crank shaft will be done using geometry optimization like different crankpin fillet radius and crankpin diameter. Finite element analysis using ANSYS and Analytical methods are done.

From this work, the maximum deformation occurs at the center of crankpin and maximum stress appears at the area between crank journal and crank cheeks. So, the value of von-mises stresses of analysis is less than the material yield stress so this design is safe. By comparing von-mises stress and deformation values, from three different materials (forged steel AISI 1045 steel, Medium carbon Steel, Ductile cast Iron) and selected Forged steel (AISI 1045steel) has higher strength and shows lower stress value (89.65MPa) than other materials. Because of the more strength, Forged steel (AISI 1045 steel) crankshaft can withstand load.

Thus, it can able to give maximum life than other selected materials. Due to the higher strength and considerable deformations, Forged steel can be used as the alternate and suitable crankshaft material for single cylinder engine.

The following conclusions can be drawn from the analysis conducted in this study:

1. The maximum static load analysis mainly focused in two crankshaft configuration, at dead center and at maximum twisting angle.

2. The maximum load occurs at the crank angle of 355 degrees for this specific engine. At this angle only bending load is applied to the crankshaft.
3. Weight can be reduced by changing the material of the current materials.
4. Value of von-mises stresses of analysis is less than the material yield stress so this design is safe.

6.2 RECOMMENDATION

The recommendations will be observed using two basic considerations:

I. To recommend that from the general analysis of crankshaft, the following terms are basic lack of this thesis, it used to for the feature work;

- Analysis due to materials fracture approaches
- Study the detail of crankshaft material properties
- Study detail the life prediction of crankshaft for improving fatigue life
- Detail study about the deformation of crankshaft
- Study vibration analysis of the crankshaft due to dynamic load
- Study the impact/ sudden load due to dynamic load
- Finding the effect of crankshaft on the engine block

II. From optimization view, the followings are basic;

- Optimize using material change or suitable material replacement
- Detail design optimization
- Optimize using composite materials
- Analysis of crankshaft can be done with other material which has high strength and low weight in future

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APPENDEX

MATLAB program used in dynamic analysis of the slider crank mechanism developed using equations

```
clc
clear
% measured weight of components
% piston                        330.87gr
% piston+pin                    417.63gr
% connecting-rod+bolts          283.35gr
% connecting-rod                244.89
% bolts                         38.50gr
% pin                           86.79gr
l1 = 36.98494e3;
l2 = 120.777e-3;
mcrank = 3.7191;
mrod = 283.35e-3;
I2 = 662523.4802e-9;
lg = 28.5827e-3;
mp = 417.63e-3;
load = xlsread('load.xls');

for theta_t = 1:145
   theta = (theta_t1)*5*pi/180;
   theta_d = 2000*2*pi/60;
   theta_dd = 0;
   beta(theta_t) = asin(l1sin(theta)/l2);
   beta_d(theta_t) = theta_d*l1*cos(theta)/l2/sqrt(1-
       (l1*sin(theta)/l2)^2);
   beta_dd(theta_t) = l1/l2*(theta_dd*cos(theta)-
     theta_d^2*sin(theta))/sqrt(1-(l1*sin(theta)/l2)^2) +
     theta_d^2*l1^2/l2^2*(cos(tta))^2*l1/l2*sin(theta)/((1-
     (l1*sin(theta)/l2)^2)^1.5);
   v_pis(theta_t) = -l1*theta_dsin(theta) -
       l1^2/l2*theta_d*sin(theta)*cos(theta)/sqrt(1-
```

```
        (l1/l2*sin(theta)^2));
    a_rod_x(theta_t) = -l1*theta_dd*sin(theta) -
        l1*theta_d^2*cos(theta) -
        theta_dd*lg*l1^2*sin(2*theta)/(l2^2*2*sqrt(1-
        (l1*sin(theta)/l2)^2)) -
        theta_d^2*lg*l1^2/l2^2*(2*cos(2*theta)*sqrt(1-
        (l1*sin(theta)/l2)^2) +185
        sin(2*theta)*l1^2/l2^2*sin(theta)*cos(theta)/sqrt(1-
        (l1*sin(theta)/l2)^2)) / (2*(1-(l1*sin(theta)/l2)^2));
    a_rod_y(theta_t) = l1*theta_dd*cos(theta) - l1*theta_d^2*sin(theta)
        - lg*l1/l2*theta_dd*cos(theta) + lg*l1/l2*theta_d^2*sin(theta);
    a_pis_x(theta_t) = -l1*theta_dd*sin(theta) -
        l1*theta_d^2*cos(theta) -
        l1^2/l2*theta_dd*sin(theta)*cos(theta)/sqrt(1-
        (l1*sin(theta)/l2)^2) -
        theta_d^2*l1^2/l2*(2*cos(2*theta)*sqrt(1-
        (l1*sin(theta)/l2)^2)+sin(2*theta)*l1^2/l2^2*sin(theta)*cos(thet
        a)/sqrt(1-(l1*sin(theta)/l2)^2))/(2*(1-(l1*sin(theta)/l2)^2));
    f_pis_x(theta_t) = mp*a_pis_x(theta_t) +
        load(theta_t,2)*1e5*pi*.089^2/4;
    f_a_x(theta_t) = mrod*a_rod_x(theta_t) + f_pis_x(theta_t);
    f_a_y(theta_t) = 1/l2*((I2*beta_dd(theta_t)-(f_a_x(theta_t)*lg +
        f_pis_x(theta_t)*(l2-
        lg))*sin(beta(theta_t)))/cos(beta(theta_t)))+1/l2*mrod*a_rod_y(t
        heta_t)*(l2-lg);
    f_local_x(theta_t) = f_a_x(thetacos(theta) +
        f_a_y(theta_t)*sin(theta);
    f_local_y(theta_t) = f_a_y(theta_t)*cos(theta) -
        f_a_x(theta_t)*sin(theta);
end
figure(2)
hold off
plot(load(:,1),f_local_x/1000,'g--')
hold on
```

```
plot(load(:,1),f_local_y/1000,'-.')
plot(load(:,1),sqrt(f_local_y.^2+f_local_x.^2)/1000,'r-')
grid
title('Force Between Piston and Connecting rod @ 2000 rpm')
xlabel('Crankshaft Angle (Degree)')
ylabel('Force (kN)')
Legend ('Axial', 'Normal', 'Magnitude')
```