JIMMA UNIVERSITY

JIMMA INSTITUTE OF TECHNOLOGY

FACULTY OF ELECTRICAL AND COMPUTER ENGINEERING

(COMMUNICATION ENGINEERING STREAM)

Performance Analysis of Low- Complexity Precoding Scheme In MU - Massive MIMO for 5G and Beyond Networks

A thesis report submitted to the school of graduate studies of Jimma University in partial fulfillment of the requirements for the degree of Masters of Science in Communication Engineering

By:

Amanuel Admassu

JIMMA, ETHIOPIA

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Jimma,Ethiopia
January 2022

Dedication

I dedicate this thesis to my very cute, almost one and half year old, son - **Nazrawi**. May he prosper in life!

Declaration

I, the undersigned, declare that this thesis is my original work, has not been presented for a degree in this or any other universities, and all sources of materials used for the thesis have been fully acknowledged.

Research Thesis Submitted by:

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Abstract

Maximum ratio combining (MRC), maximum ratio transmission (MRT) and zero-forcing (ZF) are well known precoders which have been researched out in recent studies. In this thesis, the performances of maximum ratio combining (MRC) and zero forcing (ZF) at uplink and maximum ratio transmission (MRT) and zero forcing (ZF) at downlink MU - Massive MIMO system have been analyzed and compared. This work focuses on a single-cell with multiple base station antenna serving multiple user equipments. The achievable sumrate and all the required metrics in uplink and downlink system using low complex linear precoding scheme under different scenario have been derived .

In this work two regimes are considered for SNR and it has been mathematically proved and simulated using Matlab2018. These two regimes are high SNR and low SNR. Also low complex linear precoders have been proposed. The proposed idea is to predict and use the precoder which results better performance in terms of achievable rate, spectral efficiency and energy efficiency for a given channel. In the downlink scenario we applied normalization method for equal allocation of power for both MRT and ZF precoders.

For low SNR, MRC and MRT perform better than zero forcing. For high SNR, zero forcing (ZF) performs better than MRC and MRT. From the result executed MRT have better performance than ZF when the number of user is lesser under vector normalization. Similarly, under low SNR, ZF precoders have lesser performance than MRT precoding scheme in matrix normalization.

Generally , the performance of the precoders will begin to grow up as the number of BS antenna is folded. In uplink case an achievable rate for zero forcing and MRC grows by 31.12 % and 33.4 % respectively for low power. Similarly, achievable rate for zero forcing and MRC grows by 11.48 % and 36.4 % respectively for high uplink power.In downlink case an achievable rate under vector normalization for zero forcing and MRT grows by 28.8 % and 27.1 % respectively for low BS transmitter power.Similarly, achievable rate for zero forcing and MRT grows by 11.8 % and 7.32 % respectively for high BS transmitter power.

Key words:***Achievable sum-rate, MRC ,MRT , MU-Massive MIMO, ZF***

— JiT School of Graduate Studies

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— ***Amanuel Admassu***

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Abbreviations

1G	**F**irst **G**eneration
2G	**S**econd **G**eneration
3G	**T**hird **G**eneration
4G	**F**ourth **G**eneration
5G	**F**ifth **G**eneration
6G	**S**ixth **G**eneration
AKA	**A**uthentication and **K**ey **A**greement
AMPS	**A**dvanced **M**obile **P**hone **S**ystems
BER	**B**it **E**rror **R**ate
BS	**B**ase **S**tation
CB	**C**onjugate Beamforming
CDMA	**C**ode **D**ivision **M**ultiple **A**ccess
CIR	**C**hannel **I**mpulse **R**esponse
CSI	**C**hannel **S**tate **I**nformation
DL	**D**ownlink
DPP	**D**irty **P**aper **P**recoding
EDGE	**E**nhanced **D**ata **G**SM **E**volution
FDMA	**F**requency **D**ivision **M**ultiple **A**ccess
Gbps	**G**iga **B**it **p**er **S**econd
GHz	**G**iga **H**ertz
GPRS	**G**eneral **P**acket **R**adio **S**ervice
GSM	**G**lobal **S**ystem for **M**obile **c**ommunication
HSDPA	**H**igh-**S**peed **D**ownlink **P**acket **A**ccess
HSPA	**H**igh-**S**peed **P**acket **A**ccess
HSUPA	**H**igh-**S**peed **U**plink **P**acket **A**ccess

IEEE	**I**nstitute of **E**lectrical and **E**lectronics **E**ngineers
Iid	**I**ndependent and **I**dentically Distribution
IoT	**I**nternet of **T**hings
JIT	**J**imma **I**nstitute of **T**echnology
Kbps	**K**ilo **B**it **P**er **S**econd
LTE	**L**ong **T**erm **E**volution
LTE-A	**L**TE-**A**dvanced
M2M	**M**achine-to-**m**achine
Ma-MIMO	**M**assive MIMO
MATLAB	**M**athematics **L**aboratory
Mbps	**M**ega bit per **s**econd
MF	**M**atching **F**ilters
MHz	**M**ega **H**ertz
MIMO	**M**ultiple **I**nput **M**ultiple **O**utput
MMC	**M**ultimedia **M**essage Support
MMSE	**M**inimum Mean **S**quare **E**rror
mmWave	**M**illimeter **w**ave
MRC	**M**aximum **R**atio **C**ombining
MRT	**M**aximum **R**atio **T**ransmission
MS	**M**obile **S**tation
MU-MIMO	**M**ulti **U**ser - MIMO
PZF	**P**hased-**Z**F
QoS	**Q**uality - of - **S**ervice
SDMA	**S**pace **D**ivision **M**ultiple **A**ccess
SINR	**S**ignal-to-**I**nterference plus **N**oise **R**atio
SMS	**S**hort **M**essage **S**ervice
SNR	**S**ignal-to-**N**oise **R**atio
SPA	**S**mart Pilot Assignment
SU-MIMO	**S**ingle **U**ser MIMO
TACS	**T**otal **A**ccess **C**ommunication **S**ystem
TDMA	**T**ime **D**ivision **M**ultiple **A**ccess
TH	**T**omlinson **H**arashima

THz	**T**era **H**ertz
UL	**U**plink
UMTS	**U**niversal **M**obile **T**elecommunication **S**ystem
VP	**V**ector **P**erturbation
WiMAX	**W**orldwide **I**nteroperability for **M**icrowave **A**ccess
ZF	**Z**ero **F**orcing

Notations and Symbols

$\|.\|$	$Absolute\, Value$
$\|.\|$	$Norm\, of\, Vector$
$(.)^{H}$	$Hermitian\, Transpose$
$(.)^{*}$	$Complex\, Conjugate\, Operator$
$(.)^{-1}$	$Inverse\, of\, matrix$
$(.)^{T}$	$Transpose\, of\, matrix$
$[.]^{UL}$	$Corresponds\, to\, uplink\, session$
$[.]^{ULH}$	$Corresponds\, to\, uplink\, session\, at\, high\, SNR$
$[.]^{ULL}$	$Corresponds\, to\, uplink\, session\, at\, low\, SNR$
$\mathbf{max}\{.\}$	$Maximum\, Value\, of\, a\, set$
C	$Set\, of\, complex\, number$
$\mathbf{E}\,[.]$	$Mean\, of\, the\, random\, variable$
$\mathbf{I_N}$	$Identity\, matrix\, with\, Size\, \mathbf{N}$
$\mathbf{R}$	$Set\, of\, real\, numbers$
$\mathbf{S}$	$Symbol\, Vectors$
$\mathbf{w_K^{MRC}}$	$MRC\, precoding\, employed\, by\, the\, base\, station$
$\mathbf{w_K^{ZF}}$	$ZF\, precoding\, employed\, by\, the\, base\, station$
$\mathbf{w_K^{MRT}}$	$MRT\, precoding\, employed\, by\, the\, base\, station$

Chapter 1

Introduction

1.1 Background

The rapid increase in applications such as high definition video streaming, multimedia applications and web browsing, broadband cellular, social media, etc. are providing exciting opportunities for both consumers and service providers[3]. These applications are highly data intensive and resource hungry giving rise to new kind of challenges in bandwidth delivery for mobile service operators. Multipath fading causes a random fluctuation in the received signal power in wireless communication. This random fluctuation in signal level is known as fading [44], which affects the quality and reliability of wireless communication. Therefore, the need for high data rate and high reliability is extremely challenging. One of the key technologies [13] to increase network throughput by spatially exploiting more spectrum is achieved with MIMO technology in which frequency resources are being simultaneously used to increasing demand of achievable data rates.

Massive multi-input multi-output (MIMO) is one of the key technologies in 5G [11], which can greatly boost the channel capacity, spectral efficiency, and connection density by utilizing a large number of antennas at the base station (BS). Massive MIMO equipped with a large number of antennas at the base station can communicate with multiple users simultaneously. Simultaneous communication with multiple users creates multi-user interference and degrades the throughput performance[5]. However, in practical systems, this tremendous multiplexing gain can only be provided for large signal-to-interference plus noise ratios (SINR) and for uncorrelated transmit and receive antenna arrays at both communication sides [12]. Due to space limitations, mobile designers now embed more and more antennas in small devices, which inevitably spawns non negligible correlation patterns at the antenna arrays and thus non-negligible effects on the achievable transmission rates, so the effort to exploit the spatial multiplexing gain has been shifted from Massive MIMO to multiuser massive MIMO (MU-Massive MIMO), where several users are simultaneously[22] served by a multiple-antenna base station .

With MU-Massive MIMO setups, a spatial multiplexing gain can be achieved even if each user has a single antenna. This is important since users cannot support many antennas due to the small physical size and low-cost requirements of the terminals, whereas the BS can support many antennas and have high energy efficiency, In the uplink MU-Massive MIMO, coherent combining can achieve a very high array gain which allows for substantial reduction in the transmit power of each user. In the downlink, the BS can focus the energy into the spatial directions where the terminals are located.

The purpose of MU massive MIMO systems is to exploit the spatial dimension to ensure a beam forming of the signal in the direction of the concerned user so that each user can ideally benefit from the whole allowed bandwidth at all-time [9]. This is done by a precoding of the information at the base station (BS). Precoding aims[1] at distributing users' data on the different antennas of the BS in order to perform beam forming of information toward the users. The BS computes the precoding matrix after estimating the channel impulse response (CIR) in a way to decrease the interfering part or to direct the useful energy in the direction of each user.

Precoding methods are applied during the downlink and uplink to reduce the effect of multi-user interference. In this thesis three linear precoding technique have been considered, maximum-ratio combining (MRC), maximum-ratio transmitter (MRT) and zero-forcing (ZF).

1.2 Motivation

Although massive MIMO technology is more than just an extension of MIMO technology and provides immense benefits, there are still many issues and challenges that need to be addressed. Those challenges such as pilot contamination, channel estimation, precoding, user scheduling, hardware impairments, energy efficiency and signal detection needs to be addressed and tested in a real-world environment before we can achieve its promised advantages. As it has been analyzed as future challenges in Massive MIMO, here we are motivated to conduct a study in precoding scheme.

1.3 Problem Statement

The potential of massive MIMO technology has drawn an interest among both, academicals and industrial community, given the promising improvements. These new degrees of freedom can offer when properly extended. In particular, the usage of large antenna arrays at the BS, known as massive MIMO has revealed even more notorious achievements in throughput, energy and spectral efficiency [1] with very simple linear processing techniques. We can see that data rate can grow unbounded with the number of antennas. That is why it is at the forefront of the today's research in the broadband area.

In downlink channel (broadcast channel) for massive MIMO system; the problem of Multi user Interference (MUI) has received widespread attention. MUI is an interference result from other user in the same cell when more user access to wireless link, which leads to reduce the achievable sum rate for cellular communication system.

Although the precoding techniques increases throughput and reduce interference, it increases the computational complexity of the overall system by adding extra computations.Even if linear and non-linear precoders have been proposed for massive MIMO systems, the non-linear precoders like [1] Dirty Paper Precoding (DPP), Tomlinson Harashima precoding (TH), and Vector Perturbation (VP) provide better performance, but these methods have very high computational complexity when we have large antenna system . Therefore, to achieve near-optimal performance, we should investigate the precoders having lower computational complexity to increase the performance for cellular communication system.

1.4 Objective of the thesis

1.4.1 General Objective

The main objectives of this study is to analyze the performance of the low complexity linear precoding scheme for MU- Massive MIMO system in both uplink and downlink system.

1.4.2 Specific Objective

The specific objectives of this research are:

- To evaluate the performance of MRC and ZF at different SNR values in Uplink.
- To evaluate the performance of MRT and ZF at different SNR values in downlink.
- To compare matrix and vector normalization in downlink
- To evaluate and investigate the precoder with better performance.

1.5 Methodology

This paper demonstrate the concept of using the three linear precoding schemes MRC,ZF and MRT for a multiuser uplink and downlink massive MIMO system. The performance of those precoders are analyzed and evaluated by varying SNR value in both uplink and downlink transmission.Vector and matrix normalization methods have been applied in the downlink under the assumption of base station with perfect channel state information . Channel matrix are modeled as independent complex Gaussian random variables with zero mean and unit variance.

The idea of this study originated from a need to mitigate the interference in massive MIMO systems. A scenario of multi-user massive MIMO, where the uplink transmits power is assumed to be equal for all users is considered. We consider the linear precoding techniques at the base station and study the performance of each linear precoding with different propagation environments, and different SNR.

As it has been quoted in the section (1.4), the goals of this thesis are to analyze the effects in MU-Massive MIMO uplink and downlink system with linear precoding and different channel models. To reach to our targets; firstly, we derive the optimal linear precoding through mathematical analysis. Moreover, we simulate the channel models with computer software.

The flow chart in Figure 1.1 Figure 1.2 represents the manner of applying the Maximum ratio combining, zero forcing and maximum ratio transmission linear precoding techniques for multiuser massive MIMO system.Different number of base station antennas and number of user at two different SNR value for uplink , 10 dB and -30 dBm have been simulated and analyzed the result . 0 dBm and -15 dBm are the two different SNR value for which matrix and vector normalization methods in downlink are simulated and analyzed.

1.6 Thesis Contribution

This research consists of two parts. Firstly, it gives the general overview of massive MIMO system. The performance of massive MIMO systems is analyzed in terms of achievable sum rate, spectrum efficiency and energy efficiency. Secondly, it study and compare the performance of low complexity linear precoders at different value of power ,number of user and antennas to select optimum linear precoding technique for mitigation of interference.Generally this thesis contributes a research output and opens a new research idea in finding out the challenges in the 5G and future networks.

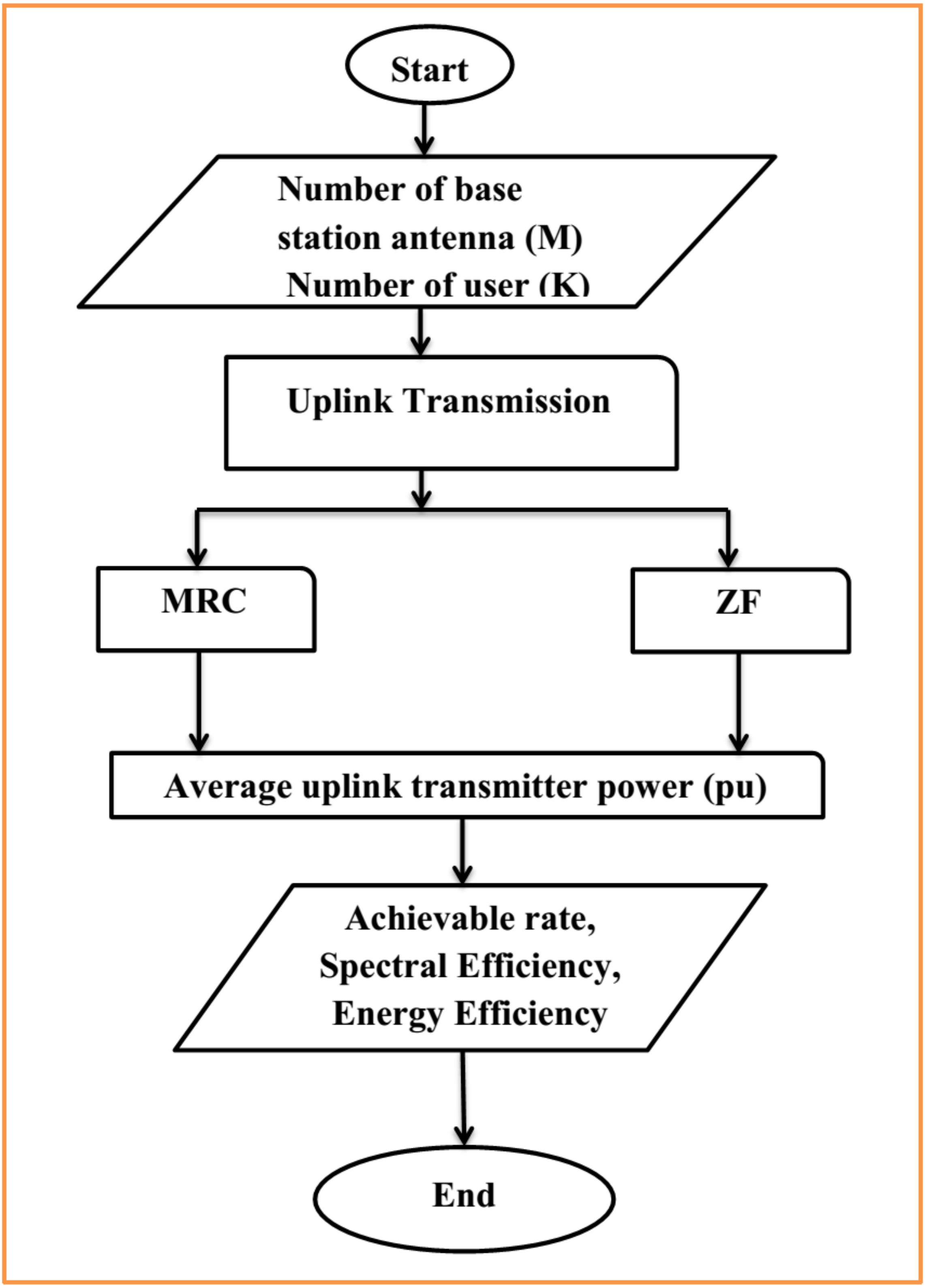

FIGURE 1.1: Methodology flow chart at uplink

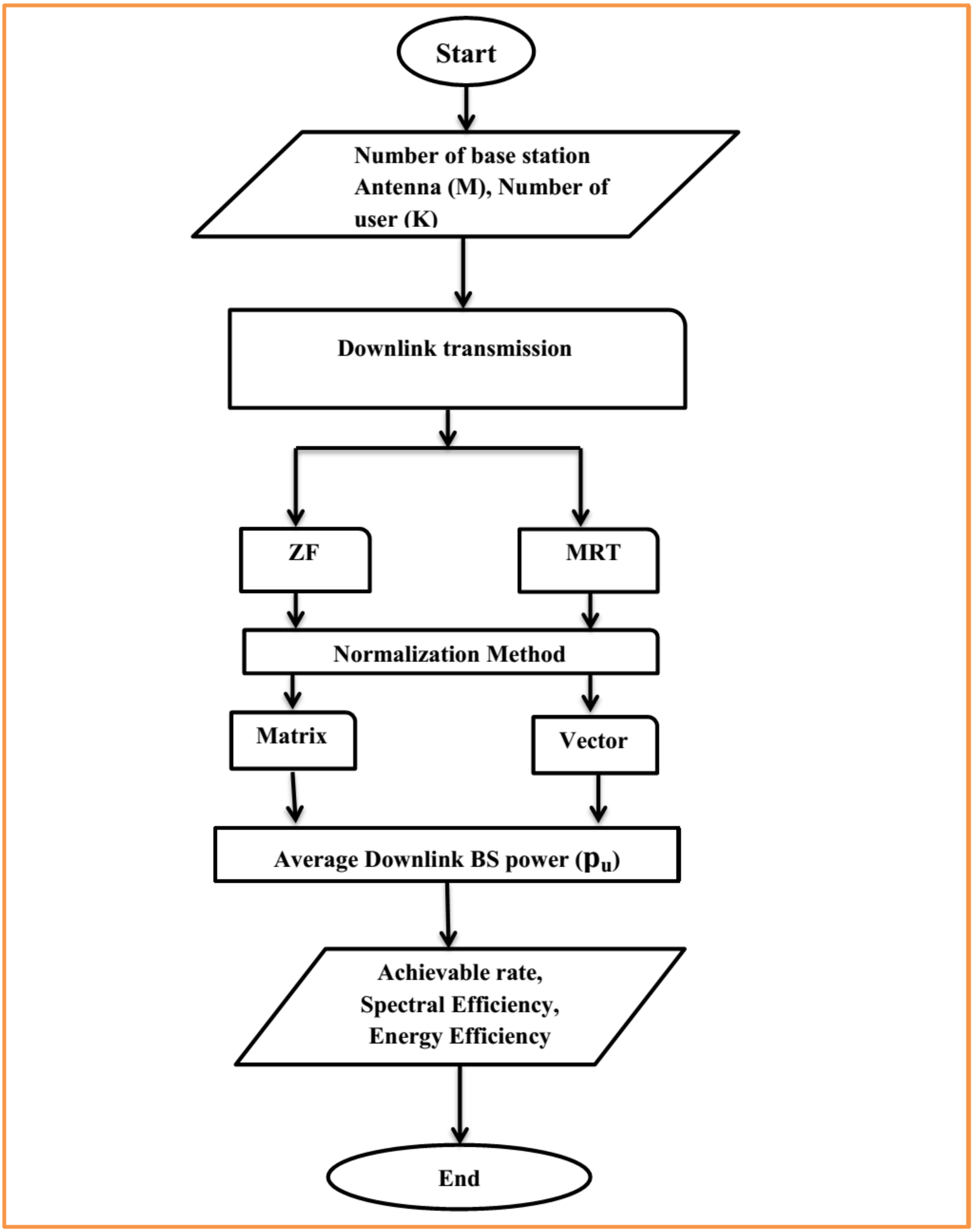

FIGURE 1.2: Methodology flow chart at downlink

1.7 Organization

This thesis work is organized into six chapters. Chapter one is an introductory section. Chapter two talks about related work by researchers and conceptual frame work.

In chapter three, overview of massive MIMO system has been discussed. Chapter four briefly discusses about system model for MU-massive MIMO and performance analysis. Chapter five is all about simulation and result discussion. Chapter six summarizes the overall view of the paper and recommends new research ideas for the future.

Chapter 2

Literature Review

2.1 Related Work

Many linear and non-linear precoders have been proposed for massive MIMO systems. Although the non-linear precoders like Tomlinson Harashima precoding (TH) [14, 15], and Vector Perturbation (VP) [16] provide better performance, these methods have very high computational complexity when we have large antenna system. The linear precoders such as Maximal Ratio Combining (MRC) [21], Zero-Forcing (ZF) [17] have lower computational complexity and can achieve near-optimal performance.Many researches focused on studying MU massive MIMO chain considering a Rayleigh channel [1, 7, 8, 9, 24, 26,].

Our work developed throughout the thesis has resulted from the following literatures:

Robin C et al. provided an extensive overview of massive MIMO systems, highlighting the key enabling technologies for 5G and beyond networks. Although massive MIMO offers immense benefits for 5G and 6G networks, the authors in [1] outlined that there are still various deployment challenges such as pilot contamination, channel estimation, precoding, user scheduling, hardware impairments, energy efficiency, and signal detection that needs to be addressed before achieving its promised advantages.

Daniel C et al.presented an overview of the basic concepts of massive multiple-input multiple-output with a focus on the challenges and opportunities, based on contemporary research.

Muaayed F et al .examined the performance of massive MIMO uplink system over Rician fading channel. The performance is estimated regarding spectral efficiency versus number of base station antennas utilizing three plans of linear detection, maximum-ratio-combining (MRC), zero forcing receiver (ZF), and minimum mean-square error receiver (MMSE). In their paper, they estimated the performance by drawing the spectral efficiency versus the number of BS antennas using MRC, ZF, and MMSE.

Mariam E et al. aimed at evaluating the performance of MU massive MIMO systems in terms of BER for the case of conjugate beam forming (CB) and zero forcing (ZF) linear precoding over a realistic 5G mmWave propagation using a statistical model of the channel. The authors in [9] focuses on the case of a downlink single-cell scenario with linear precoding as zero forcing (ZF) and conjugate beam forming (CB).

Le Liang et al. proposed a low-complexity hybrid precoding scheme to approach the performance of the traditional baseband zero-forcing (ZF) precoding (referred to as full-complexity ZF), which is considered a virtually optimal linear precoding scheme in massive MIMO systems. The proposed hybrid precoding scheme, named phased-ZF (PZF), essentially applies phase-only control at the RF domain and then performs a low-dimensional baseband ZF precoding based on the effective channel seen from baseband.

Yucheng Wu et al. did their study on Pilot contamination reduction in massive MIMO systems based on pilot scheduling . This paper proposes a pilot scheduling scheme based on the degradation to address this problem. Through computing the degradation of the users, the proposed scheduling assigns the optimal pilot sequence to the user who suffers from the greatest degradation in a greedy way. Moreover, the proposed scheme is further optimized with an extra set of orthogonal pilot sequences, which is called pilot scheduling scheme based on user grouping. Simulation results show that the target cell's achievable sum rate of the proposed scheme is much higher than the random pilot scheduling (RPS) and the smart pilot assignment (SPA) schemes.

Sinan A.et al. presented the background and advantages of future massive MIMO systems. The investigation indicates good performance due to the employed methods for the downlink channel such as user grouping and group-based feedback schemes. Furthermore, the adoption of the pattern / polarization antenna array model for massive MIMO shows an increase in the degrees of freedom of MIMO channels and thus an improved channel capacity.

Hien Q et al. studied the potential for power savings of the Massive MIMO uplink with maximum-ratio combining (MRC), zero-forcing, and minimum mean-square error receivers, under perfect and imperfect channels. Again the authors in [7], considers a physical channel model where the angular domain is divided into a finite number of distinct directions. A lower bound on the capacity is derived, and the effect of pilot contamination in this finite-dimensional channel model is analyzed. Some aspects of favorable propagation in Massive MIMO under Rayleigh fading and line-of-sight (LoS) channels have been investigated in [7].

Sammaiah T et al. analyzed the performance of Linear Precoding in Downlink Based on Polynomial Expansion on massive MIMO systems. This [40],expresses the performance of achievable sum rate linear precoding with variable signal to noise (SNR) ratio and achievable sum rate and several transmitter receiver antennas, such as imperfect CSI, fewer complex processing and inter user interference.

Colon D et al. analyzed Linear Precoding in Multi-User Massive MIMO Systems with imperfect channel state information .The author studied the performance of massive multi-user multiple-input multiple-output (MaMU MIMO) downlink systems. Several linear precoding techniques, assuming perfect channel state information at the transmitter (CSIT) side, are considered and implemented in order to evaluate their performance. They put special emphasis considering imperfect channel information and its effects over the system performance. The robustness of linear precoding techniques against channel estimation errors and hardware impairments is also evaluated.In [12] the impact of the impairments on the system bit error rate (BER) and achievable rate is investigated. Preliminary results show that linear precoders can be employed in massive MIMO systems with promising results. Particularly, the matched filter (MF) precoder, which has the lowest implementation complexity,allows to obtain good results when a large number of antennas are placed at the base station while maintaining constant power.

Taek K et al.compared the channel capacity with ZF and MMSE when the relationship between the number of transmitting antennas and the number of receiving antenna changes.In [33] ,the author applied the two types of precoding to multiple antennas in MU-MIMO downlink transmissions and the author find that their impact on capacity is different, and which one provides higher capacity is dependent on the antenna numbers. The simulation results shows the impact on capacity with different combinations of transmitting and receiving antennas. In simulation results ,[33] demonstrates that the proposed structure can achieve the capacity approaching performance of the precoding in different antenna environments. They assurned that a network has an environment that the number of receivers is larger than that of transmitters

Emmanuel M et al.analyzed and compared the regularized zero forcing (RZF) precoder, Sherman-Morrison-Woodbury (SMW)-based precoder, rapid numerical algorithms (RNA) based precoder and the truncated polynomial expansion (TPE) based precoder are compared and analyzed for massive multiple input multiple output (MIMO) wireless system in a downlink scenario. In their work, the achievable data rates and signal to noise ratio (SNR) were investigated. Also they analyzed that RZF precoder, RNA-based precoder and the SMW based precoder perform better in comparison to TPE-based precoder under good CSI quality and low ratio of transmit to receive antennas.

Andreas F et al. provided a comprehensive survey Hybrid Beamforming for Massive MIMO. The author provided a taxonomy in terms of the required channel state information (CSI), namely whether the processing adapts to the instantaneous or the average (second-order) CSI; while the former provides somewhat better signal-to-noise and interference ratio, the latter has much lower overhead for CSI acquisition.They furthermore distinguished hardware structures of different complexities and finally they point out the special design aspects for operation at millimeter wave frequencies.

Tianyang B et al. proposed a stochastic geometry framework to analyze the SINR and rate performance in a large scale uplink massive MIMO network. Based on the model, expressions are derived for spatial average SINR distributions over user and base station distributions with maximum ratio combining (MRC) and zero-forcing

(ZF) receivers. They showed that using massive MIMO, the uplink SINR in certain urban marco-cell scenarios is limited by interference. The results reveals that for MRC receivers, a super-linear (polynomial) scaling law between the numbers of base station antennas and scheduled users per cell preserves the uplink SIR distribution, while a linear scaling applies to ZF receivers. ZF receivers are shown to outperform MRC receivers in the SIR coverage and the performance gap is quantified in terms of the difference in the number of antennas to achieve the same SIR distribution.

In this thesis, we follow up the discussions on massive MIMO systems proposed in [1] and recommendation of an essential area of research for future by [1]. It is done by adding new topics that have gained attention recently in the research community such as low complexity precoding scheme in both uplink and downlink system.

Thus, it is more practical to use low complex and efficient precoders in massive MIMO.The efficient precoding technique for massive MIMO can be found out through investigations.To exmine and find out relative better performing precoders under a given antenna and user configuration,we derived simple sumrate formulas for linear precoders in both uplink and downlink scenario.Again the SNR value by which MRC and MRT should be used instead of ZF have been analytically derived.

Although the above works provide good results about the performances of the linear precoding schemes, they did not provide the comparison of MRC,ZF and MRT performances in terms of spectral efficiency,achievable rate and energy efficiency for the given transmit power at the same time, under the same conditions and in a single-cell scenario.They did not consider both uplink and downlink scenario at the same time.

In general Massive MIMO systems in multi-cell environments have been studied in [43], [44], [45], [46]. These includes pilot contamination, which becomes, in time division duplex (TDD) systems, the main capacity-limiting factor, especially when MRT is used. Where as in our our work ,we studied single-cell environment in MU-massive MIMO to analyze one of the challenge precoding .Again the authors in [26] investigated downlink performance with MRT and ZF precoder for a massive MIMO system.But they did not paid attention to normalization technique and we were unable to classify which normalization method was better.

Chapter 3

Overview of Ma-MIMO System

3.1 Massive - MIMO Concepts

Massive MIMO is the most captivating technology for 5G and beyond wireless network access era. The MIMO technologies associated with 4G/LTE network is unable to handle this huge influx in data traffic with more speed and reliability [1]. Massive MIMO is the advancement of contemporary MIMO systems used in current wireless networks, which groups together hundreds and even thousands of antennas at the base station and serves tens of users simultaneously [38, 39] and offers an immense advantage over the traditional MIMO system.

Thus, the $5G$ network is considering massive MIMO technology as a potential technology to overcome the problem created by massive data traffic and users [36, 40]. Several studies have been conducted on massive MIMO systems and their benefits [37, 41]. Massive MIMO downlink and the uplink system is shown in Figure 3.1.

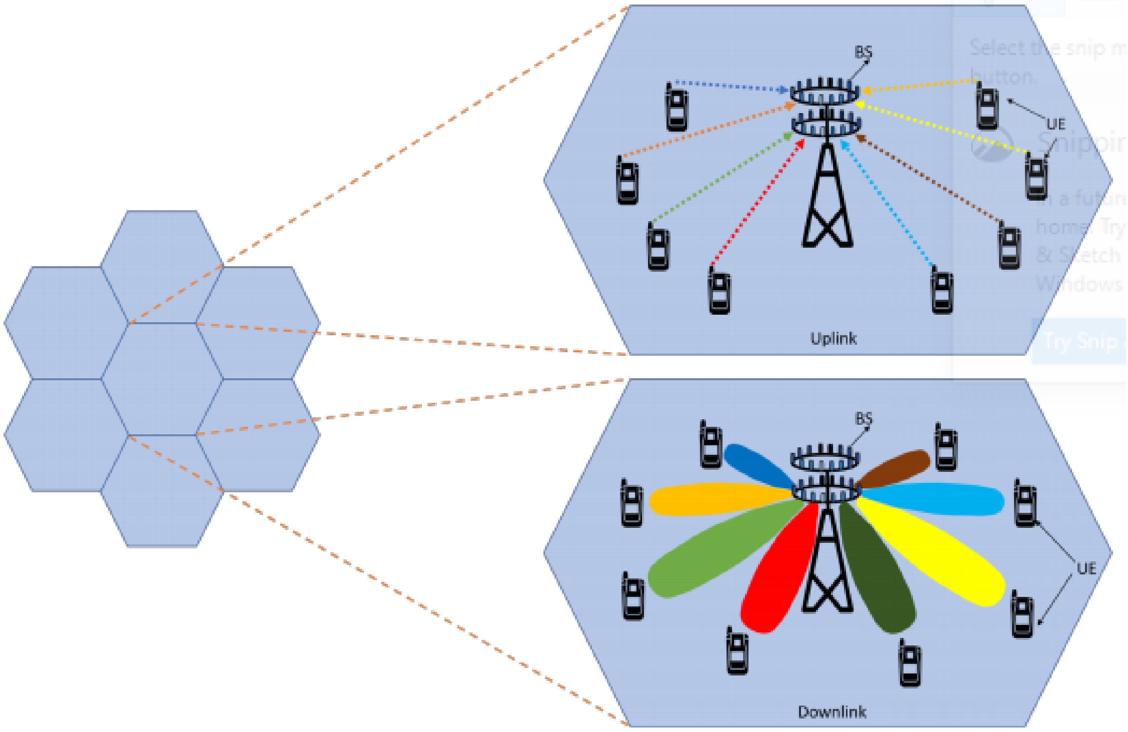

FIGURE 3.1: Massive MIMO uplink and downlink [1].

3.2 Massive MIMO Types

3.2.1 Single-User MIMO[10]

In SU-MIMO transmission only one user is served on a given time-frequency resource within a cell, possibly over multiple streams. With the simplifying assumption that out of cell interference is treated as additional Gaussian noise by the base station and users, the channel model reduces to a (distributed) point -to-point MIMO channel [32]. The typical SU-MIMO channel is shown as follows in Figure 3.2; where UT is the user terminal.

3.2.2 Multi-User Massive MIMO with Single-cell Scenario[10]

In this section, we take into consideration a single-cell MU-MIMO systems, where the BS is serving K UEs with every terminal being equipped with one antenna [10].With MU-MIMO, multiple users are served in parallel over a given time-frequency resource by means of spatial multiplexing. While in SU-MIMO the multiplexing gain is limited by the minimum of the number of transmit and receive antennas, in MU-MIMO the

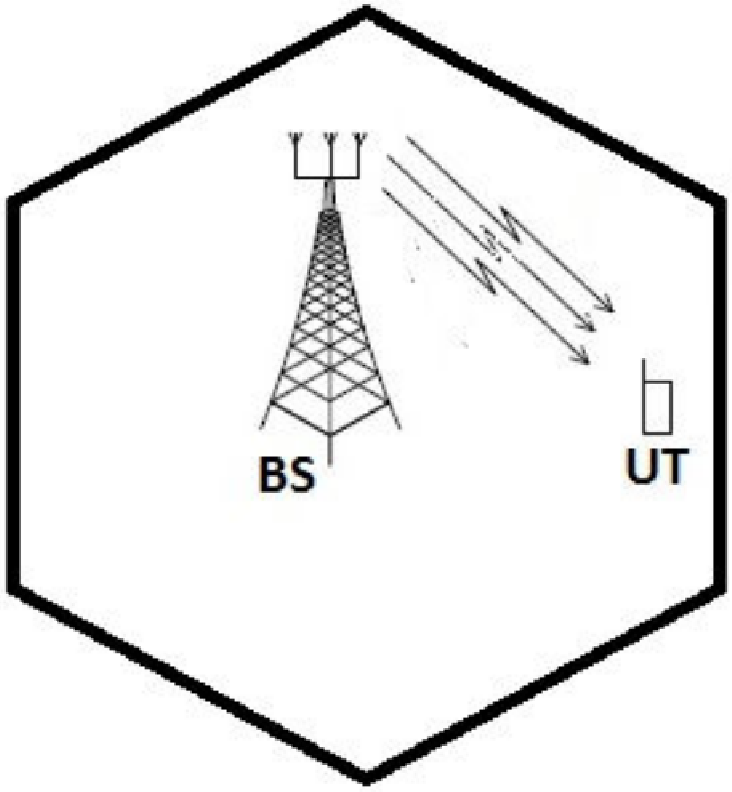

FIGURE 3.2: SU- MIMO system [10].

multiplexing gain scales with the number of transmit antennas, provided there are enough users in the cell [32]. Although multiple streams per user are possible in MU-MIMO, it has been shown that single stream transmission per user is asymptotically optimal in the number of user as shown in Figure 3.3 and that for finite number of users mostly only one stream is activated per selected user.

3.2.3 Multi-User Massive MIMO with Multi-cell Scenario[10]

In this section, we contemplate the restriction of non-cooperative cellular multiuser MIMO systems as M grows without limit[10]. For a single cell, as properly as for multi-cell MIMO, the outcome impact of letting M increase without limits is that thermal noise and small-scale Rayleigh fading vanishes.

3.3 Benefits of Massive MIMO for 5G Networks and Beyond

Some of the benefits of massive MIMO technology are [1]:

- **Spectral Efficiency**: Massive MIMO provides higher spectral efficiency by allowing its antenna array to focus narrow beams towards a user. Spectral efficiency more than ten times better than the current MIMO system used for 4G/LTE can be achieved.
- **Energy Efficiency**: As antenna array is focused in a small specific section, it requires less radiated power and reduces the energy requirement in massive MIMO systems.
- **High Data Rate**: The array gain and spatial multiplexing provided by massive MIMO increases the data rate and capacity of wireless systems.
- **Low Power Consumption**: Massive MIMO is built with ultra-lower power linear amplifiers, which eliminates the use of bulky electronic equipment in the system. This power consumption can be considerably reduced.

3.4 Challenges in Massive MIMO for 5G Networks and Beyond

The massive MIMO technology is more than just an extension of MIMO technology, and to make it a reality, there are still many issues and challenges that need to be addressed. Some of the fundamental challenges in massive MIMO systems are [1]: pilot contamination, channel estimation, precoding, user scheduling, hardware impairments, energy efficiency and signal detection.

3.5 Principle of Massive MIMO System

When more than one terminal are allowed to access identical time–frequency resource, MU-MIMO offers greater system efficiency in contrast to SU-MIMO. A MU-MIMO enables each terminal to use all available spectrum resources, improving the throughput without the need for additional (expensive) resources[38],[40]. The hardware cost involved with MU-MIMO is the need to place additional BS antennas at the locations where we wish to transmit/receive the signal. Thus, the available spatial degrees of freedom at the BS are limited by the number of antennas.

In general term a MU-Massive MIMO system refers to the system where the base station communicates with several users simultaneously. The base station and the user can be equipped with multiple antennas. The MU-Massive MIMO system enables many communications in the same time and frequency resource that called Space Division Multiple Access (SDMA) [39].

The MU- MIMO system gives the high performance in terms of the spectral efficiency and the energy efficiency [39].With the Massive MIMO technology emerging in 5G mobile communication systems, hundreds of antennas have to be deployed on the BS tower or the surface of a building. However, surfaces used for deploying massive MIMO antennas are not ideally smooth planes in most of the real scenarios. We consider a MU-Massive MIMO uplink and downlink channel where M antennas are located at the base station, and N antennas are located at the each mobile station $(MS), i = 1, 2, \cdots, K$. There are K users (MS) in the system. The total number of receive antennas is given by:

$$N_{RX} = \sum_{l=1}^{K} N_l \tag{3.1}$$

The block diagram of the system model for MU-MIMO illustrated as shown in Figure 3.3.

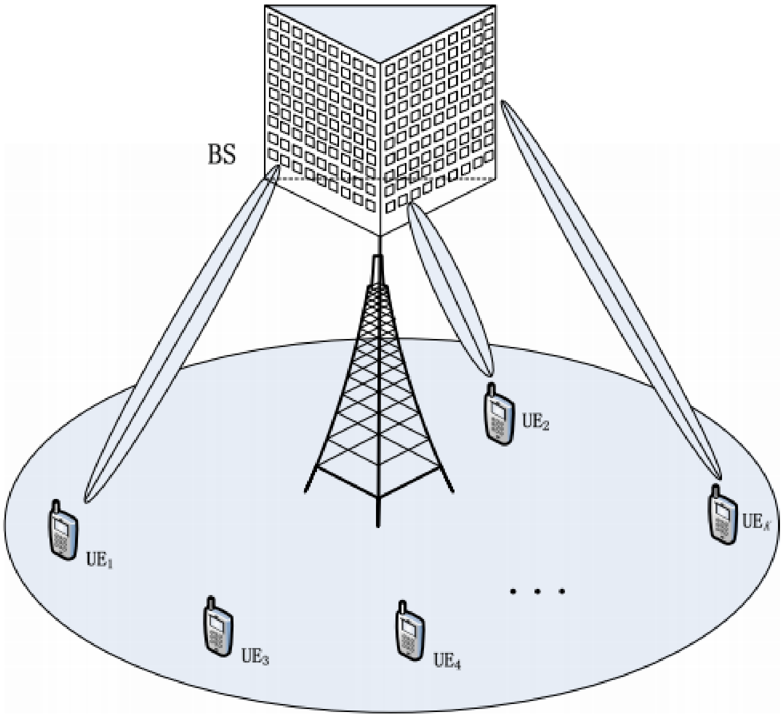

FIGURE 3.3: MU-Massive MIMO system equipped with base station (M_t antennas) serving K users (N_k receive antenna) [31].

3.6 Precoding

Precoding is a concept of beam forming which supports the multi-stream transmission in multi-antenna systems. Precoding plays an imperative role in massive MIMO systems as it can mitigate the effect created by path loss and interference, and maximizes the throughput. In massive MIMO systems, the base station estimates the CSI with the help of uplink pilot signals or feedback sent by the user terminal.

The received CSI at the base station is not uncontrollable and not perfect due to several environmental factors on the wireless channel [30]. Although the base station does not receive perfect CSI, still the downlink performance of the base station largely depends upon the estimated CSI.

Thus, the base station uses the estimated CSI and the precoding technique to reduce the interference and achieve gains in spectral efficiency. The performance of downlink massive MIMO depends upon the accurate estimation of CSI and the precoding technique employed. Figure 3.4 shows the precoding in massive MIMO systems with M - antenna base station and K-users.

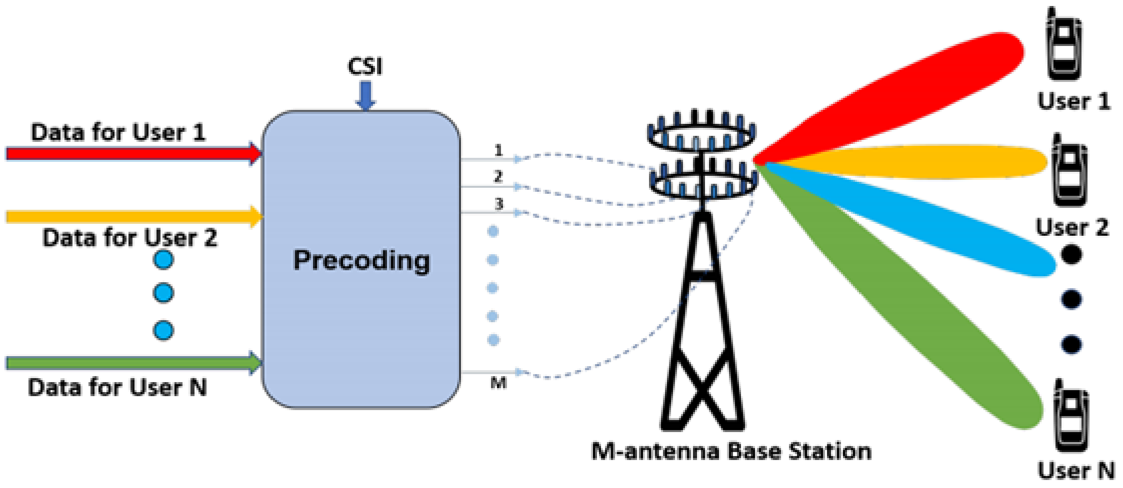

FIGURE 3.4: Precoding in Massive MIMO system with M antennas at base station with K users[1]

Although the precoding technique provides immense benefits to massive MIMO systems, it also increases the computational complexity of the overall system by adding extra computations.The computational complexity increases along with the number of antennas [1]. Figure 3.5 is the classification of precoders.Non-linear precoders like

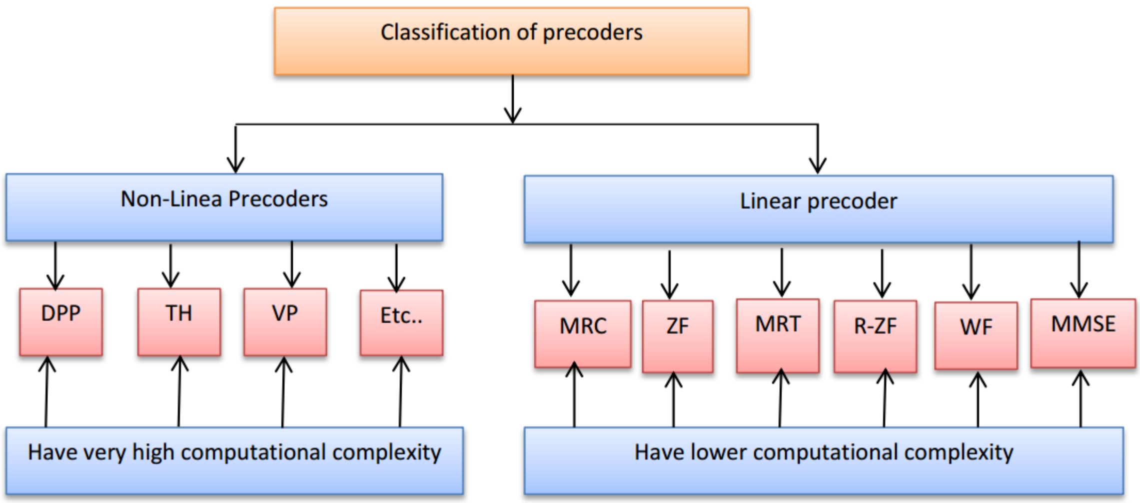

FIGURE 3.5: Classification of precoding schemes

DPP,TH,VP have very high computional complexity [1] where as linear precoders like MRC,ZF,MRT,R-ZF,WF and MMSE precoders have low computational complexity[1]. Here in this study due to its conventional complexity, we consider the linear precoding techniques, which include MRC, ZF, and MRT precoding.

Chapter 4

System Model For MU-MaMIMO

4.1 Introduction

In this section, the system model for a MU-massive MIMO system is introduced. Also in this section, the performance in terms of achievable rate, spectral efficiency and energy efficiency for different linear precoding techniques have been studied. The centralized massive MIMO system as exhibited in Figure 3.3, in which the BS is deployed M transmit antennas and there are K users being randomly distributed in the circular-shape cell is considered.

As its indicated in Figure 3.3 , the M transmitter serves to K single-antenna users at the same time frequency resources and we consider that the number of antennas is far greater than the number of users satisfying the limited condition $\mathbf{K} << \mathbf{M}$. Before all users receive the transmitted data, the BS shall use some simple linear precoding techniques pre-process, which realizes the signal term maximization and interfere term minimization as much as possible.

4.2 System Model

4.2.1 MU-Massive MIMO Uplink System

The MIMO consists of many multiple-antenna transmitters sending to a single multiple-antenna receiver, where the K users transmit signals to the BS. Uplink (or reverse

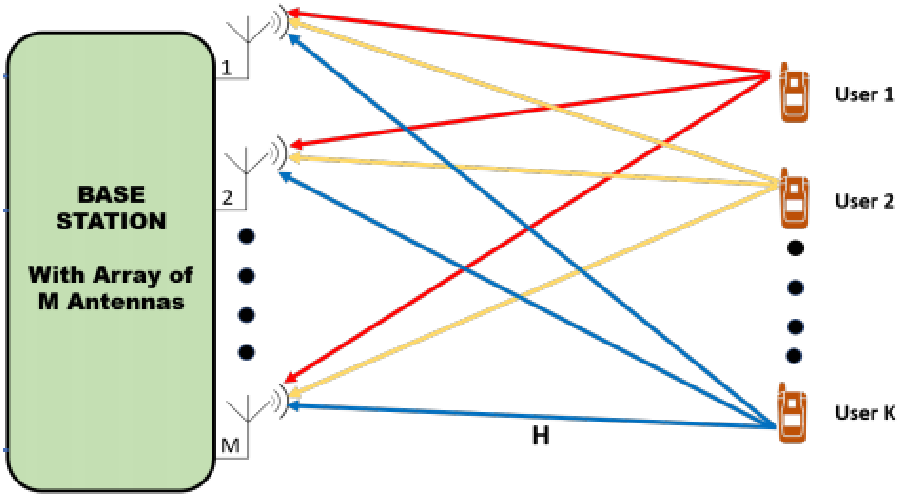

FIGURE 4.1: MU-Massive MIMO uplink Operation [1].

link) transmission is the scenario where the K users transmit signals to the BS. Let x_k, where $E\ |x_k|^2 = 1$, be the signal transmitted from the k-th user. Since K users share the same time frequency resource, the M×1 received signal vector at the BS is the combination of all signals transmitted from all K users.

TABLE 4.1: Symbols and description under uplink Transmission

No.	Symbol	Descrption
1	x_k	The signal transmitted from the k-th user
2	K	Users
3	y_k	The receiver vector at uplink
4	H	The channel vector between the user terminal and the base station
5	$n_{uplinlk}$	Addition of interference from several transmissions and the receiver noise.
6	y_k^{UL}	The signal received at the base station during uplink
7	p_u	Average uplink transmitter power (SNR)
8	$\mathbf{w_k}$	Column vector of the received combining filter for Kth user at uplink,
9	h_k	The channel vector between BS and all user
10	N	The number of antenna per user.

Then, the receiver vector at uplink is given by equation (4.1)[1]:

$$\mathbf{y} = \sqrt{\mathbf{p}_u} H\mathbf{x} + n_{uplink} \tag{4.1}$$

$$n_{uplink} = n_{Interference} + n_{noise} \tag{4.2}$$

from equation 4.3 , $\mathbf{y}_k^{UL}$ is the signal received at the base station, H is the channel vector between the user terminal and the base station, n_{uplink} is addition of interference from several transmissions and the receiver noise.

The signal received at the base station during uplink is given as[26][29][32]:

$$\mathbf{y}_k^{UL} = \underbrace{\sum_{l=1}^{N} \sqrt{\frac{\mathbf{p}_u}{N}} \mathbf{w}_k^T h_k \mathbf{x}_k}_{Desired\,signal} + \underbrace{\sum_{l=1}^{N} \sum_{n \neq k}^{K} \sqrt{\frac{\mathbf{p}_u}{N}} \mathbf{w}_k^T h_n \mathbf{x}_n}_{Interference} + \underbrace{\mathbf{w}_K^T n_k}_{Additive\,noise} \tag{4.3}$$

where $\mathbf{p_u}$ is the average signal to noise ratio (SNR), n is additive noise w_k denote the column vector of the received combining filter for K-th user at uplink, h_k denote the channel vector between BS and all user and N is the number of antenna per user.The SINR at uplink transmission is given by:

$$SINR_k = \frac{\frac{\mathbf{p}_u}{N} \sum_{l=k}^{K} \left| \mathbf{w}_k^T h_k \right|^2}{\frac{\mathbf{p}_u}{N} \sum_{n \neq k}^{K} \left| \mathbf{w}_k^T h_k \right|^2 + \left\| \mathbf{w}_k^T \right\|^2} \tag{4.4}$$

To eliminate the interference term, and to maximize the SNR, we can use the low-complexity precoding scheme.

4.2.2 MU-Massive MIMO Downlink System

Downlink (or forward link) is the scenario where the base station transmits signals to all K users. Let s ϵ $C^{M \times 1}$, where $E \left\| s_k \right\|^2 = 1$, be the signal vector transmitted from the BS antenna array.

We assume that n_k is gaussian distributed with zero mean and unit variance. The channel between the BS and the k-th user is denoted by 1×M row vector $h_k^T [k = 1, 2, 3, ..., K]$. A M×N channel matrix H between the BS and all users consists of channel vectors h_k^T. Let $\mathbf{s_k}$ represent the transmit symbol for K-th user at downlink, $\mathbf{w_k}$ represent the column vector of base station precoding and n_k is the additive white

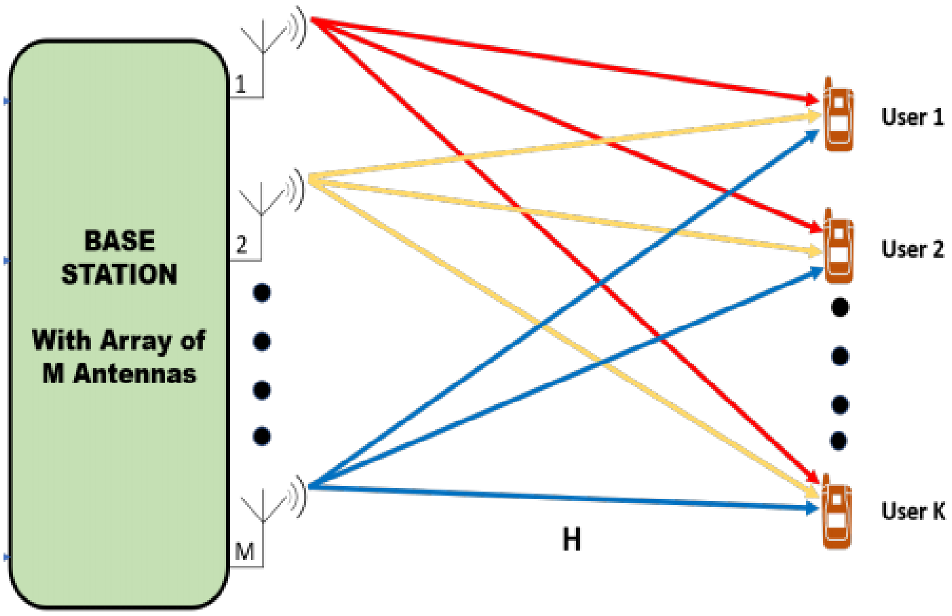

FIGURE 4.2: MU-Massive MIMO downlink operation [1].

gaussian noise vector.

TABLE 4.2: Symbols and description under Downlink Transmission

No	Symbol	Descrption
1	$\mathbf{s_k}$	Represent the transmit symbol for user at downlink
2	K	User
3	$\mathbf{y}_K$	The received signal vector of the K users at down link
4	H	The channel vector between the user terminal and the base station
5	n_k	The additive white Gaussian noise vector
6	y_k^{DL}	The signal received at the base station during downlink
7	p_b	Average downlink basestation transmitter power (SNR)
8	$\mathbf{w_k}$	The column vector of base station precoding
9	h_k^T	The channel vector between BS and all user(row vector)

The received signal vector of the K users can be written as in equation (4.5) and equation (4.6)[29] :

$$\mathbf{y}_K = \sqrt{\mathbf{p}_b} H\mathbf{x} + n_k \tag{4.5}$$

$$\mathbf{y}_K = \sqrt{\mathbf{p}_b} HW\mathbf{x} + n_k \tag{4.6}$$

where H is channel matrix, W is a precoding matrix. Then the received signal at the K-th user is expressed by[32]:

$$\mathbf{y}_k^{DL} = \underbrace{\sqrt{\mathbf{p}_b}\mathbf{w}_k h_k^T s_k}_{Desired\,signal} + \underbrace{\sqrt{\mathbf{p}_b}\sum_{\substack{l \neq k \\ l=1}}^{K} \mathbf{w}_l h_k^T s_l}_{Interference} + \underbrace{n_k}_{Noise} \tag{4.7}$$

where h_k^T is a channel vector, $\mathbf{p}_b$ denotes the average SNR across the base station. The received signal to interference plus noise ratio of the k-th user can be given by equation (4.8)[31]:

$$SINR_k^{DL} = \frac{\mathbf{p}_b \left|\mathbf{h}_\mathbf{w} \mathbf{w}_k\right|^2}{\mathbf{p}_b \sum_{\substack{l \neq k \\ l=1}}^{K} \left|\mathbf{h}_\mathbf{w} \mathbf{w}_k\right|^2 + 1} \tag{4.8}$$

which is a function of transmit precoding vector $\mathbf{w_k}$. To eliminate the interference we can use the linear precoders.

4.3 Linear precoders on Uplink Transmission

4.3.1 Performance Measurement Metrics

This section will describe the performance analysis, which includes the achievable rate, the energy efficiency and spectral efficiency. To eliminate the interference term from the received signal at the k-th user in both downlink and uplink and to maximize the SNR, we can use the low-complexity precoding scheme.

4.3.2 The Achievable Rate

The system performance can be defined by several methods. One of method to quantify the system performance is the achievable rate. The achievable rate is followed by Shannon theorem. This theory tells the maximum rate, which the transmitter can transmit over the channel [26] . This section will describe the achievable rate in uplink and downlink transmission, with the assumption that the total downlink power is fixed and equally divided among all the users. . Then, the achievable rate of k-th user for MU-Massive MIMO in both uplink and downlink system can be expressed as[26]

$$\mathbf{R}_k = E\left[\log_2(1 + SINR_k)\right] \quad (bits/S/Hz) \tag{4.9}$$

4.3.3 The Spectral Efficiency

After we calculate the achievable rate of 1 user , we calculate the spectral efficiency by multiplication between the achievable rate of 1 user and the number of users in the system [1],[26] . The spectral efficiency is given by equation 4.10 .

$$\mathbf{R}_P = K \times \mathbf{R}_k \quad (bits/S/Hz) \tag{4.10}$$

Where R_P is the spectral efficiency in $bits/S/Hz$ and R_k is the achievable rate of user K.

4.3.4 The Energy Efficiency

The energy efficiency of a system is defined as the the spectral efficiency divided by the transmit power. Generally, increasing transmit power increases the sum-rate. On the contrary, it decreases the energy efficiency[1]. The energy efficiency can be written as:

$$\eta = \frac{\mathbf{R}_P}{\mathbf{p}_{tr}} \quad (bits/J/Hz) \tag{4.11}$$

. Where $\mathbf{P}_{tr}$ is the average transmission power at the base station (J/s) for massive MIMO in 5G , $\mathbf{R}_P$ is the spectral efficiency in bits/s/Hz .

4.3.5 MRC precoding receiver

With MRC, the BS aims to maximize the received signal-to-noise ratio (SNR) of each stream, ignoring the effect of multiuser interference. In equation (4.12) , the k-th column of the MRC receiver matrix is given by [8],[12],[17],[18]:

$$\mathbf{w}_k^{MRC} = max\left(\frac{Desired\, power}{Noise\, power}\right) = max\left(\frac{\frac{\mathbf{p}_u}{N}\sum_{k=1}^{K}\left|\mathbf{w}_K^T h_k\right|^2}{\left\|\mathbf{w}_k^T\right\|^2}\right) \tag{4.12}$$

Since

$$\begin{aligned} \frac{\frac{\mathbf{p}_u}{N}\sum_{l=1}^{N}\left|\mathbf{w}_K^T h_K\right|^2}{\left\|\mathbf{w}_k^T\right\|^2} &\leq \frac{\frac{\mathbf{p}_u}{N}\sum_{k=1}^{k}\left\|\mathbf{w}_k^T\right\|^2\left\|h_k\right\|^2}{\left\|\mathbf{w}_k^T\right\|^2} \\ &= \frac{\mathbf{p}_u}{N}\sum_{k=1}^{k}\left\|h_k\right\|^2 \\ &= \frac{\mathbf{p}_u}{N}N\left\|h_k\right\|^2 \\ &= \mathbf{p}_u\left\|h_k\right\|^2 \end{aligned} \tag{4.13}$$

The signal received at the base station during uplink in MRC receiver is the received signal $\mathbf{y}_k^{UL}$ multiplied by the conjugate-transpose of the channel vector h_k , as follows

$$\mathbf{y}_{UL}^{MRC} = \mathbf{y}_k^{UL} \times \mathbf{w}_k^{MRC} = \sqrt{\mathbf{p}_u}\left\|h_k\right\|^2 x_k + \sqrt{\mathbf{p}_u}\sum_{n\neq k}^{K}\mathbf{w}_k^T h_n x_n + \mathbf{w}_k^T n_k \tag{4.14}$$

Substituting equation (4.13) into (4.4) we will get equation 4.15

$$\begin{aligned} SINR_{UL}^{MRC} &= \frac{\left\|h_K\right\|^4}{\sum_{n\neq k}^{K}\left|h_k h_n\right|^2 + \frac{1}{\mathbf{p_u}}\left\|h_k\right\|^2} \\ &= \frac{\mathbf{p_u}\left\|h_K\right\|^4}{\mathbf{p_u}\sum_{n\neq k}^{K}\left|h_k h_n\right|^2 + \left\|h_k\right\|^2} \end{aligned} \tag{4.15}$$

The proof for SINR is found on Appendix A.

For uplink transmission we consider two levels of SNR and we proved for both condition.

1) For high SNR

The sum rate for the precoding is given by:

$$\begin{aligned}\mathbf{R}_{UL}^{MRC}high &= E\left(\sum_{k=1}^{K}\log_2\left(1+SINR_k^{MRC}\right)\right)\\ &= \sum_{k=1}^{K}\log_2\left(1+\frac{E\left\|h_k\right\|^4}{\sum_{n\neq k}^{K}E\left|h_kh_n\right|^2+\frac{1}{\mathbf{p}_u}E\left\|h_k\right\|^2}\right)\end{aligned} \tag{4.16}$$

Lemma 1

$$\begin{aligned}E\left\|h_k\right\|^4 &= M^2+M\\ E\left|h_kh_n\right|^2 &= M\\ E\left\|h_k\right\|^2 &= M\end{aligned} \tag{4.17}$$

Lemma 1 of equation (4.17) is substituted into equation (4.16) and we will get (4.18).The detailed proof is found on Appendix *A*.

$$\mathbf{R}_{UL}^{MRC}high \approx K\log_2\left(1+\frac{\mathbf{p_u}(M+1)}{\mathbf{p_u}(K-1)+1}\right) \tag{4.18}$$

2) For low SNR

The sum rate for the precoding is given by:

$$\mathbf{R}_{UL}^{MRC}low = E\left(\sum_{k=1}^{K}\log_2(1+SINR_k^{MRC})\right) \tag{4.19}$$

Inserting equation (4.17) into (4.19) we will get the sum rate for low SNR.

$$\mathbf{R}_{UL}^{MRC}low \approx K\log_2(1+\frac{\mathbf{p_u}M}{\mathbf{p_u}(K-1)+1}) \tag{4.20}$$

4.3.6 ZF precoding receiver

Zero-forcing (ZF) receivers take the interuser interference into account, but neglect the effect of noise. With ZF, the multiuser interference is completely nulled out by projecting each stream onto the orthogonal complement of the inter user interference.

ZF pre-coding eliminates the interference by transmitting the signals towards the intended user with nulls in the "direction" of other users. By contrast to MRC, zero-forcing (ZF) receivers take the interuser interference into account, but neglect the effect of noise.

The ZF precoding employed by the base station is given by equation (4.21)[8]:

$$\mathbf{w}_K^{ZF} = (H^*H)^{-1}H^* = [\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, \cdot\cdot\cdot, \mathbf{w}_k] \tag{4.21}$$

Where: $\mathbf{w}_k^{ZF}$ is a precoding matrix consisting of each column vector $\mathbf{w}_k$. All more accurately, the received vector is multiplied by the pseudo- reverse of the channel matrix H

$$\begin{aligned}\mathbf{y}_{UL}^{ZF} &= \mathbf{y}_u \times \mathbf{w}_k^{ZF} \\ &= \sum_{l=1}^{N} \sqrt{\frac{\mathbf{p}_u}{N}} \mathbf{w}_{ZF} h_k \mathbf{x}_k \\ &+ \sum_{l=1}^{N} \sum_{n \neq k}^{K} \left(\sqrt{\frac{\mathbf{p}_u}{N}} \mathbf{w}_{ZF} h_n \mathbf{x}_n \right) + \mathbf{w}_\mathbf{k}^\mathbf{ZF} \mathbf{n}_\mathbf{k} \\ &= \sum_{l=1}^{N} \sqrt{\frac{\mathbf{p}_u}{N}} \mathbf{x}_k + \mathbf{w}_\mathbf{k}^\mathbf{ZF} \mathbf{n}_\mathbf{k}\end{aligned} \tag{4.22}$$

The received SINR of the k-th stream is given by:

$$\begin{aligned}SINR_k^{ZF} &= \frac{Signal\,Power}{Noise\,Power} \\ &= \frac{\frac{\mathbf{p}_u}{N} N}{\mathbf{w}_k^{ZF}} \\ &= \frac{\mathbf{p}_u}{(H^*H)^{-1}H^*}\end{aligned} \tag{4.23}$$

The sum rate will be the same for both high and low SNR value and can be written as:

$$\begin{aligned}\mathbf{R}_{ZF}^{ULL} &= \mathbf{R}_{ZF}^{ULH} \\ &= E\left(\sum_{l=1}^{N} \sum_{k=1}^{K} \log_2(1 + SINR_k) \right)\end{aligned} \tag{4.24}$$

Substituting equation (4.23) in to equation (4.24), we will get the equation (4.25) for the sum rate of zero forcing receivers.

$$\begin{aligned}\mathbf{R}_{ZF}^{ULL} &= E\left(\sum_{l=1}^{N}\sum_{k=1}^{K}\log_2(1+\frac{\mathbf{p}_u}{(H^*H)^{-1}H^*})\right)\\ &= \sum_{l=1}^{N}\sum_{k=1}^{K}\log_2(1+\frac{\mathbf{p}_u}{E\left((H^*H)^{-1}H^*\right)})\end{aligned} \tag{4.25}$$

From **Lemma 7** the power threshold to select a better receive combining filter at uplink is given by

$$\mathbf{P}_{th,UL} = \frac{1}{M-K+1} \tag{4.26}$$

If each user has larger transmit power than ,UL the solution employing ZF at BS provides a better achievable sum rate performance. Therefore from (4.25) and (4.26), we have (4.27).

$$E\left((H^*H)^{-1}H^*\right) = \frac{1}{M-K+1} \tag{4.27}$$

Inserting equation (4.27) into equation (4.25), we will get the achievable sum-rate in (4.28).

$$\begin{aligned}\mathbf{R}_{ZF}^{ULL} &= \sum_{l=1}^{N}\sum_{k=1}^{K}\log_2\left(1+\frac{\mathbf{p}_u}{\frac{1}{M-K+1}}\right)\\ &= \sum_{l=1}^{N}\sum_{k=1}^{K}\log_2(1+\mathbf{p}_u(M-K+1))\\ &\approx \log_2(1+\mathbf{p}_u(M-K+1))\end{aligned} \tag{4.28}$$

ZF is a basic signal preparing and functions admirably in interference constrained situations, however since ZF ignores the effect of noise; it works ineffectively under noise restricted situations. Compared with MRC, ZF has a higher execution multi-faceted nature because of the calculation of the pseudo-converse of the channel gain matrix [8] .

4.4 Linear precoders on Downlink Transmission

4.4.1 Normalization method

The linear precoder with zero-forcing and maximum rate transmission is known as the normalization of the vector and the matrix [32]. In particular, we consider two possible normalizations of the precoding filters for the downlink, referred to as vector and matrix normalization. We make normalize the precoding matrix for uniform power allocation over all downlink streams. To satisfy the power constraint, we consider two methods, i.e., vector/matrix normalizations [32]. The normalized transmit beam forming vectors (columns of a precoding matrix) with vector and matrix normalizations is given as: $W_K = \frac{w_l}{\left(\sqrt{K}\|w_k\|\right)}$ and $W_K = \frac{w_k}{\left(\sqrt{K}\|W\|_W\right)}$ respectively.

Vector normalization imposes equal power per downlink stream, whereas the matrix normalization relents various power streams. To more simplify, we do not consider a power optimization that could yield a complexity in massive MIMO antenna systems.

- **Vector Normalization of ZF/MRT[32]**

The received signal at the k^{th} user for vector normalization of ZF/MRT can be expressed as[32]:

$$\mathbf{y}_K = \sqrt{\mathbf{p}_b}\frac{\mathbf{w}_k}{\sqrt{K}\,\|\mathbf{w}_k\|}h_k^T S_k + \sqrt{\mathbf{p}_b}\sum_{\substack{l=1\\ l\neq k}}^{K} h_k^T \frac{\mathbf{w}_l}{\sqrt{K}\,\|\mathbf{w}_l\|} S_l + n_k \tag{4.29}$$

- **Matrix Normalization of ZF/MRT [32]**

The received signal at the k_{th} user for matrix normalization of ZF/MRT can be expressed as:

$$\mathbf{y}_K = \sqrt{\mathbf{p}_b}\frac{\mathbf{w}_k}{\|W\|_W}h_k^T S_k + \sqrt{\mathbf{p}_b}\sum_{\substack{l=1\\ l\neq k}}^{K} h_k^T \frac{\mathbf{w}_l}{\|W\|_W} S_l + n_k \tag{4.30}$$

4.4.2 MRT Precoding

MRT and ZF precoding has often been introduced for MIMO signal processing due to good quality and ease of operation. It is the counterpart of the maximal-ratio combining receiver for uplink [30]. MRT works well in the MU-MIMO system where the base station radiates low signal power to the users [26]. MF precoder is also known as maximum ratio transmission (MRT), which maximizes signal gain at the intended user. The MRT pre-coding employed by the BS is written as [29] in equation (4.31):

$$\mathbf{w}_K^{MRT} = H^* = [\mathbf{w}_1 \mathbf{w}_2 \cdots \mathbf{w}_K] \tag{4.31}$$

Where H^* is conjugate transpose (hermitian of channel matrix) of channel matrix ,$\mathbf{w}_K^{MRT}$ is a precoding matrix consisting of each column vector w_k. The received signal at the k_{th} user can be expressed as[31]:

$$\mathbf{y}_K^{MRT} = \sqrt{\mathbf{p_b}} \frac{\mathbf{w}_k}{\sqrt{K}\,\|\mathbf{w}_k\|} h_k^T S_k + \sqrt{\mathbf{p_b}} \sum_{\substack{l=1 \\ l \neq k}}^{K} h_k^T \frac{\mathbf{w}_l}{\sqrt{K}\,\|\mathbf{w}_l\|} S_l + n_k \tag{4.32}$$

Where: $\mathbf{p_b}$ is transmit power in a downlink.The received SINR for the MRT receiver is given by :

$$SINR_K^{MRT} = \frac{\mathbf{p_b} \frac{\|h_k\|^4}{\left\|\sqrt{K} h_k\right\|^2}}{\mathbf{p_b} \sum_{n \neq k}^{K} \frac{\left\|h_k^* h_n\right\|^2}{\left\|\sqrt{K} h_n\right\|^2} + 1} \tag{4.33}$$

For downlink transmission again we consider two levels of SNR and we find final equation for achievable sumrate for both condition.

1) For low SNR

The sum rate for the precoding is given by[30,32]:

$$\begin{aligned} R_{DL}^{MRT} low &= E\left(\sum_{l=1}^{N} \sum_{k=1}^{K} \log_2\left(1 + SINR_{DL}^{MRT}\right)\right) \\ &= E\left(\sum_{l=1}^{N} \sum_{k=1}^{K} \log_2\left(1 + \frac{\mathbf{p}_b \frac{\|h_k\|^4}{\left\|\sqrt{K} h_k\right\|^2}}{\mathbf{p}_b \sum_{n \neq k}^{K} \frac{\left\|h_k^* h_n\right\|^2}{\left\|\sqrt{K} h_n\right\|^2} + 1}\right)\right) \end{aligned} \tag{4.34}$$

Taking **lemma 1** of equation (4.17) into account over equation 4.34, we will get the sum-rate as equation (4.35)[30,32]:

$$\begin{aligned} R_{DL}^{MRT} low &= \sum_{\substack{l=1 \\ l\neq k}}^{N} \log_2 \left(1 + \frac{\mathbf{p_b} \frac{M}{K}}{\mathbf{p_b} \sum_{n\neq k}^{K} \frac{M}{KM} + 1}\right) \\ &\approx K \log_2 \left(1 + \frac{\mathbf{p_b} M}{\mathbf{p_b} (K-1) + K}\right) \end{aligned} \tag{4.35}$$

2) For high SNR : The sum rate for the precoding is given by:

$$\begin{aligned} R_{DL}^{MRT} high &= E\left(\sum_{l=1}^{N} \sum_{k=1}^{K} \log_2 \left(1 + SINR_{DL}^{MRT}\right)\right) \\ &= E\left(\sum_{l=1}^{N} \sum_{k=1}^{K} \log_2 \left(1 + \frac{\mathbf{p_b} \frac{E\left(\|h_k\|^4\right)}{EK\left(\|h_k\|^2\right)}}{\mathbf{p_b} \sum_{n\neq k}^{K} \frac{E\left(\left|h_k^* h_n\right|^2\right)}{EK\left(\|h_n\|^2\right)} + 1}\right)\right) \end{aligned} \tag{4.36}$$

$$\begin{aligned} R_{DL}^{MRT} high &= \sum_{l=1}^{N} \sum_{k=1}^{K} \log_2 \left(1 + \frac{\mathbf{p_b} \frac{E\left(\|h_k\|^4\right)}{KM}}{\mathbf{p_b} \sum_{n\neq k}^{K} \frac{M}{KM} + 1}\right) \\ &\approx K \log_2 \left(1 + \frac{\mathbf{p_b} (M+1)}{\mathbf{p_b} (K-1) + K}\right) \end{aligned} \tag{4.37}$$

4.4.3 ZF precoding

One of simple linear pre-coding technique is ZF precoding in which the multiuser interference can be cancelled out at each user. This pre-coding is assumed to implement a pseudo-inverse of the channel matrix. Zero forcing (ZF) precoding is another type of basic precoding technique, which eliminates the interference by transmitting the signal toward the intended user while nulling in the directions of other users. The ZF precoder is obtained by[32],[8]

$$\mathbf{w}_K^{ZF} = \left(H^H H\right)^{-1} H^H = [\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, \cdots \mathbf{w}_K] \tag{4.38}$$

Where: $\mathbf{w}_k^{ZF}$ is a pre-coding matrix consisting of each column vector w_k.The $SINR$ for ZF precoding in downlink is:

$$SINR_{DL}^{ZF} = \frac{\mathbf{p_b} \left| \frac{\mathbf{w}_k}{\sqrt{K}\|\mathbf{w}_k\|} h_k \right|^2}{\mathbf{p_b} \sum_{n\neq k}^{K} \left| \frac{\mathbf{w}_k}{\sqrt{K}\|\mathbf{w}_k\|} h_k \right|^2 + 1} \tag{4.39}$$

The sum rate is then [30,32]:

$$\begin{aligned} R_{DL}^{ZF}high &= R_{DL}^{ZF}low \\ &= E\left(\sum_{l=1}^{N}\sum_{k=1}^{K}\log_2\left(1+SINR_{DL}^{ZF}\right)\right) \\ &\approx \left(\sum_{l=1}^{N}\sum_{k=1}^{K}\log_2\left(1+\frac{\mathbf{p_b}}{K\,E\left(\|\mathbf{w}_k\|^2\right)}\right)\right) \end{aligned} \tag{4.40}$$

From **lemma 7** as stated above $E\,\|w_k\|^2 = \frac{1}{M-K+1}$

$$\begin{aligned} R_{DL}^{ZF}high &= R_{DL}^{ZF}low \\ &\approx K\,\log_2\left(1+\frac{\mathbf{p_b}\,(M-K+1)}{K}\right) \end{aligned} \tag{4.41}$$

Therefore the sumrate for zero forcing by using vector /matrix normalization methods is given by equation (4.42)[30,32]:

$$R_{ZF}Vec = R_{ZF}Mat \approx Klog_2\left(1+\frac{\mathbf{p_b}\,(M-K+1)}{K}\right) \tag{4.42}$$

The sum rate for Maximum ratio transmission precoding by using vector normalization methods is given by equation (4.43)[30,32].

Vector normalization-low SNR regime:

$$R_{MRT}Vec_{low} \approx Klog_2\left(1+\frac{\mathbf{p_b}M}{\mathbf{p_b}\,(K-1)+K}\right) \tag{4.43}$$

Vector normalization-high SNR regime

$$R_{MRT}Vec_{high} \approx Klog_2\left(1+\frac{\mathbf{p_b}\,(M+1)}{\mathbf{p_b}\,(K-1)+K}\right) \tag{4.44}$$

The sum rate for Maximum ratio transmission precoding by using matrix normalization methods is given by equation (4.44) [30,32].

Matrix normalization for low and high SNR regime

$$\begin{aligned} R_{MRT}Mat_{high} &= R_{MRT}Mat_{low} \\ &\approx Klog_2\left(1+\frac{\mathbf{p_b}\,(M+1)}{\mathbf{p_b}\,(K-1)+K}\right) \end{aligned} \tag{4.45}$$

Chapter 5

Simulation Results and Discussion

5.1 Introduction

All results discussed here are derived from the upper section of this paper. We simulated Massive MIMO system with two precoding formats MRC and ZF in Uplink and MRT and ZF in downlink .All conditions are simulated by using MATLAB software. Since we are studying about multiuser Massive MIMO, We are interested in the system where $\mathbf{M}(Antennas) >> \mathbf{K}(Users)$.

The simulation tests are done to analyze and evaluate the system performance. The performance is estimated by drawing the spectral efficiency versus the number of BS antennas /number of user(K), achievable rate versus BS power,spectral efficiency versus the number of BS antennas(M)/number of user(K) and energy efficiency versus the number of BS antennas(M)/number of user(K), using MRC and ZF for uplink and, using MRT and ZF for downlink. Our simulations are for different scenarios.

5.2 Performance comparison of the precoders in Uplink transmission

5.2.1 Achievable sumrates of MRC and ZF

- **Condition 1**: Achievable sum rate analysis (K=10 and M =100, SNR(dB) = [-30,10]

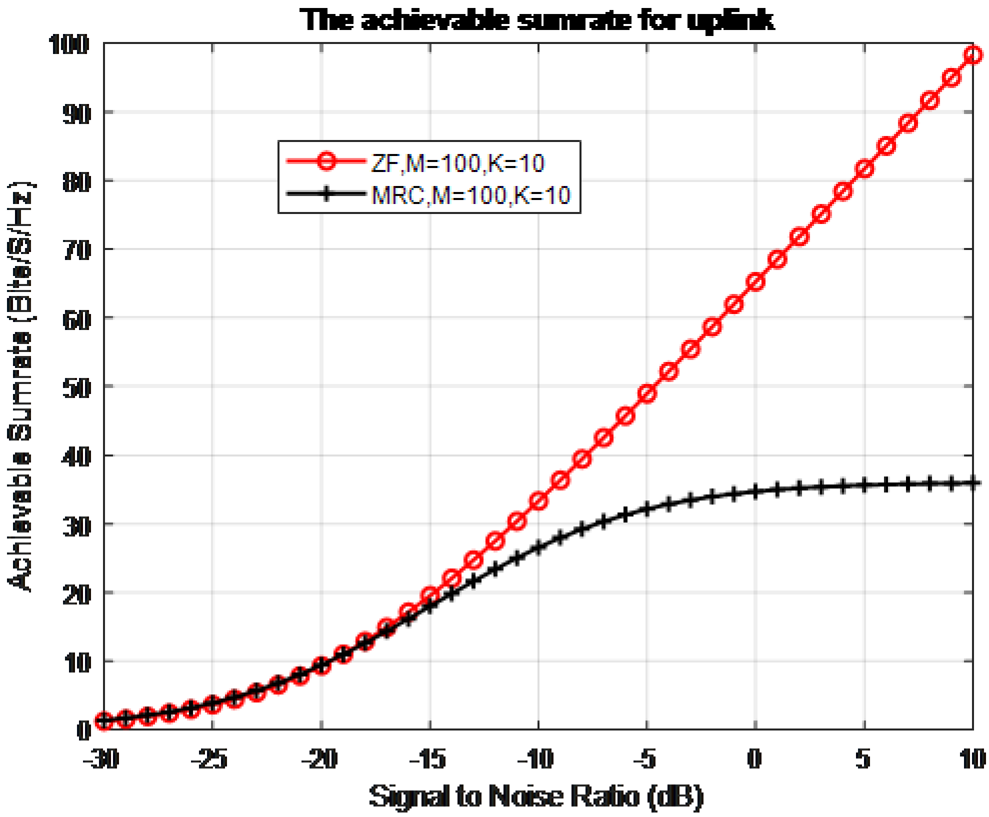

FIGURE 5.1: Achievable rate for uplink at M=100 and K=10

From the results executed on Figure 5.1 and Figure 5.2, it can be seen that the sumrate for MRC is better than sumrate for zero forcing for low SNR and sumrate for zero forcing is better than sumrate for MRC at high SNR. As it is seen from Figure 5.1 and Figure 5.2,the number of BS antenna is 100 and 200 BS antenna respectively is used for 10 users. For the same uplink transmitter power,the achievable rate increases as we fold the number of BS antenna. From mathematical analysis result in Table 5.1 it can be seen that ,the performance interms of achievable rate for zero forcing and MRC grows by 31.12 % and 33.4 % respectively for low BS power. Similarly,the performance interms of achievable rate for zero forcing and MRC grows by 11.48 % and 5.4 % respectively for high power.

As it can be seen from Figure 5.1 and Figure 5.2, the capacity difference between MRC curve and ZF curve is wide. This due to the variation of average SNR. For the given base station antenna and user configuration under low SNR, the achievable rate of both MRC and ZF is almost the same. Where as an average SNR increases the gap between achievable rate curve of MRC and ZF become wide. That means for high SNR , ZF curve leads the MRC curve.Here it can be concluded that performance in zero forcing grows faster than maximum ratio combiner when the number of BS antenna increases under low power. Where as the performance in MRC grows faster than Zero forcing when the number of BS antenna increases under high power.

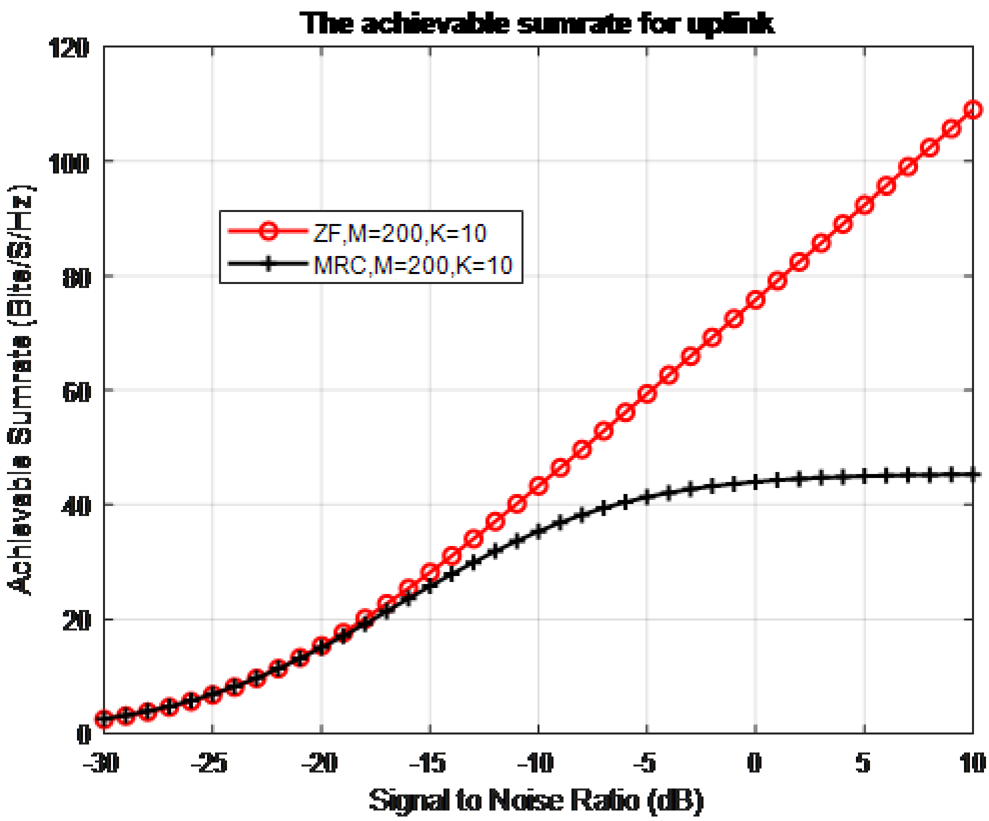

Figure 5.2: Achievable rate for uplink at M=100 and K=10

TABLE 5.1: Achievable sum rate comparison for MRC and ZF at M=100 and M=200 for uplink

Power	Low SNR(-30dBm)		High SNR(10dBm)	
Precoder	MRC	ZF	MRC	ZF
Sumrate for M=100	1.3765	1.2565	35.9680	98.3131
Sumrate for M=200	2.6208	2.5217	45.2907	109.0011
Performance change(%)	31.12	33.4	11.48	5.4

5.2.2 Spectral Efficiency of MRC and ZF

• **Condition 1**: Spectral Efficiency analysis [k = 1 to 10 and M =100, $\mathbf{p_u}$ = 10dBm]

Figure 5.3 shows spectral efficiency across the given user range according to equation

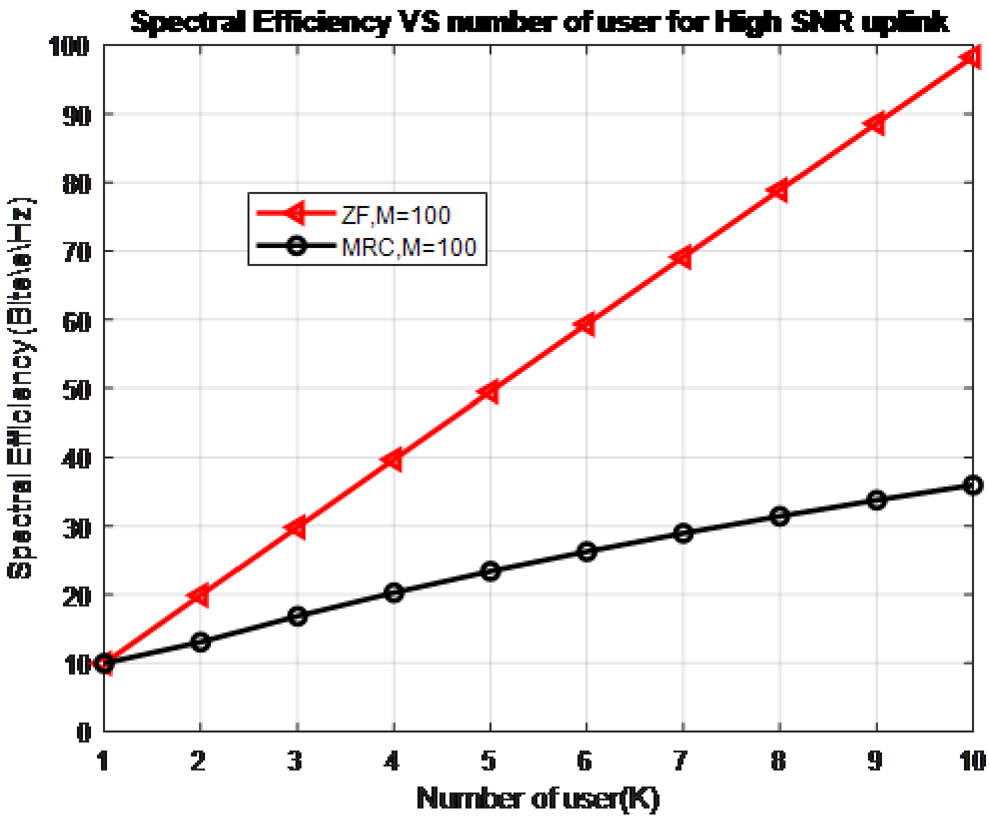

FIGURE 5.3: Spectral Efficiency comparison between MRC and ZF at high uplink transmitter power

(4.10) and equation (4.19) . For execution we applied the base station antennas, $M = 100$, the uplink power,$\mathbf{p}_u$ = 10dBm ,the user , K=1 to 10.From the result we can see that for high SNR, ZF gives better performance in terms of spectral efficiency than MRC . As it can be seen from the result in figure 5.3,the spectral efficiency curve between MRC and ZF is wide.This happened due to SNR value used.

• **Condition 2**: Spectral Efficiency analysis [K=1 to 10 (not fixed) and M =100, $\mathbf{p_u}$= -30dBm]

Figure 5.4 shows spectral efficiency across the given user range according to equation (4.10) and equation (4.20) . For execution we applied the base station antennas, M =100 , the uplink power, $\mathbf{p_u}$ = -30dBm, the user, K=1 to 10 . From the result we can see that for low SNR, MRC gives better performance in terms of spectral efficiency than ZF precoder.

For low SNR,the spectral efficiency curve between MRC and ZF is narrow.

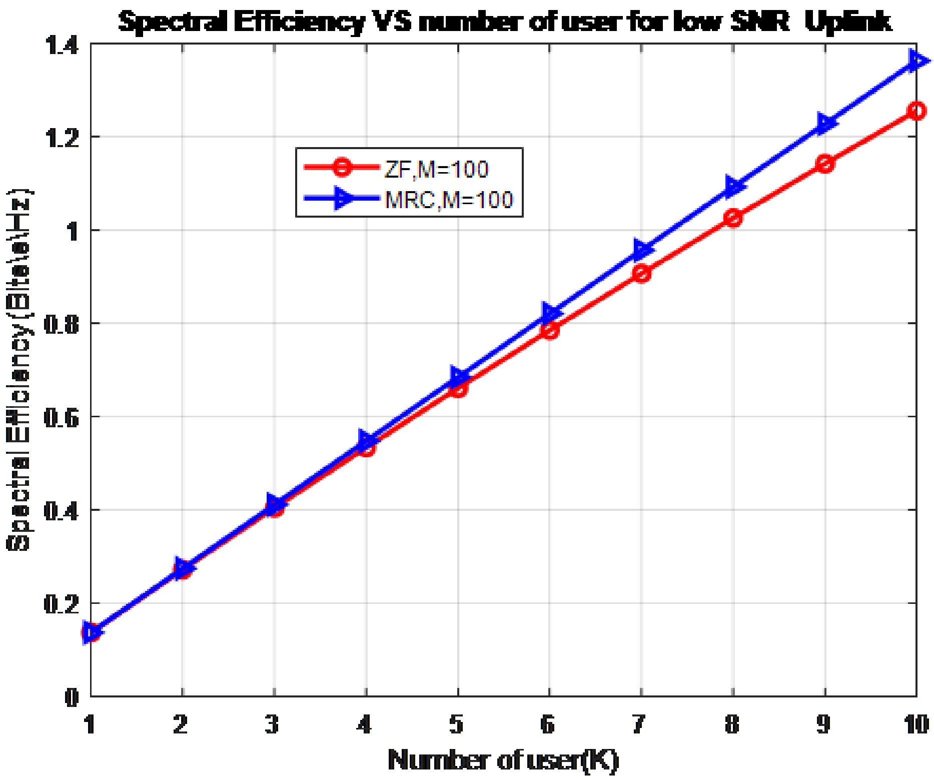

FIGURE 5.4: Spectral Efficiency comparison between MRC and ZF for uplink at low SNR

TABLE 5.2: SpectralEfficiency comparison for MRC and ZF at high and low SNR value for uplink

Number of base station antenna M= 100								
Precoder	MRC				ZF			
Number of user	3	6	9	10	3	6	9	10
Spectral Efficiency at 10dBm	16.8525	26.2727	33.7643	35.9680	29.8143	59.3598	88.6235	98.3131
Specral Efficiency at -30dBm	0.4117	0.8211	1.2282	1.3633	0.4046	0.7856	1.1428	1.2565

5.2.3 Energy Efficiency of MRC and ZF

• **Condition 1**: Energy Efficiency analysis [K=1 to 10 and M =100, BS power = -30dBm]

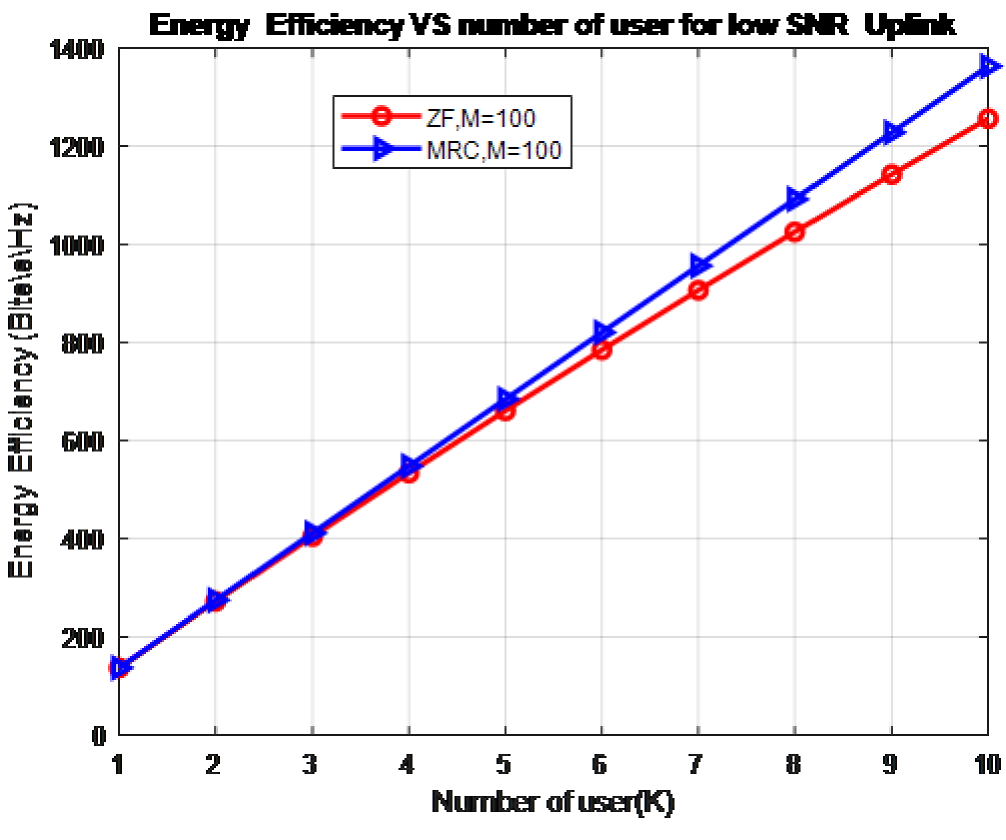

FIGURE 5.5: Energy Efficiency comparison between MRC and ZF for uplink at low SNR regime

Figure 5.5 shows the energy efficiency for the user ranging from 1 to 10 , the fixed number of base station antenna M=10 and the SNR = -30dBm . This result is based on equation 4.11 equation 4.20 and equation 4.28. From the result we can see that

MRC perform better than zero forcing at low power.

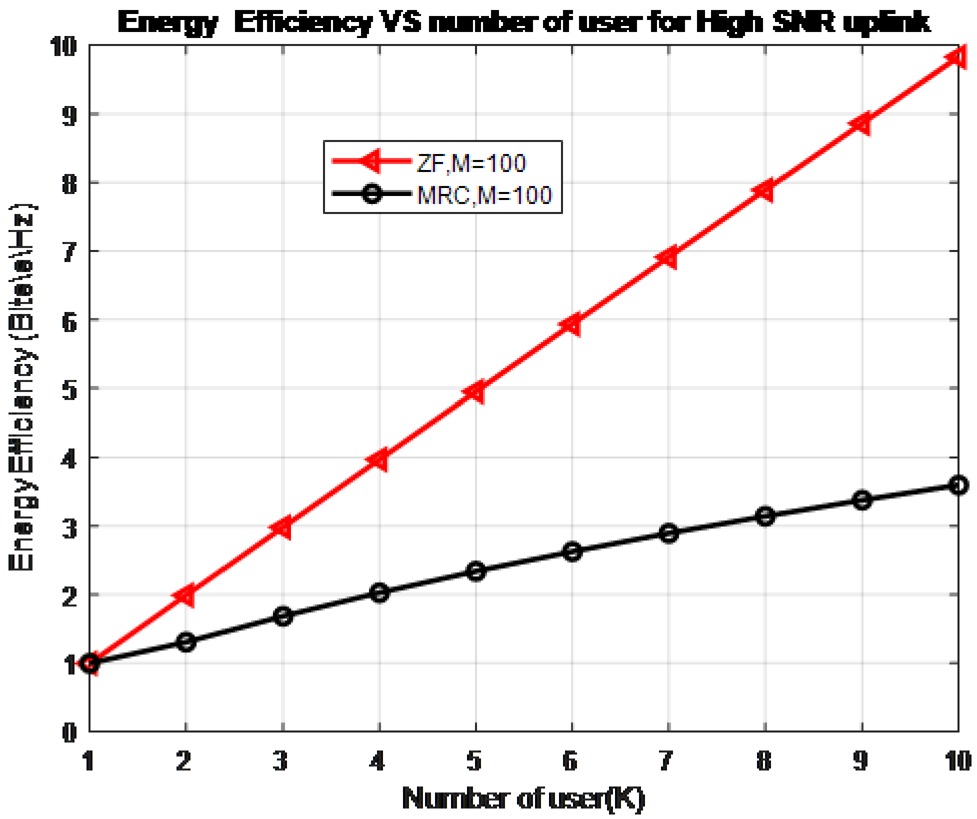

FIGURE 5.6: Energy Efficiency comparison between MRC and ZF for uplink at high SNR regime

• **Condition 2**: Energy Efficiency analysis [K=10 and M =100,BS power = 10dBm]

Figure 5.6 shows the energy efficiency for the user ranging from 1 to 10 , the fixed number of base station antenna M=100 , the average base station transmission power is 10dBm . This result is based on equation 4.11 equation 4.20 and equation 5.28. From the result we can see that zero forcing perform better at high power than MRC precoder.Under low uplink transmitter power,MRC shows better performance interms of energy efficiency than zero forcing.

TABLE 5.3: Energy Efficiency comparison between MRC and ZF for uplink

Number of base station antenna (Fixed)= 100,								
Precoder	MRC				ZF			
Number of user	3	6	9	10	3	6	9	10
Energy Efficiency at 10dBm	1.6853	2.6273	3.3764	3.5968	2.9814	5.9360	8.8624	9.8313
Energy Efficiency at -30dBm	0.4117	0.8211	1.2282	1.3633	0.4046	0.7856	1.1428	1.2565

5.3 Performance comparison of the precoders in Downlink transmission

5.3.1 Vector Normalization For MRT and ZF

Under this section, the performance of zero force (ZF) and maximum ratio transmission (MRT) in a single cell downlink multi user massive MIMO over perfect channel have been done. By considering vector normalization, different performance metrics like achievable sum rate, spectral efficiency and energy efficiency have been analyzed. The simulation metrics are the number of user, k=10 , the number of base station antenna, M = 128 and M=256 , the downlink transmission power, $\mathbf{p_b}$ = 0dBm and -15dBm .

5.3.1.1 Achievable Sumrate for ZF and MRT

Figure 5.7 and Figure 5.8 shows the sum rate across the given BS transmitter power. As it is seen from the plot, the number of BS antenna is 128 and 256 BS antenna respectively used for 10 users.The result shows that for lower number of base station antenna and low BS transmitter power, MRT will have better performance than ZF. Where as, as the number of base station antenna increases under high BS transmitter power, ZF will have better performance than MRT. MRT have better achievable sumrate at low power than high power. Where as, ZF have better achievable sumrate at high power than low power.

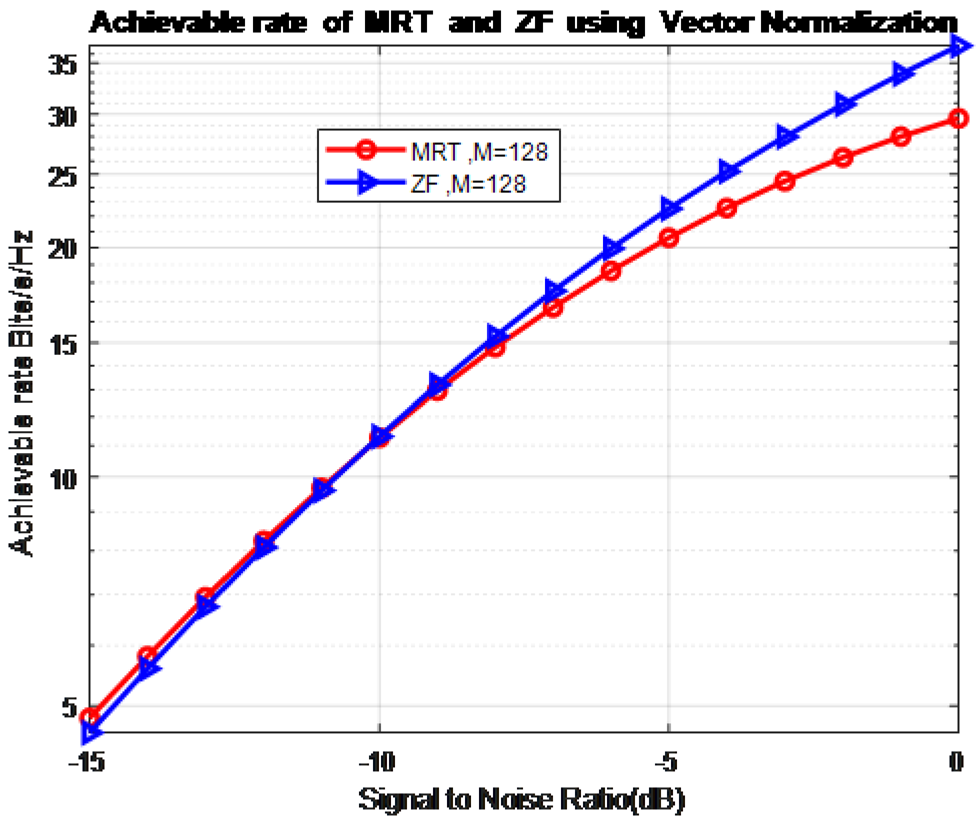

FIGURE 5.7: Achievable sumrate versus number of user curve for MRT and ZF using vector Normalization at M(fixed)=128,K(fixed)=10

The performance of MRT and the performance of ZF will begin to grow up as the number of BS antenna increases. From mathematical analysis result in Table 5.4 it can be seen that ,the performance interms of achievable rate under vector normalization for zero forcing and MRT grows by 28.8 % and 27.1 % respectively for low BS transmitter power.Similarly,the performance interms of achievable rate for zero forcing and MRC grows by 11.8 % and 7.32 % respectively for high BS transmitter power.

TABLE 5.4: Achievable sum rate comparison for MRT and ZF at M=128 and M=256 under vector normalization in downlink

BS Power	Low SNR(-15dBm)		High SNR(0dBm)	
Precoder	MRT	ZF	MRT	ZF
Sumrate for M=128	4.819	4.6081	29.6153	36.893
Sumrate for M=256	8.4013	8.3275	38.606	46.837
Performance change(%)	27.1	28.8	7.32	11.8

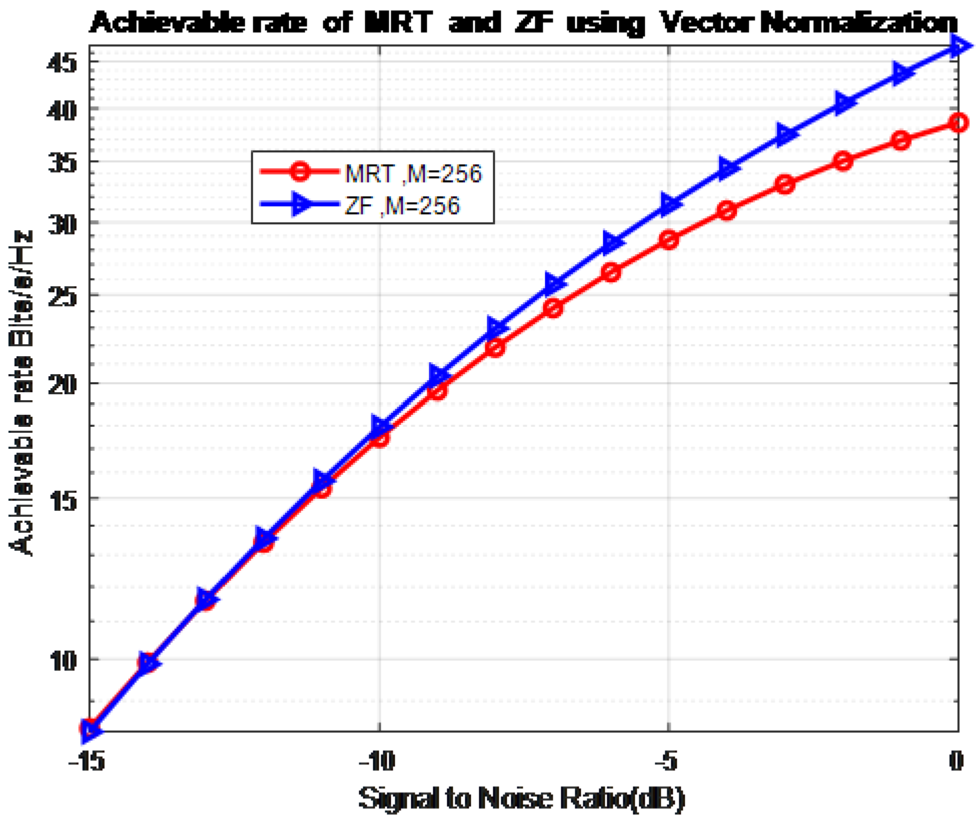

FIGURE 5.8: Achievable sumrate versus number of user curve for MRT and ZF using vector Normalization at M(fixed)=256,K(fixed)=10

5.3.1.2 Spectral Efficiency

• **Condition 1**: Plotting spectral efficiency versus number of BS Antenna curve at 0dBm and -15 dBm for ZF and MRT.

Figure 5.9 shows the spectral efficiency for the given base station antenna based on equation (4.10) , equation (4.42) ,equation (4.43) and equation (4.44). Here we consider variable number of antenna ranging from 1 to 128 and the fixed number of user , k =10. From the results executed, we can decide that as the number of base station antenna increases under high BS power, the spectral efficiency of MRT precoder fails as compared to ZF precoder. MRT shows better spectral efficiency at low power than high power. On another side, ZF precoder have better spectral efficiency for high power than low power. Generally, ZF performs better for high number of BS antenna and high BS transmitter power.

• **Condition 2**: Plotting spectral efficiency versus K users curve at 0dBm and -15 dBm for ZF and MRT.

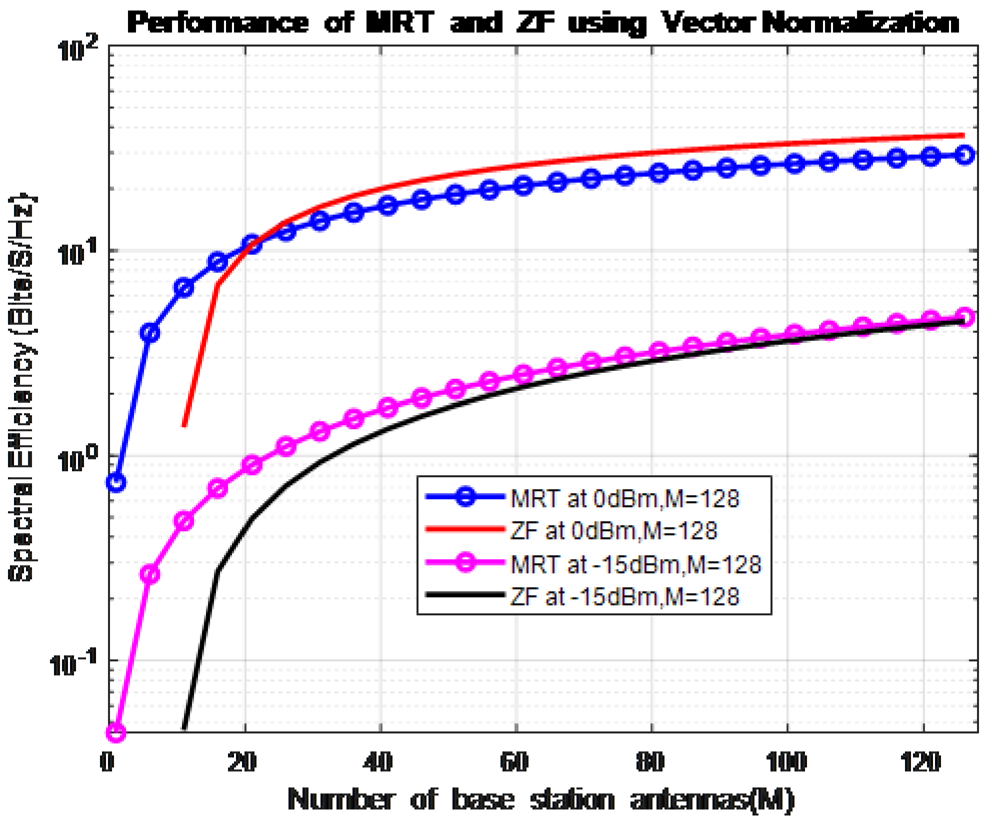

FIGURE 5.9: Spectral Efficiency versus number of base station Antenna curve for MRT and ZF using vector normalization at K(fixed)= 10,M = 1 to 128.

Figure 5.10 shows the spectral efficiency for the given user boundary based on equation (4.10) , equation (4.42) ,equation (4.43) and equation (4.44). Here we consider fixed number of base station antenna M = 128 and the variable number of user k ranging from 1 to 10 . From the results executed, we can deduce that as the number of user increases, the spectral efficiency of MRT precoder fails as compared to ZF precoder. MRT shows better spectral efficiency at low power than high power. On another side, ZF precoders have better spectral efficiency for high power than low power as shown.

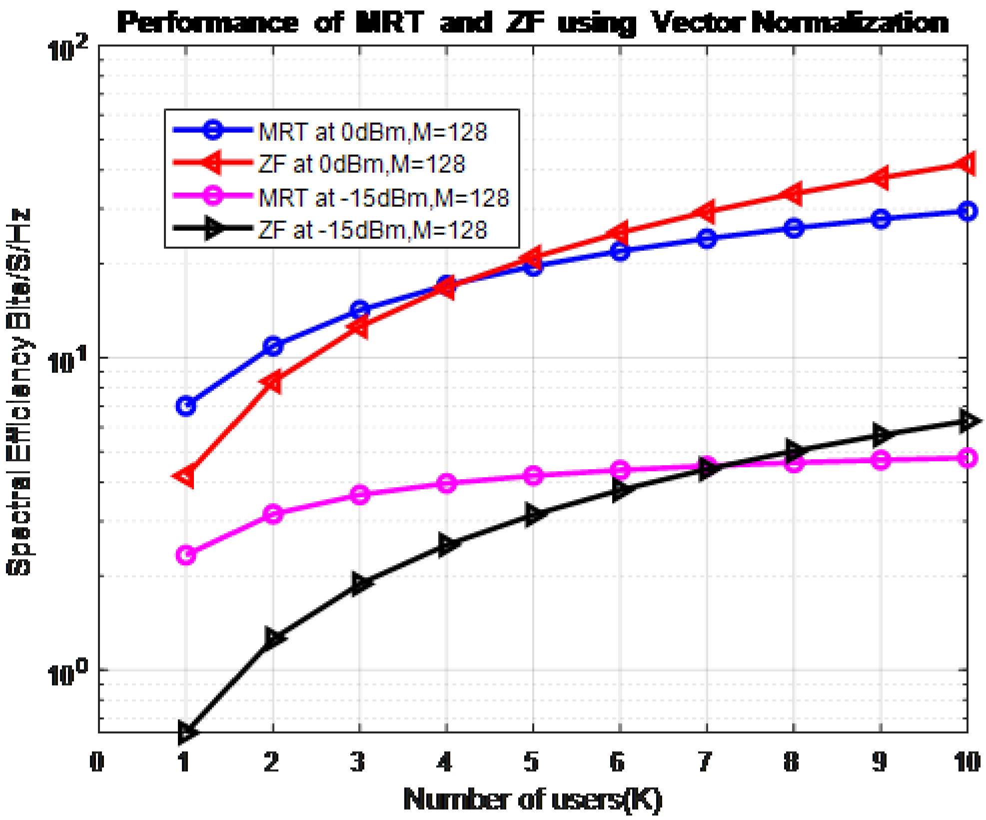

FIGURE 5.10: Spectral Efficiency versus user K curve for MRT and ZF using vector normalization at M(fixed)=128 and K (not fixed)=1 to 10.

TABLE 5.5: Spectral Efficiency comparison table for MRT and ZF precoder using vector normalization

Number of base stations (Fixed) = 128										
Precoder	ZF					MRT				
User,K	2	4	7	8	10	2	4	7	8	10
Spectral Efficiency at -15dBm	1.3	2.52	4.41	5.04	6.29	3.163	3.97	4.51	4.62	4.79
Spectral Efficiency at 0dBm	8.4	16.77	29.35	33.54	41.93	10.9	17.1	24.1	26.03	29.52

5.3.1.3 Energy Efficiency

• **Condition 1**: Plotting energy efficiency versus number of K users curve at 0dBm and -15 dBm for ZF and MRT.

Figure 5.11 shows energy efficiency for the given user boundary based on equation (4.11) , equation (4.42) ,equation (4.43) and equation (4.44). Here we consider fixed number of base station antenna M= 128 and user K ranging from 1 to 10 . From the results executed, we can deduce that as the number of user increases, the

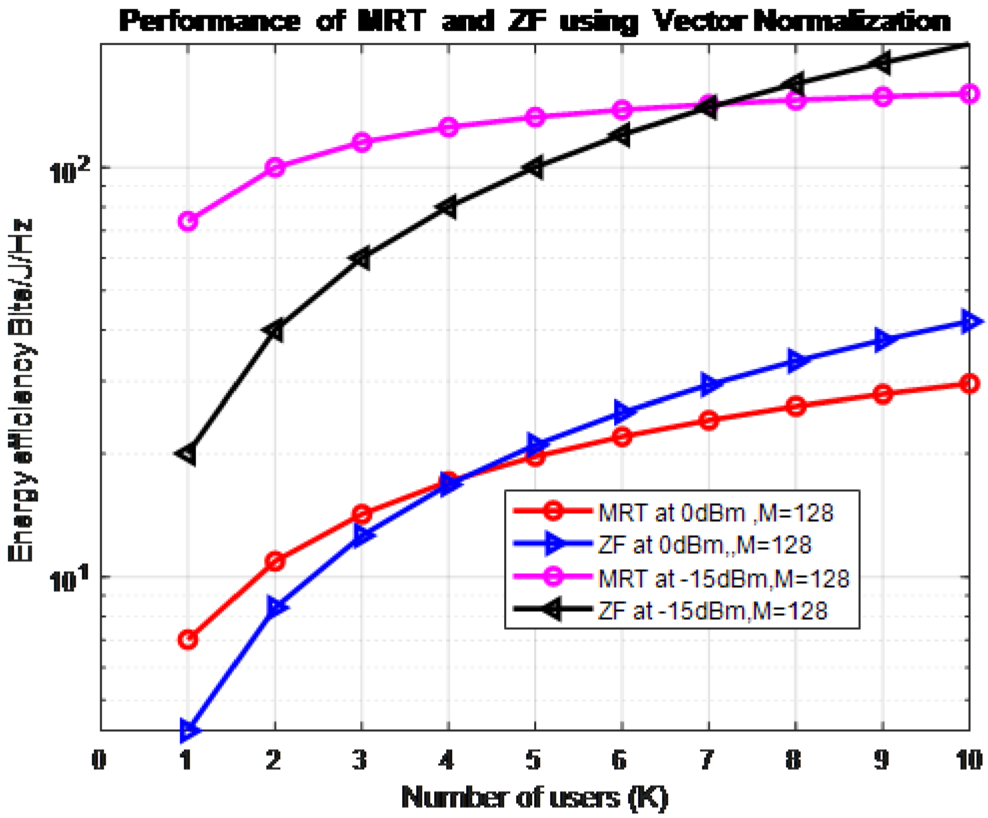

FIGURE 5.11: Energy Efficiency versus number of K users curve at 0dBm and -15 dBm for ZF and MRT.

energy efficiency of MRT precoder fails as compared to ZF precoder. MRT shows better energy efficiency at low power than high power. The assumed user is 10 , for high power the MRT shows better energy efficiency up to 4^{th} user and lesser energy efficiency after the 5^{th} user as compared to ZF precoding scheme. Similarly, for low power the MRT shows better energy efficiency up to 6^{th} user and lesser energy efficiency after the 7^{th} user as compared to ZF precoding scheme.

• **Condition 2** : Plotting Energy efficiency versus number of base station antenna curve at 0dBm and -15 dBm for ZF and MRT. Figure 5.12 shows that the energy efficiency across the given base station antenna, M range according to equation (4.10) , equation (4.42) ,equation (4.43) and equation (4.44). Here we consider the variable number of base station antenna which is from 1 to 128 , the fixed number of user, K= 10.

TABLE 5.6: Energy Efficiency comparison table for MRT and ZF precoder using vector normalization

Number of base stations (Fixed) = 128 (from figure 5.11)										
Precoder	ZF					MRT				
User,K	2	4	7	8	10	2	4	7	8	10
Energy Efficiency at -15dB	40.1	80.11	140.19	160.2	200.3	100.01	125.43	142.623	146.12	151.4
Energy Efficiency at 0dB	8.41	16.82	29.43	33.63	42.04	10.92	17.12	24.14	26.104	29.61
User K (Fixed) = 10 (from figure 5.12)										
Precoder	ZF					MRT				
BS antenna,M	10^{th}	20^{th}	30^{th}	40^{th}	50^{th}	10^{th}	20^{th}	30^{th}	40^{th}	50^{th}
Energy Efficiency at -15dB	-5.81	1.44	8.57	15.6	22.52	8.34	15.18	21.91	28.54	35.09
Energy Efficiency at 0dB	-7.37	1.4	6.78	10.71	13.8	3.96	6.59	8.81	10.74	12.44

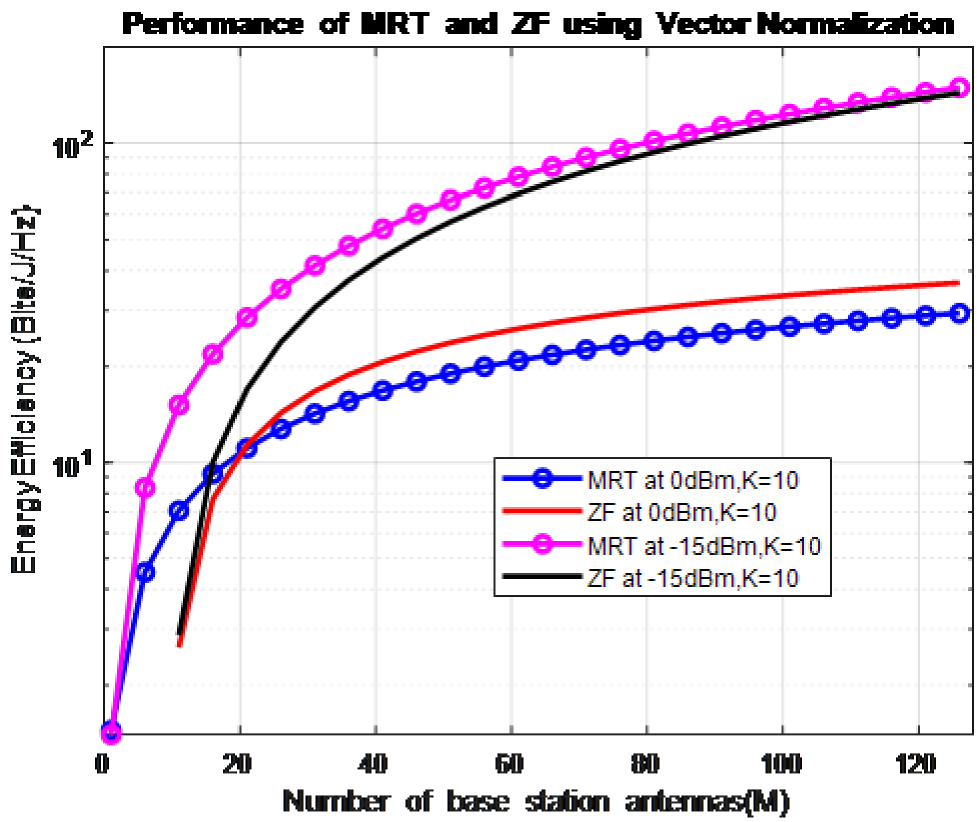

FIGURE 5.12: Energy efficiency versus number of base station antenna curve for ZF and MRT.

The result shows that for lower number of base station antenna MRT will have better energy efficiency than ZF. Whereas, as the number of base station antenna increases, the performance of MRT and the performance of ZF will begin to grow up. On another hand, MRT have better energy efficiency at low power than high power. Similarly, ZF have better achievable sum rate at high power than low power as compared to MRT precoder.

5.3.2 Matrix Normalization for MRT / ZF

Under this section, the performance of zero force (ZF) and maximum rate of transmission (MRT) in a single cell downlink multi user massive MIMO over perfect channel by considering different performance metrics like achievable sum rate, spectral efficiency and energy efficiency depending on matrix normalization have been investigated.

The simulation metrics are the number of user, K=10 , the number of base station antenna, M = 128 ,the downlink transmission power, $\mathbf{p_b}$ = 0dBm and -15dBm.

5.3.2.1 Achievable Sumrate for MRT/ZF

Figure 5.13 and Figure 5.14 shows the achievable sumrate across the given BS transmitter power according to equation (4.10) . For execution we applied the base station antennas, M =128 , the downlink power, SNR(dBm) = [-15:0] and , the user, K =10.

The performance of MRT and the performance of ZF will begin to grow up as the number of BS antenna increases. From mathematical analysis result in Table 5.7 it can be seen that ,the performance interms of achievable rate under matrix normalization for zero forcing and MRT grows by 28.8 % and 27.1 % respectively for low BS transmitter power.Similarly,the performance interms of achievable rate for zero forcing and MRC grows by 11.8 % and 7.32 % respectively for high BS transmitter power.

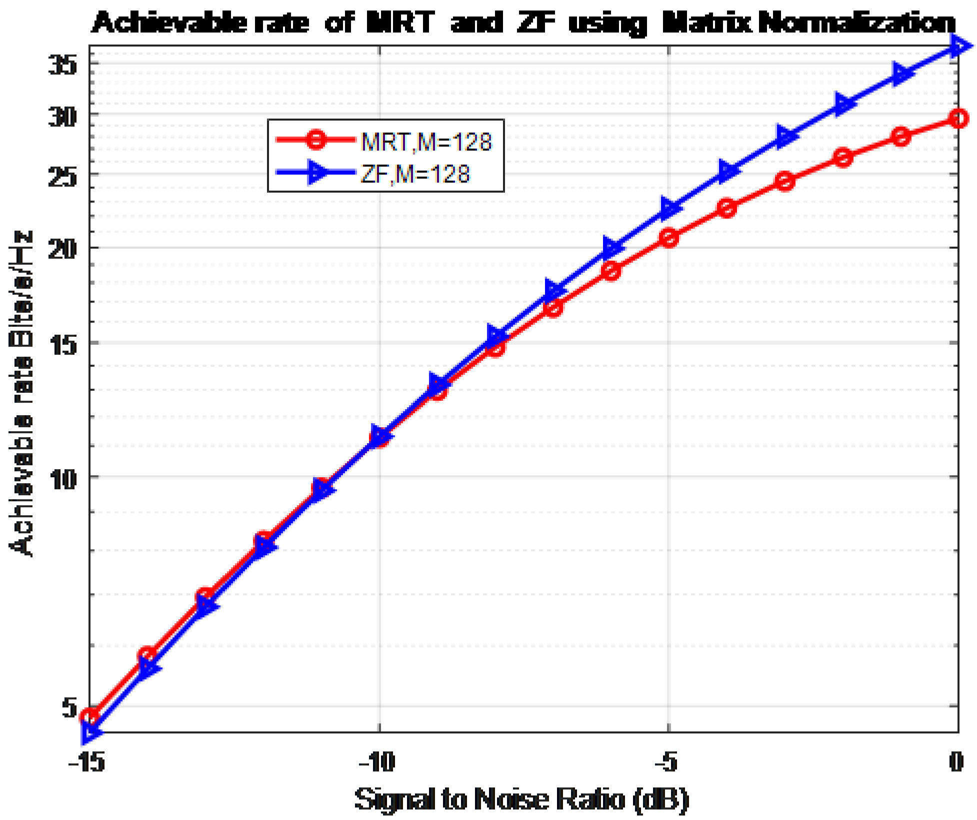

FIGURE 5.13: Achievable sum rate versus BS transmitter power curve ZF and MRT using matrix normalization

Again from the result we can see that for high SNR, ZF gives better performance than MRT. Similarly, for low SNR, MRT gives better achievable sumrate than ZF for the number of base station antenna in the range 1 to 128 . From this we can conclude that MRT will have better performance than ZF when the number of base station antenna is lesser and the given SNR is high.

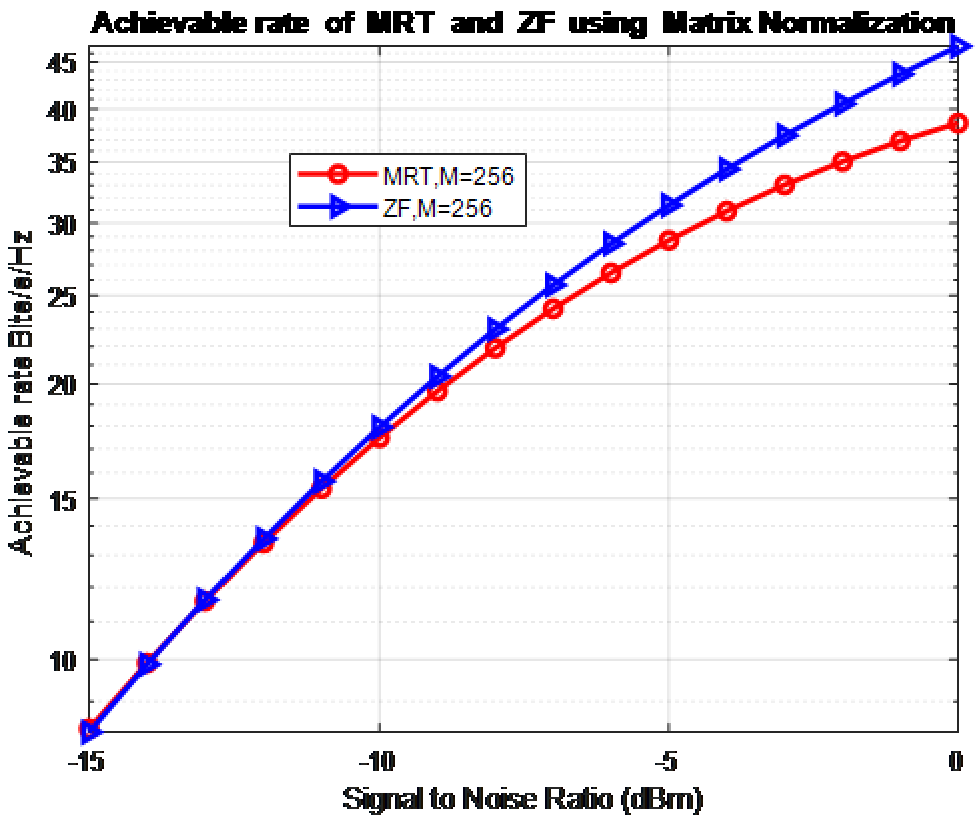

FIGURE 5.14: Achievable sum rate versus BS transmitter power curve ZF and MRT using matrix normalization

TABLE 5.7: Achievable sum rate comparison for MRT and ZF at M=128 and M=256 for under matrix normalization downlink

Power	Low SNR(-15dBm)		High SNR(0dBm)	
Precoder	MRT	ZF	MRT	ZF
Sumrate for M=128	4.819	4.6081	29.6153	36.893
Sumrate for M=256	8.4013	8.3275	38.606	46.837
Performance change(%)	27.1	28.8	7.32	11.8

5.3.2.2 Spectral Efficiency

• **Condition 1** : Plotting Spectral Efficiency versus number of K users curve at 0 dBm and -15 dBm for ZF and MRT.

Figure 5.15 shows spectral efficiency across the given user range according to equation (4.10) ,equation (4.42) and equation (4.45). For execution we applied the base station antennas, M =128 , the BS downlink power = 0dBm and -15dBm , the user, K =1 to 10 . From the result we can see that for low BS transmitter power,MRT gives

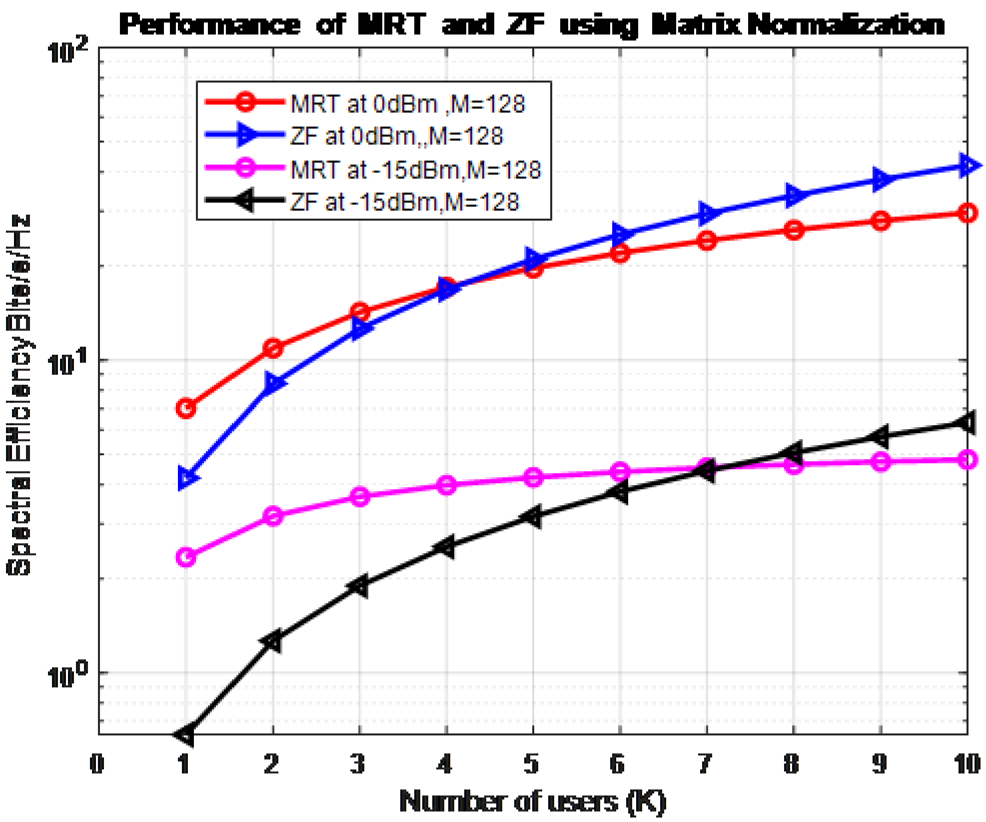

FIGURE 5.15: Spectral Efficiency versus number of k users curve for ZF and MRT

better performance than ZF for some number of users less than 7. Similarly, for high SNR, MRT gives better performance than ZF for the number of users less than five. Whereas, ZF gives better performance when number of user greater than four at high power and greater than seven at low power. From this we can conclude that MRT will have better performance than ZF when the number of user is lesser. As the number of user increases, the performance of MRT decreases or grows slowly compared to ZF.

• **Condition 2**: Plotting Spectral Efficiency versus number of base station antenna curve at 0dBm and -15 dBm for ZF and MRT.

Figure 5.16 shows the spectral efficiency for the given base station antenna range according to equation (4.10) ,equation (4.42) and equation (4.45). For execution we applied the base station antennas, from 1 to 128 , the downlink BS transmitter power = 0dBm and -15dBm , the user, K = 10 .

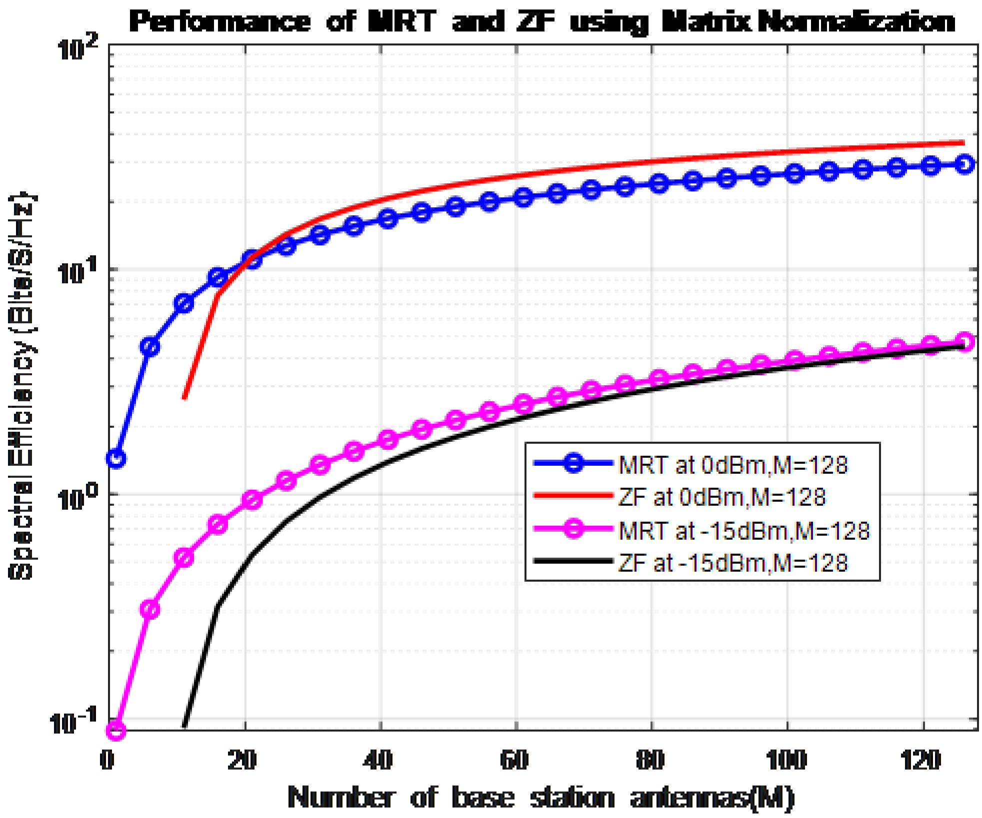

FIGURE 5.16: Spectral Efficiency versus number of base station antenna curve for ZF and MRT.

From the result we can see that for high BS power, MRT gives better performance than ZF for number of base station antenna below 20. MRT has lesser performance above 20 base station antennas than ZF. Similarly, for low SNR, MRT gives better achievable sumrate than ZF for the number of base station antenna in the range 1 to 128 . From this we can conclude that MRT will have better performance than ZF when the number of base station antenna is lesser and the given SNR is high.

5.3.2.3 Energy Efficiency

• **Condition 1** : Plotting Energy Efficiency versus number of k users curve at 0dBm and -15 dBm for ZF and MRT.

Figure 5.17 shows the energy efficiency across the given user range according to equation (4.11) ,equation (4.42) and equation (4.45). For execution we applied the base station antennas, M =128 , the downlink BS power = 0dBm and -15 dBm, the user K= 1 to 10. From the result we can see that for low SNR, MRT gives

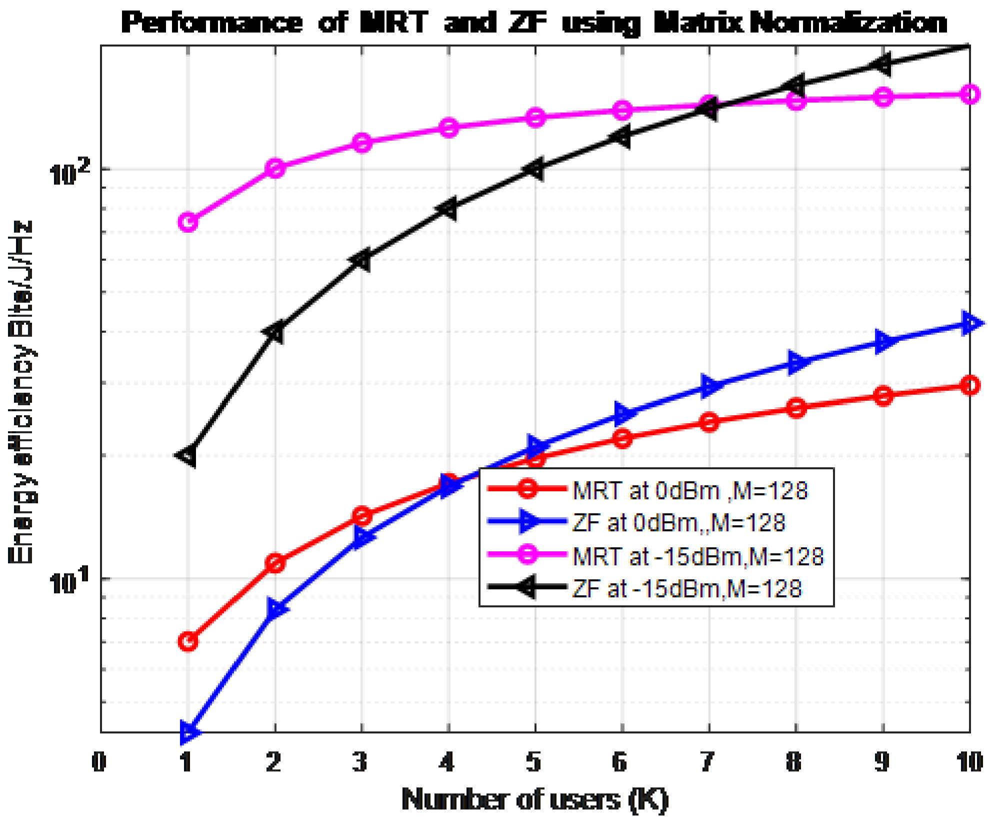

FIGURE 5.17: Energy Efficiency versus number of k users curve at for ZF and MRT.

better energy efficiency than ZF for some number of users less than seven. Similarly, for high SNR, MRT gives better performance than ZF for the number of users less than four and almost the same in the range between four to five. Whereas, ZF gives better performance when number of user greater than five at high power and greater than seven at low power. From this we can conclude that MRT will have better performance than ZF when the number of user is lesser. As the number of user increases, the performance in terms of energy efficiency of MRT decreases as compared to ZF for high SNR.

• **Condition 2** : Plotting energy efficiency versus number of base station antenna curve at 0dBm and -15 dBm for ZF and MRT.

Figure 5.22 shows energy efficiency for the given base station antenna range according to equation (4.11) ,equation (4.42) and equation (4.45). For execution we applied the base station antennas, from 1 to 128, the downlink BS power = 0dBm and -15dBm , the user, K = 10 .

From the result we can see that for high BS power, MRT gives better performance in terms of energy efficiency than ZF for number of base station antenna 1 to 20. MRT has lesser performance above 20 base station antennas than ZF. Similarly, for low BS power, MRT gives better energy efficiency than ZF for the number of base station antenna in the range 1 to 128. From this we can conclude that MRT have better performance in terms of energy efficiency than ZF when the number of base station antenna is lesser and the given SNR is high.

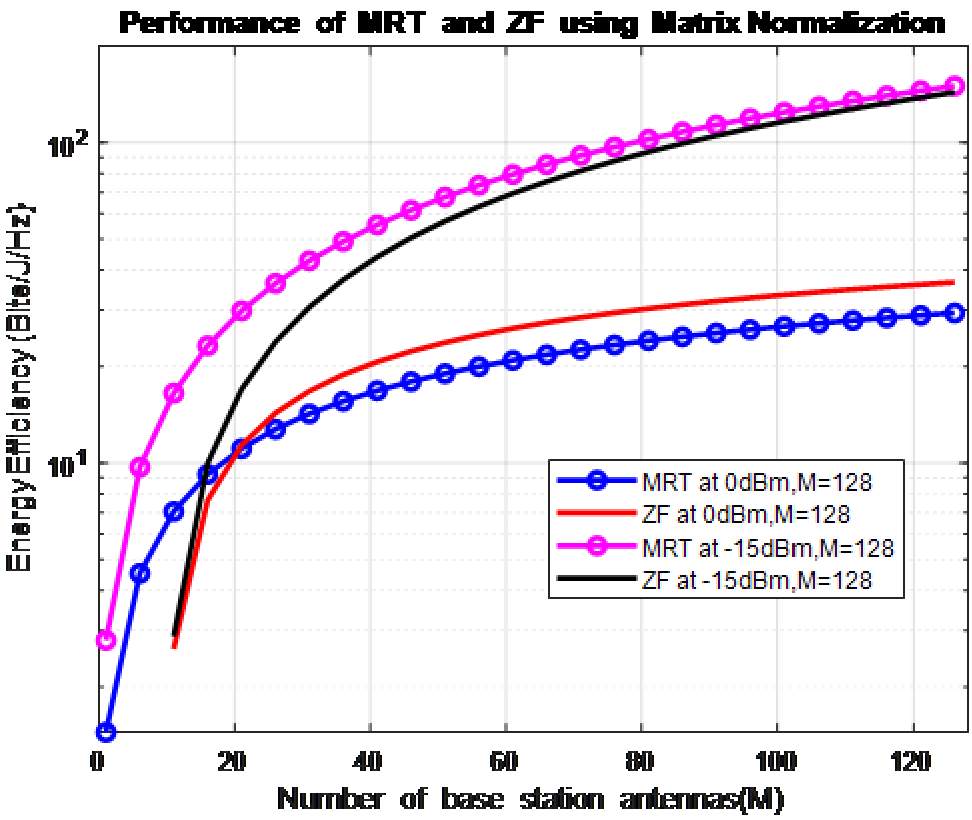

FIGURE 5.18: Energy Efficiency versus number of base station antenna curve for ZF and MRT.

Table 5.8: Energy Efficiency comparison table for MRT and ZF precoder using matrix normalization

Number of base stations (Fixed) = 128 (from figure 5.17)										
Precoder	ZF					MRT				
User,K	2	3	4	7	8	2	3	4	7	8
Energy Efficiency at -15dB	40.1	60.1	80.1	140.2	160.2	100.5	115.9	126.2	143.5	147.1
Energy Efficiency at 0dB	8.41	12.6	16.8	29.4	33.6	10.9	14.2	17.1	24.2	26.1
User K (Fixed) = 10 (from figure 5.18)										
Precoder	ZF					MRT				
BS antenna,M	15^{th}	20^{th}	25^{th}	30^{th}	35^{th}	15^{th}	20^{th}	25^{th}	30^{th}	35^{th}
Energy Efficiency at -15dB	2.876	9.9	16.9	23.9	30.7	16.5	23.3	29.9	36.4	42.8
Energy Efficiency at 0dB	2.63	7.655	11.4	14.3	16.8	7.1	9.2	11.1	12.8	14.3

5.3.3 Comparison between Matrix normalization and Vector normalization

Figure 5.19 shows the achievable sum rate across the entire BS antenna range according to equations (4.10) , equation (4.42) for ZF and equation (4.44 - 4.45) for MRT. For execution we applied the base station antennas, from 1 to 128 , the downlink BS power = 0dBm , the user, K = 10 .

Figure 5.20 shows the achievable sum rate across the entire antenna range according to equations (4.10) and equations (4.42) for ZF and equations (4.10), equations (4.42),equations (4.43) and equations (4.45) for MRT. For execution we applied the base station antennas, from 1 to 128 , the downlink power= -15dBm and the user, K = 10.

From the result,the performance of Zero forcing for vector and matrix normalization is the same. Where as,from Figure 5.20 matrix normalization gives better performance for MRT than vector normalization. Again from Figure 5.19 we can see that the performance of maximum ratio transmitter (MRT) for vector and matrix normalization is the same.

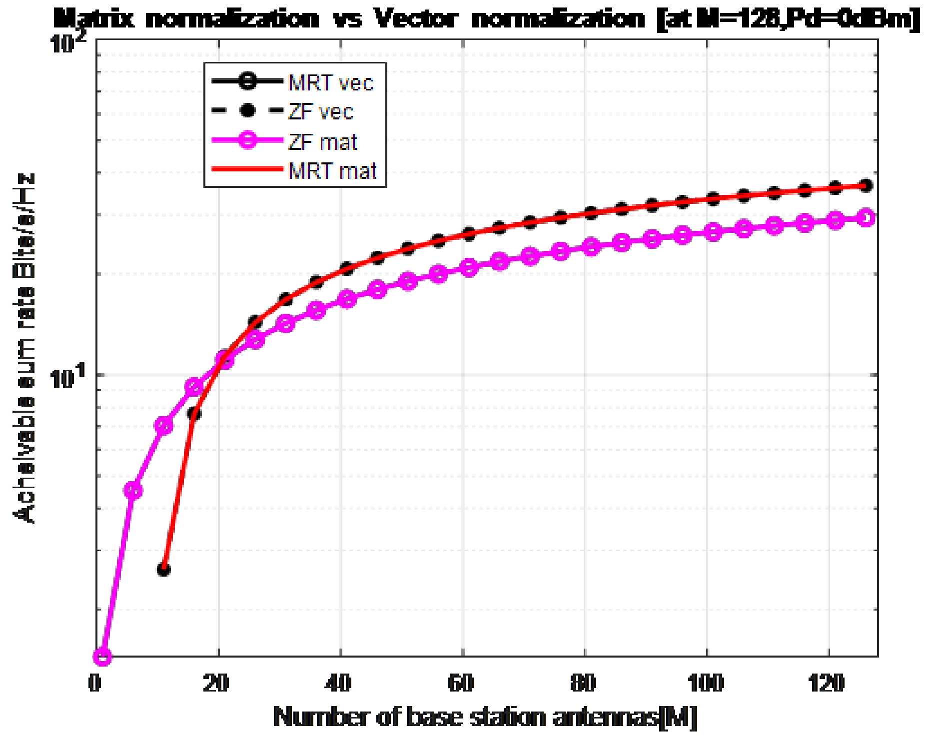

FIGURE 5.19: Matrix normalization versus vector normalization at M=128 ,Downlink BS power = -15dBm for ZF and MRT.

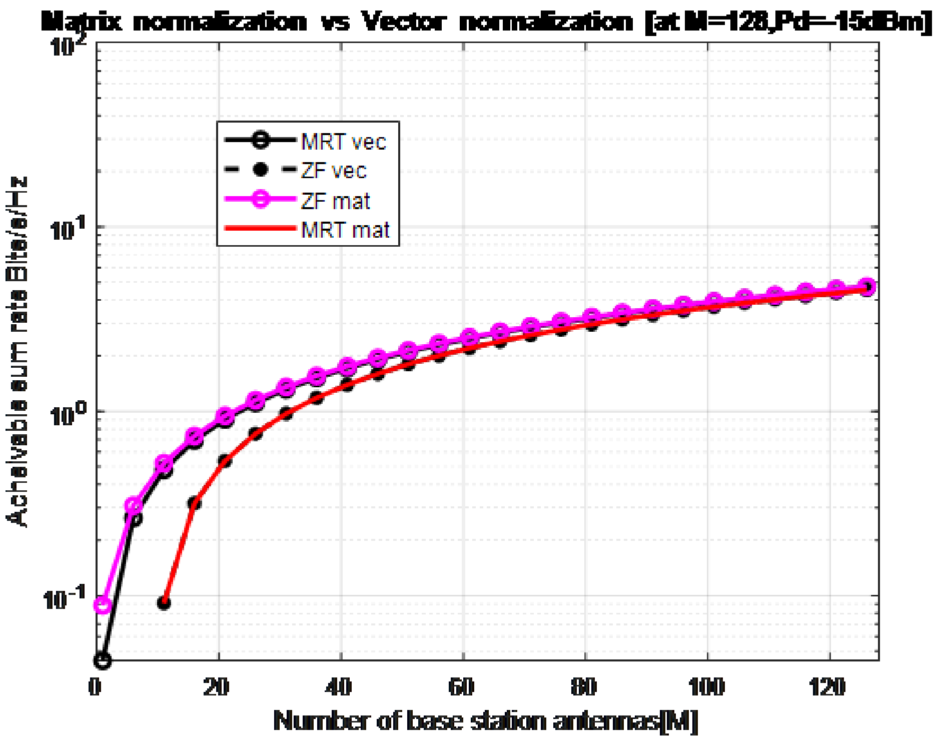

FIGURE 5.20: Matrix normalization versus vector normalization at M=128, Pb=0 dBm for ZF and MRT

Table 5.9: Performance comparison table for MRT and ZF precoder using vector and matrix normalization

User,K (Fixed) = 128								
Precoder	ZF				MRT			
Normalization Method	Matrix		Vector		Matrix		Vector	
BS antenna , M	**120^{th}**	**125^{th}**	**120^{th}**	**125^{th}**	**120^{th}**	**125^{th}**	**120^{th}**	**125^{th}**
Sumrate at 0dBm	35.48	36.1	35.48	36.1	28.39	28.91	28.39	28.91
Sumrate at -15dBm	4.2	4.37	4.2	4.37	4.43	4.59	4.4	4.56

Chapter 6

Conclusion and Future work

6.1 Conclusion

This study analyzes the performance of linear precoders ZF and MRC for uplink. Similarly, the performances of MRT and ZF for downlink scenarios have been analyzed. The analysis depends on different SNR value for both uplink and downlink case. Normalization is used in downlink scenario.A numbers of base station antennas with fixed number of users have been considered. Similarly, multiple users with fixed number of base station antenna have been also considered.

For a given antenna and user configuration, we also derive analytically the SNR level below which MRC should be used instead of ZF. Numerical simulations confirm our analytical results.

The drawback behind Zero-forcing (ZF) receivers is that it take the inter user interference into account, but neglect the effect of noise. With ZF, the multi user interference is completely nulled out by projecting each stream onto the orthogonal complement of the inter user interference.

In conclusion, ZF precoding is used for cell center users which have high SNR, and MRT is better for cell-boundary users that have low SNR for downlink MU-Massive MIMO. Similarly, therefore zero forcing precoding is used for cell center users which have high SNR, and MRC is better for cell-boundary users that have low SNR for uplink MU-MIMO . The performance of Zero forcing for vector and matrix normalization under high or low SNR is the same. Under low BS power matrix normalization gives better performance for MRT than vector normalization.

6.2 Future work

Wireless communication systems have become more and more important as they provide a flexibility and user friendly of application. The massive MIMO technology is one of the technology under wireless communication. As its name indicates a number of antennas are going to be implemented on both user side and base station side. This makes it complex and challenge full task. Here in this work we have tried to analyze one of the challenges linear precoding schemes for specified low complexity precoders.

The recommendations for those working in this area are that one should analyze and investigate the solution for:

- **Pilot contamination reduction**
- **User scheduling**
- **Hardware impairment**
- **Signal detection and Channel estimation** in multi user massive MIMO considering linear precoder.

Future researcher can also do their comparative study on linear and nonlinear precoding scheme in massive MIMO.

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Appendix A

Proofs in linear precoder

A.1 Proofs for Achievable sumrate formula in MRC precoder

The k-th column of the MRC receiver matrix is given by

$$\mathbf{w}_k^{MRC} = max\left(\frac{Desired\,power}{Noise\,power}\right) = max\left(\frac{\frac{\mathbf{p}_u}{N}\sum_{k=1}^{K}\left|\mathbf{w}_K^T h_k\right|^2}{\left\|\mathbf{w}_k^T\right\|^2}\right) \tag{A.1}$$

Since

$$\begin{aligned}
\frac{\frac{\mathbf{p}_u}{N}\sum_{l=1}^{N}\left|\mathbf{w}_K^T h_K\right|^2}{\left\|\mathbf{w}_K^T\right\|^2} &\leq \frac{\frac{\mathbf{p}_u}{N}\sum_{k=1}^{K}\left\|\mathbf{w}_K^T\right\|^2\left\|h_K\right\|^2}{\left\|\mathbf{w}_K^T\right\|^2} \\
&= \frac{\mathbf{p}_u}{N}\sum_{k=1}^{K}\left\|h_K\right\|^2 \\
&= \frac{\mathbf{p}_u}{N}N\left\|h_K\right\|^2 \\
&= \mathbf{p}_u\left\|h_K\right\|^2
\end{aligned} \tag{A.2}$$

Substituting equation (A.2) into (3.5) we will get equation A.3

$$
\begin{aligned}
SINR_{UL}^{MRC} &= \frac{\mathbf{p_u} \left|h_K\right|^2 \left|h_K\right|^2}{\mathbf{p_u} \sum_{n \neq k}^{K} \left|h_n\right|^2 \left\|h_K\right\|^2 + \left|h_k\right|^2} \\
&= \frac{\mathbf{p_u} \left\|h_K\right\|^4}{\mathbf{p_u} \sum_{n \neq k}^{K} \left|h_k h_n\right|^2 + \left\|h_k\right\|^2} \\
&= \frac{\left\|h_K\right\|^4}{\sum_{n \neq k}^{K} \left|h_k h_n\right|^2 + \frac{1}{\mathbf{p_u}} \left\|h_k\right\|^2}
\end{aligned} \tag{A.3}
$$

The achievable sum rate for two different level of SNR

- **For high SNR**

$$
\mathbf{R}_{UL}^{MRC} high = E \left(\sum_{l=1}^{N} \sum_{k=1}^{K} \log_2 \left(1 + SINR_k^{MRC} \right) \right) \tag{A.4}
$$

Inserting A.3 into A.4 we get

$$
\mathbf{R}_{UL}^{MRC} high = \sum_{k=1}^{K} \log_2 \left(1 + \frac{E \left\|h_k\right\|^4}{\sum_{n \neq k}^{K} E \left|h_k h_n\right|^2 + \frac{1}{\mathbf{p}_u} E \left\|h_k\right\|^2} \right) \tag{A.5}
$$

From **Lemma 1**

$$
\begin{aligned}
E \left\|h_k\right\|^4 &= M^2 + M \\
E \left|h_k h_N\right|^2 &= M \\
E \left\|h_k\right\|^2 &= M
\end{aligned} \tag{A.6}
$$

Inserting A.6 into A.5 we get

$$
\begin{aligned}
\mathbf{R}_{UL}^{MRC} high &= \sum_{k=1}^{K} \sum_{l=1}^{N} \log_2 \left(1 + \frac{M^2 + M}{\sum_{n \neq K}^{K} M + \frac{1}{\mathbf{P_u}} M} \right) \\
&= \sum_{k=1}^{K} \sum_{l=1}^{N} \log_2 \left(1 + \frac{M^2 + M}{M \left(K - 1\right) + \frac{1}{\mathbf{P_u}} M} \right) \\
&\approx NK \log_2 \left(1 + \frac{\mathbf{p_u}(M+1)}{\mathbf{p_u}(K-1) + 1} \right)
\end{aligned} \tag{A.7}
$$

- **For low SNR**

$$
\mathbf{R}_{UL}^{MRC} Low = E \left(\sum_{l=1}^{N} \sum_{k=1}^{K} \log_2 \left(1 + SINR_K^{MRC} \right) \right) \tag{A.8}
$$

Inserting A.3 into A.8 we get equation A.9

$$\begin{aligned}\mathbf{R}_{UL}^{MRC}Low &= E\left(\sum_{k=1}^{K}\sum_{l=1}^{N}\log_2\left(1+\frac{\mathbf{p_u}\left\|h_K\right\|^4}{\mathbf{p_u}\sum_{n\neq k}^{K}\left|h_k h_n\right|^2+\left\|h_k\right\|^2}\right)\right)\\ &= \sum_{k=1}^{K}\sum_{l=1}^{N}\log_2\left(1+\frac{\mathbf{p_u}E\left(\left|h_K\right|^2\right)}{\mathbf{p_u}\sum_{n\neq k}^{K}\frac{\left(\left\|h_k h_n\right\|^2\right)}{E\left(\left\|h_k\right\|^2\right)}+1}\right)\end{aligned} \tag{A.9}$$

Inserting equation A.6 into A.9 we will get equation A.10 which is an achievable sumrate under low SNR for MRC precoder.

$$\begin{aligned}\mathbf{R}_{UL}^{MRC}Low &= \sum_{k=1}^{K}\sum_{l=1}^{N}\log_2\left(1+\frac{\mathbf{p_u}M}{\mathbf{p_u}\sum_{n\neq k}^{K}1+1}\right)\\ &= \sum_{k=1}^{K}\sum_{l=1}^{N}\log_2\left(1+\frac{\mathbf{p_u}M}{\mathbf{p_u}\left(K-1\right)+1}\right)\\ &\approx NK\log_2\left(1+\frac{\mathbf{p_u}M}{\mathbf{p_u}\left(K-1\right)+1}\right)\end{aligned} \tag{A.10}$$

For all case M is the number of base station antenna , K is the number of user and $\mathbf{p_u}$ is the average SNR for uplink .