# JIMMA UNIVERSITY

## JIMMA INSTITUTE OF TECHNOLOGY

## FACULTY OF MECHANICAL ENGINEERING

### MSC. GRADUATE IN MANUFACTURING SYSTEMS ENGINEERING

**Analytical modeling and finite element simulation of multiple delamination for a composite beam**

**By**

**Abate Tesfaye Tafa**

**A Thesis submitted to the postgraduate studies of Jimma University in partial fulfilment of the requirement for the degree of Master of Science in Mechanical Engineering (Manufacturing systems engineering)**

**March, 2022**

**Jimma, Ethiopia**

## Declaration

I hereby declare that this thesis work: “Analytical modeling and finite element simulation of multiple delamination for a composite beam’’ is my own work and this work has not been submitted elsewhere for the award of any other degree or diploma. It is being submitted for the degree of Master of Science in manufacturing system engineering and all sources of material used for this thesis have been duly acknowledged.

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| Name of candidate | Signature | Date |

# Approval sheet

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**Jimma University**

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**Faculty of Mechanical Engineering**

**MSc. Graduate in Manufacturing System Engineering**

**MSc Thesis by:**

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# Nomenclature

$\rho_i$ Mass per unit length of beam segment i

a the length of delamination

$A_i$ cross sectional area of beam i

$EA_i$ Axial stiffness of the beam i

$EI_i$ Bending stiffness of the beam i

$\omega$ circular natural frequency

$S_i$ the ferquency domain of magnitude of $s_i$

$L_e$ length of beam

i beam segment number

p the axial force

F the column vector nodal forces

K the stiffness matrix

m the mass matrix

M bending moment

$\tau$ shear force

U columen vector of nodal displacements

N number of elements

E youngs modulus

I second moment of area

$L = \frac{L_e}{N}$ the ratio of the total length to number of element

# Abstract

This study concentrates on analytical modelling and simulation of multiple delamination for a composite beam. A detailed simulation of single, double and triple delamination has been studied by varying delamination ratio and location together with harmonic response of the beam. The works done by other researchers have been studied and presented. Matlab script file used to detect mode shapes and natural frequencies of the composite beam. The Euller-Bernulli beam is presented as beam modeling technique for analytical modeling of normal and delaminated cantilever beam depending on previous models. The simulation software used in this research is Ansys mechanical apdl. The finding of this study shows that delamination can affect the natural frequencies and mode shapes of composite beam.



**Key words:** Delamination, natural frequencies, carbon fiber reinforced polymer, anisotropy

# Chapter 1

## 1.1 Introduction

### 1.1.1 Back ground

Composites have better applications compared to traditional metals however they are very sensitive to the damages caused during their fabrication or service life. The common failure mode occurred in the layered composite is delamination. It is the separation of layers which are bonded together in composite laminate [1]. It may occur due to imperfect fabrication process, incomplete wetting, air pockets entrapped between layers, loss of adhesion between two laminates, material discontinuity, mechanical loading, or certain in service factors, such as low velocity impact by foreign particles. Due to this delamination present in the structure causes the reduction in the stiffness and strength of the composite structure. This reduction in stiffness directly affected the vibration characteristics of the composite structure, such as natural frequency and mode shapes. Reduction in stiffness results to changes in natural frequency; this reduced frequency sometimes closes to working frequency of the system it leads to resonance. Layered structures have seen greatly increased use in civil, shipbuilding, mechanical, and aerospace structural applications in recent decades, primarily due to their many attractive features, such as high specific stiffness, high specific strength, good buckling resistance, and formability into complex shapes, to name a few. The replacement of traditionally metallic structural components with laminated composites has resulted in new and unique design challenges. Metallic structures exhibit mainly isotropic material properties and failure modes. By contrast, composite materials are anisotropic, which can result in more complex failure modes [2].It's known that there are many materials which can be referred to as composites and still naturally available. Wood can be a good example in such a way that it contains fibers with association of a resinous matrix. Normally nature does a better job than human beings, but there is always shortcomings both to naturally occurring and artificial composites. Among some the main disadvantages can be anisotropy. It is an indication that some properties of the composites may exhibit different in both longitudinal and lateral directions.

Artificial composites can be manufactured by combining two or more materials. The mixing is not at a molecular level. The essence behind this method is to develop a better material that holds superior mechanical properties by reducing material defect after mixing is feasible. Based on design specification and requirement tougher, heat resistant, corrosion resistant, lighter, impact resistant and harder materials can be manufactured. For example, a material by the name ‘cermets’ is a combination of ceramics and metals. It’s applied where working at a high temperature and high speed is required such as a machine tool tip. Ceramics, polymers and filamentary metals can be combined with bulk forms of these materials can be used to manufacture carbon-reinforced plastics, metal-matrix composites and ceramic-matrix composites.

The importance of use of composite materials has been increasing consistently in different industries like civil engineering, mechanical engineering, aerospace engineering etc. due to their advantageous characteristics. One of the most remarkable properties that structures made of composites possess is their very large stiffness to weight ratio. Composites have an excellent combination of high strength and stiffness with low weight. This property among many others has flexibility to adapt different shapes, protection against corrosion, gives the possibility to look for new engineering challenges and replace the traditional materials with composites. Composite laminates are widely used in different areas due to their easy fabrication and effectiveness and because of their versatility in the orientation of the fibers. Despite of their benefits composite materials have their disadvantages. Due to their layered nature and the interaction between both materials, fibers and matrix, composite materials are prone to different failures modes, the most common of them being delamination, which can cause irreversible damage. This failure causes the separation of the layers and induces significant loss of mechanical strength. Delamination is probably the most dangerous defect in composite materials because it can appear suddenly without any notice and it keeps developing to collapse the whole structural member. Composite materials with the defect of delamination can lose up to 60% of their stiffness and still remain visibly unchanged. The development of robust techniques for detection delamination is essential to avoid such a failure. Since delamination damage leads to stiffness loss of the structure, the modal properties like natural frequencies, damping ratio and mode shapes may vary also [3].Composites normally contain a large group of materials. It’s possible to put more examples, concrete can be a familiar example to these group of materials, with sand particles and aggregate of graded sizes in a matrix of hydrated Portland cement. Generally, composites are well known and well used for many years and now a days scientists especially material scientists know how to control their properties, which simply means the form, the quantity and the distribution of the reinforcing phase can actually be altered.

Normally, properties of engineering materials should be exactly determined. Usage of composite materials in engineering or manufacturing application requires a deep understanding of the nature and properties of their structure. To use composites in a particular application, need of flexible design and prediction of some properties is highly essential. Currently properties of composites can be well predicted by use of models such as mathematical models, software based techniques, finite element analysis methods and others. But, due to the complexity of composites most of properties may not be accurately predicted. Carbon fiber reinforced polymer (CFRP) is one of the well-known and widely used composite, its expensiveness might be one of the drawback of this material. It has a superior mechanical properties with a high tensile strength and increased stiffness. It's strength and tensile properties can be compared to steel but the weight is small, which is only weigh about a quarter as much. In the present time, carbon fiber materials are overtaking aluminum in aircraft manufacturing industries [4]. In composites, the reinforcement and the matrix have their own distinct feature and exhibit different behaviors.

### 1.1.2 Reinforcement

It's the building block of a certain composite. Mechanical properties such as structural rigidity, strength and other properties comes from it. According to this study, glass-fibers were the earliest to be used as a reinforcing materials.

### 1.1.3 Matrix

It's a tie-up that joins the fibers together and gives adjustment in the required (stressed) direction. Generally, composite materials withstand loads such as tension, shear, compression and other loads by the help of this feature. In addition to that it separates the fibers from one another so that they act independently. Most of the time reinforcing fibers are brittle and hard, so that the matrix protects from damage such as abrasion or from sudden impact loads. As a result matrix materials should be weak and flexible when compared to the reinforcement. Generally, strength and moduli can be neglected in calculating composite properties.

### 1.1.4 Carbon-fiber-reinforced plastic (CFRP) and its properties

Carbon-fiber-reinforced plastic (CFRP) contains carbon-fibers and is a highly strong and light fiber reinforced material. It's applied mainly where high strength-to-weight ratio and rigidity (stiffness) is required. Major areas of application includes aerospace, ships, sport equipment, and automotive industries.

In carbon-fiber-reinforced-plastic the carbon fiber function as a reinforcement that provides its strength. The matrix is sometimes a polymer resin, such that epoxy that holds the reinforcement together.

Figure 1.1 Raw CFRP material [4]


Figure 1.2 Air bus 350 [4]

As stated before, the reinforcement gives the CFRP its rigidity and strength that can be measured by elastic modulus and stress respectively. Steel and aluminum exhibit isotropic properties and mode of failure. But, CFRP its properties vary directionally.

$$Ec = VmEm + VfEf \qquad 1.1$$

The above equation (1.1*)* is valid as long as the fibers oriented in the direction of the applied load.

Where, Ec -Total composite modulus

Vm -Volume fraction of the matrix

Vf -Volume fraction of fiber

Em -Elastic moduli of matrix

Ef -Elastic moduli of fiber

In another case, when the fibers are oriented transverse to the applied load the elastic modulus can be calculated by the following equation,

$$\text{Ec} = \left(\frac{\text{Vm}}{\text{Em}} + \frac{\text{Vf}}{\text{Ef}}\right) \qquad 1.2$$

### 1.1.5 Defects in composites

Defects are common on many practical reinforced plastic composites due to poor manufacturing process or other reasons. Composites require a very controlled manufacturing environment and tools to obtain their best feature. They exhibit anisotropic properties and complex mode of failures. It is known that composites are made up of different materials with different thermal expansion coefficients in other words during cooling there might be a tendency of developing residual stress that is sufficient to crack the brittle matrix. Some of the commonly occurring defects are delamination, misaligned or broken fibers, non-uniformity in fiber distribution, voids or pores in matrix rich region, overlapping and gaps, unbalance of fiber-volume fraction, resin cracks, due to mechanical damage around machined holes and local bond failures in adhesively bonded composite components [5].

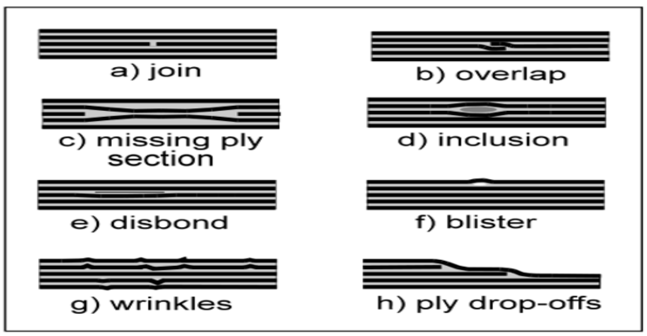



Figure 1.3 Defects in composite laminates [5]

In the present time, metals and their structure are being substituted for a new state-of-the-art materials called composites. This results in a new and unique design requirements and challenges. It is clear that metals and metallic structures exhibit mainly the same material properties and their failure modes will be much simpler to analyze and predict. By contrast, composites exhibit anisotropic material properties that can lead to a more complex failure modes. Delamination can arise from material or geometric discontinuity, loss of adhesion between two layers, due to manufacturing defects and mechanical loading can be another causing factor. Generally, if delamination is present in a certain structure it may significantly reduce the strength and stiffness of the structure. This reduction in stiffness affects the vibration characteristics of the composite structure, that is the natural frequencies and mode shapes of the system will be affected, which in general changes in the natural frequency as a result of reduction in stiffness will lead to resonance, that is if the reduced frequency is close to excitation frequency.

Practically, there are a number of non-destructive test methods that allow to detect and analyze delamination in different structures. The following are the major ones,

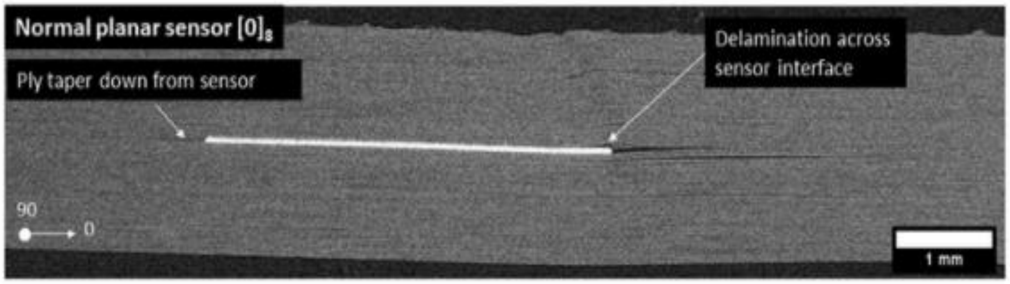
- ✓ Visual inspection(for external delamination)
- ✓ Tap testing(by detecting sound)
- ✓ Ultrasound
- ✓ Radiography, etc.

Visual inspection; as the name indicates detection of delamination by use of naked eye without using any equipment and which is applied where the delamination is visible and also if it occurs at the edge of structures.

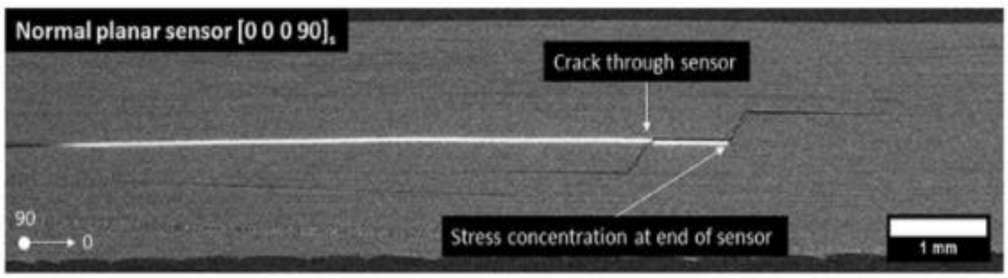
Tap testing; carried by slightly hitting the composite material by some hammer or other relatively hard object to find and analyze sound changes in the system. Generally, composite laminates gives a clear ringing sound that is an indication of a normal bond in the material, but if the sound is some kind of duller it may indicate delamination. This type of testing is generally applied for finding defects in relatively large structures, but if the size of the material is smaller it becomes difficult to apply this method. Generally, this testing method is very subjective that entirely depends on the hearing capacity and judgment of the operator.

Ultrasound; this method depends on reflected ultrasonic waves to detect, analyze and interpret delamination and other defects in composite materials and their structure. Most commonly it’s applied to find delamination in dense multi-layered composite structure.

Radiography; it’s one of the most common and widely used inspection method in composite manufacturing facilities and other testing facilities. We have already discussed that delamination type of damage is one of the most commonly occurring and highly critical damage in fiber reinforced polymers. According to many research results radiography testing is the most efficient and reliable method to detect delamination. It utilizes x-ray beam and it's oriented at a slight angle. This method is highly applied for thicker composite materials.




(a)




(b)

Figure 1.4 A) Delamination across the sensor surface [5]   B) Crack penetration through sensor [5]

## 1.2 Environmental aspects of composites

### 1.2.1 Disposal and recycling

According to this study, composites especially CFRPs when protected from the sun light their service life will be longer. For example, it's possible to mill down CFRPs but the fibers gets smaller and weaker in strength although it's possible to re-use them in applications where strength is not a priority. For instance it can be re-used in laptops and electronics.

## 1.3 Problem statement

In the present time conventional metals and alloys are being substituted by a laminated composite structures in manufacturing industries. The increase in utilization of composites allow the need of accurate modeling and prediction of failures. One of the most dangerous failure in composite laminate is delamination. Since it grows beneath the surface of the structure it's very difficult to know the condition of the structure. When delamination occurs in a system it affects the modal properties or vibration characteristics of composite structures. Delamination can be caused by one of the following; imperfect bonding, due to crack, chemical corrosion, and due to manufacturing defects such as separation of tiles or broken fibers. Generally, when delamination present in structure, the bending stiffness of the material at that cross section will decrease and that results in reduction of the natural frequency of the system. A small change in the value of natural frequency can be a great indicator for the presence of delamination. It's known that frequency is directly proportional to the square root of stiffness and a little variation in frequency mean a large failure (damage) to the stiffness of the composite structure.

## 1.4 Objective of the study

### 1.4.1 General objective

To model delamination analytically and by use of finite element simulation software for composite beam.

### 1.4.2 Specific objective

- ✓ To formulate analytical formulation, comparisons and simulate mode shapes for cantilever beam
- ✓ To compare analytical values of delaminated beam with previous analytical values for clamped-clamped beam
- ✓ To compare analytical values with finite element simulation for cantilever beam
- ✓ To study effect of delamination under various delamination ratio and location for cantilever beam
- ✓ To study effect of delamination on harmonic analysis for both normal beam and delaminated beam for cantilever beam

## 1.5 Research questions

- ✓ How to formulate analytical formulation, comparisons and simulate modes shapes for cantilever beam?
- ✓ How to compare analytical values of delaminated beam with previous analytical values for clamped-clamped beam?

- ✓ How to compare analytical values with finite element simulation for cantilever beam?
- ✓ How to study effect of delamination under various delamination ratio and location for cantilever beam?
- ✓ How to study effect of delamination on harmonic analysis for both beam normal beam and delaminated beam for cantilever beam?

## 1.6 Motivation of the study

In the current time the utilization of carbon fiber reinforced plastic (CFRP) is increasing at an alarming rate with the ability to detect and also model its associated mode of failures. Hence, besides the non-destructive testing, another alternative methods such as the computational (analytical) and finite element based software simulation methods can be very useful in places where the laboratory setup and the raw material itself is rare to find.

## 1.7 Scope of the study

The overall scope of this research concentrates on capturing the modal frequency variation and mode shape variations which includes analytical modal findings of an intact beam together with simulating its mode shapes. From previous two researchers' exact boundary condition (clamped-clamped) and material properties taken to validate the proposed analytical method. Finally, result, conclusion and recommendations are made in the last chapter.

## 1.8 Significance of the study

This study can be significantly used in areas where raw materials and laboratory equipment that used to detect condition of composite structures is rare then this study is an alternative method to determine the condition of structures. Using finite element simulation based software and analytical methods it's possible to model and predict failure modes of important structures.

## 1.9 Organization of the study

The first chapter contains, background, general and specific objectives, scope of the study, significance of the study, research questions, applications of composites and different issues related to composite structures.

The second chapter is about literature survey and research gaps.

The third chapter discusses general methodology, analytical and finite element simulation method.

The fourth chapter, comparisons and findings of the research and previous works are presented

The fifth chapter, presents conclusions, recommendations, limitations of the study and future works.

# Chapter 2

## 2.1 Literature review

In this chapter previous studies, which are directly related to the current study are thoroughly assessed. Most of the studies are journals and books which are collected from recognized sites. It's a convenient method to classify the works as analytical, finite element and experimental models for better understanding.

### 2.1.1 Analytical models

Venkate Gowda and N.Rajanna [6] Presented investigation focused on studying the effect of delamination location and estimating its effect on natural frequency of composite beams using commercial finite element software, ansys. Different ply lay ups are considered for study in both finite elements analysis and analytical formulations. The finite element results are compared with the analytical ones to validate the perfect beam design. The natural frequency is found to reduce with increase in delamination size for both cantilever and simply supported beam. Additionally, the natural frequency in simply supported composite beam is found to be higher than that in cantilever composite beam as expected due to the increased bending stiffness. Further, natural frequency of symmetric laminate is found to be higher than the cross ply laminate for both cantilever and simply supported composite beams. Wang and Gibby [7] responsible for the first delamination model, the work emphasizes on a two-layer sandwich beam (single-delamination) and the beams governed by Euler-Bernoulli beam assumption. The assumptions used is free mode assumption such that the segments of the beams that is, the lower and the upper segments were assumed to vibrate freely, which is independent of each other. So that the used model is called 'free mode' delamination model. Natural frequencies show a great reduction. K Torabi [8] a new practical procedure is presented for delamination detection in beam-like composite structures. This technique identifies the delamination axial location and its length accurately based on the only first natural frequency. An additional simply support condition besides the boundary conditions is defined and it is moved along the beam length. When this support is located on delamination, especially its tips, the frequency reduction will be noticeable. Simulating this idea in abaqus finite element software shows that the location and size of delamination can be detected accurately. To verify the numerical results, some experiments are conducted and a new fixture is designed and manufactured for simulating the additional moving support along the length of the beam. Both finite element and experimental results show the ability of the proposed method in delamination detection for different boundary conditions. Also, this new simple and applicable technique can be used even for small delamination lengths by monitoring only the first natural frequency, unlike the available methods in the literature. Mujumdar [9] the researcher put forward the constrained mode delamination model. Forecasting of the vibration characteristics for off-mid plane delamination was studied. Constrained mode simply means that, the transverse displacements of the top and bottom beams to be equal but this technique leads to increased stiffness of the system and that in turn affects the natural frequencies to be over predicted or calculated. Wang, and Gibby [7] once again Euler-Bernoulli theory used, a through width delaminations to examine free vibration of isotropic beam. Coupling of longitudinal and flexural motions of delaminated layers were examined. The assumption was free mode delamination. The relationship between delamination and natural frequency is the focus of the study. Pardoen, and Tracy [10] choose a simply supported composite beam to propose a constrained model. The results of the study shows the relationship between delamination length and its effect on beam natural frequencies. The researchers studied the relationship between delamination, vibration and natural frequencies. Hopkins and Saravanos [11] the two researchers' proposed and analyzed analytical solution for predicting mode shapes and natural frequencies of a delaminated beam, data from other literature used to validate the study. Shu [12] proposed analytical solution for a sandwich beam by considering both 'free' and 'constrained' mode. The main work concentrates on the influence of the contact modes in the same way the study used previous works to validate the result and good agreement has been reached. Shen M.H and Grady J.E [13] in another work examination and analysis of the connector conditions that is the 'rigid connector' and the 'soft connector' by investigation of local deformations near the delamination front. Jian and Chyanbin [14] assumed delamination to happen between the upper and the core of honey comb and developed 2D-model by consideration of transverse shear effect and rotary inertia for free vibration analysis. Lee and H. Lee [15] the presented model by the researchers tells that bending extension coupling and transverse shear deformation can be a powerful method for the examination of free vibration in composite beams at random boundary conditions. Sunghee Lee, Park, George Z [16] developed analytical models of beam-column single delamination and validated result by performing experiment. The assumptions are general kinematic conditions, constant slope and curvature at the delamination tip and obtained natural frequency and elastic buckling load. Ali and Mohamed [17] proposed and presented a mathematical model for partially delaminated beam, by considering lateral strains and the merit of the study is that delamination can be analyzed and modeled at any arbitrary location on composite structure. Simitses and Salaam [18] analyzed and examined a model to predict critical loads for homogeneous plates in both simply supported and clamped beam. Additionally, the researchers studied effects of delamination position, size, and other parameters on critical loads. The conclusion of the study shows the relationship between delamination and buckling load. Moshe [19]studied free vibration analysis of laminated composite. For validation purpose experimental and analytical evaluation of natural frequencies and mode shapes has been performed by the researchers. The general vibration principle used to formulate the equation of motion, and also dealt with the Coupling effect between longitudinal vibration and bending vibration. The values of natural frequencies showed reduction as delamination is induced in the structure. G. Fredric D and M.Esther [20] the research shows the effect of drilling on composite structures in the aeronautical assembly process. That is, drilling may generate delamination at the exit of the hole and this is unfavorable situation, the created delamination is directly related to the axial drill force, the characteristics of the composite material and the distribution of load along the drill edges. The paper studies load distribution along the drill edges and other points. The research proposed orthotropic analytical model to determine delamination. Then comparisons made by the researchers with other literature and experimental findings.in conclusion, results shows that delamination occurs in a mixed mode and not only in mode one. Robin [21] analytical model has been developed by the study that shows the initiation and further expansion consequently growth of delamination when composite plate is exposed to a small mass impact force. The researcher developed a solution by integrating a single integral equation, an agreement has been made between numerical solution and experimental investigations.

### 2.1.2 Finite element model

It's the most important tool for determination of vibration characteristics of a given system. It is a general systematic approach to formulate the element mass and stiffness matrices, which are constant in the frequency domain for a given system. One of the advantages of this method is it can be applied in composite systems with different geometrical arrangements and loading conditions. It's quite flexible method. Ullah [22] the research deals with a deformation behavior and damage in composite laminates due to quasi-static bending. Experimental tests are carried out to characterize the behavior of a woven CFRP material under large-deflection bending. Two-dimensional finite element (FE) models are implemented in the commercial code Abaqus/Explicit. A series of simulations is performed to study the deformation behavior and damage in CFRP for cases of high-deflection bending. Single and multiple layers of bilinear cohesive-zone elements are employed to model the onset and progression of inter-ply delamination process. Numerical simulations show that damage initiation and growth are sensitive to a mesh size of cohesive-zone elements. Top and bottom layers of a laminate experience mode-I failure whereas central layers exhibit a mode-II failure behavior. The obtained results of simulations are in agreement with experimental data. Mahapatra and Gopalakrishnan [23] studied and showed that vibration analysis of composite laminate beams by use of a method of finite element analysis. The study concentrates on asymmetric laminate beams. Ju and Lee [24] the method used in this study is Timoshenko beam theory to present and discuss the finite element method for composite beam. Comparisons made by the study with previous works and reached in a good agreement with proposed model. Hashmi [25]**,** proposed the method of weighted residuals and Galerkin finite element formulation, the exact variation of the geometry, material properties can also be modeled directly in the formulation. Alfano and Crisfeld [26] the two researchers presented a model that concentrates on singular delamination (delamination that involves four individual beams) by use of FEM and they used interface elements and an interface damage law, with the use of linear elastic fracture. K. Alnefaie [27]delamination is also a common occurrence in laminated composite plates. The study formulated a technique that determines delamination in this type of geometrical features (plates). A 3D-finite element model was introduced on the paper and different calculations were performed such as natural frequency and modal displacements for various dimensions and delamination location. Finally comparison was done by the researcher in such a way that numerical results show good agreement with experimental data and reduction in natural frequencies observed each time. R. Gadlerab [28] suggested that local flexibility changes due to the presence of delamination on composite laminated beam. The researcher used finite element method to model delamination, and determined effect of delamination length and its effect on natural frequencies. The study analyzed delaminations at different boundary conditions. Finally concludes that increase of delamination ratio increases deviation in the system natural frequencies. N. Hu a, H. Fukunaga b ,M. Kameyama Y. Aramaki b , F.K. Chang c [29] presented and discussed a FEM model based on a simple higher-order plate theory to study the vibration characteristics of delaminated composite plate for moderate thickness. Valeria and Kardomateas [30] studied delamination crack growth in laminated composites by use of experiments and finite element (FE) models. The FE models involve a cohesive layer that is used to simulate the crack propagation and deboning. Calibration of cohesive parameters from the experimental load displacement. Performed tests on cross-ply graphite/epoxy specimens under static conditions. Comparisons of crack growth and strain measurements with the FE models made in the study. Predicted models are in good agreement with the experiment. Jaehong [31] the study used a layer wise theory to explain the free vibration analysis of the beam with internal delamination. Both Hamilton's principle and a finite element method used by the study to formulate the problem. Comparison of numerical results with other works addressing the effects of the lamination size, angle, location and number of delamination on vibration frequencies of delaminated beams is also analyzed in detail. In conclusion, the paper recommends the above approach is appropriate to study modal characteristics of a delaminated composite laminate beam. Ostachowicz [32] the study proposed a damped beam with single delamination at the center. Finite element method is intensively used in the study. The study developed an algorithm based on the Newark method and that associates a Newton-Raphson based method for restating the equation of motion. The algorithm analyzes the vibration characteristics. The paper is all about the vibration responses of the beam due to harmonic and impulse excitations, at various delamination location, delamination length and the changes in the dissipation of damping energy due to the delamination. Yam, Cheng and Wong [33] studied modal characteristics of multiple layers of plates that involves delamination. Presented a more accurate three dimensional finite element model specifically for carbon fiber reinforced plastic (CFRP). Different dimensions of delamination considered to analyze natural frequency, modal strain, and modal displacement. Comparisons made between numerical results and experimental data and showed a good agreement. According to the study results obtained by this method can be a useful tool in the detection of multiple delamination in composite materials. Kyeongsik and Jim [34] studied the effect of manufacturing defects. Progressive delamination of carbon/epoxy composite laminated beam under four-point bending test conditions. Characterization and transformation wavy plies and pure resin that had flowed out were characterized from surface photography and transformed into finite element modeling by use of semi-automatic approach. The mainly applied method was a bit different approach called the cohesive zone modeling (CZM) examination of numerical results and effects of lamination sequence is also studied by the paper. Yassin, Osman and Mohammed Suleiman [35] used first order shear deformation (FSDT) theory for laminated composite beams to study composite laminate beams with natural vibration and finite element analysis is once again used to reach at a numerical solution of the governing general differential equations. The paper deals with free vibration analysis of laminated beams with rectangular cross section for various combinations of end conditions. For verification of the present model evaluation and comparisons with previous literatures is made and good agreement is reached. M. Imran, R. Khan, And S. Badshah [36] presented and discussed free vibration analysis of a laminated plate with delaminations using a layer wise theory. Derivation of equations of motion from the Hamilton's principle and a finite element method based on the first order shear deformation theory is also developed to formulate the problem. Validation of the developed models has been established by comparing the responses with those available published literature. The results of their finding are useful for detecting delamination in multi-layered composite materials. It was shown in the result that the different mode shapes of the frequency of the plate decreases if the delamination area increases. Muhammad. I, Rafiullah.K and Saeed.B [37] the study emphasize on, finite element and analytical model to analyze the influence of delamination size and stacking sequence on natural frequencies of carbon fiber reinforced plastic composite (CFRP) plate with and without delamination. Simply supported beams with associated boundary conditions are applied. Performed experiments to study and predict the vibration characteristics of CFRP delaminated composite plate. Used simulation software ABQUS that used to model and analyze the vibration response of carbon fiber reinforced polymer composite plate for the given boundary conditions and the effect of stacking sequence and delamination size was calculated, another method called the Rayleigh-Ritz method to find the natural frequencies for different delamination sizes and stacking sequences. The result of the study concludes that natural frequencies were significantly affected by the delamination sizes and stacking sequences. Stacking sequence used by the researchers of (0/90/45/90) showed higher values of natural frequencies in lower mode subjected to all-sides simply supported boundary conditions. K.Torabi, M.Shariati and M.Heidari [38] presented a new practical procedure for delamination detection in beam-like composite structures. The technique identifies the delamination axial location and its length accurately based on first natural frequency. Simulation is done in ABQUS finite element software and shows that the location and size of delamination can be detected accurately. To verify the numerical results were validated with experiments, and they prepared a new fixture and manufactured for simulating purpose of the additional moving support along the length of the beam. The study showed that both finite element and experimental results show the ability of the proposed method in delamination detection for different boundary conditions. Conclusions indicate that the new technique can be a useful tool to detect small delamination in lengths. Venkate.G and N.Rajanna [39] the concentration of the study goes to use of finite element simulation software such as (Ansys) to study and analyze the effect of single delamination on the natural frequency of the beam. Considered different ply-lay ups for research in both finite elements analysis and analytical formulations. Comparison of finite element results with the analytical results to validate beam design also performed. According to the study, the natural frequency tends to decrease with an increase in delamination size for both simply supported beam and cantilever beams. Sarat C, P.Satish, N.Guru, M.Babji [40] defined delamination and its harm in composites, also put forwarded that early detection of delamination in composites helps to control the quality of the composite structure. The study proved that delamination has the ability to deteriorate the compactness or solidness of the structure and as a result the natural frequencies drop down. The research specifically focused on single delamination of E-glass epoxy both analytically and by use of Ansys. Finally, performed harmonic analysis to see resonating and anti-resonating peaks for both healthy beam and defective beam in three beam orientation. Turon, Sarrado and Guillemot [41] the study showed that different researchers utilized the cohesive zone model (CZM) modelling method to analyze failures, and in order to simulate dis-bonding by use of finite element method. But the study suggests that some models available in the literature have not been validated correctly under mixed-mode loading conditions. That leads to an incorrect selection of the parameters of the model can result in inaccurate simulation predictions. The work updated the cohesive formulation which was used by other researchers with a mode-dependent penalty stiffness to perform accurate and somewhat reliable simulation results. Simulation performed based on different loading conditions to validate the accuracy of the new formulation. Binnur G.K [42] in the study finite element analysis method for different beam configurations with different lay-up. The Euller-Bernulli beam and classical lamination theories are the center of the research. Development a computer code on detecting the natural frequencies and mode shapes also done (Matlab). Constrained mode assumption is the focus of the study. The study focuses on location, width, beam boundary conditions and orientation of delamination and effects has been investigated. Validation done based on previous researches.

### 2.1.3 Experimental model

Tsinuel N.Geleta and Bongho Lee [43] studied the delamination behavior of L-shaped laminated composites numerically and experimentally. In finite-element modeling, cohesive zone modeling was used to simulate the delamination of plies. Cohesive elements were inserted between bulk elements at each interlayer to represent the occurrence of multiple delaminations. The laminated composite models were subjected to several types of loading inducing opening and shearing types of delamination. J. Whiteny and Hogeseden [44] performed experimental analysis that focuses on cantilever beam as a center of study and measured the fracture resistance that lies between the laminates. Experimental data used on unidirectional tape and bi-directional cloth graphite fiber reinforced polymeric matrix for comparison to assess the potential of the double cantilever beam test.Validation done by comparing matrix materials of varying toughness. Colin.R and William.B [45] conducted a nondestructive experiment to locate delamination inside a composite beam. The implemented a method operates on the fundamental displacement eigenvector which is converted in to a curvature mode shape. According to the study application of a unique, gapped smoothing damage detection method to the curvature yields delamination location. Conducted an experimental investigation of a composite beam with a manufactured delamination are also presented. Used gapped smoothing method on the experimental modal data, and successful location of the delamination was possible according to the paper. Gerard. C [46] the effects of delamination on modal characteristics of the beam is studied. Experimental modal analysis was used in measuring effect of delamination length on the first four frequencies of the simply supported beam test specimens. Also, experimental and finite element models were associated with simplified beam theory models. According to the paper presence of a delamination reduces the natural frequency of the beam. I.V Vale,Sajal.R,K.Jagtap [47] according to the study delamination has the ability to highly reduce stiffness that in turn leads to reduction of natural frequencies and mode shapes or generally the dynamic response of the structure will negatively affected. The study shows the effect of length of delamination on natural frequencies by using a piezoelectric transducer that is attached on the surface of the composites. The dynamic response is obtained by transient analysis (Ansys).Generally, the study focusses on different delamination length and locations, and in conclusion an increase in delamination length will further reduce the natural frequencies. Jakub, Sylwester and Marcelo.M [48] studied and presented experimental evaluation of influence of the elastic couplings on delamination initiation and the fracture toughness as well as propagation in carbon/epoxy composite laminates. For this purpose the mode I double cantilever beam tests according to the American Society for Testing and Materials (ASTM) Standard were performed on specimens with different delamination dimensions (interface) and specific lay-ups composition exhibiting the bending-twisting and the bending-extension couplings. The evaluation of accuracy of the different methods, the values obtained by using standardized data reduction schemes were compared with those values that are calculated by using the compliance based beam method. After all, the scanning electron microscope (SEM) analysis were also performed after the experimental tests in order to investigate a fracture surface at delamination plane. Mengyue,Zhifang and Karthik [49] due to low weight in the present day fiber reinforced polymer(FRP) are highly used in aircraft industry. Hence, it's important to detect failures early as possible before it propagates and causes another structural failure. The study focused on techniques of early detection of delamination in FRP by use of methods such as acoustic emission, fiber optic sensors, vibrational methods, specifically frequency monitoring method is used, because this method is simple to implement. The study conducted significant survey on delamination methods. Generally, results of the work allow readers to see the available structural health monitoring assessment methods.

## 2.2 Literature summary

The above studies shows that, modeling and predicting different types of failures in composite materials is a very recent and vast as well as somewhat complex study area. Carbon fiber reinforced polymers(CFRP) are currently utilized in aviation, automotive, manufacturing and other industries, perhaps utilization of these type of materials leads different researchers and industries to work on different ways to predict and model the associated failures and defects. The majority of the study conclusively shows the relationship between the vibrational parameters of a structure and its relationship with delamination and all studies indicate that delamination has the ability to decrease the stiffness or rigidity. The researchers have done by use of analytical, finite element based methods and experimentally, validation of the studies is either by comparing to previous studies or by performing experimental works and in some cases by use of simulating soft wares such as ansys or abaqus. Some papers suggested future researchers that, using simulation software is time consuming and computational methods lead to errors in predicting some quantities when the geometry of the material is complex, but researchers that conducted experimental works strongly suggested to apply experiment.

## 2.3 Research gaps

Generally, a number of researchers and scholars conducted a detailed analysis regarding on the delamination effect and how it deteriorates composite structures and how it directly affects a number of industries such as, automotive, aeronautics, ship-building, space-craft, medical and sport equipment manufacturing industries and how it generates specific design challenges and requirement of accurate failure prediction.

Wang and Gibby [7] studied the effect of delamination on natural frequency of the composite beam, but their study is limited to a constant location of delamination and only one type of delamination that is single delamination, whereas the current study analyze and present a complete analytical and finite element simulation of multiple delamination of a cantilever beam by use of a ‘free mode’ assumption delamination model by changing the location, type and length of delamination.

# Chapter 3

## 3.1 Material and method

A beam serves as a structural support for many engineering applications by withstanding bending loads. Out of diverse applications of beam the following are the major, it can be used as material handling equipment (overhead cranes, gantry, etc.), in robotics and in aerospace engineering applications (as wing of an air craft) structural support for most of machinery linkages, etc.

In structural and civil applications beams play an important role. For instance cantilever beams widely used in bridges, taking look at simply supported beam its basic model for most of bridges. Structures that are located on the surface of sea and oceans such as off-sure wind turbine blades, oil refinery terminals and towers can be built based on beam models.

Results for both intact and delaminated beam is performed by use of a software that will be presented and discussed later on this research. The concept of eigenvalues and eigenvectors is applied to determine the desired natural frequency and mode shape as the beam vibrates naturally.

## 3.2 Material and geometry

The following table shows the material properties and geometry used in the study.

Table 3.1 Material and geometry specification [20]

| Material | Carbon fiber reinforced plastic(CFRP) |
|---|---|
| L (mm) | 300 |
| B (mm) | 50 |
| D (mm) | 12 |
| Density ($\boldsymbol{Kg/m^3}$) | 1570 |
| Modulus of elasticity ($\boldsymbol{N/m^2}$) | 140 |
| Transverse modulus elasticity ($\boldsymbol{N/m^2}$) | 92 |
| Poison's ratio | 0.28 |
| Transverse poison's ratio | 0.184 |
| Area ($\boldsymbol{m^2}$) | $6 * 10^{-4}$ |
| Moment of area ($\boldsymbol{m^4}$) | $7.2 * 10^{-9}$ |

### 3.3 Research methodology

- ✓ Introducing analytical formulation and simulation: - The co-ordinate system and notation for normal and delaminated composite beam will be analyzed
- ✓ Simulation using ansys Software: - the overall work will present a verification of the presented theories using commercial software. Beam element modeling techniques will be explored, and also harmonic analysis will be conducted
- ✓ Conclusion and recommendations:-under this conclusion, recommendations and future work will be addressed

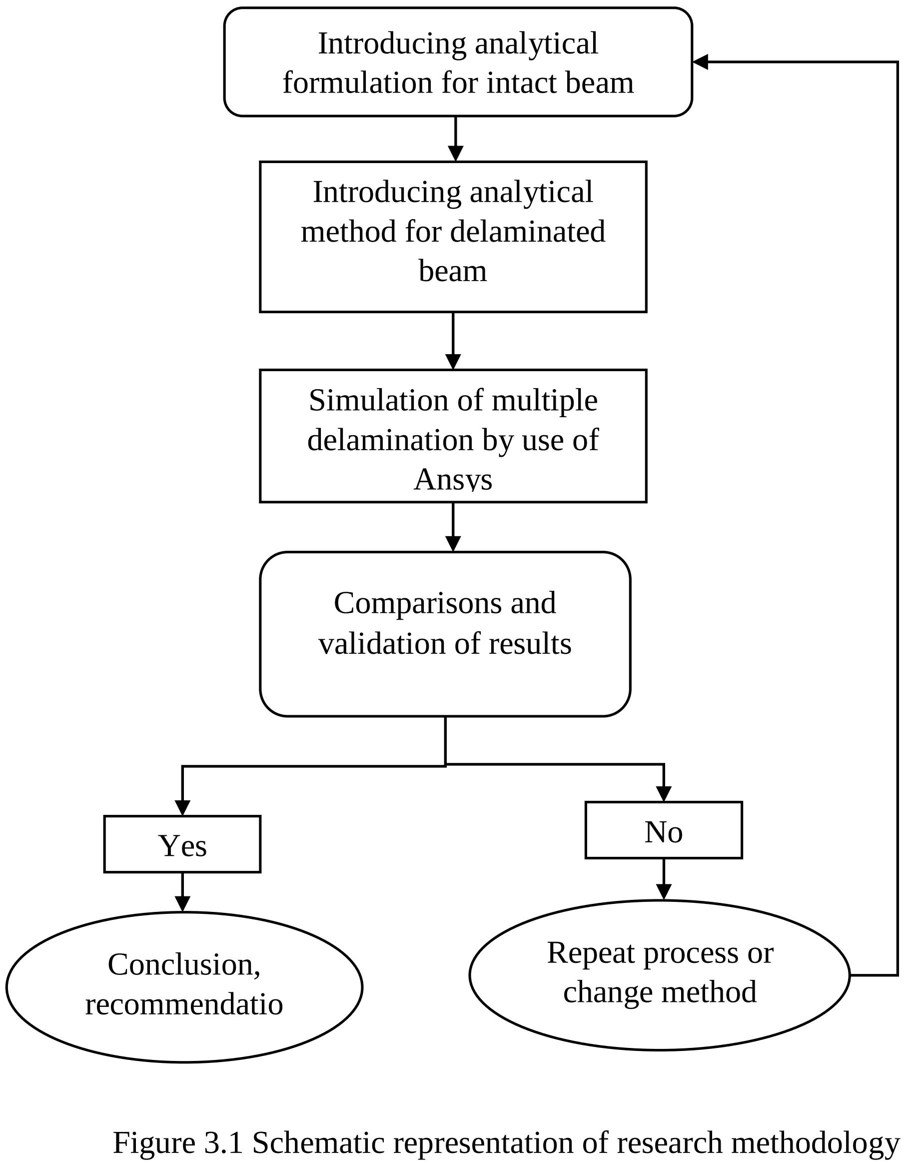



Figure 3.1 Schematic representation of research methodology

## 3.4 Analytical formulation

The classical beam theory also known as Euler-Bernoulli beam theory is a simplified theory that allows different characteristics of a beam (such as load-bearing and deflection of a beam) to be calculated.

### 3.4.1 Governing deferential equation

Determination of deferential equation for beam vibration involves forces and corresponding moments that act on the beam element.

From flexural equation the bending moment related to the radius of curvature for the above figure by the following,

$$M = EI\frac{d^{2y}}{dx^2} \qquad 3.1$$

From harmonic motion when a certain beam vibrates by its own weight and vibrates at static equilibrium its load per unit length becomes equivalent to the inertia load as the result of mass and acceleration. Since the inertia force is in the same direction as p(x), as shown in the figure we have;

$$P(x) = \rho\omega^2 y \qquad 3.2$$

Using the above equation,

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{d^2y}{dx^2}\right) - \rho\omega^2 y = 0 \qquad 3.3$$

The assumption of prismatic beam ones again applied here, flexural rigidity (EI) is constant for beam with a constant cross section and the above equation can be written as the following,

$$EI\frac{d^4y}{dx^4} - \rho\omega^2 y = 0 \qquad 3.4$$

By using the separation of variables,

$$Y(x,t) = y(x).T(t) \qquad 3.5$$

Substituting equation (3.5) to (3.4),

$$\rho AYT'' + EIYT'''' = 0 \qquad 3.6$$

From separation of variables,

$$\frac{1}{T}T'' = -\frac{EI}{\rho A} * \frac{1}{Y}Y'''' = \omega^2 \qquad 3.7$$

Can only be true if the right and the left side of the equations are equal to separation constant that is $\omega^2$.

$$T'' + \omega^2 T = 0 \qquad 3.8$$

The above equation is ode,

$$T(t) = a1\cos(\omega t) + a2\sin(\omega t) \qquad 3.9$$

For the spatial part, multiplying by ρY and divide by EI then,

$$Y'''' - \frac{\rho A\omega^2}{EI}Y = 0 \qquad 3.10$$

$$Y'''' - \beta^4 Y = 0, \qquad \text{where} \quad \beta^4 = \frac{\rho A\omega^2}{EI} \qquad 3.11$$

To find the solution try, $Y(x) = e^{sx}$

$(S^4 - \beta^4)e^{sx} = 0$ , but $e^{sx}$ is not zero for a finite values of s and x hence,

$(S^4 - \beta^4) = 0$ Is the characteristic equation

$S^4 = \beta^4$, $\quad S^2 = \pm\beta^2$….. $S = \pm\beta$ Or $\pm i\beta$

$$Y(x) = c1e^{i\beta x} + c2e^{-i\beta x} + c3e^{\beta x} + c4e^{-\beta x} \qquad 3.12$$

Substituting back into the general solution,

$$Y(x) = c1\cos(\beta x) + c2\sin(\beta x) + c3\cosh(\beta x) + c4\sinh(\beta x) \qquad 3.13$$

The above equation is the general solution for a vibrating beam.

The natural frequencies of vibration are found to be

$$\omega n = \frac{(\beta nl)^2\sqrt{EI}}{ml^4} \qquad 3.14$$

Normally $\beta n$ is dependent on the boundary condition of the system.

### 3.4.2 Boundary conditions

The forth order differential equation indicates that four separate conditions, that is the system boundary conditions. Prismatic beam EI is constant.

For this specific research cantilever beam is selected, which one end is fixed and the other is free. At the fixed side both deflection and slope are zero. In contrast at the free edge shear and bending moments are zero.

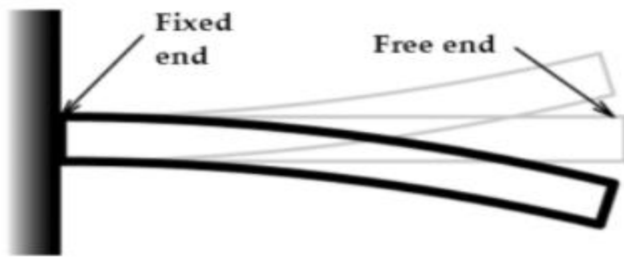



Figure 3.2 Representation of a cantilever beam

From boundary conditions, at fixed end the displacement and slope will be,

$$U@x = 0 \tag{3.15}$$

$$\frac{\partial U}{\partial x}@x = 0 \tag{3.16}$$

At the free end,

$$\frac{\partial^2 U}{\partial x^2}@x = L = 0 \tag{3.17}$$

$$\frac{\partial^3 U}{\partial x^3}@x = L = 0 \tag{3.18}$$

Introducing the boundary conditions in to the general solution;

$$(Y)@x = 0 = A + C = 0, \quad A = -C \tag{3.19}$$

$$\left(\frac{dy}{dx}\right)@x = 0 = \beta[A\sinh\beta x + B\cosh\beta x - C\sin\beta x + D\cos\beta x]@x = 0 \tag{3.20}$$

$$\beta[B + D] = 0, \quad B = -D \tag{3.21}$$

$$\left(\frac{d^2y}{dx^2}\right)@x = 1 = \beta^2[A\cosh\beta l + B\sinh\beta l - c\cos\beta l - D\sin\beta l] = 0 \tag{3.22}$$

$$A(\cosh\beta l + \cos\beta l) + B(\sinh\beta l + \sin\beta l) = 0 \tag{3.23}$$

$$\left(\frac{d^3y}{dx^3}\right)@x = l = \beta[A\sinh\beta l + B\cosh\beta l + C\sinh\beta l - D\cos\beta l] = 0 \quad 3.24$$

$$A(\sinh\beta l - \sin\beta l) + B(\cosh\beta l + \cos\beta l) = 0 \quad 3.25$$

From equation (3.24) and (3.25);

$$\frac{\cosh\beta l + \cos\beta l}{\sinh\beta l + \sin\beta l} = \frac{\sinh\beta l - \sin\beta l}{\cosh\beta l + \cos\beta l} \quad 3.26$$

$$\cosh\beta l\cos\beta l + 1 = 0 \quad 3.27$$

Different values of ($\beta il$) can be calculated that can satisfy the above equation. Hence, depending on these different roots we can find the natural frequency of the system.

Table 3.2 Different roots of natural frequencies [20]

| **Roots(i)** | $\boldsymbol{\beta i1}$ |
|---|---|
| **1.first root** | 1.8751 |
| **2.second root** | 4.6940 |
| **3.thired root** | 7.8547 |
| **4.forth root** | 10.9955 |
| **5.fifth root** | 14.1371 |

Note, for different beam orientation we have different roots of natural frequency. The above values are constant for solving natural frequency of a cantilever beam.

### 3.4.3 Theoretical (analytical) calculations for intact beam

The angular frequencies of the healthy beam can also be calculated as the following;

From equation (3.28);

$$\omega n = \frac{(\beta nl)^2\sqrt{EI}}{ml^4} \quad 3.28$$

Hence, the first angular frequency becomes,

The next step is to calculate the linear frequency (*Hz*) that becomes,

$$f = \frac{\omega}{2\Pi} \quad 3.29$$

So that, the first natural linear frequency becomes the unit will be in heartz,

$$f1 = \frac{\omega 1}{2\Pi} \quad 3.30$$

In similar fashion $\omega 2, \omega 3, \omega 4$ ... can be calculated.

Among different methods of calculating the load carrying and deflection of a certain beam, one of the method is the Euler–Bernoulli beam theory that is a simplification of the linear theory of elasticity. From classical laminate theory in order to solve problems on laminate, the total laminates divided based on the location of the mid plane, hence how lamina starts and ends based on the mid plane is critical. Mid-plane will be half of the total thickness [17].

Different researchers assume two assumptions on vibration and delamination buckling analysis. Those assumptions are the 'free mode' and 'constrained mode' assumptions, both have been studied by different researchers and they have their own distinct feature. The second assumption considers delamination at fronts. According to their study delamination fronts found in the two connectors but, closer to the rigid connector. Also experimental studies performed by different researchers suggests that the values of natural frequencies obtained by rigid connector assumptions are closer to (experimental values) than those values obtained by soft connector [46]**.**

This research concentrates on 'free mode' assumption.

- ✓ Free mode assumption; h2 >h3 when the motion is upward, or the beams 2 and 3 they vibrate freely and may have varying deformations in the transverse directions [15].

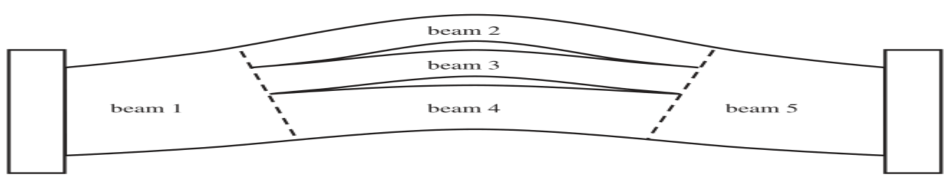



Figure 3.3 Free mode delamination model [5]

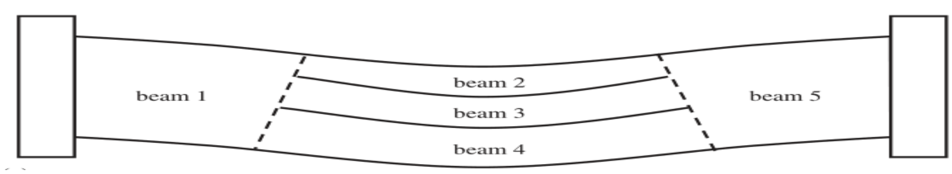



Figure 3.4 Constrained mode delamination model [5]

## 3.5 Single delamination

Among different methods of calculating the load carrying and deflection of a certain beam, one of the method is the Euler–Bernoulli beam theory that is a simplification of the linear theory of elasticity [26].

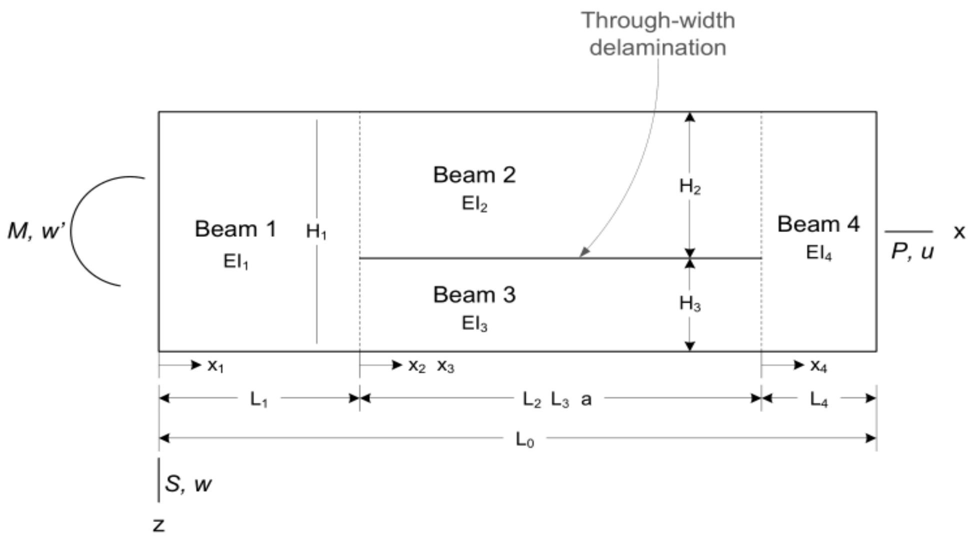



Figure 3.5 The general representation of single delamination [5]

The above figure shows the general representation of single delamination, in which it has a total length of$L_0$, $L_1$and $L_4$ indicate the length of normal (intact beam) the delamination length a and the total thickness indicated by $H_1$. This model is a general model that is applied to materials with varying properties above and below the mid-plane or isotropic materials. The top and the lower sections differentiated based on the respective subscripts. At the delamination tips that is at $x_1 = L_1$ and $x_4 = 0$ torsion, shear deformation, axial and out of plane delamination are ignored [44].

The general equation of motion for the $i^{th}$ Bernulli-beam can be formulated as the following;

$$EI_i \frac{\partial^4 Wi}{\partial x^4} + \rho_i A_i \frac{\partial^2 W_i}{\partial t^2} = 0, \quad i = 1, \dots 4 \qquad 3.31$$

The transverse displacements can be expressed by harmonic oscillations

$$w_i(t) = W_i \sin(\omega t) \qquad 3.32$$

Where, w is the displacement

W, is amplitude of the displacement

$\omega$, the circular frequency

Substituting the above two we get,

$$EI_i \frac{\partial^4 Wi}{\partial x^4} + \rho_i A_i \omega^2 W_i = 0, \quad i = 1, \dots 4 \tag{3.33}$$

The above equation can be written as a general solution to fourth order differential equation,

$$W_i(x_i) = A_i \cos\left(\lambda_i \frac{x_i}{L_i}\right) + B_i \sin\left(\lambda_i \frac{x_i}{L_i}\right) + C_i \cosh\left(\lambda_i \frac{x_i}{L_i}\right) + D_i \sinh\left(\lambda_i \frac{x_i}{L_i}\right) \tag{3.34}$$

Where, $W_i$ bending displacement

$\lambda_i$ non − dimensional frequency defined as the following,

$$\lambda_i^4 = \frac{\omega^2 \rho_i A_i}{EI_i} L_i^4 \tag{3.35}$$

Free mode delamination, the assumption behind is all the four beams treated as Euller-Bernulli beam $L_i \gg H_i, i = 4$ [4]

For simplification the following assumptions made,

- ✓ Delamination is induced in the system first
- ✓ The layers assumed to be linear elastic(Homogeneous)
- ✓ The layers are identical in densities and only relevant for ‘free mode’ model

The coefficients $A_i, B_i, C_i$ and $D_i$ evaluated to satisfy displacement continuity conditions and beam boundary conditions, this method used by many researchers [26] the other assumption made by the researchers is that after delamination, the tips remain planner.

Let’s consider the delamination tip after delamination, on the figure below no external force is applied hence, the top and bottom segments must have equal and opposite internal axial forces, $P_3 = -P_2$ to prevent inter laminar slip. Additionally, the requirement that the delamination tip faces remain planar after deformation results in, at the left delamination tip:

$$U_2(x_2 = 0) - U_3(x_3 = 0) = \frac{H_1}{2} W_1'(X_1 = L_1) \tag{3.36}$$

Where, $U_i$ is the axial displacement of beam section i, $W_1'(x_1 = L_1) = W_2'(x_2 = 0) = W_3'(X_3 = 0)$ from kinematic continuity conditions, if this is combined with the same formulation at the right delamination tip [17].

$$\left(U_3(X_3 = L_3) - U_3(X_3 = 0)\right) - \left(U_2(X_2 = L_2) - U(X_2 = 0)\right) = \frac{H_1}{2}(W_4'(X_4 = 0) - W_1'(X_1 = L_1)) \quad 3.37$$

The above assumption is made by for [26] small deformations material and geometric properties that remains constant along the length of the beam.

$$U_i(X_i = L_i) - U_i(X_i = 0) = \int_0^{L_i} \frac{P_i(x_i)}{EA_i(x_i)} dx_i = \frac{P_i L_i}{E_i A_i} \quad 3.38$$

Where, $EA_i$ is the axial stiffness of the beam segment, substituting the above two give us

$$\frac{P_2 L_2}{EA_2} - \frac{P_3 L_3}{EA_3} = \frac{H_1}{2}\left(W_4'(X_4 = 0) - W_1'(X_1 = L_1)\right) \quad 3.39$$

By using the continuity of axial forces across the tip of delamination, $P_3 = -P_2$,

$$P_3 = A * (W_4'(x_4 = 0) - W_1'(x_1 = L_1)) \quad 3.40$$

Where the parameter A, defined as the following,

$$A = \frac{H_1}{2L_2}\left(\frac{EA_2 EA_3}{EA_2 + EA_3}\right) \quad 3.41$$

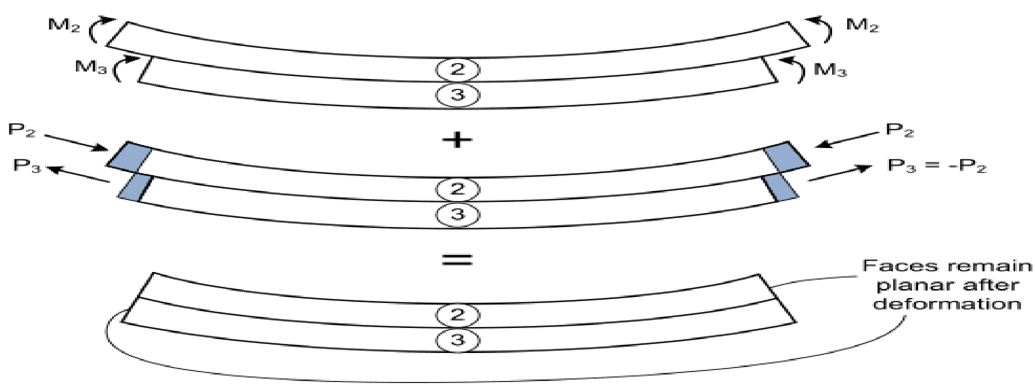



Figure 3.6 The faces of the delamination remain planar after deformation (rigid connectors) [33]

At the delamination tips, continuity of bending moments leads to

At the left of delamination tip,

$$M_1(X_1 = L_1) = M_2(X_2 = 0) + M_3(X_3 = 0) - P_2\frac{H_3}{2} + P_3\frac{H_2}{2} \quad 3.42$$

At the right tip of delamination,

$$M_4(X_4 = 0) = M_2(X_2 = L_2) + M_3(X_3 = L_3) - P2\frac{H_3}{2} + P_3\frac{H_2}{2} \quad 3.43$$

Using the above equation (3.43) which represents the internal axial force, and noting that from beam theory, bending moments and shear forces in beam segment 'i' are related to displacements, w, through $M_i = EI_iW_i''$, and $S_i = EI_iW_i'''$, respectively, can be shown that, for continuity of bending moments,

$$EI_1W_1''(X_1 = L_1) = EI_2W_2''(X_2 = 0) + EI_3W_3''(X_3 = 0) + A[W_4'(X_4 = 0) - W_1'(X_1 = L_1)] \quad 3.44$$

Where,

$$A = \frac{H_1}{2L_2}\left(\frac{EA_2EA_3}{EA_2 + EA_3}\right) \quad 3.45$$

In the same way to satisfy continuity of shear forces about the left delamination tip,

$$EI_1W_1'''(X_1 = L_1) = EI_2W_2'''(X_2 = 0) + EI_3W_3'''(X_3 = 0) \quad 3.46$$

Additionally, there exist 2 kinematic continuity conditions at each delamination tip. Again, about the left delamination tip:

Continuity of displacements,

$$W_1(X_1 = L_1) = W_2(X_2 = 0) = W_3(X_3 = 0) \quad 3.47$$

Continuity of slopes,

$$W_1'(X_1 = L_1) = W_2'(X_2 = 0) = W_3'(X_3 = 0) \quad 3.48$$

Applying the kinematic and force continuity conditions at each deamination tip gives us four equation per tip, In addition to four endpoint boundary conditions of the system, this process results in sixteen (16) equations. Applying the general solution to each beam, gives us 16 unknown coefficients, The 16 equations can be solved simultaneously, using a root finding algorithm to find the natural frequencies and mode shapes of the system. The analytical formulation for double and triple delamination is also the same, for double delamination we will have 20 equations and in the same way for triple delamination the equation further expanded in to 24 equations. In the fourth chapter results from previous researchers and present study specifically for clamped-clamped boundary condition is discussed.

## 3.6 Modal analysis by matlab

In the coming section both natural frequencies and mode shapes are calculated analytically and matlab then comparisons are made in the fourth chapter. Normally a two dimensional (2d) analysis is done by matlab (17 a) program. A computer algorithm used to do that. The cantilever beam used in matlab analysis is assumed to have a two degree of freedom that includes rotation about z axis and translation along y axis. The number of divisions (discretization) of the beam varies accordingly. Let's say the beam is divided in to 1000 equal sections of equal length. It's already mentioned that each element has 2 nodes, hence a total of 1001 nodes are there; and the system having two degree of freedom as a result the total (dof) becomes 2002. The beam is a cantilever beam as a result at the first node all the dofs are constrained (0).

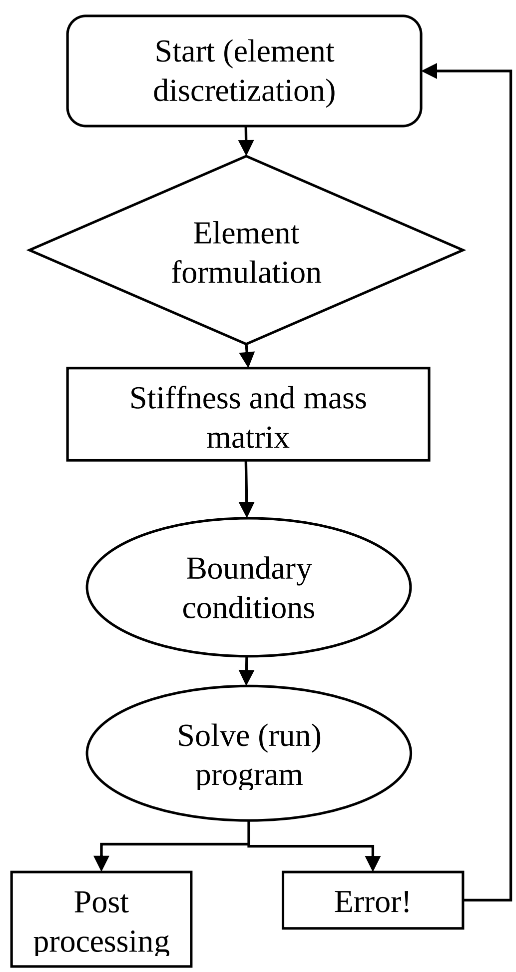



Figure 3.7 Flow chart showing MATLAB formulations

## 3.7 Method of simulation

In the present day, the advancement of software technology allow perform different simulations at a higher precision and accuracy. In mechanical engineering ansys is the most common simulating software, in fact like any other software need to set initial parameters then gradually the solution is developed.

In this particular research ansys used. The interference used is known as ‘ansys mechanical apdl’.

### 3.7.1 Ansys mechanical apdl

It can be utilized to solve structural, thermal and electrical analyses. Using this specific application it’s possible to model, analyze and solve problems.
There are some logical methods to start with this specific program;

- ✓ Creation of the analysis type
- ✓ Mechanical properties defined
- ✓ Geometry creation
- ✓ Stiffness behavior

The ansys element library provide a number of elements to choose from. Generally, element size and type is determined based on factors such as,

- ✓ Size of the model
- ✓ Structure complexity
- ✓ Curvature
- ✓ Type of analysis linear or non linear
- ✓ Structural, modal or electrical

In ansys mechanical apdl elements can be categorized in to groups, plane, solid and shell etc. for instance in the previous versions of ansys has generally lower element number compared to the recent ones.

In ansys strain varies from one linear element to another, the reason behind is the absence of a mid-side node. These elements have special name in ansys and called ‘fully integrated’ simply the act like semi-quadratic element.

Due to the presence of a mid-side node the simulation time for a quadratic element is going to be longer compared to the linear ones.

To choose the appropriate elements we can consider the following logical methods;

- ✓ Generally, it’s recommended that for bending type problems its best to select quadratic elements.
- ✓ Selecting linear elements might be erroneous for complex element shapes.
- ✓ In non-linear analysis a fine mesh is more accurate than a rough quadratic mesh even if the number of elements are the same.
- ✓ For a regularly shaped elements linear bricks can be chosen.

## 3.8 Analysis options

Various types of analysis can be performed on mechanical apdl interface. For more detailed and advanced analysis it’s possible to use the command bar to enter different orders and execute them accordingly.

### 3.8.1 Ansys modal analysis

Vibrational characteristics or natural frequencies of a certain mechanical structure can be determined by using modal analysis in ansys mechanical apdl. Based on our requirement harmonic analysis, transient and dynamic analysis can also be performed. The concentration of this research goes to natural frequencies and mode shapes.

### 3.8.2 Overall processes involved in modal analysis

1) The first step is to construct a model for the analysis,
2) Mesh control, it’s about achieving the desired mesh size and control. Element size can be specified based on the previous parameters.
3) Apply loads (if any) and get the solution, in this particular research no load is applied, since the system oscillates naturally, only the necessary boundary conditions applied.
4) Visualization of the results, the results from modal analysis can be presented as ‘Jobname.RST.’. In the coming chapter results will be presented extensively and deeply.

In this research simulation is done by ansys. By using different length of delamination for a cantilever beam. The size of delamination (delamination ratio) starts from 90mm to 270mm for single, double and triple delaminations. A total of 28 simulations (i.e. 9 for each cases and 1 for intact beam) has been performed and presented. More over the effects of delamination size as we go from single to double and triple analyzed deeply. The results of the simulation indicates the longer the delamination the more reduction in natural frequency.

## 3.9 Mesh shape and its effect

Ansys mechanical apdl provides two mesh shapes mainly, that are Hexagonal and Tetrahedral shapes. In this specific study the selected shape is hexagonal, because it's much accurate and easy to induce delamination at any location of beam and delamination ratio. In tetrahedral mesh arrangement it's difficult to insert the exact values of delamination due to the complex nature of the node arrangements but at arbitrary location and delamination ratio it's possible to model.

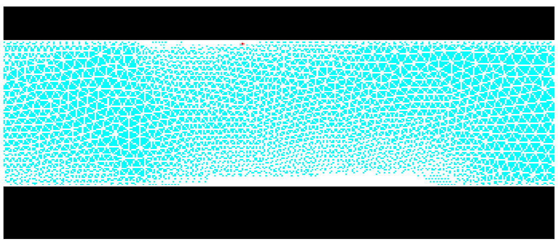


Figure 3.9 Tetrahedral mesh arrangement

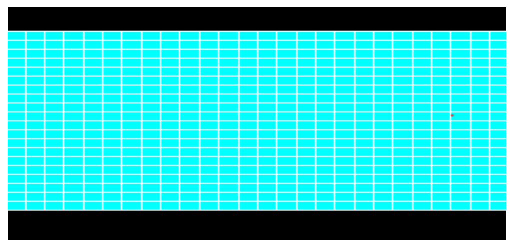


Figure 3.8 Hexagonal mesh arrangement

## 3.10 Modeling procedure used in ansys

It's possible to generalize the major and sub important steps followed in ansys as the following, there is pre-processing, solution stage and post processing.

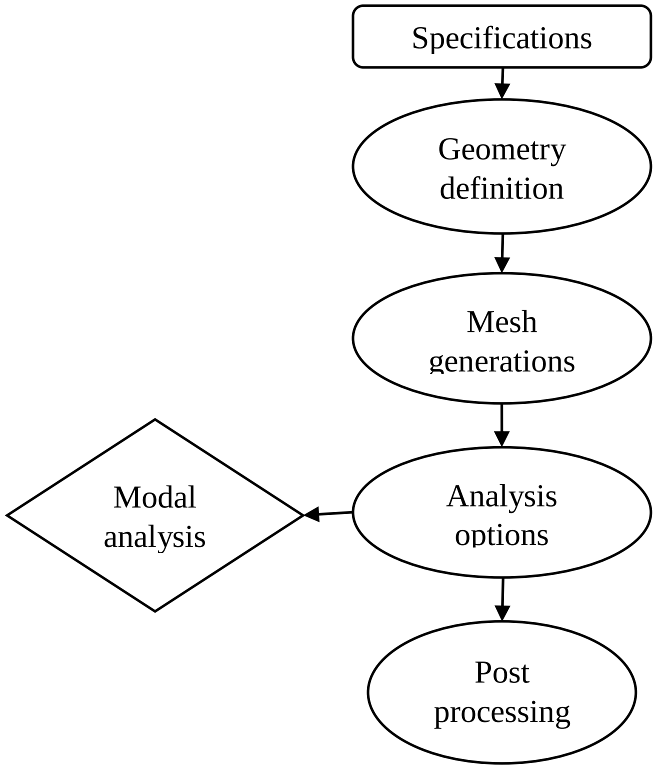



Figure 3.10 Flow chart for ansys

## 3.11 Harmonic Analysis

It's a linear dynamic analysis used to know the response of a certain system to higher frequencies excitation at specific frequencies its other name is frequency response analysis. Different mechanical components and parts are subjected to vibration loading under different conditions. For example, in automobile the chassis (structural support to the automobile) is subjected to both the vibrations that is a result of the engine and other mechanical components.

Once again recalling the concept of resonance might be helpful to fully understand harmonic analysis. It's known that in mechanical and structure sciences if the natural frequency of the system matches the frequency of the applied cyclic load there is a chance that the component might fail due to resonance. Hence, it's important to study the behavior by harmonic analysis. A 3d-hexahedral element is used in this specific study.

Generally, while analyzing harmonic conditions the maximum deflection will be observed when the applied cyclic load matches the systems natural frequency.

The introduction of harmonic analysis to this research is due to the fact that it can be a tool to identify the effect of failures (damage, crack or delamination) on beam for different beam boundary conditions and different length and location of failures). Additionally, resonating and anti-resonating peaks can be obtained from harmonic analysis.

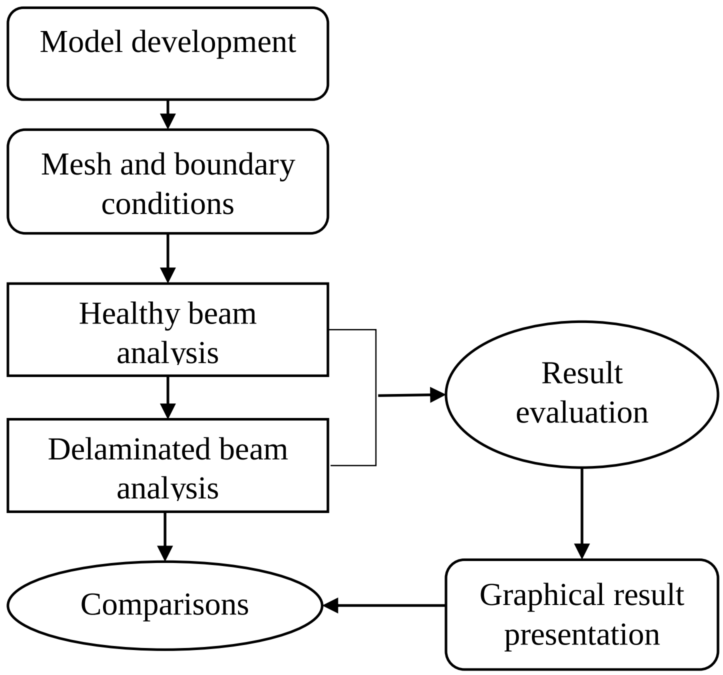



Figure 3.11 Flow chart showing harmonic analysis

Ansys is used to perform the analysis for a cantilever beam for both intact and delaminated beam. As stated above the analysis is done by use of ansys.

# Chapter 4

## 4.1 Result and discussion

### 4.1.1 Introduction

Ansys simulation method becomes convenient in detection of the location, type, and size of delamination in certain structure. Hence, in this research it allows to detect and generally simulate a defective composite cantilever beam to study its effect on natural frequencies. The results of this research shows that generally the larger the delamination length and number of delaminations, the lower the natural frequency. This research also proved that if the system has more than single delamination the natural frequency will reduce further.

The chosen beam for this particular research is a cantilever beam. In this research modal analysis is done for beams that have a rectangular cross-section with dimension of $300\text{mm} * 50\text{mm} * 12\text{mm}$ with fiber orientation of $(0°)$ for carbon fiber reinforced polymer with the help of ansys. First delamination is created and by varying its length and type across the beam for a cantilever beam different values are obtained.

The figures below shows layer stacking sequence and composite sub laminate modeling


Figure 4.1 Layer stacking sequence

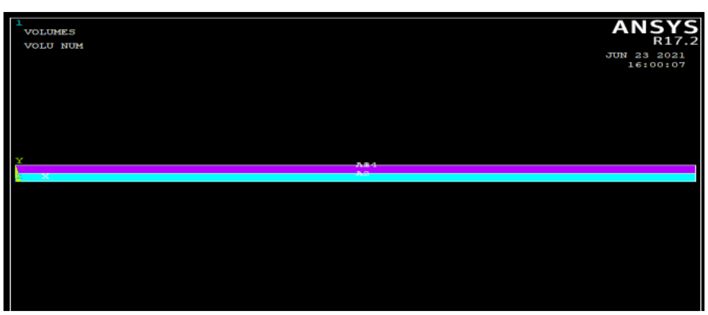



Figure 4.2 Composite beams modeled with two sub laminates

The above figure is a presentation for double beam in which that contains two individual sub-laminates that eventually made up the structure.

## 4.2 Simulation assumption, result and discussion

The simulation is performed by assuming the following assumptions;

- ✓ The selected material is isotropic and homogeneous( valid as long as the mode is 'free mode' and the deflection is small)
- ✓ Hexahedral element used
- ✓ Coupling has been omitted
- ✓ Consideration of a free mode only
- ✓ Rigid connectors used
- ✓ Element size used 0.1
- ✓ $\text{length} = 300\text{mm}, \text{width} = 50\text{mm}, \text{depth} = 12\text{mm}$
- ✓ All degree of freedom constrained(0) at the left end side
- ✓ Clamped-clamped boundary conditions to validate the proposed analytical method

### 4.2.1 Element used

In the modeling and simulation of a layered beam, SOLID185-3D-8node element was used. This research confined to the study of effects of different length of delamination on vibration characteristics such as natural frequency and mode shapes. All of the single, double and triple cases are studied and presented. The delamination modeling is presented step by step for a cantilever beam with different type and location of delamination in the coming topics.

### 4.2.2 Case study-1 modal analysis of intact cantilever beam

The first simulation is for intact beam as follows,

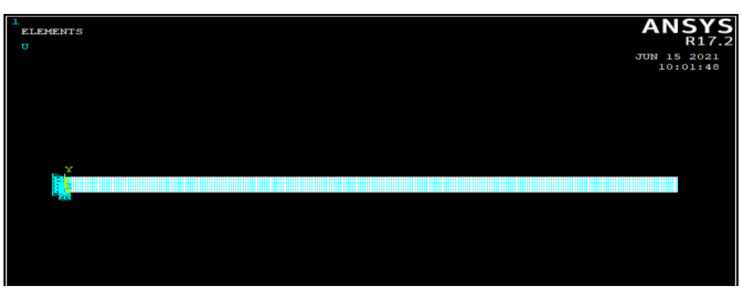



Figure 4.3 Cantilever beam representation

The values of natural frequencies obtained from Ansys is presented by the following table.

Table 4.1 Non-dimensional frequency $\lambda$ and $\lambda^2$ for intact beam

| Modes | Theoretical frequencies | | ANSYS frequencies | |
|---|---|---|---|---|
| | $\lambda$ | $\lambda^2$ | $\lambda$ | $\lambda^2$ |
| **1** | 1.8751 | 3.516 | 1.8824 | 3.5434 |
| **2** | 4.6940 | 22.034 | 4.6947 | 22.04 |
| **3** | 7.8547 | 61.697 | 7.8143 | 61.064 |
| **4** | 10.995 | 120.90 | 10.8588 | 117.914 |
| **5** | 14.136 | 199.854 | 13.8328 | 191.3488 |

Table 4.2 Modal analysis results for intact beam

| Mode number | Frequency(Hz) |
|---|---|
| **1** | 204.98 |
| **2** | 1275.0 |
| **3** | 3532.4 |
| **4** | 6821.0 |
| **5** | 11069.0 |

The first five mode shapes presented below as follows, both matlab simulation and ansys simulation graphs are identical.

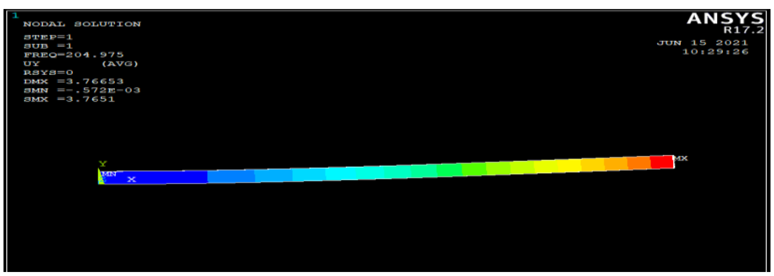



Figure 4.4 The first mode shape of a cantilever beam

The above figure shows us the first mode shape of the intact cantilever beam. At the time the mode shapes of the rest will be presented below.

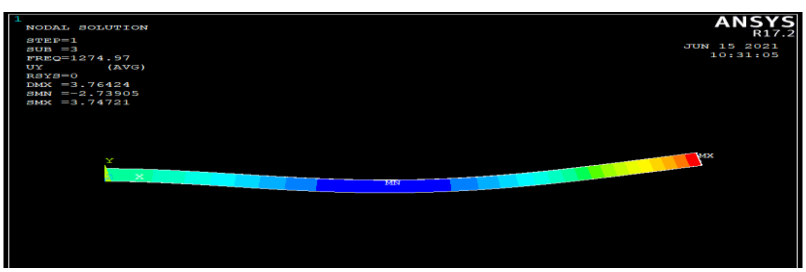



Figure 4.5 The second mode shape of the cantilever beam

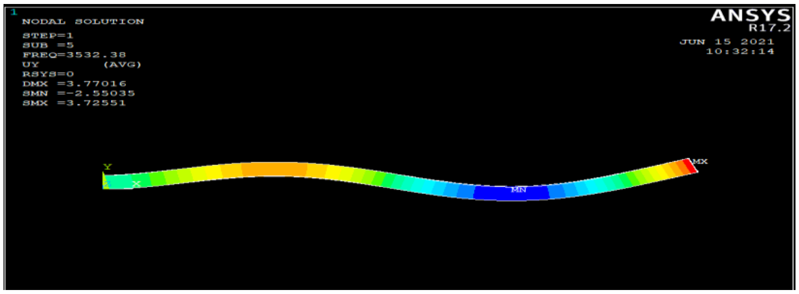



Figure 4.6 The third mode shape of the cantilever beam

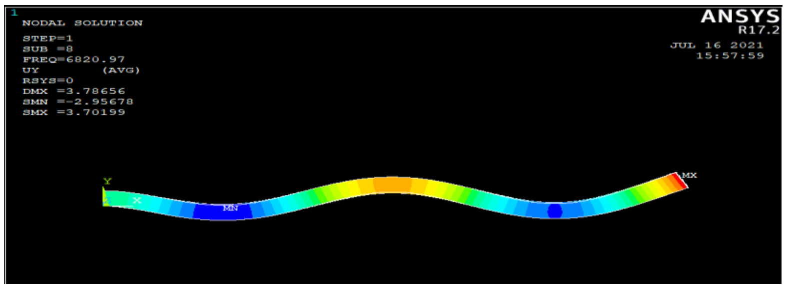



Figure 4.7 The fourth mode shape of the cantilever beam

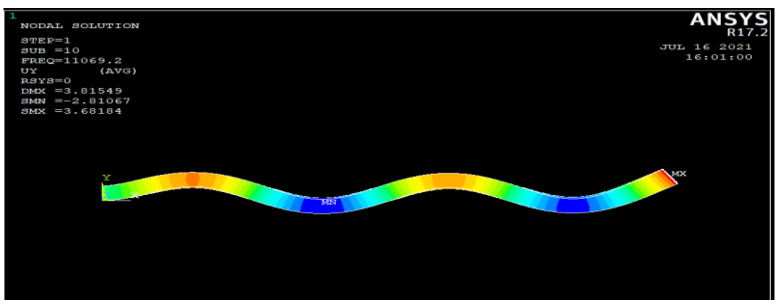



Figure 4.8 The fifth mode shape of the cantilever beam

## 4.3 Error percentage calculation

In this research error is calculated between values of theoretical and simulation values. Based on the results the necessary conclusion is drawn.

Table 4.3 Error calculation between theoretical and ansys values

| Mode | Theoretical | ANSYS | Error (%) |
|---|---|---|---|
| **1** | 203.3908 | 204.98 | 0.781 |
| **2** | 1.2746e+03 | 1275.0 | 0.031 |
| **3** | 3.5690e+03 | 3532.4 | 1.025 |
| **4** | 6.9938e+03 | 6821.0 | 2.470 |
| **5** | 1.1561e+04 | 11069.0 | 4.255 |

**Note,** it is observed that the percentage of error from mode 1 to 3 are minute when compared to a higher modes such as mode 4 and 5.

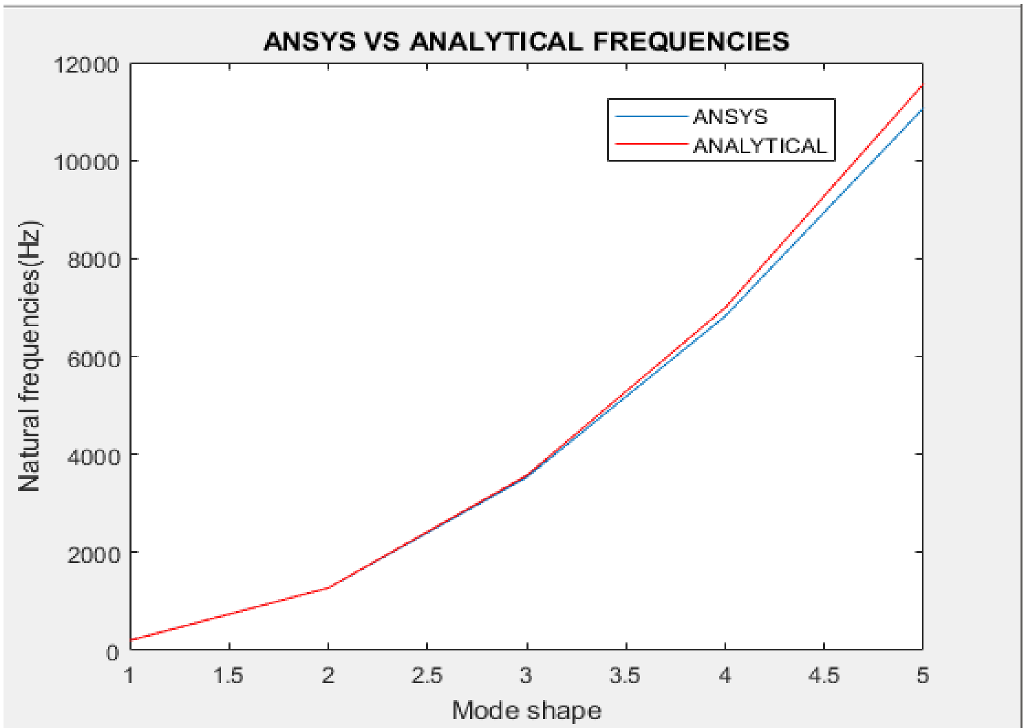



Figure 4.9 Comparison between the first five modes of frequencies

The difference of the modal frequencies between intact beam for both analytical and finite element simulation show a very small variation. Numerically, (0.781%, 0.031%, 1.025%, 2.470% and 4.255%) variations from the first mode to the fifth mode as we go down for intact beam. The present work showed that natural frequencies obtained from the intact beam is always greater than the delaminated beams.

## 4.4 Case 2-Review on clamped-clamped beam based on previous research

To confirm the accuracy and applicability of the proposed analytical and finite element simulation methods numerical checks were performed, in such a way that works done by previous researchers Were presented and compared with the present one, specifically for clamped-clamped boundary conditions and using the same geometrical and material properties (CFRP).

Table 4.4 Natural frequency parameter $\lambda^2$of single delaminated beam comparisons between different values of researchers.

| | **Researcher name** | | **Researcher name** | | **Current result** | |
|---|---|---|---|---|---|---|
| **Delam.** | Wang et.al(1) | | Nicholas Erdelyi(46) | | Present Study | |
| ***a*** | Mode-1 | Mode-2 | Mode-1 | Mode-2 | Mode-1 | Mode- 2 |
| **Intact** | 22.39 | 61.67 | 22.39 | 61.67 | 22.39 | 61.68 |
| **0.1** | 22.37 | 60.76 | 22.37 | 60.76 | 22.38 | 60.99 |
| **0.2** | 22.35 | 55.97 | 22.36 | 55.99 | 22.37 | 55.96 |
| **0.3** | 22.23 | 49.00 | 22.24 | 49.03 | 22.34 | 49.00 |
| **0.4** | 21.83 | 43.87 | 21.84 | 43.03 | 22.20 | 43.54 |
| **0.5** | 20.88 | 41.45 | 20.89 | 41.52 | 20.98 | 41.42 |
| **0.6** | 19.29 | 40.93 | 19.30 | 41.03 | 19.33 | 40.53 |

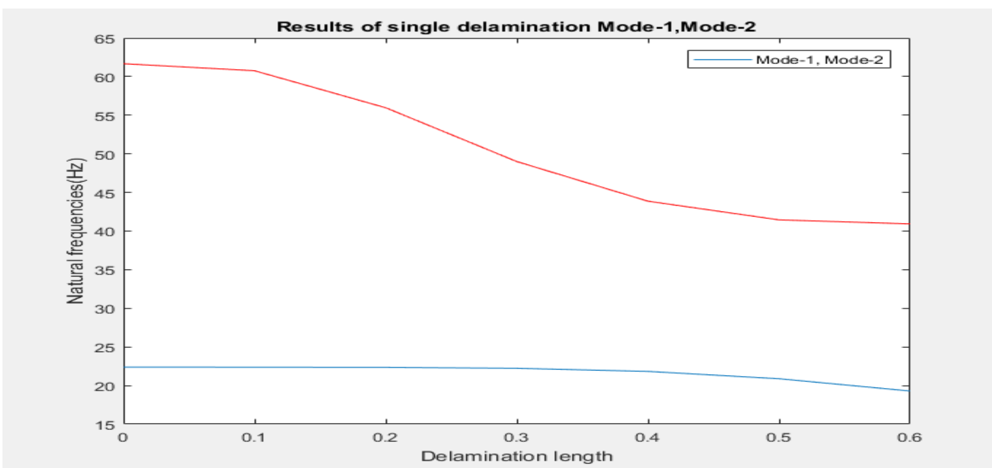



Figure 4.10 Results of single delamination for clamped-clamped boundary conditions

The first two bending modes were studied by previous researchers for specifically a clamped-clamped beam boundary condition. Variations in natural frequency is clearly observed as the length of delamination increases, thus leading to a conclusion that natural frequency drops down as length of delamination rises.

## 4.5 Case 3-Single central delamination of cantilever beam

The second case is introducing delamination with different length in to the structure and simulate it along its length for different values that is @ 30%, 60% and 90% of the beam.

**Note,** the simulation method used in this study is ‘contact deboning’ delamination modeling.

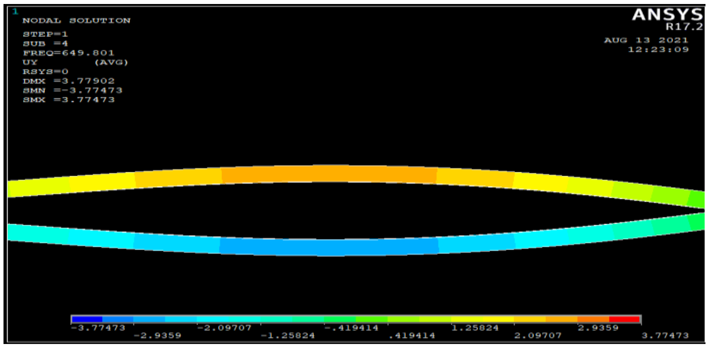



Figure 4.11 A3d-beam with single delamination (Front view)

The first five bending modes for single central delamination is drawn as the following,

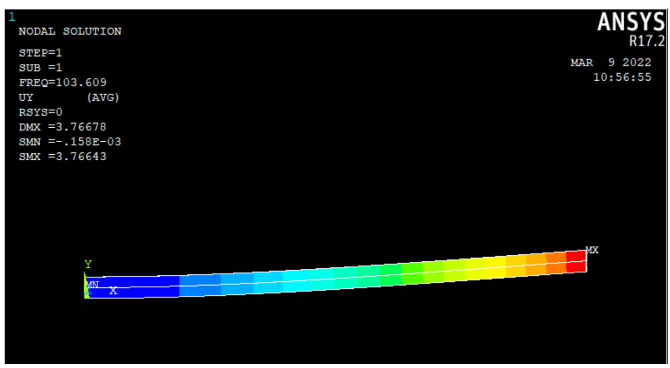



Figure 4.12 The first bending mode

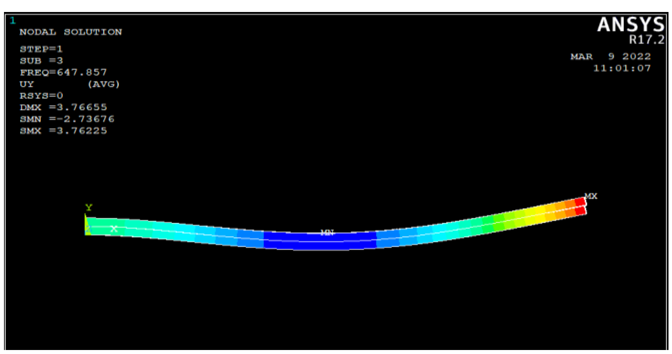



Figure 4.13 The second bending mode

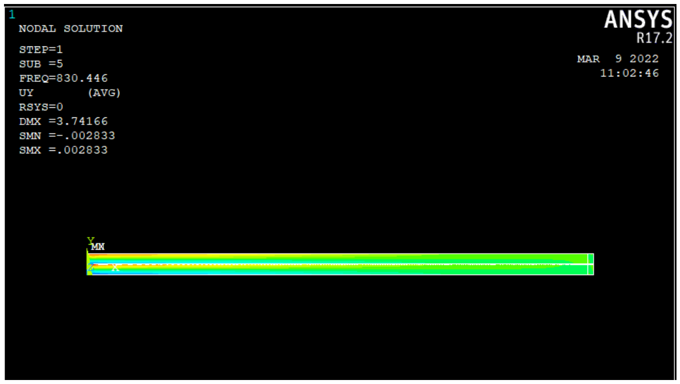



Figure 4.14 The Third bending mode

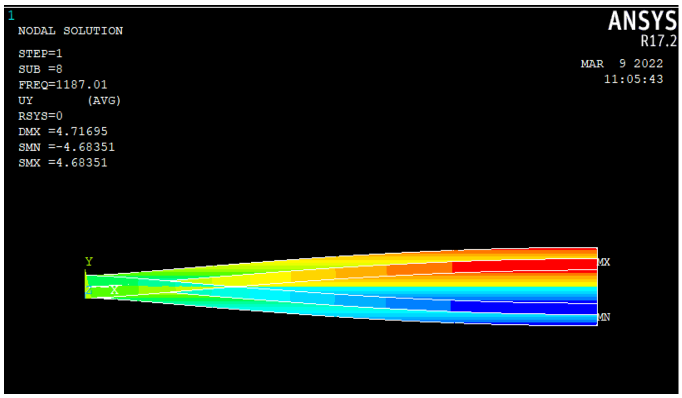



Figure 4.15 The fourth bending mode

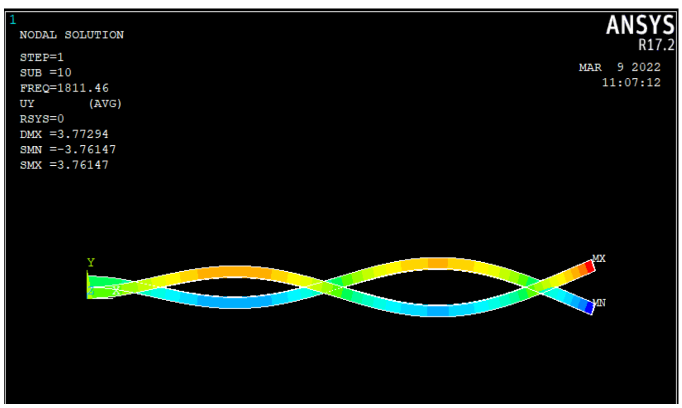



Figure 4.16 The fifth bending mode

In the following table results of cantilever beam of different delaminated beam illustrated. The delamination range varies from 0.3 $to$ 0.9 along the beam span. The results obtained are tabulated and presented using graph to illustrate the relationship between $\lambda^2$ and delamination length.

Table 4.5 Natural frequency values for analytical and finite element simulation results

| $a$ | Analytical results 1st Mode (Hz) | 2nd Mode (Hz) | 3rd Mode (Hz) | 4th Mode (Hz) | 5th Mode (Hz) | Finite element simulation results 1st Mode (Hz) | 2nd Mode (Hz) | 3rd Mode (Hz) | 4th Mode (Hz) | 5th Mode (Hz) |
|---|---|---|---|---|---|---|---|---|---|---|
| **0.3** | 99.64 | 640.52 | 816.39 | 1167.62 | 1792.86 | 103.60 | 647.86 | 830.00 | 1186.6 | 1811.2 |
| **0.6** | 95.25 | 641.44 | 812.30 | 1162.60 | 1791.91 | 103.55 | 647.85 | 830.44 | 1185.0 | 1810.4 |
| **0.9** | 94.23 | 640.28 | 809.66 | 1160.58 | 1791.80 | 103.00 | 647.85 | 830.15 | 1183.2 | 1810.2 |

Table 4.6 Non-dimensional frequencies for single delamination

| $a$ | ANSYS central-single delamination($\lambda^2$) 1st Mode | 2nd Mode | 3rd Mode | 4th Mode | 5th Mode |
|---|---|---|---|---|---|
| **0.3** | 1.7919 | 11.1995 | 14.355 | 20.51 | 31.31 |
| **0.6** | 1.7900 | 11.1995 | 14.355 | 20.48 | 31.29 |
| **0.9** | 1.7805 | 11.1885 | 14.350 | 20.43 | 31.29 |

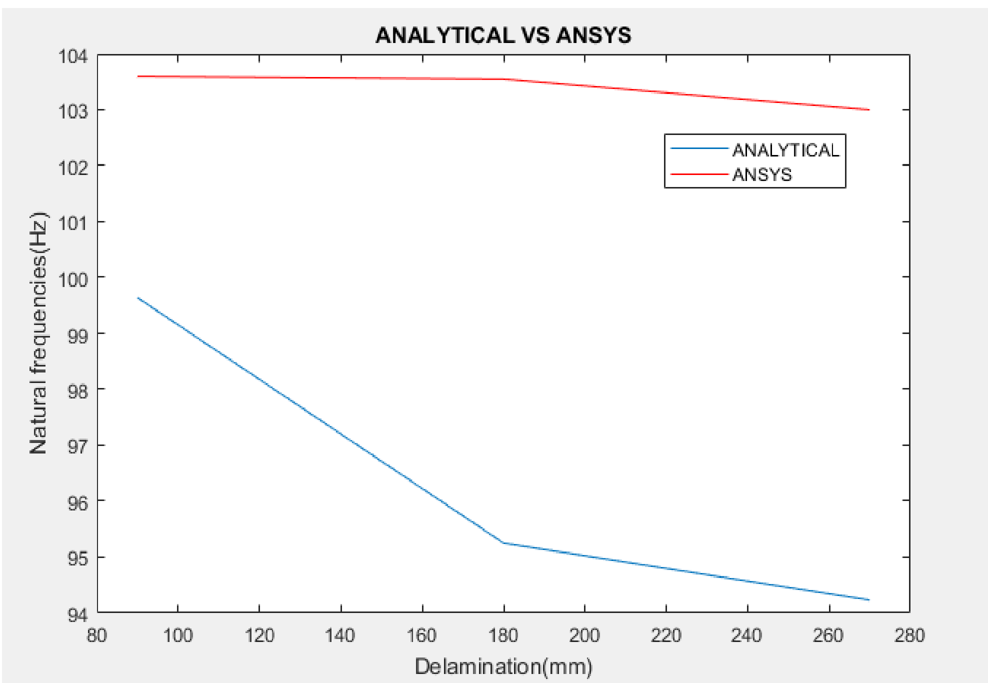



Figure 4.17 First mode analytical and ansys frequencies vs delamination length

The effects of different length of delamination is run from 90mm, 180mm and 270mm. at each case results has been interpreted accordingly. The above figure shows the relationship between the natural frequency and delamination length for the first bending modes in both analytical and ansys results at each case natural frequencies show reduction, but analytical results show progressive reduction when compared to simulation result.

### 4.5.1 Result discussion for central single-delamination

In the following section, a cantilever beam with homogeneous properties that contains two layers of delaminated beam with different delamination ratio is discussed. The delamination in this case is induced at the center, at the same time about the mid portion of the beam, $(x_1 = x_4)$, the length of the induced delaminations varies from $(30\%, 60\% \text{ to } 90\%)$ of the beam length$(90 \leq a \leq 270)$.

The simulation showed that delamination highly affects the beam at its introduction @ 30% delamination ratio the first mode natural frequencies reduces about 49.45% from the intact beam, the second natural frequency drops by 49.18% in addition other results can be calculated. The values illustrated in the table can be shown by use of a graph as shown below.

**Note,** the beam natural frequencies highly affected at the introduction of delamination, then the reduction is minimum.

## 4.6 Case 4-double central delamination of cantilever beam

In another case introducing delamination length in to the structure and simulate it along its length for different vales that is @ $30\%,\ 60\%\ and\ 90\%$ of the beam. The delamination is now double.

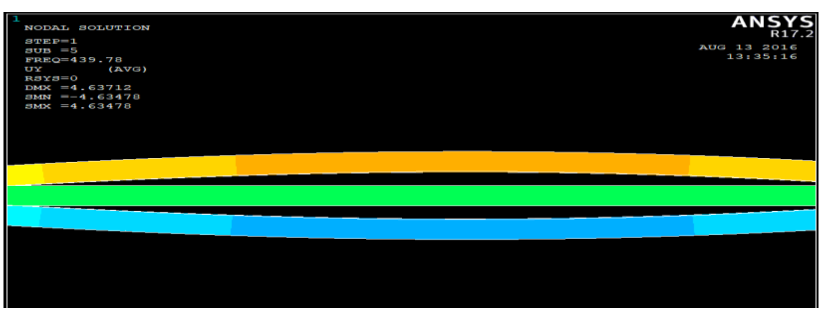



Figure 4.18 A3D- beam with central delamination (front view)

The first five bending modes for double delamination is drawn as the following,

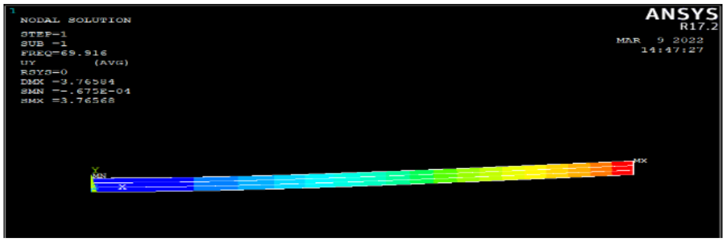



Figure 4.19 The first bending mode

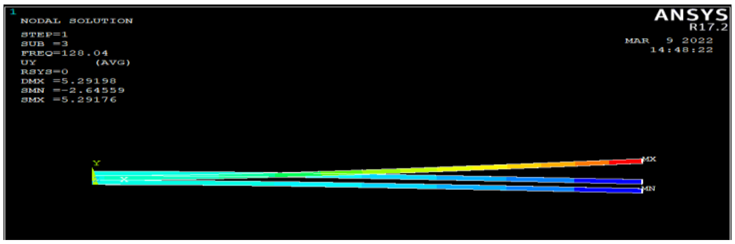



Figure 4.20 The second bending mode

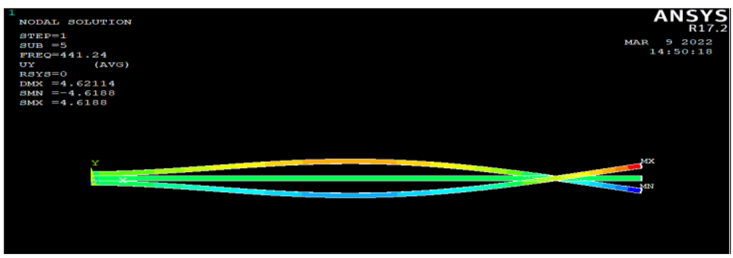



Figure 4.21 The third bending mode

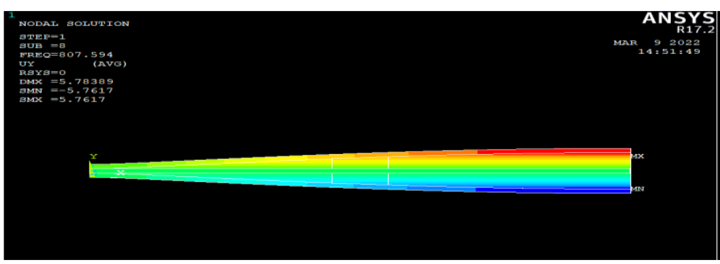



Figure 4.22 The fourth bending mode

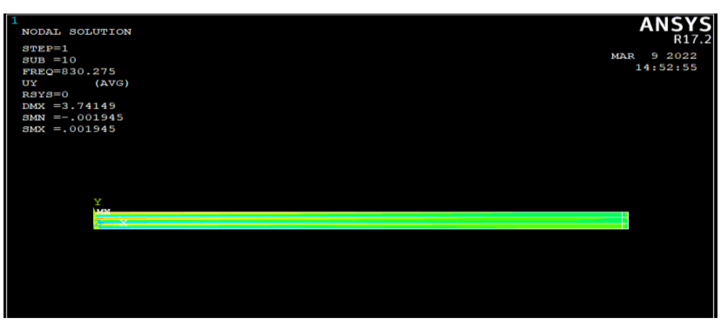



Figure 4.23 The fifth bending mode

The values of natural frequencies obtained from ansys is presented by the following table.

Table 4.7 Natural frequency values for analytical and finite element simulation results

| $a$ | Analytical results | | | | | Finite element simulation results | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1st Mode (Hz) | 2nd Mode (Hz) | 3rd Mode (Hz) | 4th Mode (Hz) | 5th Mode (Hz) | 1st Mode (Hz) | 2nd Mode (Hz) | 3rd Mode (Hz) | 4th Mode (Hz) | 5th Mode (Hz) |
| **0.3** | 62.89 | 119.82 | 420.25 | 782.96 | 813.08 | 69.91 | 122.63 | 439.78 | 807.10 | 830.28 |
| **0.6** | 61.82 | 104.53 | 419.60 | 781.61 | 812.64 | 69.90 | 109.45 | 438.98 | 806.35 | 830.28 |
| **0.9** | 61.28 | 81.60 | 417.80 | 779.96 | 808.32 | 69.90 | 83.60 | 438.29 | 805.57 | 830.28 |

Table 4.8 Ansys non-dimensional frequency results for double- central delamination

| $a$ | ANSYS central-double delamination($\lambda^2$) | | | | |
|---|---|---|---|---|---|
| | 1st Mode | 2nd Mode | 3rd Mode | 4th Mode | 5th Mode |
| **0.3** | 1.2085 | 2.1198 | 7.6024 | 13.952 | 14.3529 |
| **0.6** | 1.2085 | 1.8920 | 7.5886 | 13.939 | 14.3529 |
| **0.9** | 1.2085 | 1.445 | 7.5766 | 13.925 | 14.3529 |

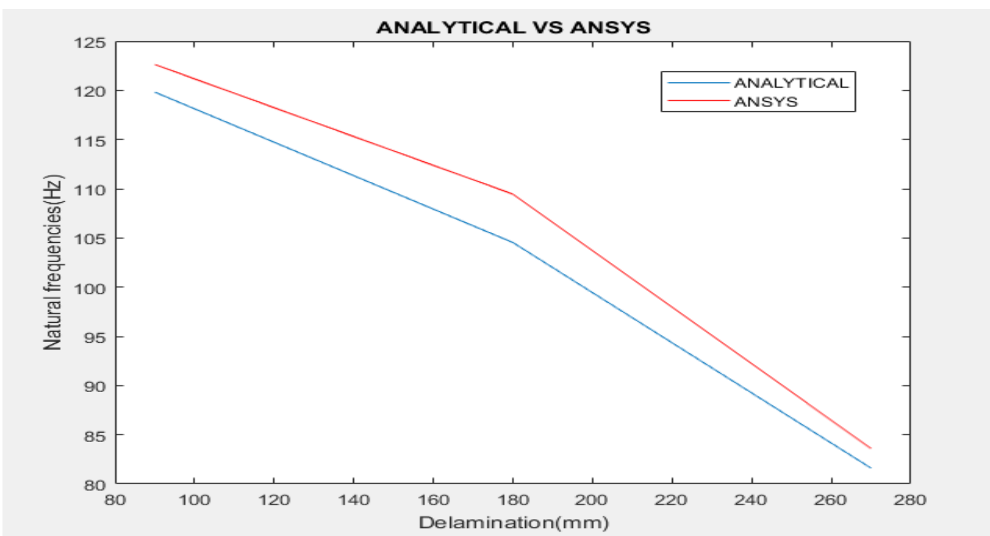



Figure 4.24 Second mode analytical and ansys frequencies vs delamination length

The above figures shows the relationship between natural frequencies and delamination for double central delamination, both analytical and ansys results proves the reduction in natural frequencies due to the rise in length of delamination. The variations in natural modes between the two results is somewhat close when compared to single delamination values.

### 4.6.1 Result discussion for double-delamination

In this sub section, a cantilever beam with the same homogeneous conditions, but with three layers of delaminated beam with each different delamination ratio is done. Delamination starts from the center and varies accordingly.

Table 4.9 Over all presentation of modal frequencies for double delamination

| $a$ | Finite element simulation results | | | | |
|---|---|---|---|---|---|
| | $1^{st}$ Mode (Hz) | $2^{nd}$ Mode (Hz) | $3^{rd}$ Mode (Hz) | $4^{th}$ Mode (Hz) | $5^{th}$ Mode (Hz) |
| **0.3** | 69.91 | 122.63 | 439.78 | 807.10 | 830.28 |
| **0.6** | 69.90 | 109.45 | 438.98 | 806.35 | 830.28 |
| **0.9** | 69.90 | 83.60 | 438.29 | 805.57 | 830.28 |

Values of natural frequencies generally drops as the length and number of delamination increases as can be observed from the tables. But we can see the effects of delamination as we introduce it initially, hence, @ 30% double delamination ratio the first natural frequencies reduces about 65.89% for the finite element simulation value, when compared to the intact beam. In the same way, the second natural frequencies drops by 90.38%.

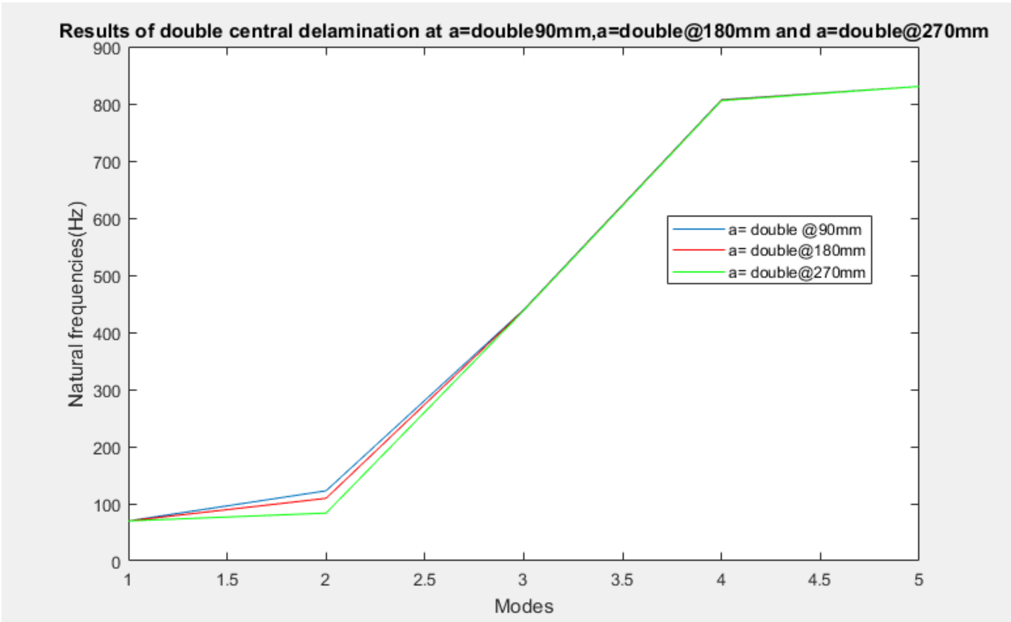



Figure 4.25 Double delamination at different delamination length

**Note,** the first and the fifth modes show a significant reduction at introduction of delamination then they remain constant, whereas the second, third and fourth modes show a significant reduction in natural modes as length of delamination increases.

## 4.7 Case study 5- triple central delamination

The fourth case is introducing delamination with different length in to the structure and simulate it along its length for different values that is from $30\%$ ,$60\%$ $to$ $90\%$ of the beam this time the type of delamination is triple.

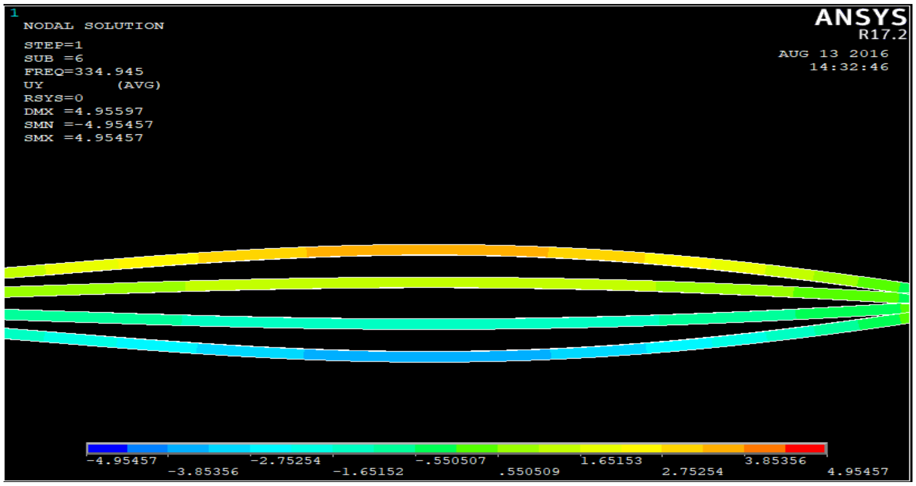



Figure 4.26 A3d-beam with triple delamination (front view)

The following are the first five bending modes for central triple delamination.

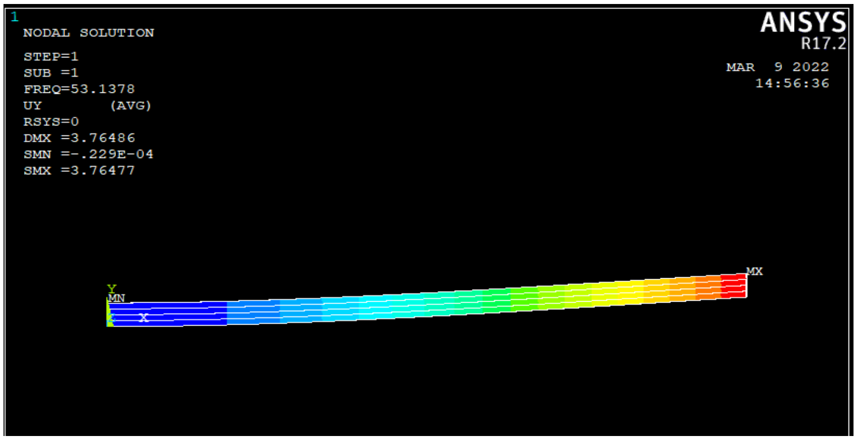



Figure 4.27 The first bending mode

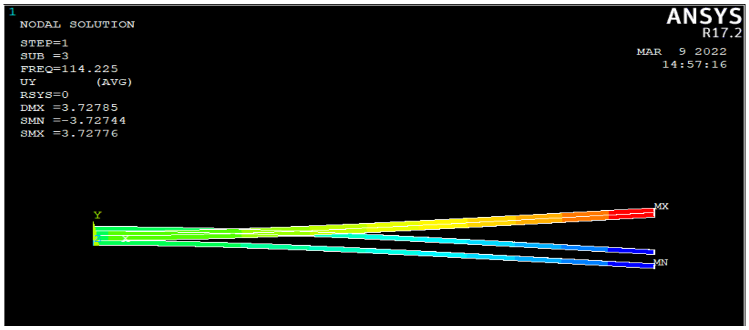



Figure 4.28 The second bending mode

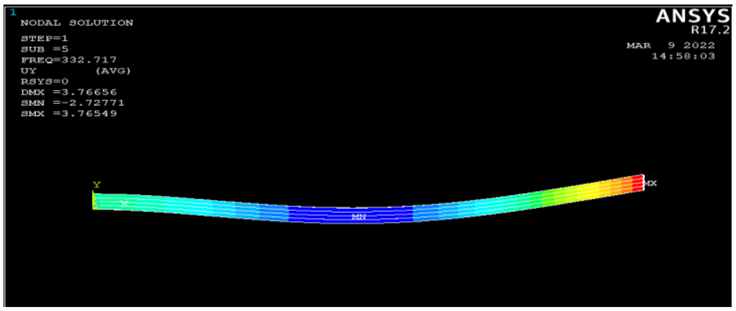



Figure 4.29 The third bending mode

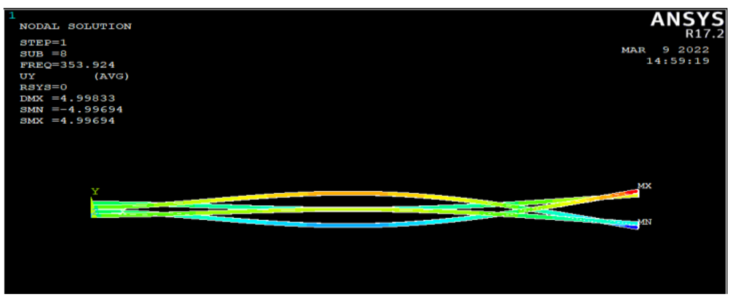



Figure 4.30 The fourth bending mode

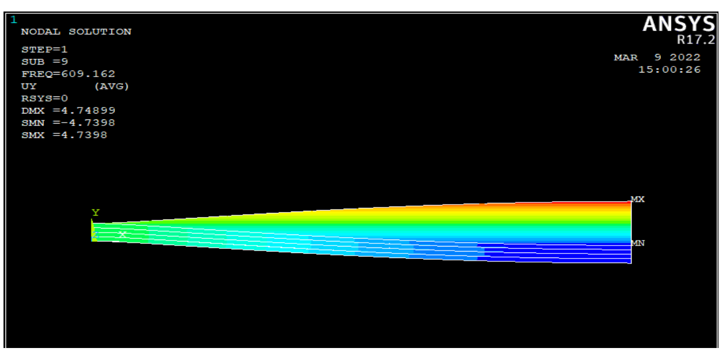



Figure 4.31 The fifth bending mode

Table 4.10 Natural frequency values for analytical and finite element simulation results

| $a$ | Analytical results | | | | | Finite element simulation results | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1st Mode (Hz) | 2nd Mode (Hz) | 3rd Mode (Hz) | 4th Mode (Hz) | 5th Mode (Hz) | 1st Mode (Hz) | 2nd Mode (Hz) | 3rd Mode (Hz) | 4th Mode (Hz) | 5th Mode (Hz) |
| **0.3** | 49.96 | 104.90 | 317.48 | 328.60 | 596.66 | 53.134 | 108.70 | 332.72 | 345.81 | 610.90 |
| **0.6** | 48.83 | 95.93 | 314.89 | 326.78 | 596.43 | 53.128 | 95.216 | 332.71 | 341.43 | 610.11 |
| **0.9** | 44.82 | 65.83 | 313.21 | 320.23 | 595.08 | 53.124 | 68.410 | 332.70 | 336.88 | 609.29 |

The non-dimensional values can be tabulated as the following below.

Table 4.11 Ansys non-dimensional frequency results for triple- central delamination

| $a$ | ANSYS central-triple delamination($\lambda^2$) | | | | |
|---|---|---|---|---|---|
| | 1st Mode | 2nd Mode | 3rd Mode | 4th Mode | 5th Mode |
| **0.3** | 0.9185 | 1.879 | 5.751 | 5.977 | 10.56 |
| **0.6** | 0.9185 | 1.645 | 5.730 | 5.902 | 10.54 |
| **0.9** | 0.9184 | 1.182 | 5.711 | 5.823 | 10.53 |

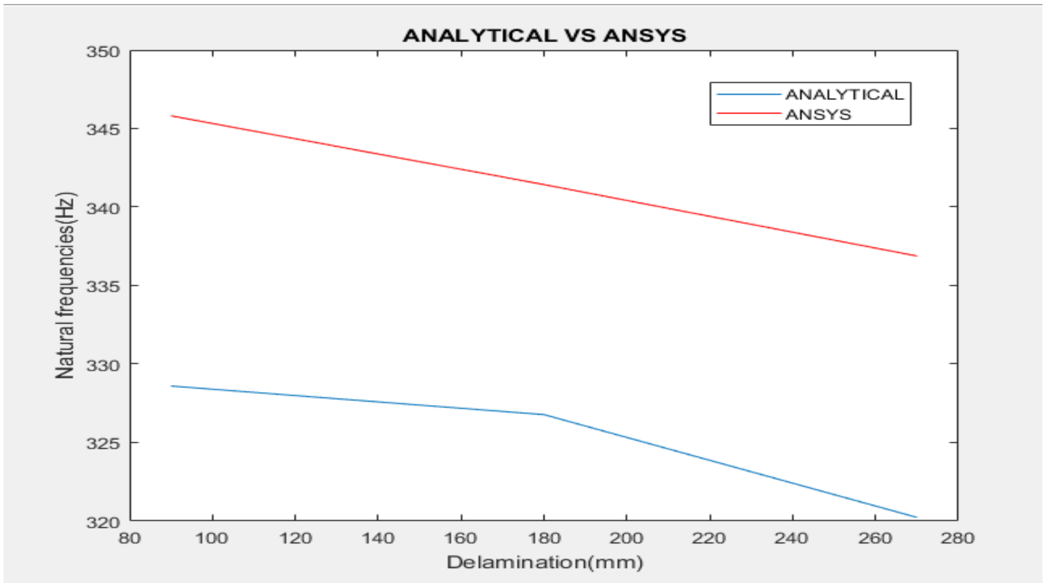



Figure 4.32 Fourth mode analytical and ansys frequencies vs delamination length

The effects of different length of delamination is run from 90mm, 180 and 270mm. at each case results has been interpreted accordingly. In the same way to previous cases from the above figure we can observe the relationship between natural frequencies and delamination length such that increase in type and length of delamination further deteriorates natural frequencies of the composite beam.

### 4.7.1 Result discussion for Triple-delamination

Delamination starts from the center and varies accordingly. In the same way the induced delamination varies from$(90 \leq a \leq 270)$.

Table 4.12 Over all presentation of triple delamination

| $\boldsymbol{a}$ | **Finite element simulation results** | | | | |
|---|---|---|---|---|---|
| | 1st Mode (Hz) | 2nd Mode (Hz) | 3rd Mode (Hz) | 4th Mode (Hz) | 5th Mode (Hz) |
| **0.3** | 53.134 | 108.70 | 332.72 | 345.81 | 610.90 |
| **0.6** | 53.128 | 95.216 | 332.71 | 341.43 | 610.11 |
| **0.9** | 53.124 | 68.410 | 332.70 | 336.88 | 609.29 |

Values of natural frequencies generally drops as the length and number of delamination increases as can be observed from the tables. But we can see the effects of delamination as we introduce it initially, hence, @ 30% delamination ratio the first natural frequencies will reduce by 74.08% from the intact an also the value of the second natural frequency further reduces by 91.47% , other results show reduction in natural frequencies as we increase the length and change the type of delamination.

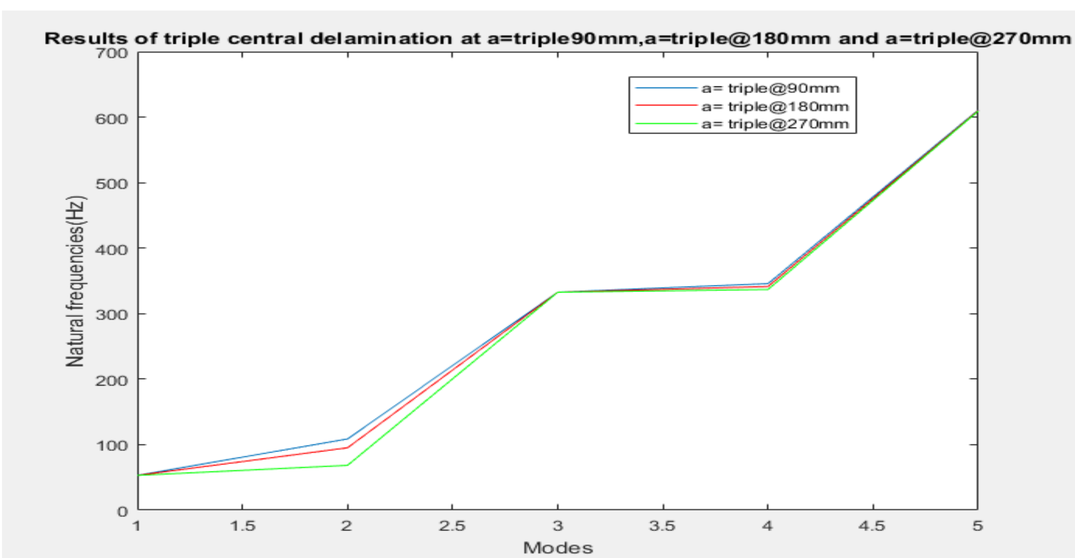



Figure 4.33 Triple delamination at different delamination length

**Note,** the above figure demonstrates that, in all cases natural frequencies showed reduction the first and the fifth modes show the lowest reduction when compared to the rest. The second and the fourth modes show a progressive reduction in natural frequencies in this specific case.

## 4.8 Result comparisons on central delamination

In the following section the results for single, double and triple delamination discussed by use of a graph. As presented below in the figure that the values of natural frequencies will significantly drop down as we go from single to multiple delamination. For this case, the delamination at 90mm for each cases will be compared. The blue line shows single delamination, the red line indicates double delamination and the green stands for triple delamination.

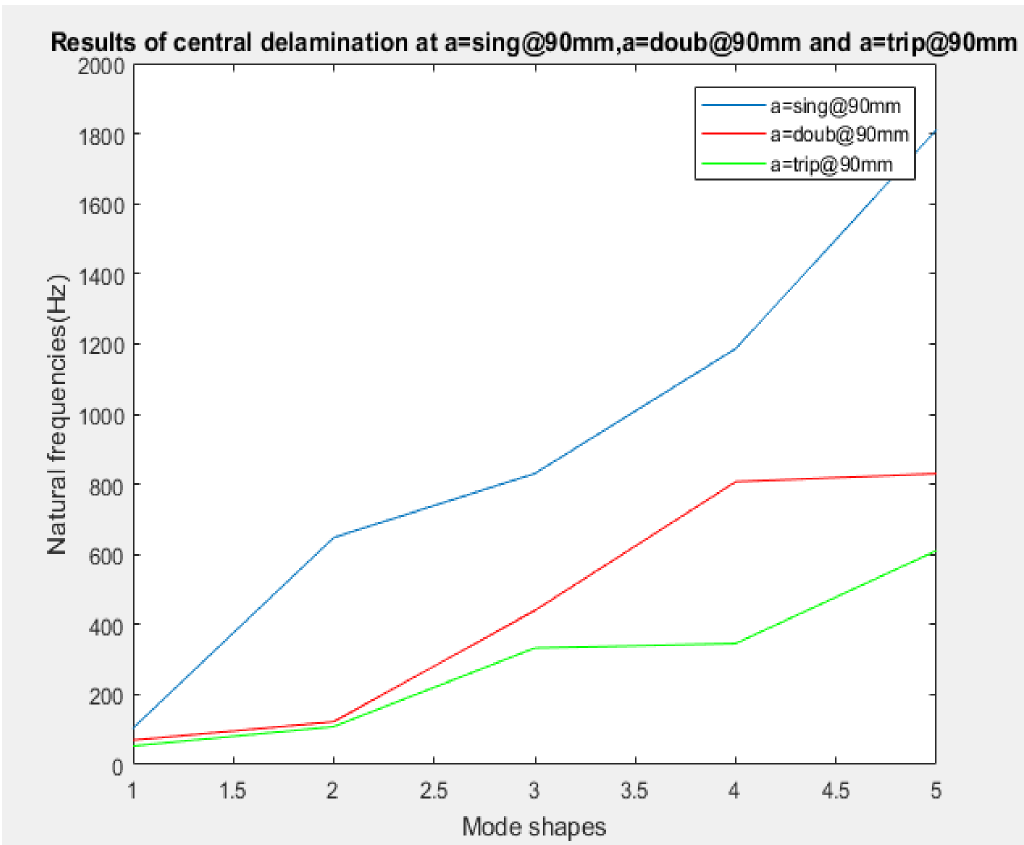



Figure 4.34 Result comparisons for single, double and triple central delamination

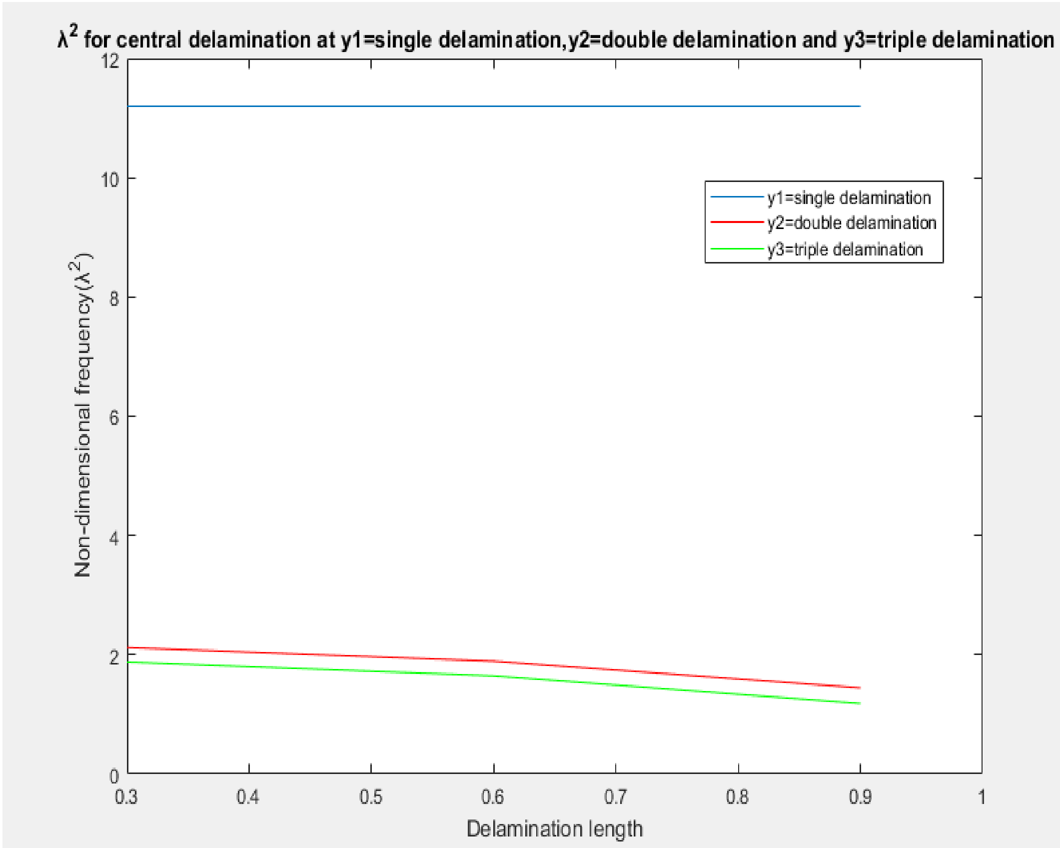



Figure 4.35 Non dimensional frequencies for central delamination

As observed from the tabulated results and the graphs that increasing the length and type of delamination significantly affects the non-dimensional frequencies.

## 4.9 Case-6 Free-end single delamination

In this research the position of delamination has been changed from center, free end and fixed end position of the beam, in order to understand and evaluate the effect of delamination location on natural frequencies. As a result the simulation is done for all kind of delamination by varying its length.

### 4.9.1 Cantilever beam free-end delamination

In the following table results of cantilever beam with free-end delamination of different delamination length and type is discussed. The delamination range varies from 0.3 to 0.9 along the beam span. The results obtained are tabulated and presented using graph to illustrate the relationship between $\lambda^2$ and delamination length.

Table 4.13 Ansys non-dimensional frequency results for free-end single delamination

| $a$ | ANSYS free-end single delamination($\lambda^2$) | | | | |
|---|---|---|---|---|---|
| | 1st Mode | 2nd Mode | 3rd Mode | 4th Mode | 5th Mode |
| **0.3** | 1.7897 | 1.8533 | 11.1913 | 14.36 | 20.459 |
| **0.6** | 1.7897 | 1.7947 | 11.1913 | 14.36 | 20.459 |
| **0.9** | 1.7800 | 1.7947 | 11.1913 | 14.36 | 20.459 |

In the same way to previous conditions the observed results showed us that changing the location of delamination also affects the beams natural frequencies, similar conclusions can also be made here, as delamination is changed from single to double and triple the natural frequencies negatively affected.

Table 4.14 Free end single delamination values

| $a$ | Finite element simulation results | | | | |
|---|---|---|---|---|---|
| | 1st Mode (Hz) | 2nd Mode (Hz) | 3rd Mode (Hz) | 4th Mode (Hz) | 5th Mode (Hz) |
| **0.3** | 103.53 | 107.21 | 647.39 | 830.70 | 1183.50 |
| **0.6** | 103.53 | 103.82 | 647.39 | 830.70 | 1183.50 |
| **0.9** | 103.53 | 103.53 | 647.39 | 830.70 | 1183.50 |

The results proved delamination highly affects the beam at its introduction @ 30% delamination ratio the first mode natural frequencies reduces about 49.49% from the intact beam, the second natural frequency drops by 91.59% in addition other results can be calculated. The values illustrated in the table can be shown by use of a graph as shown below.

### 4.9.2 Free end double delamination

In this case we introduce delamination at different length in to the structure and simulate it along its length for different vales that is from 30% ,60% to 90% of the beam this time the type of delamination is double. Results are tabulated bellow.

Table 4.15 Ansys non-dimensional frequency results for free-end double delamination

| $a$ | ANSYS free-end double delamination($\lambda^2$) | | | | |
|---|---|---|---|---|---|
| | 1st Mode | 2nd Mode | 3rd Mode | 4th Mode | 5th Mode |
| **0.3** | 1.2083 | 1.3141 | 7.564 | 7.733 | 13.922 |
| **0.6** | 1.2083 | 1.2169 | 7.564 | 7.620 | 13.922 |
| **0.9** | 1.2083 | 1.2083 | 7.564 | 7.5647 | 13.921 |

Table 4.16 Free end double delamination values

| $a$ | Finite element simulation results | | | | |
|---|---|---|---|---|---|
| | 1st Mode (Hz) | 2nd Mode (Hz) | 3rd Mode (Hz) | 4th Mode (Hz) | 5th Mode (Hz) |
| **0.3** | 69.90 | 76.02 | 437.60 | 447.36 | 805.38 |
| **0.6** | 69.90 | 70.40 | 437.60 | 440.81 | 805.34 |
| **0.9** | 69.90 | 69.90 | 437.60 | 437.60 | 805.33 |

As we can observe from the results that delamination highly affects the beam at its introduction that is @ 30% delamination ratio the first mode natural frequencies reduces about 65.89% from the intact beam, the second natural frequency drops by 94.03% in addition other results can be calculated. The values illustrated in the table can be shown by use of a graph as shown below.

**Note**, from the above value we can clearly observe that the increase in length of delamination affects the second mode shape and the fourth mode shape, when compared to the rest.

### 4.9.3 Free end triple delamination

In this case we introduce triple delamination at different length in to the structure and simulate it along its length for different vales that is from 30% ,60% to 90% of the beam this time the type of delamination is triple. In the same way results are tabulated bellow. The diagrammatic representation is followed as the following.

Table 4.17 Ansys non-dimensional frequency results for free end triple delamination

| $a$ | ANSYS free-end triple delamination($\lambda^2$) | | | | |
|---|---|---|---|---|---|
| | 1st Mode | 2nd Mode | 3rd Mode | 4th Mode | 5th Mode |
| **0.3** | 0.9182 | 1.025 | 1.4030 | 5.8090 | 6.087 |
| **0.6** | 0.9182 | 0.9324 | 0.9672 | 5.7703 | 5.8611 |
| **0.9** | 0.9182 | 0.9182 | 0.9182 | 5.7513 | 5.7513 |

Table 4.18 Free end triple delamination values

| $a$ | Finite element simulation results | | | | |
|---|---|---|---|---|---|
| | 1st Mode (Hz) | 2nd Mode (Hz) | 3rd Mode (Hz) | 4th Mode (Hz) | 5th Mode (Hz) |
| **0.3** | 53.12 | 59.31 | 81.16 | 336.04 | 352.12 |
| **0.6** | 53.12 | 53.64 | 55.95 | 333.80 | 339.05 |
| **0.9** | 53.12 | 53.12 | 53.12 | 332.70 | 332.71 |

As we can observe from the results that delamination highly affects the beam at its introduction that is @ 30% delamination ratio the first mode natural frequencies reduces about 74.08% from the intact beam, the second natural frequency drops by 95.35% in addition other results can be calculated. The values illustrated in the table can be shown by use of a graph as shown below.

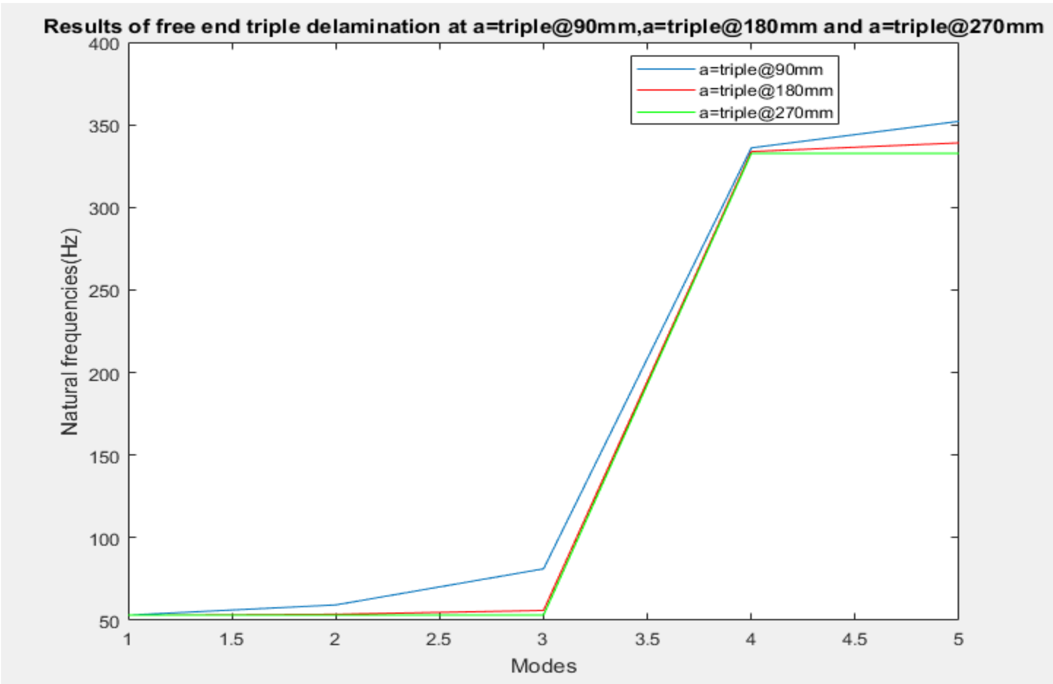



Figure 4.36 Results for free end triple delamination

The above result clearly show that when the ratio of delamination increases the respective mode shapes falls down considerably.

**Note,** all the values of natural modes show considerable amount of reduction except, the first mode.

### 4.9.4 Free end delamination result conclusion

In the following section the results for single, double and triple delamination discussed by use of a graph. As presented below in the figure that the values of natural frequencies will significantly drop down.

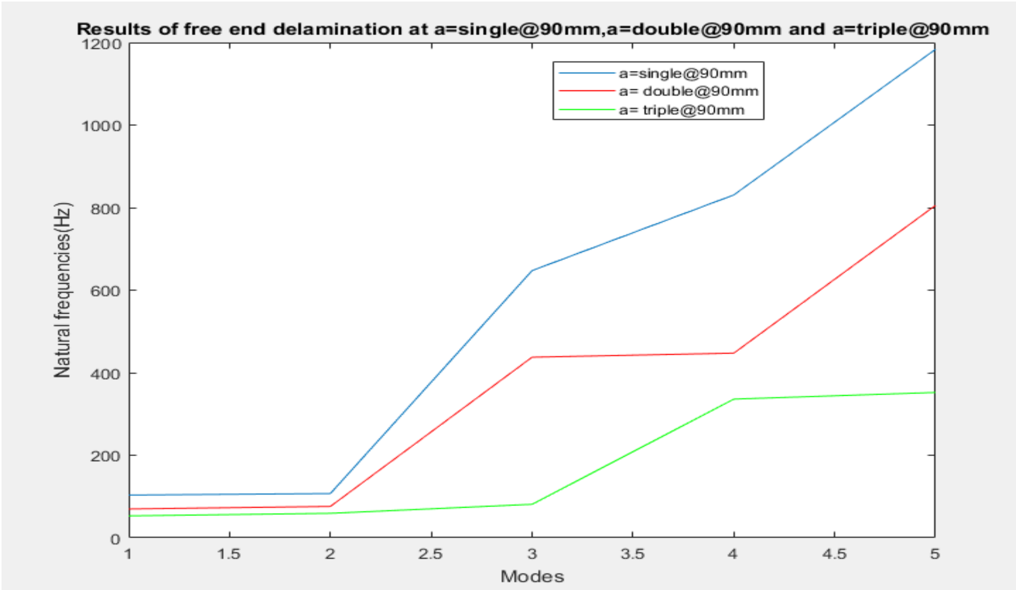



Figure 4.37 Result comparisons between single, double and triple free end delamination

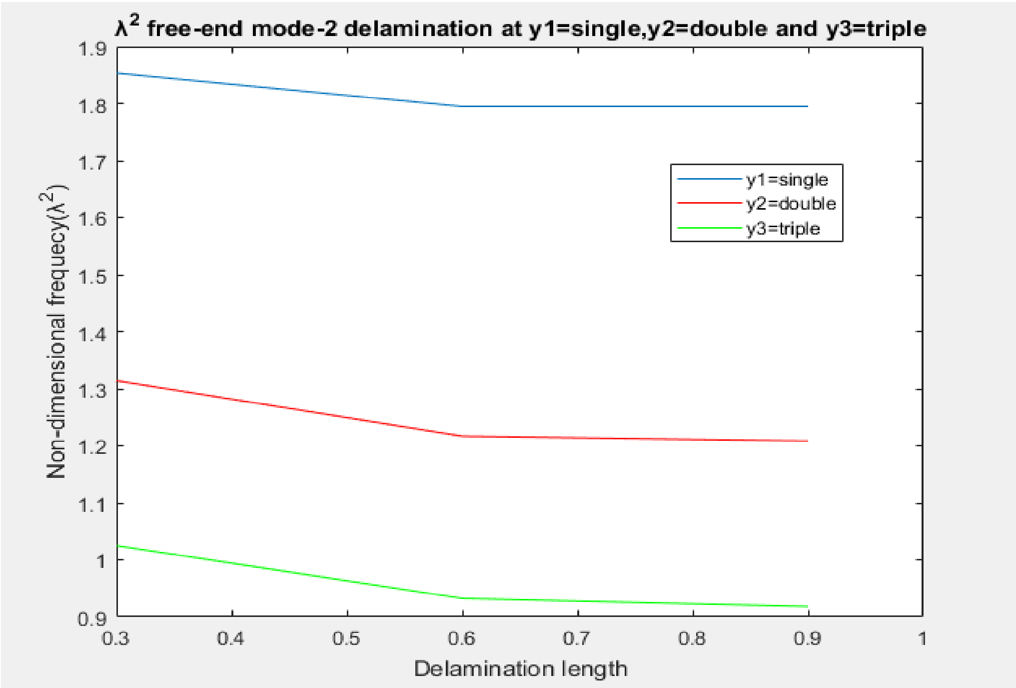



Figure 4.38 Non dimensional frequency result comparisons

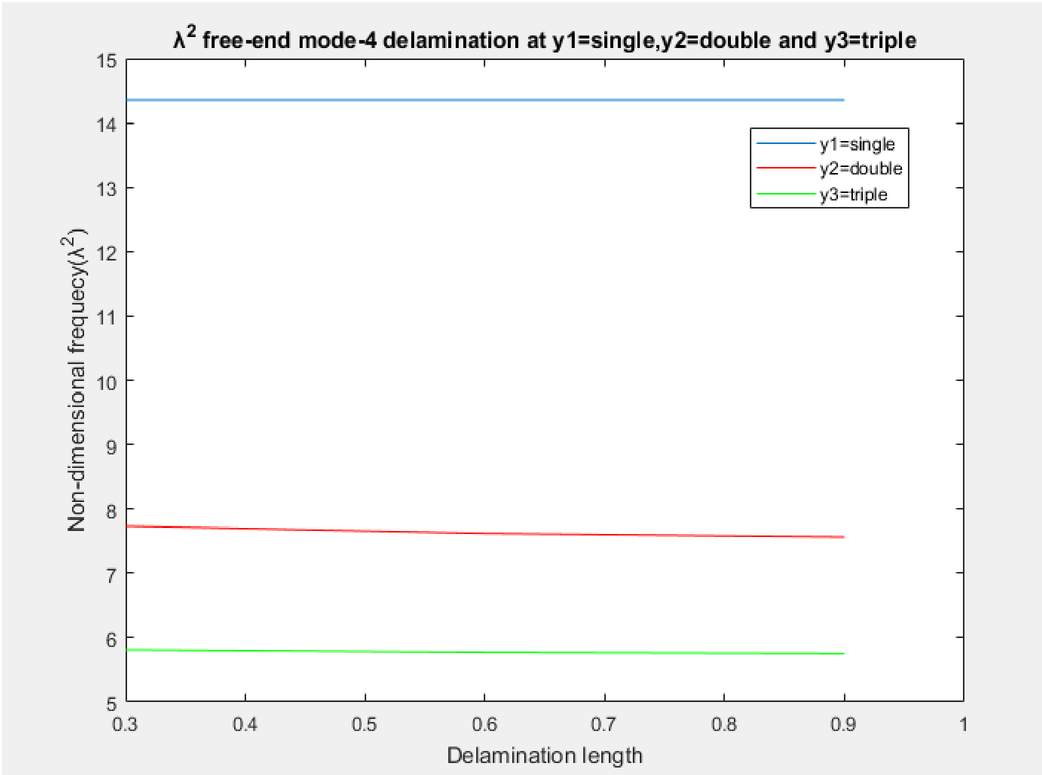



Figure 4.39 Mode-4 non dimensional frequency result comparisons

## 4.10 Case-7 Fixed end delamination

### 4.10.1 Fixed end single delamination

The other interest of this study is when the delamination location is changed to other locations in the beam such as the fixed end. By the same procedure different types of delaminations at different delamination length is the simulated on ansys apdl. The results obtained are tabulated and presented using graph to illustrate the relationship between $\lambda^2$ and delamination length.

Table 4.19 Ansys non-dimensional frequency results for fixed-end single delamination

| $\boldsymbol{a}$ | ANSYS fixed-end single delamination($\lambda^2$) | | | | |
|---|---|---|---|---|---|
| | 1st Mode | 2nd Mode | 3rd Mode | 4th Mode | 5th Mode |
| **0.3** | 1.79109 | 2.1923 | 11.1995 | 14.355 | 20.4849 |
| **0.6** | 1.79109 | 2.1508 | 11.1995 | 14.355 | 20.4849 |
| **0.9** | 1.79109 | 1.9366 | 11.1995 | 14.355 | 20.4849 |

Table 4.20 Ansys natural frequency results for fixed-end single delamination

| $a$ | Finite element simulation results | | | | |
|---|---|---|---|---|---|
| | 1st Mode (Hz) | 2nd Mode (Hz) | 3rd Mode (Hz) | 4th Mode (Hz) | 5th Mode (Hz) |
| **0.3** | 103.60 | 126.82 | 647.86 | 830.45 | 1185.0 |
| **0.6** | 103.60 | 124.42 | 647.85 | 830.45 | 1185.0 |
| **0.9** | 103.60 | 112.03 | 647.85 | 830.45 | 1185.0 |

As can be observed from the results that delamination highly affects the beam at its introduction that is @ 30% delamination ratio the first mode natural frequencies reduces about 49.46% from the intact beam, the second natural frequency drops by 90.05% in addition other results can be calculated. The values illustrated in the table can be shown by use of a graph as shown below.

**Note,** the above result shows us that, in the case of fixed end single delamination the second mode shape is highly affected than the rest mode shapes.

### 4.10.2 Fixed end double delamination

In this sub topic the results of fixed end double delamination is thoroughly discussed and conclusions are made.

Table 4.21 Ansys non-dimensional frequency results for fixed end double delamination

| $a$ | ANSYS fixed-end double delamination($\lambda^2$) | | | | |
|---|---|---|---|---|---|
| | 1st Mode | 2nd Mode | 3rd Mode | 4th Mode | 5th Mode |
| **0.3** | 1.2085 | 1.6308 | 7.5647 | 7.7794 | 13.923 |
| **0.6** | 1.2085 | 1.5886 | 7.5647 | 7.6555 | 13.923 |
| **0.9** | 1.2085 | 1.3661 | 7.5647 | 7.6219 | 13.923 |

Table 4.22 Double delamination at fixed end

| $a$ | Finite element simulation results | | | | |
|---|---|---|---|---|---|
| | 1st Mode (Hz) | 2nd Mode (Hz) | 3rd Mode (Hz) | 4th Mode (Hz) | 5th Mode (Hz) |
| **0.3** | 69.90 | 94.34 | 437.60 | 450.02 | 805.44 |
| **0.6** | 69.90 | 91.90 | 437.60 | 442.85 | 805.41 |
| **0.9** | 69.90 | 79.03 | 437.60 | 440.91 | 805.35 |

Similarly from the results that delamination highly affects the beam at its introduction that is @ 30% delamination ratio the first mode natural frequencies reduces about 65.89% from the intact beam, the second natural frequency drops by 92.60% in addition other results can be calculated. The values illustrated in the table can be shown by use of a graph as shown below.

**Note,** the above result shows us that, in the case of fixed end double delamination the second, fourth and the fifth modes are highly affected compared to the rest of the mode values. The values of the first and the third mode are somewhat constant, and the fifth mode values decreases at a small rate.

### 4.10.3 Fixed end triple delamination

In this sub topic the results of fixed end triple delamination is thoroughly discussed and conclusions are made. By the same procedure different types of delaminations at different delamination length is the simulated on Ansys apdl. Let’s consider the diagrammatic representation of the delamination.

Table 4.23 Ansys non-dimensional frequency results for fixed-end triple delamination

| $\boldsymbol{a}$ | **ANSYS fixed-end triple delamination($\lambda^2$)** | | | | |
|---|---|---|---|---|---|
| | 1st Mode | 2nd Mode | 3rd Mode | 4th Mode | 5th Mode |
| **0.3** | 0.9182 | 1.3343 | 2.5107 | 5.8255 | 6.170 |
| **0.6** | 0.9182 | 1.2935 | 2.3717 | 5.7824 | 5.9335 |
| **0.9** | 0.9182 | 1.07697 | 1.6189 | 5.7708 | 5.8685 |

Table 4.24 Ansys natural frequency results for fixed-end triple delamination

| $\boldsymbol{a}$ | **Finite element simulation results** | | | | |
|---|---|---|---|---|---|
| | 1st Mode (Hz) | 2nd Mode (Hz) | 3rd Mode (Hz) | 4th Mode (Hz) | 5th Mode (Hz) |
| **0.3** | 53.12 | 77.19 | 145.24 | 336.99 | 356.95 |
| **0.6** | 53.12 | 74.83 | 137.20 | 334.50 | 343.24 |
| **0.9** | 53.12 | 62.30 | 93.65 | 333.83 | 339.48 |

The results indicate delamination highly affects the beam at its introduction that is @ 30% delamination ratio the first mode natural frequencies reduces about 74.08% from the intact beam, the second natural frequency drops by 93.95% in addition other results can be calculated. The values illustrated in the table can be shown by use of a graph as shown below.

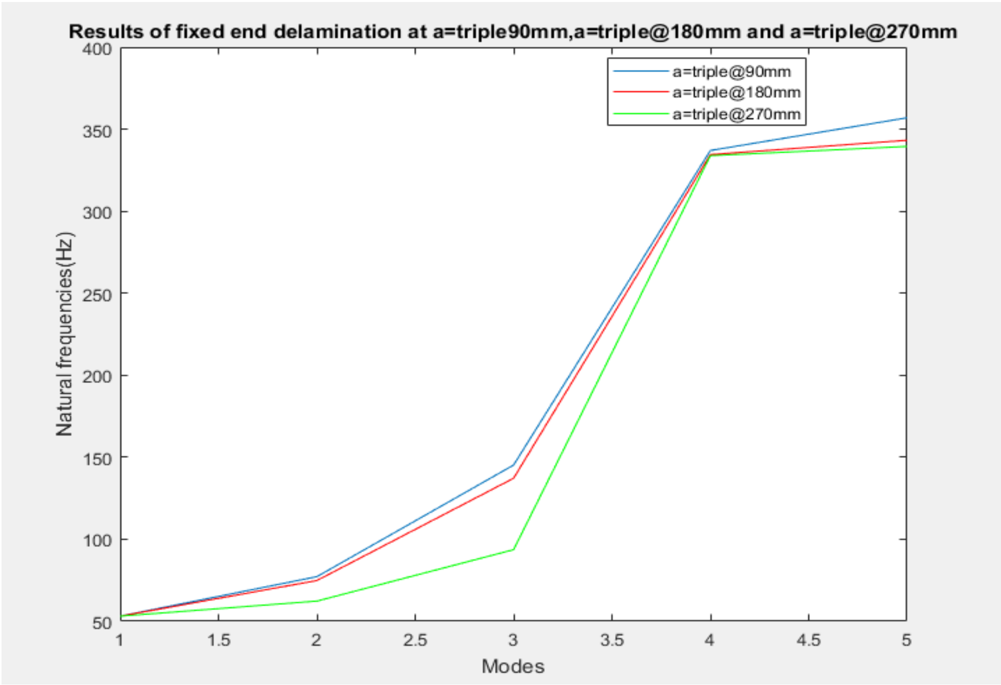



Figure 4.40 Results of fixed-end triple delamination

**Note,** the result clearly shows that for every natural modes except the first mode, significant reduction in modes observed.

### 4.10.4 Fixed end delamination result conclusion

In the following section the results for single, double and triple delamination discussed by use of a graph. As presented below in the figure that the values of natural frequencies will significantly drop down. For this case, the delamination at 90mm for each cases will be compared.

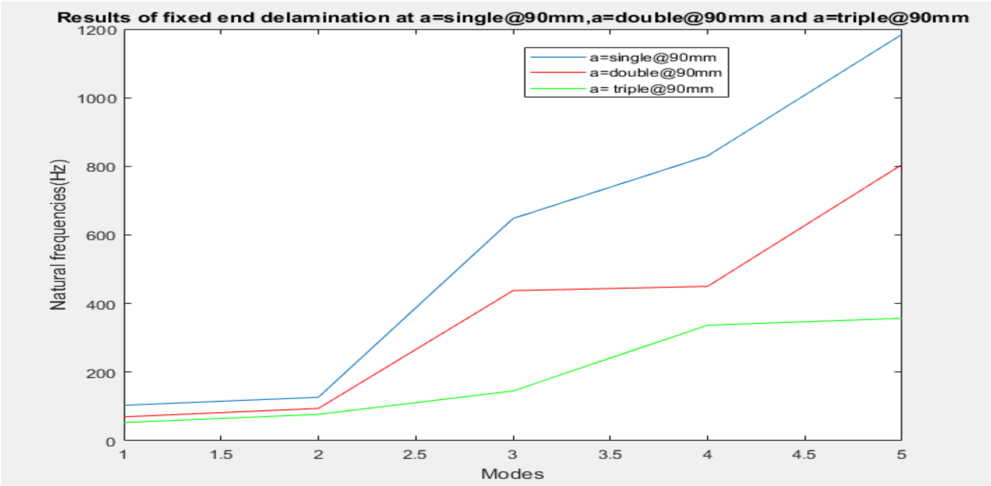



Figure 4.41 Fixed end result comparisons

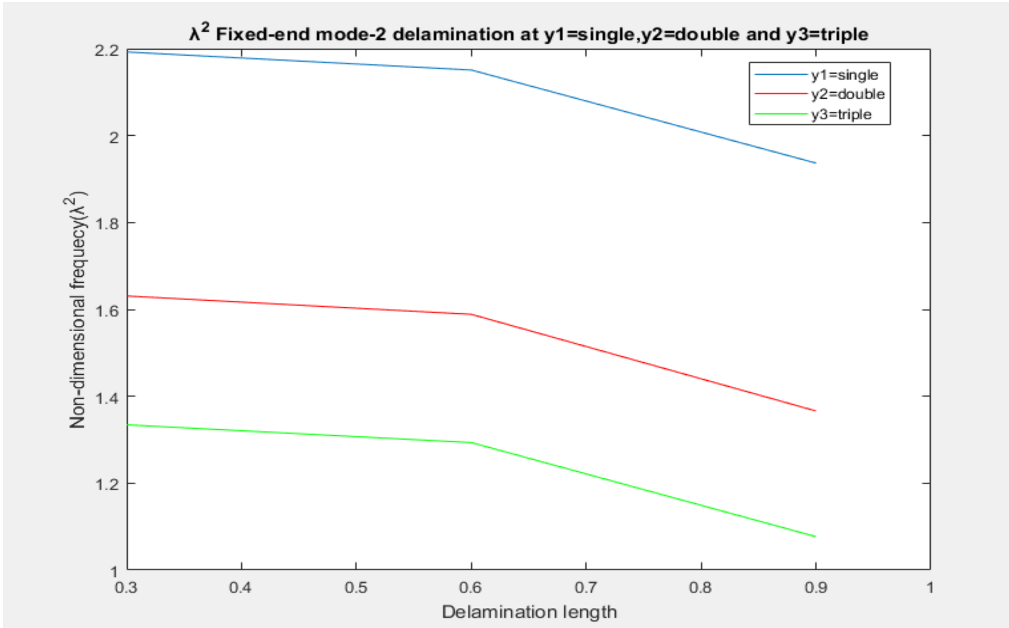



Figure 4.42 Mode-2 non dimensional frequency

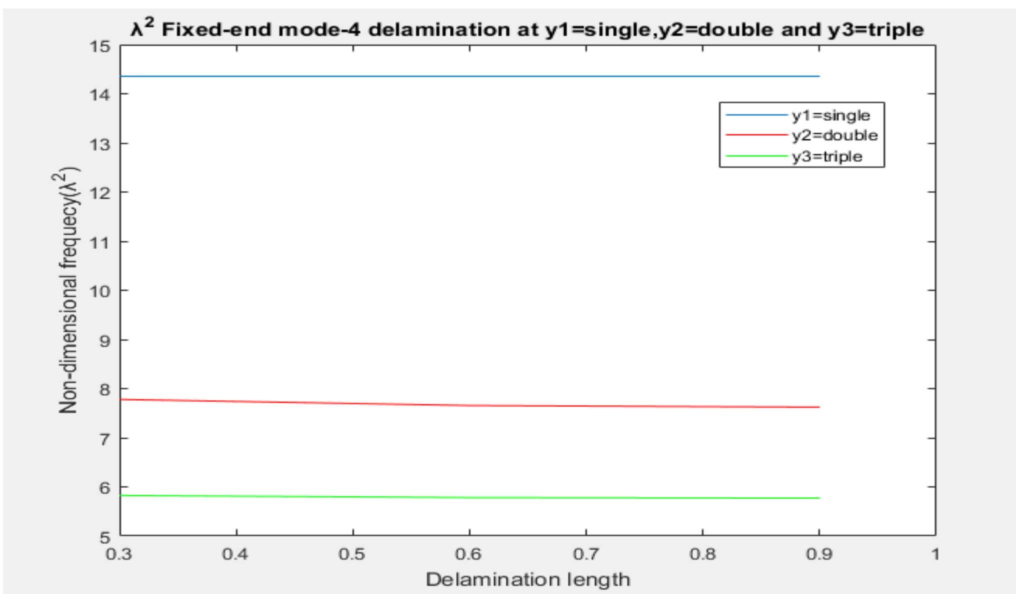



Figure 4.43 Mode-4 non dimensional frequency

In the same way to previous conditions, the observed results showed us that changing the location of delamination also affects the beams natural frequencies differently, in all conditions of delamination one thing is observable that is any type of introduction of delamination in beams will significantly affect the stiffness of the beam that eventually can be a cause for failure.

## 4.11 Effect of delamination location on natural frequencies

Three locations were discussed in this study and results at each case has been interpreted and compared, now let's consider the effect of location of delamination on the beam natural frequency. Let's see the comparisons of the central delamination and free-end delamination at the introduction of delamination, the results are tabulated below.

### 4.11.1 Central delamination

The results of central delamination is tabulated below the table, the obtained results are for only 90mm delamination.

Table 4.25 Central delamination comparisons by percentage reduction

| Central delamination (30%) | central delamination comparisons by percentage reduction | | | | |
|---|---|---|---|---|---|
| | $1^{st}$ Mode (%) | $2^{nd}$ Mode (%) | $3^{rd}$ Mode (%) | $4^{th}$ Mode (%) | $5^{th}$ Mode (%) |
| **single** | 49.45 | 49.18 | 76.5 | 82.60 | 83.64 |
| **double** | 65.89 | 90.38 | 87.55 | 88.16 | 92.45 |
| **triple** | 74.08 | 91.47 | 90.58 | 94.93 | 94.48 |

### 4.11.2 Free-end delamination

The results of free-end delamination is tabulated below the table, the obtained results are for only 90mm delamination.

Table 4.26 Free-end delamination comparisons by percentage reduction

| Free-end delamination (30%) | Free-end delamination comparisons by percentage reduction | | | | |
|---|---|---|---|---|---|
| | $1^{st}$ Mode (%) | $2^{nd}$ Mode (%) | $3^{rd}$ Mode (%) | $4^{th}$ Mode (%) | $5^{th}$ Mode (%) |
| **single** | 49.45 | 91.59 | 81.67 | 87.82 | 89.30 |
| **double** | 65.89 | 94.03 | 87.61 | 93.44 | 92.73 |
| **triple** | 74.08 | 95.34 | 97.70 | 95.07 | 96.81 |

As can be observed from the figures that in all cases of delamination (central vs free-end), delamination certainly affects the beam when it's located at the end rather than the centered, although delamination causes a great natural frequency reduction at the center but it's more at the end, in both cases the first natural frequencies remains un-changed but others show reduction.

### 4.11.3 Fixed-end delamination

The last position is the fixed-end location, in the same way the reduction in natural modes is calculated and tabulated below the following table.

Table 4.27 Fixed-end delamination comparisons by percentage reduction

| Fixed-end delamination (30%) | Fixed-end delamination comparisons by percentage reduction | | | | |
|---|---|---|---|---|---|
| | 1st Mode (%) | 2nd Mode (%) | 3rd Mode (%) | 4th Mode (%) | 5th Mode (%) |
| **single** | 49.45 | 90.05 | 81.67 | 87.82 | 89.30 |
| **double** | 65.90 | 92.60 | 87.61 | 93.40 | 92.72 |
| **triple** | 74.08 | 93.94 | 95.88 | 95.05 | 96.77 |

The above table clearly shows that, the values of natural frequencies affected by type of delamination and when delamination is present at fixed end, according to the figures it is the second position to affect natural frequencies. Somewhat, close to free end delamination but it's more likely to prevent the reduction of the beam natural frequency. The results are closer together at this delamination ratio, consequently need to change the ratio of delamination furthermore is necessary.

Let's compare free-end triple and fixed-end triple delamination to observe the change even more at different delamination ratio.

Table 4.28 Free-end delamination comparisons by percentage reduction

| Free-end delamination (60%) | Free-end delamination comparisons by percentage reduction | | | | |
|---|---|---|---|---|---|
| | 1st Mode (%) | 2nd Mode (%) | 3rd Mode (%) | 4th Mode (%) | 5th Mode (%) |
| **Triple** | 74.08 | 95.80 | 98.41 | 95.10 | 96.93 |

Table 4.29 Fixed-end delamination comparisons by percentage reduction

| Fixed-end delamination (60%) | Fixed-end delamination comparisons by percentage reduction | | | | |
|---|---|---|---|---|---|
| | 1st Mode (%) | 2nd Mode (%) | 3rd Mode (%) | 4th Mode (%) | 5th Mode (%) |
| **Triple** | 74.08 | 94.13 | 96.11 | 95.09 | 96.89 |

The above two tables show us that natural frequency of the beam affected more when it's located at the free end than the fixed end, since the percentage reduction is generally more when compared to the equivalent fixed-end delamination case.

To further observe the reduction of natural frequency let's close look at 90% triple delamination at free-end and fixed-end.

Table 4.30 Free-end delamination comparisons by percentage reduction

| Free-end delamination (90%) | Free-end delamination comparisons by percentage reduction | | | | |
|---|---|---|---|---|---|
| | 1st Mode (%) | 2nd Mode (%) | 3rd Mode (%) | 4th Mode (%) | 5th Mode (%) |
| **Triple** | 74.08 | 95.83 | 98.49 | 95.12 | 96.99 |

Table 4.31 Fixed-end delamination comparisons by percentage reduction

| Fixed-end delamination (90%) | Fixed-end delamination comparisons by percentage reduction | | | | |
|---|---|---|---|---|---|
| | 1st Mode (%) | 2nd Mode (%) | 3rd Mode (%) | 4th Mode (%) | 5th Mode (%) |
| **Triple** | 74.08 | 95.11 | 97.34 | 95.10 | 96.93 |

**Note,** the results on the above tables can be another concluding evidence that generally the natural frequencies of a beam is more likely to decrease, when delamination occurs at the free end. Putting it chronologically the values of natural frequencies tend to decrease more when delamination is at the free end, the second position is the fixed-end location and finally the least location of delamination is at center and at this specific position the reduction of the natural frequencies are somewhat lower compared to the other two.

## 4.12 Result discussion on harmonic analysis

### 4.12.1 Results for normal beam

The following figure shows the frequency response of the cantilever beam with given conditions and as can be observed from the figure that the resonating peak lies between the natural frequencies from (200 up to 250Hz).

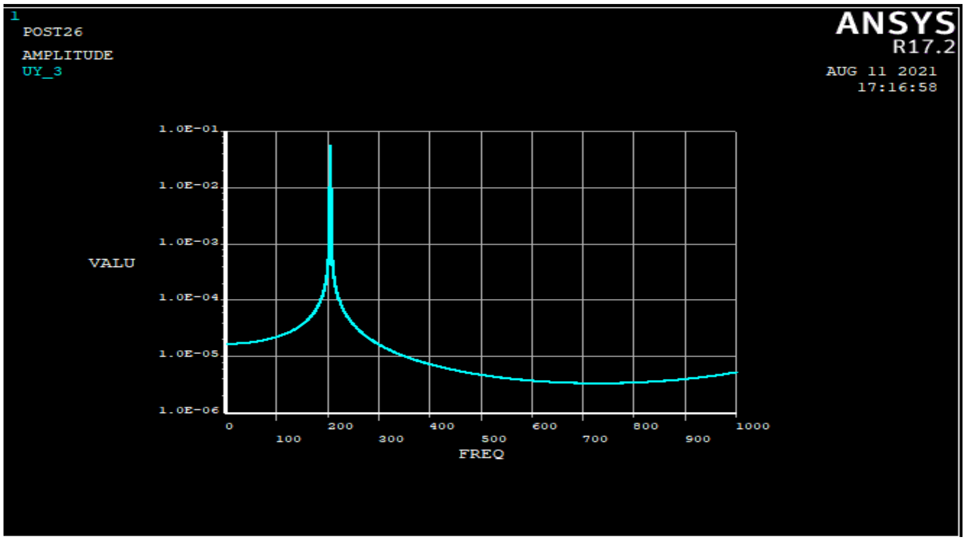



Figure 4.44 Results for intact beam

### 4.12.2 Results for delaminated beam

The harmonic response for beam with 90mm delamination is also done by this research and by the same way result has been plotted, as shown below in the figure. Unlike, the first case increase in resonating peaks and anti-resonating peak observed due to the induced delamination.

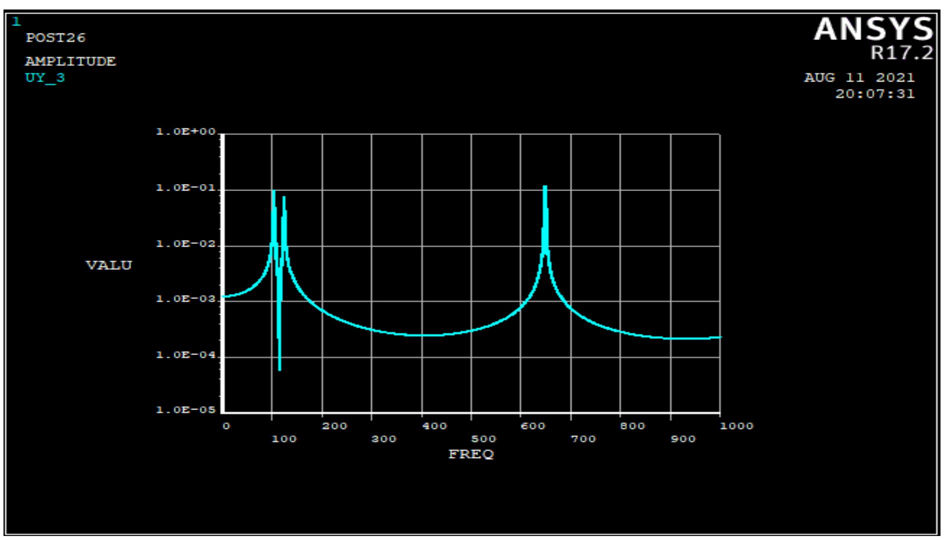



Figure 4.45 Results for delaminated beam

Therefore, while delamination is present in a certain beam, different peaks and anti-peaks observed due to resonance.

# Chapter 5

## 5.1 Conclusion, recommendation and limitation of the study

### 5.1.1 Conclusion

As already discussed in the research that using the finite element simulation and analytical modeling method the effect of delamination on vibration parameters of a composite beam is studied. The finite element simulation was performed by use of Ansys. Analytical modal frequencies for intact beam and delaminated beam is also discussed in detail by use of Euler Bernoulli beam assumptions. Mode shape and natural frequencies of a beam without delamination is also calculated by both methods and presented. Comparisons of natural frequencies as we go from normal beam up to delaminated beam was also discussed. The results generally indicate that delamination length and type of delamination present in a certain composite beam can negatively affect the vibrational characteristics of the beam and that in turn reduce the overall stiffness and integrity of the system that gradually can be a cause for failure.

- ✓ The difference of the modal frequencies between intact beam for both analytical and finite element simulation show a very small variation. Numerically, (0.781%, 0.031%, 1.025%, 2.470% and 4.255%) variations from the first mode to the fifth mode as we go down for intact beam. The finding indicates that increasing delamination length in both clamped-clamped boundary condition and cantilever beam decreases the non-dimensional frequencies. The present work showed that natural frequencies obtained from the intact beam is always greater than the delaminated beams.
- ✓ Works of two previous researchers was presented to check the applicability and reliability of the proposed analytical model and good agreement is found for specific case in which the same geometrical and material properties to validate the results and agreement is achieved that are presented by table and graphs.
- ✓ The comparisons between analytical values with the finite element simulation shows that in both analysis natural frequencies shows considerable reduction.
- ✓ Effect of delamination under different delamination ratio and location shows that increasing delamination affects natural frequencies and location of delamination has a direct relationship with vibration, in which the simulation done on different location of the

cantilever beam showed that generally the natural frequencies of a beam is more likely to decrease when delamination occurs at the free end. Putting it chronologically, the values of natural frequencies tend to decrease more when delamination is at the free end, the second position is the fixed-end location and finally the least location of delamination is at center and at this specific position the reduction of the natural frequencies are somewhat lower compared to the other two.

- ✓ The results of harmonic analysis demonstrate the frequency response of both intact and delaminated beam and it shows us the different resonating and anti-resonating peaks.

The present work proved that it’s possible to predict the natural frequencies of some important structures by inducing delamination in mechanical structures in this case beams without conducting experimental analysis. Generally, this can give some advantages, for instance if the material is too expensive this method can be very handy.

### 5.1.2 Recommendation for future works

Depending on results of the current study and literature survey the following points recommended for future studies.

- ✓ Need of expanding and widening the modeling and the simulation method is necessary.
- ✓ The mesh size used in this research is uniform for all the three cases, hence if the mesh sizes re-meshed a more accurate result can be obtained.
- ✓ The simulation method can be applied to other cases such as clamped-clamped beam, free-free beam and simply supported also with a little modification external force (f) can be applied at any location on the beam.
- ✓ Using analytical method to predict the natural frequencies for complex geometries and multiple delamination can cause error and hence, using alternative finite element analysis or dynamic finite element method is advantageous but requires good programming skill.
- ✓ By changing the configurations of a defective system (beam, plate, column) modeling with alternative boundary conditions can be studied.
- ✓ It’s also possible to model delamination by introducing different modes of vibration such as (forced, damped etc.).
- ✓ Plates and shells having multiple delaminations can also be a point of interest.
- ✓ More importantly, performing experimental tests adds more value and reliability.

### 5.1.3 Limitation of the study

Generally, this research is confined to the simulation of multiple delamination for a cantilever beam. According to this study and through literature survey, there are a wide range of issues that can affect natural frequency of beams. Such as; envelop or non-envelop delaminations and types of applied loads, crack, fiber orientation, fiber imperfections due to manufacturing defects, voids in structures etc. In order to deal with all time and resource can be a limiting factor. But, the main limiting factor is a resource constraints that is in order to perform experiments finding the raw material and a lab setup can be challenging and also in simulating models a computer with a better memory and capability also be useful.

# References


[1] Sarat Chandra Yanapu,P.Satish Reddy,N.Guru Murthy,M Babji, "Detection of Delamination in A Composite Beam By Using Vibration Analysis," vol. 5, pp. 226-229, 2018.

[2] Nicholas H.Erdelyi and Seyed M. Hashemi, "A Dynamic Stiffness Element for FreeVibration Analysis of Delaminated Layered Beams," vol. 5, pp. 1-8, 2019.

[3] Jakub Rzeczkowsk,Sylwester Samborski,and Marcelo de Moura, "Experimental Investigation of Delamination in Composite Continuous Fiber-Reinforced Plastic Laminates with Elastic Couplings," vol. 1, pp. 2-17, 2020.

[4] A. Turon, E.V. González, C. Sarrado, G. Guillamet, P. Maimí, "Accurate simulation of delamination under mixed-mode loading using a cohesive model with a mode-dependent penalty stiffness," vol. 1, pp. 506-511, 2018.

[5] Robin and Olsson, "Analytical model for delamination growth during small mass impact on plates," vol. 4, pp. 84-92, 2015.

[6] Venkate Gowda,N.Rajanna, "Investigating the effects of delamination location and size on the vibration behaviour of laminated composite beams," vol. 4, pp. 44-51, 2017.

[7] Wang J., Liu Y., and Gibby J, "vibration of split beams," vibration of split beams, vol. 84, pp. 491-502, 1982.

[8] K Torabi,M Shariati-Nia,and M Heidari-Rarani, "Moving support technique for delaminatoin detection in laminated composite beams using the first natural frequency," vol. 1, pp. 1-13, 2017.

[9] Mujumdar and S. Suryanarayan., "Flexural vibrations of beams with delamination," vol. 3, p. 441–461, 2019.

[10] Tracy and G.C. Pardoen, "Effect of delamination on the natural frequencies of composite laminates," vol. 12, p. 1200–1215, 2016.

[11] Saravanos and D.A. Hopkins, "Effects of delaminations on the damped characteristics ofcomposite laminates analytical and experiments," vol. 5, p. 977–993, 2015.

[12] Shu, "Vibration of Sandwich Beams with Delaminations," vol. 1, pp. 13-28, 2020.

[13] M.H. and Grady, "Free Vibrations of Delaminated Beams," vol. 6, pp. 1361-1370, 2018.

[14] Shiun Hu and Chyanbin, "Free Vibration of Delaminated Composite Sandwich Beams," vol. 3, pp. 1911-1917, 2015.

[15] Lee and K. H. Lee, "Free-vibration analysis of composite beams," vol. 1, pp. 715-730, 2016.

[16] Sunghee Leea,Taehyo Park,George Z. Voyiadjis, "Vibration analysis of multi-delaminated beams," vol. 34, p. 647–659, 2018.

[17] Ali Mahieddine and Mohammed Ouali, "Modeling and analysis of beams with delamination," vol. 1, pp. 335-444, 2010.

[18] Simitses and S. Sallam, "Effect of Delamination of Axially Loaded homogenous laminated plates," vol. 1, pp. 1437-1444, 2020.

[19] Shen, and J. E. Grady, "Free Vibrations of Delaminated Beams," vol. 6, pp. 1361-1366, 2018.

[20] G. Fredric D and M.Esther, "New analytical model for delamination of CFRP during drilling," vol. 24, p. 332–343, 2017.

[21] O. Robin, "Analytical model for delamination growth during small mass impact on plates," vol. 47, pp. 2884-2892, 2018.

[22] H. Ullah,Harland,and T. Lucas, "Finite-element modelling of bending of CFRP laminates," vol. 52, p. 147–156, 2019.

[23] Mahapatara and Gopalakrishnan, "Finite element analysis of free vibration and wave propagation in asymetric composite beams with structural discontunities," vol. 55, pp. 23-36, 2019.

[24] J, Lee, "Free vibration analysis of delaminated composite beams," vol. 74, pp. 121-129, 2020.

[25] Hashami and S.M, "Free Vibrational Analysis of Rotating Beam-like Structures," vol. 1, pp. 1-13, 2019.

[26] Alfano and Crisfeld, "Finite element interface models for the delamination analysis of laminated composites: Mechanical and computational issues," vol. 50, p. 1701–1736, 2019.

[27] K. Alnefaie, "Finite element modeling of composite plates with internal delamination," vol. 90, pp. 21-27, 2019.

[28] R. Gadlerab, "The effect of delamination on the naturalfrequencies of a laminated composite beam," vol. 1, p. 283–292, 2018.

[29] N. Hu a;, H. Fukunaga b , M. Kameyama b , Y. Aramaki b , F.K. Chang c, "Vibration analysis of delaminated composite beams and plates using a higher-order FE method," vol. 1, p. 1479–1503, 2020.

[30] Valeria La Saponara, Hanifah Muliana ,Rami Haj-Ali ,George A. Kardomateas, "Experimental and numerical analysis of delamination growth in double cantilever laminated beams," vol. 69, p. 687–699, 2015.

[31] L. Jaehong, "Free vibration analysis of delaminated composite beams," vol. 74, pp. 121-129, 2000.

[32] W. Ostachowicz and A, "Vibration of a laminated beam with a delamination including contact effects," vol. 11, p. 157–171, 2014.

[33] Yam, Wei, L, and W.O. Wong, "Numerical analysis of multi-layer composite plates with internal delamination," vol. 82, p. 627–637, 2016.

[34] Kyeongsik Woo,Biruk F. Nega, Douglas S. Cairns, Jim Lua, "Delamination behavior of L shaped composite beam with manufacturing defects," vol. 1, pp. 55-65, 2020.

[35] Yassin Osman; Osama Mohammed Elmardi Suleiman, "Free Vibration Analysis of Laminated Composite Beams using Finite Element Method," vol. 3, pp. 1-17, 2017.

[36] Muhammad Imran, Rafiullah khan, and Saeed Badshah, "Vibration Analysis of Cracked Composite Laminated Plate," vol. 38, pp. 693-704, 2019.

[37] Muhammad imran, Rafiullah khan, and Saeed Badshah, "Vibration Analysis of Cracked Composite Laminated Plate," vol. 38, pp. 693-704, 2019.

[38] K. Torabi, M. Shariati-Nia, M. Heidari-Rarani, "Experimental and theoretical investigation on transverse vibration of delaminated cross-ply composite beams," vol. 10, pp. 1-11, 2016.

[39] C. Venkate Gowda, N. Rajanna, N. G.S. Udupa, "Investigating the effects of delamination location and size on the vibration behaviour of laminated composite beams," vol. 4, pp. 10944-10951, 2017.

[40] Sarat c, P. Satish Reedy N.Guru Murthy, M. Babji, "Detection of Delamination in A Composite Beam By Using Vibration analysis," vol. 6, pp. 1-4, 2018.

[41] Turon,sandro and Gulliemot, "Delamination of Sandwich Composites," vol. 1, pp. 22-54, 2010.

[42] Kiral, Binnur G ren, "Free Vibration Analysis of Delaminated Composite Beams," vol. 16, pp. 209-223, 2009.

[43] Tsinuel N.Geleta and Bongho Lee , "Delamination Behavior of L-Shaped Laminated Composites," vol. 5, pp. 1-12, 2018.

[44] Whitney, Browning and Hoogsteden , "A Double Cantilever Beam Test for Characterizing Mode I Delamination of Composite Materials," vol. 3, pp. 22-33, 2016.

[45] Colin P. Ratcliffe and William , "Vibration Technique for Locating Delamination in a Composite Beam," vol. 36, pp. 1-4, 2017.

[46] Gerard c and Parden , "Effect of Delamination on the Natural Frequencies of Composite Laminates," vol. 5, pp. 1-13, 2020.

[47] V. Tate a,Sajal Roy b, K. R.Jagtap , "Delamination Detection of Composite Cantilever Beam Coupled With Piezoelectric Transducer Using Natural Frequency Deviation," vol. 97, p. 1293 – 1304, 2014.

[48] Jakub,sylwester and Marcelo, "Experimental Investigation of Delamination in Composite Continuous Fiber-Reinforced Plastic Laminates with Elastic Couplings," vol. 1, pp. 1-17, 2020.

[49] Mengyue He, Zhifang Zhang , and Karthik Ram Ramakrishnan, "Delamination Identification for FRP Composites with Emphasis on Frequency-Based Vibration Monitoring-A Review," vol. 12, pp. 213-243, 2018.

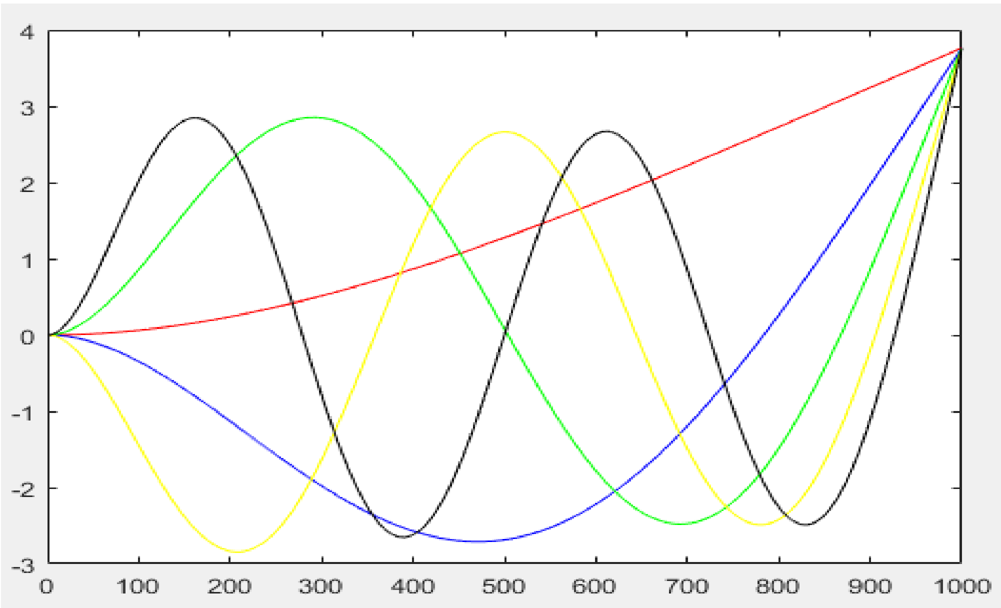
# Appendix A

The following script file used to detect the mode shapes of a vibrating cantilever beam, both matlab and ansys results provide the same result.

```
% date: 06/19/21
% by: (source, MATHWORKS)
% Finite element calculation for normal cantilever beam
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% defining material properties and dimensions
Clear all;
Close all;
clc;
% description of the inputs to the program
N=1000; % Number of elements
ρ=1570; % Density
E=140e9; % Young Modulus
A=0.05*0.012; % Cross sectional area
I=(0.05*(0.012^3))/12; % Second moment of area
Le=0.3; % Length
L=Le/N;
% Stiffness Matrix formulation
k=(E*I/(L^3))*[12, 6*L, -12, 6*L; 6*L, 4*L^2, -6*L, 2*L^2; -12, -6*L, 12, -6*L; 6*L, 2*L^2, -6*L, 4*L^2];
k1=(E*I/(L^3))*[24, 0, -12, 6*L; 0, 8*L^2, -6*L, 2*L^2; -12, -6*L, 24, 0; 6*L, 2*L^2, 0, 8*L^2];
 % Mass Matrix formulation
m=(rho*A*L/420)*[156, 22*L, 54, -13*L; 22*L, 4*L^2, 13*L, -3*L^2; 54, 13*L, 156, -22*L; -13*L, -3*L^2, -22*L, 4*L^2];
m1=(rho*A*L/420)*[312, 0, 54, -13*L; 0, 8*L^2, 13*L, -3*L^2; 54, 13*L, 312, 0; -13*L, -3*L^2, 0, 8*L^2];
 for i=1:4
   for j=1:4
          K(i,j)=k(i,j);
          K(2*N-2+i,2*N-2+j)=k(i,j);
   end
   end
for n=1:(N-2)
   for i=1:4
        for j=1:4
        K(i+2*n,j+2*n)=k1(i,j);
```

```
        end
    end
end
for i=1:4
    for j=1:4
            M(i,j)=m(i,j);
            M(2*N-2+i,2*N-2+j)=m(i,j);
    end
end
for n=1:(N-2)
    for i=1:4
        for j=1:4
        M(i+2*n,j+2*n)=m1(i,j);
        end
    end
end
% setting the Boundary conditions applied for a cantilever beam
    K(1,:)=[];
    K(1,:)=[]; % Second row
    K(:,1)=[];
    K(:,1)=[]; % Second column
    M(1,:)=[];
    M(1,:)=[]; % Second row
    M(:,1)=[];
    M(:,1)=[]; % Second column
    % Finding Eigenvectors and eigenvalues
[V,D]=eig(K,M);
Omega=sqrt(D);
f=omega/(2*pi);
 % solution
f1=((1.8751^2)/(2*pi*Le^2))*sqrt((E*I)/(rho*A))
f2=((4.6940^2)/(2*pi*Le^2))*sqrt((E*I)/(rho*A))
f3=((7.8547^2)/(2*pi*Le^2))*sqrt((E*I)/(rho*A))
f4=((10.9955^2)/(2*pi*Le^2))*sqrt((E*I)/(rho*A))
f5=((14.1371^2)/(2*pi*Le^2))*sqrt((E*I)/(rho*A))
f(1,1)
 % the first five modes of vibration
```

```
for i=1:(size(V,1)/2)
    X(i,1)=V(2*i-1,1);
    X(i,2)=V(2*i-1,2);
    X(i,3)=V(2*i-1,3);
    X(i,4)=V(2*i-1,4);
    X(i,5)=V(2*i-1,5);
end
plot(X(:,1),'red','lineWidth',0.5); hold on;
plot(X(:,2),'blue','lineWidth',0.5); hold on;
plot(X(:,3),'green','lineWidth',0.5);
plot(X(:,4),'yellow','lineWidth',0.5);
plot(X(:,5),'black','lineWidth',0.5);
% % % %Result % % %
```




%%%%%%%%%end program%%%%%%%%%%%

# Appendix B

%%ANSYS MACRO FOR 3D DELAMINATION SIMULATION%%

Definition of elements

/PREP7

ET, 1, SOLID185

Definition of material properties

MPTEMP, 1, 0

MPDATA, DENS, 1,

MPDATA, PRXY, 1, 1,

MPDATA, EX,

MPDATA, MU, 2, 0

MSHAPE, 0,3D

MSKKEY, 1

VMESH, Y

Generate target surface

NSEL, S, VOL2NOTTOM1

CM,-TARGET, NODE

TYPE, 2

ESLN, S, 0

ESURF

CMSEL, S, _ELEMCM

Generate the contact surface

NSEL, S, VOLTOP1

```
CM, _CONTACT, NODE

TYPE, 3

ESLN, S, 0

ESURF

ALLSEL

CONTACT PAIR CREATION –END

END

FINISH

/SOL

MODAL SIMULATION

ANTYPE, 2

MODOPT, LANB, 10

EQSLV, SPAR

MXPAND, 10, 0

LUMPM, 0

MODOPT, LANB, 10, 0, 0, OFF

/STATUS, SOLU

SOLVE

FINISH

/PREP

/POST1

%%%%%%%%END%%%%%%%%%
```

# Appendix C

## Delamination modeling procedures used in the study

**Specification requirement**: need of selection of type of analysis that need to be performed, in this study, the beam is analyzed as a structural component. Hence,

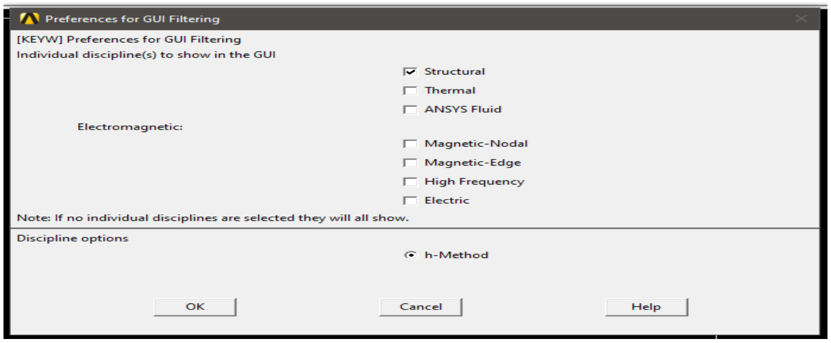



Figure 0.1 Menus from preferences

**Preprocessor**: it's the place to insert the requirement for analysis including element type, material properties, mesh tools, the modeling, if there are loads etc.

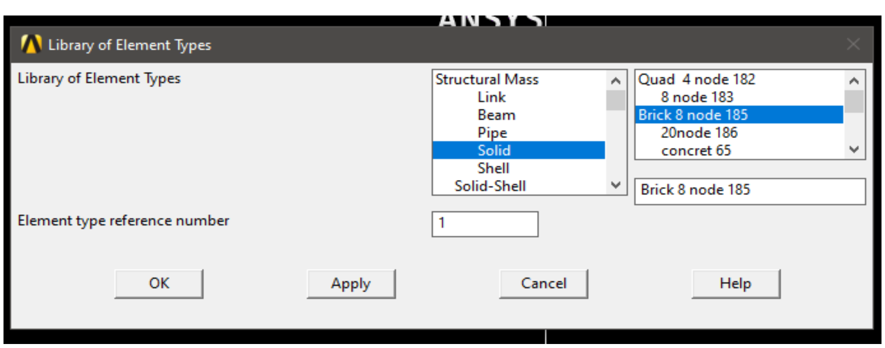



Figure 0.2 ANSYS mechanical apdl showing Element types

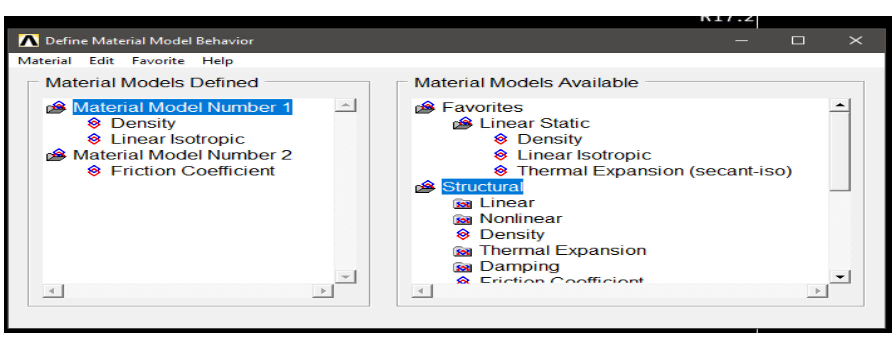



Figure 0.3 Defining material models

The above figure shows us the material models used in the research for single delamination. Under the sectioning portion the orientation of the laminates are given accordingly,

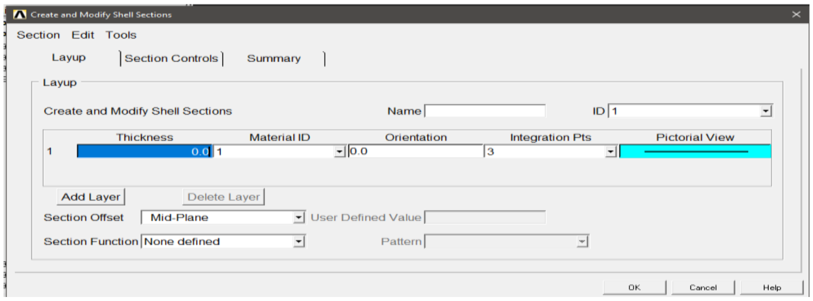



Figure 0.4 Creating modeling and sections

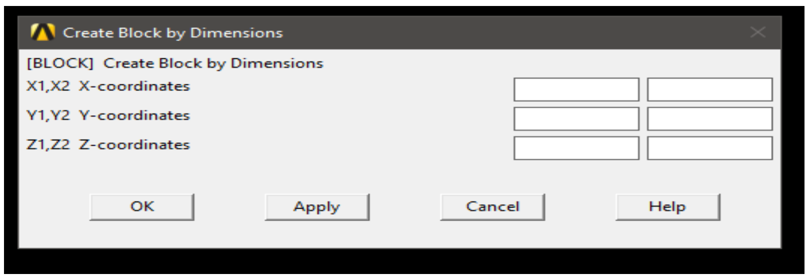



Figure 0.5 Modeling block by dimension

The above fig shows us the dimension (modeling) section.

The next step will be creating the mesh, in this research a fine research with element size o.1 is used.

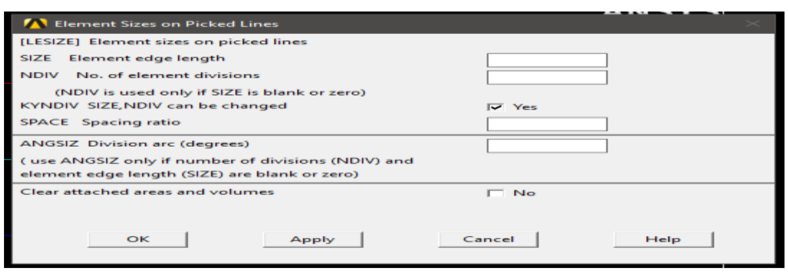



Figure 0.6 Selection of Element size

After the modeling, the next step is to select surface areas of the beam where delamination need to be created.

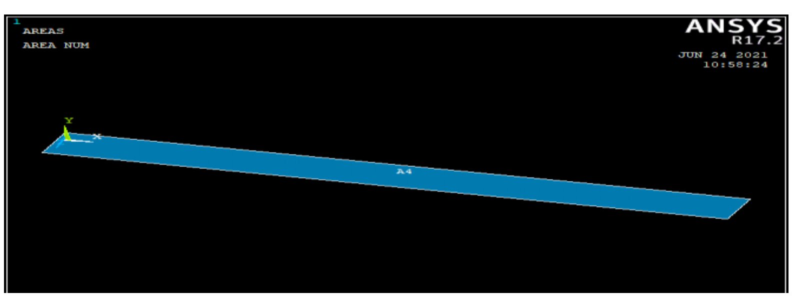



Figure 0.7 The bottom area of the beam where we want to create delamination

Then the nodes will be selected,

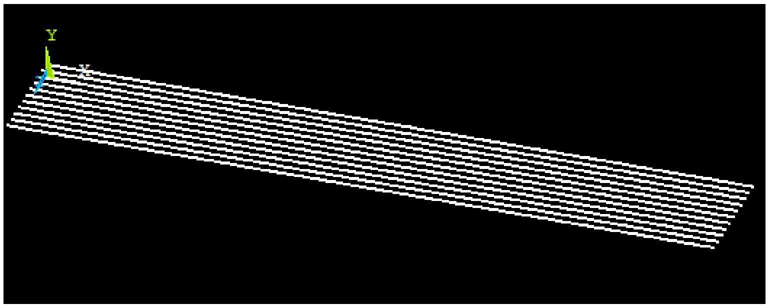



Figure 0.8 Random selection of nodes

After selecting the nodes where delamination is given then we will create contact surface between the top layer and the bottom top layer, and delamination generates where nodes are not selected.

From modeling, we select create and then contact pair, 00

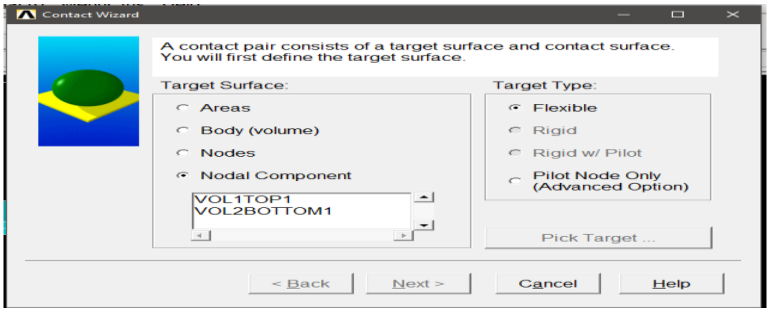



Figure 0.9 Surface contact creation window

The bottom is the target surface and the top is the contact surface. When the contact is created the colures at the top which are in green and yellow will be transformed in to yellow.

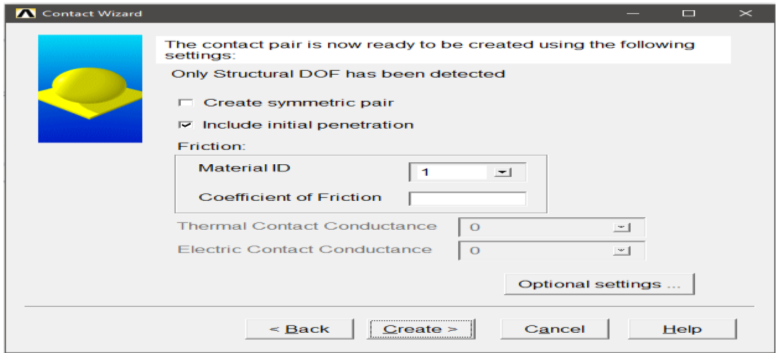



Figure 0.10 The contact pair window showing formation of contact pair

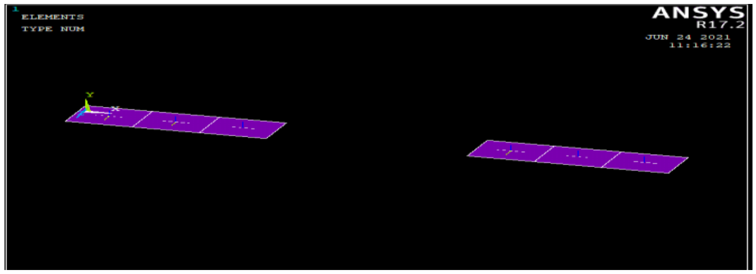



Figure 0.11 The generated delamination

The above figure shows us the amount of delamination generated in this specific example. The next step is modeling the modal analysis and applying the appropriate boundary conditions.

Now apply the boundary conditions and all degree of freedom at the beginning is zero for a cantilever beam.

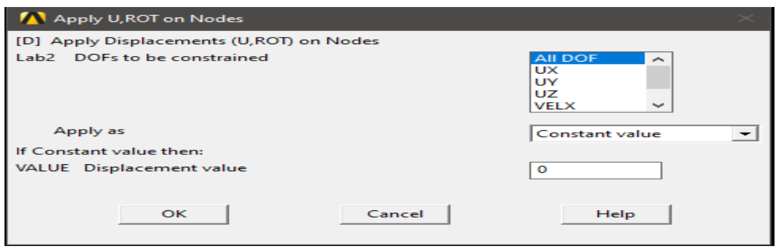



Figure 0.12 Setting the cantilever beam boundary conditions

The final step is to perform the modal analysis;

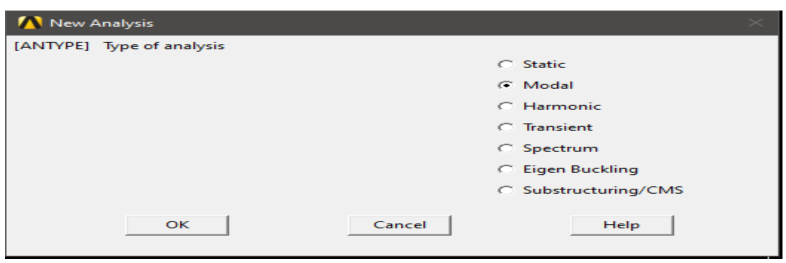



Figure 0.13 Selection of analysis types

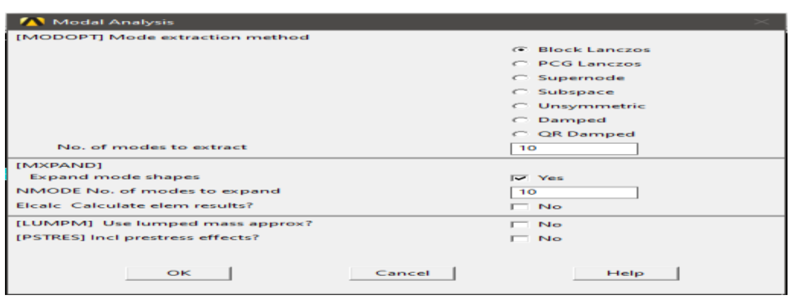



Figure 0.14 Modal analysis extraction

Then finally go to solve and hit current load step then solution is done.

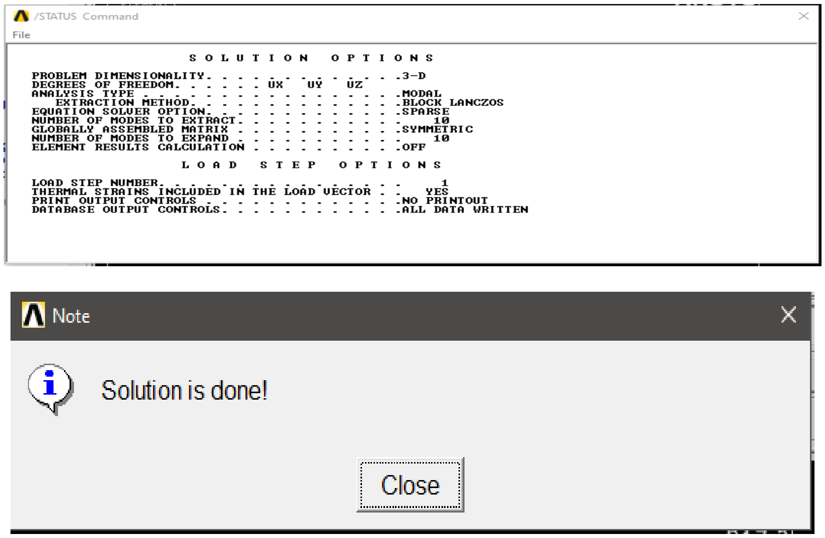



Figure 0.15 Solve and followed by end process

Then end and post processing and result interpretation will take place accordingly

# Appendix D

## Harmonic analysis modeling procedures used in the study

A Cyclic load of 100N and the frequency range (0-1000Hz) is taken in the study to analyze the harmonic conditions of the cantilever beam.

The first step is defining the problem on ansys mechanical apdl. After completing the beam modeling techniques harmonic analysis takes place. From the command selection of the desired analysis takes place.

- ✓ Solution>analysis type>Harmonic

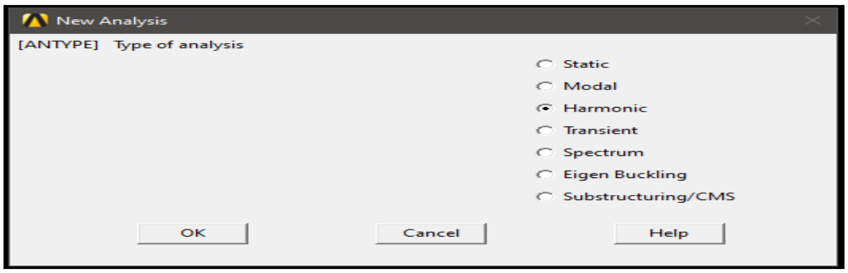



Figure 0.1 Type of analysis

The next step is application of the necessary boundary conditions in to the problem.

- ✓ Applying the constraints
  Solution>defining loads >apply >structural>all nodes 0

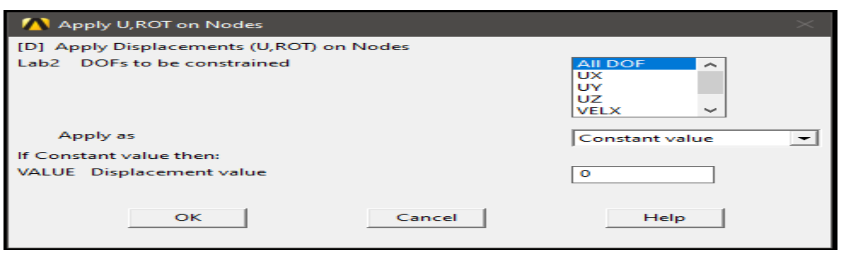



Figure 0.2 Restraining the boundary conditions for cantilever beam

The cyclic load is then applied in the –y axis.

- ✓ Applying the cyclic loading
  Solution>apply force>key point>-100

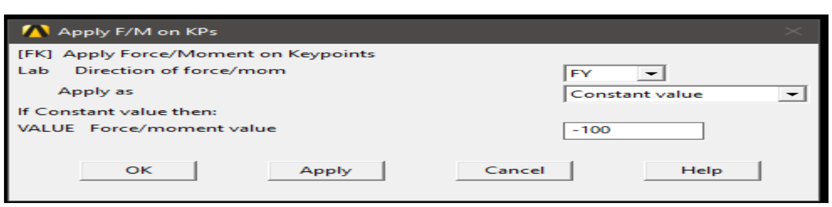



Figure 0.3 Applying cyclic load

Then setting the frequency range takes place, also another options will be selected depending upon the problem.

- ✓ Setting the frequency range
  Solution>Time/frequency>Frequency and sub steps

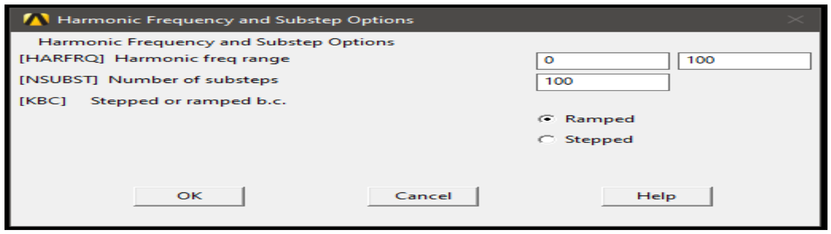



Figure 0.4 The harmonic frequency and sub steps

The last step is to solve the current analysis and waiting for the solution.

- ✓ Solve
  Solution > current load step> solve

  Post processing

- ✓ Open the post processing
  General post processer> Time histpost processor> variable viewer

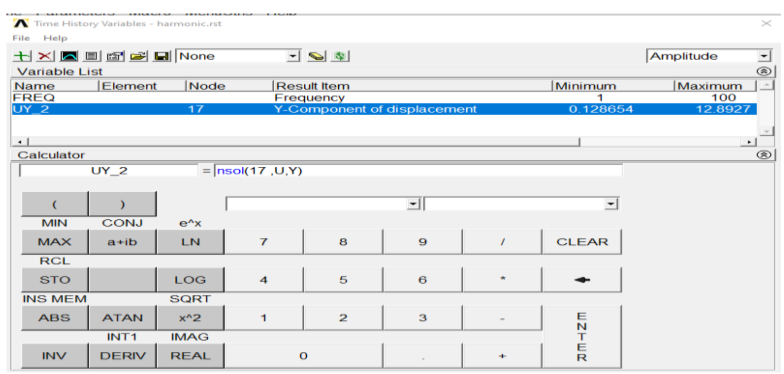



Figure 0.5 Selection of the displacement values

- ✓ Plotting displacement vs frequency

The final step is, from the general post processer window plotting the displacement vs frequency graph and interpreting it accordingly.

## Appendix E

# Samples of element types adapted from ansys (ANSYS, INC 17.2)

| Element Order | 2D Solid | 3D Solid | 3D Shell | Line Elements |
|---|---|---|---|---|
| Linear | PLANE42<br>PLANE182 | SOLID45<br>SOLID185 | SHELL63<br>SHELL181 | BEAM3/44<br>BEAM188 |
| Quadratic | PLANE82/183<br>PLANE2 | SOLID95/186<br>SOLID92/187 | SHELL93 | BEAM189 |

SOLID185 is used for 3-D modeling of solid structures. It is defined by eight nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions. The **element** has plasticity, hyper elasticity, stress stiffening, creep, large deflection, and large strain capabilities.

SOLID185 is available in two forms:

- ✓ Homogeneous Structural Solid (KEYOPT(3) = 0, the default)
- ✓ Layered Structural Solid (KEYOPT(3) = 1)

SOLID185 Homogeneous Structural Solid Element Description

SOLID185 Structural Solid is suitable for modeling general 3-D solid structures. It allows for prism, tetrahedral, and pyramid degenerations when used in irregular regions. Various element technologies such as B-bar, uniformly reduced integration, and enhanced strains are supported.

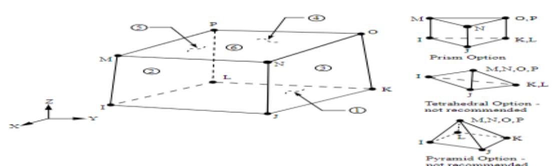



Figure 0.1 Solid185 Homogeneous structural solid geometry

SOLID185 Layered Structural Solid Element Description

Use SOLID185 Layered Solid to model layered thick shells or solids. The layered section definition is given by ANSYS section (SECxxx) commands. A prism degeneration option is also available.

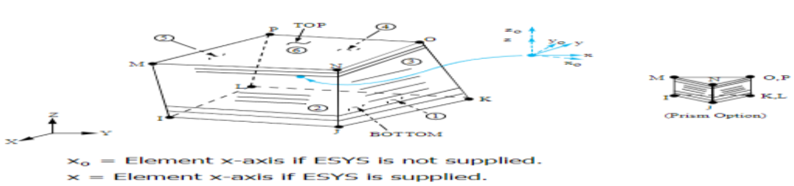



Figure 0.2 Solid185 layered structural solid geometry (adapted from ansys, Inc. 17.2)

# Appendix F

Mode shapes of delaminated beams

Single delamination mode shapes

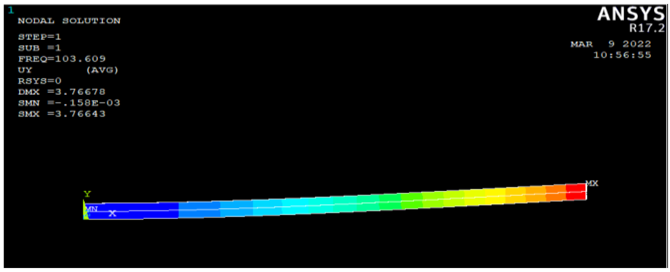



Figure 0.1 Mode one

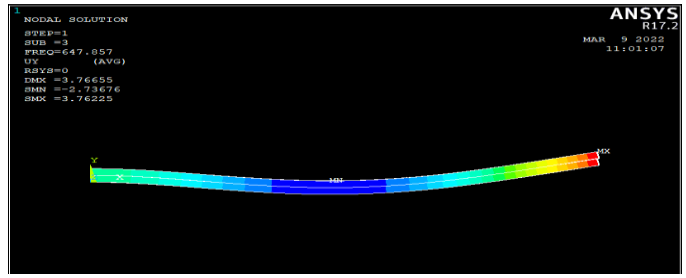



*Figure 0.2 Mode two*

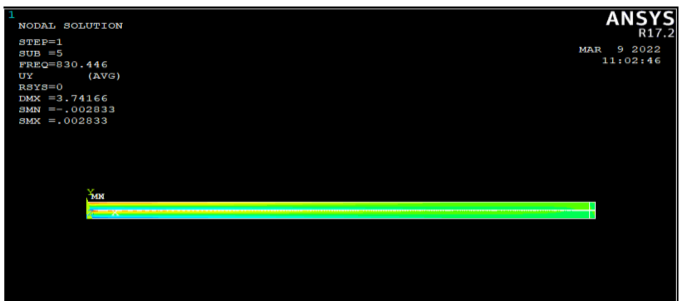



*Figure 0.3 Mode three*

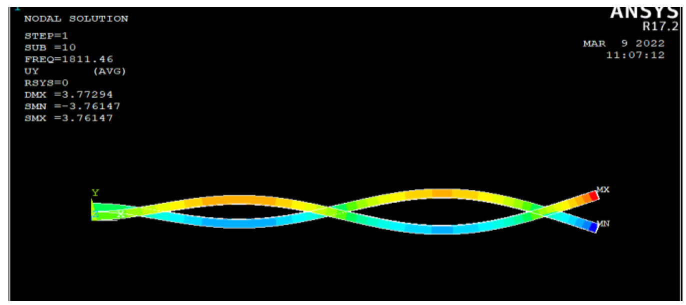



*Figure 0.4 Mode four*

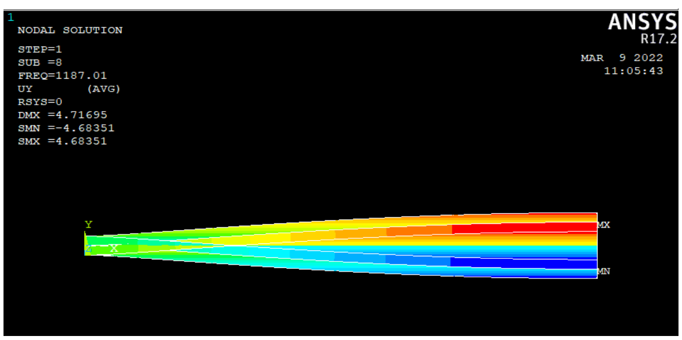



*Figure 0.5 Mode five*

# Appendix G

Central double delamination mode shapes

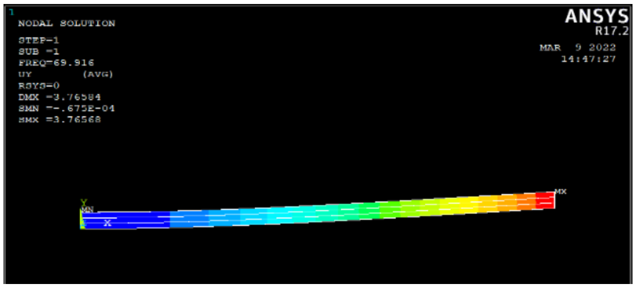



*Figure 0.1 Mode one*

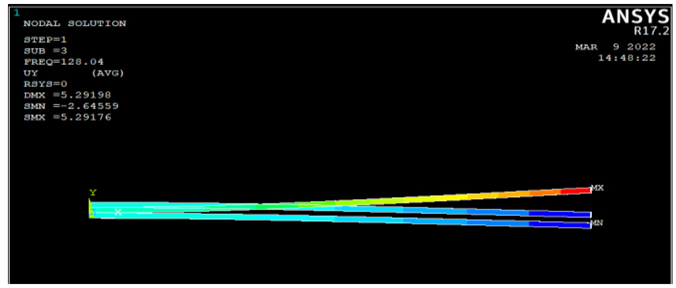



*Figure 0.2 Mode two*

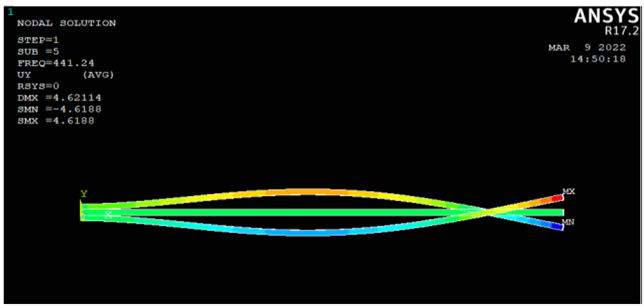



*Figure 0.3 Mode three*

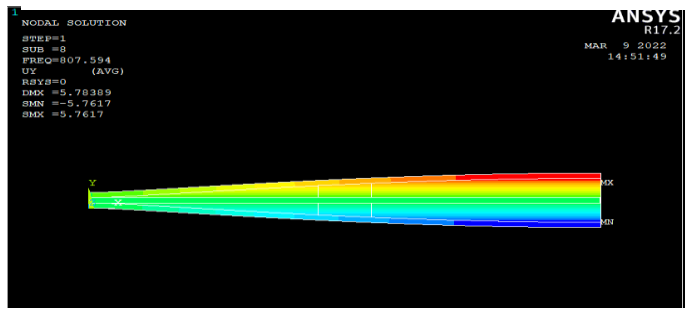



*Figure 0.4 Mode four*

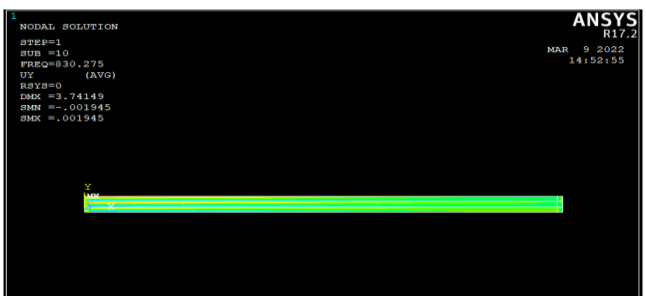



*Figure 0.5 Mode five*

# Appendix H

Central triple delamination mode shapes

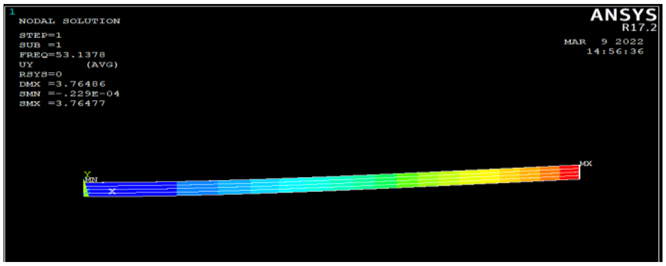



*Figure 0.1 Mode one*

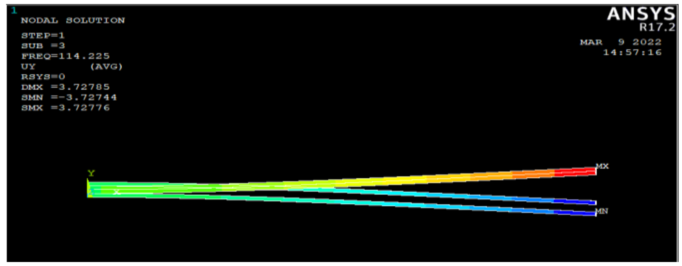



*Figure 0.2 Mode two*

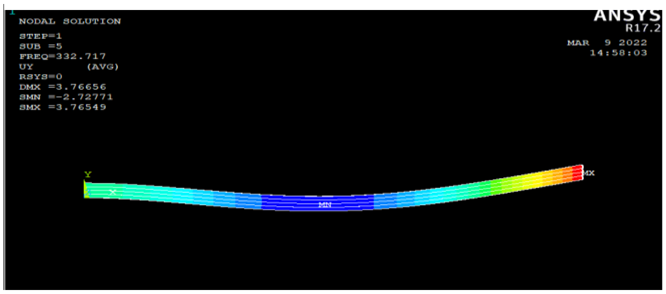



*Figure 0.3 Mode three*

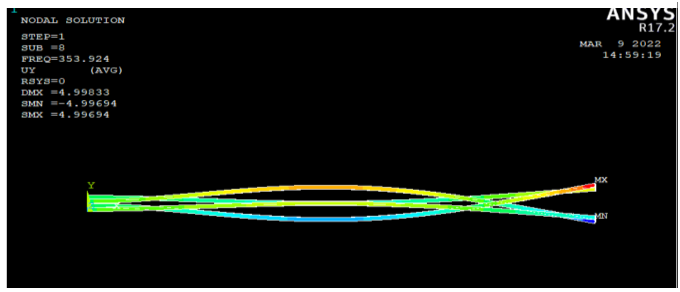



*Figure 0.1 Mode four*

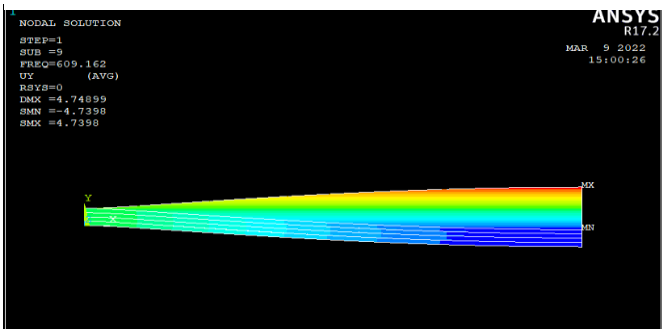



*Figure 0.5 Mode five*