

# Vehicle Dynamics Modeling and Simulation with Control Using Single Track Model

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**Abstract**— One of the most important points to consider when designing and developing vehicles is vehicle dynamics control systems. The most important models that are used in the study of the vehicle control system is called the single-track model of the vehicle. To give this study more importance, two control methods were used, namely the Fuzzy PID method and the Linear quadratic control. Two important parameters that give an indication of the vehicle's performance are the yaw rate and the vehicle body sideslip angle. The results of this study showed that the use of the two control methods is effective in controlling the vehicle's yaw rate under different road conditions.

**Keywords**— Yaw rate, Fuzzy PID control, LQR, vehicle dynamics, single-track model

## I. INTRODUCTION

Vehicle Dynamic Control (VDC) became popular in the 1990s. When the vehicle is steering or under the lateral force, this is a new active safety control device that improves manoeuvrability and stability by regulating and managing the longitudinal force of the wheel [1]. The yaw rate is an important variable that shows vehicle efficiency when maneuvering; it is the most important variable for the road vehicle stability system to recognize [2]. When studying vehicle dynamics and control, another important variable to consider is the vehicle body sideslip angle, where sideslip control is used either for vehicle stabilization or only in emergency situations, particularly if the tire-road friction coefficient calculation is not accurate [3].

The single-track model is the most basic mathematical model for explaining vehicle dynamics and is commonly used in simulations [4]. Figure 1 depicts a basic track model, also known as a bicycle model in vehicle dynamics studies. This model is flat and has three degrees of freedom. The vehicle's center of mass is said to be its projection onto the road surface, and the vehicle body is symmetrical with itself. The axles were removed and replaced with single wheels. The model assumes that the vehicle's left and right wheels generate equal lateral forces and that the axle slip angle is the only factor that determines the vehicle's description. This means that the vehicle description is identical to that of a single-track vehicle. The model also assumes that the longitudinal driving forces generated by the vehicle's left and right wheels are identical. The lack of vehicle overturning is a serious downside. This model will be used twice: once as a linear vehicle model to design the desired values of yaw rate and sideslip angle, and then as a nonlinear vehicle model (actual vehicle) to design the desired values of yaw rate and

sideslip angle based on the previous assumptions and what this study requires.

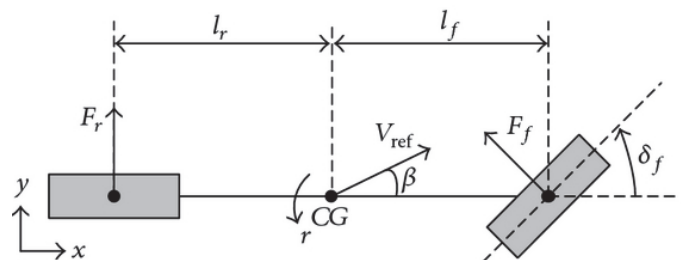


Fig. 1. Single track model (bicycle model).

Many vehicle models and control methods for yaw rate control and handling have been proposed in vehicle dynamics and control studies, such as Fuzzy PID controller [5,9], artificial neural network control [6,9]. Improving vehicle efficiency and control by adding steering system compliance parameters and accuracy has increased to 12 percent and 8% for steady state and transient response, respectively [7]. Model predictive control (MPC) is useful for developing an electrical vehicle path-tracking controller [8], as well as vehicle yaw rate control using PID control technology [10].

## II. TRACK MODEL

The single-track model is a classic model that represents the most basic model for accurately predicting the lateral dynamics of a road vehicle. It is useful in non-extreme conditions, such as when the longitudinal speed is constant and the sideslip angle is minimal. On the same axle, the two tires are clumped together, resulting in one front and one rear tire, as seen in figure 2. Table 1 contains descriptions of the symbols on the figure.

The equations of motion based on the free-body diagram in Figure 2 can be written using Newton's second law, and the model can be implemented using Matlab software and the motion equations.

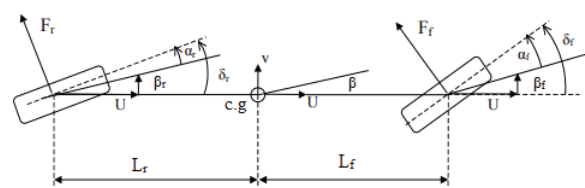


Fig. 2. The single-track model's free body diagram.

To complete this project, a single-track Simulink model using Matlab should be implemented, as shown in Figure 3. This model has six parts: Vehicle Dynamics, Steering Angle Projection, Tire Model, Kinematics/Geometry, Trajectory Calculations, and Simple Driver Model.

The steering angle of the front tire and the steering angle of the rear tire are the input signals in figure 3. Since the rear steering angle is assumed to be zero, there is only one input signal: the front steering angle, which is represented by phase steering and at lane change maneuvers as seen in figures 4 and 5.

TABLE 1. NOMENCLATURE USED

Parameter	Symbol	Parameter	Symbol
Center of gravity	c.g.	$x_o, y_o$	Inertial frame of reference
Vehicle velocity at c.g.	V	$x, y$	Chassis fixed frame of reference
Vehicle side slip angle	$\beta$	$V_f$	Velocity at center of front tire
Distance between front axle and c.g.	$l_f$	$V_r$	Velocity at center of rear tire
Distance between rear axle and c.g.	$l_r$	$\alpha_f$	Front tire side slip angle
Front tire lateral force	$F_f$	$\alpha_r$	Rear tire side slip angle
Rear tire lateral force	$F_r$	$\beta_f$	Front tire velocity angle with X axis
Front tire steering angle	$\delta_f$	$\beta_r$	Rear tire velocity angle with X axis
Rear tire steering angle	$\delta_r$	$\psi$	Yaw angle
yaw rate	$r$		

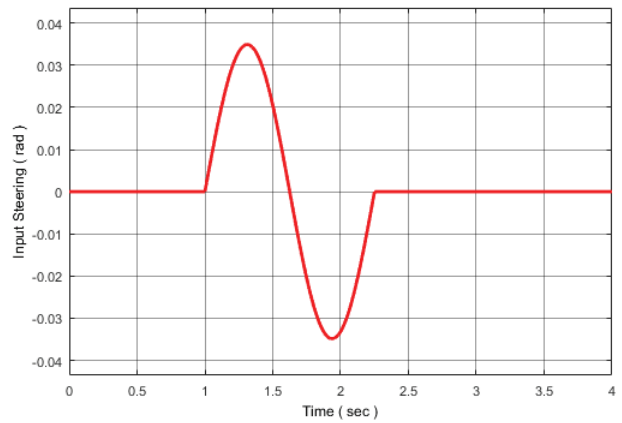


Fig. 5. The steering input of vehicle as a lane-change maneuver.

Figures 6 and 7 display two Matlab Simulink models, with figure 6 representing the vehicle dynamics system in the linear state (Linear track model or reference model), which is used in the design of controllers, and figure 7 representing the vehicle dynamics system in the non-linear state (actual vehicle model).

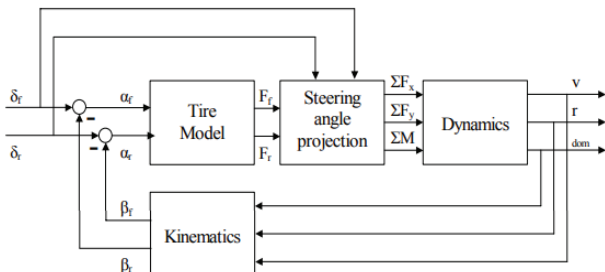


Fig. 3 Vehicle Dynamic Structure Block Diagram.

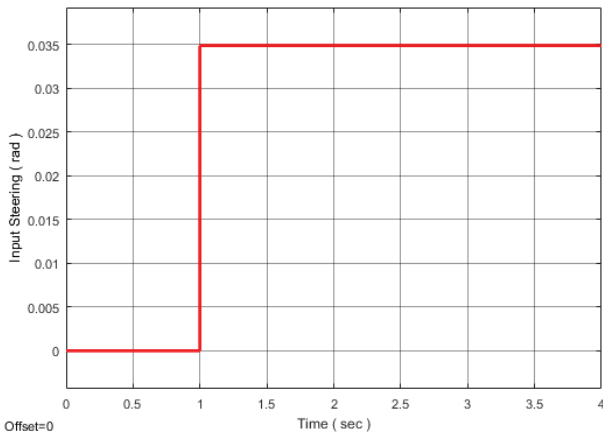


Fig. 4. The steering input of vehicle as a step angle.

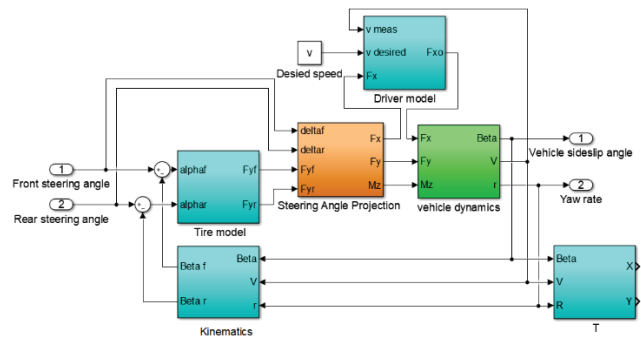


Fig. 6. The nonlinear single-track model in Simulink.

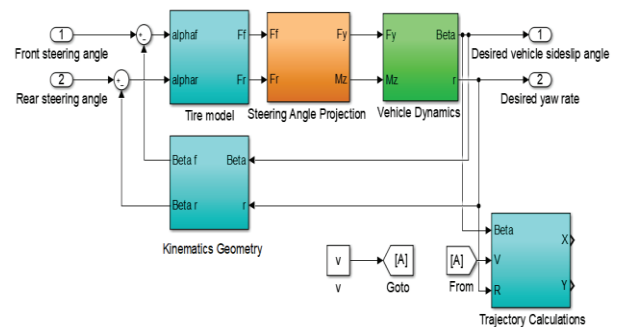


Fig. 7. Simulink model of the linear single-track model

### III. TECHNIQUES AND STRATEGIES FOR CONTROL

#### A. LQR Vehicle stability control

In the context of several studies in the field of vehicle dynamics, the LQR approach to vehicle control is commonly used. It was used in a quarter-car model, a half-car model, and a full-sized vehicle model. When using the success factor matrix, the LQR method's strength is that it can be weighted according to the engineers' expectations and ambitions, as well as other constraints. When performance index factors are taken into account with this approach type, an optimal result can be achieved.

To improve passenger comfort and improve road handling, the LQR approach for vehicle dynamics has been suggested and proposed.

When designing a LQR controller (also known as the gain matrix), the best control vector is  $u(t)$ , which reduces the quadratic cost function. The quadratic cost function has the following mathematical formula:

$$J = \int_0^{\infty} (x(t)^T Q x(t) + u(t)^T R u(t)) dt \quad (1)$$

Where,

The state vector and the power vector are  $x$  and  $u$ , respectively.

The designer must choose the appropriate values for both  $R$  and  $Q$  in order to find the appropriate gain matrix using MATLAB software.

Figure 8 portrays the configuration of the state variable feedback.

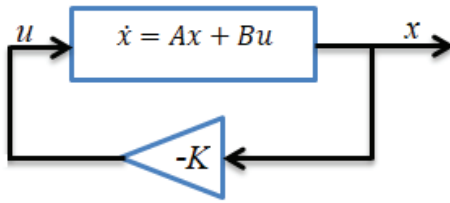


Fig. 8. The state variable feedback configuration.

As an example, a suitable linear full-state feedback control law may be used.

$$u(t) = -Kx(t) \quad (2)$$

Where  $K$  denotes the LQR controller's state feedback gain matrix, which is defined as:

$$K = R^{-1} B^T P \quad (3)$$

To estimate the matrix  $P$ , the next Algebraic Riccati Equation (ARE) is used.

$$A^T P + AP + PBR^{-1} B^T P + Q = 0 \quad (4)$$

Figure 9 displays the Simulink model for the control system, which includes the LQR controller.

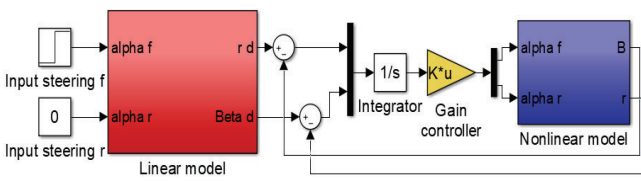


Fig. 9. A vehicle's LQR control system.

### B. Fuzzy PID Controller

The Fuzzy PID control is made up of three main parts: 1: fuzzification, 2: fuzzy rules and inference, 3: defuzzification. The configuration of the vehicle control system, which includes a fuzzy PID controller is shown in Figure 5.

Figures 10 and 11 display the configuration of the controller and the block diagram of the desired control scheme, which

includes the reference model, actual model of the vehicle, and fuzzy PID controller.

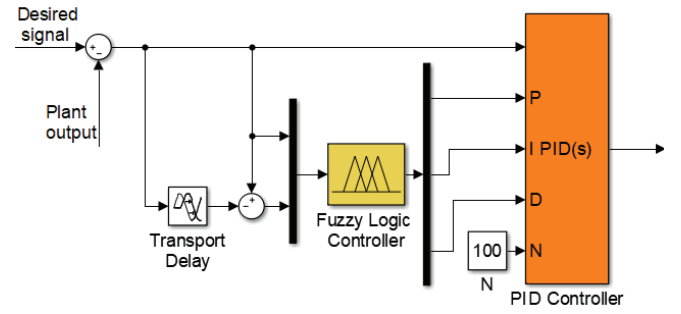


Fig. 10. Structure of a fuzzy PID controller.

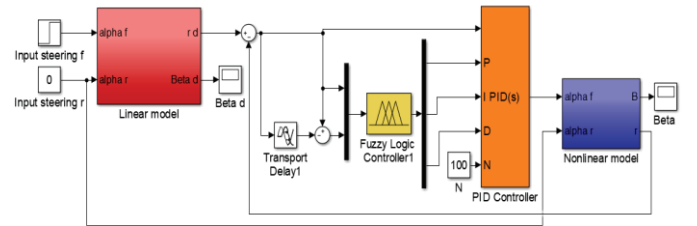


Fig. 11. A vehicle's fuzzy PID control scheme.

## IV. SIMULATION RESULTS

A simulation investigation is carried out in order to assess the efficiency of the built control system. The efficiency and complex behaviors of the control system using Matlab/Simulink and some control methods. We assume the vehicle travels at a constant speed of 70 km/hr and that the road's friction coefficient is ignored.

One driving condition is performed, which is the step steering input, which is set as a step signal with a value of five degrees and set as a lane change maneuver in the case of the single-track model.

Different control techniques, such as fuzzy PID control and Linear Quadratic Regulator, were used in this work, as well as modeling and simulations for vehicle stability (LQR).

Figures (13-15) display a comparison of vehicle yaw rate and vehicle sideslip angle in the absence of regulation and with the above-mentioned control techniques.

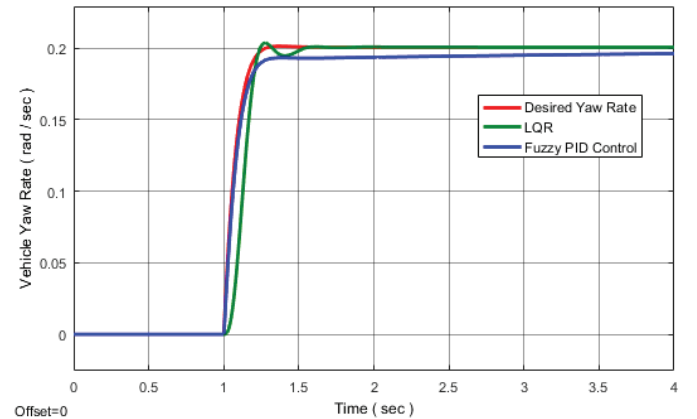


Fig. 12. Vehicle yaw rate at step steering.

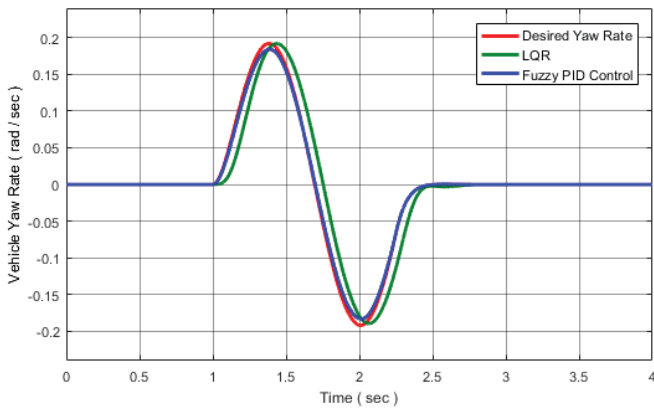


Fig 13. Vehicle yaw rate at lane change maneuver.

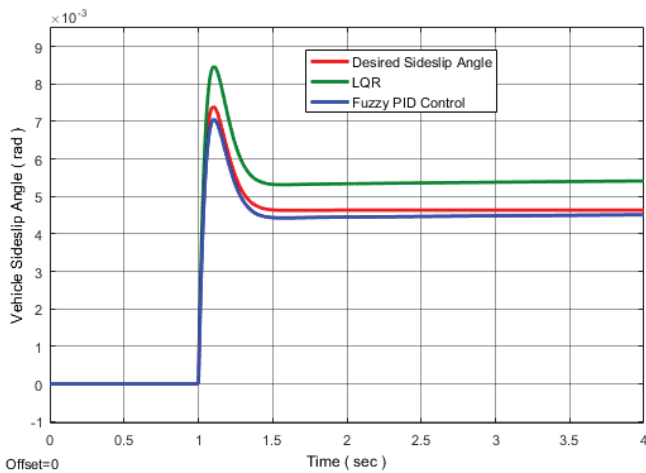


Fig 14. Vehicle body sideslip angle at step steering.

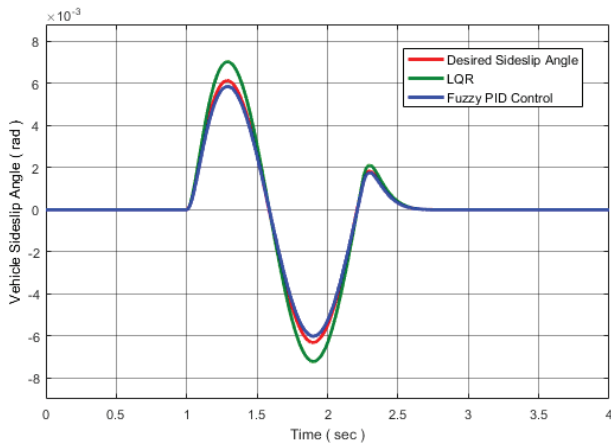


Fig 15. Vehicle body sideslip angle at lane change maneuver.

## V. CONCLUSION

Based on the simulation findings in this paper, it can be concluded that both the fuzzy PID controller and the LQR control solution boost vehicle handling and stability, confirming the efficacy of the control systems. The results of this study showed that both controllers produced positive results, as the vehicle's yaw rate increased as a result of using step steering or lane change maneuver steering, but simulation results show that the proposed control system with fuzzy PID controller can boost both the vehicle's yaw rate and sideslip angle better than the linear quadratic regulator.

## REFERENCES

- [1] Li, Hui-min, Xiao-bo Wang, Shang-bin Song and Hao Li. "Vehicle Control Strategies Analysis Based on PID and Fuzzy Logic Control." *Procedia Engineering* 137 (2016): 234-243.
- [2] Mmin Tolga Emirler, Kerim Kahraman, Mutlu Őentrk, Bilin Aksun Gven, Levent Gven, and BarıŐ Efendiođlu. "Vehicle Yaw Rate Estimation Using a Virtual Sensor." *International Journal of Vehicular Technology*, Vol. 2013, <https://doi.org/10.1155/2013/582691>.
- [3] Lu, Qian, P. Gentile, A. Tota, A. Sorniotti, P. Gruber, F. Costamagna and J. D. Smet. "Enhancing vehicle cornering limit through sideslip and yaw rate control." *Mechanical Systems and Signal Processing* 75 (2016): 455-472.
- [4] K Weigel-Milleret and W GrzegoŐek, "Modelling and simulation of narrow car dynamic." 2018 IOP Conf. Ser.: Mater. Sci. Eng. 421 022038.
- [5] A. Ali Ahmed and M. Emheisen, "Analysis of Vehicle Handling Using a Simple Track Model of Automobile," 2019 19th International Conference on Sciences and Techniques of Automatic Control and Computer Engineering (STA), Sousse, Tunisia, 2019, pp. 130-133, doi: 10.1109/STA.2019.8717244.
- [6] B. Aalizadeh and A. Asnafi, "Integrated bees algorithm and artificial neural network to propose an efficient controller for active front steering control of vehicles," *International Journal of Automotive and Mechanical Engineering* 13(2):3476-3491, 2016, DOI: 10.15282/ijame.13.2.2016.15.0287.
- [7] P Hejtmnek, O ŐavoŐ, and P PorteŐ, "Evaluation of vehicle handling by a simplified single track model," *Electronical technical journal of technology, engineering and logistic in transport*, Number 2, Volume VIII, July 2013.
- [8] J. Nilsson, J. Fredriksson and A. C. E. Ődblom, "Reliable Vehicle Pose Estimation Using Vision and a Single-Track Model," in *IEEE Transactions on Intelligent Transportation Systems*, vol. 15, no. 6, pp. 2630-2643, Dec. 2014, doi: 10.1109/TITS.2014.2322196.
- [9] A. A. Ahmed and A. F. Saleh Alshandoli, "Using Of Neural Network Controller And Fuzzy PID Control To Improve Electric Vehicle Stability Based On A14-DOF Model," *2020 International Conference on Electrical Engineering (ICEE)*, Istanbul, Turkey, 2020, pp. 1-6, doi: 10.1109/ICEE49691.2020.9249784.
- [10] Ahmed AA, Abunada ARH. Vehicle Yaw Rate Control Based on Fuzzy PID Control Technology. *J Adv Res Mech Engi Tech* 2018; 5(3&4): 17-23.