

OPTICAL PROPERTIES OF GALLIUM ARSENIDE CYLINDRICAL QUANTUM WIRE,WITH APPLIED MAGNETIC FIELD.

By

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JIMMA UNIVERSITY DEPARTMENT OF PHYSICS

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Abstract

In this study we describe the third harmonic generation of GaAs cylindrical quantum wire with applied magnetic field. The ground and exited state energy eigenvalue of the system are calculated by solving the schrodinger equation. For study the nonlinear optical properties of cylindrical quantum wire, the density matrix formalism is employed, Using this formalism, the third order nonlinear absorption coefficient and refractive index are solved analytically. The graphical analysis is made to interpret the result. The finding show that as the magnitude of static magnetic field increases the magnitude of the third order non linear absorption coefficient and refractive index also increases. Moreover, the maxima of the non linear absorption coefficient and refractive index shifts to wards higher energy (frequency) as the magnitude of static magnetic field increases. This important properties are useful for designing different optoelectronic device.

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Chapter 1 Introduction

1.1 General Background

In the past few years, the low dimensional semiconductor structure like quantum wire, quantum wells and quantum dots have aroused, much attention in potential application to high performance device. These semiconductor structure are theoretically predicted to offer superior optical and electrical characteristics. In this structure the charge carriers are confined in one,two and three dimensions. Efforts are being made, to control the confinement of charge carriers (electron and holes) in these structure. These man made semiconductor materials of low dimensionally exhibit fascinateing new electronic and optical properties, that permits improvement in the performance of electronic device[1]. This thesis is based on theoretical investigation of optical properties of Gallium Arsenide (GaAs) cylinderical quantum wire with applied magnetic field. The properties, studied were optical absorption coefficient (AC) and change in refractive index (CRI) of quantum wire. This thesis appropriately investigate the energy Eigen value of GaAs cylindrical quantum wire with applied magnetic field to show, how magnetic field has an effect in low dimensional semiconductor structure system. Thus the energy Eigen value is solved for different value of magnetic field and frequency. The energy eigen value depends on graund state and exited state energies. Quantum wires are extremely narrow device, having application in a nanoelectronics, optoelectronics, and Biotechnology, due to quantum confinement. This geometrical construction, of cylindrical quantum wire, semiconductor with applied magnetic field is uses one or two photon absorption of light. Regarding GaAs material, the general properties in a periodic table, GaAs is, a compound of semiconductor alloy of gallium and arsenic. GaAs is group III/V semiconductor. Because it is a compound of material out of group III and group V, of a periodic table. This compound is different from the properties of silicon. GaAs is a direct semiconductor, because it doesn't require a change in momentum, for an electron transition from the valance band to conduction band. However It is a direct band gap semiconductor, it needs the change of momentum. Thus required for the transition of electron, from valence band to conduction band. Optical properties of low dimensional structure are determined by the quantum size effect and excitation. The confinement of quasi-particles, in a nanostructure, leads to enhancement of oscillators, strength of excisions [2]. The interest, in the study of optical properties of confined quantum system, has increased with recent progress, in semiconductor technology. One of the most interesting properties of this structure, is the possible occurrence of the intersband, optical transition, that are characterized by small energy spectrum between sub band level. large value of dipoles transition matrix elements and the possibility of achieving resonance condition. It is well known, that nonlinear optical properties such as optical absorption and refractive index change, have a great potential for device application like far-infrared laser amplifier, far infrared photo detector, high speed electro optical modulation and optical device [2].

1.2 Statement of the problem

The linear and nonlinear optical properties of low dimensional semiconductor, structure have been studied, in the last few years. This is due to their potential application, in optoelectronic and photooptic devices. The study of this structure, has got considerable attention, due to novel electronic properties and device applications. It is clear that these applications strongly depend on their electronic and optical properties. Therefore the change in electronic state and modification of electronic wave function, and in different factors are very important. In many new quantum phenomena, such as Nanostructure cylindrical surface, shows the physical interesting properties. In this regard external factor such as electric field, magnetic field and hydrostatic pressure, modify the electronic state. The physical properties of low dimensional structure, in the semiconductor structure.concerening the problem of the research, could be studied. In the low dimensional semiconductor, we could have able to show the optical properties such as linear and nonlinear optical properties, and how the influence of magnetic field has an effect on optical properties of cylindrical quantum wire.

1.3 Research questions

- How could we calculate, the energy Eigen value of GaAs, cylindrical quantum wire with applied magnetic field?
- How one could determine, third order non linear absorption coefficient of GaAs,cylindrical quantum wire with applied magnetic field?
- How could we describe, the third order, non linear refractive index of GaAs , cylindrical quantum wire, with the applied magnetic field?

1.4 Objectives

1.4.1 General Objective

General objective of this study.

• Investigation on optical properties of GaAs cylindrical Quantum wire, with applied magnetic field.

1.4.2 Specific Objectives

The study answers;-

- Investigation in the energy Eigen value of GaAs cylindrical Quantum wire, With applied magnetic field;
- determine the third order, nonlinear absorption coefficient of GaAs, cylindrical quantum wire with applied magnetic field;
- describe the third order, nonlinear refractive index of GaAs, cylindrical quantum wire with applied magnetic field;

1.5 Significance of the study

The study is more significancant for other researcher as a reference source, helps in the area of literatures review.

1.6 The scope

The study is delimited to determining, the energy Eigen value, the linear and third order nonlinear absorption coefficient and refractive index of GaAs, cylindrical quantum wire.

Chapter 2 Literature Review

2.1 Quantum wire

The investigation of semiconductor low dimensional, structure has a key role in development of Nanotechnology and future Nanoscale devices. The recent advanced techniques is possible, for the fabrication of quantum wire structures where electron and holes are confined in two dimension [3]. Quantum wires in its nature under nanostructure, such as low dimensional quantum dots, quantum well and quantum wire. Quantum wires are one dimensional system with the extended direction along Z axis [4]. Quantum wires are one dimensional system, with the extended direction along Z axis which is governed by Hamiltonian equation [2].

$$\widehat{H} = \frac{-\hbar^2}{2m^*} \nabla^2 + V(x, y) \tag{2.1.1}$$

Where m^* , The effective mass of electron, V(x, y), is the confinement potential. The concept of quantum wire investigated for the first time in the year 1995. The growth of semiconductor technology could fabricate quantum wire in nanosize wire. Various shape, of cross sectional quantum wire and quantum dots are high optical quality with high frequency and strong nonlinear behavior. Quantum wires hetrostructure are, still

strongly dependent on the fabrication technology. In recent years, quantum wires have attracted a greater attention due to their potential application in optoelectronic device and fundamental physics investigation.

2.2 Cylindrical quantum wire

Cylindrical quantum wires are a nanostructure, cylindrical surface and dimensionally reduced, having model of quantum cable structure [18]. These can be made in either forms of the coaxial nanowire such as, GaAs system [19]. The confining potential of cylindrical quantum wire is very similar to quantum cable. The energy spectrum and transport properties of electrons, in a low dimensional system are reviewed. Generally believed that the quantum effects become, significant as the system dimensionally reduced in semiconductor, example, confining electron in 2D plane or Zero, dots give rises [20]. The prediction of semiconductor quantum wire. can be important in high speed device application [24]. The model and formulation for the motion of electron along cylindrical quantum wire in the redial direction is quantized [29]. In CQW, using the effective mass of electron approximation, the Schrodinger Equation governing, the motion of electron with energy E. read as [9].

$$\left[\frac{-\hbar^2}{2m^*}\nabla^2 + V(\rho)\right]\psi(\rho, \phi, z) = E\psi(\rho, \phi, z)$$
(2.2.1)

Where m^* , is effective mass of electron, V(r), potential in the radial direction.

2.2.1 Optical properties

Optical and electronic properties of low dimensional semiconductor structure are important parts of the Modern physics of semiconductors [23]. In the last decades scientist have investigated a number of similar systems. The absorption of light in parabolic quantum wire, located in electric and the magmatic field was theoretically studied [27]. Theoretical investigation of the absorption coefficient for direct and indirect transition when electron scattering occurs at impurity center was obtained [5].

2.2.2 The absorbtion of electromagnetic radiation in quantum wire

In recent year the optical properties in a bulk semiconductor as well as low dimensional system have been investigated. The linear absorption has been studied in normal bulk semiconductor in two dimensional and quantum wire [21]. The nonlinear absorption of strong electromagnetic wave (EMW) was coincided in normal bulk semiconductor quantum wells, doping superlatises. [11]. Nonlinear optical absorbtion has been studied in quantum dots. However, in cylindrically quantum wires, the nonlinear absorbtion of a strong EMW is open, for studying in one dimensional systems [25]. The motion of electrons is restricted in two dimensions as they can freely move in one dimension [22]. The confinement of electron in these system changes the electron mobility remarkably [12]. This result in a number of new phenomena. The nonlinear absorption of strong electromagnet wave (EMW) in quantum wire will be different from the nonlinear absorption of strong EMW in bulk semiconductors and two dimensional nanostructure system. [16]. Optical electronic properties of low dimensional semiconductor structure are, important parts of modern physics [28]. The expression of the absorption coefficient was abstained, with the used wave function in the zero radius potential, with the dependence of AC, on frequency have the peak of Gaussian type [1]. The display of spectrum quantum wire, on the magnetic field and absorption coefficient depends on photon energy. [13].

2.2.3 Effects of magnetic and electric field on optical properties of semiconductor quantum wire

The effects of external electric and magnetic field on electron are energy, spectrum and its probability density. Theoretical investigation of the influence of magnetic and electric field on energy spectrum and of electron in cylindrical layer is the cause of electric and magnetic field [14]. The Schrodinger equation, is used to solve the energy eigen value with applied electric and magnetic field, of electron. In spherical layer, under the external magnetic field applied perpendicular to the Nanowire axis causes the optical phonon confinement [26]. These optical phonon confinement, as well as, electric field, on the energy momentum relation, leads to polaron quantum confinement. This effect and properties of cylindrical quantum wire with applied magnetic field, follow the Schrodinger equation as [6].

$$\frac{1}{2m^*}(-i\hbar\nabla + \frac{e}{c}A)\psi + V(\psi) = E(\psi)$$
(2.2.2)

where m^* =effective mass A=vector potential

V=The scalar potential

$$A = \frac{1}{2}B\rho e\phi$$

Introducing the cyclotron frequency

$$\omega_c = \frac{eB}{m_c}$$
$$A = \frac{BxR}{2}$$

In this thesis the effects of magnetic field for one dimensional quantum wire. These effect can be seen, Whenever the applied magnetic field is parallel to the Z axis of the wire direction. The spectra of a wire plotted as function of magnetic field and photon energy for both parabolic and inverse parabolic confinement.

2.2.4 Density matrix formalism on quantum wire

The density matrix formulation is used to study optical properties. Density matrix formulation is capable of treating effect such as collision of atomic resonance, the wave function is needed for the effect, for a number of, related reason, i.e nonlinear effect [17]. Density matrix formulation allow, to describe the width of atomic resonance. Quantum mechanical state, describe all the physical properties of the system, in terms of the wave function [7]. $\psi(r, t)$, appropriate to the state, The Schrodinger equation yield;

$$\left[-i\hbar\frac{\partial}{\partial t}\psi_s(r,t)\right] = \hat{H}_s\psi_s(r,t) \tag{2.2.3}$$

Where \hat{H}_s denote the Hamiltonian operator of the system we assume that \hat{H}_s can be represented as

$$\hat{H} = \frac{\hbar^2}{2m^*} \nabla^2 + v(r)$$
(2.2.4)

Where \hat{H} , is the Hamiltonian for free atom and v(r), represented the interaction energy. This is always requirement, when a direct solution of the Schrodinger equation, is attempted, because losses, would lead to non-Hamiltonian operator. In reality, losses are always present, for anatomic system including the possibility that, an exited atom decays spontaneously, emitting a photon energy. in such mechanism, We will introduce the density matrix formulation which allow, for losses, in a straight forward manner writing [8]. This is known to be in particular we distinguish 's' we describe, all the physical properties, interims of the wave function. Thus $\psi_s(r,t)$, is appropriate state for the wave function obeys the schrodinger equation.

$$\left[-i\hbar\frac{\partial}{\partial t}\psi_s(r,t)\right] = \hat{H}_s\psi_s(r,t) \tag{2.2.5}$$

Where \hat{H}_s denote the Hamiltonian operator of the system we assume that \hat{H}_s can be represented as

$$\hat{H} = \frac{\hbar^2}{2m^*} + V(r)$$
(2.2.6)

Where \hat{H} , is the Hamiltonian for free atom and $\vec{V}(r)$, represented the interaction energy. With in this approach we want to show, the optical properties we want to show the third order non linear optical interaction. This is described by susceptibility, $x^{(2)}$. In order to describes optical non linearity, then considering how the dipole moment per unit volume or polarization, P(t), of material system E(t), of an applied magnetic field. In this case variational linear optics and polarization depends on die electric field strength. In this manor it is described by the following relation.

$$\vec{P}(t) = \varepsilon_o X^{(1)} \vec{E}(t). \tag{2.2.7}$$

where the constant of proportionality, \check{x}^1 . is known as the linear, susceptibility and ε_0 is the permittivity of free space. In nonlinear optics, the optical response can often be described by the polarization. $\overrightarrow{p}(t)$ as a power series in the field strength $\overrightarrow{E}(t)$ as :

$$\overrightarrow{p}(t) = \varepsilon_0[\chi^{(1)}\overrightarrow{E}(t) + \chi^{(2)}\overrightarrow{E}^2(t) + \chi^{(3)}\overrightarrow{E}^3(t) + \dots]$$
(2.2.8)

$$\overrightarrow{P}^{3}(t) = \varepsilon_0 \chi^{(3)} \overrightarrow{E}^{3}(t)$$
(2.2.9)

as the third-order nonlinear polarization. The third-order nonlinear relatione to the density matrix is given by

$$\rho_{nm}^{(3)} = \exp[-(\omega_{nm} + \gamma_n m)t] \int_{-\infty}^t \frac{-i}{\hbar} [\hat{V}, \hat{\rho}^2]_{nm} \exp(i\omega_{nm} + \gamma_{nm})t' dt' \qquad (2.2.10)$$

where the commutator can be represented explicitly as

$$[\hat{V}, \hat{\rho}^2]_{nm} = -\sum_{\nu} (\mu_{n\nu} \rho_{\nu m}^{(2)} - \rho_{n\nu)}^{(2)} \mu_{\nu m}).\tilde{E}(t)$$
(2.2.11)

Expressions for $\rho_{\nu m}^{(2)}$ and $\rho_{n\nu}^{(2)}$ are very complicated, we use the abbreviated notation introduced there:

$$\rho_{\nu m}^{(2)} = \sum_{l} \sum_{pq} K_{\nu lm} \exp{-i(\omega_p + \omega_q)t}$$
(2.2.12)

where $K_{\nu m l}$ has been displayed explicitly. We also represent the electric field as

$$\tilde{E}(t) = \sum_{r} E(\omega_r) e^{-i\omega_r t}$$
(2.2.13)

The commutator thus becomes

$$[\hat{V}, \hat{\rho}^{2}]_{nm} = -\sum_{\nu l} \sum_{pqr} [\mu_{n\nu} \cdot E(\omega_{r})] K_{\nu m l} e^{-i(\omega_{p} + \omega_{q} + \omega_{r})t} + \sum_{\nu l} \sum_{pqr} [\mu_{\nu m} \cdot E(\omega_{r})] K_{n\nu l} e^{-i(\omega_{p} + \omega_{q} + \omega_{r})t}$$
(2.2.14)

The nonlinear polarization oscillating at frequency $\omega_p + \omega_q + \omega_r$ is given by

$$P(\omega_p + \omega_q + \omega_r) = N \langle \mu(\omega_p + \omega_q + \omega_r) \rangle$$
(2.2.15)

where N=Numbr density of electron' We express the nonlinear polarization in terms of the third-order susceptibility defined by

$$P_k(\omega_p + \omega_q + \omega_r) = \epsilon_0 \sum_{hij} \sum_{pqr} \chi_{kjih}^{(3)}(\omega_p + \omega_q + \omega_r, \omega_r, \omega_q, \omega_p) \times E_j(\omega_r) E_i(\omega_q) E_h(\omega_p)$$
(2.2.16)

where ω_p , ω_q and ω_r are permutation frequencies.

Chapter 3 Materials and Methodology

3.1 Material

Materials the intensive survey of literature and published article. Books, Thesis Dissertation have been carried out. Further more Matmatica softwares, and computer, have been used for accomplishment of this work.

3.2 Methodology

3.2.1 Analytical

we apply analytical method to calculate the energy eigen value and wave function of ground and exited state of GaAs. cylindrical quantum wire. These was solved by applying schrodinger equation. More over, the third order nonlinear absorption coefficient and refractive index are derived analytically using density matrix formalism.

3.2.2 Graphical

Graphical method is, a fundamental method used to interpret the result of third order nonlinear absorption coefficient and refractive index that have been used to solve the intended research problem.

Chapter 4

Result and Discussion

4.1 Analytical formulation of energy eigen value and optical properties of GaAs, cylindrical quantum wire.

The Hamiltonian, for charge carriers in cylindrical GaAs, quantum wire with applied magnetic field in a two dimensional parabolic confining potential, along Z direction is given by:

$$\hat{H} = \frac{1}{2m^*} [-i\hbar\nabla - e\vec{A}]^2 + \frac{1}{2}m^*\omega_0^2(x^2 + y^2): \qquad (4.1.1)$$

where \vec{A} , is the vector potential as $\vec{B} = \nabla \times \vec{A}$. is symmetric gage, of the vector potential. $\vec{A} = \frac{-1}{2}\vec{B}y\bar{x} + \frac{1}{2}\vec{B}x\bar{y}$, which gives, $\vec{B} = \vec{B}z$. m^* , is the effective mass of electron in GaAs semiconductor, ω_o is the parabolic confining potential frequency. equation(4.2.1) can be expressed as

$$\hat{H} = \frac{-\hbar^2}{2m^*} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) + \frac{eB}{2m^*} i\hbar \left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) + \frac{e^2B^2}{8m^*} (x^2 + y^2) + \frac{1}{2}m^*\omega_o^2(x^2 + y^2)$$

$$\tag{4.1.2}$$

$$\hat{H} = \frac{-\hbar^2}{2m^*}\nabla^2 + \frac{\omega_c}{2}(-L_z) + \frac{1}{2}m^*[\omega_c^2 + \frac{\omega_o^2}{4}](x^2 + y^2)$$
(4.1.3)

$$\hat{H} = \left[\frac{-\hbar^2}{2m^*}\nabla^2 + \frac{1}{2m^*}\Omega^2(x^2 + y^2)\right] - \frac{1}{2}\omega_c L_z \tag{4.1.4}$$

$$\vec{A} = \frac{-1}{2}B_y \vec{X} + \frac{1}{2}B_x \vec{Y} + 0\vec{Z}$$

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{B} = B_z$$

$$\hat{H} = \left[\frac{-\hbar^2}{2m^*}\frac{d^2}{dx^2} - \frac{\hbar^2}{2m^*}\frac{d}{dy^2} + \frac{\hbar^2}{2m^*}\frac{d^2}{dz^2} + \frac{1}{2}m^*\Omega^2(x^2 + y^2)\right] - \frac{1}{2}\omega_o L_z$$

$$\hat{H} = \left[\frac{-\hbar^2}{2m^*}\frac{d^2}{dx^2} - \frac{\hbar^2}{2m^*}\frac{d^2}{dy^2} + \frac{1}{2}m^*\Omega^2(x^2 + y^2)\right] - \frac{\hbar^2}{2m^*}\frac{d^2}{dz^2} - \frac{1}{2}\omega_o L_z$$

$$\hat{H} = \hat{H}_o - \frac{\hbar^2}{2m^*}\frac{d^2}{dz^2} - \frac{1}{2}\omega_c L_z = \hat{H}_\Omega = \frac{1}{2}\omega_c L_z \text{ where } \Omega = \sqrt{\omega_o^2 + \frac{\omega_c^2}{4}} \text{, is normalize oscillatory}$$
frequency $\omega_c = \frac{eB}{m^*}$ is the cyclotron frequency and

 $L_z = -ih(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x})$, is the angular momentum operator from fundamental quantum mechanics, it is understood that the angular momentum operator. L_z , comute with the Hamiltonian H is the eygen function of H is almost similar to H_{Ω} . The eigen function of $\psi_{nl,k}$ and eigen energies $E_{nl,k}$, satisfies schrodinger equation, $\hat{H}_{\psi nl,k} = \varepsilon_{nl,k}\psi_{nl,k}$ and described by;-

$$\psi_{nl,k} = \psi_{nl}(x,y)U_c(r)e^{ikz} \tag{4.1.5}$$

Solving the schrodinger equation with this eigen function gives the eigen value of discrete and continues energy.

$$\varepsilon_{nl,k} = E_{nl} + \frac{\hbar^2 K^2}{2m^*} \tag{4.1.6}$$

where, $U_c(r)$, is the periodic part of the Blotch function in the conduction band, and K, is the wave vector in the z, direction. How ever our interest in the confinement region where, the energy eigen value and discret to this the schrodinger equation.

$$\hat{H}_o = E_{nl}\psi_{nl} \tag{4.1.7}$$

using cylindrical coordinate the value of a function ψnl is given by

$$\psi_{n,l}(\rho) = \sqrt{\frac{2m^*\Omega n!}{\hbar(n+|m|)!}} e^{\frac{-\rho^2}{2}} \rho^{|m|} L_n^{|m|}(\rho^2) \frac{1}{\sqrt{2\Pi}} e^{im\theta}$$
(4.1.8)

where ρ is the dimensionless variable $\sqrt{\frac{m^*\Omega}{\hbar}r}$, with $r = \sqrt{x^2 + y^2} \le a$ a, is a radius of a cylindrical wire: $n = 0, 1, 2, 3, \dots, m = 0 \pm 1 \pm 2 \dots$ and $\mathcal{L}_n^{(m_i)}$, are generalized leaguer polynomials.

$$<\psi_{nl}(r)\mid \frac{-\hbar^2}{2m^*} [\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r})] + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}] + \frac{1}{2}m^*\Omega^2 r^2\mid \psi_{nl}(r)> = E_{nl}(r)\psi_{nl}(r) \quad (4.1.9)$$

using dimensionless variable this equation is written as;-

$$<\psi_{nl}(\rho)\mid\frac{-1}{2}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\frac{\partial}{\partial\rho})\right]+\frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}\right]+\frac{1}{2}\rho^2\mid\psi_{nl}(\rho)>=E\psi_{n,l}(\rho)$$
(4.1.10)

where $\varepsilon = \frac{E}{\hbar\Omega}$, solving schrodinger equation. (4.2.10) are obtain, the energy eigen value.

$$E_{nm} = (2n + |m| + 1)\hbar\Omega + \frac{1}{2}\hbar\omega_c m$$
(4.1.11)

For the value of high magnetic field, the cyclotron motion is dominant and electron do not feel. The effect of parabolic confinement, In such case, equation(4.2,11) may be expressed as;-

$$E_{nm} = (2n + |m| + 1)\hbar\frac{\omega_c}{2} \tag{4.1.12}$$

The energy for aground state $E_{0,0} = \hbar \omega$, and energy for exiting state $E_{0,1} = 2\hbar \omega_o$, $\Delta E = E_{0,1} - E_{0,0}$

$$E_{0,0} = \hbar \sqrt{\omega_o^2 + \frac{e^2 B^2}{4m^*}} \tag{4.1.13}$$

Ground state energy.

$$E_{0,1} = 2\hbar(\omega_o^2 + \frac{e^2 B^2}{4m^{*2}})^{\frac{1}{2}} - \frac{\hbar eB}{2m^*}$$
(4.1.14)

Exited state energy. $\Delta E = E_{0,1} - E_{0,0} = \hbar (\omega_o^2 + \frac{e^2 B^2}{4m^{*2}})^{\frac{1}{2}} - \frac{\hbar e B}{2m^{*}}$;

 ΔE =The difference between Exited state and ground state energy.

The energy eigen value for some arbitrary value of frequency and magnetic field for GaAs.

 $\hbar = 1.05 \times 10^{-34} J,$

 $m^* = 0.067mo \ m_o = \text{mass of free electron}$,

 $m_o = 1.6\times 10^{-19}c$

for B = 0. from the table, which is described for different value of magnetic field,

Table 4.1: The energy eigen value, for some arbitrary value of frequency.

$\omega_o(s^{-1})$	E0,0	E0,1	ΔE
1.0×10^{12}	$1.05 \times 10^{-22} J$	$2.1 \times 10^{-22} J$	$1.05 \times 10^{-22} J$
1.5×10^{12}	$1.5751 \times 10^{-22} J$	$3.15 \times 10^{-22} J$	$1.575 \times 10^{-22} J$
2×10^{12}	$2.1 \times 10^{-22} J$	$4.2 \times 10^{-22} J$	$2.1 \times 10^{-22} J$
2.5×10^{12}	$2.625 \times 10^{-22} J$	$5.25 \times 10^{-22} J$	$2.625 \times 10^{-22} J$
3.0×10^{12}	$3.15 \times 10^{-22} J$	$6.3 \times 10^{-22} J$	$3.15\times 10^{-22}J$

from the table described for different value of frequency as ω_o , increases the change in energy(photon) increases.

Table 4.2: The energy eigen value, for some arbitrary value of magnetic field.

B(T)	E0,0	E0,1	ΔE
0	$1.05 \times 10^{-22} J$	$2.1 \times 10^{-22} J$	$1.05 \times 10^{-22} J$
4	$5.664 \times 10^{-22} J$	$1.689 \times 10^{-21} J$	$1.123 \times 10^{-21} J$
8	$1.12 \times 10^{-21} J$	$3.35 \times 10^{-21} J$	$2.23 \times 10^{-21} J$

as the magnetic field increase, the change in energy(photon)increase.

4.2 Nonlinear optical properties of GaAs cylindrical quantum wire using density matrix formalism

In this section the third order non linear refractive index and absorption coefficient are calculated by using density matrix formalism. Density matrix formalism, is preferred as it is capable of treating effects, such as collision of atomic resonance, that can not be treated by simple theoretical formalism based on atomic wave wave function, let as assume that the charge caries in cylindrical GaAs quantum wire are exited by the laser field.

$$\vec{E(t)} = Ee^{i\omega t} + Ee^{-i\omega t} = E_o cos\omega t \tag{4.2.1}$$

$$\vec{E}(t) = \check{E}e^{iwt} + \check{E}e^{-iwt} \tag{4.2.2}$$

where ω , is the frequency of the external field with polarization field, normal to the quantum wire. The time dependent equation of the density matrix operator is $\dot{\varrho}_{ij}$, or $\frac{\partial \rho}{\partial t}$

$$\dot{\varrho}_{ij} = (i\hbar)^{-1} [\hat{H}_0 - er \overrightarrow{E}(t), \varrho]_i j - \gamma_{ij} (\varrho - \varrho^{(0)})_i j.$$

$$(4.2.3)$$

where \hat{H}_0 is unperturbed Hamiltonian, e is the electronic charge, $\varrho^{(0)}$ is the unperturbed density matrix and γ_{ij} is the relaxation rate representing the damping due to the electro-phonon interaction or collision between electrons. The value of the relaxation rate is assumed $\gamma_{ij} = \gamma = \frac{1}{T}$, with T is the relaxation time. For analysis of equation(4.3.2) the usual iterative method is used.

$$\varrho(t) = \sum_{n} \varrho^{(n)}(t) \tag{4.2.4}$$

with

$$\dot{\varrho}_{ij}^{(n+1)} = (i\hbar)^{-1} \{ [\hat{H}_0, \varrho^{(n+1)}]_{ij} - i\hbar\gamma_{ij}\varrho_{ij}^{(n+1)} \} - (i\hbar)^{-1} [er, \varrho^{(n)}]_{ij}\check{E}(t)$$
(4.2.5)

The electric polarization of the parabolic cylindrical quantum wire due to the time dependent electric field $\check{E}(t)$ can be expanded with respect to equation(4.3.3). We consider only the first three lowest orders, and the second order is usually zero due to the symmetry condition.

$$\overrightarrow{p}(t) = \epsilon_0 \chi^{(1)} \check{E} e^{-iwt} + \epsilon_0 \chi^{(2)}_{\omega} \check{E}|^2 \check{E} e^{-iwt} + \epsilon_0 \chi^{(3)}_{3\omega} \check{E}^3 e^{-3iwt} + cc \qquad (4.2.6)$$

where $\chi^{(1)}$, $\chi^{(2)}_{\omega}$, and $\chi^{(3)}_{3\omega}$ are the linear second order and third order harmonic order generation susceptibilities respectively, ϵ_0 is the permittivity of free space. The electronic polarization of the n^{th} order is given by

$$\overline{P}^{(n)}(t) = \frac{1}{V} Tr(\varrho^{(n)}\mu)$$
(4.2.7)

where μ is the transition dipole moment. The transition dipole moment is given by

$$\hat{\mu} = \langle \varphi_{nm} | er | \varphi_{nm} \rangle \tag{4.2.8}$$

'V', is the volume of the interaction and Tr denotes, trace of the summation over the diagonal element of the matrix $\rho^{(n)}\hat{\mu}$. The analytical expression for, $\chi^{(1)}$ and $\chi^{(3)}$ are described by;

$$\chi^{(1)} = \frac{1}{\epsilon_0} \frac{N|\mu_{nm}|^2}{E_{nm} - \hbar\omega - i\hbar\Gamma_{nm}}$$
(4.2.9)

$$\chi^{(3)} = \frac{1}{\epsilon_0} \frac{N|\mu_{nm}|^2 E^2}{E_{nm} - \hbar\omega - i\hbar\Gamma_{nm}} [\frac{4|\mu_{nm}|^2}{(E_{nm} - \hbar\omega)^2 + (\hbar\omega)^2}] - \frac{(\mu_{nn} - \mu_{mm})^2}{(E_{nm} - i\hbar\Gamma_{nm})(E_{nm} - \hbar\omega - i\hbar\Gamma_{nm})}$$
(4.2.10)

The electric susceptibility $\chi(\omega)$ is connected with the change in refractive index as;-

$$\Delta n(\omega) = Re \frac{\chi(\omega)}{2n_r} \tag{4.2.11}$$

where n_r is the medium refractive index.

$$\Delta n^{(1)}(\omega) = \frac{1}{2n_r} |\mu_{nm}|^2 N[\frac{E_{nm} - \hbar\omega}{(E_{nm} - \hbar\omega)^2 + (\hbar\Gamma_{nm})^2}]$$
(4.2.12)

and

$$\Delta n^{(3)}(\omega) = \frac{-\mu c}{4n_r \epsilon_0} |\mu_{nm}|^2 \frac{NI}{[(E_{nm} - \hbar\omega)^2 + (\hbar\Gamma_{nm})^2]^2} \times [4(E_{nm} - \hbar\omega)|\mu_{nm}|^2 \quad (4.2.13)$$

 $-\frac{(\mu_{nn}-\mu_{mm})}{(E_{nm})^{2}+(\hbar\Gamma_{nm})^{2}}] \times \{(E_{nm}-\hbar\omega)E_{nm}[E_{nm}-\hbar\omega-(\hbar_{\Gamma}nm)^{2}]-\hbar_{\Gamma}nm)^{2}(2E_{nm}-\hbar\omega)^{2}\},\$ where $I = \frac{2n_{r}}{\mu c}|\check{E}(\omega)|^{2}$, c is the speed of light in free space, N is the carrier density in the system , μ is the permeability of the system and E_{nm} , is the energy difference of the two systems, and Γ_{nm} , is the inverse of relaxation rate. Similarly the absorption coefficient can be given by

$$\alpha^{1}(\omega) = \omega \sqrt{\frac{\mu}{\epsilon_{r}}} \frac{|\mu_{nm}|^{2} N \hbar \Gamma_{nm}}{(E_{nm} - \hbar \omega)^{2} + (\hbar \Gamma_{nm})^{2}}$$
(4.2.14)

$$\alpha^{3}(\omega) = -\omega \sqrt{\frac{\mu}{\epsilon_{r}}} (\frac{I}{2\epsilon_{0}n_{r}c}) \frac{|\mu_{nm}|^{2} N\hbar\Gamma_{nm}}{[(E_{nm} - \hbar\omega)^{2} + (\hbar\Gamma_{nm})^{2}]^{2}} \times \{4|\mu_{nm}|^{2}$$
(4.2.15)

$$-\frac{(\mu_{nm}-\mu_{mm})^{2}[3E_{nm}-4E_{nm}\hbar\omega+\hbar^{2}(\omega^{2}-\Gamma_{nm}^{2})]}{E_{nm}^{2}+(\hbar\Gamma_{nm})^{2}}$$

4.2.1 Numerical results and discussions

In this study we are going to describe the third, harmonic generation of GaAs cylindrical quantum wire with applied magnetic field. The parameter used in this numerical work are $\rho_v = 10^{22} m^{-3}$, the damping constant $\gamma = 3 \times 10^{12}/s$, the medium refractive index, $n_r = 3.25$, the effective mass, $m^* = 0.0675mo$, where m_o is the mass of free electron, and the transition dipole moment= $1.2 \times 10^{-28}C.m$ In Figure 4.1 : the change in third order absorption coefficient is shown as a function of incident photon energy. $\hbar \omega$, for three different value of the static magnetic field, $\vec{B} = 0T$, $\vec{B} = 4T$ and $\vec{B} = 8T$ with confining frequency $\omega_o = 1 \times 10^{12} rad/s$.



Figure 4.1: Absorbation coefficient versus photon energy

From figure 1, it can be observed that the three resonant peaks appear at $\hbar\omega = 0.55 \times 10^{-21} J$, $\hbar\omega = 1.1 \times 10^{-21} J$, and $\hbar\omega = 2.22 \times 10^{-21} J$, respectively and that the resonant peak shift to the right side of the curve and that its magnitude increases with static magnetic field, \vec{B} increasing. This is due to the lateral confinement to the election from the applied static magnetic field.

In Figure 4.2, the change in third order non linear refractive index, is described as a function of incident photon energy . $\hbar\omega$, for three different value of the static magnetic

field, $\vec{B} = 0T$, $\vec{B} = 4T$, and $\vec{B} = 8T$, with the confining frequency $\omega_0 = 1 \times 10^{12} rad/s$. From Figure 2 one can see that the change in third order non linear refractive index,



Figure 4.2: change in refractive index versus photon energy

 $\Delta n^{(3)}/n_{nr}$, related to the magnitude of the static magnetic field. As the static magnetic field increases, the change in third order non linear refractive index have been magnified in magnitude, and also shifts to wards higher energies.

Chapter 5 Conclusion

In this study the energy igen value of GaAs cylindrical quantum wire with applied magnetic field are obtained solving the schrodinger equation analytically. Then the third order nonlinear absorption and refractive index changes are large and affect the magnitude of linear absorption coefficient and refractive index. The change in third order nonlinear absorption coefficient and refractive index versus photon energy are drawn graphically for different value of static field. The finding clearly shows that as the magnitude of static magnetic field increase, the magnitude of the change in third order nonlinear absorption coefficient and refractive index also increases. Moreover, the resonant peak of the third order nonlinear absorption coefficient, and refractive index coefficient shifts towards higher frequency or energy as, the magnitude of static magnetic field increases. This results is may be due to the lateral coefficient of electrons from static magnetic field.

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Name of Student: Mulugeta Sime ID No. SSMSC S32848/08

Graduate Program: Summer, MSc.

1. Course Work Performance

Course	Course Title	Cr. hr	Number	Rank **	Remark
Code			Grade		
Phys799	MSc. Thesis	6	70.83	Good	

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- 2. Board of Examiners decision Mark in one of the boxes. Pass × Failed If failed, give reasons and indicate plans for re-examination.
- 3. Approved by: Name and Signature of members of the examining Board, and Deans, SGS

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