# Particle Tri-axial Rotor Model Calculation of Low-lying state of Three Odd-Mass Isotopes of Indium

A thesis submitted to post graduate program at the department of physics college of natural sciences, Jimma University, in partial fulfillment for the requirements of degree of masters of science in Physics

(Nuclear Physics)

By : **Jiregna Amenu**

:

**Advisor :Teklemariam Tesemma (PhD)**

Jimma, Ethiopia

January, 2021

DECLARATION

I hereby declare that this MSc thesis is my original work and has not been presented for a degree in any other university, and that all sources of material used for the thesis have been duly acknowledged.

Name: Jiregna Amenu

Signature: ——————

This MSc thesis has been submitted for examination with my approval as university advisor.

Name: Teklemariam T. (PhD)

Signature: ————————

Place and date of submission:

Jimma University

Department of Physics

January , 2021

# Contents

# List of Figures

# List of Tables

## Acknowledgment

First of all,I would like to thank almighty God for letting me to accomplish this study. Secondly, I would like to express my deepest gratitude to my advisor and instructor, Dr.Teklemariam Tessema for developing this masters thesis and for inspirational guidance,unreserved support and critical comments on the thesis work by sharing his every rich experience in the end of Nuclear physics. And also I would like to express my great respect for Dr.Teklemariam Tessema,for his valuable guidance how to develop Fortran based computer code during this work.Also I would like to thank my co-advisor Mr.Gashew Dajene, who gave me moral and material support.Finally, my most heart felt thank goes to all my friends and members of physics department of Jimma University.

## Abstract

In this thesis the title Particle Tri-axial Rotor Model based on Calculation of the level energy,gamma energy and intensity branching ratio for Odd isotopes of Indium ,the behavior of Odd Isotopes of Indium ( 105-In, 107-In and 111-In) have been studied. By taking the particle Tri-axial rotor model with variable $\kappa$ and $\mu$ we systematically investigate the level energy, gamma energy and gamma branching ratio of the indium isotopes 105-In, 107-In and 111-In. The calculated energy spectra agree quite well with experimental data. The obtained results indicate that the rotation aligned bands observed in 105-In, 107-In and 111-In. The work mainly focused on the Level Energy, gamma energy,gamma branching ratio and gamma transition type of odd isotopes of Indium.

The energy ground state and high exited energies of states were calculated by using FORTRAN based computer code developed during this work. The calculated values were compared with values given in previous work for the purpose of validity.

# 1

# Introduction

## 1.1 Back ground of study

One of the aims of nuclear physics is to calculate the energies and quantum numbers of nuclear bound states.In atomic physics, one can do this starting from first principles.In fact,Coulombs law (or more generally the equations of electromagnetism) determines the interactions between electrons and nuclei.The Shell model of the nucleus was first proposed by Maria Goeppert- Mayer in 1948.In this she is the one of the only two women who has received Nobel Prize for physics for her path breaking discovery Model, the nuclei of atoms are also divided into different shells and the nucleons (protons and neutrons) are distributed into these shells in the same manner that the electrons are distributed in the different shells of the atoms.With this nuclear shell model the stability of the certain nucleus are justified.More importantly it is observed that the nuclei associated with the magic numbers of protons are more stable with respect to other nuclei.The magic numbers:2,8,20,28,50,82,126,184.The basic model for the description of nuclear properties is the shell model.The strong nuclear force binds together protons and neutrons.Each individual nucleon moves in the average potential generated by all the others.The states of a single nucleon in the average potential cluster together into layers or shells, much like the single particle states in atoms.This type of model of non-interacting particles in a mean potential is often called the independent particle model.Nuclear structure studies the properties of nuclei in isolation (for interactions between nuclei and radiation, see nuclear reactions), such as nuclear mass, characteristic energy levels, and radioactive decay modes.[1]

Nuclei are named by the element that they belong to (chemical symbol and/or atomic charge number Z, which is equal to the number of protons in the nucleus), the nuclear mass number A (which is the sum of the number of protons and the number of neutrons), and sometimes an isomeric state of increased energy [1].The neutron number N=A-Z is also Thousands of nuclei have been observed in nature or in the laboratory.About 300 of them are stable, but most are radioactive and decay into stable isotopes or other radioactive isotopes with some characteristic half life. (The half life is the time required for half of an initial sample of radioactive isotope to decay away. For a half life of one year, half will be gone in 1 year, three fourths in two years, seven tights in three years, and so on.) Theory says that there are many more possible nuclides, for a total of perhaps 5000 species, but these additional isotopes have such short half lives that they have not been observed in the laboratory.However, even these fleeting nuclei have their effects in processes like stellar nucleo synthesis.The shell model is partly analogous to the atomic shell model which describes the arrangement of electrons in an atom, in that a filled shell results in greater stability. When adding nucleons (protons or neutrons) to a nucleus, there are certain points where the binding energy of the next nucleon is significantly less than the last one. This observation, that there are certain magic numbers of nucleons: 2, 8, 20, 28, 50, 82, 126 which are more tightly bound than the next higher number, is the origin of the shell model and the shells for protons and for neutrons are independent of each other.Some semi magic numbers have been found, notably Z = 40 giving nuclear shell filling for the various elements; 16 may also be a magic number.In order to get these numbers,the nuclear shell model starts from an average potential with a shape something between the square well and the harmonic oscillator. To this potential a spin orbit term is added.Even so, the total perturbation does not coincide with experiment, and an empirical spin orbit coupling must be added with at least two or three different values of its coupling constant, depending on the nuclei being studied.Nevertheless, the magic numbers of nucleons, as well as other properties,can be arrived at by approximating the model with a three-dimensional harmonic oscillator plus a spin-orbit interaction [1].

A more realistic but also complicated potential is known as Woods–Saxon potential. These Potentials can give the level scheme and structure information of nuclei in low, medium and heavy mass region.The theoretical calculation of the structure of nuclei in deformed mass region remains active research area for the nuclear physics community.The Nucleus Indium lies in this region,Between the region $30 < A < 260$ the variation in $\epsilon$ small that for the bulk of nuclei the binding fraction may be taken approximately as constant. This shows that the nuclear forces are saturated and the calculation of its shell structure and states might give additional information why nuclei get deformed in this regions .Therefore, the work was aimed to calculate the level energy, gamma transition energy and gamma branching ratios of the low-lying states of Odd-A Indium isotopes ($^{105}In,^{107}In,^{111}In$)[2]

## 1.2 Statement of the Problem

An electron shell may be thought of as an orbit followed by electrons around an atomic nucleus.Because each shell can contain only a fixed number of electrons, each shell is associated with a particular range of electron energy and thus each shell must fill completely before electrons can be added to an outer shell.In this thesis, similar procedure will be applied for the nucleons energy level in the nucleus and calculation of the low lying states of the Indium will be performed.To the knowledge we didn't find the low-lying states calculation for Indium isotopes using particle tri-axial rotor model calculation.Therefore, the work was aimed to calculate the level energy, gamma transition energy and gamma branching ratios of the low-lying states of odd-A Indium isotopes ($^{105}In,^{107}In,^{111}In$)[2].

## 1.3 Basic Research Questions

- What are the calculated values of level energy and gamma transition probabilities of low-lying states of odd-mass isotopes of Indium ($^{105}In,^{107}In,^{111}In$)?
- What agreement is there between the calculated values and the experimentally measured value?

## 1.4 Objective

### 1.4.1 General objective

The general objective of this study was to find the lowl-ying states of three odd mass Indium isotopes by using particle tri-axial rotor model calculation.

### 1.4.2 Specific Objective

The specific objectives of this thesis were:

- To calculate the nuclear states of odd isotopes of Indium (In) of low-lying states of Indium isotopes($^{105}In,^{107}In,^{111}In$)changing Nilson parameters to get best related values with compae to experimental value.
- To search for experimental data for the odd mass indium isotopes from the Evaluated Nuclear Structure Data Files (ENSDF)
- To check how the calculated level scheme agrees with literature.

## 1.5 Significance of the study

Calculating theoretical data for low-lying states on level energies, gamma transition energies and gamma intensity branching ratio of three odd mass Indium isotopes by theoretical calculations is the main relevance of this research. In addition this study can be used as a reference to develop the work further to higher rotational bands of odd mass Indium isotopes and also could be used as a reference on similar calculations.

## 1.6 Limitation of the study

There were limitations while working wioth this thesis.Some of the limitation were:

- Due to time constraint the study was limited to three odd mass Indium isotopes $^{105}In,^{107}In,^{111}In$.
- Non availability of some experimental data fo comparison.

- Less access to internet and lack of related work for reference in the library.
- Covid-19 Pandemic

# 2

# Theoretical Background

## 2.1 Nuclear shell structure

One of the most obvious indications for the existence of a shell structure was obtained by comparison with the analogous picture of electrons moving in the atom. In studying the ionization energy as a function of the number of electrons (as a function of Z), a clear indication for 'magic' numbers at 2, 10, 18, 36, 54, 86 was obtained.The behavior can be explained since,going from atom Z to Z + 1, the charge of the nucleus increases.The ionization energy will thus increase on the average and in a rather smooth way when filling the electron shells. When starting to fill a new electron shell, the last electron occurs in a less strongly bound orbit but a screening effect of the central, nuclear charge by a number of filled electron orbit presents an effective smaller Coulomb potential. So, the electron ionization energy reflects rapid variations whenever an electron shell is closed [3].So,even though a number of distinct differences between the atomic nucleus and the electron motion in an atom appear, the presence of an average nuclear potential acting on nucleons in an averaged way is very clear. In the final section of this chapter we shall concentrate more on the precise arguments and methods used to define and determine such an average single-particle potential starting from the two-body nuclear interaction.In the shell model using Harmonic oscillator potential well, the nucleon numbers corresponding to the shell closures are calculated as: 2, 8, 20, 40, 70, 112 and 168.Here also the experimentally observed magic numbers above 20 are not reproduced [3].

## 2.2 Nuclear Models

A goal of nuclear physics is to account for the properties of nuclei in terms of mathematical models of their structure and internal motion.Three important nuclear models are the Liquid Drop Model, the Shell Model (developed by Maria Goeppert-Mayer and Hans Jensen), which emphasizes the orbits of individual nucleons in the nucleus, and the Collective Model (developed by Age Bohr and Ben Mottelson), which complements the shell model by including motions of the whole nucleus such as rotations and vibrations [3].The LDM treats the nucleus as a liquid. Nuclear properties, such as the binding energy, are described in terms of volume energy, surface energy,comprehensibility,etc.Parameters that are usually associated with a liquid. This model has been successful in describing how a nucleus can deform and undergo fission.The nuclear shell model is similar to the atomic model where electrons arrange themselves into shells around the nucleus.The least-tightly bound electrons (in the incomplete shells) are known as valence electrons because they can participate in exchange or rearrangement, that is, chemical reactions [4].The shell structure is due to the quantum nature of electrons and the fact that electrons are fermions–particles of half-integer spin.A group of bosons all tend to occupy the same state (usually the state with the lowest energy),whereas fermions with the same quantum numbers do just the opposite: they avoid each other.Consequently the fermions in a bound system will gradually fill up the available states: the lowest one first, then the next higher unoccupied state, and so on up to the valence shell [4].The Collective Model emphasizes the coherent behavior of all of the nucleons. Among the kinds of collective motion that can occur in nuclei are rotations or vibrations that involve the entire nucleus. In this respect, the nuclear properties can be analyzed using the same description that is used to analyze the properties of a charged drop of liquid suspended in space. The Collective Model can thus be viewed as an extension of the Liquid Drop Model; like the Liquid Drop Model, the Collective Model provides a good starting point for understanding fission [4].The Shell Model and the Collective Model represent the two extremes of the behavior of nucleons in the nucleus. More realistic models, known as unified models.However, if we consider a nuclear shell model, with a flat bottom potential, we would get the [4]:

### 2.2.1 Nuclear Shell Model

The basic model for the description of nuclear properties is the shell model.The strong nuclear force binds together protons and neutrons.Each individual nucleon moves in the average potential generated by all the others.The states of a single nucleon in the average potential cluster together into layers or shells, much like the single particle states in atoms. This type of model of non-interacting particles in a mean potential is often called the independent particle model. The most important piece of experimental information on shell structure is the existence of magic numbers. Nuclei having magic numbers of both neutrons and protons have been found to have spherical equilibrium shapes and special stability.Since the total binding energy of the nuclei having magic numbers are large, a larger energy is required to separate a single nucleon.A few special characters of this type of nuclei are:higher energy of the lowest excited states and large number of isotopes or isotones with the same magic number of proton(neutrons).The lower magic numbers are the same for protons and neutrons, namely 2, 8, 20, 28,50 and 82, whereas the next number 126 is established only experimentally for neutrons.Theoretically one would expect additional magic number 114 for protons and 184 for neutrons leading to super-heavy nuclei but these have not been confirmed in experiment.A phenomenological shell model thus is based on the Schrodinger equation for the single-particle levels [5].

$$-\frac{h^2}{2M} + V_(r) = E_i\Psi(r) \tag{2.1}$$

with a prescribed potential V(r).Inside heavier nuclei, this potential should be relatively constant.This is to explain the constant density suggested by the fact that the nuclear radius behaves as $R = r0A^{1/3}$ but goes to zero quite rapidly outside the nuclear surface.Assuming the potential V (r) is spherically symmetrical there are three types of potentials are followed.They are,

- the Woods-Saxon potential
- the harmonic-oscillator potential
- the square-well potential

$$H = -\frac{h^2}{2M}\Delta^2 + V_{ws}(r,\phi) + V_s + V(r)l.s[5] \tag{2.2}$$

Where$V_{ws}$ = wood-saxon potential,$V_{ls}$ = spin-orbital potential and$V_c$ = Coulumb potential. This equation is separable into radial and angular coordinates and therefore the solutions can be written as:

$$\frac{h^2}{2M}\Psi(x,t) = i\frac{h}{at}\Psi(x,t) \tag{2.3}$$

Here, n,l and m is the radial quantum number, l the orbital angular momentum and m-the eigenvalue of its z-component, $I_z$.Thus yielding many results analytically.Its eigenvalue is simply given by [4]:

$$< l.s >= \frac{1}{2}(l+s)^2 - l^2 - s^2 \tag{2.4}$$

$$< l.s >= j^2 - l^2 - s^2 = [j(j+1) - l(l+1) - s(s+1)](\frac{\hbar^2}{2}) \tag{2.5}$$

$$Ej = l + \frac{1}{2} - E_j = l\frac{1}{2} = \hbar^2(l + \frac{1}{2}) \tag{2.6}$$

The factor connecting these effects is exactly 2 for harmonic oscillator potential: it is also 2 for a square well potential. It is this grouping of levels that provides the shell structure required of any central potential useful for real nuclei. If we recall that each energy level has 2(21 + 1) degenerate m states, then, by the Pauli principle, each nl level can contain 2(2l + 1) particles [3,4]. Therefore, if we imagine filling such a potential well with fermions, each group or shell can contain, at most, the specific numbers of particles indicated in the figure. Hence, such a potential automatically gives a shell structure rather than, say, a uniform distribution of levels.Unfortunately, except for the lowest few, these shells do not correspond to the empirical magic numbers. Therefore, while the harmonic oscillator potential is a reasonable first order approximation to the effective nuclear potential, it must be modified to be useful. It was in fact, the monumental achievement of Meyer and , independently, of Haxel,Jensen and Suess, to concoct a simple modification to the harmonic oscillator potential that enabled it to reproduce the empirical magic numbers. Therefore, the addition of an l2 term is equivalent to a more attractive potential at larger radii and comes closer to the desired effect of a more constant interior potential. The relation of the single particle levels produced by a Simple Harmonic Oscillator potential, (SHO) along with the addition of an l2 term is illustrated in the middle panel of Fig.2.1, which shows how the 2n + 1 degeneracy of the SHO levels is broken as high angular momentum levels are brought down in energy.It is clear that neither of these alternatives yet produce the magic numbers observed experimentally. So another additional term is introduced, the so called spin orbit coupling force. With a spin-orbit component , the force felt by a given particle differs according to whether its spin and orbital angular momenta are aligned parallel or anti-parallel. If the parallel alignment is favored, and if the form of the spin-orbit potential is taken as [5]:

$$V(r) = -V(r) l.s \tag{2.7}$$

so that it affects higher l values more [5]

### 2.2.2 The Collective Model

The atomic nucleus can be regarded as a system of Z protons and N neutrons, which move independently in a central potential generated by all nucleons together. This system has in principle many degrees of freedom, which complicates a theoretical description. Nuclear models, for example based on the idea of the shell model enable to reduce the number of degrees of freedom. In a description of a single-closed-shell nucleus (e.g. Z =50,82), the nucleons in the open shell generally account well for the nuclear properties at low excitation energies; the nucleons in the closed shell are considered as an inert core.The nuclei with one or two nucleons more or less, e.g. Z ± 1,Z ± 2, can then be included by adding particles or holes to the closed-shell core. The possibility of this relatively simple approach motivates experimental investigations on nuclei in the vicinity of those closed shells. Collective nuclear excitation are a common feature of nuclei in between closed shells; in particular collective rotations occur in non-spherical nuclei (e.g. rare earth nuclei).Recently, collective rotations have been observed in' single-closed-shell nuclei too. In the even mass Sn isotopes (Z=50),rotational bands have been observed 2I , which are explained by assuming that the central po-

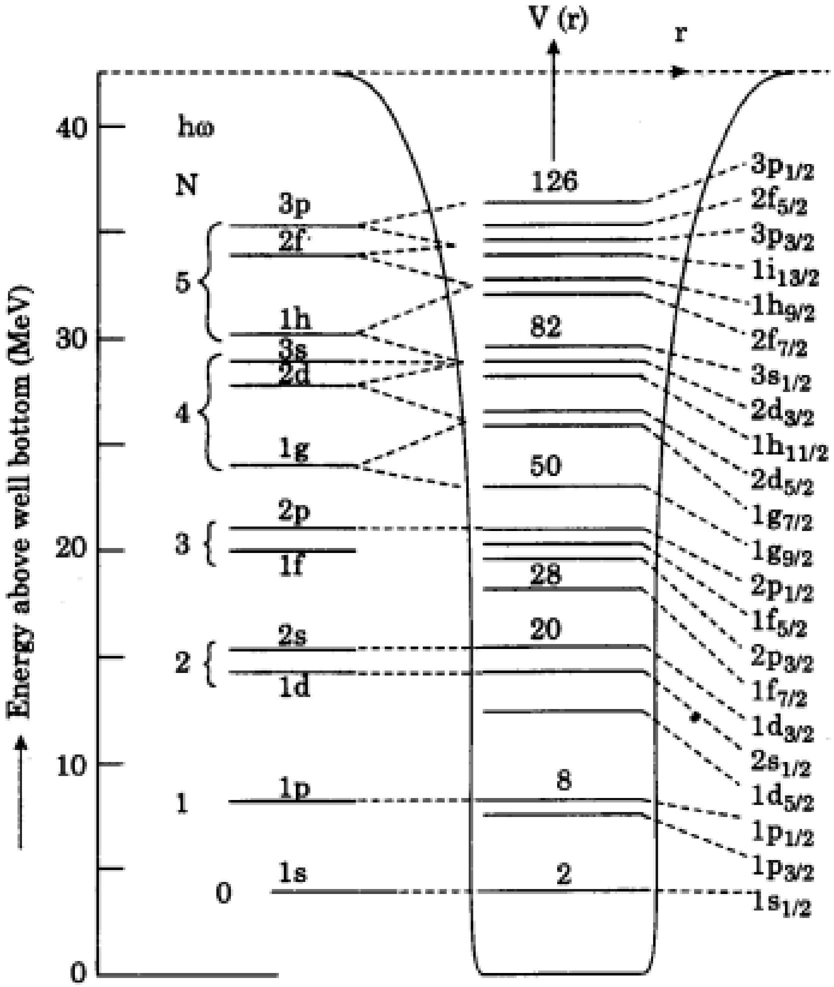



Figure 2.1: Single-particle levels in a potential with Spin-Orbit coupling

tential, which is spherical in the ground state, is deformed at higher excitation energies. In this situation, the energy required for the excitation of a proton across the Z=50 closed shell has decreased considerably.The so-called proton particle-hole excitations of the closed shell constitute the basis of the rotational bands. As protons or neutrons are added to a closed shell nucleus the nucleus may shift to oblate,prolate or tri-axial shape.In these instances, the spherical shell model is inadequate and collective models, such as that proposed by Bohr and Mottelson must instead be brought to bear. Again, for nuclei that are in regions of strong deformation,these models have had a wide array of successes. It is well known the rotational bands in deformed nuclei and their electromagnetic transitions are fundamental manifestations of nuclear collective modes. This is also true for the neutron deficient isotopes which lie in the mass region below the closed Z = 82 shell and in the neutron shell above N = 82 where slightly deformed rotational structure has been observed. Hence in these regions a deformed potential is a good assumption justified by the fact that nuclear deformation appears to be present although it may easily be susceptible to changes induced by various excitation.The Nilsson modified potential can be written as [6]:

$$V = \frac{M}{2}[w_x^2(x^2 + y^2) + w_z^2 z^2] \tag{2.8}$$

where the first term is the an isotropic harmonic oscillator with oscillator frequencies:

$$\omega_x^2 = \omega_y^2 = \omega_0^2(1 + \frac{2}{3})\sigma \tag{2.9}$$

The parameter$\sigma$ which is named the parameter of deformation by Nilson can be considered as the measure of elongation of the potential along the nuclear z-axis and is related to the previously mentioned $\beta$ parameter to the last term is added to produce a more realistic potential that accounts for the fact that for high l-values the nucleons experience a deeper potential as compared to the harmonic oscillator and consequently the energy levels of higher l-values are shifted down in In odd-mass atomic nuclei with quadruple-octupole shapes the coupling of the collective rotations and vibrations to the single-particle (s.p.) motion of the unpaired nucleon provides a split parity-doublet structure of the spectrum . It is known that the octupole deformation of the core can induce respective deformation in the s.p. potential . In this situation, known as a strong coupling, the state of the odd nucleon appears without good parity. On its turn the parity-mixed s.p. state couples to the core state in a way that the total state of the nucleus remains with a good parity. The relevant wave function of the total state can be constructed if a particular model assumption for the behaviour of the core with respect to the reflection asymmetric (octupole) degree of freedom is made [7].

**Electric Quadrupole Moment(Q)**: While µ reflects more the individual orbital occupancy, the electric quadrupole moment, Q, is more sensitive to collective behaviour. More importantly, it is related to the nuclear deformation. Nonetheless, it is possible to define a single-particle quadruple moment [6,7]

$$Q\ \mathrm{s.p} = -e_{p.n}\langle r^2\rangle \tfrac{2j-1}{2j+2} \tag{2.10}$$

where j is the angular momentum of the valence nucleon, e p,n is the electric charge of either the proton or the neutron and hr j 2 i is the mean-squared radius of that nucleon in orbital j.The latter two make up the intrinsic quadrupole moment, $Q$ s.p $= -e_{p.n}\langle r^2\rangle \frac{2j-1}{2j+2}$ while the whole of Eq.(2.12) is also referred to as the spectroscopic quadruple moment, i.e. what is observed . It will vanish for a j = 1/2 state,even though $Q_0 \neq 0$ Furthermore, according to this definition Q s.p. is always zero for a neutron since it does not carry any charge [9].

### 2.2.3 Particle Tri-axial Rotor Model (P T R M)

This model was first suggested in 1953 by A. Bohr and B. R. Mottelson and is described in several textbooks, for example [9].The model treats an odd-A nucleus in terms of the last odd particle coupled to the core which describes the collective degrees of freedom.The particle plus rotor Hamiltonian may be written as:

The tri-axial rotor model was introduced for the description of nuclear data by Davydov and Filippov . Historically, irrotational flow has been used to describe the inertia tensor.We make no assumptions about the inertia tensor and leave it general.The general Hamiltonian of an asymmetric, triaxial rotor.The rotor is first developed to represent near symmetric nuclei as a limit of the general form. This limit provides excitation bands (non-zero body projections on the symmetry axis, e.g., for L = 0, I = 0, 2, 4, 6, . . . and for l = 2,I =2,3,4,5,6, . . .) but it provides no mixing between them. However, asymmetry in the electric quadruple (as opposed to the inertia) can provide transitions between the bands.This limit of the general rotor model has a diagonal Hamiltonian and unmixed rotational states.Mikhailov theory is used and expanded upon to analyze the precision data in the $^{166}Er$ study. This limit of the rotor is used in describing nearsymmetric nuclei that have minimal mixing between bands.Mikhailov theory is useful for describing branching ratios between bands of different spin, I + .Therefore, it provides a means for relating relative intensities are the body fixed operator we can however,rewrite the Hamiltonian in terms of the rising and lowering operators $I^+$ and $I^-$ respectively.The simple model consisting of a particle coupled to a rotor provides an approximate description of many of the properties of the lowlying bands of odd-A (Odd-A Indium) nuclei and at the same time, illustrates various general features of rotating systems. In the particle tri-axial rotor model, the hamiltonian of an odd-A nucleus is [11]:

$$V = \frac{M}{2}[w - x^2(x^2 + y^2) + w^2 - zz^2] \tag{2.11}$$

$$w^2x = w^2y = \omega(\sigma)(1 + \frac{2}{3}\sigma) \tag{2.12}$$

$$w^2z = \omega(\sigma)(1 - \frac{4}{3}\sigma) \tag{2.13}$$

$$H = H_0 + H_1 - \omega_{jx} \qquad where, H_0 istrixial Nilson Hamiltonian \tag{2.14}$$

$$H_0 = \frac{p2}{2M} + \frac{1}{2}[\omega_x X^2 + \omega_y Y^2 + \omega_z Z^2] \tag{2.15}$$

$$\text{E} = (N + \frac{3}{2}).\hbar.\Omega.(\sigma) + \kappa\hbar.\omega.f \tag{2.16}$$

The Nilsson model represents the self-consistent potential by an axially symmetric oscillator potential with spin-orbit coupling. The single particle$\omega.f$(Nilsson Orbitals) are obtained by solving Schridinger equation with this potential [12].

## 2.3 The Rotational Nuclear Motion

Many empirical observations such as low-lying rotational states and very large electric quadruple moments of even Z-even N nuclei that lie in the mass region $150 \leq A \leq 190$ have confirmed the existence of permanent nuclear deformationin nuclei far from closed shells.In the collective model the nucleonic motion is quantized by assuming that the even-even core is an incompressible nuclear matter in the form of a deformed liquid drop.A distinct feature is that the coherent motion of all nucleons contributes in the total angular momentum of the system.Another important property of collective rotational motion is that the angular momentum of the odd nucleon is no longer a conserved quantum number and for the conservation of the total angular momentum of the system the core must have an angular momentum coupled to the single particle angular momentum [12].

This is explained within the particle-plus-rotor model which was proposed by Bohr and Mottelson by means of the coupling of a few valence nucleons outside a rotating rigid core .For an axially symmetric rotor the total Hamiltonian consists of intrinsic and collective parts.The energy levels of rotational states in odd-A nuclei are obtained by [12]:where $eK$ is the single-particle energy,$h\sqrt{I(I+1)}$is the total angular momentum of the nucleus, K is the angular momentum component along the symmetry axis and J is the component of the moment of inertia perpendicular to the symmetry axis.The intrinsic property of the system as being invariant to rotation by an angle 180∘about an axis perpendicular to the symmetry axis gives rise to two-fold degenerate Ω states that are filled pairwise.The ground state band in even-even nuclei has positive parity and K = 0.The energies then reduce to [13]:

$$E = \frac{h^2}{2J}[I(I+1)] \tag{2.17}$$

Such low-lying rotational states are characterized by spin sequence I = 0, 2, 4, 6, ....In odd-A nuclei parity and angular momentum in the band head is

$$K\pi = \Omega\pi \tag{2.18}$$

corresponding to the odd nucleon. One can also discuss different degrees of coupling of the odd nucleon to the collective axially symmetric rotor [13]. In the strong coupling

limit (deformation alignment) the orientation of the rotating deformed core is a leading factor to determine the motion of the valence nucleons. This is the case when the deformation is large and the Coriolis force is weak and consequently the large quadruple deformation causes the odd nucleon to couple to the deformed core (K = Ω). The spin values of the rotational states are then given as$I = K, K + 1, K + 2, ...(K = W)$. In the decoupling limit (rotation alignment) the Coriolis force largely dominates the motion of the valence nucleon and the angular momentum of the band head is not necessarily the same as the K value. This is the case for nuclei with high-j orbitals and low-Ω values where the Coriolis force favors the alignment of the angular momentum j of the odd particle with the rotating core. The spin sequence of the band members is then given by$I = J, j + 2, j + 4, ...$ and the energies of the rotational states can be calculated by considering the projection of j on the rotational axis (denoted as$J_x$).The complete alignment of j along the rotation axis, i.e.$J = J_x$ generates the lowest-lying rotational band which is often termed as a favored band [13].
The spin values of a rotational band with less alignment is

I = J ,j + 1,j + 3, . . .unfavored band. (2.19)

This approach has been successful in describing the rotational bands in well-deformed odd-A nuclei as well as the back bending phenomena as a consequence of breaking time-reversed nucleon nucleon pairs due to the Coriolis interactions with the rotating core. In a heavy ion fusion evaporation experiment, where a large amount of angular momentum is transferred to the nucleus, a stable nuclear deformation could be characterized by coherent movement of many nucleons. Such collective rotational excitation are experimentally observed over a wide range of nuclei. The rules governing the angular momentum couplings of protons and neutrons in odd-odd nuclei were studied by Nordheim in 1950 who proposed two coupling rules known as strong and weak rules .Later Brennan and Bernstein performed an empirical analysis over a large range of odd-odd nuclei and replaced the Nordheim rules with new revised rules .In brief, for configurations in which both the odd- proton and neutron

$$Jgs = jp - jp for Jp = lp + \frac{1}{2} and Jn = ln + \frac{1}{2} \quad (2.20)$$

$$Jgs = jp - jn for Jp = lp + \frac{1}{2} and Jn = ln + \frac{1}{2} \qquad (2.21)$$

However,in the case of an odd-even nucleus with many nucleons outside the core the complexity of the coupling of the valence nucleons to the core usually prohibits a clear description of angular momentum coupling scheme,neutron are particles (or holes) the spins of the lowest states,$J_{gs}$ can be obtained [13].

## 2.4 The Nuclear Angular momentum Assignment

The original indication that nuclei have spin came from atomic spectroscopy.Two sources of such structure,one due to the present of isotopes,the other has been found in spectral lines of nucleus of single isotope such as bismuth 209 Bi,the explanation of the last hyperfine structure spectral lines,is suggested by Pauli [12,13].Just like nucleons inside the nucleus,the nucleus of the atom also spins about an axis and possesses a nuclear angular momentum .

In the application of quantum mechanics to the nucleus,we label every nucleon (proton or neutron) with the corresponding quantum numbers l, s and j.The total angular momentum of a nucleus having (A) nucleons would then be the vector sum of the total angular momentum of all nucleons j and it is often called the intrinsic angular momentum or nuclear spin and is represented by the symbol:

J = l + s

$$< l.s >= \frac{1}{2}(l+s)^2 - l^2 - s^2 \qquad (2.22)$$

$$< l.s >= j^2 - l^2 - s^2 = [j(j+1) - l(l+1) - s(s+1)](\frac{\hbar^2}{2}) \qquad (2.23)$$

where$J = l\frac{\hbar^2}{2}$ for l is parallel to s and for l is anti-parallel to s $J = -(l+1)\frac{\hbar^2}{2}$ even-A nuclei $J = integer, evenZ{-}evenN, J = 0 oddZ{-}oddN, J = integer, odd - Anuclei J = oddhalfinteger, evenZ{-}oddN J = oddhalfinteger oddZ{-}evenN J = oddhalfinteger$ We next include a spin–orbit interaction. The first six shells, described by the new quantum numbers, are level 0 (n = 0):2 states (j = 1/2) Even parity.

level 1 (n = 1):2 states (j = 1/2) + 4 states (j = 3/2) =Odd parity.

level 2 (n = 2):2 states (j = 1/2) + 4 states (j = 3/2) + 6 states (j = 5/2) =12. Even parity.

level 3 (n = 3): 2 states (j = 1/2) + 4 states (j = 3/2) + 6 states (j = 5/2) + 8 states (j = 7/2) =20. Odd parity.

level 4 (n = 4): 2 states (j = 1/2) + 4 states (j = 3/2) + 6 states(j = 5/2)+8 states(j = 7/2) + 10states (j = 9/2)= 30. Even parity.

level 5 (n = 5):2 states(j = 1/2) + 4 states (j = 3/2) + 6 states (j = 5/2) + 8 states (j = 7/2) + 10 states (j = 9/2) + 12 states (j = 11/2) = 42. Odd parity.

where for every j there are 2J+1 different states from different values of mj [13,14].

**Degeneracy** :is the state with equal energy D = 2(2l+1)=2J+1.Gamma Transition selection rules :The multi-pole 'l' is related to gamma angular momentum.Angular momentum must be conserved in the gamma decay.Possible 'l'

$|Jf - Ji| < l < |Jf + Ji|$

The static electric quadruple and magnetic dipole moments have been computed for the ground state.The electric quadruple transition rate is given by the off-diagonal matrix element of the same operator whose diagonal matrix element gives the quadruple moment.We may expect then, to obtain information about the charge shape of the nucleus from a knowledge of the electric quadruple transition rate.The reduced quadruple transition rate may be=linked to the quadruple deformation [13,14].

## 2.5 Nuclear Parity Assignment

In addition to their magnetic and electric properties,nuclei have certain properties which are not obviously physical in nature. Among them is the parity.To a good approximation,the wave function of a nucleus may be expressed as the product of a function of the space coordinates and a function depending only on the spin orientation.While when reflection changes the sign of the spatial part of the wave function,the motion of the nucleus is said to have odd parity.The motion of a nucleus is said to have even parity if the spatial part of its wave function is unchanged when the space coordinates (x, y, z) are replaced by (-x, -y, -z), or r to -r i.e., reflection of the nucleus. While when reflection changes the sign of the spatial part of the wave function,the motion of the nucleus is said to have odd parity.In relation to the value of the orbital angular momentum L and the parity of the energy state donated is by $\pi$ [14].

The parity of a state is determined by: the orbital angular momentum quantum number as,$\pi$= (1).A quantum state of a nucleus is defined by the parity, its energy and its total angular momentum of the constituent nucleons $j$ written conventionally as:Example Even nuclei $Z - even, N - even$ will be $J = 0+$ its ground state.For nuclei farther from the magic numbers one must add the assumption that due to the relation between the strong nuclear force and angular momentum, protons or neutrons with the same $n$ tend to form pairs of opposite angular momenta [15].

Therefore, a nucleus with an even number of protons and an even number of neutrons has 0 spin and positive parity.A nucleus with an even number of protons and an odd number of neutrons (or vice versa) has the parity of the last neutron (or proton), and the spin equal to the total angular momentum of this neutron (or proton).By last we mean the properties coming from the highest energy level.In the case of a nucleus with an odd number of protons and an odd number of neutrons one must consider the total angular momentum and parity of both the last neutron and the last proton. The nucleus parity will be a product of theirs, while the nucleus spin will be one of the possible results of the sum of their angular momentum (with other possible results being excited states of the nucleus) $\pi = (-1)^l$ its said to be Electric moment and

$\pi = (-1)^{l-1}$

it becomes Magnetic moment [16].

## 2.6 The Band Selection Rules

Table 2.1: Gamma transition selection rules

| Multipolarity | Angular momentum 'l' | $Parity' \prod'$ | Multipolarity | Angular mo'l | $Parity' \prod'$ |
|---|---|---|---|---|---|
| M1 | 1 | + | E1 | 1 | - |
| M2 | 2 | - | E2 | 2 | + |
| M3 | 3 | + | E3 | 3 | - |
| M4 | 4 | - | E4 | 4 | + |
| M5 | 5 | + | E5 | 5 | - |

# 3

# Materials and Methods

## 3.1 Materials

During the work calculations was performed based on the Fortran Program based Nuclear Structure Codes called ASYRMO ,PROBAMO and GAMPYN using a laptop computer.Experimental data was downloaded from different sites with an internate connection of Jimma University Digital Library . Data were stored and transferred using flash disks,pen , white papers and printers were used.

## 3.2 Methodology

### 3.2.1 Experimental data

Different reference materials have been visited for the purpose of finding experimental data. The experimental data uploaded in the cite ENSDF (Evaluated Nuclear Structure Data files) have been used.

### 3.2.2 Computational Method

Using a computational code, Particle Tri-axial Rotor model code numerical calculation of level energies, B(M1) and B(E2) transition probabilities and gamma branching ratios have been made. Different input data were generated and the data which givesthe best fit with the experimental data was chosen.The output obtained during the best fit input data have been used for analysis.Finally,the result were compared with the experimental data.The PTRM code has three parts;

1. Program Gampn :Calculates the deformed single particle orbitals,their eigenval-

ues,eigenvectors and the s,p matrix elements for Asyrmo.Calculates single particle energies and modified oscillator(Nilson) Potential parameters such as(k,$\mu$ and etc).

2. Program Asyrmo :It accept input data from Gampn and number of Z, number of A,I min,I max and so on.It diagonalise the particle plus tri axial rotor hamiltonian in strong coupling basis, with the single particle matrix elements.

3. Program Probamo : Calculates both diagonal and off- diagonal matrix elements . Calculates $\frac{m1}{E2}$ matrix elements ,static moments,transition rates and mixing ratios in the particle -Rotor formalism from the energy eigenvectors supplied by Asyrmo. [17,18]

### 3.2.3 Analytic Method

The resulting output of level energies, gamma transition energies and intensity branching ratio from the particle triaxial rotor model code, had been presented in table and graphs. Finally, the results were compared with the experimental data.

# 4

# Result and Discussion

As we were discused in the literature review, the most interesting feature relevant to our PTRM calculations is the numbering among the positive parity orbitals.The quasi-crossing of the two orbitals occurs very close to the optimal value of $\gamma$ found in the PTRM calculations.As the calculations shows and as it would be discussed below, the orbitals dominate the structure of the low-lying states. The PTRM calculated low-lying states of 105In compared with experimental data obtained from ENSDF web page displayed in Table 4.1 was shown in the Table 4.2 and plotted in the level diagram Fig.4.2.

Table 4.1: Experimental value of Band(A) J=1 band based on 9/2 In-105 ground state

| $J_i$ | $E_i$ | $J_f$ | $E_f$ | $E_\gamma$ | $I_\gamma$ |
|---|---|---|---|---|---|
| $21/2_+$ | 2937.9 | 19/2+ | 2097.9 | 840 | 100 |
| $19/2_+$ | 2097.9 | 17/2+ | 1826.3 | 271.5 | 100 |
| $17/2+$ | 1826.3 | 13/2+ | 1341.63 | 484.7 | 100 |
| $13/2_+$ | 1341.63 | 11/2+ | 992 | 349.6 | 35.7 |
| $11/2_+$ | 992 | 9/2+ | 0.0 | 992 | 100 |

### 4.1.Low-lying states for $^{105}$In

The PTRM calculated low-lying states of $^{105}$In compared with experimental data obtained from ENSDF(Evaluated Nuclear Structure Data Files) web page displayed in Table 4.1 was shown in the table 4.2 and plotted in the level diagram Fig 4.2.

**Table 4.2 PTRM calculated data compared with experimental.**

| $J_i$ | $E_i$ | | $J_f$ | $E_f$ | | $E_\gamma$ | | $I_\gamma$ | |
|---|---|---|---|---|---|---|---|---|---|
| | Exp | calc | | Exp | Calc | Exp | calc | Exp | Cal |
| 21/2+ | 2937.9 | 2936.9 | 19/2+ | 2097.8 | 2096.8 | 840 | 840.1 | 100 | 100 |
| | | | 17/2+ | 1826.3 | 1826.3 | 1111.6 | 1110.6 | 100 | 100 |
| 19/2+ | 2097.8 | 2096.8 | 17/2+ | 1826.6 | 1826.6 | 271.5 | 270.5 | 100 | 100 |
| | | | 13/2+ | 1341.63 | 1341.5 | 756.27 | 755.3 | 100 | 100 |
| 17/2+ | 1826.3 | 1826.3 | 13/2+ | 1341.63 | 1341.5 | 487 | 484.8 | 100 | 100 |
| | | | 11/2+ | 992 | 992.4 | 834.3 | 833.9 | 100 | 100 |
| 13/2+ | 1341.63 | 1341.5 | 11/2+ | 992 | 992.4 | 349.6 | 349.1 | 35.7 | 35.6 |
| | | | 9/2+ | 0.0 | 0.0 | 992 | 992.4 | 100 | 100 |
| 11/2+ | 992 | 992.4 | 9/2+ | 0.0 | 0.0 | 992 | 992.4 | 100 | 100 |

### 4.1.2.Experimental Data for $^{105}$In

The values of level band of energy,gamma transition energy and intensity branching ratio of low-lying states of 105-In taken from ENSDF(Evaluated Nuclear Structure Data File) which was evaluated by Jean Blachot in 2009 and were given in the table 4.1 .The level scheme diagram of the experimental values of low-lying states of the rotational band of 105-In were plotted in the Fig 4.2.

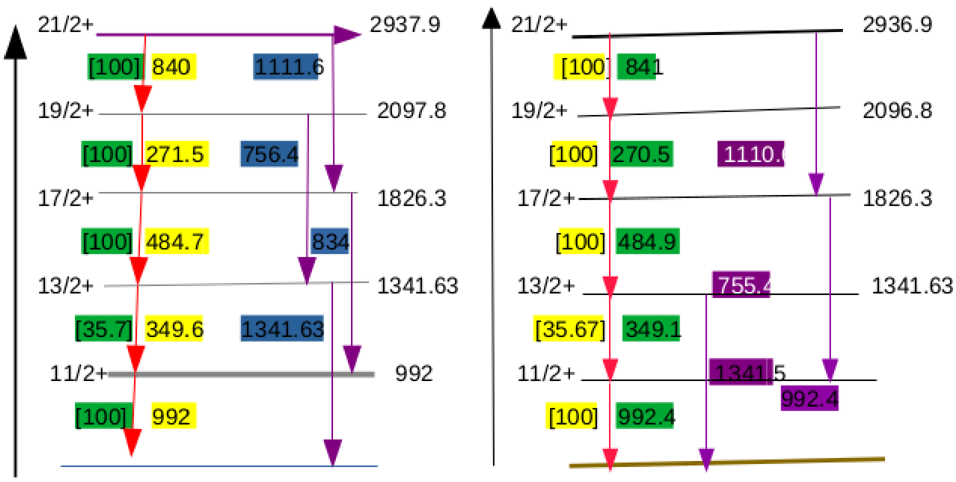
### 4.1.3. Comparison between the experimental and PTRM Data for the Isotope of In-105

For both the experimental and PT RM calculation In this work the rotational band has ground state angular momentum 9/2 and are both plus parity. This shows the PTRM calculation becomes acceptable for further analysis of the structure of the In-105.The comparison between the PTRM calculation and experimental values were agree in level energy,gamma transition energy and intensity branching ratio.

As has been mentioned,table 4.2 the PTRM calculated level energy for all the level energies in the band 9/2 ground state agree with experimental value compared to the energies in the other level.

Gamma transition energies of PTRM has the same as values to the experimentally obtained values except for the transition from level 17/2+ to 13/2+ in which PTRM calculation gives 349.1KeV and experimental energy was 349.6KeV. And the intensity branching ratio was the same.

Fig 4.2 (a) and (b) represents the energy level of an Indium-105 nucleus whose range indicates the energy levels. The lowest level called the ground state energy, represents the nucleus at rest. If raised to any of the higher levels, the nucleus immediately emits one or more characteristic gamma rays, lowering its energy sufficiently to allow it to regain ground state.




**(a) Experimental work** [7] (b) **Theoretical work**

**Fig 4.2: Energy levels of Indium $^{105}$In Scheme(plus parity)**

Table 4.2.1 indicated the comparison between experimental and PTRM calculation of level energy and gamma transition energy for 105-In.

| $J_i$ | Ei(kev) | | Jf | $E_f$ (kev) | | E (kev) | |
|---|---|---|---|---|---|---|---|
| | Exp | Calc | | Exp | Calc | Exp | Calc |
| 11/2+ | 992 | 992.4 | 9/2+ | 0.0 | 0.0 | 992 | 992.4 |
| 13/2+ | 1341.63 | 1341.5 | 11/2+ | 992 | 992.4 | 349.6 | 349.1 |
| 17/2+ | 1826.3 | 1826.3 | 13/2+ | 1341.63 | 1341.5 | 484.74 | 484.9 |
| 19/2+ | 2097.8 | 2096.9 | 17/2+ | 1826.3 | 1826.3 | 271.5 | 270.5 |
| 21/2+ | 2937.9 | 2936.9 | 19/2+ | 2097.8 | 2096.8 | 840 | 841 |

Table 4.2 .1 Comparison between the Experimental and PTRM calculation results of band of 105 positive parity from 9/2 ground state rotational In.

## 4.2 Low-lying states for $^{107}$In

The following Table 4.3 (Fig 4.3 a and b) indicates the information about level energy ,gamma transition energy and intensity branching ratio of Indium 107-In, at the parity of 9/2+ the ground state (at rest). According to the exited energy with angular momentum + 19/2 .The Energy level (E level) differences between the calculated and experiment of positive parity state of 107-In,from literature is 2004.0KeV. And the same exited state energy of 107-In, during this work is 2001.1KeV. The energy difference between the two is 2.9KeV . The experimental and theoretical values of level energy, gamma transition energy and intensity branching ratios of those states of the band are displayed in table below.

**4.2.1 Experimental Values**

According to the written by S.Lavskovi in (2008) the values of level band of energy and gamma transition energy of the low-lying states of the of 107-In were taken from ENSDF( Evaluated Nuclear Structure Data Files) which was evaluated and were given in the Table 4.3.The level scheme diagram of the experimental Values of the low-lying states of the rotational band of 107-In were plotted in the Fig . 4.3.

**4.2.2 PTRM Calculation result for 107-In**

Using a nuclear structure computational code Particle Tri-axial Rotor Model (PTRM),the level energies,gamma transition energies and intensity branching ratio of the low-lying states of the rotational band of In were calculated. The output were displayed in Table 4.3. The PTRM calculated level scheme was wrote as the above.

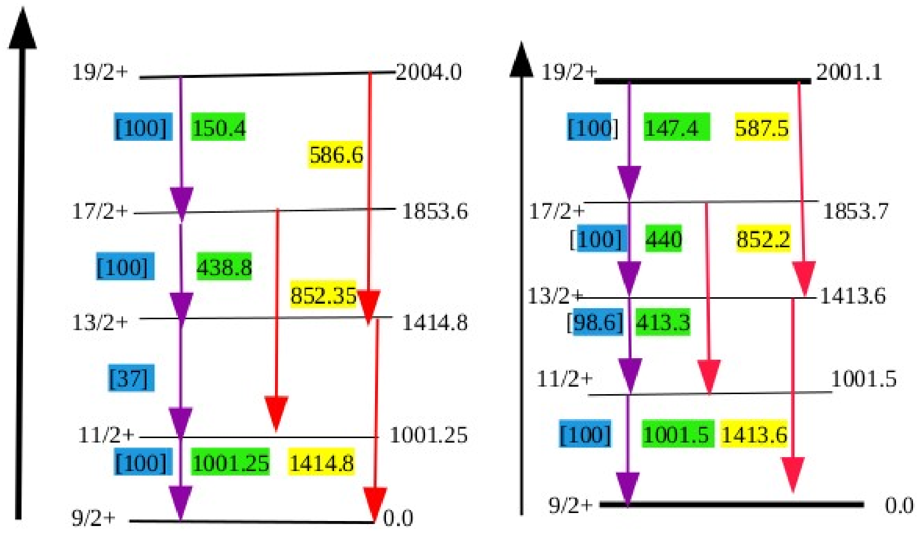
**4.2.3 Comparison between the Experimental and PTRM calculation results of 107In.**

As the above also for both the experimental and PT RM calculation In this work the rotational band has ground state angular momentum 9/2 and are both plus parity. This shows the PTRM calculation becomes acceptable for further analysis of the structure of the In-107.The comparison between the PTRM calculation and experimental values were agree in level energy,gamma transition energy and intensity branching ratio. As has been mentioned,table 4.3 the PTRM calculated level energy for all the level energies in the band 9/2 ground state agree with experimental value compared to the energies in the other level.Gamma transition energies of PTRM has the same as values to the experimentally obtained values except for the transition from level 19/2+ to 17/2+ in which PTRM calculation gives 147.4KeV and experimental energy was 150.4KeV and their intensity branching ratio were the same.

| $J_i$ | $E_i$ | | $J_f$ | $E_f$ | | $E_\gamma$ | | $I_\gamma$ | |
|---|---|---|---|---|---|---|---|---|---|
| | Exp | Calc | | Exp | Calc | Exp | Calc | Exp | Calc |
| 19/2+ | 2004.0 | 2001.1 | 17/2+ | 1853.6 | 1853.7 | 150.4 | 147.4 | 100 | 100 |
| | | | 13/2+ | 1414.8 | 1413.6 | 589.2 | 587.5 | 100 | 100 |
| 17/2+ | 1853.6 | 1853.7 | 13/2+ | 1414.8 | 1413.6 | 438.8 | 440 | 100 | 100 |
| | | | 11/2+ | 1000.25 | 1001.5 | 438.8 | 440 | 100 | 100 |
| | | | 11/2+ | 1001.25 | 1001.5 | 413.5 | 413.3 | 37 | 98.6 |
| 13/2+ | 1414.8 | 1413.6 | 9/2+ | 0.0 | 0.0 | 1414.8 | 1413.6 | 100 | 100 |
| 11/2+ | 1001.25 | 1001.5 | 9/2+ | 0.0 | 0.0 | 1001.25 | 1001.5 | 100 | 100 |

**Table 4.3 :Experimental and Theoretical values of rotational band of 107-In**

Fig 4.3 (a) and (b) represents the energy level of an Indium-107 nucleus whose range indicates the energy levels,gamma transition energy and intensity branching ratio.The lowest level called the ground state energy.




**(a) . Experimental work [7]** **(b). Theoretical work**

**Fig 4.3: Energy levels of Indium $^{107}$In Scheme(plus parity)**

Table 4.3.1 Represents the level energies and gamma transition energies between calculated and literature of 107-In.

| $J_i$ | Ei(kev) | | Jf | $E_f$(kev) | | $E_\gamma$(kev) | |
|---|---|---|---|---|---|---|---|
| | Exp | Calc | | Exp | Calc | Exp | Calc |
| 11/2+ | 1001.25 | 1001.5 | 9/2+ | 0.0 | 0.0 | 1001.25 | 1001.5 |
| 13/2+ | 1414 | 1413.6 | 11/2+ | 1001.25 | 992.4 | 349.6 | 349.1 |
| 17/2+ | 1853.6 | 1853.6 | 13/2+ | 1414.8 | 1413.6 | 413.5 | 413.3 |
| 19/2+ | 2004.0 | 2001.5 | 17/2+ | 1853.6 | 1853.6 | 150.4 | 147.4 |

**Table 4.3.1 Comparison between experimental and theoretical level energies ,gamma transition energies of 107-In.**

**4.3 Low-lying states of $^{111}$In.**

As we mentioned below$^{111}$In were identified experimentally. But for this study the low-lying states were taken low-lying rotational band,that is,band of 111-In positive parity from low-lying states of 111-In ground state . The experimental and theoretical values of level energy, gamma transition energy and intensity branching ratios of those states of the band are discused below.

| $J_i$ | $E_i$ | | $J_f$ | $E_f$ | | $E_\gamma$ | | $I_\gamma$ | |
|---|---|---|---|---|---|---|---|---|---|
| | Exp | Calc | | Exp | Calc | Exp | Calc | Exp | Calc |
| 13/2+ | 2580.85 | 2581.7 | 9/2+ | 1752.60 | 1753.6 | 828 | 828.1 | 100 | 100 |
| | | | 5/2+ | 1101.80 | 1101.50 | 1479.05 | 1480 | 100 | 100 |
| 9/2+ | 1752.60 | 1753.6 | 5/2+ | 1101.80 | 1101.50 | 651 | 652 | 100 | 100 |

**Table 4.4 :Experimental and Theoretical value of 111-In ground state.**

**4.3.1 Experimental Values**

The values of level band of energy and gamma transition energy of the low-lying states of the of 111-In were taken from ENSDF ( Evaluated Nuclear Structure Data Files) which was evaluated by Jean Blachot in 2009 and were given in the Table 4.4.The level scheme diagram of the experimental Values of the low-lying states of the rotational band of 111-In were plotted in the Fig. 4.4 .

**4.3.2 PTRM Calculation result for 111-In**

Interpreting **a** nuclear structure computational code Particle Tri-axial Rotor Model (PTRM),the level energies,gamma transition energies and intensity branching ratio of the low-lying states of the rotational band of In were calculated. The output were displayed in Table 4.4.The PTRM calculated level scheme was wrote as the above.

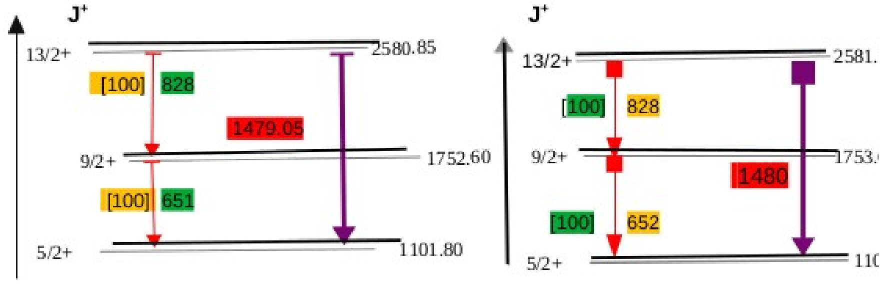
**4.3.3. Comparison between the experimental and PTRM Data for the Isotope of In-111**

For both the experimental and PT RM calculation In this work the rotational band has ground state angular momentum 9/2 and are both plus parity.This shows the PTRM calculation becomes acceptable for further analysis of the structure of the In-111.The comparison between the PTRM calculation and experimental values were agree in level energy,gamma transition energy and intensity branching ratio.

As has been mentioned,table 4.4 the PTRM calculated level energy for all the level energies in the band 9/2 ground state agree with experimental value compared to the energies in the other level.

Gamma transition energies of PTRM has the same as values to the experimentally obtained values except for the transition from level 13/2+ to 9/2+ in which PTRM calculation gives 828.1KeV and experimental energy was 828KeV. And the intensity branching ratio was the same.

Fig 4.4 indicates the energy level of Gamma decay scheme in Indium isotope 111-In. The excited nucleus can emit gamma rays with many different energies, depending on which excited and final states are involved. According to the excited energy with angular momentum and positive parity state of 111-In from literature is 2580.85KeV .The same momentum excited state energy of 111-In, during this work is 2581.7KeV with similar angular momentum and positive parity.




**Experimental work[7]** **Theoretical work**

**Fig 4.4 : Energy levels of Indium $^{111}$In Scheme(plus parity)**

Table 4.4 indicates the PTRM calculated level energy of the experimental values and the PTRM calculated level energy is nearly the same for energy level. But,at the level energy 13/2+ the experimental value 2580.85kev where the theoretical values was 2581..7kev .

| $J_i$ | Ei(kev) | | Jf | $E_f$(kev) | | $E_\gamma$(kev) | |
|---|---|---|---|---|---|---|---|
| | Exp | Calc | | Exp | Calc | Exp | Calc |
| 5/2+ | 1101.80 | 1101.50 | 9/2+ | 0.0 | 0.0 | 1101.80 | 1101.5 |
| 9/2+ | 1752.6 | 1753.6 | 5/2+ | 1101.80 | 1101.5 | 651 | 652 |
| 13/2+ | 2580.85 | 2581.7 | 9/2+ | 1752.6 | 1753.6 | 828 | 828 |

**Table :4.4.1 Comparison between Experimental and PTRM calculated values of level energy and gamma transition energy for 111-In**

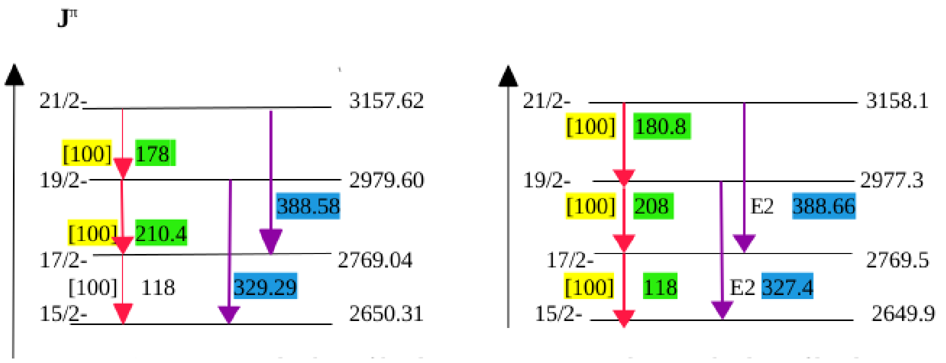
**4.4.Low-lying states negative parity of $^{111}$In**

According to the literature work negative parity of this diagram the J=21/2 state energy of 111-In were 3158KeV and theoretical values were 3157KeV.

Fig 4.5 (a) and (b) represents the energy level of an Indium-111 nucleus whose range indicates the energy levels.

| $J_i$ | $E_i$ | | $J_f$ | $E_f$ | | $E_\gamma$ | | $I_\gamma$ | |
|---|---|---|---|---|---|---|---|---|---|
| | Exp | calc | | Exp | Calc | Exp | Calc | Exp | Cal |
| 21/2- | 3157 | 3158 | 19/2- | 2979.60 | 2977.3 | 178 | 180 | 100 | 100 |
| | | | 17/2- | 2769.04 | 2769.5 | 388.58 | 388.5 | 100 | 100 |
| 19/2- | 2979.6 | 2977.3 | 17/2- | 2769 | 2769.5 | 210 | 207.8 | 100 | 100 |
| | | | 15/2- | 2650.31 | 2649.9 | 329.29 | 327.4 | 100 | 100 |
| 17/2- | 2769.04 | 2769.5 | 15/2- | 2650.31 | 2649.9 | 118 | 119 | 100 | 100 |

**Table 4.5 PTRM calculated data compared with experimental of 111-In negative parity state.**




(a) Experimental work [7] (b) Theoretical work

Fig 4.5: Energy levels of PTRM calculated data compared with experimental of 111-In negative parity state.)

# 5

# Conclusion

## 5.1 Conclusion

In this work, the results show that,the energy level of Gamma decay scheme in Indium isotope 105In.The exited nucleus can emit gamma rays with many different energies depending on which exited and final states are involved.According to the exited energy and positive parity state of 105In from literature is energy with angular momentum $21/2^{+}$ 2937.9KeV.The same exited state energy of 105In, during this work is 2936.9KeV.The energy difference between the two is 1KeV .The energy level of an Indium 107In nucleus (product of the decay of the Indium 107), whose ranges indicate the energy levels the nucleus can reach. The lowest called the ground state, represents the energy of a nucleus at rest.If raised to any of the higher levels,the nucleus immediately emits one or more characteristic gamma rays, lowering its energy sufficiently to allow it to regain the ground state.These energies are calculated by taking the difference between the (very accurately measured) departure and arrival energies.The information of Gamma information (energy level of Gamma decay) in Indium 107In, the energy level of gamma information at the parity of $9/2^{+}$ the energy level is 0.0 its ground state (at rest) .The exited nucleus can emit gamma rays with many different energies depending on which exited and final states are involved. According to the exited and the Energy level (E level) differences between energy with angular momentum $21/2^{+}$ the calculated and experiment of positive parity state of Indium 111In from literature is 2580.85 KeV.And the same exited state energy of Indium 111In, during this work is 2581.7KeV. The energy difference between the two is 1KeV .

# Bibliography


[1] Maria Goeppert Mayer. *On Closed Shells in Nuclei*. II. Phys. Rev., 75 (1949) 1969–1970.

[2] A. R. Edmonds, *Angular Momentum in Quantum* Mechanics, 3rd printing, Princeton University Press, Princeton, New Jersey (1974)

[3] Casten, R. F. (1990). *Nuclear structure from a simple perspective*. Brookhaven National Laboratory, Oxford, Oxford University Press.

[4] A. Bohr and B. R. Mottelson, *Nuclear Structure* Vol. 11, *Nuclear Deformations*, W. A. Benjamin, Inc., Reading, Massachusetts (1975) (2003).

[5] S.Lavskovi,Z.Elekes NDS 161,I(2009) 1-April-2009

[6] Jean-Louis, (2004), James Rich ,Michel Spiro *Fundamentals In Nuclear Physics,* Sprig, France

[7] Jean Blachot(2008) NDS 109,1383(2008) 1-Mar-2008

[8] John M. Blatz,(1908) Victor F. *Weisskepf Theoretical Nuclear Physics* , Springer-Verlag, Berlin.

[9] Girija. K.K.(2012) *"Studies of shape changes of deformed nuclei and its effects on cluster emission"*

[10] David Haliday.2nd.ed. Rev.ed.of *Introductory Nuclear physics,*(1955) kim Hup. Lee.printing Co.pte Ltd-Singapore

[11] A. Abbas, Mod. Phys. Lett. A 20, 2553 (2005).

[12] *Nuclear structure* vol.II, word scientific, 1999:*Nuclear structure* vol.II, word scientific, 1999

[13] R. C. Nayak and L. Satpathy, At. Data and Nucl. Data Tables 72, 213 (1999).

[14] Krane, K. S. (1988). *Introductory nuclear physics* (Rev. ed.). New York, NY: John Wiley.

[15] Verma and S.C. Gupta V.K. Mittal,R.C. Verma and S.C. Gupta. *Introduction to nuclear and particle physics fourth ed.*

[16] H. Schuler and T. Schmidt, Z.H. Schuler and T. Schmidt,

[17] Z.Phys.94(1935) 457 Physics Dept, Box 5051

[18] Nuclear. Phys. A307 (1978) 189.