Mathematical Modeling and Stability Analysis of Corruption Dynamics

A Thesis Submitted to the Department of Mathematics, College of Natural Sciences, Jimma University in Partial Fulfilment for the Requirement of the Degree of Masters of Science in Mathematics

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Declaration

Here, I submit a thesis entitled "**Mathematical Modeling and Stability Analysis of Corruption Dynamics"** for the award of degree of Master of Science in Mathematics. I, the undersigned declare that, this study is original and it has not been submitted to any institution elsewhere for the award of any academic degree or the like, where other sources of information have been used, they have been acknowledged.

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The work has been done under the supervision and approval of advisor and Co-Advisor

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Acknowledgment

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Abstract

In this thesis, Mathematical model of for corruption dynamics was developed based on compartmental approach. The thesis incorporates the following fruitful findings. Boundedness and positivity of the solution of the model were proved. The model was linearized at equilibrium point. Basic reproduction number was also calculated by using next generation matrix. The local and global stability conditions of the model were also well investigated for both disease free and endemic equilibrium points. Furthermore, sensitivity analysis of the model parameters was also carried out. Finally, in order to verify the applicability of the result MATLAB simulation was implemented and agrees with the analytical result.

Key words: Mathematical model, equilibrium point, basic reproduction number,

Local stability, global stability, sensitivity analysis.

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CHAPTER ONE

INTRODUCTION

1.1 Background of the study

Mathematical modeling has a great role in solving problems that are happened in our daily activities. This is due to describing and explaining real world phenomena researching into the key question about the observed world and testing corresponding ideas.

Mathematical modeling has always been an important activity in science and engineering. The formulation of qualitative questions about an observed phenomenon as mathematical problems was the motivation for and an integral part of the development of mathematics from the very beginning. Challenge in modeling is not to obtain the most extensive describing model but to produce the simplest model possible which includes the main features of the observed phenomenon (De vries, 2001). Real world phenomena that encompass causality between two or more variable values when modeled produce a function as a mathematical model for the studied phenomena (Takaci *et al.*, 2010). Corruption is an illegal activity carried out for private gain and benefit, by misuse of authority or power by public(government) or private (company) office holders (Bahoo, 2019). Different types of corruption are documented in the literature and include public corruption (Coervo, 2016), private corruption (Pontell, 2007), pervasive corruption, and arbitrary corruption (Egdum, 2017). Corruption can originate from either the demand side or the supply side (Heimann, 1998). In general, corruption faces a major threat to the rule of law, democracy and human rights, fairness, and social justice, hinders economic development, and brings market economies at risk for their proper and fair functioning. It is a serious problem in all countries in the world mainly for developing countries (Milnar, 2017). Even though most countries have anticorruption policies or strategies that are being made to control corruption, it remains an epidemic in the society. Mathematical modeling of such problem was investigating by different scholars. For example; the study by (Abdulrahman, 2014) proposed and analyzed a deterministic model for corruption in a population. They computed the basic reproduction number (BRN), corruption-free equilibrium point, and endemic equilibrium point. The study by (Lemecha, 2018) proposed a mathematical model for corruption by considering awareness created by anticorruption and counseling in jail.

The author modeled by using four compartment but its not contain recovery compartment. The existence of unique corruption-free and endemic equilibrium points was investigated, and the basic reproduction number was computed. The study by (Haileyesus, 2020) proposed an optimal control theory was used to study the effectiveness of all possible combinations of two corruption preventive measures, namely, (i) campaigning about corruption through media and advertisement and (ii) exposing the corrupted individuals to jail and giving punishment. The model consists of five compartments but punished compartment is ignored.

The study by (Oscar Danford *et al*, 2020) this studies intends to determine the dynamics of corruption in Tanzania. To achieve this, compartmental model, based on ordinary differential equations is presented, and solutions determined using fourth order Runge-Kutta method to find the best way to help the government in controlling corruption. The author uses four compartments but punished compartment is missed. The study by (Jaderick, 2017) investigating the evolution of corruption in the public service is studied through the dynamics of interaction between the public servants and the citizens they serve. The interaction dynamics is defined by a system of ordinary differential equations. This model consists of three compartments but punished compartment is very important compartment it does not exist.

However, there is still a room for improvement to propose a new mathematical model which involves the situation in our country. As a result, this study aims to investigate mathematical modeling and analysis of corruption dynamics based on the following schematic flow diagram:

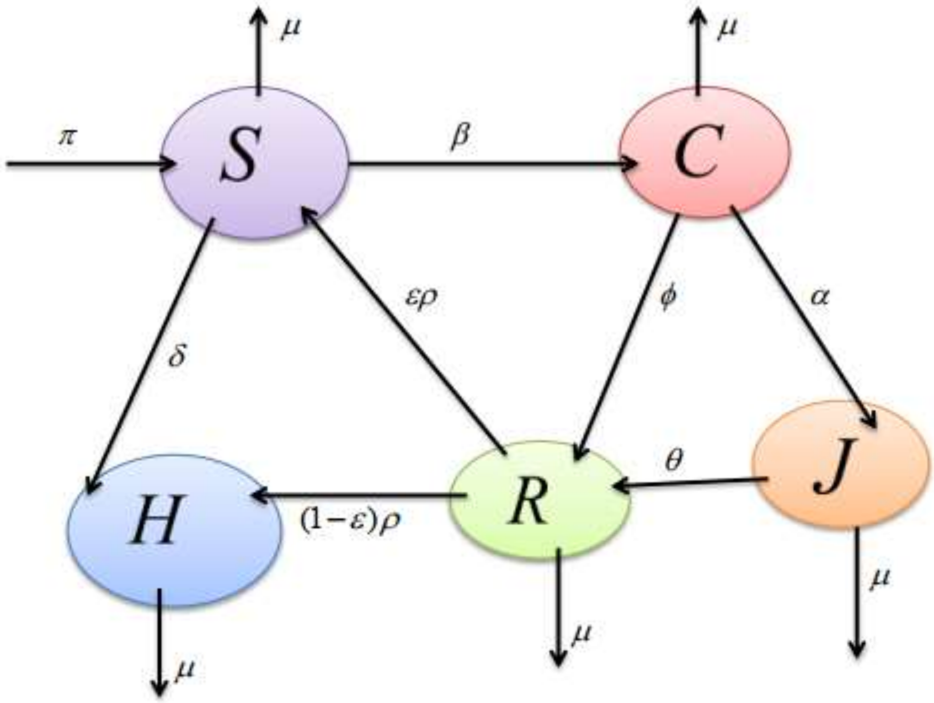

Figure 1: Compartmental flow diagram for the transmission dynamics of corruption

1.2 Statement of the problem

In recent life the way of controlling and minimizing corruption is becomes an important issues for my country. Many scholars conducted research on corruption mathematical model with control measures and obtained relevant results as pointed out in the introductory part. However, to the best knowledge of the researcher there is still a room for improvement for mathematical modeling and analysis of corruption dynamics. Therefore, this study intends to determine the dynamics of corruption in Ethiopia. Consequently, this study attempts to focus on the following problems:

- Boundedness and positivity of the solution of model,
- Local stability condition of model represented,
- Global stability condition of model,
- Sensitivity analysis of the model.

1.3 Objectives of the Study

1.3.1. General Objective

The general objective of this study is to investigate mathematical modeling and analysis of corruption dynamics which is represented by fig. (1).

1.3.2 Specific objectives

The specific objectives of the study are:

- To prove boundedness and positivity of the solution of model,
- To determine local stability condition of model represented,
- To determine global stability condition of model,
- To carryout sensitivity analysis.

1.4 Significance of the Study

This study provides the way to reduce corruption from society by recommending appropriate conditions based on parameters involved in the model.

1.5 Delimitation of the Study

This study is delimited to mathematical modeling and analysis of corruption dynamics represented by fig. (1).

CHAPTER TWO

LITERATURE REVIEW

Corruption is a social problem that persists in most organizations (Corruption Perception Index, 2018), particularly in the government where public servants are perceived with influence or power of some kind and are expected to provide some service to the citizens they serve. Transparency International adopted the technical definition by (Eicher, 2009)

Corruption in the public service results in revenue losses for the government, degradation of social justice, violation of human rights, and exploitation of vulnerable people (Verma and Sengupta, 2015).Corruption distorts the rule of law and weakens the institutional foundations on which economic growth depends (World Bank, 2014) and thus identified among the greatest obstacles to economic and social development. Corruption degrades national security, economic prosperity and international reputation. It is a major cause of poverty around the world, especially in Africa .It is the deep rooted cause of instability and conflict as witnessed in the present situation in Ethiopia. Corruption could be "characterized as disease" inherent to public power and an indication of bad governance (Tiihonen, 2003).

Mathematical modeling is the process by which we obtain and analyze the model. This process includes introducing the important and relevant quantities or variables involved in the model, making model-specific assumptions about those quantities, solving the model equations by some method, and then comparing the solutions to real data and interpreting the results. Numerous authors have over the past two decades used mathematical models to evaluate the effects of corruption on national development (Abdulrahman , 2014) A corruption prevention model was formulated by Khan (Khan, 2000) and showed that the complete prevention of corruption is possible if the ratio between rate of dismissal and rate of corruption is equal to one. (Athithan *et al*, 2018) develop an SIR model for the corruption dynamics. Furthermore, they extended the model to include optimal control with a single optimal control strategy.

They concluded that the level of corruption in society can be reduced if efforts to control corruption are increased and put in place through media/punishments. As investigation of (Blanchard *et al*,2005) the features of corruption propagation differs from classical epidemic processes due to its dependence on the threshold value of the local transition probabilities and the mean field dependence of the corruption process. A non-corrupt individual gets infected with high probability if the number of corrupt individuals in the social neighborhood exceeds a certain threshold value whereas in the case of mean field dependence an individual can get corrupt because there is a high prevalence in the society even in the absence of corruption in the local neighborhood.

CHAPTER THREE

3. METHODOLOGY

3.1 Study area and Period

The study was conducted in Jimma University under the department of mathematics from August, 2021 January, 2022 G.C.

3.2 Study design

This study employed mixed-design (documentary review design and experimental design) on mathematical modeling represented by fig. (1)

3.3 Source of Information

The relevant source of information for this study were books published articles and related studies from internet.

3.4 Mathematical Procedures

This study will be conducted based on the following procedures:

1. Proving positivity and boundedness of solution for model represented by fig. (1),
2. Determining equilibrium points of the model represented by fig. (1),
3. Linearizing the model represented by fig. (1),
4. Determining local stability condition of the model,
5. Carrying out sensitivity analysis of the model,
6. Verifying the applicability of the result using MATLAB simulation.

CHAPTER FOUR

RESULT AND DISCUSSION

4.1 Preliminaries

Definition 4.1: Consider non-linear system $\frac{dx}{dt} = f(x)$, where $f : R^n \to R^n$. A point $x^* \in R^n$ is an equilibrium point if $\frac{dx}{dt}(x^*) = f(x^*) = 0$

Definition 4.2: For a linear system $\frac{dx}{dt} = AX$ the stability of equilibrium point can be completely determined by location of eigenvalues of *A*. This is expressed as follows;

I. If the all eigenvalues of the Jacobian matrix have real parts less than zero, then the linear system is locally asymptotically stable and

II. If at least one of the eigenvalue of Jacobian matrix has real part greater than zero, then the system is unstable (Khalil, 2002).

Definition 4.3: Let x^* is an equilibrium point and a scalar function $V : D \to R$ is said to be:

1. Positive definite function if $V(x^*) = 0$ and $V(x) > 0$ for all $x \in D - \{x^*\}$
2. Negative definite function if $V(x^*) = 0$ and $V(x) < 0$ for all $x \in D - \{x^*\}$

Theorem 4.1: Lyapunov Stability Theorem (Khalil, 2002)

Let $x = x^*$ be an equilibrium point of non-linear system of $\frac{dx}{dt} = f(x)$, $f : D \to R^n$.

Suppose $V : D \to R$ be continuously differentiable function such that:-

I. $V(x^*) = 0$

II. $V(x) > 0$ for all $x \in D - \{x^*\}$

III. $\frac{dV(x)}{dt} \le 0$ for all $x \in D - \{x^*\}$ (Domain D excluding x^*).Then $x = x^*$ is stable.

Theorem 4.2: (Globally asymptotically stable)

Let $x = x^*$ be an equilibrium point of non-linear system of $\frac{dx}{dt} = f(x)$, $f : D \to R^n$.

Let $V : D \to R$ be continuously differentiable function such that:-

1. $V(x^*) = 0$
2. $V(x) > 0$ for all $x \in D - \{x^*\}$ (Domain D excluding x^*)
3. $\frac{dV(x)}{dt} < 0$ for all $x \in D - \{x^*\}$ (Domain D excluding x^*)

Then $x = x^*$ is globally asymptotically stable.

Theorem 4.3: (Gronwall Inequality) (Perko, 2000)

Let $x(t)$ be a function that is satisfying the following differential inequality

$\frac{dx}{dt} \le ax + b;\ x(0) = x_0$ where a, b are constants. Then, for all $t \ge 0$ we have:

$$x(t) \le x_0 e^{at} + \frac{b}{a}(e^{at} - 1);\ a \ne 0 \text{ and } x(t) \le x_0 + bt;\ a = 0$$

Theorem 4.4: LaSalle invariance princip**le (LaSalle, 1976)**

Suppose that $x^* = 0$ is an equilibrium point of system an autonomous dynamical system, and V is a Lyapunov function on some neighborhood U of $x^* = 0$. If $x_0 \in U$ has its forward trajectory bounded with limit points in U, and M is the largest invariant set of $E = \{x^* \in U : V(x^*) = 0\}$, then the solution $\phi(x_0, t) \to M$ as $t \to \infty$.

4.2 Formulation of Mathematical Model

The total population is divided into five sub-populations: Susceptible individuals (denoted by S) are those who are not corrupted but there is a possibility to be corrupted. Corrupted individuals (denoted by C) are those who are committing corruption. After some period of time those corrupted individuals passes to the jailed compartment (denoted by J) are those taken to prison for corrective measures. Recovered individuals (denoted by R) are those which recovered from the disease. Honest individuals are denoted by H are those who never involved in corruption activities or decided to quit committing corruption after corrective measures. From schematic flow diagram in Figure (1), we have the following model.

$$\left.\begin{aligned}
\frac{dS}{dt} &= \pi - \beta SC - (\delta + \mu)S + \varepsilon\rho R \\
\frac{dC}{dt} &= \beta SC - (\alpha + \phi + \mu)C \\
\frac{dJ}{dt} &= \alpha C - (\theta + d_1 + \mu)J \\
\frac{dR}{dt} &= \phi C + \theta J - (\varepsilon + \mu)R \\
\frac{dH}{dt} &= \delta S + (1 - \rho)\varepsilon R - \mu H
\end{aligned}\right\} \tag{4.1}$$

Subjected to initial conditions

$$S(0) = S_0 > 0\,,\ \ C(0) = C_0 > 0\,,\ \ J(0) = J_0 > 0\,,\ \ R(0) = R_0 > 0\,,\ H(0) = H_0 > 0 \tag{4.2}$$

Table 1: Description of parameters of the model (1)

Parameters	Description
π	Influx rate
ε	The rate at which recovered individual becomes Susceptible individual
β	Contact rate of susceptible individuals
α	Punishment rate of corrupted individuals
ρ	Proportion of recovered individuals who joins Susceptible individuals
μ	Natural mortality rate
θ	The rate at which jailed individuals becomes recovered individuals
d_1	disease induced death rate of corrupted individuals in prison
δ	The rate at which Susceptible individuals becomes Honest individuals
ϕ	The rate at which Corrupted individuals becomes recovered individuals

4.3 Positivity of the Solutions of the Model

Since the state variables involved in model (4.1) represents human population, it is needful to show that all the state variables are also positive for all time $t \geq 0$.

Theorem 1: All the state variables $S(t), C(t), J(t), H(t)$ and $R(t)$ of model (4.1) subjected to initial conditions (4.2) remain positive for all time $t \geq 0$.

Proof: From the first equation of Eq. (4.1),

$$\frac{dS}{dt} = \pi - \beta SC - (\delta + \mu)S + \varepsilon\rho R$$

$$\frac{dS}{dt} \geq -(\beta C + (\delta + \mu))S$$

$$S(t) \geq S_0 e^{\int -(\beta C + (\delta + \mu))dt} > 0 \tag{4.3}$$

From the second equation of Eq. (4.1),

$$\frac{dC}{dt} = \beta SC - (\alpha + \mu)C$$

$$\frac{dC}{dt} \geq -(\alpha + \mu)C$$

$$C(t) \geq C_0 e^{-(\alpha+\phi+\mu)t} > 0 \tag{4.4}$$

From the third equation of Eq. (4.1),

$$\frac{dJ}{dt} = \alpha C - (\theta + d_1 + \mu)J$$

$$\frac{dJ}{dt} \geq -(\theta + d_1 + \mu)J$$

$$J(t) \geq J_0 e^{-(\theta+d_1+\mu)t} > 0 \tag{4.5}$$

From the fourth equation of Eq. (4.1),

$$\frac{dR}{dt} = \phi C + \theta J - (\varepsilon + \mu)R$$

$$\frac{dR}{dt} \geq -(\varepsilon + \mu)R$$

$$R(t) \geq R_0 e^{-(\varepsilon+\mu)t} > 0 \tag{4.6}$$

From the last equation of Eq. (4.1),

$$\frac{dH}{dt} = \delta S + (1-\rho)\varepsilon R - \mu H$$

$$\frac{dH}{dt} \geq -\mu H$$

$$H(t) \geq H_0 e^{-\mu t} > 0 \tag{4.7}$$

From Eqs. (4.3)- (4.7), all the state variables are positive and hence the theorem is proved.

4.4 Boundedness of the Solutions of the Model

Theorem 2: All the solution of $S(t), C(t), J(t), H(t)$ and $R(t)$ of model (4.1) which initiate in R_+^5 are uniformly bounded.

Proof: Let $\left(S(t), C(t), J(t), H(t), R(t)\right)$ be positive solution of the model (4.1) with initial condition (4.2).

Let $W(t) = S(t) + C(t) + J(t) + R(t) + H(t)$,

$$\frac{dW}{dt} = \frac{dS}{dt} + \frac{dC}{dt} + \frac{dJ}{dt} + \frac{dR}{dt} + \frac{dH}{dt}$$

After some mathematical simplification,

$$\frac{dW}{dt} < \pi - \mu W(t) \tag{4.8}$$

Applying Gronwall's inequality on Eq. (4.8),

$$0 < W(t) \le \frac{\pi}{\mu}\left(1 - e^{-\mu t}\right) + W(0)\, e^{-\mu t}$$

Applying limit sup as $t \to \infty$, $\lim_{t\to\infty} S\,\mathrm{up} W(t) < \lim_{t\to\infty} S\,\mathrm{up} \frac{\pi}{\mu}\left(1 - e^{-\mu t}\right) + \lim_{t\to\infty} S\,\mathrm{up}\, w(0)\, e^{-\mu t}$

$$0 < W(t) < \frac{\pi}{\mu} \tag{4.9}$$

As a result, all the solutions of model (4.1) that initiated in R_+^5 are attracted to the region

$$\Omega = \left\{ (S(t), E(t), I(t), R(t), C(t)) \in R_+^5 : W(t) = S(t) + C(t) + J(t) + R(t) + H(t) < \frac{\pi}{\mu} \right\}$$

Which is the feasible solution set for the model (4.1) and all the solution set is uniformly bounded in it and hence the theorem is proved.

4.5 Equilibrium points of the Model

To find equilibrium points,

$$\frac{ds}{dt} = \frac{dC}{dt} = \frac{dJ}{dt} = \frac{dR}{dt} = \frac{dH}{dt} = 0$$

$$\left.\begin{aligned} &\pi - \beta SC - (\delta + \mu)S + \varepsilon\rho R = 0 \\ &\beta SC - (\alpha + \phi + \mu)C = 0 \\ &\alpha C - (\theta + d_1 + \mu)J = 0 \\ &\phi C + \theta J - (\varepsilon + \mu)R = 0 \\ &\delta S + (1-\varepsilon)\rho R - \mu H = 0 \end{aligned}\right\} \tag{4.10}$$

From the second equation of Eq. (4.10),

$$C = 0 \text{ or } S = \frac{\alpha + \phi + \mu}{\beta}$$

For $C = 0$, the following equilibrium point is obtained

$$E_0 = \left(\frac{\pi}{\delta + \mu}, 0, 0, 0, \frac{\delta \pi}{\mu(\delta + \mu)} \right)$$

This equilibrium point is called corruption free equilibrium point (CFEP).

For $S = \dfrac{\alpha + \phi + \mu}{\beta}$, after some mathematical manipulation the following are obtained.

Endemic equilibrium point: $E_1 = \left(S^*, C^*, J^*, R^*, H^* \right)$,

where $$S^* = \frac{\alpha + \phi + \mu}{\beta},$$

$$C^* = \frac{(\beta\pi - (\delta + \mu)(\alpha + \phi + \mu))(\varepsilon + \mu)(\theta + d_1 + \mu)}{\beta\left[(\varepsilon + \mu)(\theta + d_1 + \mu)(\alpha + \phi + \mu) - \alpha\rho\varepsilon\theta - \varepsilon\rho\phi(\theta + d_1 + \mu)\right]},$$

$$J^* = \frac{\alpha}{(\theta + d_1 + \mu)} C^*$$

$$R^* = \frac{(\alpha\theta + \phi(\theta + d_1 + \mu))}{(\varepsilon + \mu)(\theta + d_1 + \mu)} C^*$$

$$H^* = \frac{\delta}{\mu\beta}(\alpha + \phi + \mu) + \frac{\varepsilon}{\mu}\left(\frac{(1-\rho)\alpha\theta + (1-\rho)\phi(\theta + d_1 + \mu)}{(\varepsilon + \mu)(\theta + d_1 + \mu)} \right) C^*$$

Theorem 3: The endemic equilibrium point is positive if the following condition is satisfied.

$$\left.\begin{aligned} &(i) \;\; \beta\pi > (\delta + \mu)(\alpha + \phi + \mu) \\ &(ii) \; (\varepsilon + \mu)(\theta + d_1 + \mu)(\alpha + \phi + \mu) > \alpha\rho\varepsilon\theta + \varepsilon\rho\phi(\theta + d_1 + \mu) \end{aligned}\right\} \qquad (4.11)$$

Proof: Since all parameters of the model are positive, $S^* = \dfrac{\alpha + \phi + \mu}{\beta}$, $S^* > 0$

$$C^* > 0, \quad C^* = \frac{(\beta\pi - (\delta+\mu)(\alpha+\phi+\mu))(\varepsilon+\mu)(\theta+d_1+\mu)}{\beta\left[(\varepsilon+\mu)(\theta+d_1+\mu)(\alpha+\phi+\mu) - \alpha\rho\varepsilon\theta - \varepsilon\rho\phi(\theta+d_1+\mu)\right]} > 0$$

$$\left.\begin{array}{l} (i)\ \ \beta\pi > (\delta+\mu)(\alpha+\phi+\mu) \\ (ii)\ (\varepsilon+\mu)(\theta+d_1+\mu)(\alpha+\phi+\mu) > \alpha\rho\varepsilon\theta + \varepsilon\rho\phi(\theta+d_1+\mu) \end{array}\right\}$$

If C^* is positive, then J^*, R^* and H^* are positive. Hence, the proof is completed.

4.6 Basic Reproduction Number

The basic reproduction number, denoted by R_0, is defined as number of secondary infections appears from one infected individual. The basic reproduction number of the system is calculated by applying the next generation matrix method. The next-generation matrix is used to derive the basic reproduction number, for a compartmental model of the spread of infectious diseases. The infected subsystem of the model is:

$$\left.\begin{array}{l} \dfrac{dC}{dt} = \beta SC - (\alpha+\phi+\mu)C \\ \dfrac{dJ}{dt} = \alpha C - (\theta+d_1+\mu)J \end{array}\right\} \tag{4.12}$$

$$T = \begin{pmatrix} \dfrac{\beta\pi}{\delta+\mu} & 0 \\ 0 & 0 \end{pmatrix}, \qquad V = \begin{pmatrix} \alpha+\phi+\mu & 0 \\ -\alpha & \theta+d_1+\mu \end{pmatrix}$$

T represents transmission matrix and V represents transition matrix.

$$V^{-1} = \begin{pmatrix} \dfrac{1}{\alpha+\phi+\mu} & 0 \\ \dfrac{\alpha}{(\alpha+\phi+\mu)(\theta+d_1+\mu)} & \dfrac{1}{\theta+d_1+\mu} \end{pmatrix}$$

$$TV^{-1} = \begin{pmatrix} \dfrac{\beta\pi}{(\alpha+\phi+\mu)(\delta+\mu)} & 0 \\ 0 & 0 \end{pmatrix}$$

The spectral radius of TV^{-1} is $\frac{\beta\pi}{(\alpha+\phi+\mu)(\delta+\mu)}$.

This spectral radius is called basic reproduction number. As a result,

$$R_0 = \frac{\beta\pi}{(\alpha+\phi+\mu)(\delta+\mu)}$$

4.7 Local Stability Analysis CFEP

Theorem 4: The corruption free equilibrium point $E_0 = \left(\frac{\pi}{\delta+\mu}, 0, 0, 0, \frac{\delta\pi}{\mu(\delta+\mu)}\right)$ is locally asymptotically stable if and only if $R_0 < 1$.

Proof: The Jacobian matrix of the model evaluated at corruption free equilibrium point is:

$$J = \begin{pmatrix} -(\delta+\mu) & \frac{-\beta\pi}{\delta+\mu} & 0 & \varepsilon\rho & 0 \\ 0 & \frac{\beta\pi-(\alpha+\phi+\mu)(\delta+\mu)}{(\delta+\mu)} & 0 & 0 & 0 \\ 0 & \alpha & -(\theta+d_1+\mu) & 0 & 0 \\ 0 & \phi & \theta & -(\varepsilon+\mu) & 0 \\ \delta & 0 & 0 & (1-\rho)\varepsilon & -\mu \end{pmatrix}$$

The eigenvalues of J are:

$$\lambda_1 = \frac{\beta\pi-(\alpha+\phi+\mu)(\delta+\mu)}{(\delta+\mu)} \quad , \quad \lambda_2 = -(\delta+\mu) \quad , \quad \lambda_3 = -(\theta+d_1+\mu) \quad ,$$
$$\lambda_4 = -(\varepsilon+\mu) \quad , \quad \lambda_5 = -\mu$$

Since all parameters of the model are positive, $\lambda_2, \lambda_3, \lambda_4, \lambda_5$ are all negative.

For $\lambda_1 < 0 \Rightarrow \frac{\beta\pi-(\alpha+\phi+\mu)(\delta+\mu)}{(\delta+\mu)} < 0 \Rightarrow R_0 < 1$. Hence, the proof completed.

4.8 Local Stability Analysis EEP

Theorem 5: The endemic equilibrium point $E_1 = \left(S^*, C^*, J^*, R^*, H^*\right)$ is locally asymptotically stable if and only if the following conditions are satisfied.

$$\left.\begin{array}{l}(i)\ \ R_0 > 1 \\ (ii)\ \ (\varepsilon+\mu)(\theta+d_1+\mu)(\alpha+\phi+\mu) > \alpha\rho\varepsilon\theta + \varepsilon\rho\phi(\theta+d_1+\mu) \\ (iii)\ \alpha_1\alpha_2 - \alpha_3 > 0 \\ (iV)\ \alpha_3(\alpha_1\alpha_2-\alpha_3)+\alpha_1(\alpha_5-\alpha_1\alpha_4) > 0 \\ (v)\ \ \alpha_3(\alpha_1\alpha_2-\alpha_3)(\alpha_1\alpha_4-\alpha_5)-\alpha_1(\alpha_5-\alpha_1\alpha_4)^2-\alpha_5(\alpha_3-\alpha_1\alpha_2) > 0\end{array}\right\} \quad (4.14)$$

Proof: The Jacobian matrix of the model evaluated at endemic equilibrium point is:

$$J = \begin{pmatrix} -\beta C^* - (\delta+\mu) & -(\alpha+\phi+\mu) & 0 & \varepsilon\rho & 0 \\ \beta C^* & 0 & 0 & 0 & 0 \\ 0 & \alpha & -(\theta+d_1+\mu) & 0 & 0 \\ 0 & \phi & \theta & -(\varepsilon+\mu) & 0 \\ \delta & 0 & 0 & (1-\rho)\varepsilon & -\mu \end{pmatrix}$$

The characteristic equation of the Jacobian matrix is given by:

$$\lambda^5 + \alpha_1\lambda^4 + \alpha_2\lambda^3 + \alpha_3\lambda^2 + \alpha_4\lambda + \alpha_5 = 0 \quad , \quad (4.13)$$

where $\alpha_1 = \beta C^* + \delta + 4\mu + \theta + d_1 + \varepsilon$

$$\alpha_2 = \beta C^*(\alpha+\phi+\mu) + (\beta C^*+\delta+\mu)(\theta+d_1+\mu) + (\beta C^*+\delta+\mu)(\varepsilon+\mu)$$
$$\mu(\beta C^*+\delta+\mu) + (\theta+d_1+\mu)(\varepsilon+\mu) + \mu(\theta+d_1+\mu) + \mu(\varepsilon+\mu)$$

$$\alpha_3 = \beta C^*\alpha(\alpha+\phi+\mu) + \beta C^*(\alpha+\phi+\mu)(\varepsilon+\mu) + \beta C^*\mu(\alpha+\phi+\mu) + (\beta C^*+\delta+\mu)(\theta+d_1+\mu)(\varepsilon+\mu)$$
$$+\mu(\beta C^*+\delta+\mu)(\varepsilon+\mu) + \mu(\beta C^*+\delta+\mu)(\theta+d_1+\mu) + \mu(\theta+d_1+\mu)(\varepsilon+\mu) - \beta C^*\phi\varepsilon\rho$$

$$\alpha_4 = \beta C^*(\alpha+\phi+\mu)(\theta+d_1+\mu)(\varepsilon+\mu) + \beta C^*\mu(\alpha+\phi+\mu)(\theta+d_1+\mu)$$
$$\beta C^*\mu(\alpha+\phi+\mu)(\varepsilon+\mu) + \beta C^*\mu(\alpha+\phi+\mu)(\varepsilon+\mu) + \mu(\beta C^*+\delta+\mu)(\theta+d_1+\mu)(\varepsilon+\mu)$$
$$-\beta C^*\varepsilon\rho\left(\alpha\theta + \phi(\theta+d_1+\mu) + \phi\mu\right)$$

$$\alpha_5 = \beta C^*\left((\alpha+\phi+\mu)(\theta+d_1+\mu)(\varepsilon+\mu) - \varepsilon\rho(\alpha\theta+\phi(\theta+d_1+\mu))\right)$$

Remark: All the coefficients characteristic equation given by Eq. (4.13) are positive.

Proof: Since all parameters of the model are positive and condition given by Eq. (4.11), α_1 and α_2 are positive. Since all parameters of the model are positive, condition given by Eq. (4.11) and $1-\rho>0$ α_3 and α_4 are positive. Since all parameters of the model are positive and condition given by Eq. (4.11), α_5 is positive.

Finding eigenvalues of characteristic equation given by Eq. (4.13) is not straightforward. Hence, talking about local stability conditions requires another technique called Routh-Hurwitz stability criterion. Applying this technique on characteristic equation given Eq. (4.13):

$$
\begin{array}{c|ccc}
\lambda^5 & 1 & \alpha_2 & \alpha_4 \\
\lambda^4 & \alpha_1 & \alpha_3 & \alpha_5 \\
\lambda^3 & b_1 & b_2 & b_3 \\
\lambda^2 & c_1 & c_2 & c_3 \\
\lambda & d_1 & d_2 & \\
\lambda^0 & g_1 & &
\end{array}
$$

where $b_1=\dfrac{\alpha_1\alpha_2-\alpha_3}{\alpha_1}$, $b_2=\dfrac{\alpha_1\alpha_4-\alpha_5}{\alpha_1}$,

$$c_1=\frac{\alpha_3(\alpha_1\alpha_2-\alpha_3)+\alpha_1(\alpha_5-\alpha_1\alpha_4)}{(\alpha_1\alpha_2-\alpha_3)},\ c_2=\alpha_5$$

$$d_1=\frac{(\alpha_3(\alpha_1\alpha_2-\alpha_3)(\alpha_1\alpha_4-\alpha_5)-\alpha_1(\alpha_5-\alpha_1\alpha_4)^2-\alpha_5(\alpha_3-\alpha_1\alpha_2)^2)(\alpha_3(\alpha_1\alpha_2-\alpha_3)+\alpha_1(\alpha_5-\alpha_1\alpha_4))}{\alpha_1(\alpha_3(\alpha_1\alpha_2-\alpha_3)+\alpha_1(\alpha_5-\alpha_1\alpha_4))}$$

$$g_1=\alpha_5$$

As a result, when conditions $(iii)-(v)$ are satisfied, the first column of Routh array have the same sign. Hence, the theorem proved by Routh-Hurwitz stability criterion.

4.9 Global Stability Analysis of CFEP

Theorem 6: The CFEP given by $E_0=\left(\dfrac{\pi}{\delta+\mu},0,0,0,\dfrac{\delta\pi}{\mu(\delta+\mu)}\right)$ is globally asymptotically stable when the following three conditions are satisfied.

$$\begin{cases} (i)\ \delta < 2\mu \\ (ii)\ \delta S < 6\mu \\ (ii)\ R_0 < \dfrac{\mu}{\alpha+\phi+\mu} \end{cases} \tag{4.15}$$

Proof: Consider the following Lyapunov function

$$V(S,C,J,R,H) = \left(S - S^* - S^*\ell n\frac{S}{S^*}\right) + C + J + R + \left(H - H^* - H^*\ell n\frac{H}{H^*}\right)$$

$$(i)\ V(S,C,J,R,H) > 0$$
$$(ii)\ V(S^*,0,0,0,H^*) = 0$$

Differentiating V with respect to t along the solution of Eq. (4.1) gives

$$\frac{dV}{dt} = \left(\frac{S-S^*}{S}\right)\frac{dS}{dt} + \frac{dC}{dt} + \frac{dJ}{dt} + \frac{dR}{dt} + \left(\frac{H-H^*}{H}\right)\frac{dH}{dt}$$

$$\frac{dV}{dt} = \left(\frac{S-S^*}{S}\right)\left(\pi - \beta SC - (\delta+\mu)S + \varepsilon\rho R\right) + \left(\beta SC - (\alpha+\phi+\mu)C\right) + \left(\alpha C - (\theta + d_1 + \mu)J\right)$$
$$+\left(\phi C + \theta J - (\varepsilon+\mu)R\right) + \left(\frac{H-H^*}{H}\right)\left(\delta S + (1-\rho)\varepsilon R - \mu H\right)$$

$$\frac{dV}{dt} = \left(\frac{S-S^*}{S}\right)\left(\pi - \beta SC - (\delta+\mu)(S-S^*) - (\delta+\mu)S^* + \varepsilon\rho R\right) + \left(\beta SC - (\alpha+\phi+\mu)C\right)$$
$$+\left(\alpha C - (\theta + d_1 + \mu)J\right) + \left(\phi C + \theta J - (\varepsilon+\mu)R\right) + \left(\frac{H-H^*}{H}\right)\left(\delta(S-S^*) + (1-\rho)\varepsilon(R-R^*) - \mu(H-H^*)\right)$$

$$\frac{dV}{dt} = \left(1 - \frac{S^*}{S}\right)\left(-\beta SC + \varepsilon\rho R\right) - \frac{(\delta+\mu)}{S}(S-S^*)^2 + \left(\beta SC - (\alpha+\phi+\mu)C\right)$$
$$+\left(\alpha C - (\theta+d_1+\mu)J\right) + \left(\phi C + \theta J - (\varepsilon+\mu)R\right) + \frac{\delta}{H}(S-S^*)(H-H^*) + (1-\rho)\varepsilon(R-R^*)\left(1-\frac{H^*}{H}\right) - \frac{\mu}{H}(H-H^*)^2$$

$$\frac{dV}{dt} = -\beta SC + \varepsilon\rho R + \beta CS^* - \frac{\varepsilon\rho}{S}S^*R - \frac{(\delta+\mu)}{S}(S-S^*)^2 + \beta SC - (\alpha+\phi+\mu)C$$
$$+\left(\alpha C - (\theta+d_1+\mu)J\right) + \left(\phi C + \theta J - (\varepsilon+\mu)R\right) + \frac{\delta}{H}(S-S^*)(H-H^*)$$
$$+(1-\rho)\varepsilon R - (1-\rho)\varepsilon R^* - \frac{(1-\rho)\varepsilon}{H}H^*R + \frac{(1-\rho)\varepsilon}{H}H^*R^* - \frac{\mu}{H}(H-H^*)^2$$

$$\frac{dV}{dt} = \beta CS^* - \frac{\varepsilon\rho}{S}S^*R - \frac{(\delta+\mu)}{S}(S-S^*)^2 - \mu(C+J+R) - d_1J + \frac{\delta}{H}(S-S^*)(H-H^*) - (1-\rho)\varepsilon R^*$$
$$-\frac{(1-\rho)\varepsilon}{H}H^*R + \frac{(1-\rho)\varepsilon}{H}H^*R^* - \frac{\mu}{H}(H-H^*)^2$$

$$\frac{dV}{dt} = (\beta CS^* - \mu C) - \frac{\varepsilon\rho}{S}S^*R - \frac{(\delta+\mu)}{S}(S-S^*)^2 - \mu(J+R) - d_1J + \frac{\delta}{H}(S-S^*)(H-H^*)$$
$$-\frac{(1-\rho)\varepsilon}{H}H^*R - \frac{\mu}{H}(H-H^*)^2$$

$$\frac{dV}{dt} = C(\frac{\beta\pi}{\delta+\mu} - \mu) - \frac{\varepsilon\rho}{S}S^*R - \frac{(\delta+\mu)}{S}(S-S^*)^2 - \mu(J+R) - d_1J + \frac{\delta}{H}(S-S^*)(H-H^*)$$
$$-\frac{(1-\rho)\varepsilon}{H}H^*R - \frac{\mu}{H}(H-H^*)^2$$

$$\frac{dV}{dt} \le C(\frac{\beta\pi}{\delta+\mu} - \mu) - \frac{\varepsilon\rho}{S}S^*R - \frac{(\delta+\mu)}{S}(S-S^*)^2 - \mu(J+R) - d_1J + \frac{\delta}{2H}(S-S^*)^2 + \frac{\delta}{2H}(H-H^*)^2$$
$$-\frac{(1-\rho)\varepsilon}{H}H^*R - \frac{\mu}{H}(H-H^*)^2$$

$$\frac{dV}{dt} \le -(\mu - \frac{\beta\pi}{\delta+\mu})C - \frac{\varepsilon\rho}{S}S^*R - \frac{1}{2HS}(2(\delta+\mu) - \delta S)(S-S^*)^2 - \frac{1}{2H}(2\mu-\delta)(H-H^*)^2$$
$$-\frac{(1-\rho)\varepsilon}{H}H^*R - \mu(J+R) - d_1J$$

When the three conditions given by Eq. (4.15) are satisfied $\frac{dV}{dt} < 0$.

$\frac{dV}{dt} = 0$, when $(S, C, J, R, H) = (S^*, 0, 0, 0, H^*)$

This fact indicates that the largest invariant set where $\frac{dV}{dt} = 0$ is the singleton $E_0 = (S^*, 0, 0, 0, H^*)$. Thus, by *LaSalle's* invariance principle, the CFEP is globally asymptotically stable.

4.10 Global Stability Analysis of EEP

Theorem 7: The EEP given by $E_1 = (S^*, C^*, J^*, R^*, H^*)$ is globally asymptotically stable when the following three conditions are satisfied.

$$\begin{cases} (i)\ \left(\dfrac{\alpha}{J}(C-C^*)\right)^2 + \dfrac{4\varepsilon\rho(\theta+d_1+\mu)(S-S^*)(R-R^*)}{JS} < 0 \\ (ii)\ \left(\dfrac{\theta}{R}(J-J^*)\right)^2 + \dfrac{4\delta(\varepsilon+\mu)(S-S^*)(H-H^*)}{RH} < 0 \\ (ii)\ \left(\dfrac{(1-\rho)\varepsilon}{R}(R-R^*)\right)^2 + \dfrac{4\mu\phi(C-C^*)(R-R^*)}{RH} < 0 \end{cases} \tag{4.16}$$

Proof: Consider the following Lyapunov function

$$V(S,C,J,R,H) = \int_{S^*}^{S}\left(\frac{x-S^*}{x}\right)dx + \int_{C^*}^{C}\left(\frac{x-C^*}{x}\right)dx + \int_{J^*}^{J}\left(\frac{x-J^*}{x}\right)dx + \int_{R^*}^{R}\left(\frac{x-R^*}{x}\right)dx + \int_{H^*}^{H}\left(\frac{x-H^*}{x}\right)dx$$

$(i)\ V(S,C,J,R,H) > 0$

$(ii)\ V(S^*,C^*,J^*,R^*,H^*) = 0$

Differentiating V with respect to t along the solution of Eq. (4.1) gives

$$\frac{dV}{dt} = \left(\frac{S-S^*}{S}\right)\frac{dS}{dt} + \left(\frac{C-C^*}{C}\right)\frac{dC}{dt} + \left(\frac{J-J^*}{J}\right)\frac{dJ}{dt} + \left(\frac{R-R^*}{R}\right)\frac{dR}{dt} + \left(\frac{H-H^*}{H}\right)\frac{dH}{dt}$$

$$\begin{aligned}\frac{dV}{dt} &= \left(\frac{S-S^*}{S}\right)\left(\pi - \beta SC - (\delta+\mu)S + \varepsilon\rho R\right) + \left(\frac{C-C^*}{C}\right)\left(\beta SC - (\alpha+\phi+\mu)C\right) \\ &\quad + \left(\frac{J-J^*}{J}\right)\left(\alpha C - (\theta+d_1+\mu)J\right) + \left(\frac{R-R^*}{R}\right)\left(\phi C + \theta J - (\varepsilon+\mu)R\right) \\ &\quad + \left(\frac{H-H^*}{H}\right)\left(\delta S + (1-\rho)\varepsilon R - \mu H\right)\end{aligned}$$

$$\frac{dV}{dt}=\left(\frac{S-S^*}{S}\right)\left(\pi-\beta S(C-C^*)-BSC^*-(\delta+\mu)(S-S^*)-(\delta+\mu)S^*+\varepsilon\rho(R-R^*)+\varepsilon\rho R^*\right)$$
$$+\left(\frac{C-C^*}{C}\right)\left(\beta C(S-S^*)+\beta CS^*-(\alpha+\phi+\mu)(C-C^*)-(\alpha+\phi+\mu)C^*\right)$$
$$+\left(\frac{J-J^*}{J}\right)\left(\alpha(C-C^*)+\alpha C^*-(\theta+d_1+\mu)(J-J^*)-(\theta+d_1+\mu)J^*\right)$$
$$+\left(\frac{R-R^*}{R}\right)\left(\phi(C-C^*)+\theta(J-J^*)-(\varepsilon+\mu)(R-R^*)\right)$$
$$+\left(\frac{H-H^*}{H}\right)\left(\delta(S-S^*)+(1-\rho)\varepsilon(R-R^*)-\mu(H-H^*)\right)$$

$$\frac{dV}{dt}=\left(\frac{S-S^*}{S}\right)\left(\pi-\beta S(C-C^*)-\beta C^*(S-S^*)-\beta S^*C^*-(\delta+\mu)(S-S^*)-(\delta+\mu)S^*+\varepsilon\rho(R-R^*)+\varepsilon\rho R^*\right)$$
$$+\left(\frac{C-C^*}{C}\right)\left(\beta C(S-S^*)+\beta S^*(C-C^*)+\beta S^*C^*-(\alpha+\phi+\mu)(C-C^*)-(\alpha+\phi+\mu)C^*\right)$$
$$+\left(\frac{\alpha}{J}\right)(C-C^*)(J-J^*)-\frac{(\theta+d_1+\mu)}{J}(J-J^*)^2+\frac{\phi}{R}(C-C^*)(R-R^*)+\frac{\theta}{R}(J-J^*)(R-R^*)$$
$$-\frac{(\varepsilon+\mu)}{R}(R-R^*)^2+\left(\frac{\delta}{H}\right)(S-S^*)(H-H^*)$$
$$+\left(\frac{(1-\rho)\varepsilon}{H}\right)(R-R^*)(H-H^*)-\frac{\mu}{H}(H-H^*)^2$$

$$\frac{dV}{dt}=\frac{-\beta C^*}{S}(S-S^*)^2-\frac{(\delta+\mu)}{S}(S-S^*)^2+\frac{\varepsilon\rho}{R}(S-S^*)(R-R^*)$$
$$+\left(\frac{\alpha}{J}\right)(C-C^*)(J-J^*)-\frac{(\theta+d_1+\mu)}{J}(J-J^*)^2+\frac{\phi}{R}(C-C^*)(R-R^*)+\frac{\theta}{R}(J-J^*)(R-R^*)$$
$$-\frac{(\varepsilon+\mu)}{R}(R-R^*)^2+\left(\frac{\delta}{H}\right)(S-S^*)(H-H^*)$$
$$+\left(\frac{(1-\rho)\varepsilon}{H}\right)(R-R^*)(H-H^*)-\frac{\mu}{H}(H-H^*)^2$$

$$\frac{dV}{dt}=\frac{-1}{S}\left(\beta C^{*}+(\delta+\mu)\right)(S-S^{*})^{2}-\frac{(\theta+d_{1}+\mu)}{J}(J-J^{*})^{2}+\left(\frac{\alpha}{J}\right)(C-C^{*})(J-J^{*})+\frac{\varepsilon\rho}{R}(S-S^{*})(R-R^{*})$$
$$-\frac{(\varepsilon+\mu)}{R}(R-R^{*})^{2}+\frac{\theta}{R}(J-J^{*})(R-R^{*})+\left(\frac{\delta}{H}\right)(S-S^{*})(H-H^{*})$$
$$-\frac{\mu}{H}(H-H^{*})^{2}+\left(\frac{(1-\rho)\varepsilon}{H}\right)(R-R^{*})(H-H^{*})+\frac{\phi}{R}(C-C^{*})(R-R^{*})$$

When the three conditions given by Eq. (4.16) are satisfied $\frac{dV}{dt}<0$.

$\frac{dV}{dt}=0$, when $(S,C,J,R,H)=(S^{*},C^{*},J^{*},R^{*},H^{*})$

This fact indicates that the largest invariant set where $\frac{dV}{dt}=0$ is the singleton $E_{1}=(S^{*},C^{*},J^{*},R^{*},H^{*})$. Thus, by LaSalle's invariance principle, the EEP is globally asymptotically stable.

4.11 Sensitivity Analysis

Sensitivity analysis is carried out on the basic parameters, to identify their effect to the transmission of the corruption. To go through sensitivity analysis, we applied the normalized forward sensitivity index definition. The Normalized forward sensitivity index of a variable, p, that depends differentiable on a parameter, p, is defined as: $\Lambda_{p}^{R_0}=\frac{\partial R_0}{p}x\frac{p}{R_0}$ for p represents all the basic parameters.

$$\Lambda_{\pi}^{R_0}=\frac{\partial R_0}{\partial\pi}x\frac{\pi}{R_0}=1>0 \quad , \quad \Lambda_{\phi}^{R_0}=\frac{\partial R_0}{\partial\phi}x\frac{\phi}{R_0}=\frac{-\phi}{\alpha+\phi+\mu}<0$$

$$\Lambda_{\beta}^{R_0}=\frac{\partial R_0}{\partial\beta}x\frac{\beta}{R_0}=1>0 \quad , \quad \Lambda_{\mu}^{R_0}=\frac{\partial R_0}{\partial\mu}x\frac{\mu}{R_0}=\frac{-\mu\left(\alpha+\phi+\delta+2\mu\right)}{(\alpha+\phi+\mu)(\delta+\mu)}<0$$

$$\Lambda_{\alpha}^{R_0}=\frac{\partial R_0}{\partial\alpha}x\frac{\alpha}{R_0}=\frac{-\alpha}{\alpha+\phi+\mu}<0 \quad , \quad \Lambda_{\delta}^{R_0}=\frac{\partial R_0}{\partial\delta}x\frac{\delta}{R_0}=\frac{-\delta}{\delta+\mu}<0$$

The sensitivity indices of the basic reproductive number with respect to main parameters revealed that, those parameters that have positive indices are (π, β) show that they have great impact on expanding the corruption in the community if their values are increasing. Also those parameters in which their sensitivity indices are negative $(\phi, \mu, \alpha, \delta)$ have an effect of minimizing the burden of the corruption in the community as their values increase. Therefore, this study recommends for stakeholders to work on decreasing the positive indices and increasing negative indices parameters.

4.12 MATLAB Simulation

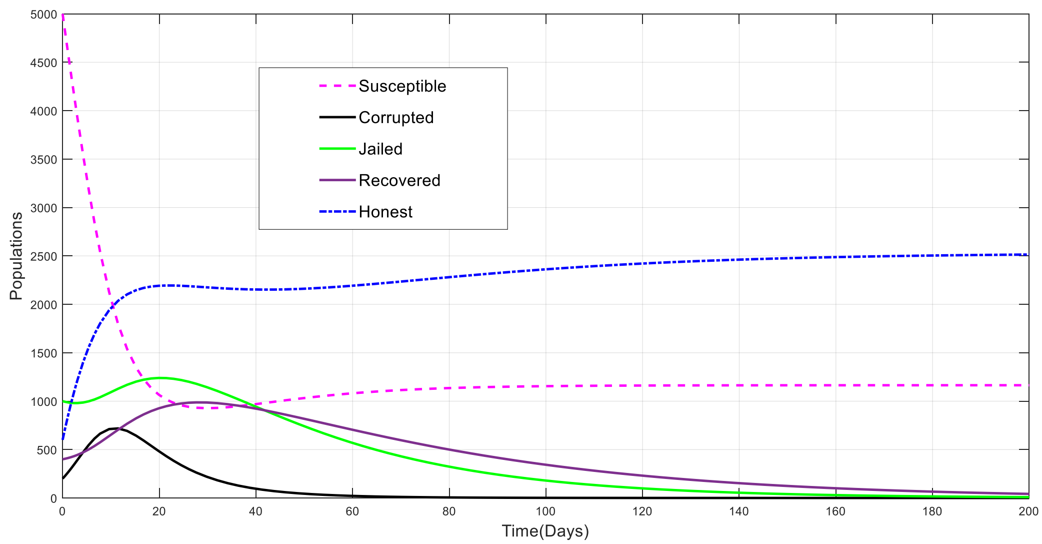

Figure 2: Graph of corruption dynamics verses time

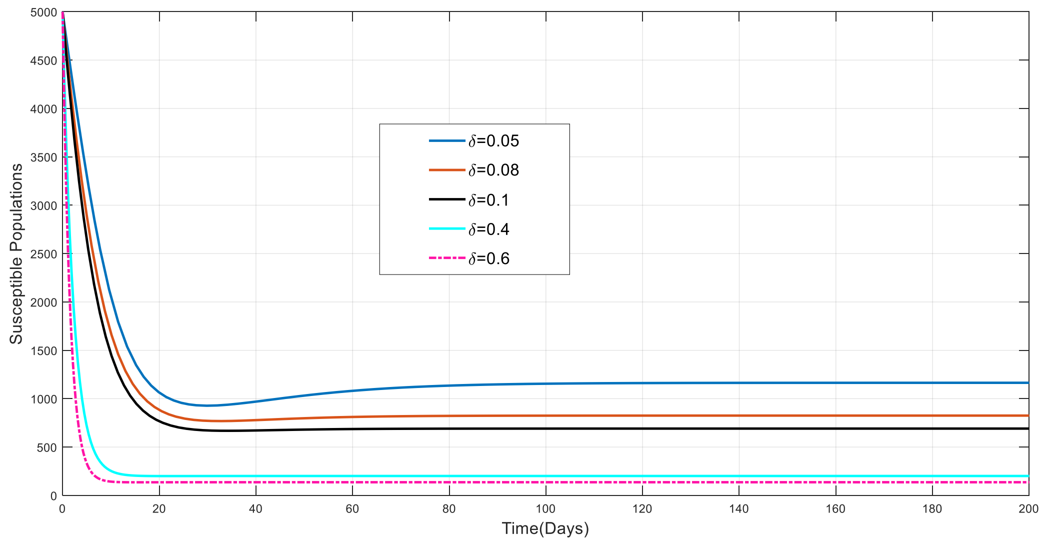

Figure 3: Graph of Susceptible population for different values of δ

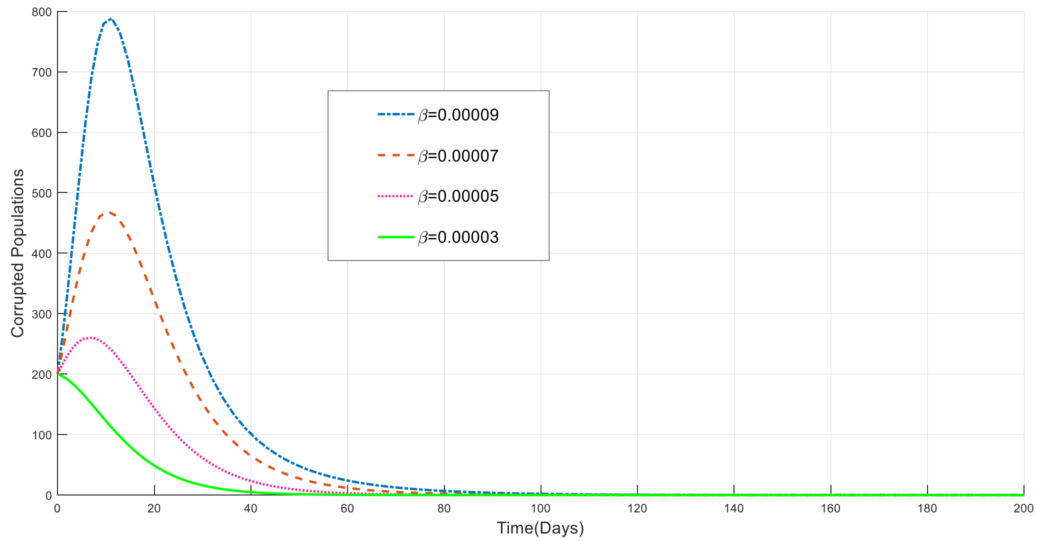

Figure 4: Graph of corrupted population for different values of β

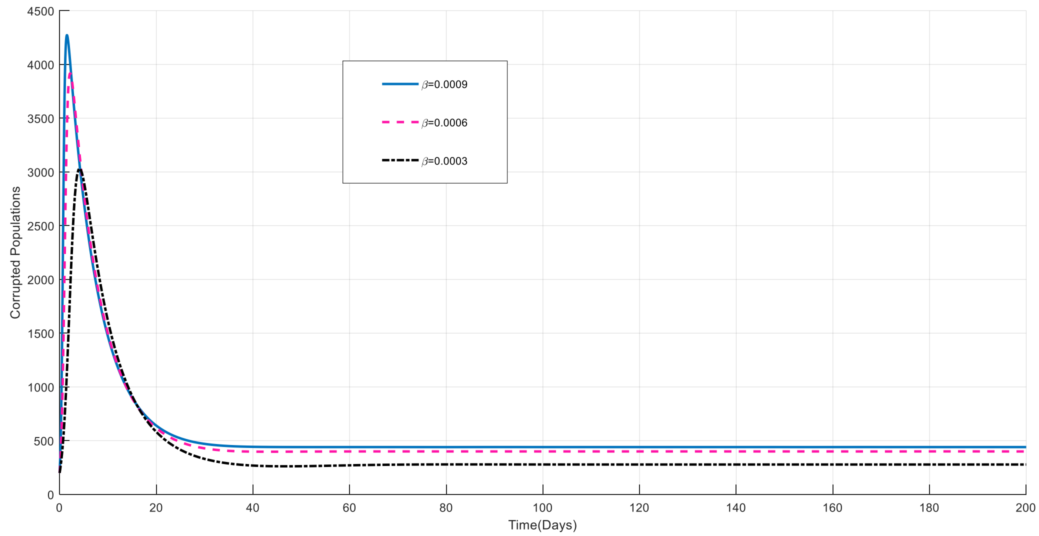

Figure 5: *Graph of corrupted population for different values of* α

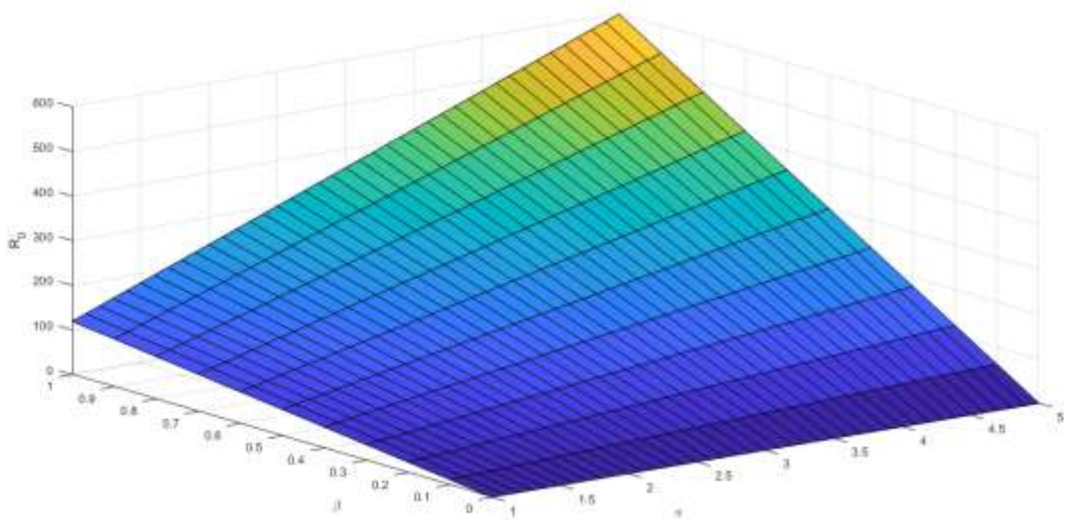

Figure 6: Graph of Basic Reproduction Number verses π and β

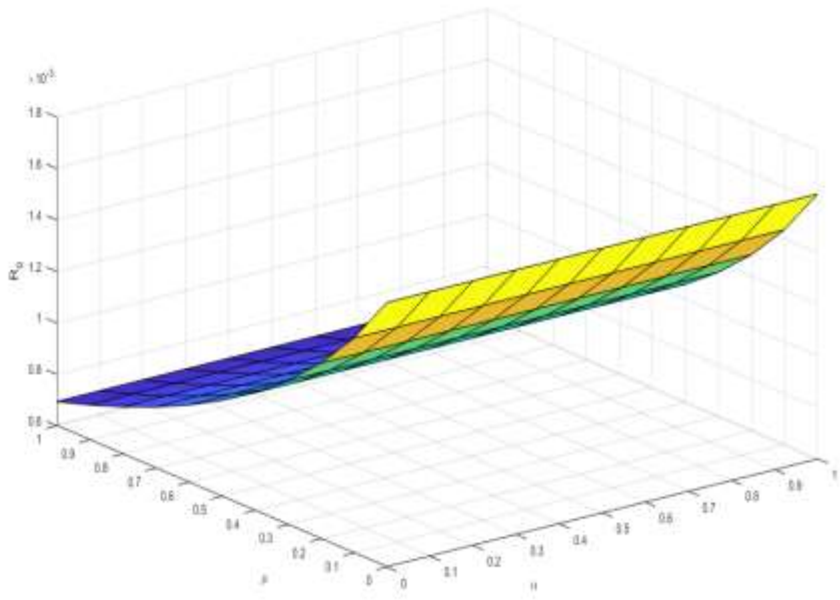

Figure 7: Graph of Basic Reproduction Number verses α and δ

4.13 Discussions

Figure 2 indicates that the graph of corruption dynamics in different compartment verses time. It revealed the fact that equilibrium point is locally asymptotically stable when the basic reproduction number is less than one. Figure 3 indicates graph Susceptible population for different values of rate at which susceptible population becomes honest population. As the rate at which susceptible population becomes honest population increases the susceptible population decreases. Figure 7 depicts that graph of Basic Reproduction Number verses influx rate and contact rate. It revealed the fact that increasing influx rate and contact rate has the capacity to increase the basic reproduction number which in turn increases the burden of corruption. Figure 8 depicts that graph of Basic Reproduction Number verses the rate at which susceptible population becomes honest population and transmission rate from corrupted to jailed. It revealed the fact that increasing those parameters has the capacity to decrease the basic reproduction number which in turn lowers the burden of corruption.

CHAPTER FIVE

5. CONCLUSION AND FUTURE SCOPE

5.1 Conclusion

The findings of this thesis are concluded as follows.

- New mathematical model for corruption dynamics were developed,
- Qualitative analysis like boundedness and positivity of the model were proved,
- Equilibrium points of the model (disease free and endemic equilibrium points) were calculated ,
- Basic reproduction number was calculated by using next generation matrix,
- The local and global stability conditions of the model were also well investigated for both disease free and endemic equilibrium points,
- Furthermore, sensitivity analysis of the model parameters was also carried out,
- Finally, in order to verify the applicability of the result MATLAB simulation was implemented and agrees with the analytical result.

5.2 Future Scope

One can conduct the following further investigation on this area of study.

- Refinement of the mathematical model by incorporating other important factor,
- Optimal control analysis of the model is also further investigation,
- It is also possible to extend the model to fractional order derivative to make new analysis with new result,
- Furthermore, introducing time delay into the model and conducting qualitative analysis like bifurcation, global stability, existence of periodic solution and limit cycle is also future scope of the study.

References

Abdulrahman, “Stability analysis of the transmissiondynamics and control of corruption,” Pacific Journal of Science and Technology, vol. 15, no. 1, pp. 99–113, 2014.

Abdulrahman, Stability Analysis of theTransmission Dynamics and Control of Corruption,The pasfic journal of Science and Technology, Vol. 15, No. 1, 2014, pp 99-113.

Ashok Mondal et al,2018” Stability and Bifurcation Analysis of a Delayed Three Species Food Chain Model with Crowley-Martin Response Function” Vol. 13, Issue 2 (December 2018).

Athithan, M. Ghosh, and X.-Z. Li, “Mathematical modelingand optimal control of corruption dynamics,” Asian-EuropeanJournal of Mathematics, vol. 11, no. 6, article 1850090, 2018

Bahoo, I. Alon, and A. Paltrinieri, “Corruption in international business: a review and research agenda,” International Business Review, vol. 29, no. article 101660, 201[2] A. Cuervo-Cazurra, “Corruption in international business,”Journal of World Business, vol. 51, no. 1, pp. 35–49, 2016.

Blanchard, P., Krüger, A., Krueger, T., Martin, P. (2005). *The epidemics of corruption*. arXiv preprint physics/0505031.

De Vrie , G. ((2001). what is Mathematical Modeling, Department of Mathematical Sciences, University of Alberta, Alberta

Driver, R. D. Ordinary and Delay Differential Equations, Springer Verlag, 1977.

Duygu Arugaslan,2015” Dynamics of a harvested logistic type model with delay and piecewise constant argument"J. Nonlinear Sci. Appl. 8 (2015), 507–517

Egudam, F. Oguntolu, and T. Ashezua, “Understandingthe dynamics of corruption using mathematical modeling approach,” International Journal of Innovative Science, Engineering & Technology, vol. 4, no. 4, pp. 2348–7968, 2017

Eicher S. 2009. Corruption in International Business: The Challenge of Cultural and Legal Diversity. 5.Gower Pabico/Modeling Corruption as a Contagious Disease

Haileyesus,2020 “Mathematical Modeling, Analysis, and Optimal Control of Corruption Dynamics” University of Gondar, Gondar, Ethiopia.

Heimann and N. Z. Boswell, “The oecd convention: milestoneon the road to reform,” in New Perspectives on Combating Corruption, pp. 65–74, Transparency International,Washington, 1998.

Jaderick p.pabico 2017” Modeling corruption as contagious disease University of the Philippines Los Banos *jppabico@up.edu.ph

Khalil, H. K., & Grizzle, J. W. (2002). **Nonlinear systems** (Vol. 3). Upper Saddle River, NJ: Prentice hall.

Khan, “The corruption prevention model,” Journal of Discrete Mathematical Sciences and Cryptography, vol. 3, no. 1-3, pp. 173–178, 2000

LaSalle, J. P. (1976). Stability theory and invariance principles. In *Dynamical systems* (pp. 211-222). Academic Press

Lemecha, “Modelling corruption dynamics and its analysis,”Ethiopian Journal of Sciences and Sustainable Development,vol. 5, no. 2, pp. 13–27, 2018

Milner-Smyth, “Consequences of corruption,” HR Future,vol. 2017, no. Jul 2017, pp. 30-31, 201.

Nelson shawn, .(2013). Population” Modeling with Delay Differential Equations” Rochester Institute of Technology.

Oscar Danford et al,2020” Mathematical Modelling and Analysis of Corruption Dynamics with Control Measures in Tanzania” *Journal of Mathematics and Informatics Vol. 19, 2020, 57-79*

Perko, L. (2013). **Differential equations and dynamical systems** (Vol. 7). Springer Science & Business Media, USA.

Syamsuddin Toaha,2006"Stability Analysis And Maximum Profit Of Logistic Population Model With Time Delay And Constant Effort Of Harvesting"vol.3 no_1

Tiihonen, S. Ed. (2003). The history of corruption in central government. IOS Press 10.World Bank (2014). Fraud and corruption awareness handbook. A handbook for civil servants involved in public procurement.

Verma P and Sengupta S. 2015. Bribe and punishment: An evolutionary game-theoretic analysis of bribery. PLoSONE 10(7):e0133441. doi:10.1371/journal.pone.0133441

Wayker, "Mathematical modelling: a study of corruption in the society of India," International Journal of Scientific and Engineering Research, vol. 4, no. 7, pp. 2303–2318, 2013.