

Study of light and heavy-duty vehicle suspension model performance under different road profiles

Cite as: AIP Conference Proceedings 2413, 020015 (2022); <https://doi.org/10.1063/5.0079102>
Published Online: 23 June 2022

A. Johnson Santhosh, Biniyam Ayele Abebe, N. Ashok, et al.



[View Online](#)



[Export Citation](#)

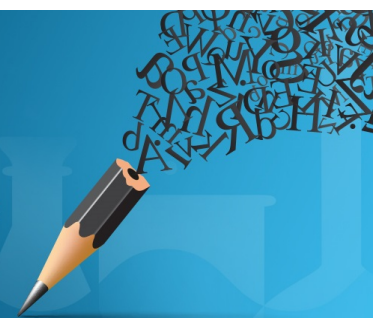


Author Services

English Language Editing

High-quality assistance from subject specialists

[LEARN MORE](#)



Study of Light and Heavy-Duty Vehicle Suspension Model Performance under Different Road Profiles

A.Johnson Santhosh ^{1,a)},Biniyam Ayele Abebe ^{2,b)}, N.Ashok^{3,c)},Murugan Ponnusamy^{1,d)}, K.Srinivasa Rao^{1,e)}, and Fakada Dabalo Gurmesa^{1,f)}

¹ Faculty of Mechanical Engineering, Jimma Institute of Technology, Jimma University, Jimma, Ethiopia

²Department of Mechanical Engineering, College of Engineering & Technology, Wolkite Univeristy,Ethiopia

³Faculty of Mechanical & Production Engineering, Arba Minch University, Ethiopia.

^{a)} Corresponding author: johnsonsanthosh@gmail.com

^{b)}biniyamaye83@gmail.com

^{c)}ashok2488@rediffmail.com

^{d)}mu0105@gmail.com

^{e)}srinivasaokancharla8@gmail.com

^{f)}ff.dabalo@gmail.com

Abstract. Ride comfort and Road holding execution of the vehicle are the most critical factors in the suspension system performance study. Most of the time researchers assume the numerical model of vehicle suspension is a linear and detailed comparative study with the non-linear model that has not been done in previous researches. Hence, the numerical model for both linear and non-linear quarter vehicle suspension is developed in this research work to study the importance of nonlinearity suspension segments. The dynamic outcomes such as sprung and unsprung mass displacement, velocity, acceleration are calculated by conducting the simulation for light-duty and heavy-duty vehicles under sinusoidal and unit step road input based on developed numerical models and program codes. The outcome from this research paper shows that an extensive distinction has been seen in the ride comfort and street holding execution of the linear and non-linear car suspension framework. Simulation results show that there is an increase in ride comfort execution and road holding execution of the vehicle in the event of non-linear suspension than the typical linear suspension. In this way, it tends to get close for a sinusoidal road excitation the non-linear suspension consistently acquires preferred execution over the linear suspension. Considering the nonlinearity suspension segments is more important during modeling and designing of the vehicle suspension system.

Keywords: Quarter vehicle linear suspension, Non-linear suspension, Ride comfort, Road holding execution, Numerical model, Suspension Parameters, Suspension performance criteria

INTRODUCTION

In this paper, a basic two-degree of freedom spring-mass-damper framework as appeared by Figure.1(a) & (b) represents the quarter car suspension model. It comprises one upper mass (sprung mass), one lower mass (unsprung mass), two springs, and one stun absorber[1].

The street unsettling influence from different street contribution, for instance, single step street profile, sinusoidal street profile with pitching, flinging, and mixed model excitation are causing off-kilter drive and uproar in the vehicle body, so it is fundamental to examine the vibrations of the vehicle by considering those sorts of street inputs[2]- [3]. The driver-seat forces and suspension system of the seat play a major role in the comfort of the driver [4]- [5].

A vehicle suspension is a non-linear system in genuine terms since it comprises of adaptable suspension tires and different parts, which have non-linear properties, for example, non-linear spring and damper, yet most of the researchers considered as a linear system for the matter of straightforwardness of numerical displaying of it. Along these lines, the disorganized reaction may happen as the vehicle moves over a street [6].

A comparative study for quarter car 2 DOF system model shows the good performance in Non-linear model and it is validated using MATLAB-SIMULINK model with theoretical results [7]. Implementation of simulation methods in a non-linear active suspension system helps to achieve the optimal suspension performance [8–11]. The simulation methods are helping to get optimal ride comfort under various conditions [12–14].

Most of the time researchers have assumed the numerical model of vehicle suspension is a linear and detailed comparative study concerning the non-linear model has not been done in previous researches [15–18]. Hence, the numerical model for both linear and non-linear quarter vehicle suspension is developed in this research work to study the importance of nonlinearity suspension segments under different road input profiles.

NUMERICAL MODELING OF LINEAR QUARTER CAR PASSIVE SUSPENSION SYSTEM

In this paper, the body connection and upper control arm considered as the sprung mass (M_s). While the tire and wheel get together, center point gathering and lower control arm approximated as unsprung mass (M_{us}). The free-body outlines of M_s and M_{us} are indicated in Figure 1. Let us take, the mass M_s moves quicker in a vertical way than the mass M_{us} , at that point the lengthening of the spring K is given by $X_1 - X_2$. The heading of the force applied by the spring K on the mass M_s is descending, as it will, in general, reestablish to the un-disfigured position. Because of Newton's third law of motion, the force applied by the spring K on the mass M_{us} has a similar extent, however, it acts the other way.

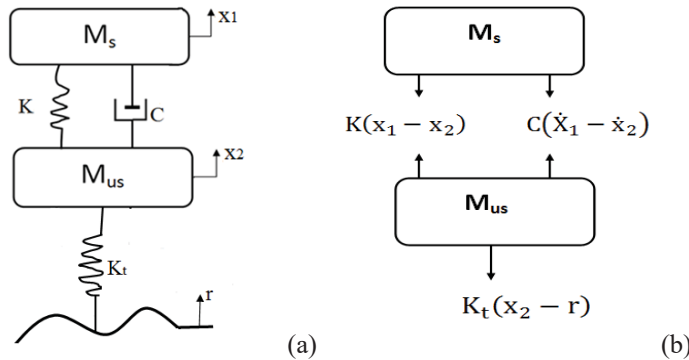


FIGURE 1.(a) Quarter car passive suspension model in Linear; (b) Free body diagram

Figure.1 represents the model characteristics of the linear front quarter vehicle passive suspension. It displayed the spring-mass-damper frameworks and Free body Diagram of quarter car passive suspension model.

In a linear framework, Hook's law typically communicates the statement of a spring force: $F_s = KX$, where F_s is the spring force K is the spring stiffness, X is the spring displacement. Coulomb's law of friction communicates the statement of damping force for moderate speed: $F_d = C\dot{x}$. F_d represents the damping force, C represents the linear damping, \dot{x} representing the relative velocity.

Utilizing the linear formulation of spring force, damping force, and tire spring force and by applying Newton's second law of motion for both sprung masses, M_s and un-sprung mass, M_{us} the conditions of two masses movement may be represented by the following equations (1) and (2).

$$M_s \ddot{x}_1 + K(x_1 - x_2) + C(\dot{x}_1 - \dot{x}_2) = 0 \quad (1)$$

$$M_{us} \ddot{x}_2 + K_t(x_2 - r) - C(\dot{x}_1 - \dot{x}_2) - K(x_1 - x_2) = 0 \quad (2)$$

State-Space Formation of Quarter-Car Suspension Framework

Let the states of the system defined as the following state variables:

$y_1 = x_1$, $y_2 = \dot{x}_1$, $y_3 = x_2$ and $y_4 = \dot{x}_2$ this implies that:

$$\begin{aligned}\dot{y}_1 &= \dot{x}_1 \\ \dot{y}_2 &= \ddot{x}_1 \\ \dot{y}_3 &= \dot{x}_2 \\ \dot{y}_4 &= \ddot{x}_2\end{aligned}$$

Therefore, the system of the first-order differential equation becomes;

$$\dot{y}_1 = y_2 \quad (3)$$

$$\dot{y}_2 = -\frac{K}{M_s}(y_1 - y_3) - \frac{c}{M_s}(y_2 - y_4) \quad (4)$$

$$\dot{y}_3 = y_4 \quad (5)$$

$$\dot{y}_4 = \frac{K}{M_{us}}(y_1 - y_3) - \frac{c}{M_{us}}(y_2 - y_4) - \frac{K_t}{M_{us}}(K_t - r) \quad (6)$$

NUMERICAL MODELING OF THE NON-LINEAR QUARTER VEHICLE SUSPENSION SYSTEM

In the event of the genuine vehicle suspension framework expecting it as non-linear model. Since, it is having more advantages and results accuracy in dynamic conditions. In this paper, the non-linear quarter vehicle suspension framework largely comprises coil springs having fourth-degree polynomial stiffness, suspension damping having cubic nonlinearities and quadratic tire stiffness nonlinearities. Quarter car passive suspension model in Non-linear condition is represented in Figure.2.

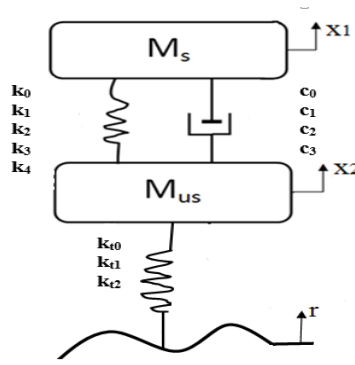


FIGURE 2. Quarter car passive suspension model in Non-linear condition

By applying Newton's second law[19] of movement for both sprung mass (M_s) and unsprung mass, (M_{us}); the conditions of movement of the two masses are given by:

$$\begin{aligned}M_s \ddot{x}_1 + k_4(x_1 - x_2)^4 + k_3(x_1 - x_2)^3 + k_2(x_1 - x_2)^2 + k_1(x_1 - x_2) + c_3(\dot{x}_1 - \dot{x}_2)^3 \\ + c_2(\dot{x}_1 - \dot{x}_2)^2 + c_1(\dot{x}_1 - \dot{x}_2) = 0\end{aligned} \quad (7)$$

$$\begin{aligned}M_{us} \ddot{x}_2 - k_4(x_1 - x_2)^4 - k_3(x_1 - x_2)^3 - k_2(x_1 - x_2)^2 - k_1(x_1 - x_2) - c_3(\dot{x}_1 - \dot{x}_2)^3 - c_2(\dot{x}_1 - \dot{x}_2)^2 - \\ c_1(\dot{x}_1 - \dot{x}_2) + k_{t2}(x_2 - r)^2 + k_{t1}(x_2 - r) = 0\end{aligned} \quad (8)$$

To limit the second-order differential equation to an arrangement of first-order differential equations utilizing the state space approach, let the state of the framework defined as the accompanying state factors: $y_1 = x_1$, $y_2 = \dot{x}_1$, $y_3 = x_2$ and $y_4 = \dot{x}_2$ this implies that:

$$\begin{aligned}\dot{y}_1 &= \dot{x}_1 \\ \dot{y}_2 &= \ddot{x}_1 \\ \dot{y}_3 &= \dot{x}_2 \\ \dot{y}_4 &= \ddot{x}_2\end{aligned}$$

Therefore, the system of the first-order differential equation for equation (7) and (8) becomes;

$$\dot{y}_1 = y_2 \quad (9)$$

$$\dot{y}_2 = -\frac{1}{M_s} [k_4(y_1 - y_3)^4 + k_3(y_1 - y_3)^3 + k_2(y_1 - y_3)^2 + k_1(y_1 - y_3) + c_3(y_2 - y_4)^3 + c_2(y_2 - y_4)^2 + c_1(y_2 - y_4)] \quad (10)$$

$$\dot{y}_3 = y_4(11)$$

$$\dot{y}_4 = \frac{1}{M_{us}} [k_4(y_1 - y_3)^4 + k_3(y_1 - y_3)^3 + k_2(y_1 - y_3)^2 + k_1(y_1 - y_3) + c_3(y_2 - y_4)^3 + c_2(y_2 - y_4)^2 + c_1(y_2 - y_4) - k_{t2}(y_3 - r)^2 - k_{t1}(y_3 - r)](12)$$

Road Excitation Modeling

Road disturbance or excitation is the road profile or the road elevation along the road and includes everything from smooth roads to potholes and any up and down as shown in Fig. 3. (a) & (b). The road profile is highly affected the ride comfort and safety of the passengers.



FIGURE 3. (a) Sinusoidal road excitations; (b) unit step road excitations[7]

Sinusoidal Road Input

The sporadic road profile that is appeared in Fig. 3 (a) considered as a sine wave spoke to by the condition:

$$y(t)=A*\sin(\omega*t) \quad (13)$$

Where, y = road surface excitation at time t , A = amplitude of sine wave, and ω =radial recurrence of the wheel ignoring harsh road. The velocities of the vehicle and spiral recurrence of the wheels are related by the accompanying equation:

$$\omega = \frac{2\pi v}{\lambda} \quad (14)$$

The velocity of the car pass over the sinusoidal road excitation is assumed as 9.5 km/hr and the wavelength of the sinusoidal road excitation is assumed as 2m. Based on Equation 14, which is equal to 30rad/s.

Unit Step Road Input

The irregular road profile that is appeared in Fig. 3 (b) is approximated by a unit step road input as given by the formula [20]:

$$Y(t) = \begin{cases} 0, & t < 1 \\ 0.15, & t > 1 \end{cases}$$

$Y(t)$ speaks to Road surface disturbance at time t and 0.15m amplitude.

For the two degrees of freedom quarter vehicle model, MATLAB codes are developed to analyze sprung and unsprung mass removal, velocity, and acceleration for the car travel over a sinusoidal and unit step road input. Input suspension parameters shown in Tables 1 is used in MATLAB for a linear and non-linear car suspension system with road input of sinusoidal and unit step conditions. Table 1 shows the input parameters for analysis of a linear quarter car suspension system for both types of vehicles obtained from the respected works of literature [21]. The amplitude of the rod excitation and simulation time is assumed as 0.15m and 5 seconds respectively.

TABLE 1. Linear suspension parameter for light-duty (car) and heavy-duty (bus) vehicles [22]

Suspension Parameters	Symbol	Car	Bus	Units
Sprung Mass	M_s	370	2050	kg
Unsprung Mass	M_{us}	40.5	100	kg
Suspension Spring Stiffness	K	21500	400000	N/m
Damping Coefficient	C	1400	5000	N.s/m
Coefficient of Tire Stiffness	K_t	190900	2000000	N/m
Road Excitation Amplitude	A	0.15	0.15	M
Simulation Time	t	5	5	Second
Fixed Step Size	t_{step}	0.005	0.005	Second

RESULT AND DISCUSSION

The result and discussion of this study are based on the sinusoidal and unit step road irregularity and simulation parameters for both linear and non-linear quarter car passive suspension models of light-duty and heavy-duty vehicles that are given in Table 1. The sinusoidal and unit step road has been input into two models; the first one is the linear quarter car model and the other one is the non-linear quarter car model with 4th-degree suspension spring nonlinearity, 3rd-degree suspension damper nonlinearity, and quadratic tire nonlinearity. In this research, the amplitude of the road 0.15m has considered to study the effect of the non-linear suspension and the linear suspension on the vertical vibration of the vehicle.

Comparison of the simulated results from MATLAB for linear and non-linear quarter car passive suspension model as shown in Figure. 4 and 5. Each of these figures consists of four different parametric plots which show sprung mass removal (displacement), velocity, acceleration, and unsprung mass removal for two different types of vehicles. It is obvious from the figures that there are good agreements between the two sets of results.

Simulation Result for Light-Duty Vehicles under Sinusoidal Road Input

The simulation of the mathematical model for a light-duty quarter car suspension system has produced the displacement, velocity, and acceleration of the sprung mass and displacement of unsprung mass in the case of sinusoidal road excitation.

The graphical result in Figure. 4 (a); shows that the reduction of the sprung mass vertical displacement is approximately 50% in the case of non-linear suspension as compared with linear suspension. This occurs because the non-linear suspension model considered all the nonlinearities on the flexible components of the quarter car suspension systems. Figure. 4 (b) shows that sprung mass vertical velocity is limited by 58.2% if there should arise an occurrence of non-linear suspension as contrasted with and linear suspension framework. From Figure. 4 (c) is conceivable to see that the sprung mass vertical velocity is limited by 66.4% in the event of non-linear suspension as contrasted and linear suspension, which yields better ride comfort performance of the vehicle when it proceeds onward the sinusoidal road profile. Likewise Figure. 4 (d) shows that unsprung mass uprooting is almost 37.2% which is less if there should be an occurrence of non-linear suspension than linear suspension framework; this suggests better street holding performance. Although, the ride comfort and street holding are negating boundaries, there is an increase in ride comfort execution and road holding execution of the vehicle in the event of non-linear suspension than the typical linear suspension. In this way, it tends to get close for a sinusoidal road excitation the non-linear suspension consistently acquires preferred execution over the linear suspension.

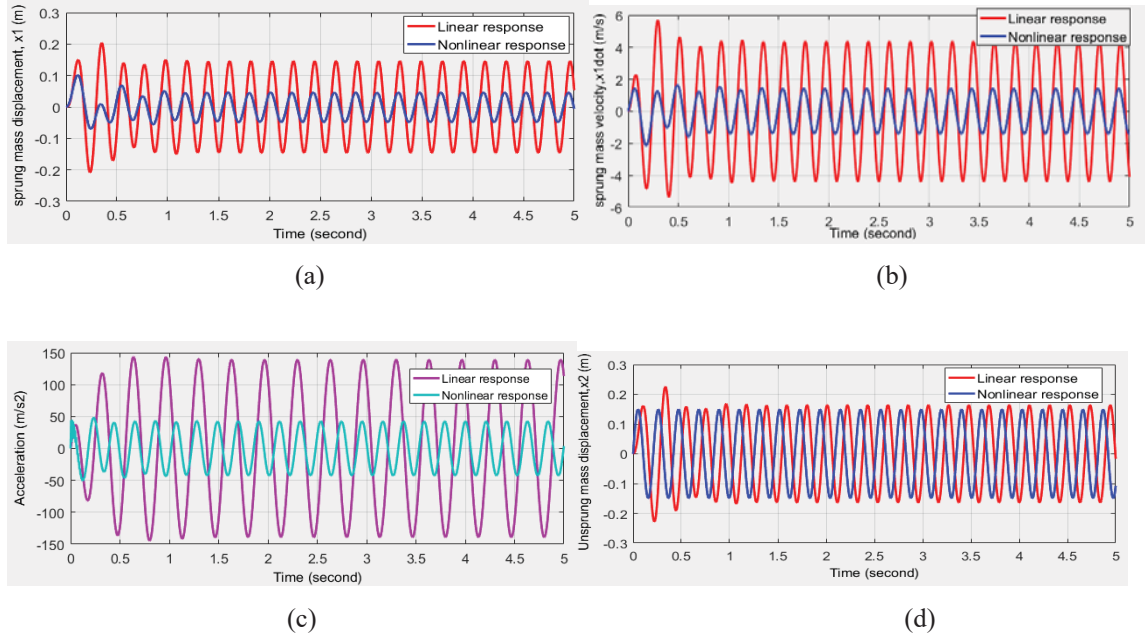


FIGURE 4. Simulation Result for Light-Duty Vehicles under Sinusoidal Road Input

Comparative quantitative numerical values of the peak amplitude of the quantities sprung mass displacement, velocity, acceleration, and unsprung mass displacement are shown in Table 2. It is evident from Figure. 4 that non-linear suspension gives lower amplitude for sprung mass displacement, velocity, and acceleration than the commonly used linear suspension model. Besides, unsprung mass displacement of the non-linear model is being lower even though amplitude is only slightly lower than the linear model.

TABLE 2. Quantitative numerical values of peak amplitude for both models under sinusoidal road input

Suspension performance criteria	For the linear suspension model	For the non-linear suspension model	Percentage of variation
Sprung Mass Displacement (m)	0.2	0.1	50%
Unsprung Mass Displacement (m)	0.24	0.15	37.2%
Sprung Mass Velocity (m/s)	5.85	2.45	58.2%
Sprung Mass Acceleration (m/s ²)	149	50	66.4%

Simulation Result for Light-Duty Vehicles under Unit Step Road Input

To concentrate in detail on the impact of road contributions on the vehicle suspension framework it is essential to include another street information, for example, a unit step street contribution with a similar amplitude of 0.15m. So that, a unit step street input is applied as per Figure. 5 and its impact on linear and non-linear suspension model of light-duty vehicle are analyzed. For unit step street input the presentation distinction between the linear and non-linear suspension framework regarding the sprung mass displacement, velocity and acceleration, and unsprung mass displacement has been examined under Figure. 5 (a - d) respectively.

Figure. 5 (a) shows that the sprung mass displacement reduced by 8.3% from supplanting an ordinary linear suspension with a non-linear suspension. From Figure. 5 (b) seen that sprung mass vertical velocity is decreased by 44% if there should arise an occurrence of non-linear suspension as contrasted and linear suspension. To look at the ride comfort exhibitions of the two suspensions model it is smarter to see the vertical acceleration of the sprung mass as appeared in Figure. 5 (c). The impact of the non-linear suspension decreases the acceleration of the sprung mass from 67m/s² to 49m/s² or 26.86%, which is a moderately better ride comfort execution.

As it is shown in Figure. 5 (d) the road holding execution of the non-linear suspension additionally improved. Even though, the vertical displacement of unsprung mass higher at starting point yet, it settles quicker than the linear suspension.

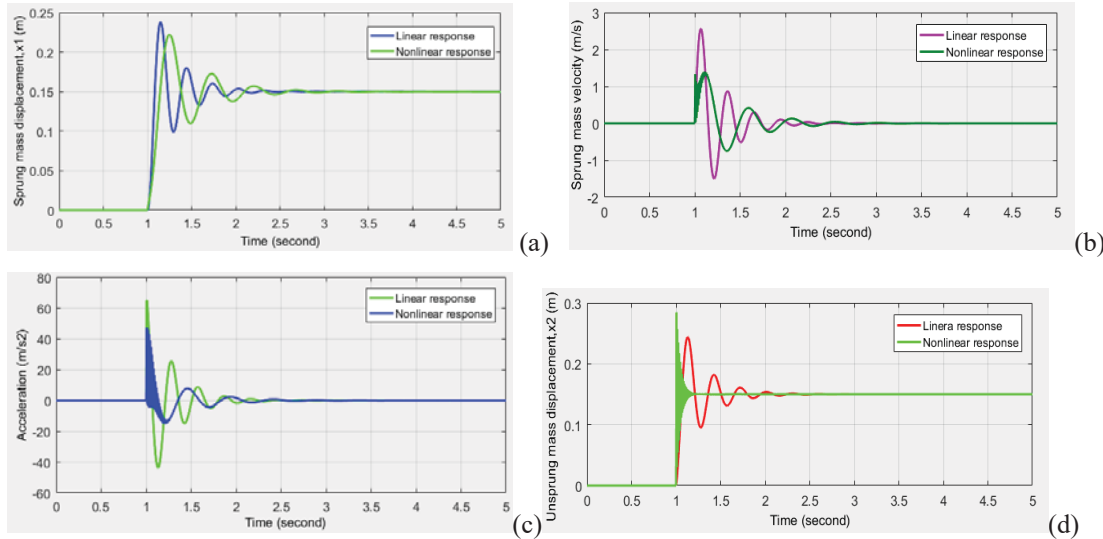


FIGURE 5. Simulation Result for Light-Duty Vehicles under Unit Step Road Input

Table 3 shows the quantitative numerical values of peak amplitude for both models under unit step road input. It is possible to observe that the non-linear suspension model has better performance than the linear suspension model. In general Figure 6 and 7, indicates the non-linear car suspension system performed better than that of the conventional linear car suspension system for light-duty vehicles.

TABLE 3. Quantitative numerical values of peak amplitude for both models under unit step road input

Suspension Performance Criteria	For the linear suspension model	For nonlinear suspension model	Percentage of variation
Sprung Mass Displacement (m)	0.24	0.22	8.3%
Unsprung Mass Displacement (m)	0.25	0.29	13.8%
Sprung Mass Velocity (m/s)	2.5	1.4	44%
Sprung Mass Acceleration (m/s ²)	67	49	26.86%

These Figures illustrate clearly the effectiveness of absorbing vibrations in the non-linear suspension system over linear suspension system. Better ride comfort performance is achieved due to the less sprung mass acceleration and the minimum wheel suspension deflection (unsprung mass displacement) over the linear system.

Simulation Result for Heavy-Duty Vehicles under Sinusoidal Road Input

To detail study the effect of the nonlinear suspension it is important to use different vehicles with different suspension parameters under the same sinusoidal and unit step road input with the fixed amplitude of 0.15 m acting as the road input. For this case, a heavy-duty vehicle specifically a bus is selected for the comparative study of the linear and non-linear suspension system. The impact of non-linear suspension is studied and compared with the linear suspension system which is given in Figure. 6 and 7 for the heavy-duty vehicle bus.

The vibration experienced by the sprung mass displacement is lower in the case of nonlinear suspension which is shown in Figure. 6 (a). The graph of sprung mass vertical acceleration shown in Figure. 6 (b) shows that reduction in

corresponding amplitudes in the case of non-linear suspension is more than linear suspension which indicates an improvement in ride comfort. On ascertaining values from the graphs, discovered that there is a decline in sprung mass acceleration roughly by 66.25% shows the improvement in ride comfort. Additionally, the road holding execution or unsprung mass displacement is reduced by 85% in the non-linear case as appeared in Figure. 6 (c).

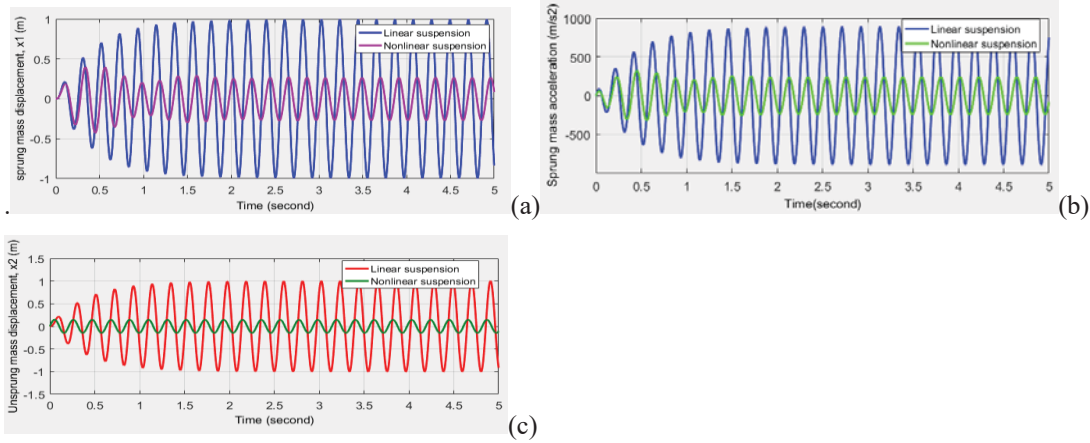


FIGURE 6. Simulation Result for Heavy-Duty Vehicles under Sinusoidal Road Input

Simulation Result for Heavy-Duty Vehicles under Unit Step Road Input

To detail study the impact of road inputs on the car suspension system for another road input i.e., unit step road input with the same amplitude of 0.15m. For unit step road input the comparative analysis between various factors and conditions are illustrated under Figure. 7 (a)-(c) respectively.

It is evident from Figure. 7 (a)-(c), that the non-linear suspension model gives a lower amplitude for sprung mass displacement and acceleration and unsprung mass displacement compared to linear suspension. On ascertaining values from the graphs, discovered that there is a decline in sprung mass acceleration approximately by 43.47% that is corresponding to improvement in ride comfort. Also, the road holding performance or unsprung mass displacement is reduced by 40.74% as shown in Figure. 7 (c).

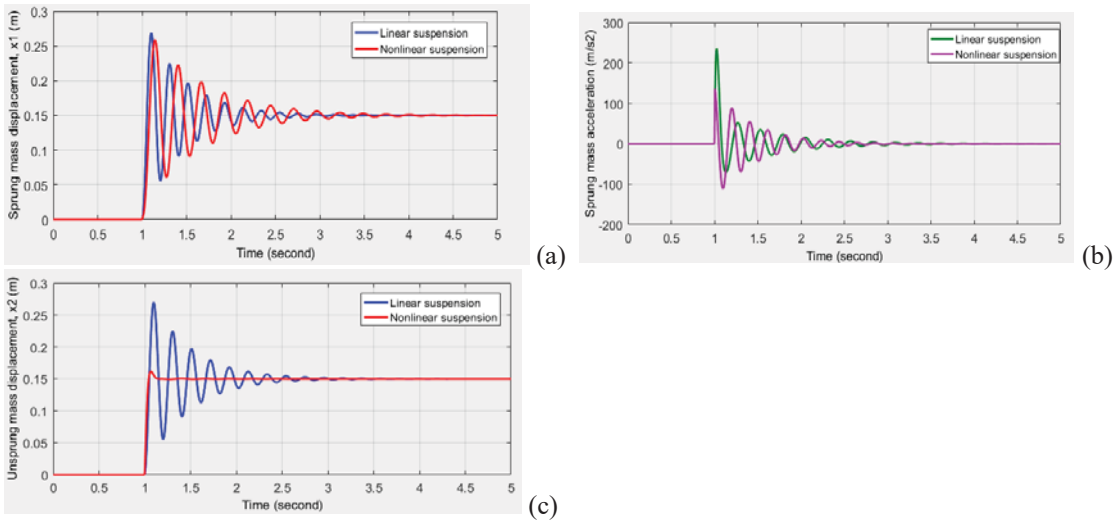


FIGURE 7. Simulation Result for Heavy-Duty Vehicles under Unit Step Road Input

CONCLUSION

The outcome from this research paper shows that an extensive distinction has been seen in the ride comfort and road holding execution of the linear and non-linear car suspension framework. Most of the time researchers for the matter of straightforwardness of the numerical model of vehicle suspension is assumed as linear Model. Hence, the numerical model of both linear and non-linear quarter vehicle suspension is developed in this research work to study the importance of nonlinearity suspension segments. The amplitude of the road 0.15m has considered studying the effect of the non-linear suspension and the linear suspension on the vertical vibration of the vehicle for this study. The detailed analysis is carried out under Sinusoidal Road Input and Unit Step Road Input conditions for two different types of vehicles (Light Duty Vehicle Car and Heavy Duty Vehicle Bus) for the selected input suspension parameters. Light duty vehicle car suspension performance criteria's such as sprung mass displacement (m) , unsprung mass displacement (m) , velocity (m/s), acceleration (m/s^2) is reduced 50%, 37.2%, 58.2%, 66.4% under sinusoidal road input and reduced 8.3% , 13.8%, 44%, 26.86% under unit step road input for non-linear suspension model when compared to linear suspension model.

In the case of heavy-duty vehicle suspension, vibration experienced by the sprung mass displacement is lower in the case of nonlinear suspension, and reduction in corresponding amplitudes in the case of non-linear suspension is more than linear suspension which indicates an improvement in ride comfort. Sprung mass acceleration(m/s^2)reduced by 66.25%& 43.47%shows the better improvement in ride comfort and road holding execution or unsprung mass displacement (m) is reduced by 85% & 40.74% for the non-linear suspension model under sinusoidal road input and unit step road input conditions.

Hence, it is concluded that comparison of the linear and non-linear quarter car passive suspension model and heavy-duty model simulation results shows that there is an increase in ride comfort execution and road holding execution of the vehicle in the event of non-linear suspension than the typical linear suspension. In this way, it tends to get close for a sinusoidal road excitation the non-linear suspension consistently acquires preferred execution over the linear suspension. Considering the nonlinearity suspension segments is more important during modeling and designing of the vehicle suspension system.

REFERENCES

- [1] B.A. Abebe, J. Santhosh, A.A. Ahmed, P. Murugan, N. Ashok, Non-Linear Mathematical Modelling for Quarter Car Suspension Model, 11 (2020) 536–544.
- [2] k. S.R.& a. P. T. Ram Mohan Rao, G. Venkata Rao, Analysis of Passive and Semi Active Controlled Suspension Systems for Ride Comfort in an Omnibus Passing Over a Speed Bump, *Int. J. Res. Rev. Appl. Sci.* (2010).
- [3] H.A. Hamersma, P.S. Els, Vehicle suspension force and road profile prediction on undulating roads, *Veh. Syst. Dyn.* (2020). <https://doi.org/10.1080/00423114.2020.1774067>.
- [4] R. Desai, A. Guha, P. Seshu, Modelling and simulation of an integrated human-vehicle system with non-linear cushion contact force, *Simul. Model. Pract. Theory.* 106 (2021) 102206. <https://doi.org/10.1016/j.simpat.2020.102206>.
- [5] Y. Yao, G. Li, G. Wu, Z. Zhang, J. Tang, Suspension parameters optimum of high-speed train bogie for hunting stability robustness, *Int. J. Rail Transp.* (2020). <https://doi.org/10.1080/23248378.2019.1625824>.
- [6] Q. Zhu, M. Ishitobi, Chaos and bifurcations in a nonlinear vehicle model, *J. Sound Vib.* (2004). <https://doi.org/10.1016/j.jsv.2003.10.016>.
- [7] A.G. Mohite, A.C. Mitra, Development of Linear and Non-linear Vehicle Suspension Model, in: *Mater. Today Proc.*, 2018. <https://doi.org/10.1016/j.matpr.2017.11.697>.
- [8] S.D. Nguyen, B.D. Lam, S.B. Choi, Smart dampers-based vibration control – Part 2: Fractional-order sliding control for vehicle suspension system, *Mech. Syst. Signal Process.* (2021). <https://doi.org/10.1016/j.ymssp.2020.107145>.
- [9] M. Ghoniem, T. Awad, O. Mokhiamar, Control of a new low-cost semi-active vehicle suspension system using artificial neural networks, *Alexandria Eng. J.* (2020). <https://doi.org/10.1016/j.aej.2020.07.007>.
- [10] S.A. Chen, J.C. Wang, M. Yao, Y.B. Kim, Improved optimal sliding mode control for a non-linear vehicle active suspension system, *J. Sound Vib.* (2017). <https://doi.org/10.1016/j.jsv.2017.02.017>.
- [11] G.D. Shelke, A.C. Mitra, V.R. Varude, Validation of simulation and analytical model of nonlinear passive

- vehicle suspension system for quarter car, in: *Mater. Today Proc.*, 2018. <https://doi.org/10.1016/j.matpr.2018.06.288>.
- [12] S. Liu, T. Zheng, D. Zhao, R. Hao, M. Yang, Strongly perturbed sliding mode adaptive control of vehicle active suspension system considering actuator nonlinearity, *Veh. Syst. Dyn.* (2020). <https://doi.org/10.1080/00423114.2020.1840598>.
- [13] S. Fazeli, M.R. Jahed-Motlagh, A. Moarefianpur, An adaptive approach for vehicle suspension system control in presence of uncertainty and unknown actuator time delay, *Syst. Sci. Control Eng.* 9 (2021) 117–126. <https://doi.org/10.1080/21642583.2020.1850369>.
- [14] T.D. Lewis, Y. Li, G.J. Tucker, J.Z. Jiang, Y. Zhao, S.A. Neild, M.C. Smith, R. Goodall, N. Dinmore, Improving the track friendliness of a four-axle railway vehicle using an inertance-integrated lateral primary suspension, *Veh. Syst. Dyn.* (2021). <https://doi.org/10.1080/00423114.2019.1664752>.
- [15] K. Kia, P.W. Johnson, J.H. Kim, The effects of different seat suspension types on occupants' physiologic responses and task performance: implications for autonomous and conventional vehicles, *Appl. Ergon.* 93 (2021) 103380. <https://doi.org/10.1016/j.apergo.2021.103380>.
- [16] D. Cantero, P. McGetrick, C.W. Kim, E. O'Brien, Experimental monitoring of bridge frequency evolution during the passage of vehicles with different suspension properties, *Eng. Struct.* 187 (2019) 209–219. <https://doi.org/10.1016/j.engstruct.2019.02.065>.
- [17] L. Tu, D. Ning, S. Sun, W. Li, H. Huang, M. Dong, H. Du, A novel negative stiffness magnetic spring design for vehicle seat suspension system, *Mechatronics.* 68 (2020) 102370. <https://doi.org/10.1016/j.mechatronics.2020.102370>.
- [18] M.J. Mahmoodabadi, N. Nejadkourki, Optimal fuzzy adaptive robust PID control for an active suspension system, *Aust. J. Mech. Eng.* (2020). <https://doi.org/10.1080/14484846.2020.1734154>.
- [19] L.M. Jugulkar, S. Singh, S.M. Sawant, Analysis of suspension with variable stiffness and variable damping force for automotive applications, *Adv. Mech. Eng.* (2016). <https://doi.org/10.1177/1687814016648638>.
- [20] M.P. Nagarkar, G.J. Vikhe Patil, Multi-objective optimization of LQR control quarter car suspension system using genetic algorithm, *FME Trans.* (2016). <https://doi.org/10.5937/fmet1602187N>.
- [21] Ubaidillah, K. Hudha, H. Jamaluddin, Simulation and experimental evaluation on a skyhook policy-based fuzzy logic control for semi-active suspension system, *Int. J. Struct. Eng.* (2011). <https://doi.org/10.1504/IJSTRUCTE.2011.040783>.
- [22] T. Ram Mohan Rao, G. Venkata Rao, Parametric sensitivity analysis of a heavy duty passenger vehicle suspension system, *J. Eng. Appl. Sci.* (2009).