JIMMA UNIVERSITY
SCHOOL OF GRADUATE STUDIES
JIMMA INSTITUTE OF TECHNOLOGY
FACULTY OF CIVIL AND ENVIRONMENTAL ENGINEERING
STRUCTURAL ENGINEERING STREAM

ASSESSMENT ON EXTERNAL POST TENSIONING TO STRENGTHEN REINFORCED CONCRETE T-BEAM

A Thesis submitted to the School of Graduate Studies of Jimma University in Partial Fulfilment of the Requirements for the Degree of Master of Science in Civil Engineering (Structural Engineering).

By: Assefa Haile

March, 2019

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By: Assefa Haile

Advisor: Eng. Elmer C. Agon

Co-Advisor: Eng. Habtamu G/Medhin

March, 2019

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DECLARATION

I declare that the work which is being presented in this Research entitles “Assessment on external post tensioning to strengthen reinforced concrete T-beam” is comprises my own work and it has not been presented for a degree in any other university.

Assefa Haile ____________ _____/______/______

Researcher **Signature** **Date**

This thesis has been submitted for examination with my approval as university supervisor.

Eng. Elmer C. Agon (Asso.Prof) ____________________ _____/_______/_______

Advisor **Signature** **Date**

Eng. Habtamu G/Medhin(Msc) ____________________ ____/________/_______

Co-Advisor **Signature** **Date**

ABSTRACT

External posttensioning was become one of the most attractive techniques for strengthening reinforced concrete structures and the technique is widely used in the construction of various engineering structures and also as one of the most efficient approaches for strengthening of both new and existing structures. In an external prestressing system, the prestressing tendons are placed outside the concrete section and the prestressing force is transferred to concrete through end anchorages.

This research is carried out to investigate the externally post-tensioning technique for strengthening RC T-beams. In this research, ten identical T-section RC beams having the same dimensions and material properties were modeled and analyzed based on nonlinear finite element method applying two mid-third concentrated loads by using Abaqus software was investigated by varying the independent variables up to failure. Nine of these beams are strengthened by using external tendons, while the remaining beam is kept without strengthening as a control beam. Two external strands of 12mm diameter were fixed at each side of the web of the strengthened beams and located at depth of 200mm, 225mm and 243mm from top fiber of the section (d_{ps})*. So that the depth of strands to overall depth of the section ratio* ($d_{ps}/h=0.8$), ($d_{ps}/h=0.9$) *and* ($d_{ps}/h=0.972$) *were used in this research thesis.*

Eccentricity and strengthened length has significant effect on external post tensioning to strengthen of reinforced concrete beam, as depth of tendons increases and the distance of tendons from beam face decreases, the ultimate load carrying capacity of the beam increases. The load carrying capacity of beam increases as the depth of strand increases. It is concluded that the load carrying capacity for depth of strand 0.8h, 0.9h and 0.972h and strengthened length ratio of 0.83L,0.667L and 0.5L are 40.75%, 51.12% and 62.44% respectively compared to the control beam.

The parameter studied was carried out by considering concrete compressive strength, span to strengthened length ratio, depth of strand (eccentricity) and distance of strand from the face of the concrete beam(horizontal eccentricity).

Key words: Abaqus, post-tensioning, T-section Beams, Strengthening, Externally, Tendons

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First, I would like to gratefully thank the Almighty God for his divine help. I am sincerely grateful to my advisor **Eng. Elmer C.Agon** for his continuous support, suggestions and immeasurable contribution to my thesis. He always provides me guidance and feedback in this thesis and my co-advisor **Eng. Habtamu G/Medhin** for all their limitless efforts in guiding me through my work; without their help, I would not have completed my thesis.

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Finally, I am sincerely and endless thankful to my friends, Dawit Fufa and my family for their love, encouragement and support provided throughout my study.

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ACRONYMS AND LIST OF SYMBOLS

3D-FEM	Three-dimensional finite element modeling
NFEA	Non-linear Finite Element Analysis
ACI	American Concrete Institute
EBCS	Ethiopian Building Code Standard
EN	European code standard
JIT	Jimma Institute of Technology
PST	Posttensioning
RC	Reinforced concrete
C3D8R	8-node linear brick, reduced integration, hourglass
T3D2	2-node linear 3-D truss
CDP	Concrete compression damage plasticity
E_{cm}	Longitudinal Modulus of Elasticity of concrete
f_{cm}	Mean value of concrete cylinder compressive strength
f_{ck}	Characteristic compressive cylinder strength of concrete at 28 days
σ_c	Compressive Stress of concrete
σ_t	Tensile stress of Concrete
ε_c	Compressive strain of concrete
ε_{cl}	Compressive strain of concrete at peak point
${\varepsilon_t}^{pl}$	Plastic tensile strain of Concrete

${\varepsilon_t}^{ck}$	Strain at Concrete Crushing
d_t	Concrete tension damage parameters
d_c	Concrete compression damage parameters
μ	Viscosity parameters
β	Dilation angle
d_{ps}	Depth of strand from concrete top fiber
E_s	Modulus of Elasticity of steel
f_y	Steel yield stress
h	Over all depth of the cross-section
L_s	Partially Strengthened length

CHAPTER ONE

INTRODUCTION

1.1 Background

External posttensioning was initially developed for strengthening bridge structures, but this day it is used for strengthening both existing and new built structures. External posttensioning has become one of the most attractive techniques for strengthening reinforced concrete structures and the technique is widely used in the construction of various engineering structures and also as one of the most efficient approaches for strengthening reinforced concrete structures. In an external posttensioning system, the prestressing tendons are placed outside the concrete section and the prestressing force is transferred to concrete through end anchorages.

External posttensioning was defined as a prestress introduced by tendons located outside a section of a structural member, only connected to the member through deviators and end-anchorages. The main advantages using this technique are; higher utilization of small sectional areas, ease in inspection of the tendons and in their replacement and low friction losses. This type of prestressing can be applied to both new and existing structures that need to be strengthened due to several reasons such as: changes in use, deficiencies in design or construction phase, increased loading, progressive aging of concrete, corrosion of internal steel reinforcement and structural degradation.

Concrete structural components exist in buildings and bridges in different forms. Understanding the response of these components during loading is crucial to the development of an overall efficient and safe structure. In Prestressed concrete members, stresses are induced during the construction in such a way that they can resist stresses caused by externally applied loads. Prestressed concrete is most suitable for long span structural elements like beams and girders, where larger bending moment results in greater depth of beam or girder. (Chouragade, M., 2013).

In most of the cases of prestress beam, tendons are located with eccentricities towards the soffit of beams to counteract the sagging bending moments due to transverse loads. Consequently, the concrete beam develops a camber on the application or transfer of prestress. Since the bending moment at every section is product of the prestressing force and eccentricity, the tendon profile itself will represent the shape of B.M.D. (Krishna Raju N. 2011). There are a number of approaches for the study of the behavior of concrete structures, experimental, numerical, theoretical, and others. Finite Element Analysis (FEA) is a numerical one which provides a tool that can accurately simulate the behavior of concrete structures. (Joshuva, N. R.,et al., 2014).

The use of external prestressing as a means of strengthening or rehabilitating existing structures has been used in many countries since the 1950s. It has been found to provide an efficient and economical solution for a wide range of bridge types and conditions. The technique is growing in popularity because of the speed of installation and the minimal disruption to traffic flow. (Ng C. 2003).

The purpose of this research was to implement and verify a procedure in ABAQUS to check behaviour of reinforced concrete T-section beam externally strengthened steel tendons by using Finite Element method

1.2 Statement of the problem

There is a significant and growing need for the strengthening of RC structures. Poor reinforcement detailing, design errors, and general wear and tear can affect structural performance. Corrosion of reinforcement, attack by chemicals or pollution, overloading, and others can lead to loss of strength. Beam strengthening by external prestressing is to be carried out to regain the prestress lost. An external prestressing technique is being widely used for enhancing strength of the existing reinforced concrete beams to carry the required loading or enhanced loading but also for new structure. If end blocks are not strong enough to withstand the additional prestressing forces due to the proposed external prestressing, it is susceptible to distress.

Recently most of the structures are huge and big size in cross section which is cost and occupy more spaces. Dimensions of the concrete section can be reduced due to less space needed for internal reinforcement when the steel tendon was externally used. This research is stated with strengthening of reinforced concrete T-sections beams externally by post tensioning using non-linear finite element analysis.

1.3. Research questions

This study mainly focused on the following research questions:

- What is the increment on the load carrying capacity of beam with and without strengthening?
- How depth of strand (eccentricity) affects the strength of the T-section beam?
- Where is the location of end anchorages that strengthened RC beams externally?
- What are the numerical results obtained from finite element analysis compared to the test result?

1.4 Objectives

1.4.1 General objective

- Assessment on the behaviour (Responses) of reinforced concrete beams strengthened externally by posttensioning using non-linear finite element analysis subjected to concentrated loading.

1.4.2 Specific Objectives

- To investigate the load carrying capacity of the RC, T-sections beam strengthened externally posttensioning subjected to concentrated load.
- To determine the effects of eccentricity on externally post tensioning beam.
- To investigate installation of end anchorages at a distance from supports with governed purpose of strengthening.
- To compare and verify the results with previous experimental study.

1.5. Significance of the study

Placing prestressed tendons outside a structural member is an effective strengthening method to improve load carrying capacity. It requires a relatively small work effort and can be made very cost effective in comparison to other strengthening methods. Dimensions of the concrete section can be reduced due to less space needed for internal reinforcement. The study will increase the understanding of the structural responses associated with reinforced concrete T-beam strengthen externally with steel tendons which is the most widely used and also has a significant effect on the construction industry and economy of the country.

1.6 Scope and Limitations

Since verification is a vital part of numerical analysis, familiar cross-sections shall be studied where experimental results are expected to be obtained. Hence, a broad class of sections was not incorporated in the study. The scope of this study was limited to investigate the effect of eccentricity, depth of steel tendons, Cable profile and strengthened length to determine the structural static properties such as ultimate strength, concrete compression damage, tension damage and deflections under the concentrated loads. For that, T-sections reinforced concrete beams are taken for the analysis. The study was conducted on non-linear finite element program ABAQUS CAE v6.14-1. The numerical study was verified by previously done experimental results.

CHAPTER TWO

RELATED LITERATURE REVIEW

This chapter presented the past research works on reinforced concrete beams strengthened by posttensioning externally and related investigations world-wide. The main purpose of a literature review is to establish the academic and research areas that are relevant to the subject under study. Literature review was carried out to comprehend the linear and nonlinear behaviour of post tensioned reinforced-concrete beams and the applicability of the finite element software packages in simulating the behavior of the beam.

2.1 General

From the previous research done the principle of prestressing is the application of an axial load combined with a hogging bending moment to increase the flexural capacity of a beam and improve the cracking performance. It can also have a beneficial effect on shear capacity (Dally, 1997).

External post-tensioning involves the installation of anchors to the structure with minimal disturbance to the existing structure. The strengthening method allows the owner to continue using the buildings and only a small portion is closed for a short time while external tendons are installed. A well-known example of such a project is the rehabilitation of the Pier 39 Garage in San Francisco as reported by Aalami and Swanson (1988). The existing parking structure suffered severe cracking at the roof and leaking problem due to insufficient protection of internal unbonded tendons. The existing beams were strengthened by external post-tensioning; the anchors, deviators and precast members were fixed at night, and the external tendons were precut and pulled into their final position during the day shift. Most stressing works were accomplished by jacking from the outside of the building. This project demonstrates that the rehabilitation procedures through external post-tensioning can be carried out with practically no interruption to the regular functioning of the building.

Composite steel-concrete beams prestressed with high strength external tendons have demonstrated many advantages as compared with plain composite beams: Increase in ultimate moment capacity of structure, Enlarge the range of elastic behaviour before yielding for the structure with the introduction of internal stresses. The stresses can then oppose the moment generated by the loading. The amount of structural steel used in construction, based on yield strength alone, can be significantly reduced by the use of high-strength tendons, thereby reducing the cost of construction.(Dally, 1997).

A composite beam can be prestressed, using a jack, by the tensioning high-strength tendons connected at both ends to brackets or anchorages that are fixed to the composite beam. Prestressing a composite beam can introduce internal stresses into the member cross sections that can be defined for different purposes. Such induced stresses can then counteract the external loads applied on the structure. Prestressing can be carried out for simple-span or continuous span composite beams. In the positive moment region, the steel beam is usually prestressed before the concrete is cast because the negative moment induced by prestressing may be used to counteract the positive moments caused by the concrete's self-weight. In the negative moment region, the steel beam and concrete deck can also be prestressed either separately or jointly along the top flange before or after casting of the deck. (Saadatmanesh, 1989).

Scordelis, A. C., (1984), discussed analytical models and an efficient numerical procedure for the material and geometric nonlinear analysis of reinforced and prestressed concrete rigid frames, slabs, panels, and three-dimensional solids. Time dependent effects due to load history; temperature history; creep, shrinkage and aging of the concrete; and relaxation of the prestressing steel were included in the analysis. Fanning, P., (2001), presented dedicated numerical models for the nonlinear response of concrete under loading. Appropriate numerical modeling strategies were recommended and comparisons with experimental load-deflection responses were discussed for ordinary reinforced beams and post-tensioned concrete beams. Kim,U.,et al.(2010), suggested sophisticated 3-D finite element model for simulating the nonlinear flexural behavior of unbonded post-tensioned beams to compare analysis results with experimental results and investigated the effects of various prestressing forces on the flexural behavior of post-tensioned beams. Kasat, A. S. & Varghese, V., (2012), discussed a study of prestressed concrete beams using finite element analysis to understand the response of prestressed concrete beams due to transverse loading.

Ma et al. (1999) provided the details of optimized anchorage zone based on analytical and full-scale experimental studies. Aparicio et al. (2000) conducted ultimate load analysis with a nonlinear finite element numerical model and measured the thermal stresses and calculated the anchorage diaphragms due to the hydration heat of concrete. Stoll et al.(2000) studied experimentally two full-scale bridge beams made up of high strength concrete and fiber reinforced polymer products for prestressing and shear reinforcement. Miyamoto et al.(2000) investigated the dynamic behavior of prestressed composite girder bridges, strengthened with external tendons. Aparicio et al. (2002) presented the results of a test program on externally prestressed concrete beams. Five monolithic and three segmental beams were tested in bending and in combined bending and shear. Choy et al.(2002) studied the shear transfer mechanism of prestressed concrete encased steel beams.

Ghallab and Beeby(2005) described various factors that can influence the increase in the ultimate stress in steel external prestressing tendons. Chen,S (2005) conducted experiments on four groups of prestressed steel-concrete composite beams with external tendons in negative moment regions. Lou and Xiang(2006) proposed a numerical model based on the finite element method incorporating an arc length solution algorithm for materially and geometrically nonlinear analysis of concrete beams prestressed with external tendons. Czaderski and Motavalli(2007) presented the details of a 40-year old full scale concrete bridge girder with prestressed carbon fiber reinforced polymer plates anchored using the gradient method. Youakim and Karbhari(2007) presented a simple method to calculate the long-term prestress loss and the long-term change in concrete stresses in continuously prestressed concrete members with either carbon fiber reinforced polymer tendons.

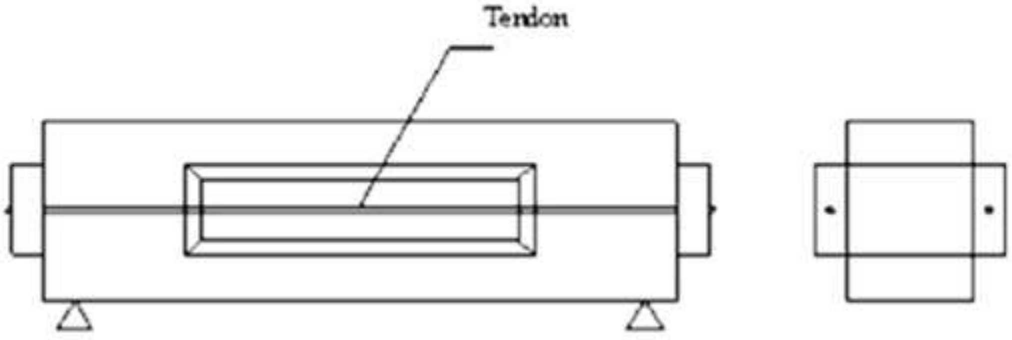


Figure 2.1 Anchoring of external prestressing at the ends. (Chen,S., 2005)

In strengthening reinforced concrete beams by external prestressing, the external tendons are anchored mostly to the ends of the girder if the end regions are accessible for anchoring. Refer to Fig.2.1. (Chen,S., 2005)

In this method, the external force is transferred to the member in the compression mode. It may create bursting tensile stresses in the end block. If the end blocks are not strong, it may lead to cracking. Steel brackets with through-bolts along with laterally prestressed anchor system is the another method to anchor external prestressing tendons to the end block/web (Fig.2.2). This technique is widely employed in the field for retrofitting prestressed concrete girders. In this method, transfer of prestress to the member will take place through friction.

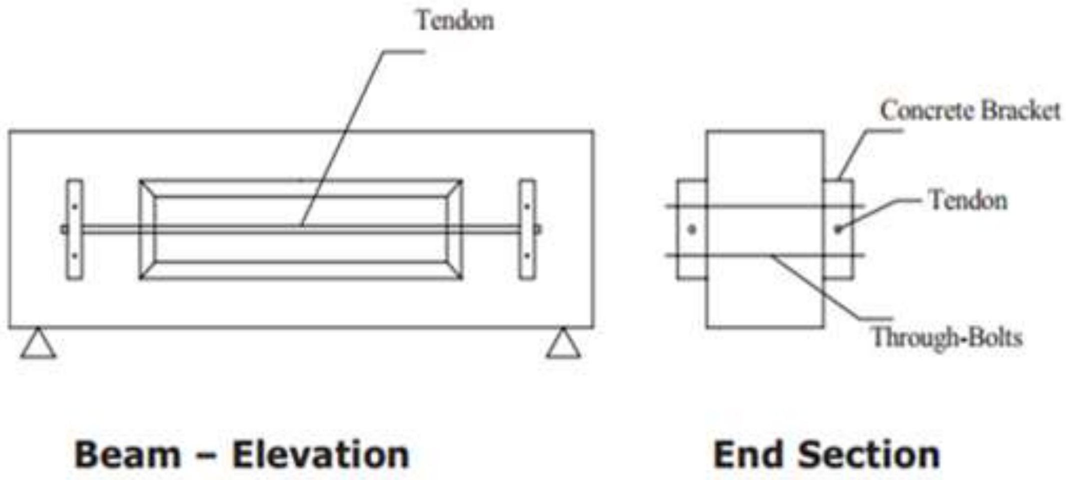


Figure 2.2 Anchoring of external prestressing using through bolts. (Chen S., 2005).

The writers propose an innovative method of side steel bracket with shear key anchors. In the proposed method, the transfer of external prestressing force will be in shear mode. Sleeve-type expansion anchors have been proposed to anchor the external prestressing tendons on the sides of the end block of the prestressed girders through steel brackets as shown in (Fig.2.3). (Chouragade, M., 2013).

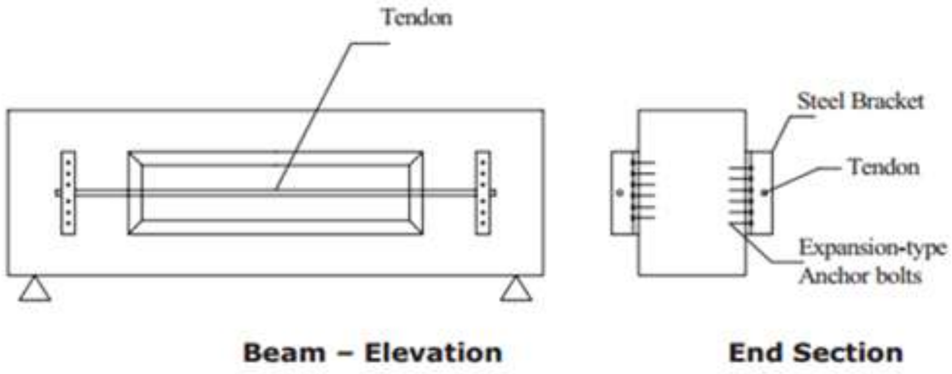


Figure 2.3Anchoring of external prestressing using expansion-type anchor bolts.

The number of bolts and anchorage depth into the concrete block depends upon the magnitude of required external force. The required transverse prestressing force is less compared to the value obtained by the second method. Fouling with the existing reinforcement in the end block does not arise due to shorter anchorage depth of anchor bolts. In this method, required external prestressing force will be transferred at a greater number of points (through bolts), developing more uniform stress distribution in the end block. This method is mainly suitable for continuous span and suspended span bridges. Hence, this method is a simple and safe solution for the anchorage of additional longitudinal prestressing tendons. Tensile and shear forces will be developed in the bolts due to external load. (Rama Chandra Murthy, 2009).

2.2 Tendon Stress at Ultimate Limit State

In external post-tensioning, tendons are fixed outside of concrete sections, and attached to the beam by anchors and deviators at discrete locations. The assumption of perfect bond between the tendons and surrounding concrete as in the beams prestressed with internal bonded tendons is no longer valid, as the relative displacements of concrete and external tendons are not prevented. The tendon stress at any load level in the response history depends on the global deformation of the whole structure. This makes the tendon stress member dependent rather than section dependent. Thus the ultimate tendon stress and consequently, the flexural capacity of the member should be evaluated through a nonlinear analysis of the beam-tendon system. (Naaman and Alkhairi, 1991)

2.3 Serviceability Requirement of Beams Strengthened with External Tendons

External post-tensioning can be used to enhance the load-carrying capacity of the strengthened beam; meanwhile, the serviceability requirements of the strengthened beam have to be satisfied in terms of deflection and maximum crack width. (Tan and Ng, 1997)

2.4 Continuous Beams Strengthened with External Tendons

Earlier works on beams strengthened with external tendons were focused on simple-span beam; studies on continuous beams were seen only during the last decade. Aparicio and Ramos (1996) noticed that the majority of Europe bridge codes were too conservative due to ignoring the increase in tendon stress beyond the effective stress while the American codes recommended unreasonably high stress increase up to yielding. Based on a finite element study on the externally prestressed concrete bridges, they proposed tendon stress increase, Δf_{ps}, for continuous bridges as follows:

1. For continuous monolithic box girder bridges: Δf_{ps} varies from 20 to 90 MPa. The increase in tendon stress depends on the span-to-depth ratio and the prestressing tendon length between the anchorages. In the study, the tendons anchored in every span registered highest tendon stress increase, followed by the tendons anchored in every two spans. The tendons anchored in every three spans had the lowest tendon stress increase.
2. For continuous segmental box girder bridges: Δf_{ps} =39 MPa

Du (2000) tested two double-span continuous beams strengthened with external tendons; the tests included a preloading process. The tests showed that, external prestressing reduced the beam deflection and crack width during the preloading process. At ultimate, external post-tensioning increased the flexural capacity of beam up to 60% and 124%. Beam strengthened with tendons singly-draped at loading point showed better control of crack width and higher increase in load-carrying capacity than the beam with straight tendons.

Tan and Tjandra (2002) investigated two-span continuous beams prestressed with external steel tendons and FRP tendons. It was shown that, the localized tendons anchored within beam spans were effective in enhancing the flexural performance of strengthened beams. Tendons provided over mid-span were more effective in reducing the beam deflection and crack width, and increasing the load-carrying capacity than those provided over interior support. Pattern loading reduced the beam capacity, caused higher crack width and larger deflection. Beams strengthened with FRP tendons were similar to those with the steel tendons as far as the load-carrying capacity, deflection and maximum crack width are concerned.

Harajli et al. (2002) tested nine continuous beams strengthened with external tendons. Except for one beam which failed in shear-type failure, all other beams failed in flexural mode by forming a collapse mechanism. Beams with straight tendons showed lower load-carrying capacity due to the severe second-order effect; on the other hand, beams with draped tendons registered a higher load-carrying capacity. It was shown that the span-to-depth ratio had a significant effect on the tendon stress increase for beams loaded with a single concentrated load due to a short plastic hinge length. When the plastic hinge length increased, the span-to-depth ratio had negligible effect.

2.5 Shear Deficiency in Beams Strengthened With External Tendons

Based on the study of simple-span beam strengthened with external tendons, Tan et al. (1997) found that the gain in the shear strength was not commensurate with the gain in the flexural strength, and the strengthened beam may failed by shear. Due to its sudden and catastrophic nature, shear failure should be avoided. The study showed that, decreasing the concrete strength or the amount of transverse reinforcement leads to shear type failure. When appropriate concrete strength and amount of shear reinforcement were provided, the beam would fail in flexure, even for the shear span-to-effective depth ratio as low as 2.5.

Due to the continuity in the beams, high shear and high moment occurs at the interior supports, making continuous beams more susceptible to shear deficiency. Tan and Tjandra (2003) conducted experimental and analytical studies on this problem. It was concluded that shear capacity governed the degree of strengthening and ignoring this would lead to undesirable shear-type failure. Adopting draped or parabolic profile reduced the high-shear zone, particularly if deviators could be provided near or exactly at the interior support. The parametric studies indicated that continuous beams with low concrete strength and span-to-effective depth ratios were more susceptible to shear-type failure.

2.6 Design Approach for Beams Strengthened With External Tendons

The design for beams strengthened with external tendons starts with structural analysis, in which load factors are applied to both the dead load and imposed load to get the design load, and moments at critical section are obtained and compared with the respective flexural capacities.

The behavior of beams strengthened with external tendons is conceptually the same as beams with internal unbonded tendons. The flexural capacity of the strengthened beams can be determined based on conventional section analysis. However there are two major differences between the two systems. Tan, K.H. and Kong, D. C. (2008)

1. In beams with internal unbonded tendons, the internal tendons remain in contact with the surrounding concrete; hence the eccentricity remains unchanged. However, for beams with external tendons, the tendons are free to displace relative to the beam axis when load are applied, giving rise to second-order effects.
2. Internal unbonded tendons can provide dowel action and increase the shear capacity of the beam, but external tendons cannot offer this action.

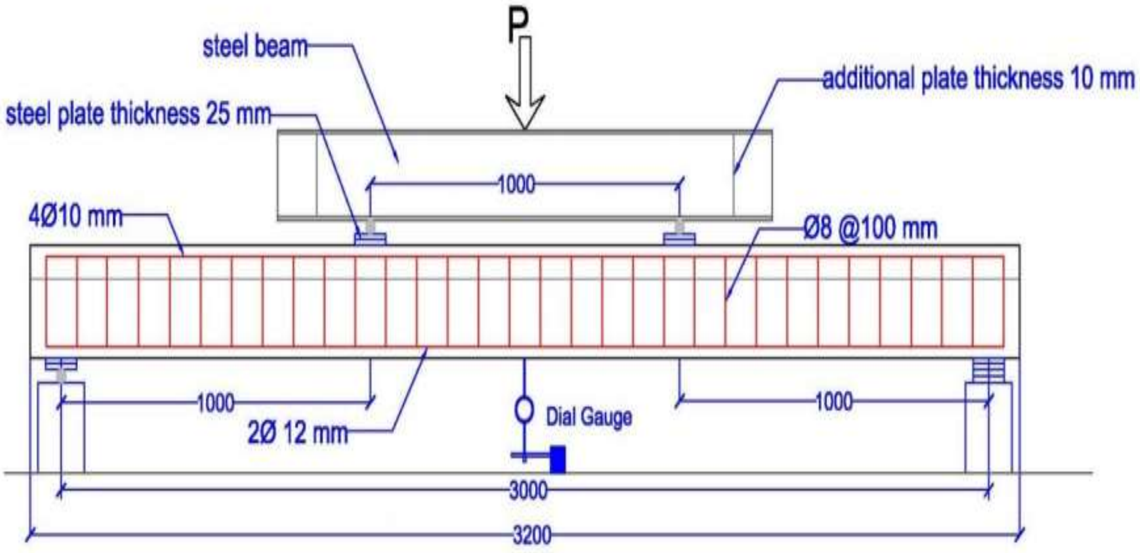
To make it possible to compute the flexural strength in accordance with the code procedure, ACI code (ACI 2008) requires that the second-order effect be minimized by attaching the tendons to the concrete members and maintain the eccentricity throughout the full range of expected member deflection.

2.7 Previous Experimental Studies on externally strengthened RC beams

Four RC T-beam having effective length equal to (3000 mm) and overall length of (3200 mm) were casted with cross sectional dimensions of h_f =75 mm, b_f=350 mm, b_w= 150 mm and h=250 mm. One of them is considered as control beam (Reference beam) without strengthening while; the other three beams were partially strengthened by using two external tendons. The main parameter conducting in this research is the strengthening length ratio (Ls/L) which is equal to the length of strengthening region (Ls) divided to the length of beam (L). In this research the strengthening ratios were 0.83, 0.67 and 0.50 for AT-1, AT-2 and AT-3 respectively. Beams reinforcements have been designed according to ACI Code as under-reinforced section i.e. ($\rho_w < \rho_{max}$). 2ϕ12mm deformed bars was used for longitudinal reinforcements in tension with effective depth of 220mm (510MPa yield stress). 4ϕ10mm deformed bars was used for reinforcement in compression with a depth of 30mm (580MPa yield stress). Stirrups of ϕ8@100mm deformed bars (540MPa yield stress) have been used in all beams. Cubic compressive strength of concrete at 28 days was 30MPa for all beams. (AbdulMuttalib I. Said, 2015)

2.7.1 Jacking Process

Two stress-relieved strands were used by fixing into two sides of the web of the strengthening beams. Diameter of strand was 12mm (The ultimate strength and the modulus of elasticity were 1860MPa and 190000MPa respectively). Fixing strands on the T-beams was used by manufactured equipment's from steel plates in each ends of the two strands. Fixing equipment anchored on two sides of concrete beams by drilling holes and used four steel bolts having diameter of 16mm. To reduce the second order effect on the behaviour of external prestressed beams, one deviator was used in mid span of all strengthened beams. Prestressing techniques conducted by install two strands on each side in a symmetrical manner to avoid any lateral deformation, which is avoided by jacking two strands simultaneously by using balance device. The maximum initial post-tensioning prestressed stress was 600MPa (AbdulMuttalib I. Said, 2015). The beam layouts and experimental setup by AbdulMuttalib I. Said is given by the following figure.



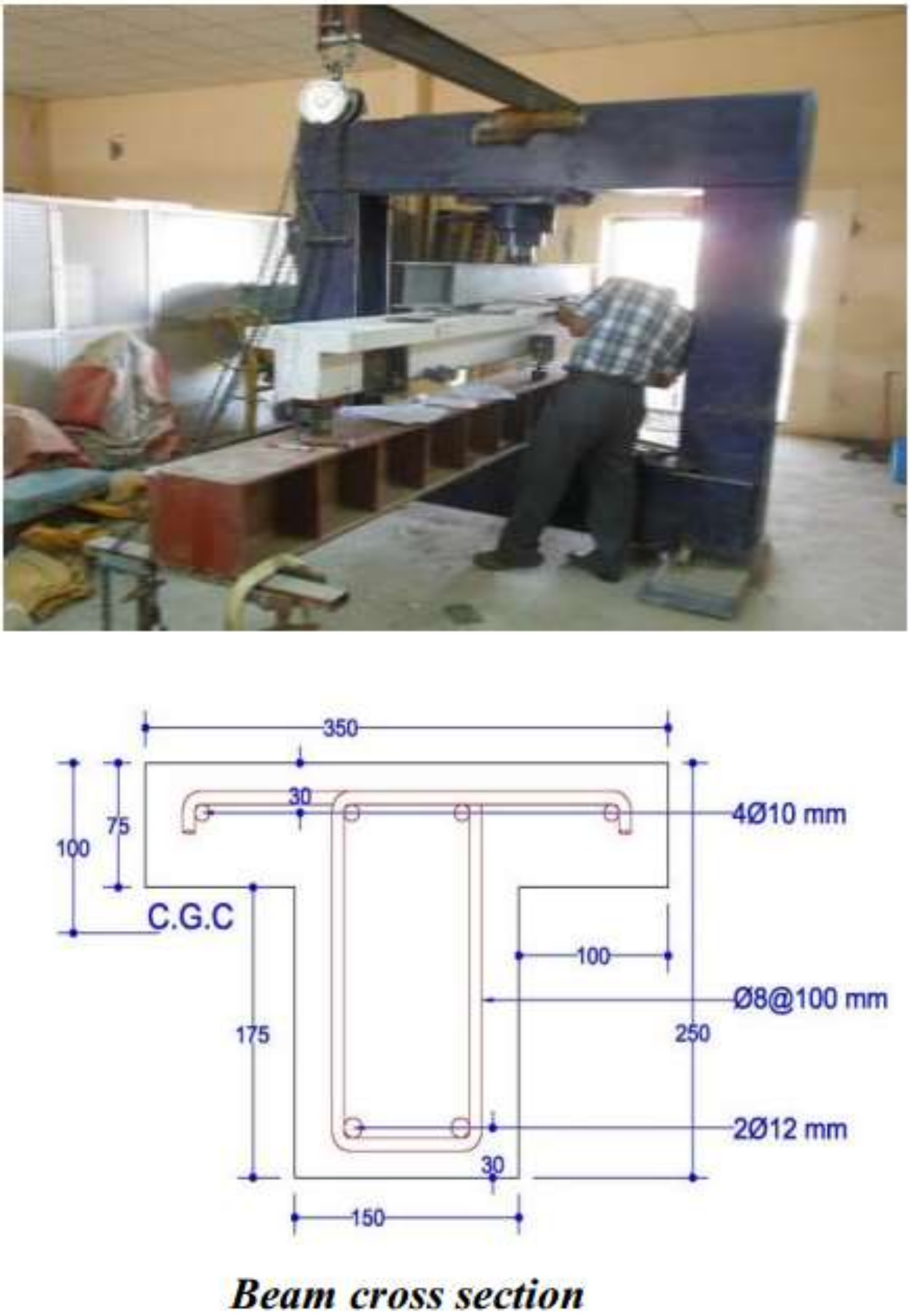


Figure 2.4 Layout and setup of typical tested beam (AbdulMuttalib I. Said, 2015).

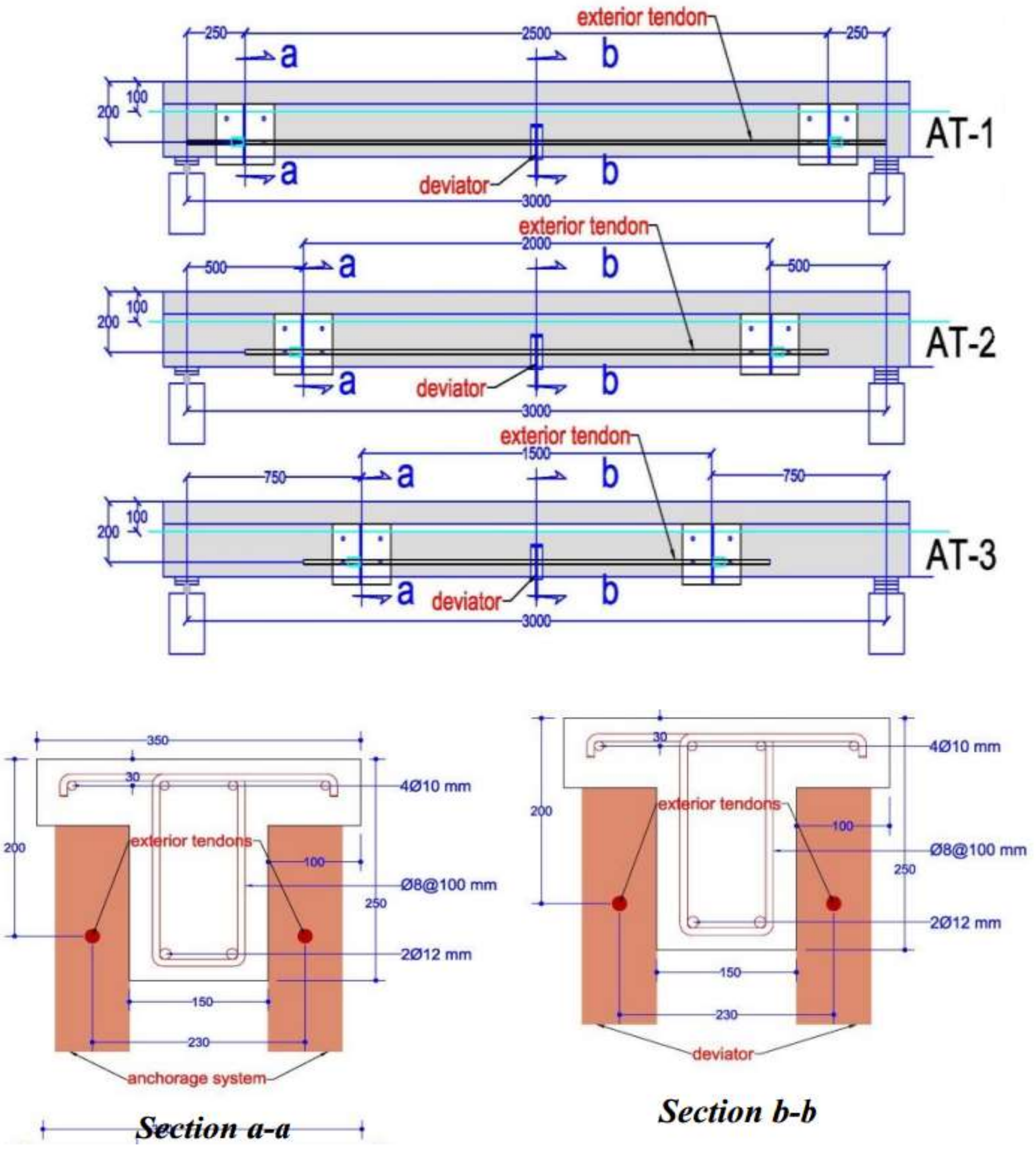


Figure 2.5 Strengthened beams layout (AbdulMuttalib I. Said, 2015).

2.7.2 Deviators and second-order effects

To reduce second order effect, one deviator was used at mid span for all strengthened beams. This deviator was fabricated from a plate of 6 mm thickness surrounding beam web in the bottom and welded to perpendicular stiffener plates on both sides as shown in Figure 2.6.

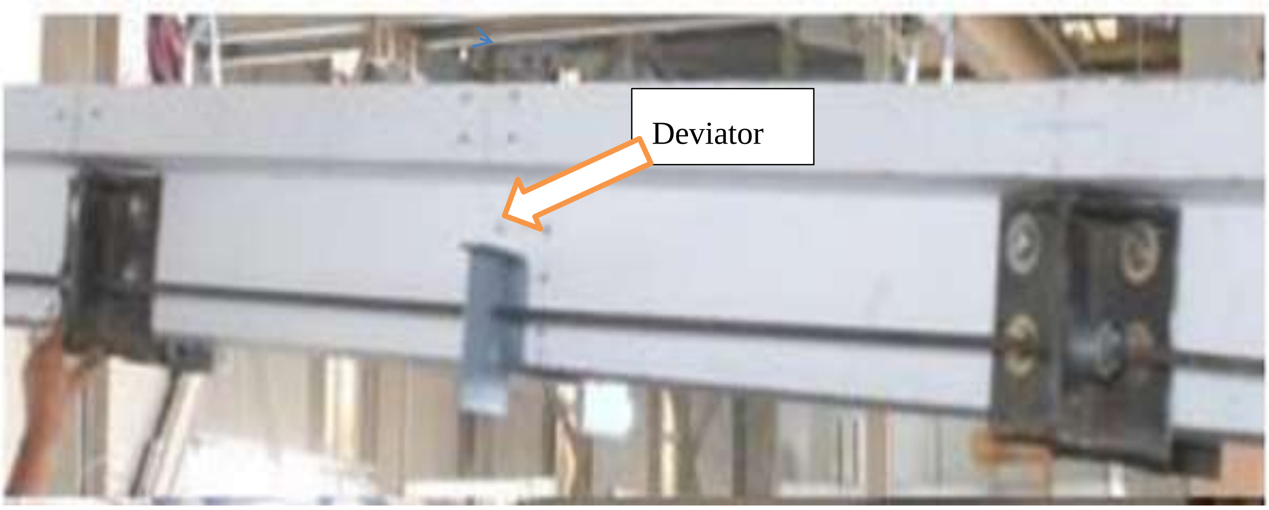


Figure 2.6 Deviator. (AbdulMuttalib I. Said, 2015)

As the tendons are placed outside the structure the connection to the structure are at deviators and anchorage. Between those points the tendons are free to move relative to the section of the structure. If deviators are not used the second-order effects due to changing tendon eccentricity, see figure 2.7, lead to a lower load carrying capacity, (Tan and Ng 1997). The use of deviators along the span of the structure can effectively reduce those effects, tests conducted by Tan and Ng (1997) showed that a single deviator at the section of maximum deflection resulted in satisfactory service and ultimate load behaviour.

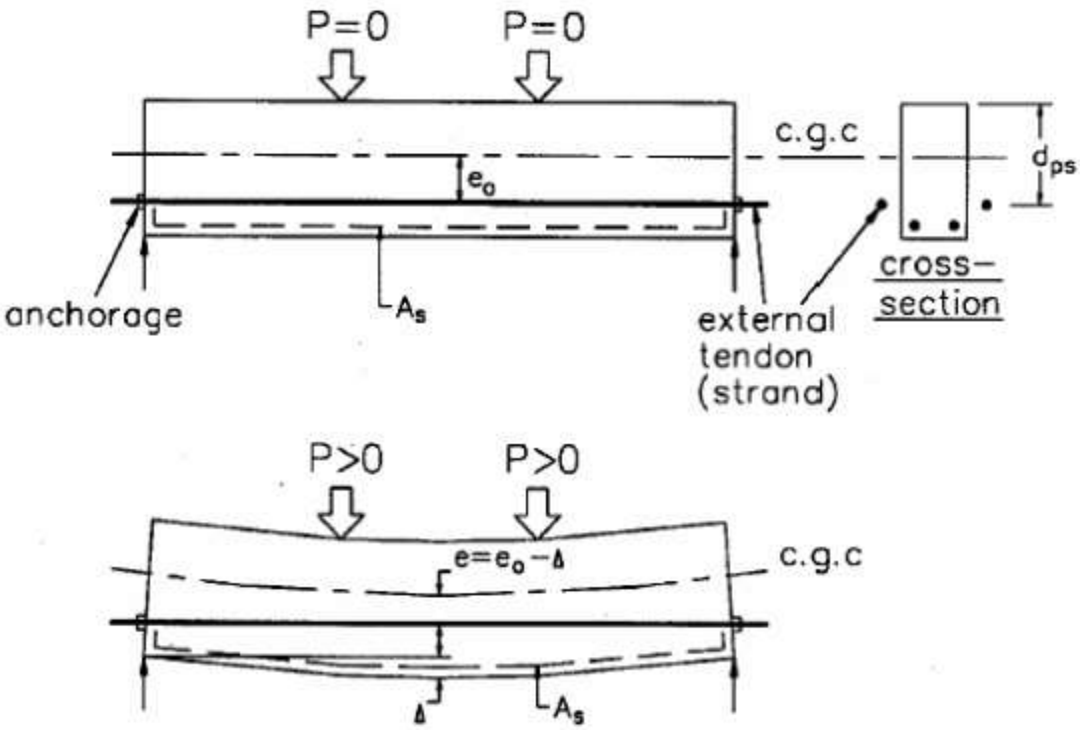


Figure 2.7 Second-order effects without deviators, Tan and Ng (1997).

This section will be discussing the results obtained from the experimental results. The main variables were the strengthened length to span length ratio for straight tendons at a constant depth of strand (0.8h). The beams have been strengthening partially along the span and the results have been compared in order to determine the significance of the under consideration variables, AbdulMuttalib I. Said (2015).

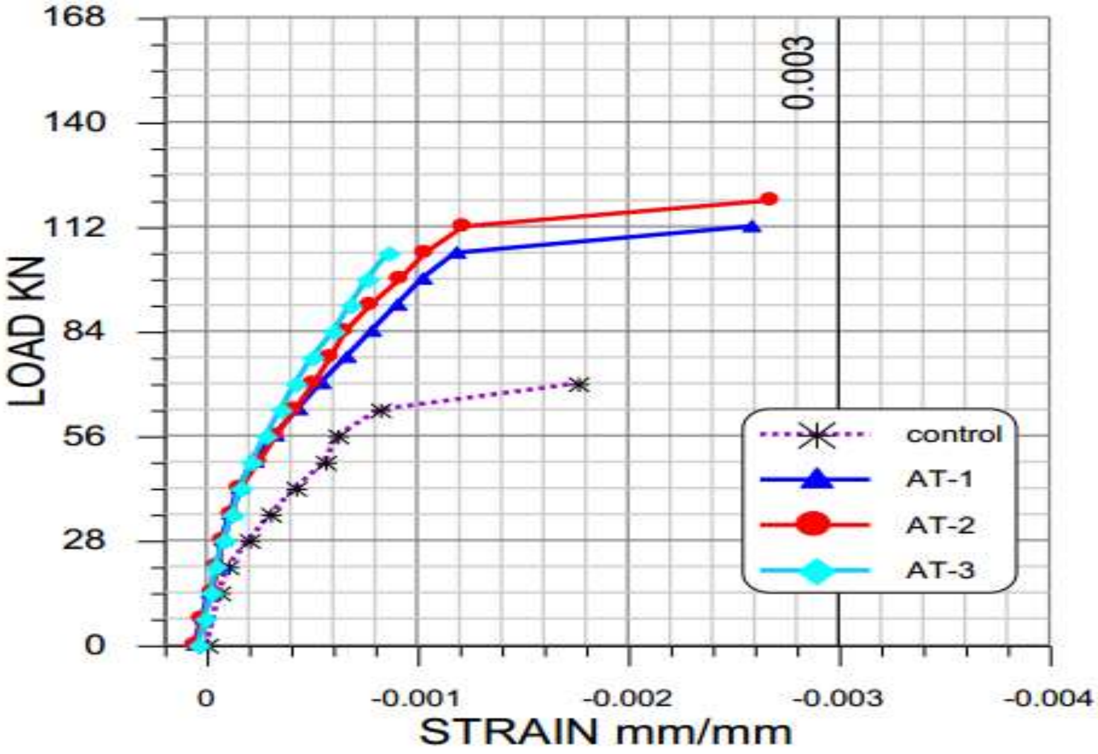


Figure 2.8 Strain at top fiber of concrete at mid-span section of tested beams.

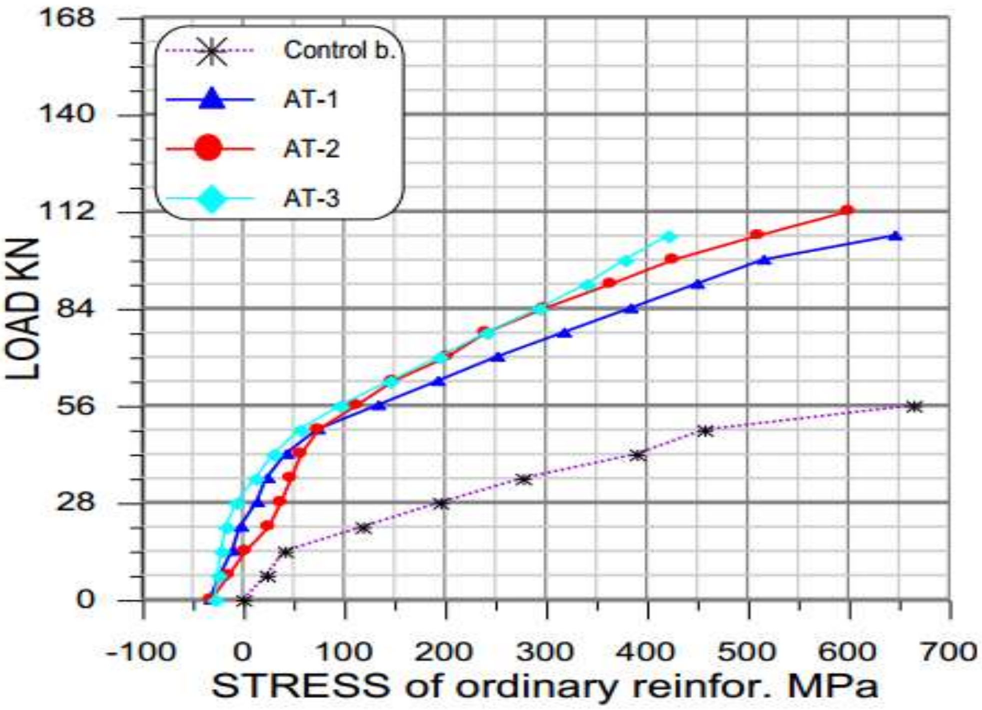


Figure 2.9 Stress of ordinary steel rebar’s of tested beams, AbdulMuttalib I. Said (2015).

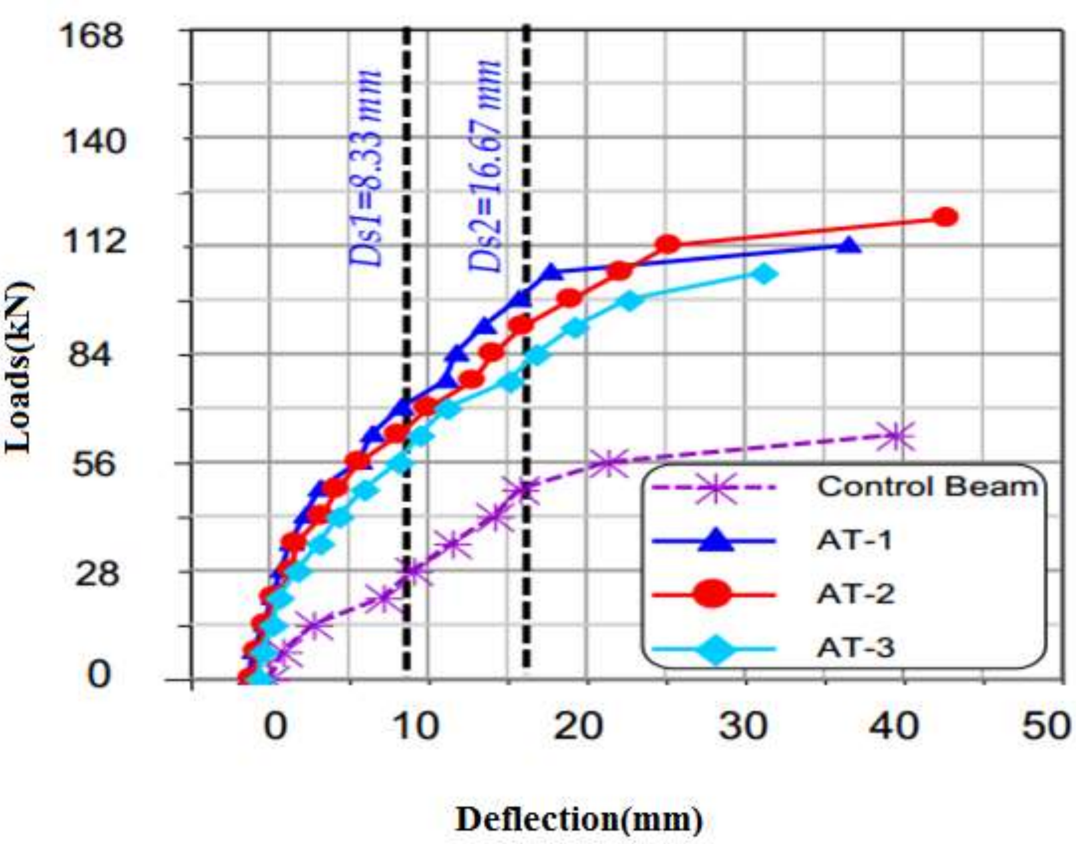


Figure 2.10 Load- mid span deflection responses for tested beams, AbdulMuttalib I. Said (2015).

The mode failure was predicted by measuring concrete strains at mid-span section of beams, fitting with straight lines and drawn for yield and ultimate conditions. Stresses in the ordinary reinforcement were calculated as strains at steel level, (d_s), multiplied by steel modulus of elasticity (E_s) but not greater than yield stress.

From previous experimental studied results by Abdulmuttalib I. Said, failure load capacity for control beam was 63kN when ordinary reinforcement has been yielded at section in pure bending moment zone. While the partially strengthening beam, AT-1, AT-2 and AT-3, are failed at load capacity equal to 112kN, 119kN and 105kN respectively. AbdulMuttalib I. Said (2015).

CHAPTER THREE

RESEARCH METHODOLOGY

This chapter presents and describes the approaches and techniques that the researcher will use to collect data and investigate the research problem. It includes the research design, sample size and selection, sampling techniques and procedure, data collection methods, data quality controls such as validations and reliability, procedure of data collection, data analysis and the process to build the model and the process to run the nonlinear analysis using ABAQUS software**.** The implementation of nonlinear material laws in finite element analysis codes is generally tackled by the software development industry in one or more ways. A number of powerful finite element software packages like ABAQUS, LISA and ANSYS have been becoming commercially available. In order to meet the objectives of this research, the reinforced concrete T-section beams externally strengthened using steel tendon was modeled using ABAQUS Version 6.14-1.

3.1 General Research Overview

The research design is based on sampling procedure which is essential for the study of the effect of posttensioning RC beams strengthened externally. It was modeled and analyzed by using finite element analysis of ABAQUS CAE v6.14-1. This research was systematically investigated to find answers to the effect associated with the RC beam incorporation to the external posttensioning. Generally, ten different models were conducted on the finite element software.

- Nine RC T-section Beams with externally strengthened by posttensioning tendons.
- One RC beams without post-tensioning (control beam).

On the other hand, it will process for collecting, analyzing and interpreting information to provide a recommendation to the research findings.

Comprehensive and detailed reviewing of literatures was done so as to validate the findings with the previously conducted experimental investigations for girder externally strengthened by post-tensioning. ABAQUS CAE v6.14-1 integrates every aspect of the engineering design process in one easy and intuitive environment.

3.2 Study Variables

3.2.1. Dependent Variables

The dependent variables, which are to be observed and measured to determine the effect of the independent variables, are listed below.

- Load carrying capacity of post tensioning reinforced concrete beam.
- Deflection and ultimate strength responses of external post tensioning reinforced concrete T-section beam.

3.2.2. Independent Variables

The independent variables, which are to be measured and manipulated to determine its relationship to observed phenomena, are selected and listed below.

- Concrete strength
- Cable Profile
- Eccentricity
- External distance of tendons from the web of the beam.
- Prestressing force

3.3. Data collection process

Both primary and secondary data was used in this research. The primary data for this research was the input material properties data in ABAQUS analysis result relative to the variables previously enumerated and the secondary data, which is used for comparison and validation purposes; experimental result was taken from literature review.

3.4 Finite Element Methods

Finite element method has been utilized for many applications in Engineering, in linear or nonlinear approach. Finite elements and nodes define the basic geometry of the physical structures being modeled in ABAQUS. Each element in the model represents a discrete portion of the physical structure, which is, in turn, represented by many interconnected elements. Elements are connected to one another by shared nodes. The coordinates of the nodes and the connectivity of the elements; that is, which nodes belong to which elements included the geometry of models. The collection of all the elements and nodes in a model is called the mesh. Generally, the mesh will be only an approximation of the actual geometry of the structure. The element type, shape, and location, as well as the overall number of elements used in the mesh, affect the results obtained from a simulation.

The Finite Element Method (FEM) is a numerical technique used to perform Finite Element Analysis (FEA) of any given physical phenomenon. The description of the laws of physics for space and time-dependent problems are usually expressed in terms of nonlinear relationships which cannot be solved with analytical methods, instead, an approximation can be constructed, typically based upon different types of discretization. These discretization methods approximate the real problem to the numerical models and the finite element method (FEM) is used to compute such approximations.

Nonlinear analysis in structural engineering include:- Strength analysis, Stability analysis, progressive failure analysis, safety and serviceability assessment of existing infrastructure, a shift towards high performance materials and more efficient utilization of structural components, direct use of NFEA in design for both ultimate load and serviceability limit states, simulation of materials processing and manufacturing, to understand basic structural behaviour, to test the validity of proposed material models.

Newton iterations are used to enforce equilibrium. The incremental-iterative procedure that advances the solution while satisfying the global equilibrium equations at each iteration “i”, within each time (load) step “n+1”, is governed by the incremental equations. Newton’s method is the most rapidly convergent process for solution of problems in which only one evaluation of the residual is made in each iteration. Indeed, it is the only method, provided that the initial solution is within the ball of convergence, in which the asymptotic rate of convergence is quadratic.

3.4.1 Abaqus modeling and analysis procedures

Abaqus/CAE is the Complete Abaqus Environment that provides a simple, consistent interface for creating Abaqus models, interactively submitting and monitoring Abaqus jobs, and evaluating results from Abaqus simulations. Abaqus offers a wide range of capabilities for simulation of linear and nonlinear applications. Problems with multiple components are modelled by associating the geometry defining each component with the appropriate material models and specifying component interactions. In a nonlinear analysis Abaqus automatically chooses appropriate load increments and convergence tolerances and continually adjusts them during the analysis to ensure that an accurate solution is obtained efficiently. A complete Abaqus analysis usually consists of three distinct stages: pre-processing, simulation, and post processing.

The concrete damaged plasticity model in Abaqus; provides a general capability for modeling concrete in all types of structures, uses concepts of isotropic damaged elasticity in combination with isotropic tensile and compressive plasticity to represent the inelastic behavior of concrete; used for plain concrete, even though it is intended primarily for the analysis of reinforced concrete structures; can be used with rebar to model concrete reinforcement; is designed for applications in which concrete is subjected to monotonic, cyclic, and/or dynamic loading under low confining pressures; consists of the combination of non-associated multi-hardening plasticity and scalar (isotropic) damaged elasticity to describe the irreversible damage that occurs during the fracturing process; can be used in conjunction with a viscoelastic regularization of the constitutive equations in Abaqus/Standard to improve the convergence rate in the softening regime; and requires that the elastic behavior of the material be isotropic and linear.

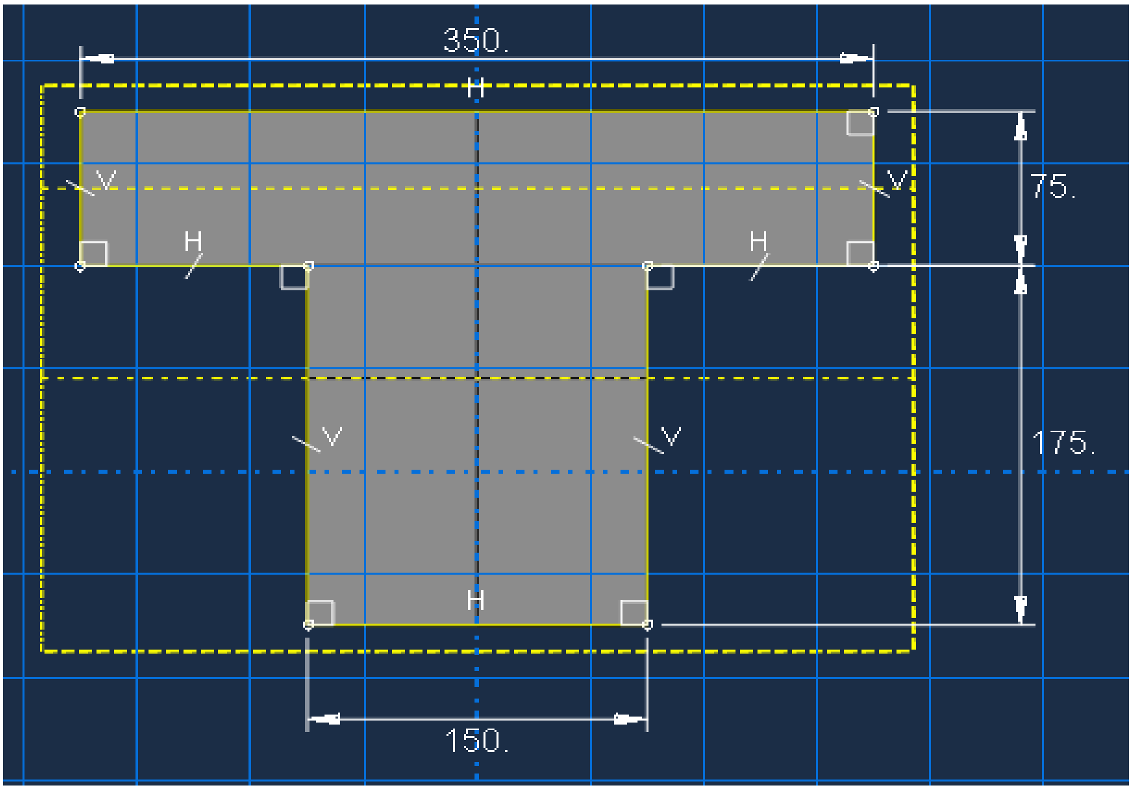
3.4.2 Element types

Beam and truss elements are generally triangular or quadrilateral with a node at each corner. However, elements have been developed that include an additional node on each side, this gives triangle elements with six nodes and quadrilateral elements with eight nodes. Since the only places where the forces are accurately calculated are at the nodes (they are interpolated at other positions), the accuracy of the model is directly related to the number of nodes. By introducing more nodes into an element, the accuracy of the results is increased; alternatively, the number of elements can be reduced for the same number of nodes, so reducing computational time.

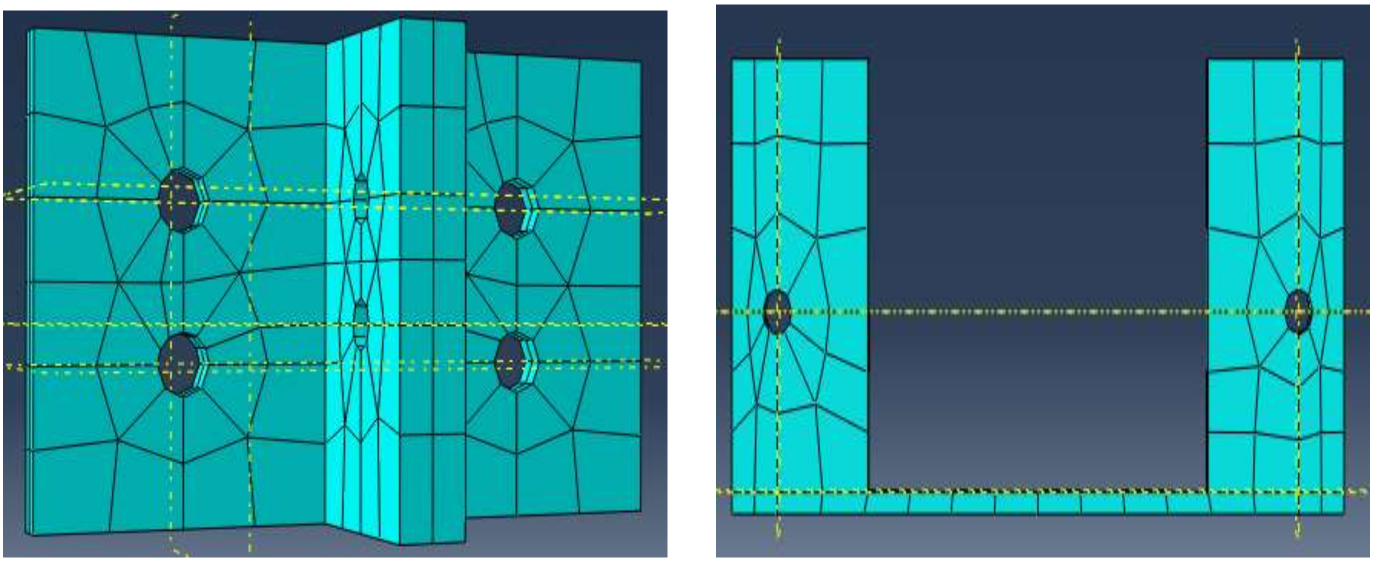
The mesh element for modeling of concrete and steel loading and bearing plates is 8-node brick element with three translation degrees of freedom at each node with reduced integration (C3D8R). The reinforcement bar is being defined by truss element which is called T3D2 (3Dimensional-2Node truss element) for analysis.

3.4.3 Geometry

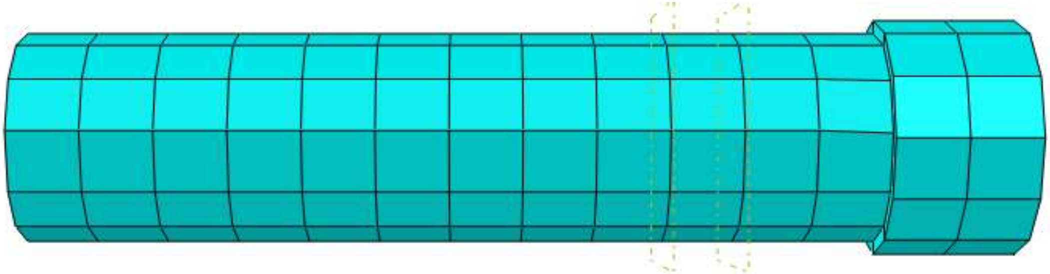
Reinforced concrete T-beams of different strengthened length and different depth of external strands are modeled using ABAQUS software. Ten reinforced concrete T-beams having effective length equal to (3000mm) and overall length of (3200mm) were modeled with cross sectional dimensions of h_f =75 mm, b_f=350 mm, b_w= 150 mm and h=250 mm. One of them is considered as control beam (Reference beam) without strengthening while; the other nine beams were partially strengthened by using two external tendons. Two plates with dimensions of 200 by 200mm and thickness of 10 mm have been bent to form an angle-shape section with unequal legs of 120mm and 80mm for end anchorage systems. Fixing equipment anchored on two sides of concrete beams by drilling holes and used four steel bolts having diameter of 16mm. To reduce the second order effect on the behaviour of external prestressed beams, one deviator was used in mid span of all strengthened beams. Tie constraint was used to connect deviator to beam and tendons to end anchorages.



a) Beam cross-section (All dimensions are in mm)



b) Anchorage c) Deviator



d) Shear stud as a steel bolt

Figure 3.1 Modeled geometry for beam cross-section, anchorage, steel bolt and deviator

3.4.4 Mesh Design (techniques)

The Mesh module contains tools that allow us to generate a finite element mesh on an assembly created within Abaqus/CAE. Various levels of automation and control are available so that the researcher can create a mesh that meets the needs of the research objectives during analysis.

Finite elements and nodes define the basic geometry of the physical structure being modeled in Abaqus. Each element in the model represents a discrete portion of the physical structure, which is, in turn, represented by many interconnected elements. In finite element analysis, elements are connected to one another by shared nodes. The coordinates of the nodes and the connectivity of the elements, that is, which nodes belong to which elements comprise the model geometry. The collection of all the elements and nodes in a model is called the mesh. Generally, the mesh will be only an approximation of the actual geometry of the structure.

The element type, shape, and location, as well as the overall number of elements used in the mesh, affect the results obtained from a simulation. The greater the mesh density (i.e., the greater the number of elements in the mesh), the more accurate the results. As the mesh density increases, the analysis results converge to a unique solution, and the computer time required for the analysis increases. The solution obtained from the numerical model is generally an approximation to the solution of the physical problem being simulated.

3.4.5 Loading

The Load module in Abaqus allows specifying loads, boundary conditions, and predefined fields. Loads and boundary conditions are step-dependent objects, which mean that we must specify the analysis steps in which they are active; some predefined fields are step dependent, while others are applied only at the beginning of the analysis.

The concentrated compression load was assigned at the top face of the steel plate. The results of the software analysis were showed that the amount of loading is constant in each beam. However, ABAQUS would apply the load in the incremental manner until the analysis stopped.

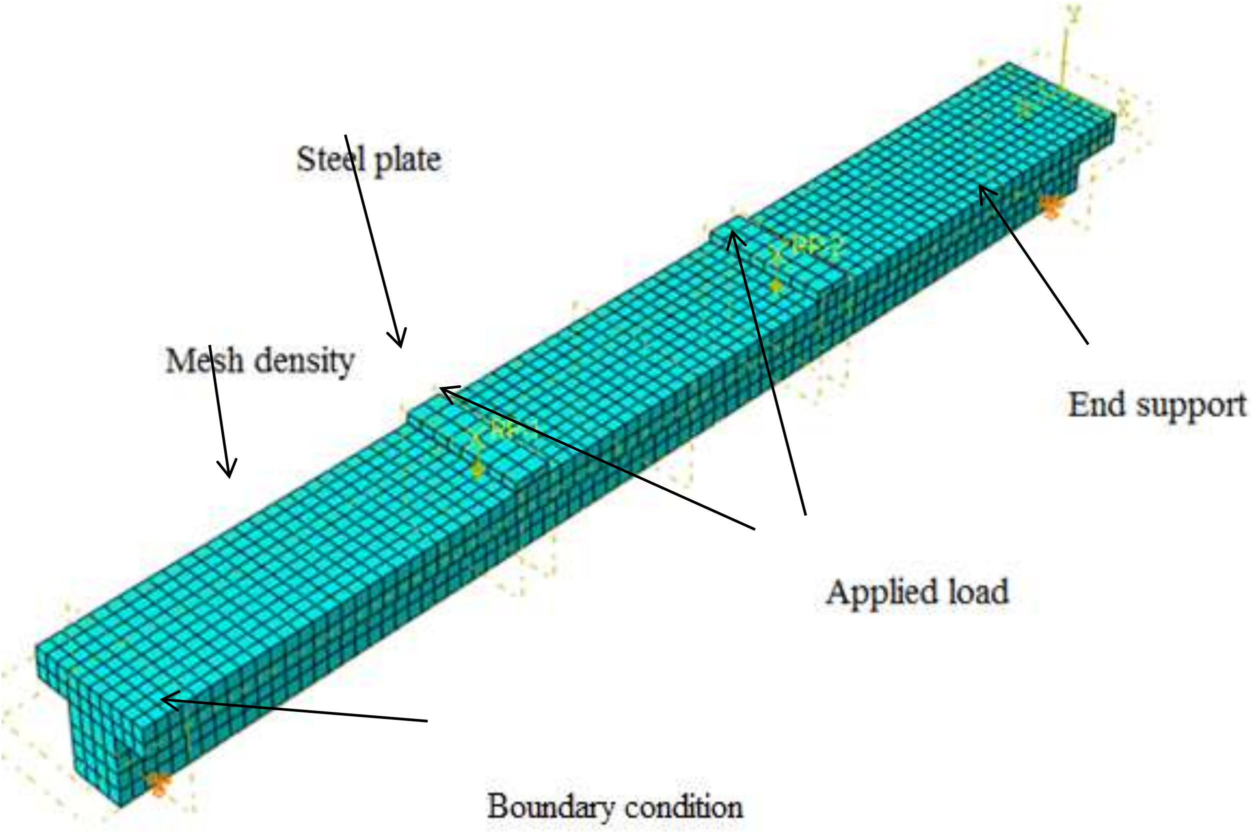


Figure 3.2 Generated mesh, applied load and boundary condition for control beam model

3.5 Material property

Material properties for all parameters of study must be specified. High-quality material data are often difficult to obtain, particularly for the more complex material models, the validity of the Abaqus results are limited by the accuracy and extent of the material data. Material nonlinearity occurs when the stress- strain relationship ceases to be linear by plastic yielding and strain hardening. Materially nonlinear effects arise from nonlinear constitutive model (that is, progressively disproportionate stresses and strains). Common examples of nonlinear material behaviour are the plastic yielding of metals, the ductile fracture of granular composites such as concrete or time-dependent behaviour such as creep ABAQUS incorporates a variety of nonlinear constitutive models, covering the behaviour of the more common engineering materials. Material natural properties under load and their combinations are accurately modelled.

To understand the material property, characteristic and behaviour under loading or other condition is fundamental to understanding the performance of structural reinforced concrete. This reinforced structure made of two materials with different property, namely concrete and steel. Steel materials consider a homogenous material and concrete material are heterogeneous combination of three material cement, sand and aggregates. This section presents the concrete and reinforcing steel material models that are elected in this study. Material properties for the constituent model are described below.

3.5.1 Properties of concrete

Reinforced concrete is a complex material, consisting of reinforcing steel, aggregates, water, cementious material, admixtures, and probably voids and un-hydrated cement. The properties of concrete are affected significantly by the different types of aggregate and by the varying proportions of the constituent materials. The properties of concrete are also affected by workmanship, weather, curing conditions and age of loading. The concrete beam was modelled using an 8-node linear brick, reduced integration, hourglass (C3D8R) element type and Hex element shape used structured for meshing.

Development of a material model for the behaviour of concrete is not a straightforward task. Concrete is a quasi-brittle material and has different behaviour in compression and tension. The tensile strength of concrete is typically 8 to 15% of the compressive strength (Shah, et al. 1995). The increase in strain in one direction increases the equivalent strain in another direction.

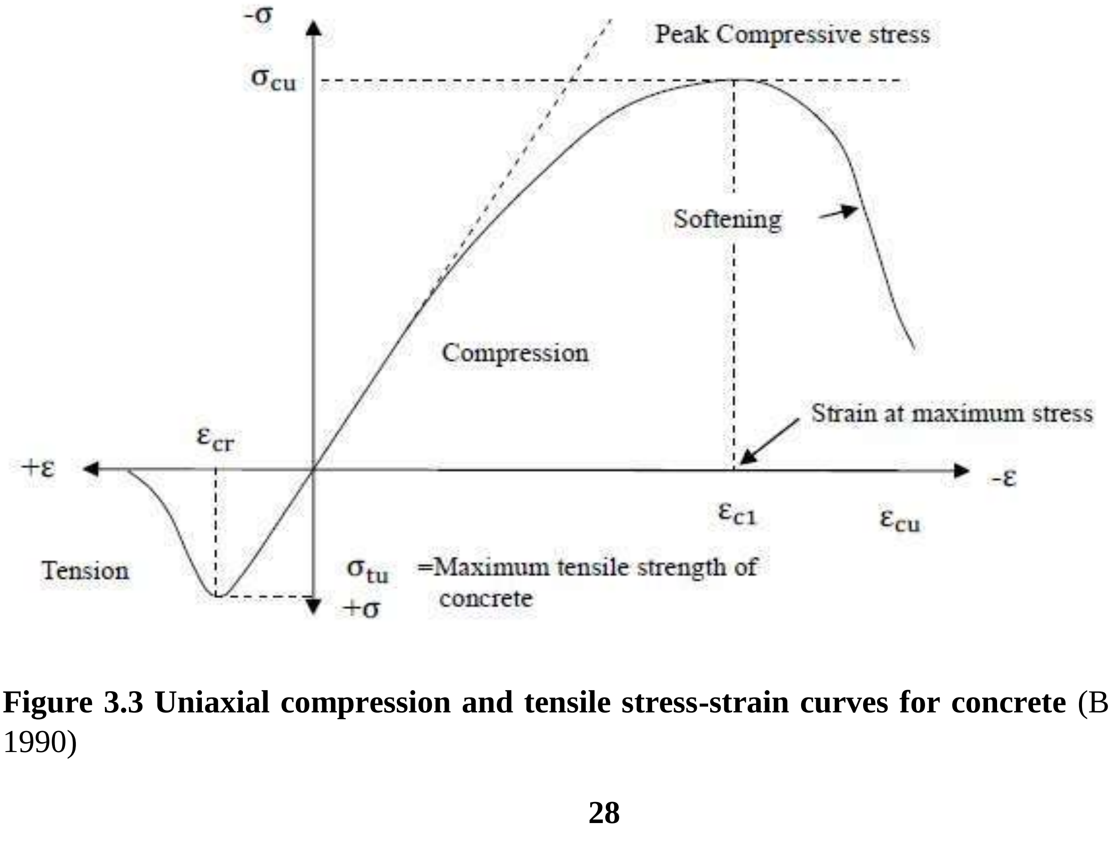


Figure 3.3 Uniaxial compression and tensile stress-strain curves for concrete (Bangash, 1990)

The mechanical behaviour of concrete is nonlinear in both tension and compression. In compression, the stress-strain curve for concrete is linearly elastic up to 30% of the maximum 40% compressive stress f_c'. Above this point, the stress increases gradually up to the maximum compressive strength. After it reaches the maximum compressive strength σ_{cu}, the curve descends into a softening region, and eventually crushing failure occurs at an ultimate strain ε_{cu}. In tension, the stress-strain curve for concrete is approximately linearly elastic up to the maximum tensile strength. After this point, the concrete cracks and the strength decreases gradually to zero (Bangash, 1990). Concrete exhibits a large number of micro cracks, especially, at the interface between coarser aggregates and mortar, even before subjected to any load. The presence of these micro cracks has a great effect on the mechanical behaviour of concrete, since their propagation during loading contributes to the nonlinear behaviour at low stress levels and causes volume expansion near failure. Many of these micro cracks are caused by segregation, shrinkage or thermal expansion of the mortar. Some micro cracks may develop during loading because of the difference in stiffness between aggregates and mortar. Since the aggregate-mortar interface has a significantly lower tensile strength than mortar, it constitutes the weakest link in the composite system. This is the primary reason for the low tensile strength of concrete.

3.5.1.1 Stress-Strain Curves for Concrete

Concrete is a lot more difficult to model in a finite element package. Concrete does not have a homogenous makeup. Concrete itself is a composite structure. It consists an aggregate material that is interlocked together and bound with cement. Aggregate interlock is complex and inconsistent, adding complexity to the theoretical modelling of concrete. Adding to the complexity is the difference that concrete curing or vibration makes to the strength of the concrete. Therefore, an adequate concrete model needs to be utilized to ensure that the required reliability is obtained.

3.5.1.2 Concrete in Compression

Concrete in compression was modelled as an elastic–plastic material with strain softening. The stress–strain relationship for concrete in uniaxial compression. Concrete damaged plasticity model is available in ABAQUS for concrete material model, which simulates crushing under compression and cracking under tension with tension stiffening and shear capacity of cracked concrete. Elastic state of concrete was determined by equation 3.1 mentioned on Euro code with Poison’s ratio of 0.2(EBCS EN 1992-1-1:2013).

$$E_{cm} = 22(0.1f_{cm})^{0.3} \tag{3.1}$$

The concrete stress in compression was assumed to be linear up to 40% of its maximum compressive stress. Beyond this point, the concrete stress was represented as a function of strain as shown in equation 3.2.

$$\sigma_c = \frac{E_c \varepsilon_c}{1+\left(\frac{\varepsilon_c}{\varepsilon_{cl}}\right)^2} \tag{3.2}$$

Concrete was modelled under the “Concrete compression damage plasticity”(CDP) option, when creating a material. A stress strain curve needs to be generated to define the compressive behavior of concrete. The compressive strength of the concrete used in this model was shown on the figure below.

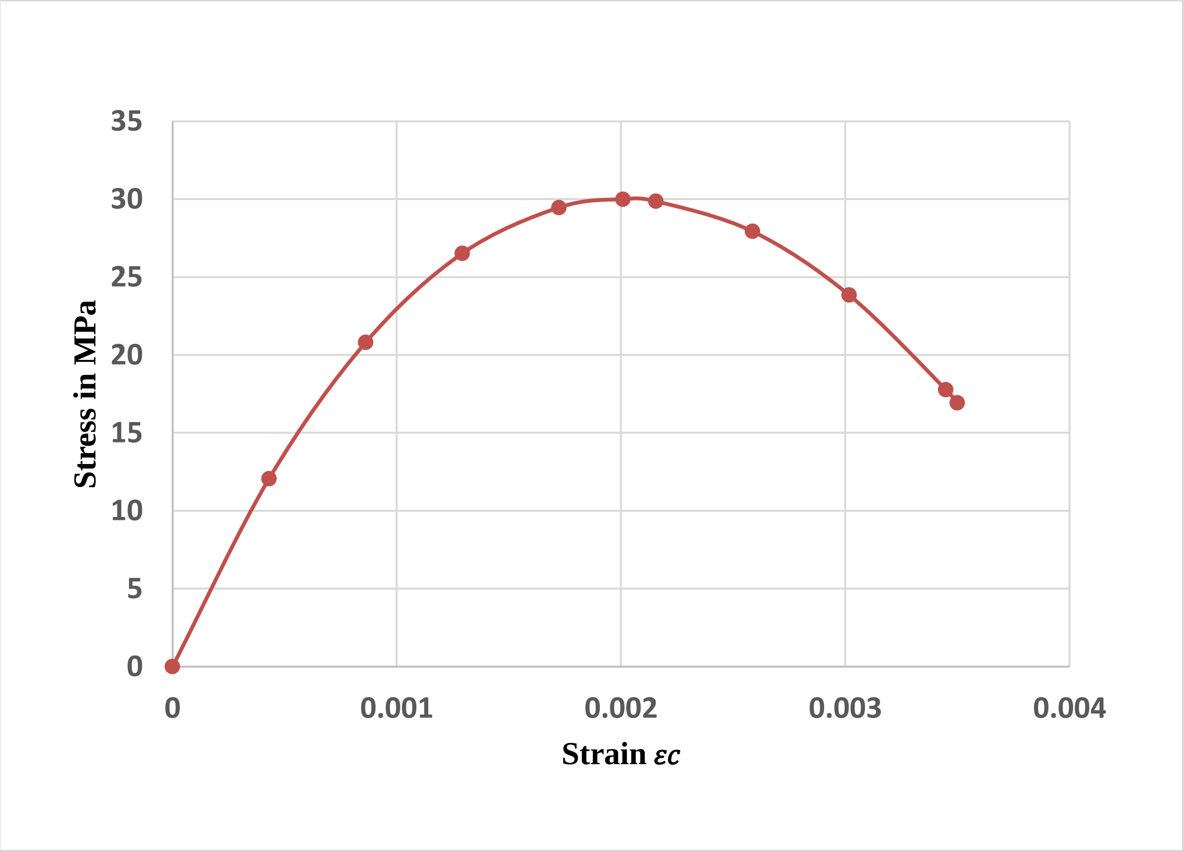


Figure 3.4 Stress-strain curves for concrete in compression

The above representation was calculated using the following uniaxial compression relationship.

.
$$\frac{\sigma_c}{f_{cm}} = \frac{\kappa\eta-\eta^2}{1+(\kappa-2)\eta} \tag{3.3}$$

$$\kappa = 1.05 * E_{cm}|\varepsilon_{cl}|/f_{cm} \tag{3.4}$$

$$\eta = \frac{\varepsilon_c}{\varepsilon_{cl}} \tag{3.5}$$

$$f_{cm} = f_{ck} + 8(Mpa) \tag{3.6}$$

In this case, the value of $\boldsymbol{\varepsilon_c}$ was taken as 0.002, within the elastic region; the Young's modulus was taken as 30.558 MPa and the Poison's ratio taken was 0.2.

It is assumed that the uniaxial stress-strain curves can be converted into stress versus plastic strain Curves in Abaqus for concrete damage plasticity model. Compressive stress data are provided as a tabular function of inelastic (or crushing) strain. Positive (absolute) values should be given for the compressive stress and strain. The stress strain curve can be defined beyond the ultimate stress, into the strain-softening regime. Hardening data are given in terms of an inelastic strain, instead of plastic strain.

Abaqus will issue an error message if the calculated plastic strain values are negative and/or decreasing with increasing inelastic strain, which typically indicates that the compressive damage curves are incorrect. In the absence of compressive damage, inelastic strain and plastic strain are equal.

3.5.1.3 Concrete in Tension

Tension damage parameters in the cracking behavior of concrete were developed with strain responses by Equation 3.7

$${\varepsilon_t}^{pl} = {\varepsilon_t}^{ck} - \frac{d_t}{(1-d_t)}\frac{\sigma_t}{E_o} \tag{3.7}$$

$$\text{Where, } \varepsilon_t = \frac{f_t}{E_c} \tag{3.8}$$

Concrete in tension was modeled by using the "concrete tension damage" option. The yield stress-cracking strain curve for C-30 concrete according to EBCS EN 1992-1-1:2013, graphically drawn as figure 3.5 below.

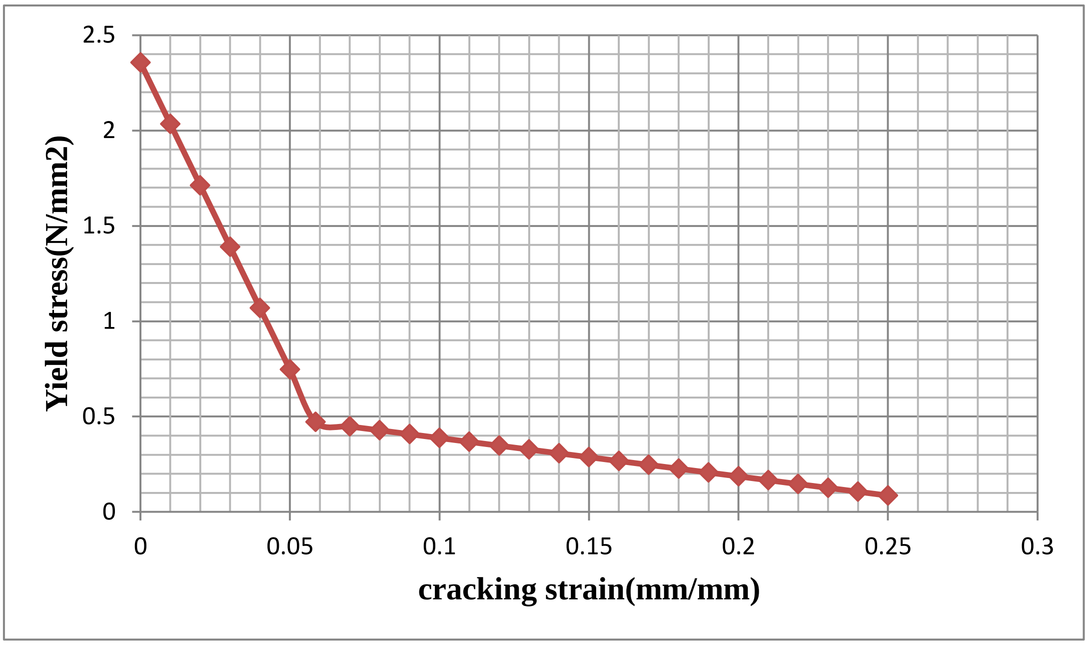


Figure 3.5 Yield stress-cracking strain curve for C-30 concrete

The material input data (properties of concrete), compressive strength, density, and elasticity properties are given in table 3.1 below.

Table 3.1 Material data input (concrete properties)

Concrete grade	C30	Concrete Plasticity Parameters	
Concrete density	2.4E-009		
Concrete elasticity		γ(Eccentricity)	0.1
E(GPa)	30.588	β(dilation angle)	36
ν	0.2	K(Second stress invariant ratio)	0.67
		σ_{b0}/σ_{c0} (stress ratio)	1.16
		μ(Viscosity parameters)	0.00001

Table 3.2 Concrete damaged plasticity

concrete damaged plasticity			
Compression damage		Tension damage	
Damage Parameter(dc)	Inelastic Strain	Damage Parameter(dt)	Displacement
0	0	0	0
0	3.63643E-05	0.13664553	0.01
0	0.000181511	0.27329106	0.02
0	0.000425809	0.40993659	0.03
0	0.000760728	0.54658212	0.04
0	0.00102836	0.68322765	0.05
0.004537427	0.001178691	0.8	0.05854564
0.069031154	0.001672944	0.809782419	0.07
0.205150067	0.002237444	0.818322765	0.08
0.407367365	0.00286677	0.826863111	0.09
0.435990282	0.002946843	0.835403456	0.1
		0.843943802	0.11
Compressive behavior		0.852484147	0.12
Yield stress	Inelastic Strain	0.861024493	0.13
12.07133818	0	0.869564839	0.14
20.81516597	3.63643E-05	0.878105184	0.15
26.52613735	0.000181511	0.88664553	0.16
29.46511454	0.000425809	0.895185876	0.17
30	0.000760728	0.903726221	0.18
29.86387719	0.00102836	0.912266567	0.19
27.92906538	0.001178691	0.920806912	0.2
23.84549799	0.001672944	0.929347258	0.21
17.77897906	0.002237444	0.937887604	0.22
16.92029154	0.00286677	0.946427949	0.23
		0.954968295	0.24
		0.963508641	0.25

Table 3.3 Concrete Tensile behavior

Tensile behavior	
Yield stress	Cracking Strain
2.355427323	0
2.033568708	0.01
1.711710093	0.02
1.389851478	0.03
1.067992863	0.04
0.746134249	0.05
0.471085465	0.05854564
0.448043687	0.07
0.427927523	0.08
0.40781136	0.09
0.387695196	0.1
0.367579033	0.11
0.34746287	0.12
0.327346706	0.13
0.307230543	0.14
0.287114379	0.15
0.266998216	0.16
0.246882052	0.17
0.226765889	0.18
0.206649726	0.19
0.186533562	0.2
0.166417399	0.21
0.146301235	0.22
0.126185072	0.23
0.106068908	0.24
0.085952745	0.25

3.5.2 Steel Reinforcement Modeling

Reinforcement comes in different types and shapes. Those most commonly used are deformed circular cross-sectional bars. The spiral deformation pattern on the bars strengthens the mechanical bond between the bars and concrete. The properties of reinforcing steel, unlike concrete, are generally not dependent on environmental conditions or time. Thus, the specification of a single stress-strain relation is sufficient to define the material properties needed in the analysis of RC structures.

Rebar's are typically used with metal plasticity models to describe the behaviour of the rebar material and are superposed on a mesh of standard element types used to model the concrete. Since the steel reinforcement is modelled as a one dimensional element, only a one-dimensional stress-strain relation for steel is required. Figure 3.3 shows the typical uniaxial stress-strain relation for reinforcement used in the analysis. As can be seen from this figure, it is linear elastic up to the steel yield stress f_y the stress is then assumed to be constant with increasing steel strain. The stress-strain relation in compression is assumed to be the same as the one in tension. Poisson's ratio of 0.3 was assumed for steel reinforcement.

The material coefficients of steel used in Abaqus linear elastic material model modulus of elasticity E_S = 210 GPa, Poisson's ratio of ν = 0.3 and density of ρ = 7850 kg/m^3.

3.5.2.1 Uniaxial stress-strain model for steel reinforcement in tension

The reinforcing steel is defined as an isotropic material. In the plastic properties, the stress potential model was selected and Von Mises yield surface was tried for the plastic behavior of the steel. In Abaqus reinforcement in concrete structures is typically provided by means of rebars, which are one-dimensional rods that can be defined singly or embedded in oriented surfaces. Rebars are typically used with metal plasticity models to describe the behavior of the rebar material and are superposed on a mesh of standard element types used to model the concrete (Simulia 2013)).

All the analyses in this study used the embedded region alternative and therefore this section will only explain how to model reinforcement in this manner. The material is defined as an ideal elasto–plastic material, i.e. No hardening behavior for those experiments that does not have the fracture tensile strength and including the hardening behaviour if ultimate tensile strength was available. This is done by using the elastic and plastic material models in Abaqus. The input parameters should be set according to the reinforcement used in the structure. The required input parameters are Young's modulus, Poisson's ratio, yield strength and the ultimate strength with ultimate strain recommended according to EBCS EN 1992. Normally it is sufficient to model the reinforcement with truss elements, for which the only input parameter needed is the cross–sectional area of the reinforcement.

3.6 General studies on model of external posttensioning beam

The flexural behavior of reinforced concrete (RC) T-beams strengthened with external prestressed tendons is investigated. The objective of the experimental investigation was to examine effectiveness and feasibility of strengthening using external post-tensioning technique to increase flexural capacity and improve serviceability of RC beams. Effectiveness of locally strengthened beams with external prestressed tendons have been investigated in terms of strengthening ratio which equal to length of strengthening region (length of tendon or distance between anchors) divided by length of the beam. Effects of tendons depth levels relative to beam top fiber expressed as depth ratio, which equal to depth of the strand to height of the section.

3.6.1 Details of Specimens

Ten RC T-beam having effective length equal to (3000 mm) and overall length of (3200 mm) were casted with cross sectional dimensions of h_f =75 mm, b_f=350 mm, b_w= 150 mm and h=250 mm. One of them is considered as control beam (Reference beam) without strengthening while; the other three beams were partially strengthened by using two external tendons. The main parameter conducting in this research is the strengthening length ratio (Ls/L) which is equal to the length of strengthening region (Ls) divided to the length of beam (L). In this research the strengthening ratios were 0.83, 0.67 and 0.50 for AT-1, AT-2 and AT-3 respectively. Beams reinforcements have been designed according to ACI Code as under-reinforced section i.e. ($\rho_w < \rho_{max}$). 2ϕ12mm deformed bars was used for longitudinal reinforcements in tension with effective depth of 220mm (510MPa yield stress). 4ϕ10mm deformed bars was used for reinforcement in compression with a depth of 30mm (580MPa yield stress). Stirrups of ϕ8@100mm deformed bars (540MPa yield stress) have been used in all beams. Cubic compressive strength of concrete at 28 days was 30MPa for all beams.

Table 3.4 Properties of the strengthened beams.

Beam designation	Strengthening length(Ls) mm	(Ls/L) Ratio	Configuration of strand	Depth of strand (dps) mm	(dps/h) Ratio
AT-0	Control Beam(Reference beam)				
AT-1	2500	0.83	Straight	200	0.8
AT-2	2000	0.67	Straight	200	0.8
AT-3	1500	0.50	Straight	200	0.8

The other six beams are modeled by varying depth of strand, strengthened length and distance of strand horizontally from the face of the beams.

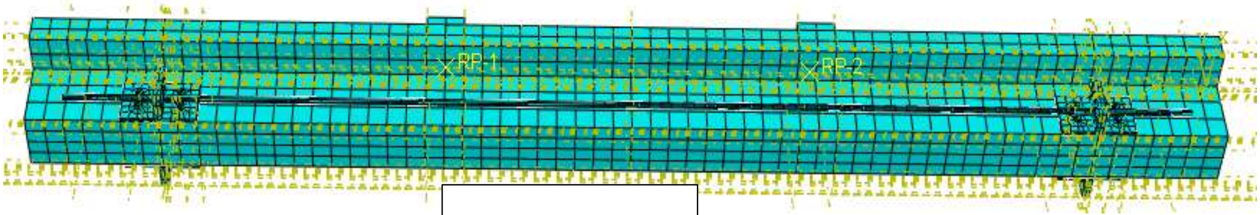
Table 3.5 Properties of the strengthened beams for constant length.

Beam designation	Strengthening length(Ls) mm	(Ls/L) Ratio	Configuration of strand	Distance of Tendons from beam face(mm)	Depth of strand (dps) mm
AT-0	Control Beam(Reference beam)				
AT-1	2500	0.83	Straight	40	200
BT-1	2500	0.83	Straight	40	243
BT-2	2500	0.83	Straight	25	243

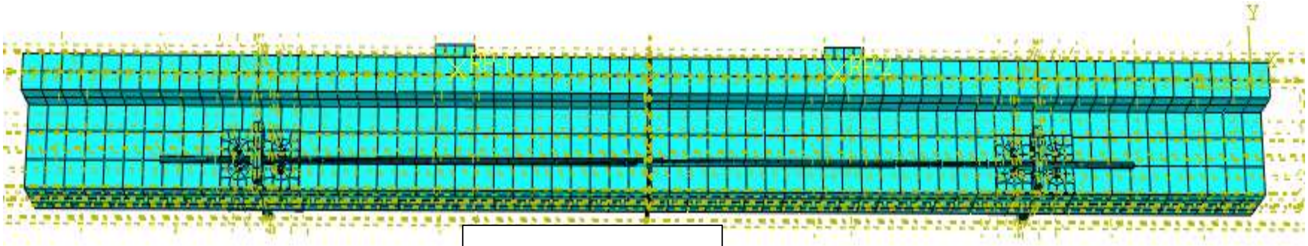
Table 3.6 Properties of the strengthened beams varying the depth of strand.

Beam designation	Strengthening length(Ls) mm	(Ls/L) Ratio	Configuration of strand	Distance of Tendons from beam face(mm)	Depth of strand (dps) mm
AT-0	Control Beam(Reference beam)				
AT-1	2500	0.83	Straight	40	0.8h
AT1-3	2500	0.83	Straight	40	0.9h
AT1-4	2500	0.83	Straight	40	0.972h

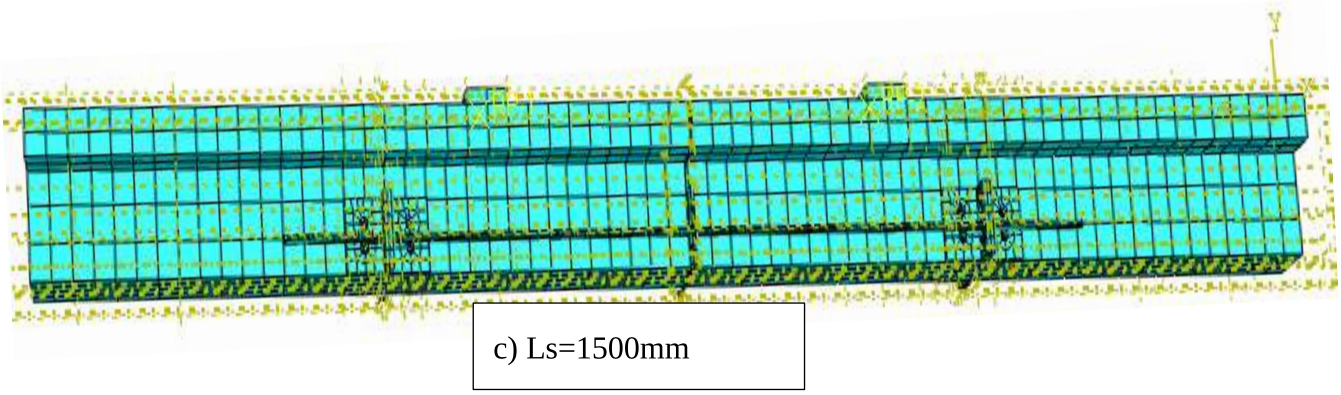
The model was created graphically using Abaqus/CAE. The model and mesh generated for the strengthened beam listed in (Table 3.4) above is expressed by the following figure. (Figure 3.6).



a) Ls=2500mm



b) Ls=2000mm



c) Ls=1500mm

Figure 3.6 Modeling of beam for different cases, AT-1(a), AT-2(b) and AT-3(c)

The above properties of beams were modelled using finite element analysis and the result and its discussion was discussed in chapter four.

CHAPTER FOUR

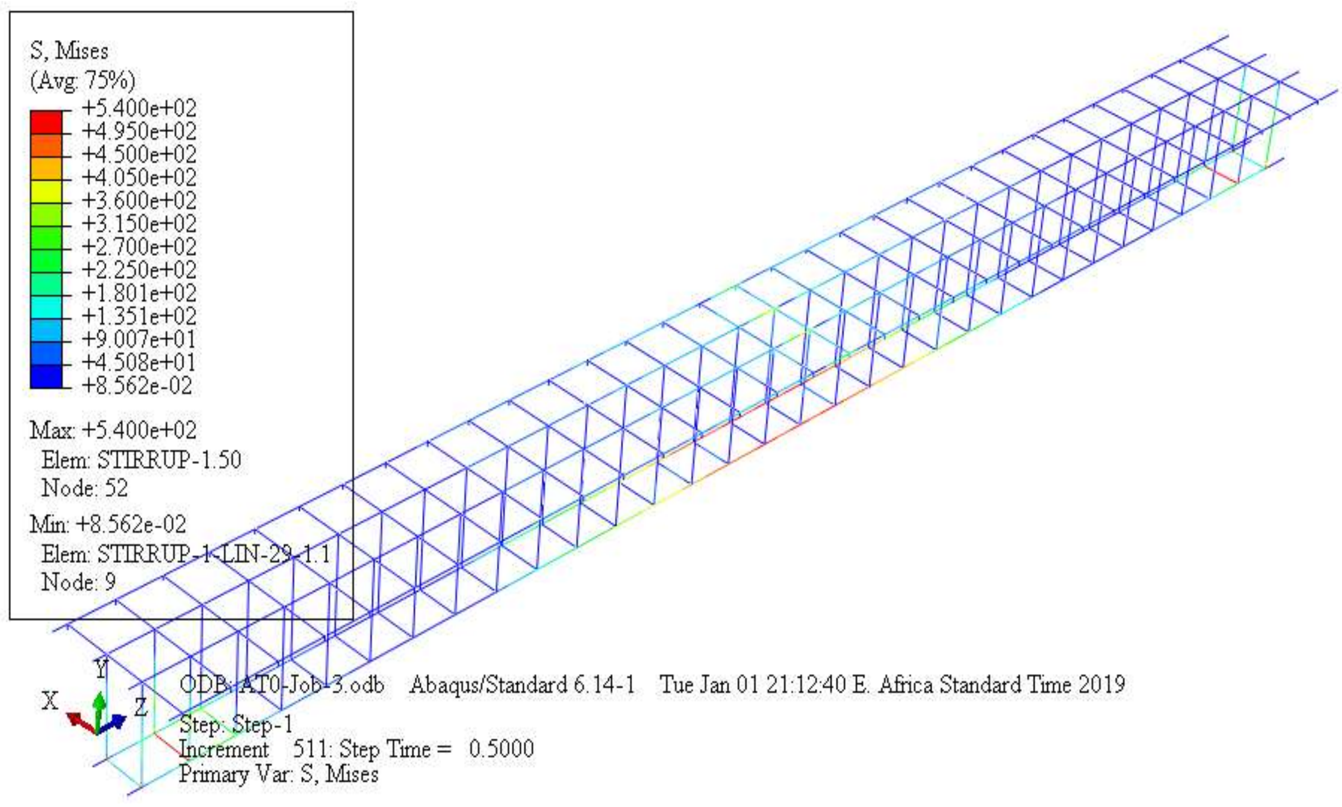
ANALYSIS, RESULT AND DISCUSSIONS

This chapter presents the flexural behavior of reinforced concrete (RC) T-section beams strengthened with external prestressed tendons to meet the objective of the study using finite element analysis (ABAQUS). The results obtained were discussed with respect to each independent variable by comparing their load deflection values drawn graphically.

4.1 Results of Finite Element Analysis

The complete finite element analysis results of control beam (reference beam), including maximum deflection along Y-axis (u_2), Mise stress (steel yield stress), compression damage and tension damage obtained from the analysis is graphically discussed as the following figures.

According to the finite element analysis results, figure 4.1 shows the maximum stress distribution was occurred at the mid span and end support of the beam bottom fiber. From the contour observed the shear failure was at the end support and tensile failure was at mid span.



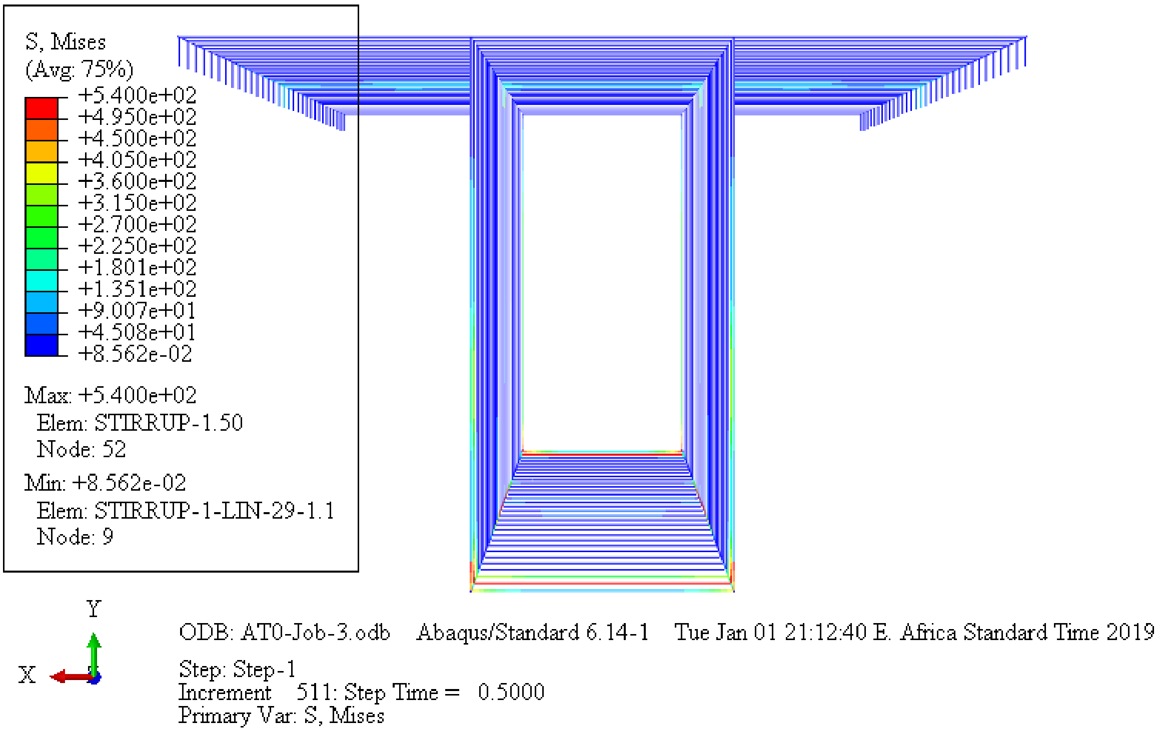


Figure 4.1 Steel yield stresses for control beam

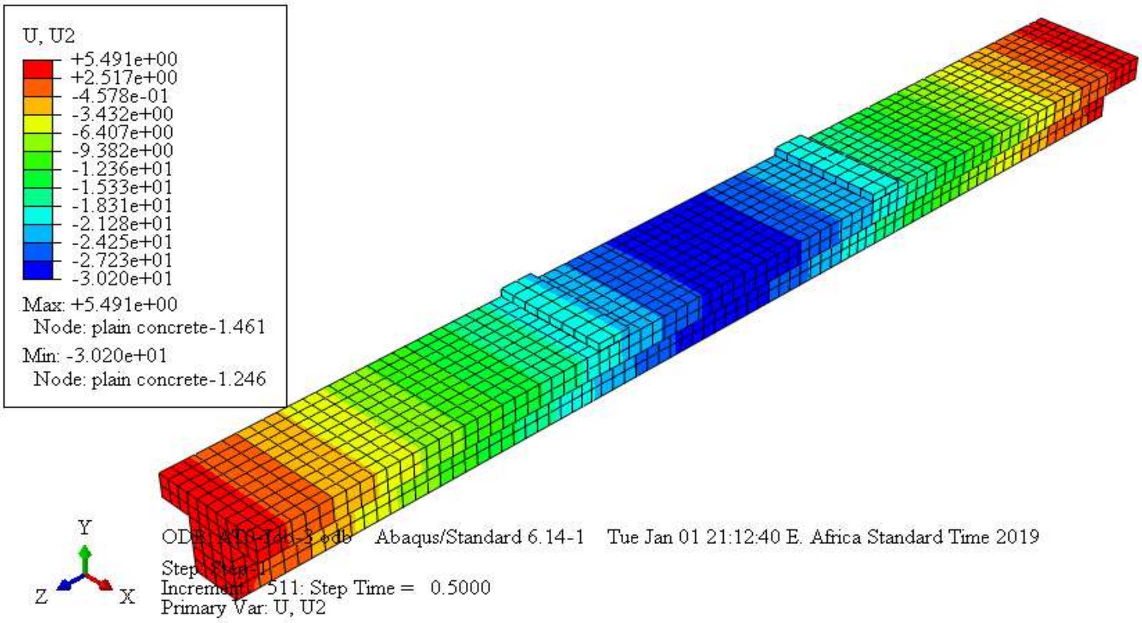


Figure 4.2 Displacement results of control beam

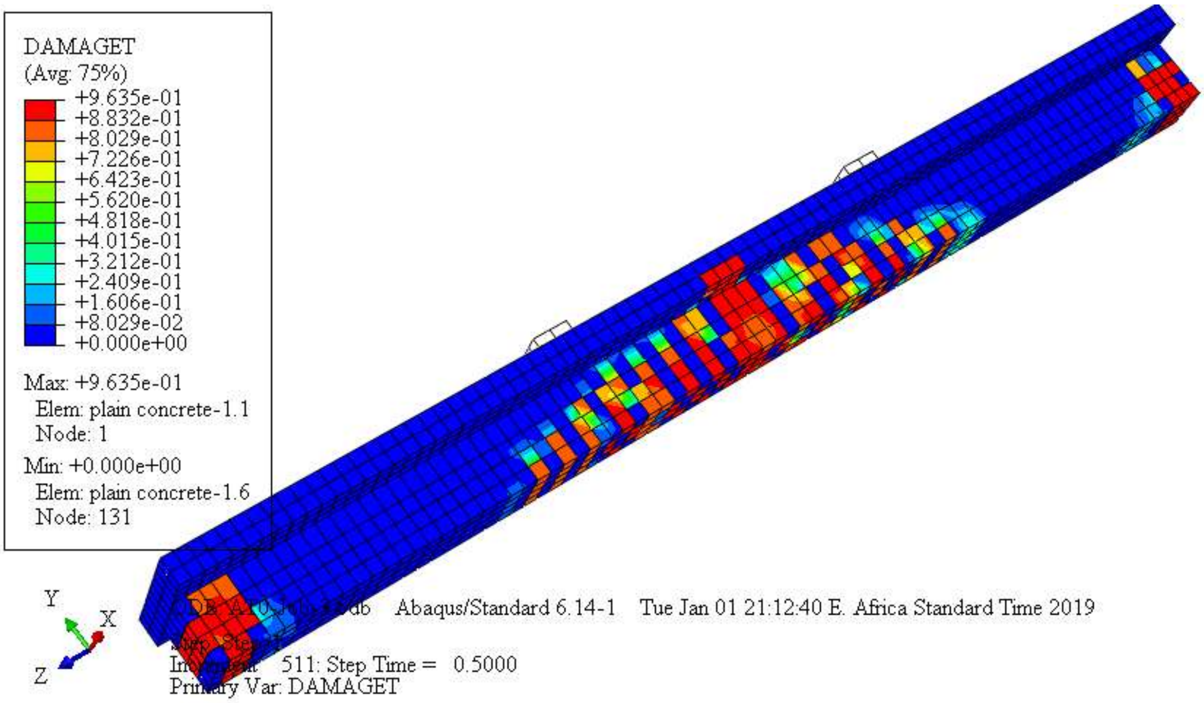


Figure 4.3 Tension damage for control beam

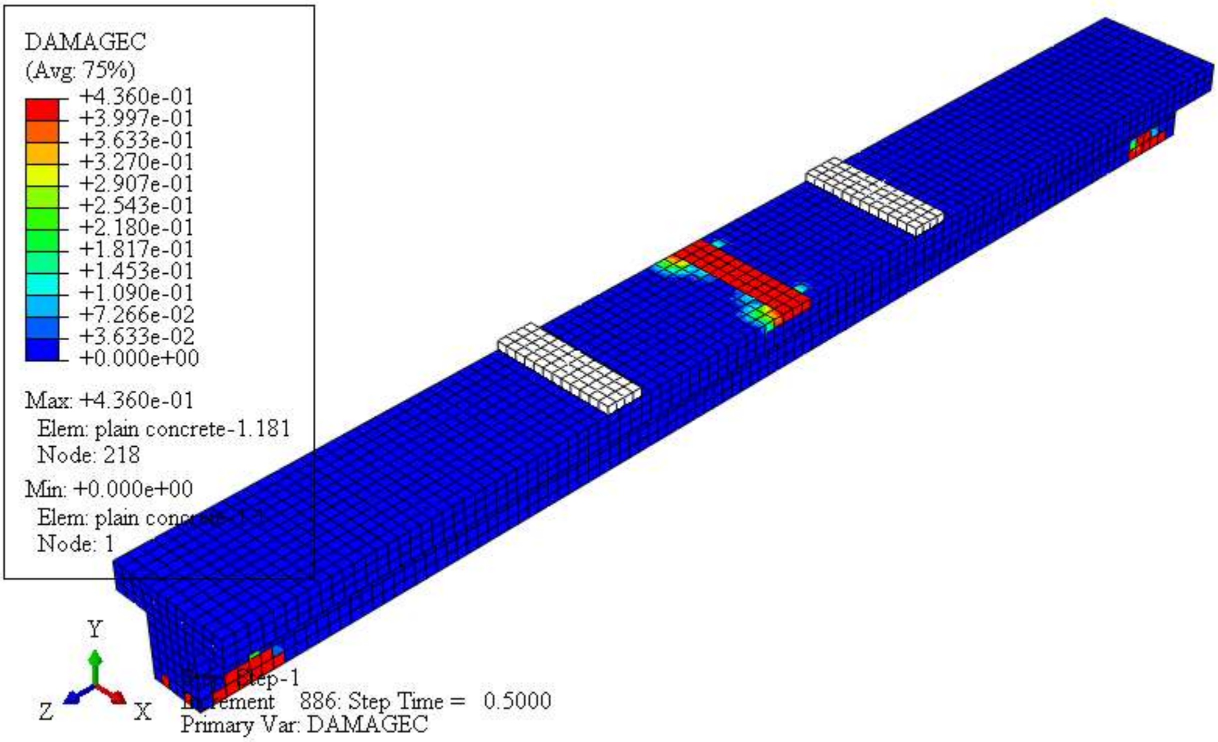


Figure 4.4 Compression damage results for control beam

4.2 Discussion on the results of Mesh-size effect for control beam

The Mesh module contains tools that allow us to generate a finite element mesh on an assembly created within Abaqus/CAE. Various levels of automation and control are available to create a mesh that meets the needs of analysis result. The element type, shape, and location, as well as the overall number of elements used in the mesh, affect the results obtained from a simulation. The greater the mesh density (i.e. the greater the number of elements in the mesh), the more accurate the results. As the mesh density increases, the analysis results converge to a unique solution, and the computer time required for the analysis increases. Fig 4.5 shows the effects of different mesh size for control beam.

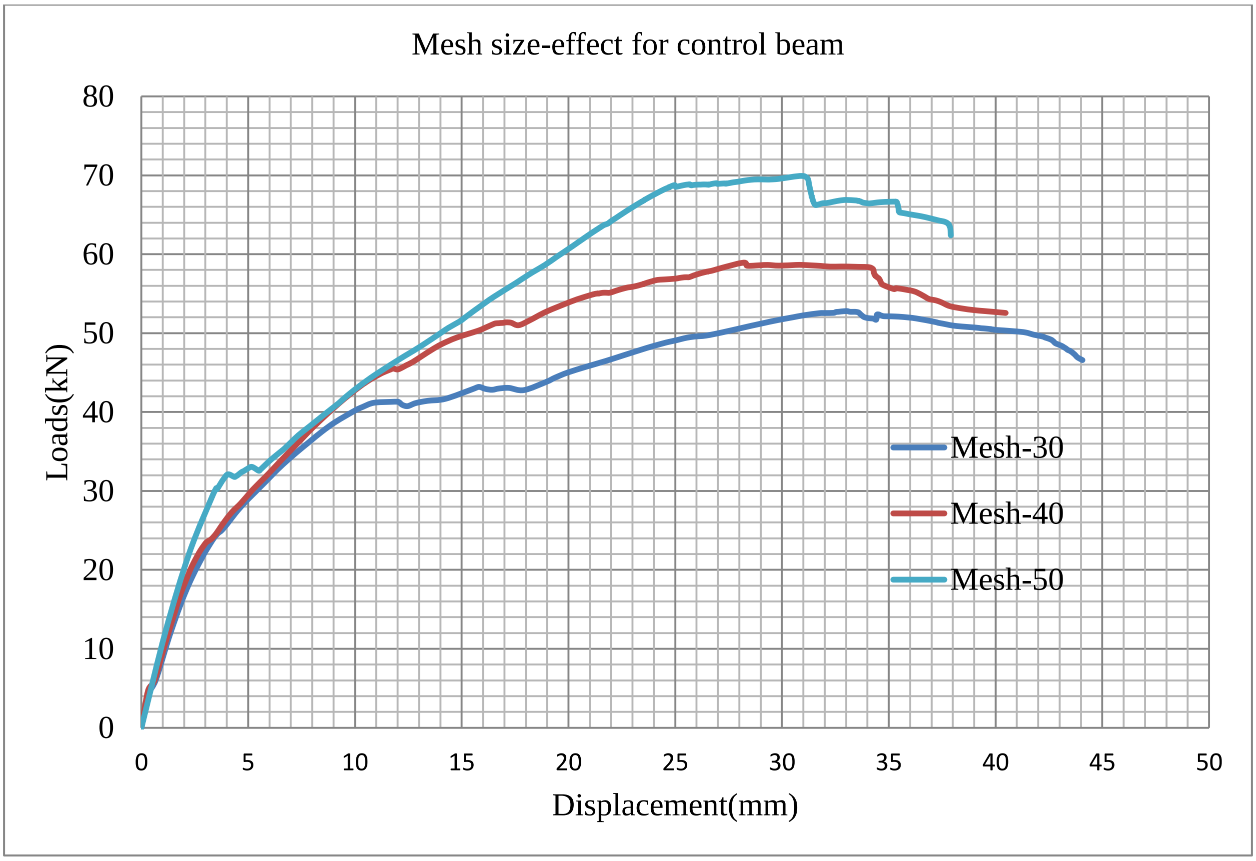


Figure 4.5 Mesh size effect of Control beam

The model was loaded as a simply supported Reinforced-concrete beam with a two point concentrated load at one-third of the effective length of the beam. The concentrated load in the model was applied on 25mm thick steel plate in order to distribute the load stress evenly on the concrete beam. Deflection was monitored at the bottom of the reinforced concrete beam at mid-span and the load applied was automatically increased up to the point of failure.

4.3 Model Validation

In order to validate the proposed model, comparison with experimental results was made. The model validation was done by the comparison of load-deflection data from the experiment results by (AbdulMuttalib I. Said, 2015). The comparison between the model and test results was good agreement in terms of the load–deflection responses over their entire loading profiles until failure. Failure load capacity (the ultimate load carrying capacity) was obtained when ordinary reinforcement has been yielded at section in pure bending moment zone. The beam tested by (AbdulMuttalib I. Said, 2015), obtained an ultimate load of **63kN** and the ultimate load obtained from the finite element model was **58.9kN** (from mesh size 40mm). Comparing those values the percentage error is calculated below as:

$$\boldsymbol{Difference}(\%) = \left(\mathbf{1} - \frac{\mathbf{58.9}}{\mathbf{63}}\right) * \mathbf{100} = \mathbf{6.5}\%$$

Comparing the results from experimental and numerical results, the model were good agreement with the experimental result with percentage error (6.5%). The loads-deflections curves for both experimental and numerical result was graphically expressed in (Figure 4.6) below.

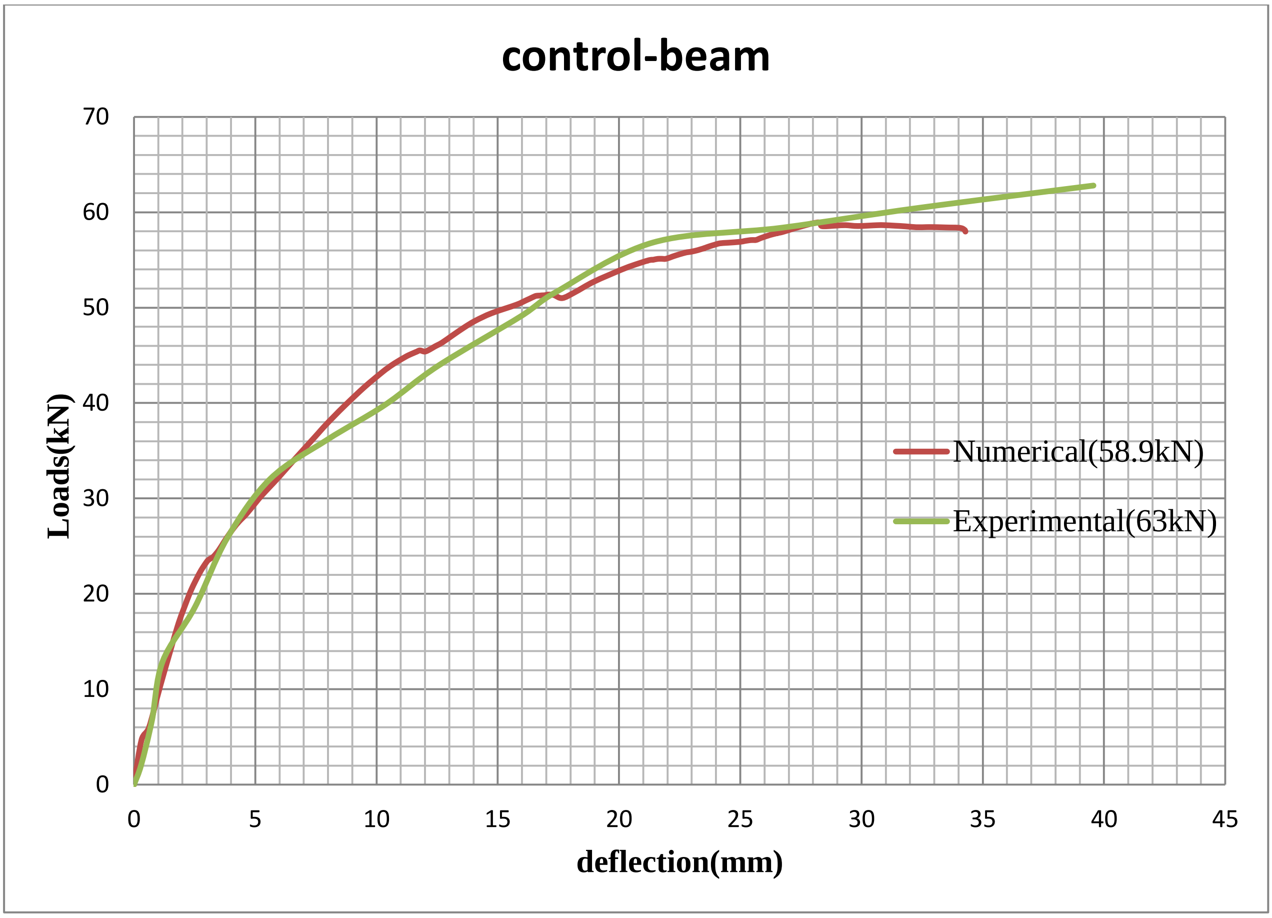


Figure 4.6 Loads-displacement responses of control beam.

4.4 Discussion on the results of Concrete compressive strength

Concrete strengths used in this finite element analysis are 25MPa and 30MPa for control beam and 30Mpa is used for the others beams. From (Figure 4.7) it shows that the slope of load-displacement curve gradually increases with increasing concrete compressive strength. The ultimate loads are numerically increases from 54kN to 58.9kN, with the variation of 9% by changing the concrete compressive strength from C-25 to C-30.

As the concrete compressive strength increase the load carrying capacity also increase and the concrete compressive strength have significant effect on increasing the ultimate load carrying capacity of beam, for a particular reinforcement ratio. Figure 4.7 shows the Load-displacement responses by varying concrete strength for control beam.

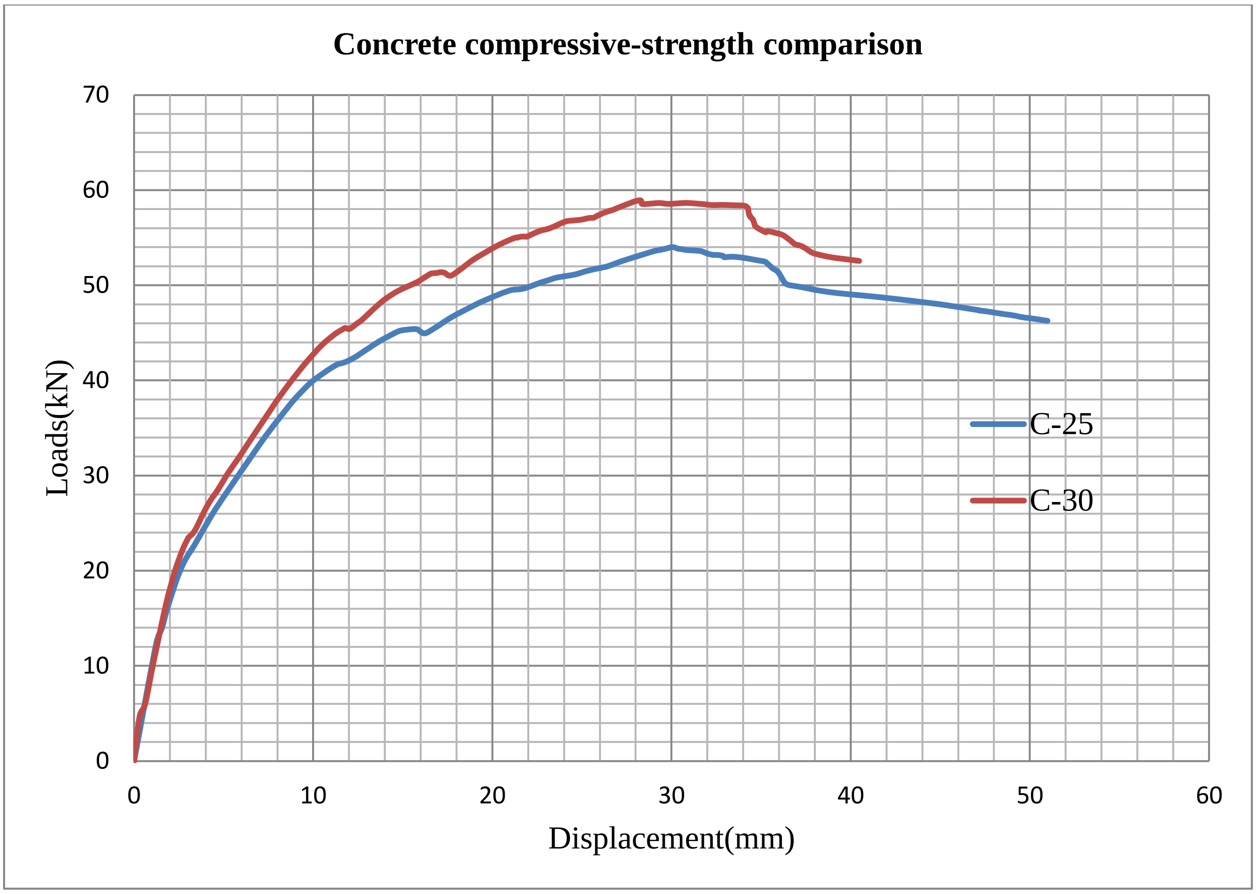
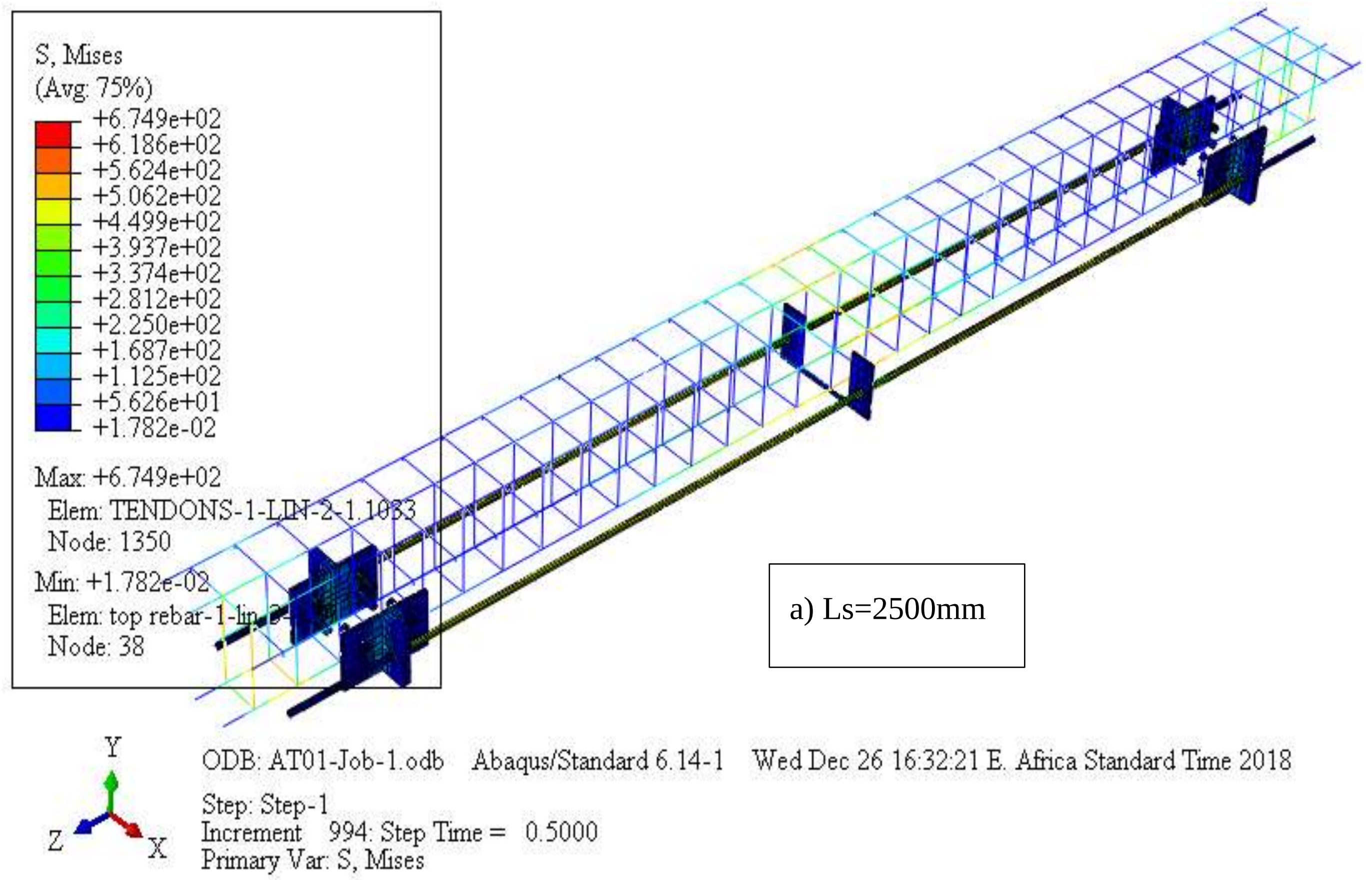


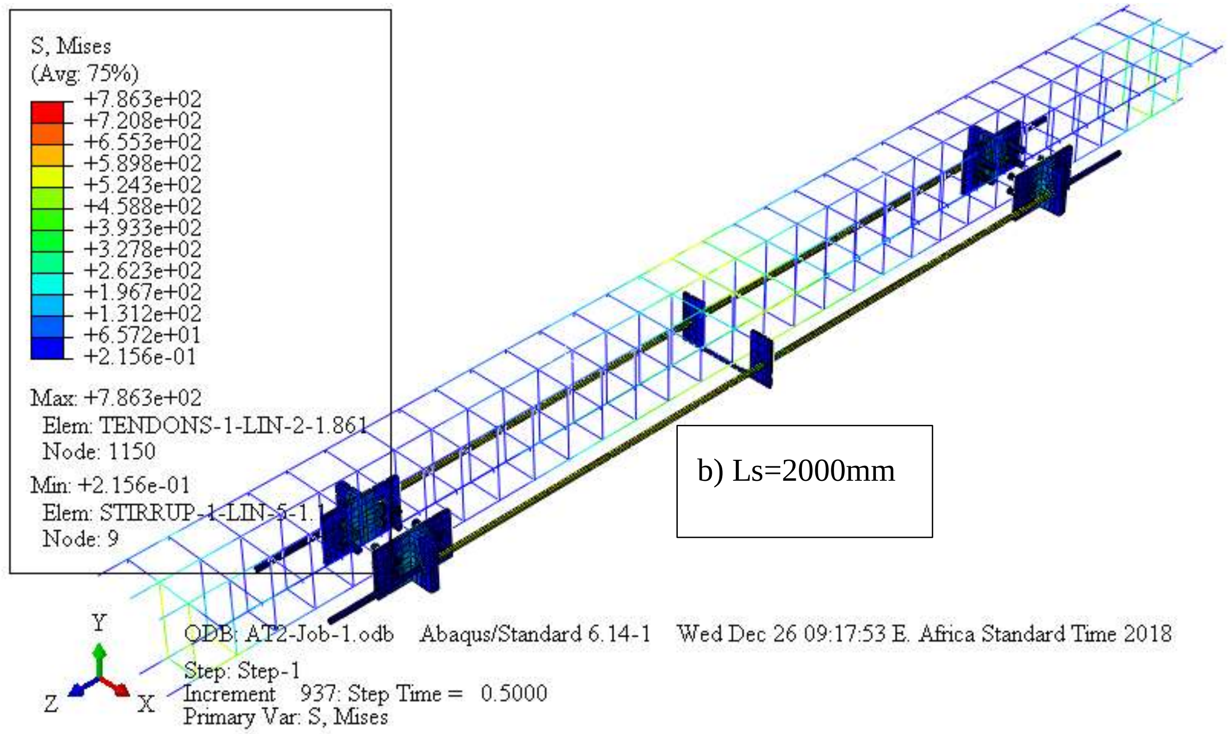
Figure 4.7 Load-displacement responses by varying concrete strength

4.5 Effects of strengthened length ratio (L_s/L)

The finite element analysis result shows, the loads-displacement responses of the beam is increase with reducing the strengthen length partially at some distances from the end supports. The steel yield stress for depth of strand 0.8h for different length of strengthened was shown by (Figure 4.8).



a) Ls=2500mm



b) Ls=2000mm

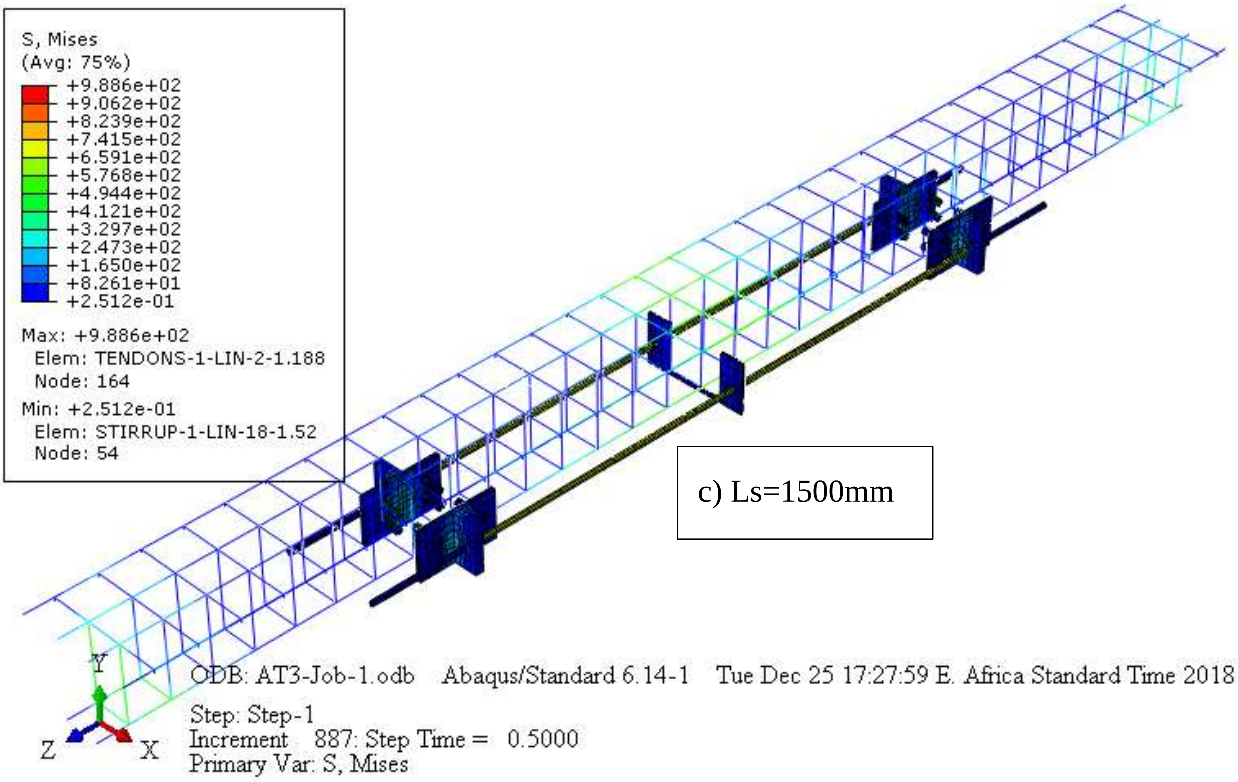


Figure 4.8 Steel yield stress of beam for different cases, AT-1(a), AT-2(b) and AT-3(c)

Table 4.1 Ultimate load results for constant tendons depth ratio, dps=0.8h.

Beam designation	strengthened Length(mm)	Ultimate Load(kN)		Depth of Tendons from top fiber(mm)	Tendons from face of beam(mm)	Difference in Percentage (%)
		Experimental	FEA			
Control-beam	-----	63	58.9	-----	-----	6.51
AT-1	2500	112	77.15	0.8h	40	29.17
AT-2	2000	119	79.7	0.8h	40	30.09
AT-3	1500	105	81.82	0.8h	40	22.08

The above table can be expressed graphically (Figure4.9) by drawing the loads-displacement responses for constant strand depth ratio of 0.8h, where h (=250mm) is the overall depth of the T-beam, by varying the length of strengthened beam partially along the sides of web from both sides.

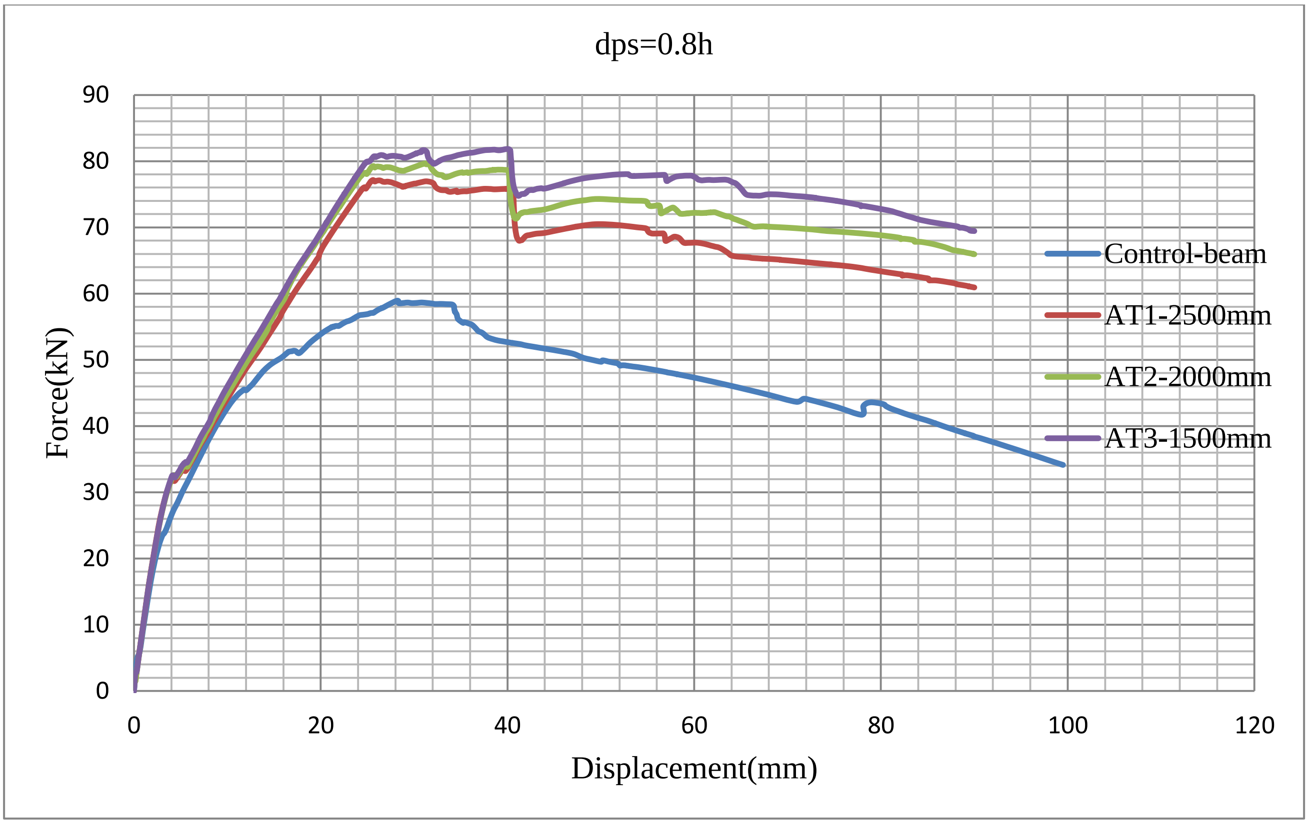


Figure 4.9 Load-displacement curve for constant tendons depth ratio, dps=0.8h

4.6 Effects of tendons depth to strengthen beams externally

The result obtained from the finite element analysis shows that, as the depth of tendons increase, the ultimate load capacity of the beam to carry the load also increase. The end anchorage is fixed at a distance of 250mm from the center of end support in both sides with the strengthen length of 2500mm for AT-1, AT1-3, and BT-1 with the depth of strand 0.8h,0.9h and 0.972h respectively. The analysis result is given by the following (Table4.2). The tendons are connected from both sides at the distance of 40mm from the face of the beam for AT-1, AT1-3, BT-1 and 25mm for BT-2.

Table 4.2 Ultimate load results for constant strengthened length, Ls=2500mm.

Beam designation	strengthened length(mm)	Ultimate Load(kN)	Depth of tendons from top fiber(mm)	Center of Tendons from sides beam	load Carrying capacity increase in percentage
Control-beam		58.90			
AT-1	2500	77.15	0.8h	40	30.98
AT1-3	2500	79.36	0.9h	40	34.74
BT-1	2500	81.84	0.972h	40	38.95
BT-2	2500	82.90	0.972h	25	40.75

The flexural capacity of the beam with constant length of strengthened was increased by increasing the depth of strand and decreasing the distance of strand from the face of the beam. Table 4.2 shows that the load carrying capacities of the beam is 30.98% , 34.74%, 38.95% and 40.75% for AT-1, AT1-3, BT-1 and BT-2 respectively, compared to the control beam by varying the depth of tendons 0.8h,0.9h and 0.972h. Figure 4.10 shows the load-displacement curve for Ls=2500mm by varying the depth of strand.

From Table 4.2 above the result of BT-2 governs our objectives with respect to the load carrying capacity of the beam by 40.75% compared to the control beam. Therefore depth of strand 0.972h and external tendon distance from the face of the beam 25mm was used in the next analysis result.

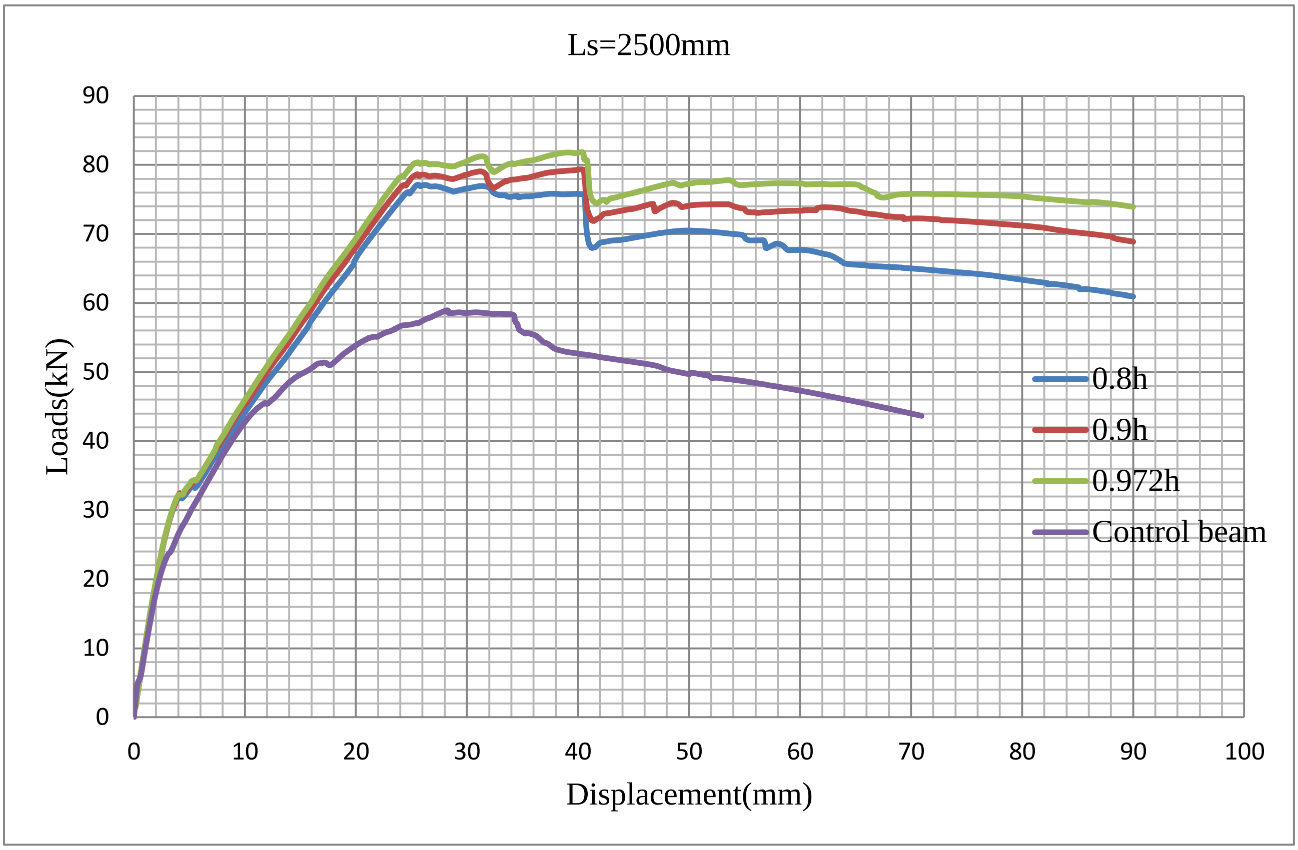


Figure 4.10 Loads deflection curve for constant strengthened length, Ls=2500mm.

Table 4.3 Ultimate load results for constant tendons depth ratio, d_{ps}=0.972h.

Beam designation	strengthened length(mm)	Ultimate Load results (kN), from FEA	Depth of tendons from top fiber (mm)	Center of Tendons from side of beam (mm)	load Carrying capacity increase in percentage
Control-beam		58.9			
BT-2	2500	82.90	0.972h	25	40.75
BT-3	2000	89.01	0.972h	25	51.12
BT-4	1500	95.68	0.972h	25	62.44

The load carrying capacities of the beam also increases as the depth of the strand from the top fiber of the beam increase and location of the strand closest to the beam from the outside. From (Table 4.3) above the results shows that the load carrying capacity for BT-2, BT-3 and BT-4 are increase by 40.75%, 51.12% and 62.44% respectively comparing to the control beam.

Figure4.11 Shows, the loads-deflection curves for beam partially strengthened with different length externally at depth of strand 0.972h.

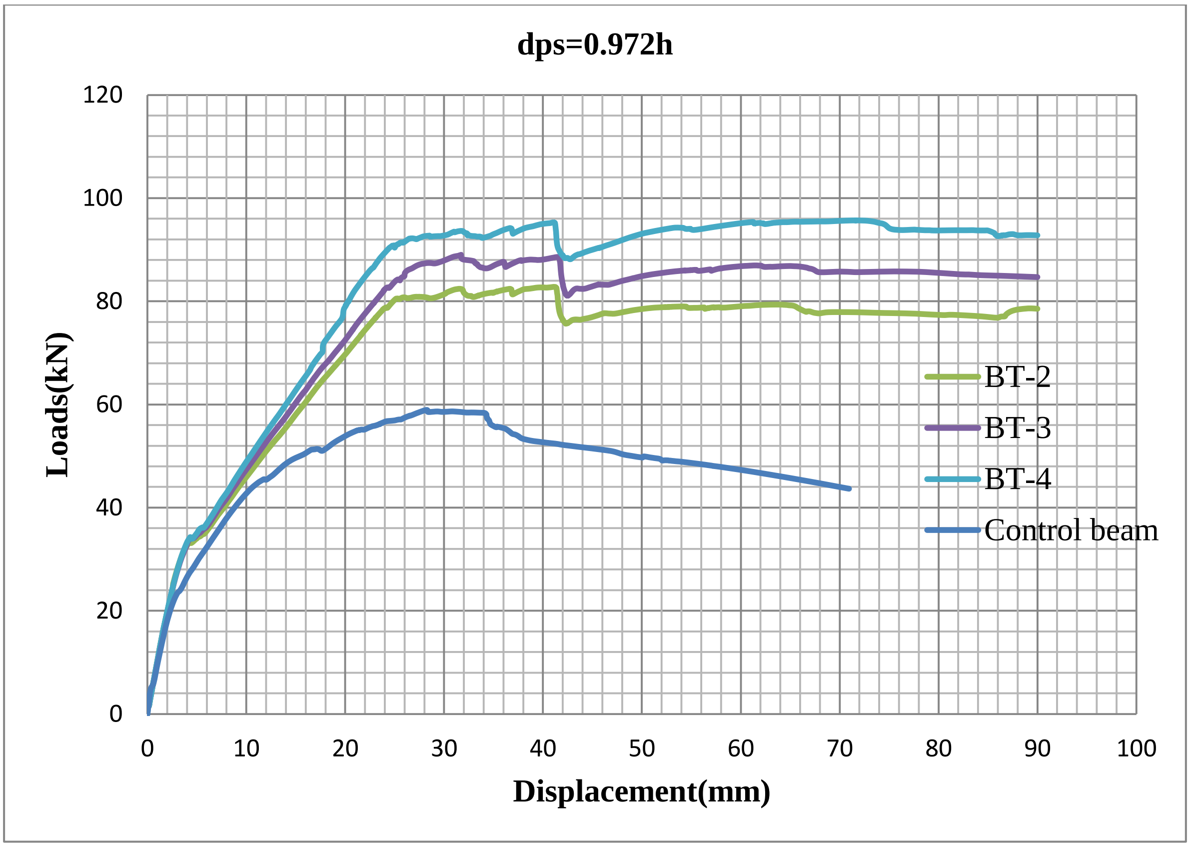


Figure 4.11 Load-displacement curve for constant tendons depth ratio, dps=0.972h.

Generally, from the above all Finite element analysis results assessment, the strengthened beam BT-4 (Ls=1500mm) at strand depth 0.972h from top fiber of reinforced concrete T-beam governs the objectives of this research with the load carrying capacity of 62.44% compared to the control beam.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1 CONCLUSION

The numerical modeling to simulate behavior of reinforced concrete T-beam externally strengthened by post tensioning based on nonlinear finite element method under concentrated loads by using Abaqus software has been investigated in this thesis. The parameter studied has been carried out by considering concrete compressive strength, span to strengthened length ratio, depth of strand (eccentricity) and distance of strand from the face of the concrete beam(horizontal eccentricity). The finite element analyses of ten T-beams are investigated in this study and the following conclusion was made from this research investigations.

- Generally, using external tendons for strengthening RC beams is effective to increase beams load carrying capacities. Comparing the RC beams strengthened externally using posttensioning to the beam without strengthening; the ultimate load carrying capacity was increase by 62.44%.
- Eccentricity has significant effect on external post tensioning to strengthen of reinforced concrete T-beam, as depth of tendons increases and the distance of tendons from face beam decreases the ultimate load carrying capacity of the beam increase.
- Strengthened length to span ratio has a significant effects on the ultimate load carrying capacity of the beam, thus carrying capacity for 0.83L, 0.667L and 0.5L are 40.75%, 51.12% and 62.44% respectively compared to the control beam.
- Relating the ultimate load result from previous experimental (63kN) and obtained Finite element analysis result (58.9kN), the Finite element analysis was a good agreement with Experimental result with a percentage error of 6.5%.

5.2 RECOMMENDATION

Based on the finding of this study, the following recommendations were forwarded.

- Further Finite element analysis for external posttensioning for existing reinforced concrete beam by considering the effect of creep coefficient and age of concrete.
- In this research, only straight profile steel tendon was considered, the other researcher may considering the draped or trapezoidal profile and compare the results with this study.
- Simply supported (single span beam) was considered in this study, the other researcher may consider the continuous beam externally strengthened by posttensioning.

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Appendix-A

Load-deflection values in tabular form for different sets beams.

Table A.1Tabular data for comparison of mesh size for control beam.

Control Beam					
Mesh 30		Mesh 40		Mesh 50	
Disp(mm)	Force(N)	Disp(mm)	Force(N)	Disp(mm)	Force(N)
0	0.000	0	0	0	0
0.338020444	4177.688	0.3109336	4688.429688	1.125	12140.80664
0.676040888	6060.703	0.6218671	6011.227539	2.25	22189.58398
1.183071494	10495.982	1.0882676	10405.91016	3.375	29678.36914
1.373208046	12114.827	1.787868	16526.93555	3.480469	30224.86328
1.658412695	14325.756	2.1376684	19014.01953	3.506836	30366.36133
1.765364647	15114.431	2.2251184	19592.5625	3.533203	30221.67773
1.925791979	16263.440	2.3125684	20144.9375	3.572754	30346.27344
1.985952735	16675.844	2.4437435	20886.61914	3.63208	30601.98047
2.076193333	17266.240	2.6405063	21883.2207	3.721069	30994.66992
2.211553812	18120.215	2.7142923	22233.76563	3.854553	31538.2207
2.414595366	19308.055	2.8249714	22707.4043	4.054779	32114.02344
2.617636681	20401.299	2.9909897	23335.03516	4.355118	31778.31445
2.820677757	21442.809	3.0532467	23520.69922	4.430202	31865.79102
3.023719072	22439.324	3.1466322	23687.73828	4.505287	31989.58984
3.328280926	23754.215	3.2867103	23934.49609	4.617914	32236.07813
3.404421568	24049.412	3.4968271	24581.35547	4.660149	32322.875
3.480561972	24313.564	3.7069447	25415.42383	4.723502	32438.93555
3.594772577	24655.535	3.9170613	26217.87695	4.818532	32558.95898
3.766088247	25022.533	4.1271787	26955.45703	4.961074	32803.35938
4.023062706	25838.941	4.3372955	27595.91602	5.17489	33053.52344
4.408524036	27196.330	4.6524711	28427.92969	5.495613	32562.16992
4.79398489	28387.920	5.1252356	29897.83008	5.575794	32720.07031
5.179445744	29454.061	5.3025212	30411.59961	5.655974	32920.01563
5.32399416	29837.912	5.5684505	31138.95703	5.776246	33233.50391
5.54081583	30435.520	5.9673457	32225.13867	5.956653	33690.76172
5.622124195	30651.078	6.5656867	33940.84766	6.227262	34328.14453
5.744086266	30982.439	7.4631987	36456.83984	6.633178	35221.15625
5.927030087	31482.084	7.6875763	37098.17969	6.785396	35600.99219
6.201445103	32241.215	7.9119544	37725.57813	7.013723	36186.375
6.613068104	33327.719	8.2485218	38608.6875	7.356214	37068.29688
7.230503082	34799.793	8.7533731	39888.33984	7.86995	38176.02734
7.847937107	36184.609	9.5106478	41690.28906	8.640554	39845.23828
8.465370178	37541.430	10.267924	43303.07422	9.218509	41078.10547
9.08280468	38736.352	10.457244	43663.40625	9.435241	41604.19141
10.00895596	40206.016	10.741222	44160.55469	9.76034	42343.9375
10.35626316	40641.234	11.16719	44803.41797	10.24799	43377.94531
10.87722301	41173.328	11.326928	45018.66406	10.97946	44782.69922
11.65866375	41289.059	11.566534	45288.51563	12.07667	46679.99609

11.95170307	41309.281	11.656387	45384.10938	12.48812	47355.36719
12.06159401	41260.965	11.791165	45513.75	13.1053	48377.40234
12.22642803	40902.238	11.993334	45396.55469	14.03107	50042.26563
12.47368145	40753.941	12.195502	45615.48438	14.37823	50673.70313
12.84455967	41134.758	12.39767	45917.92969	14.89898	51471.5
13.40087795	41421.652	12.700922	46327.96094	15.09426	51855
14.23535538	41676.676	13.155801	47130.375	15.38718	52453.58594
15.48707104	42877.863	13.610678	47915.89063	15.82656	53318.85547
15.80000019	43184.984	14.065557	48611.35938	16.48563	54569.77734
16.1129303	42933.789	14.747875	49418.23438	17.47423	56240.56641
16.42585945	42821.402	15.771352	50308.16797	17.72138	56677.30078
16.73879051	42979.859	16.155159	50751.21875	17.96853	57111.60156
17.20818138	43059.316	16.299084	50919.16406	18.33926	57745.70703
17.9122715	42780.598	16.514973	51175.05078	18.89534	58636.14063
18.96840858	43831.703	16.595932	51241.05078	19.45143	59673.54297
19.36445808	44347.742	16.717369	51260.80469	19.59046	59927.48828
19.95853615	45002.734	16.762909	51269.98438	19.72948	60174.73828
20.84964752	45756.957	16.831219	51288.1875	19.93801	60555.16797
21.74076271	46466.969	16.933681	51297.99609	20.14655	60936.64063
22.63187599	47238.203	17.087376	51370.00391	20.35508	61328.35938
23.52299309	48002.605	17.317917	51315.79688	20.56361	61721.59766
23.7457695	48187.500	17.663731	51002.49609	20.77215	62111.26563
24.07993698	48449.340	18.182451	51618.35547	21.08495	62692.32813
24.58119011	48816.402	18.960531	52717.41797	21.55414	63517.59375
24.76915932	48931.137	20.127651	54010.21094	21.59813	63594.22266
25.05111504	49100.621	20.565321	54425.42969	21.66411	63700.17578
25.156847	49170.961	20.729448	54569.10156	21.76308	63797.21094
25.31544495	49278.051	20.975639	54772.01953	21.8002	63830.1875
25.55334473	49423.988	21.067957	54846.67578	21.85587	63900.38281
25.9101944	49565.754	21.102579	54873.86719	21.93938	64077.74609
26.44546318	49691.609	21.154509	54913.125	22.06464	64316.39453
27.24837112	50124.133	21.159378	54916.67188	22.25253	64645.67188
28.45273399	50867.754	21.166681	54922.11719	22.53437	65150.67969
28.56564522	50937.590	21.169418	54924.14063	22.95712	65888.67188
28.73500824	51037.973	21.173525	54927.28516	23.37988	66582.49219
28.79851913	51075.051	21.179689	54932.14844	23.40631	66627.07813
28.89378738	51130.395	21.188931	54939.33594	23.41621	66641.89844
29.03668594	51216.379	21.202795	54949.79688	23.43108	66665.46094
29.25103378	51342.730	21.223589	54963.94922	23.43665	66674.28906
29.57256317	51528.422	21.254786	54981.76563	23.44501	66687.39844
30.05485535	51784.363	21.301575	55002.65625	23.44814	66692.21875
30.53713989	52020.254	21.319122	55009.1875	23.45285	66699.39063
31.01942825	52259.969	21.34544	55015.73047	23.45461	66702.03125
31.74286461	52528.359	21.35531	55017.89844	23.45726	66705.89063
32.01415253	52548.984	21.370115	55013.94141	23.45825	66707.32813
32.42108154	52568.914	21.392321	55010.17578	23.45974	66709.41406
32.5228157	52677.902	21.40065	55011.30078	23.46197	66712.23438

32.62454987	52698.145	21.40377	55012.85938	23.46532	66716.16406
32.7771492	52746.605	21.408457	55015.96094	23.47034	66723.99219
32.83437347	52761.898	21.415483	55020.26563	23.47787	66736.33594
32.92021179	52780.660	21.426022	55026.64844	23.48917	66754.67188
32.95239639	52786.551	21.441833	55036.10938	23.49341	66761.45313
33.00068283	52793.234	21.465548	55049.67969	23.49976	66770.74219
33.01878738	52794.832	21.501118	55068.47266	23.5093	66784.55469
33.04594803	52792.746	21.554476	55092.48047	23.5236	66808.25781
33.05613327	52790.711	21.634512	55117.78906	23.54505	66843.92188
33.07140732	52784.816	21.754566	55118.70703	23.55309	66857.1875
33.09432602	52772.000	21.934649	55118.00781	23.55611	66862.24219
33.12869644	52738.160	22.204767	55353.79688	23.56063	66870.30469
33.1802597	52707.859	22.474888	55579.26172	23.56742	66883.10938
33.25760269	52699.664	22.745008	55770.07813	23.5776	66902.71094
33.37361145	52703.445	22.846304	55816.76172	23.59287	66928.51563
33.41711426	52701.906	22.998249	55873.71094	23.61578	66964.25781
33.48237228	52684.836	23.226162	56009.90625	23.65014	67017.14844
33.50684357	52675.586	23.31163	56074.1875	23.70168	67096.44531
33.54354858	52654.492	23.439835	56170.00391	23.77898	67214.42188
33.59860992	52594.867	23.487909	56206.03516	23.89495	67387.5625
33.68119812	52373.578	23.505938	56219.44922	23.93843	67453.25781
33.80508423	52094.195	23.507627	56220.60547	24.00366	67548.08594
33.81669617	52073.355	23.510164	56222.57422	24.10151	67684.21875
33.83411789	52048.035	23.513966	56225.89063	24.24827	67899.82813
33.86025238	52018.301	23.51967	56231.38281	24.46842	68196.32813
33.89944839	51983.539	23.528227	56240.22656	24.55098	68300.27344
33.95824814	51943.336	23.541059	56253.67969	24.67481	68441.67188
34.04643631	51899.188	23.560314	56273.75	24.72125	68498.60156
34.17873383	51870.188	23.589191	56302.77344	24.72234	68501.78125
34.22834396	51857.563	23.632507	56343.32031	24.72397	68505.70313
34.30275726	51818.789	23.697483	56400.69922	24.72642	68513.8125
34.41437912	51714.758	23.794947	56481.41016	24.73009	68522.08594
34.44228363	52328.730	23.941141	56591.98438	24.7356	68526.64844
34.47019196	52379.633	24.160433	56741.39844	24.74387	68528.45313
34.51205063	52370.504	24.489372	56802.96875	24.75626	68543.57031
34.52774811	52364.223	24.98278	56897.13672	24.77486	68567.97656
34.5512886	52350.813	25.167807	56978.51563	24.80276	68603.66406
34.58660889	52290.492	25.237194	57009.66406	24.84459	68653.61719
34.63958359	52236.672	25.341272	57050.69922	24.90736	68718.30469
34.71905136	52165.844	25.380302	57064.38281	24.93089	68738.92188
34.83824921	52134.246	25.438847	57079.74219	24.9662	68757.04688
35.01704788	52132.805	25.526661	57085.30859	24.97943	68754.21094
35.28524399	52111.219	25.658388	57098.34375	24.99929	68584.72656
35.68753433	52043.590	25.855974	57301.14844	25.02908	68546.04688
36.29097748	51859.969	26.152353	57556.50391	25.07376	68557.79688
37.19613647	51400.094	26.263494	57641.01172	25.14078	68598.10938
37.25270844	51362.234	26.430206	57752.65234	25.24131	68660.01563

37.3092804	51311.250	26.492723	57790.64453	25.27901	68681.88281
37.39414215	51269.129	26.516169	57804.03906	25.33556	68715.23438
37.52143097	51206.973	26.518368	57805.62891	25.42039	68762.53125
37.7123642	51103.910	26.521664	57807.27344	25.50521	68804.25
37.99876022	50960.566	26.526609	57810.19141	25.59004	68832.50781
38.42835999	50838.281	26.534025	57814.53125	25.59798	68834.83594
39.07275009	50703.453	26.545155	57820.98438	25.60097	68835.69531
39.31440353	50626.246	26.561842	57830.51953	25.60544	68837.04688
39.40502167	50606.375	26.58688	57843.62109	25.60712	68837.67188
39.54094696	50578.336	26.62443	57860.64063	25.60963	68838.5625
39.74483871	50517.645	26.680761	57887.5	25.61341	68839.86719
39.76395416	50511.504	26.765257	57947.14844	25.61482	68839.60938
39.79262543	50497.828	26.891996	58046.07031	25.61695	68840.60156
39.79441833	50498.371	27.082106	58195.42188	25.62013	68841.89844
39.79621124	50500.020	27.367273	58408.60547	25.62491	68843.07031
39.7989006	50499.531	27.795023	58710.15234	25.63207	68845.47656
39.80293274	50487.414	27.955431	58819.67578	25.64282	68848.84375
39.80897522	50481.043	28.196039	58933.95313	25.65894	68853.45313
39.81804657	50479.609	28.218599	58936.62891	25.68312	68858.57813
39.83165359	50478.762	28.25243	58929.41016	25.7194	68750.60156
39.85206604	50474.746	28.265123	58924.66016	25.77381	68732.46875
39.88269043	50464.027	28.284155	58910.11328	25.79421	68748.15625
39.92861557	50442.520	28.29129	58902.75391	25.82482	68762.57031
39.99750519	50409.707	28.301998	58884.46094	25.85542	68775.25781
40.10083771	50390.828	28.306011	58875.92188	25.88603	68785.6875
40.25584412	50364.973	28.312033	58855.41406	25.93194	68798.33594
40.48834991	50321.039	28.321066	58664.40625	26.0008	68810.28125
40.83711624	50246.668	28.334614	58585.4375	26.02662	68813.30469
40.96789551	50218.859	28.354942	58551.78516	26.06536	68816.57031
41.16407776	50168.332	28.385427	58536.57813	26.07988	68817.32031
41.2376442	50147.527	28.431158	58529.03125	26.08533	68817.51563
41.34799576	50112.066	28.49975	58529.55078	26.08737	68818.30469
41.51351929	50018.531	28.602646	58538.92969	26.09044	68819.32031
41.57559204	49970.836	28.756979	58566.04297	26.09503	68820.28125
41.66869736	49893.906	28.988489	58603.58984	26.10193	68820.88281
41.80835724	49790.719	29.335749	58642.98438	26.11227	68821.24219
42.01784897	49678.504	29.856642	58553.79688	26.12778	68821.32813
42.09640503	49638.379	30.63798	58641.28906	26.15105	68820.32813
42.21424866	49562.742	30.930979	58644.97266	26.18595	68821.32813
42.25844574	49523.336	31.370478	58589	26.2383	68830.92188
42.32472229	49441.617	31.809982	58521.04688	26.31683	68841.36719
42.42414856	49358.012	32.249485	58428.23047	26.43462	68839.03906
42.52357864	49260.234	32.688988	58439.76563	26.47879	68833.95313
42.62300873	49142.340	32.853802	58441.11719	26.49535	68833.16406
42.66028595	49065.199	33.101021	58434.26563	26.50156	68832.86719
42.71621704	48940.402	33.471848	58406.75391	26.51088	68832.34375
42.80010986	48718.215	33.610909	58398.21094	26.51438	68832.10938

42.9259491	48578.699	33.819496	58392.27734	26.51569	68832.0625
43.11470413	48360.844	33.89772	58388.71875	26.51765	68832.5625
43.18548203	48240.305	34.015057	58369.76953	26.5206	68834.26563
43.29166031	48064.977	34.059055	58360.37109	26.52502	68835.80469
43.33148193	47976.910	34.125057	58322.67188	26.53165	68829.99219
43.39120865	47859.512	34.149807	58303.78906	26.5416	68799.53906
43.48078918	47762.020	34.186928	58259.54688	26.55653	68801.75781
43.51437759	47702.426	34.200851	58239.80078	26.56212	68804.82031
43.56476974	47610.055	34.221737	58203.00391	26.57052	68808.96094
43.58366776	47578.754	34.229565	58187.92188	26.58311	68814.32031
43.60256195	47534.793	34.241314	58162.76172	26.602	68830.99219
43.62146378	47494.184	34.258934	58128.73438	26.63034	68853.28906
43.64981079	47423.336	34.265541	58103.51953	26.67284	68883.38281
43.69232941	47330.480	34.275452	58041.90625	26.73659	68920.70313
43.75610352	47151.000	34.279163	58013.20703	26.83221	68966.30469
43.78002548	47073.234	34.28056	58001.22266	26.86808	68980.82813
43.81589508	46978.133	34.28265	57980.91406	26.92187	68995.88281
43.8697052	46870.742	34.283436	57972.76563	26.94204	68995.24219
43.95042419	46753.234	34.284611	57959.13281	26.9723	68913.4375
44.07149887	46583.883	34.286377	57932.19141	27.01768	68919.35156
		34.289021	57885.67578	27.08576	68932.07031
		34.292992	57816.02344	27.18787	68952.35156
		34.298943	57710.05469	27.22617	68959.90625
		34.307877	57611.44531	27.28361	68967.44531
		34.32127	57506.96875	27.30515	68969.47656
		34.341362	57401.01563	27.33746	68972.5
		34.341831	57398.30859	27.34958	68970.73438
		34.342541	57394.57031	27.36775	68937.42969
		34.343597	57388.65234	27.39501	68946.42969
		34.345188	57379.3125	27.4359	68968.70313
		34.347569	57366.75391	27.49724	68999.65625
		34.351147	57351.21094	27.58925	69043.85938
		34.35651	57331.01172	27.72726	69117.10938
		34.364559	57304.36719	27.93428	69187.14063
		34.376629	57270.44531	28.24481	69336.73438
		34.381153	57258.41406	28.55534	69435.35938
		34.387943	57241.85156	28.86587	69483.89844
		34.390488	57235.76953	29.33166	69459.75
		34.391441	57233.51953	29.44811	69470.42188
		34.392872	57230.10156	29.56456	69485.03125
		34.39502	57224.88281	29.60823	69490.96875
		34.398247	57217.23828	29.67373	69501.75
		34.403076	57206.24219	29.77198	69523.03125
		34.410328	57190.08203	29.91936	69575.04688
		34.4212	57163.91406	30.14043	69663.10156
		34.437515	57120.18359	30.47204	69798.39063
		34.453827	57088.28125	30.59639	69848.13281

34.470139	57057.83203	30.78292	69904.25781
34.494602	57008.66406	30.85287	69920.11719
34.531307	56937.81641	30.95779	69915.82813
34.586361	56707.84375	30.99714	69907.64063
34.668938	56228.64063	31.05615	69878.34375
34.792812	56035.72266	31.14468	69739.10156
34.978607	55830.73047	31.15298	69724.48438
35.257313	55575.42969	31.15609	69718.69531
35.326992	55699.22266	31.16076	69709.52344
35.396667	55678.6875	31.16776	69694.64844
35.501179	55644	31.17039	69688.76563
35.657951	55580.58203	31.17433	69679.63281
35.893105	55461.47656	31.18023	69664.54688
35.981293	55421.68359	31.1891	69638.41406
36.11356	55351.05469	31.20239	69588.50781
36.311977	55166.01172	31.22234	69466.91406
36.609596	54742.82422	31.22981	69409.78125
36.721203	54563.99219	31.24103	69270.76563
36.888618	54317.01563	31.25786	68990.77344
37.056026	54224.78125	31.28309	68626.875
37.223438	54137.94922	31.32095	68178.29688
37.474552	53888.89844	31.33042	68070.5625
37.851223	53425.57422	31.33988	67964.03125
38.416241	53129.78125	31.35408	67800.08594
38.981255	52918.3125	31.3594	67734.25
39.546261	52782.38281	31.3614	67708.92969
40.393784	52576.78906	31.36439	67668.97656
40.473236	52556.86328	31.36888	67602.04688

Table A.2 Tabular data (loads-deflection) of strengthened length for dps=200mm

AT-1		AT2		AT3	
Disp	Force	Disp(mm)	Force(N)	Disp(mm)	Force(N)
0	0	0	0	0	0
0.070313	1970.95	0.070313	2008.842	0.140625	2521.556
0.140625	1472.903	0.140625	1460.957	0.28125	2902.557
0.246094	2540.488	0.246094	2513.256	0.421875	4315.698
0.404297	4151.243	0.404297	4104.975	0.632813	6443.664
0.641602	6566.699	0.641602	6494.67	0.949219	9634.275
0.997559	10188.88	0.997559	10076.79	1.423828	14415.22
1.531494	15567.16	1.531494	15400.04	1.601807	16094.35
1.73172	17362.51	1.73172	17177.1	1.868774	18427.5
1.806805	18022.3	1.806805	17829.35	1.968887	19272.26
1.919432	18987.78	1.919432	18784.54	2.119057	20528.21
2.088372	20412.6	2.088372	20192.7	2.344311	22388
2.341783	22521.01	2.341783	22277.04	2.428781	23074.93
2.531841	24026.99	2.436812	23041.85	2.555486	24058.77
2.579356	24477.15	2.472448	23321.35	2.682192	24999.91
2.62687	24827.78	2.525902	23730.89	2.808897	25905.17
2.698142	25348.15	2.606083	24330.85	2.935603	26754.98
2.80505	26085.05	2.726354	25201.47	3.062308	27547.47
2.911958	26761.14	2.906761	26428.36	3.252366	28691.81
3.018865	27421.22	3.177371	28083.02	3.537454	30214.23
3.179227	28357.22	3.583286	30202.74	3.965084	32066.57
3.419769	29602.43	3.989201	31864.44	4.125445	32572.86
3.660311	30669.58	4.09068	32188.33	4.365988	32233
3.900853	31520.93	4.242898	32521.58	4.60653	32714.25
3.991057	31768.42	4.299979	32618.92	4.847072	33240.98
4.126362	32031.29	4.385602	32527.14	4.937275	33456.18
4.329319	31697.42	4.514037	32323.36	4.971102	33551.98
4.633756	32237.64	4.706687	32607.04	5.021842	33692.22
4.938192	32889.49	4.995665	33210.82	5.097951	33885.11
5.014302	33063.82	5.104031	33482.87	5.126491	33953.18
5.128465	33283.91	5.144668	33577.68	5.169303	34045.93
5.171276	33355.25	5.205624	33702.75	5.23352	34153.34
5.235493	33440.15	5.297058	33850.04	5.329845	34237.26
5.331819	33519.19	5.331346	33897.03	5.353927	34382.86
5.476307	33208.28	5.382777	33954.78	5.378008	34408.23
5.620796	33383.82	5.402064	33978.37	5.414131	34434.53
5.765284	33679.94	5.430995	34012.8	5.468314	34498.8
5.909772	34030.32	5.47439	34058.09	5.549588	34586.61
6.126504	34601.84	5.539483	34060.51	5.6715	34472.19
6.451603	35409.51	5.563893	34052.01	5.854369	34878.25
6.776702	36314.04	5.600508	34010.44	6.128671	35583.34
7.101801	37179.84	5.65543	33858.63	6.540123	36641.72
7.589449	38248.38	5.710352	33856.44	7.157303	38415.59
8.077097	39364.23	5.765275	33944.22	8.083073	40652.25

8.199009	39767.09	5.847659	34149.13	8.256656	41302.13
8.320921	40100.24	5.971234	34476.85	8.430237	41813.13
8.503789	40586.82	6.156597	34943.08	8.690609	42594.73
8.686658	41010.56	6.434642	35674.01	9.081168	43589.83
8.869526	41461.21	6.851708	36822.31	9.471726	44650.22
9.143827	42171.19	7.477308	38395.13	9.862287	45646.52
9.555281	43160.46	7.633707	38906.76	10.44812	47067.64
10.17246	44551.92	7.790107	39247.57	11.32688	49220.65
10.4039	45076.04	8.024708	39801.32	12.64502	52318.12
10.75107	45878.71	8.376608	40757.39	12.97455	53073.13
11.27181	47057.13	8.508571	41145.14	13.30409	53802.84
12.05293	48798.29	8.706514	41659.81	13.79839	54967
12.83405	50360.91	8.904458	42131.04	13.98375	55409.25
13.61517	51950.41	9.102403	42659.62	14.2618	56052.75
14.39628	53707.84	9.399318	43417.26	14.67886	57062.48
15.56796	56278.81	9.844691	44488.05	15.30446	58565.03
15.86088	57145.97	10.51275	46046.88	15.53906	59035.41
16.1538	57853.09	10.76327	46645.3	15.77366	59654.04
16.26365	58108.52	11.13906	47532.34	15.83231	59807.72
16.42841	58475.8	11.70273	48850.07	15.85431	59864.28
16.4902	58607.75	12.54825	50736.34	15.86256	59884.68
16.58288	58822.3	13.39376	52493.64	15.87493	59916.15
16.61764	58902.25	14.23927	54432.43	15.87957	59927.51
16.66977	59021.73	14.45065	55357.51	15.88652	59945.61
16.68932	59067.25	14.66203	55920.25	15.88913	59952.41
16.71865	59135.14	14.9791	56707.34	15.89305	59962.57
16.76263	59237.37	15.4547	57798.74	15.89892	59977.52
16.82861	59389.15	16.1681	59375.37	15.90112	59982.96
16.92759	59613.17	16.34645	60329.1	15.90443	59990.69
17.07604	59946.22	16.52481	60939.81	15.90566	59993.39
17.29873	60436.07	16.79233	61709.02	15.90752	59996.48
17.63276	61140.87	16.89265	61986.52	15.91031	59999.97
18.13381	62160.84	17.04313	62384.82	15.91449	60008.46
18.3217	62540.89	17.19362	62760.08	15.92076	60023.77
18.39216	62681.96	17.3441	63136.85	15.93016	60048.64
18.49784	62890.89	17.56983	63666.86	15.94427	60086.73
18.65638	63201.44	17.90841	64419.77	15.96543	60143.52
18.89418	63681.2	18.41629	65474.25	15.99718	60227.45
19.25088	64428.6	19.17812	66994.6	16.04479	60351.94
19.38465	64728.16	19.36857	67421.13	16.11621	60537.14
19.58529	65162.37	19.55903	67849.13	16.22334	60813.33
19.78593	65562.15	19.84471	68477.57	16.38404	61221.72
19.83609	65913.11	19.95184	68708.39	16.62508	61822.55
19.88625	66103.64	20.11254	69029.51	16.71547	62044.57
19.9615	66331.68	20.15271	69108.27	16.85106	62370.86
20.07436	66657.41	20.19289	69183.37	17.05444	62850.5
20.18722	66969.23	20.25315	69300.93	17.10529	62969.96
20.30008	67244.32	20.34354	69490.23	17.15614	63087.93

20.46938	67625.54	20.43393	69709.4	17.2324	63266.46
20.53286	67766.25	20.45653	69765.02	17.34681	63530.68
20.62809	67975.06	20.47913	69814.55	17.38971	63632.07
20.77093	68287.61	20.51303	69887.88	17.45406	63778.93
20.9852	68755.7	20.56387	69998.71	17.55058	63995.31
21.30659	69446.92	20.58294	70038.19	17.69538	64310.03
21.42711	69702.62	20.59009	70052.19	17.91256	64772.88
21.60789	70081.15	20.60081	70076.63	17.99401	64943.62
21.87907	70643.55	20.6169	70114.22	18.11617	65186.25
21.98076	70853.45	20.64103	70167.76	18.29942	65569.25
21.9903	70872.72	20.67723	70247.35	18.5743	66149.22
22.00459	70901.69	20.73153	70366.14	18.98661	67037.38
22.02604	70947.93	20.75189	70410.61	19.39892	67872.51
22.05822	71016.64	20.75952	70429.05	19.81124	68818.56
22.10648	71118.91	20.77098	70457.12	20.22355	69811.18
22.17888	71267.36	20.78816	70497.08	20.37817	70183.91
22.28747	71489.23	20.81392	70556.44	20.43615	70321.18
22.45036	71820.84	20.85258	70643.02	20.52312	70524.39
22.69469	72317.73	20.91056	70770.89	20.55574	70598.41
22.78632	72507.78	20.99753	70960.9	20.56797	70625.36
22.87794	72697.92	21.12799	71244.06	20.58631	70666.48
22.96957	72883.47	21.32368	71662.59	20.59319	70681.38
23.10701	73161.08	21.61721	72284.21	20.59577	70686.63
23.31316	73575.23	21.72728	72515.9	20.59674	70688.51
23.33249	73613.38	21.8924	72862.75	20.59819	70691.52
23.36148	73670.1	21.95431	72992.62	20.60037	70696.19
23.40497	73755.05	21.97753	73041.27	20.60363	70703.3
23.4702	73888.84	21.98624	73059.49	20.60853	70714.14
23.53543	74020.41	21.9993	73086.8	20.61588	70730.57
23.60066	74151.33	22.0042	73088.35	20.6269	70755.63
23.6985	74345.32	22.01155	73101.27	20.64343	70793.23
23.84526	74629.91	22.0143	73107.27	20.66822	70849.49
24.06542	75060.49	22.01843	73109.43	20.70541	70932.23
24.28557	75485.55	22.02463	73113.72	20.7612	71053.66
24.289	75494.55	22.03393	73131.44	20.84488	71239.7
24.29416	75504.53	22.04787	73160.25	20.97041	71532.71
24.3019	75519.16	22.0688	73203.57	21.15869	71974.61
24.31351	75541.77	22.10018	73268.27	21.44112	72621.21
24.33093	75576.08	22.14725	73366.74	21.86476	73578.93
24.35705	75627.2	22.21785	73515.22	22.50022	74974.74
24.39623	75702.84	22.32377	73739.66	22.51511	75007.44
24.455	75809.7	22.48263	74075.26	22.53745	75054.35
24.47704	75848.63	22.72093	74570.48	22.53955	75058.03
24.5101	75901.36	23.07837	75313.92	22.54269	75063.83
24.55969	75956.09	23.21242	75588.68	22.5474	75073.9
24.60928	76007.09	23.41348	76009.26	22.54858	75077.27
24.65887	76039.59	23.71508	76615.78	22.54976	75079.74
24.67746	76053.16	24.01667	77221.83	22.55153	75083.46

24.70536	76064.31	24.09207	77371.77	22.55418	75089.29
24.7472	76029.66	24.20517	77595.96	22.55815	75098.14
24.80996	75842.63	24.24758	77650.8	22.56412	75111.38
24.87272	75909.83	24.3112	77768.34	22.57306	75131.11
24.93549	76034.49	24.31716	77763.98	22.58648	75160.58
25.02963	76241.25	24.32611	77777.92	22.60661	75204.7
25.06493	76317.5	24.32695	77773.63	22.6368	75270.55
25.11789	76431.54	24.32821	77768.48	22.68209	75369.07
25.19732	76600.82	24.33009	77766	22.75003	75516.73
25.31647	76841.2	24.33292	77767.77	22.85193	75738.65
25.36115	76923.92	24.33717	77772.15	23.00479	76068.11
25.42817	77018.43	24.34354	77779.52	23.23407	76558.55
25.5287	77112.34	24.3531	77792.53	23.32005	76741.55
25.5664	77137.2	24.36743	77813.84	23.3523	76809.85
25.59468	77147.13	24.38892	77847.66	23.40066	76912.93
25.63709	77130.45	24.42116	77900.98	23.47321	77066.85
25.65299	77121.03	24.46953	77981.58	23.58203	77296.33
25.67685	77095.98	24.48766	78010.61	23.62283	77380.67
25.71264	77046.09	24.51487	78050.22	23.63814	77412.62
25.76631	76931.88	24.55567	78083.13	23.64387	77423.74
25.81999	76934.89	24.61688	78143.32	23.65248	77440.88
25.87367	76953.16	24.63984	78165.55	23.65571	77447.05
25.92735	76993.58	24.67427	78202.63	23.65692	77449.2
26.00786	77043.48	24.72592	78231.77	23.65738	77449.9
26.12864	77075.93	24.73076	78232.85	23.65806	77451.02
26.24941	77095.02	24.73802	78235.05	23.65908	77452.8
26.37018	77070.82	24.74891	78239.1	23.66061	77455.75
26.49096	77016.05	24.75981	78242.54	23.66291	77460.3
26.67212	76884.09	24.77071	78240.16	23.66636	77467.31
26.94386	76870.5	24.78705	78233.44	23.67153	77477.94
27.04577	76899.02	24.81156	78239.46	23.67928	77493.98
27.19862	76896.73	24.84833	78213.45	23.69092	77518.09
27.4279	76845.16	24.90348	78045.78	23.70837	77554.34
27.65718	76762.91	24.95863	78102.46	23.73454	77608.59
27.88647	76628.63	25.01378	78209.56	23.7738	77689.69
28.11575	76532.39	25.09651	78391.31	23.77749	77699.34
28.45968	76325.45	25.12753	78457.67	23.78301	77713.33
28.54566	76282.56	25.17407	78558.09	23.79129	77732.62
28.56715	76269.54	25.2206	78660.58	23.80371	77759.82
28.5994	76247.43	25.26714	78759.41	23.82235	77799.58
28.61149	76239.05	25.33694	78902.84	23.8503	77858.42
28.62963	76224.79	25.44164	79090.09	23.89223	77946.08
28.63643	76218.9	25.54635	79213.23	23.90795	77978.8
28.64663	76215.09	25.57252	79234.59	23.90943	77981.88
28.66193	76207.67	25.61178	79256.19	23.90998	77983.05
28.68488	76190.78	25.62651	79260.02	23.91081	77984.76
28.71931	76156.99	25.64859	79257.38	23.91205	77987.36
28.77096	76114.13	25.68172	79238.69	23.91392	77991.26

28.84843	76137.61	25.73142	79176.38	23.91672	77997.11
28.96464	76187.8	25.74384	79159.78	23.92091	78005.84
28.9719	76192.8	25.75627	79136	23.92721	78018.96
28.97917	76198.26	25.7749	79079.96	23.93666	78038.61
28.98189	76199.33	25.78189	79051.1	23.95082	78068.07
28.98291	76199.48	25.79237	79028.22	23.97207	78112.38
28.98444	76199.77	25.80809	79043.19	24.00395	78178.94
28.98674	76200.31	25.83168	79072.3	24.05176	78277.97
28.9876	76199.86	25.86706	79103.84	24.12348	78425.7
28.98889	76199.85	25.92012	79131.51	24.23106	78644.28
28.99083	76199.55	25.99972	79177.55	24.39243	78971.65
28.99277	76202.98	26.11912	79187.9	24.45294	79094.14
28.99471	76201.1	26.29821	79156.89	24.47563	79140.14
28.99762	76201.77	26.36538	79139.23	24.50967	79208.3
29.00198	76204.14	26.39056	79131.48	24.52244	79233.8
29.00852	76208.06	26.42834	79117.44	24.54159	79271.41
29.01834	76214.14	26.43542	79117.11	24.5703	79327.22
29.03306	76223.11	26.44605	79113.63	24.61338	79409.79
29.05515	76235.53	26.46198	79115.74	24.67801	79531.34
29.08828	76251.91	26.47792	79110.15	24.70224	79576.05
29.13797	76274.42	26.49386	79101.36	24.73859	79642.63
29.14263	76276.5	26.51777	79087.19	24.73944	79644.07
29.14962	76279.52	26.55363	79064.2	24.74072	79646.29
29.1601	76283.7	26.60742	79024.8	24.74264	79649.84
29.17583	76290.31	26.6881	78955.72	24.74551	79655.16
29.19941	76299.98	26.80913	79010.69	24.74982	79662.88
29.23479	76314.11	26.99067	79085.41	24.75629	79674.47
29.28785	76335.78	27.26298	79079.51	24.766	79692.38
29.36745	76367.96	27.53529	79024.1	24.78055	79718.56
29.48685	76415.19	27.8076	78889.59	24.80239	79756.61
29.66594	76479.26	28.21606	78695.98	24.83514	79808.84
29.93459	76574.97	28.25436	78670.75	24.84742	79827.34
30.03534	76606.98	28.3118	78635.33	24.85202	79834.02
30.18645	76615.58	28.36924	78612.58	24.85893	79844.31
30.33756	76681.45	28.42668	78598.27	24.8693	79859.43
30.48867	76731.36	28.44822	78597	24.88484	79877.27
30.71534	76803.66	28.48054	78592.03	24.90815	79897.91
31.05534	76912.35	28.529	78580.98	24.94313	79914.66
31.18285	76941.95	28.54717	78577.8	24.99559	79935.66
31.3741	76949.3	28.57444	78574.05	25.01526	79947.44
31.66098	76885.09	28.61533	78563.42	25.04477	79949.56
31.7327	76864.38	28.67666	78538.78	25.08904	79889.24
31.80442	76834.98	28.76868	78539.1	25.09319	79887.43
31.912	76777.75	28.90669	78543.98	25.09941	79884.58
31.95234	76753.35	29.0447	78615.49	25.10875	79884.06
32.01285	76705.66	29.18272	78685.56	25.11575	79888.59
32.03555	76685.65	29.38973	78784.47	25.12275	79894.8
32.06958	76648.68	29.70026	78944.47	25.13326	79906.37

32.12064	76549.36	30.16606	79185.13	25.14901	79925.57
32.19723	76331.2	30.86475	79533.99	25.17265	79954.98
32.22595	76255.86	31.12675	79622.05	25.2081	79978.6
32.26903	76160.52	31.22501	79649.14	25.26128	80003.36
32.33365	76049.23	31.26186	79655.95	25.34104	80119.06
32.43058	75930.07	31.27567	79657.58	25.46069	80353.81
32.57598	75799.45	31.28085	79651.3	25.64016	80659.11
32.72137	75713.31	31.28862	79643.2	25.70746	80729.85
32.86677	75642.03	31.29154	79638.09	25.80841	80729.22
33.08487	75613.83	31.29591	79632.2	25.83365	80717.95
33.16665	75608.43	31.29755	79629.3	25.85889	80684.23
33.28933	75609.07	31.30001	79625.15	25.89675	80605.84
33.33533	75609.48	31.30093	79623.23	25.9346	80641.45
33.40434	75607.86	31.30231	79620.06	25.97246	80687.14
33.43022	75606.36	31.30439	79614.03	25.98666	80693.08
33.46903	75599.73	31.3075	79595.67	26.00795	80709.58
33.47631	75578.55	31.31217	79550.78	26.03989	80730.41
33.48723	75568.2	31.31684	79521.24	26.08781	80731.61
33.49541	75560.53	31.32151	79509.42	26.15968	80773.16
33.5077	75544.59	31.32851	79509.53	26.26748	80848.52
33.52612	75521.25	31.33901	79518.41	26.30791	80870.76
33.55375	75493.09	31.35477	79531.72	26.36855	80896.14
33.5952	75457.34	31.3784	79546.09	26.39129	80903.04
33.65738	75416.19	31.41386	79556.98	26.4254	80908.32
33.75064	75379.97	31.46703	79559.05	26.43819	80909.73
33.78562	75371.7	31.54679	79541.09	26.44299	80910.16
33.83808	75363.88	31.66644	79472.01	26.45018	80910.77
33.91677	75366.54	31.84591	78911.55	26.45288	80910.91
34.03481	75389.15	32.02539	78580.07	26.45389	80911.14
34.21186	75440.53	32.07025	78508.43	26.45541	80911.42
34.27826	75460.83	32.08708	78481.62	26.45769	80911.28
34.37785	75497.83	32.11232	78444.36	26.4611	80911.55
34.47745	75541.73	32.15017	78391.59	26.46622	80912.13
34.50235	75552.63	32.20696	78316.13	26.47391	80914.02
34.52724	75556.78	32.22826	78288.22	26.48543	80915.64
34.53347	75558.02	32.23624	78277.82	26.50272	80917.39
34.53503	75547.99	32.24822	78262.36	26.52001	80916.23
34.53736	75532.38	32.26619	78239.63	26.53729	80913.55
34.54086	75512.05	32.29314	78207.02	26.56322	80908.48
34.54611	75479.9	32.33356	78161.89	26.60212	80898.8
34.54808	75465.9	32.3942	78103.15	26.66047	80879.45
34.55104	75446.6	32.48516	78028.13	26.68235	80870.73
34.55547	75427.94	32.6216	77952.46	26.69056	80867.45
34.56212	75389.22	32.82627	77913.89	26.70286	80862.49
34.56876	75361.88	32.90301	77904.89	26.72132	80854.45
34.57541	75348.25	33.01813	77878.01	26.74902	80840.97
34.58538	75337.24	33.02533	77877.96	26.79055	80817.87
34.60034	75328.9	33.03252	77876.8	26.85286	80777.58

34.62277	75323.93	33.03522	77873.48	26.94632	80705.2
34.65642	75324.48	33.03927	77865.87	27.0865	80628.16
34.70689	75333.27	33.04534	77854.88	27.29679	80719.98
34.78261	75353.46	33.05445	77838.16	27.61221	80798.84
34.89618	75384.25	33.0681	77813.05	27.92763	80789.76
35.06654	75419.65	33.08176	77792.22	28.24306	80725.59
35.32208	75443.92	33.09542	77773.13	28.55848	80674.2
35.34603	75446.55	33.10054	77762.71	28.67677	80631.61
35.38197	75449.23	33.10823	77748.74	28.85419	80527.05
35.39544	75448.09	33.11975	77730.34	29.12033	80533.56
35.40049	75449.44	33.13704	77707.49	29.51954	80728.27
35.40808	75448.39	33.16297	77676.57	29.91875	80980.37
35.41945	75447.05	33.20187	77640.48	30.01855	81040.91
35.4365	75446.46	33.26022	77604.11	30.16825	81129.3
35.46209	75445.41	33.34774	77580.49	30.18229	81137.11
35.50046	75444.93	33.35594	77580.79	30.20334	81149.01
35.55802	75442.5	33.36825	77578.64	30.21123	81153.31
35.57961	75442.9	33.38671	77577.22	30.21419	81154.77
35.5877	75443.28	33.4144	77577.73	30.21863	81156.9
35.59074	75442.56	33.45594	77583.07	30.22529	81160.41
35.59529	75441.99	33.51825	77599.53	30.22779	81161.58
35.60212	75441.59	33.6117	77633.92	30.22873	81161.93
35.61236	75441.41	33.75189	77699.82	30.23013	81162.36
35.62773	75442.22	33.89208	77775.46	30.23224	81160.3
35.65078	75444.11	34.03226	77853.4	30.23435	81161.51
35.68535	75449.06	34.17245	77929.45	30.23646	81158.16
35.73722	75463	34.38274	78044.97	30.23962	81158.3
35.81501	75485.95	34.69816	78183.6	30.24436	81161.65
35.9317	75514.31	34.81644	78227.56	30.25147	81166.52
36.10674	75552.93	34.99387	78277.55	30.26214	81173.89
36.3693	75593.89	35.06041	78293.45	30.27815	81183.83
36.76313	75679.7	35.08535	78299.48	30.30215	81198.09
37.05851	75749.02	35.12278	78314.22	30.33816	81217.76
37.16927	75771.44	35.12629	78312.93	30.39218	81245.31
37.19696	75772.84	35.13155	78308.77	30.4732	81282.89
37.22466	75777.25	35.13945	78298.94	30.59473	81335.49
37.23504	75781.61	35.15129	78282.77	30.77703	81374.41
37.25062	75786.16	35.16905	78260.25	30.79412	81543.61
37.27398	75788.27	35.19569	78242.36	30.81121	81576.59
37.29735	75796.16	35.23566	78232.93	30.82831	81576.18
37.32071	75801.02	35.29561	78236.05	30.85394	81607.01
37.35576	75806.58	35.38554	78253.91	30.87958	81630.94
37.40833	75812.99	35.52042	78283.13	30.90521	81646.41
37.48719	75819.57	35.571	78291.26	30.94367	81658.27
37.60547	75824.19	35.64687	78300.64	31.00135	81662.74
37.78289	75821.44	35.67533	78303.67	31.08787	81650.57
38.04903	75804.08	35.71801	78306.63	31.12031	81638.46
38.14884	75797.98	35.78202	78211.19	31.13248	81630.63

38.29854	75784.92	35.84604	78226.21	31.13704	81628.13
38.52309	75744.03	35.91006	78249.18	31.14389	81624.2
38.85992	75752.39	35.97408	78272.02	31.14645	81622.37
39.36517	75790.23	36.0701	78302.25	31.15031	81619.37
39.55464	75798.23	36.21415	78342.86	31.15175	81618.2
39.83884	75817.69	36.4302	78398.88	31.15391	81616.1
39.86549	75820.83	36.75429	78457.27	31.15716	81610.95
39.86798	75822.37	37.24043	78499.17	31.16203	81579.45
39.87173	75820.34	37.42272	78503.23	31.1669	81561.46
39.87735	75817.73	37.69617	78514.42	31.17178	81554.12
39.88578	75813.37	38.10635	78612.2	31.17909	81552.85
39.89843	75809.3	38.13198	78621.66	31.19005	81555.38
39.9174	75803.33	38.15762	78622.27	31.20649	81557.88
39.94585	75795.48	38.19608	78625.03	31.23116	81555.72
39.98853	75786.45	38.25376	78630.36	31.26816	81533.2
40.00453	75783.42	38.34028	78643.29	31.32366	81458.27
40.02854	75779.57	38.37272	78648.52	31.34447	81421.42
40.06455	75777.26	38.38489	78650.51	31.37569	81323.33
40.10056	75777	38.38603	78651.05	31.3874	81275.8
40.13657	75775.86	38.38774	78650.42	31.39618	81231.69
40.19058	75778.52	38.39031	78648.37	31.40935	81132.84
40.2716	75781.44	38.39416	78646.89	31.4291	80975.42
40.30199	75781.17	38.39993	78646.94	31.45873	80780.94
40.31338	75780.94	38.40859	78646.67	31.50318	80581.62
40.33047	75780.27	38.42159	78647.65	31.56986	80383.36
40.33207	75779.17	38.44107	78649.02	31.59486	80318.46
40.33448	75777.78	38.47031	78651.74	31.63236	80236.86
40.33809	75776.65	38.51416	78657.43	31.64643	80207.75
40.34349	75775.48	38.53061	78659.34	31.6517	80196.99
40.3516	75773.31	38.55527	78662.73	31.65961	80181.02
40.36377	75770.31	38.56452	78663.93	31.67148	80157.83
40.38202	75765.43	38.5784	78665.13	31.68335	80136.38
40.40939	75755.58	38.59921	78666.7	31.69521	80112.94
40.45046	75732.49	38.63042	78672.33	31.71301	80078.39
40.46586	75721.66	38.63335	78672.75	31.73971	80031.12
40.48896	75701.69	38.63774	78672.91	31.77976	79980.06
40.49762	75693.02	38.63939	78673.11	31.83983	79925.17
40.51061	75678.05	38.64186	78673.17	31.86236	79906.17
40.51548	75672.1	38.64556	78672.42	31.89615	79879.48
40.52279	75662.35	38.64695	78671.42	31.90883	79869.03
40.52348	75661.55	38.64904	78670.22	31.92784	79849.84
40.52374	75661.11	38.65216	78669.94	31.93496	79841.4
40.52412	75660.44	38.65685	78672.45	31.94565	79821.77
40.52469	75659.73	38.66388	78675.55	31.96169	79779.8
40.52556	75658.38	38.67443	78678.67	31.98575	79724.19
40.52686	75656.47	38.69025	78681.98	32.02184	79661.27
40.52882	75653.06	38.71398	78685.83	32.07596	79613.23
40.53174	75647.2	38.74958	78690.7	32.15715	79603.1

40.53613	75638	38.80298	78696.83	32.27894	79666.63
40.54272	75623.25	38.80799	78697.37	32.40073	79762.86
40.55259	75600.27	38.8155	78697.97	32.52252	79868.82
40.56741	75556.61	38.81831	78698.17	32.7052	80035.7
40.57297	75538.95	38.82254	78698.05	32.70626	80036.48
40.5813	75507.34	38.82887	78697.33	32.70667	80036.52
40.58443	75494.61	38.83838	78698.81	32.70727	80036.65
40.5856	75489.7	38.85263	78701.54	32.70817	80037.09
40.58604	75487.95	38.87402	78704.59	32.70953	80037.91
40.5867	75484.98	38.90609	78707.55	32.71156	80038.07
40.58694	75483.92	38.95421	78710.12	32.7146	80038.64
40.58731	75482.38	38.97225	78710.64	32.71918	80040.97
40.58769	75480.81	38.99931	78710.67	32.72604	80045.45
40.58806	75479.29	39.00946	78710.54	32.73632	80053.13
40.58862	75476.8	39.01327	78710.53	32.75176	80065.03
40.58945	75472.9	39.01898	78710.42	32.7749	80082.9
40.5907	75466.28	39.02754	78709.53	32.80962	80108.9
40.59258	75455.01	39.04039	78705.73	32.86171	80146.66
40.5954	75436.59	39.05965	78708.01	32.93982	80200.46
40.59962	75406.01	39.08855	78708.84	33.05701	80272.22
40.6012	75394	39.1319	78708.31	33.23277	80361.13
40.60358	75374.23	39.19693	78707.39	33.29869	80392.77
40.60714	75337.08	39.29448	78706.81	33.39756	80436.6
40.61249	75261.92	39.44078	78703.16	33.54586	80490.7
40.62051	75131	39.66024	78679.25	33.76832	80547.53
40.62351	75080.02	39.74254	78676.96	34.10202	80651.38
40.62803	74992.3	39.86599	78672.63	34.60255	80869.76
40.63253	74880.79	39.91228	78666.82	34.63383	80885.17
40.63704	74749.34	39.98172	78635.13	34.66512	80897.65
40.64381	74529.09	40.00777	78618.44	34.71204	80914.77
40.65396	74165.27	40.04682	78574.38	34.78242	80937.41
40.66919	73591.83	40.06147	78553.96	34.888	80966.21
40.69202	72792.57	40.06697	78545.72	34.9276	80976.48
40.70058	72492.77	40.07521	78531.89	34.98699	80992.68
40.71343	72092.03	40.0783	78526.59	35.07607	81023.04
40.72627	71717.91	40.07946	78524.57	35.16516	81053.09
40.72948	71623.08	40.08119	78521.48	35.25424	81078.5
40.7343	71487.73	40.08184	78520.3	35.38787	81114.05
40.73551	71455.32	40.08282	78518.47	35.58831	81164.66
40.73671	71421.94	40.08429	78515.46	35.88897	81234.85
40.73851	71372.55	40.08649	78510.1	36.18962	81273.92
40.74123	71300.92	40.08979	78501.11	36.49028	81341.45
40.74529	71198.26	40.09473	78487.52	36.79095	81440.66
40.74681	71160.28	40.10216	78465.91	37.24193	81571.83
40.7491	71104.81	40.1133	78429.73	37.41105	81613.61
40.75253	71024.94	40.11747	78415.21	37.66474	81666.66
40.75767	70912.34	40.11904	78409.7	38.04526	81688
40.76538	70757.43	40.11962	78407.75	38.42578	81726.94

40.77696	70551.46	40.12051	78404.45	38.52091	81731.4
40.79432	70288.63	40.12084	78403.29	38.6636	81726.63
40.82036	69962.41	40.12096	78403.18	38.87765	81666.48
40.85942	69550.81	40.12115	78403.3	39.19871	81644.88
40.91801	69059.45	40.12143	78401.45	39.68031	81781.05
41.00589	68505.2	40.12184	78399.59	39.80071	81807.71
41.13772	68169.49	40.12247	78397.05	39.92111	81822.34
41.20364	68049.62	40.12341	78393.34	39.9324	81824.65
41.22012	68020.47	40.12482	78387.29	39.93663	81823.77
41.22166	68017.77	40.12694	78377.04	39.93821	81822.91
41.22397	68013.81	40.13012	78360.29	39.9406	81821.93
41.22485	68012.33	40.13487	78332.57	39.94417	81820.73
41.22615	68010.15	40.14201	78277.41	39.94952	81820.31
41.2281	68007.13	40.15273	78119.43	39.95756	81821.14
41.23104	68003.09	40.16344	77922.23	39.96962	81822.23
41.23544	67997.8	40.17416	77711.13	39.9877	81823.38
41.24203	67990.83	40.19024	77398.16	40.01482	81824.55
41.25193	67983.57	40.19626	77281.02	40.0555	81824.21
41.26678	67976.6	40.2053	77109.13	40.11652	81813.89
41.28905	67974.23	40.20756	77067.55	40.1394	81807.85
41.32246	67984.3	40.20982	77023.99	40.17372	81792.52
41.37257	68015.54	40.21321	76957.01	40.22521	81738.06
41.44773	68059.63	40.21448	76931.24	40.24452	81707.48
41.56048	68083.67	40.21639	76892.31	40.26382	81669.15
41.7296	68357.14	40.21924	76829.41	40.26865	81658.66
41.89872	68607.84	40.22211	76758.15	40.27589	81641.17
42.06784	68763.66	40.22496	76677.35	40.27861	81634.29
42.32153	68831.53	40.22926	76544.95	40.28268	81623.06
42.70205	68942.91	40.23569	76340.75	40.28421	81618.89
43.08257	69047.3	40.24535	76038.63	40.28478	81617.4
43.46309	69087.22	40.25983	75616.13	40.28564	81614.88
44.03387	69175.72	40.28155	75090.67	40.28596	81614
44.89005	69424.77	40.30327	74655.95	40.28645	81612.56
46.1743	69793.41	40.32499	74297.67	40.28717	81610.41
46.49537	69879.43	40.35757	73883.13	40.28825	81606.97
46.81644	69963.45	40.40644	73424.29	40.28989	81601.23
47.29803	70100.78	40.42477	73257.41	40.29234	81591.88
47.34318	70112.31	40.45226	73030.22	40.29601	81576.98
47.36012	70117.6	40.4935	72726.26	40.30151	81552.42
47.36647	70118.95	40.55535	72344.23	40.30976	81507.95
47.37599	70121.01	40.64813	71861.63	40.32215	81325.45
47.39027	70124.3	40.68292	71692.84	40.34072	80972.73
47.39563	70125.27	40.69597	71630.2	40.34769	80838.46
47.40366	70126.47	40.69719	71625.05	40.35814	80637.34
47.41572	70118.27	40.69842	71620.05	40.37381	80378.62
47.4338	70119.55	40.69964	71614.51	40.39733	80056.56
47.46092	70126.05	40.70147	71606.06	40.40614	79934.26
47.5016	70135.83	40.70423	71593.67	40.40945	79887.22

47.56262	70150.55	40.70835	71575.93	40.41069	79869.24
47.65415	70173.24	40.71455	71551.48	40.41255	79841.45
47.79145	70205.67	40.72383	71518.5	40.41534	79791.88
47.99739	70253.13	40.73777	71477.17	40.41813	79733.1
48.30632	70315.33	40.75867	71429.62	40.42092	79665.21
48.7697	70394.53	40.79002	71383.04	40.4251	79552.08
49.46476	70484.98	40.83704	71347.04	40.43138	79373.58
50.50736	70469.14	40.90757	71306.8	40.43373	79305.97
52.07127	70308.55	40.91418	71305.89	40.43726	79204.43
53.63517	70056.41	40.92411	71304.78	40.44256	79053.33
53.73291	70037.64	40.92504	71304.52	40.4505	78829.12
53.83066	70021.73	40.92643	71304.09	40.46243	78527.56
53.85509	70015.08	40.92852	71303.59	40.46689	78421.41
53.89174	70009.53	40.93166	71303.44	40.46856	78382.39
53.94673	70002.07	40.93637	71304.38	40.47108	78325.38
54.0292	69990.25	40.94343	71307.37	40.47485	78244.33
54.1529	69973.83	40.95403	71316.09	40.4805	78131.48
54.33846	69950.73	40.96991	71334.15	40.48899	77977.89
54.61681	69905.9	40.99374	71368.59	40.50171	77775.84
54.72118	69882.71	41.0295	71429.3	40.5208	77517.1
54.76033	69872.59	41.0429	71452.68	40.54943	77194.34
54.81904	69853.02	41.04792	71461.41	40.56017	77078.18
54.9071	69789.16	41.05547	71474.8	40.56118	77067
54.91536	69781.39	41.05829	71479.84	40.56155	77062.84
54.92775	69772.31	41.05935	71481.67	40.56211	77056.49
54.93239	69766.85	41.05975	71482.37	40.56297	77047.19
54.93936	69759.2	41.06034	71483.19	40.56424	77033.4
54.94197	69755.38	41.06124	71484.57	40.56615	77009.03
54.94589	69749.41	41.06258	71486.83	40.56902	76973.43
54.94736	69747.01	41.0646	71490.48	40.57332	76926.66
54.94956	69743.4	41.06761	71496.32	40.57977	76861.48
54.95287	69736.91	41.07215	71504.8	40.58944	76766.03
54.95783	69724.51	41.07894	71518.77	40.60395	76630.81
54.96527	69702.56	41.08913	71539.97	40.62572	76452.39
54.97643	69665.38	41.10442	71572.34	40.65837	76230.64
54.99316	69593.81	41.12735	71621.88	40.70734	75967.48
55.0099	69529.88	41.16175	71698.58	40.7808	75656.38
55.02664	69474.04	41.21334	71813.05	40.891	75295.5
55.05175	69406.62	41.29073	71965.79	40.93232	75165.45
55.08941	69331.6	41.40681	72084.95	40.9943	74974.58
55.1459	69257.59	41.45036	72121.55	41.08728	74857.8
55.23064	69177.8	41.45443	72125.28	41.11052	74839.86
55.31538	69143.52	41.45596	72126.69	41.11633	74834.48
55.40011	69093.61	41.45826	72128.55	41.12505	74828.27
55.52722	69063.7	41.45912	72129.13	41.12832	74826.16
55.71788	69059.87	41.46041	72129.37	41.13322	74823.28
56.00387	69072.6	41.46235	72126.03	41.14057	74819.37
56.43285	69084.06	41.46428	72125.39	41.14333	74818.04

56.47306	69086.08	41.46622	72122.8	41.14747	74815.92
56.53339	69086.75	41.46913	72122.98	41.15367	74793.46
56.62388	69087.55	41.47348	72125.09	41.15988	74777.02
56.65781	69086.17	41.48002	72129.17	41.16608	74770.23
56.70871	69081.76	41.48983	72136.02	41.17539	74763.78
56.78505	68949.31	41.50454	72147.75	41.17888	74761.52
56.80415	68897.05	41.5266	72166.42	41.18019	74760.66
56.82323	68818.98	41.55969	72193.73	41.18215	74759.54
56.84233	68713.55	41.5721	72203.59	41.1851	74758.33
56.87095	68424.26	41.57676	72207.33	41.18951	74757.08
56.9139	67977.12	41.58374	72212.73	41.19614	74756.73
56.97832	67939.83	41.58636	72214.86	41.20608	74757.91
57.04274	67976.97	41.59028	72217.55	41.22099	74763.18
57.10716	68024.45	41.59176	72218.19	41.2359	74771.8
57.20379	68101.61	41.59396	72214.55	41.25081	74780.95
57.34874	68214.75	41.59728	72209.22	41.27318	74798.52
57.56615	68385.08	41.60225	72207.62	41.30672	74830.62
57.89228	68597.28	41.6097	72209.87	41.35704	74885
58.38147	68393.75	41.62088	72214.84	41.43252	74961.23
58.87065	67696.94	41.63766	72223.69	41.54574	75029.48
59.35984	67661.23	41.66282	72237.6	41.71556	75061.05
59.84903	67688.64	41.70056	72257.77	41.77925	75057.74
60.33821	67685.78	41.75717	72283.26	41.80313	75049.55
61.07201	67521.04	41.84209	72306.64	41.80537	75048.57
61.80579	67244.91	41.96945	72308.11	41.80873	75047.27
61.98923	67181.84	42.16051	72318.41	41.81377	75046.95
62.05802	67157.71	42.44709	72426.69	41.82133	75048.46
62.16121	67117.23	42.87697	72507.85	41.83266	75053.56
62.31599	67063.32	43.52179	72612.41	41.84966	75064.91
62.54817	67001.57	43.7636	72656.93	41.87517	75091.67
62.89643	66802.36	44.12631	72758.23	41.91342	75137.25
63.24468	66495.54	44.48902	72889.21	41.9708	75211.1
63.26645	66474.29	44.85173	73036.18	41.99232	75241.54
63.2991	66446.03	45.39579	73273.38	42.00039	75253.14
63.34808	66401.66	45.59982	73358.05	42.00341	75257.19
63.36645	66383.8	45.90585	73481.17	42.00795	75262.56
63.39399	66356.99	46.36491	73649.27	42.01476	75272.61
63.43531	66316.63	46.82396	73796.76	42.02497	75287.81
63.4973	66255.67	47.28302	73912.99	42.04029	75310.52
63.52054	66233.19	47.45517	73952.09	42.06327	75344.37
63.55541	66199.82	47.71338	74006.68	42.09774	75394.47
63.6077	66146.86	47.7618	74017.95	42.14945	75462.38
63.62732	66126.24	47.77995	74019.9	42.227	75536.94
63.65673	66094.59	47.80719	74023.41	42.34333	75605.16
63.66776	66082.42	47.84804	74027.33	42.51782	75638.29
63.68431	66064.05	47.90931	74037.91	42.53418	75643.22
63.69052	66057.05	48.00123	74054.46	42.54032	75642.45
63.69983	66046.6	48.1391	74076.02	42.54951	75642.7

63.71378	66030.33	48.34591	74100.84	42.55297	75642.73
63.72775	66012.85	48.65612	74169.22	42.55814	75641.85
63.73124	66008.27	49.12144	74255.33	42.56009	75640.88
63.73647	66001.42	49.81942	74289.94	42.56081	75640.4
63.74433	65990.82	50.86637	74222.59	42.5619	75638.83
63.75611	65975.39	52.43683	74098.7	42.56355	75634.81
63.77378	65952.5	53.02574	74049.48	42.566	75634.13
63.80029	65918.86	53.90912	74023.7	42.56969	75636.42
63.84004	65867.26	54.24039	74018.72	42.57521	75639.45
63.89968	65801.58	54.36461	74013	42.5835	75641.71
63.98913	65749.91	54.55094	73998.48	42.59594	75642.79
64.12332	65701.44	54.62082	73989.29	42.61459	75642.26
64.3246	65640	54.62738	73988.7	42.64257	75638.97
64.62652	65584.96	54.6372	73987.67	42.68455	75629.45
65.0794	65546.22	54.65194	73984.63	42.74751	75632.93
65.75871	65479.42	54.67405	73980.76	42.84195	75673.41
65.8224	65475.27	54.70722	73974.84	42.98361	75749.84
65.84628	65471.44	54.75696	73963.3	43.19609	75837.33
65.8821	65466.91	54.83158	73933.48	43.51484	75890.41
65.89553	65465.04	54.85957	73919.56	43.63436	75913.33
65.91569	65461.53	54.90154	73888.44	43.81364	75841.52
65.94591	65456.58	54.91728	73873.62	43.88087	75842.9
65.95724	65454.5	54.92908	73858.62	43.98173	75862.79
65.97424	65450.84	54.94678	73824.26	44.133	75901.7
65.99975	65440.47	54.96007	73793	44.35991	75974.2
66.038	65423.05	54.97999	73712.43	44.70028	76114.7
66.09538	65405.11	54.99991	73641.49	45.21083	76309.89
66.18146	65389.5	55.01983	73578.86	45.97665	76631.01
66.31057	65374.24	55.04972	73502.59	46.04845	76666.41
66.50423	65356.73	55.06092	73475.88	46.05518	76666.87
66.79472	65331	55.06512	73466.09	46.06528	76673.51
66.90366	65320.57	55.07143	73451.74	46.06907	76674.79
67.06707	65302.79	55.07379	73446.55	46.07474	76676.47
67.31217	65281.69	55.07734	73438.5	46.08326	76676.35
67.31361	65282.18	55.08265	73425.72	46.09178	76681.95
67.31576	65281.62	55.09063	73406.05	46.1003	76686.66
67.31657	65281.14	55.10259	73379.39	46.11308	76692.91
67.31779	65280.74	55.12054	73345.38	46.13224	76701.62
67.31824	65280.43	55.14746	73305.98	46.161	76714.43
67.31892	65279.91	55.18785	73264.99	46.20412	76733.48
67.31918	65279.86	55.24843	73229.41	46.26881	76761.04
67.31956	65279.63	55.33929	73207.72	46.36585	76801.44
67.32014	65279.09	55.47559	73208.77	46.5114	76860.52
67.321	65278.64	55.68005	73237.07	46.72972	76946.05
67.32229	65278.18	55.98672	73298.66	47.05721	77062.05
67.32423	65277.67	56.10173	73318.59	47.54845	77227.58
67.32714	65276.64	56.14486	73324.77	48.2853	77436.58
67.33151	65276.03	56.20955	73330.27	49.39058	77648.91

67.33806	65275.37	56.27423	73286.79	51.04851	77907.48
67.34789	65274.58	56.2904	73261.33	51.46299	77959.77
67.36263	65273.68	56.31467	73190.41	51.87747	77997.27
67.38474	65272.42	56.35105	73016.88	52.49919	78030.93
67.41790	65270.61	56.38744	72607.52	52.65461	78037
67.46765	65267.87	56.42383	72159.22	52.81005	78034.95
67.54227	65263.79	56.43747	72112.27	52.86833	78031.91
67.65419	65257.7	56.45794	72099.7	52.95576	78022.74
67.82209	65247.51	56.48864	72108.85	53.08691	77860.45
68.07393	65230.27	56.53469	72138.1	53.21805	77787.06
68.45168	65199.59	56.60378	72190.95	53.3492	77760.28
68.59335	65186.76	56.7074	72275.43	53.54591	77756.39
68.80583	65166.27	56.86283	72402.94	53.56435	77758.91
68.88552	65157.93	56.92112	72449.42	53.59202	77757.31
69.00504	65145.8	57.00854	72518.85	53.59893	77761.73
69.04986	65141.46	57.04133	72545.09	53.60585	77758.96
69.11709	65135.38	57.04441	72547.39	53.61622	77757.78
69.12341	65133.44	57.04902	72551.71	53.63178	77758.66
69.13285	65134.79	57.05593	72557.61	53.65512	77759.13
69.13641	65134.73	57.0663	72566.23	53.69013	77760.15
69.14172	65134.31	57.06727	72567.12	53.74265	77762.27
69.14372	65134.03	57.06764	72567.37	53.82142	77765.16
69.14671	65133.23	57.06819	72567.77	53.93959	77771.15
69.15119	65132.67	57.06901	72568.47	54.11684	77780.62
69.15288	65132.43	57.07024	72569.56	54.3827	77791.77
69.1554	65132	57.07208	72571.27	54.7815	77813.11
69.15919	65130.1	57.07485	72573.86	55.37971	77842.26
69.16297	65127.9	57.07901	72577.64	56.277	77900.35
69.16676	65124.77	57.08524	72582.91	56.61349	77922.18
69.17243	65121.06	57.09458	72590.59	56.73967	77924.88
69.18095	65116.31	57.10861	72601.94	56.78699	77923.96
69.19373	65111.06	57.12964	72618.74	56.80473	77922.8
69.2129	65106.16	57.16118	72643.79	56.83135	77918.06
69.24165	65100.99	57.2085	72681.14	56.87128	77890.99
69.28478	65093.99	57.27948	72734.81	56.93117	77693.05
69.34946	65084.55	57.38595	72810.7	56.95362	77583.45
69.44651	65071.82	57.54565	72917.87	56.98731	77222.22
69.59206	65054.5	57.7852	72968.64	57.03784	76996.95
69.81038	65030.05	58.14452	72556.44	57.08837	76999.7
70.13787	64992.68	58.27927	72342.45	57.09153	77000.73
70.6291	64932.66	58.48139	72067.16	57.09627	77003.41
71.36596	64834.83	58.55719	72053.52	57.09804	77003.86
72.47124	64682.44	58.67088	72038.17	57.10071	77004.53
74.12917	64462.43	58.84142	72044.19	57.1047	77006.23
74.54365	64411.81	59.09723	72080.63	57.1107	77009.45
74.64726	64398.37	59.48094	72139.15	57.11969	77015.03
74.68613	64392.66	59.62484	72158.44	57.13318	77024.83
74.70069	64390.75	59.84067	72181.48	57.15342	77039.93

74.72255	64388.55	59.92161	72187.55	57.18377	77063.6
74.72801	64388.66	60.04301	72194.99	57.2293	77100.41
74.73347	64390.56	60.08855	72197.34	57.29759	77156.79
74.73895	64386.88	60.15684	72199.94	57.40002	77241.4
74.74714	64385.09	60.20806	72193.96	57.55368	77358.78
74.75944	64383.04	60.25928	72187.09	57.78418	77515.41
74.77788	64380.63	60.3105	72183.1	58.12991	77687.98
74.80553	64377.04	60.38733	72181.92	58.64851	77765.35
74.84703	64371.66	60.41615	72183.02	58.77815	77774.61
74.90927	64363.54	60.45935	72183.22	58.9078	77794.29
75.00264	64351.39	60.47556	72183.17	59.10228	77807.4
75.14268	64332.77	60.48164	72183.29	59.39399	77809.3
75.35275	64304.93	60.49076	72183.38	59.50338	77813.2
75.37244	64302.95	60.50443	72183.27	59.66747	77808.76
75.37429	64303.54	60.52494	72182.73	59.72900	77802.68
75.37706	64302.72	60.54545	72182.20	59.75208	77799.09