



**TEMPERATURE EVOLUTION OF  
NEUTRON STARS**

**By**

**Tagesse Teka**

**Advisor: Tolu Biressa(PhD)**

**Co-advisor: Abdisa Tessema**

**SUBMITTED IN PARTIAL FULFILLMENT OF THE  
REQUIREMENTS FOR THE DEGREE OF  
MASTER OF SCIENCE IN PHYSICS  
(ASTROPHYSICS)**

**AT**

**JIMMA UNIVERSITY**

**JIMMA, ETHIOPIA**

**JULAY 2014**

**©Copyright by Tagesse Teka, 2014**

**School of Graduate Studies**  
**Jimma University**  
**College of Natural Sciences**  
**MSc. Thesis Approval Sheet**

The undersigned hereby certify that they have read and recommend to the College of Natural Sciences for acceptance a thesis entitled **Temperature Evolution of Neutron Stars** the by requirements **Tagesse Teka** for the degree in of **Master of Science in Physics(Astrophysics)**

_____	_____	_____
Name of the Chairperson	Signature	Date
_____	_____	_____
Name of Major Advisor	Signature	Date
_____	_____	_____
Name of Internal Examiner	Signature	Date
_____	_____	_____
Name of External Examiner	Signature	Date

**SCHOOL OF GRADUATE STUDIES**

# JIMMA UNIVERSITY

date : \_\_\_\_\_

Author: **Tagesse Teka**

Title: **Temperature Evolution of Neutron Stars**

Department: **Physics**

Degree: **MSc.**

Convocation: **May**

Year: **2022**

Permission is herewith granted to Jimma University to circulate and to have copied for non-commercial purposes, at its discretion, the above title upon the request of individuals or institutions.

---

Signature of Author

THE AUTHOR RESERVES OTHER PUBLICATION RIGHTS, AND NEITHER THE THESIS NOR EXTENSIVE EXTRACTS FROM IT MAY BE PRINTED OR OTHERWISE REPRODUCED WITHOUT THE AUTHOR'S WRITTEN PERMISSION.

THE AUTHOR ATTESTS THAT PERMISSION HAS BEEN OBTAINED FOR THE USE OF ANY COPYRIGHTED MATERIAL APPEARING IN THIS THESIS (OTHER THAN BRIEF EXCERPTS REQUIRING ONLY PROPER ACKNOWLEDGEMENT IN SCHOLARLY WRITING) AND THAT ALL SUCH USE IS CLEARLY ACKNOWLEDGED

# Contents

<b>Table contents</b>	<b>ii</b>
<b>List of figures</b>	<b>iv</b>
<b>List of tables</b>	<b>v</b>
<b>Acknowledgment</b>	<b>vi</b>
<b>Abstract</b>	<b>1</b>
<b>1 Introduction</b>	<b>2</b>
1.1 Background and Literature Review . . . . .	2
1.2 Statement of the problem . . . . .	3
1.3 Objectives . . . . .	4
1.3.1 General objective . . . . .	4
1.3.2 Specific objectives . . . . .	4
1.4 Methodology . . . . .	4
<b>2 Introduction to neutron stars</b>	<b>6</b>
2.1 Neutron star progenitor and formation . . . . .	8
2.2 Structure of neutron stars . . . . .	9
2.2.1 Outer Crust . . . . .	11
2.2.2 Inner Crust . . . . .	12
2.2.3 Outer core . . . . .	12
2.2.4 Inner core . . . . .	13

2.3	Observation and properties of neutron stars . . . . .	14
<b>3</b>	<b>Thermal evolution of neutron stars</b>	<b>18</b>
3.1	Tolman-Oppenheimer-Volkoff equations . . . . .	18
3.2	Thermal heating and energy balance process . . . . .	19
3.3	Neutrino and photon cooling . . . . .	27
<b>4</b>	<b>Result and discussion</b>	<b>29</b>
4.1	Theoretical results . . . . .	29
4.1.1	Neutrino vs photon cooling . . . . .	29
4.2	Observational data and their implications . . . . .	30
<b>5</b>	<b>Summary and conclusion</b>	<b>33</b>
	<b>References</b>	<b>33</b>

# List of Figures

2.1	neutron star structure . . . . .	13
4.1	The LogLog plots of temperature vs time for neutrino and photon emission for isolated neutron stars. <b>Blue:</b> neutrino emission curve. <b>Orange:</b> photon emission curve. . . . .	30
4.2	The luminosity vs temperature plot of neutrino and photon emissions for isolated neutron stars. <b>Blue:</b> neutrino emission. <b>Orange:</b> photon emission. . . . .	30
4.3	Temperature vs age distribution of NSs in black-body and hydrogen atmospheres. <b>Left:</b> Environment is in black body atmosphere. <b>Right:</b> Environment is in hydrogen atmosphere. . . . .	31
4.4	Temperature vs age distribution of NSs in black-body and hydrogen atmospheres. <b>Left:</b> Environment is in black body atmosphere. <b>Right:</b> Environment is in hydrogen atmosphere. . . . .	32

# List of Tables

# Acknowledgments

First of all, I would like to thank the Almighty God. I am deeply indebted to my Advisor Dr. Tolu Biressa for his guidance, constructive comments, consistent support and thanks for providing me an interesting topic. Without his encouragement and guidance, this work would never have happened. Thank you for your endless patience. Then my deepest thank goes to my wife, Medhanit Assefa and my family for their love and support over the years. I also thank Kembata Tembaro Zon, special Tembaro Wored who sponsored me. Finally, I would like to thank my friends and class mates who we work together. In addition to this, I would like to appreciate and thank Mr. Solomon Sayebo who supported me.



# Abstract

Thermal evolution of neutron stars(NSs) is one of the active research areas in astrophysics. In this thesis we focused on the temperature evolution of the NSs where energy conservation and transformation, mainly derived from the General relativistic Tolman-Oppenheimer-Volkoff (TOV) based evolutionary models were considered. In the energy transformation process, cooling of the star via electromagnetic emission in terms of luminosity is used in tracking the evolutionary scenario of the star. Mathematica 13 was used for the computation and also for observational data process in the analysis. The plots of the theoretical results by pattern are similar with that of the observational data. However, a detailed analysis with fine data sets and boundary conditions are needed in the future for better analysis and progress.

**Keys:** Neutron Star, Cooling, Temperature, Luminosity, Compact objects, Tolman-Oppenheimer-Volkoff.

# Chapter 1

## Introduction

### 1.1 Background and Literature Review

When a star has exhausted all its fuel, the gas pressure of the hot interior can no longer support its weight and the star dies by collapsing to a denser state, then a compact star (also called compact object (CO)) is formed [1]. If the star is of low mass the end product will be a white dwarf (WD) who is supported by electron-degeneracy pressure against further collapse by gravity. If the star is of intermediate mass, the product will be a neutron star (NS) who is supported by neutron-degenerate pressure. When a very high mass star runs out of fusion it completely collapses to singularity called black hole is formed (BH).

The compactness of these stars give them many extreme properties which make them relevant for the high energy astrophysics, emission of X-rays and Gamma-rays. They exhibit peculiar properties that cannot be created in normal laboratories and involve phases of matter that are not yet well understood. So, their astrophysical studies are active research areas to understand the associated relativistic phenomena both theoretically and observationally.

For instance NSs, exhibit a variety of astrophysical manifestations. They are considered as stable environment in where extreme physical conditions of density, temperature, gravity, and magnetic fields are realized simultaneously [2]. They are born hot in supernova

explosions and with subsequent cooling are driven from neutrino emission in their interior to emission of photons at their surfaces [3]. Thus, they are natural laboratories to study the properties of matter and the surrounding plasma under such extreme limits. They probe the properties of cold matter at extremely high densities and have considered to test bodies for theories of general relativity[4] and the physics of matter at very high densities and temperatures.[5].

However, still the proper composition of these compact objects remains uncertain. In the case of NS, a violent process as a supernova is expected to have a very high heat content shortly after birth, resulting in temperatures of  $T \sim 10^{11}k$  that challenges cooling calculations. NS cooling is important tool to probe the inner structure of neutron stars. A neutron star initially loses thermal energy through neutrino emission, but this process is taken over by surface photon radiation about  $10^5$ yr after birth. The observations of surface thermal emission from a neutron star have the potential to provide constraints on its inner structure, dense matter inside it and magnetic field properties[6].

## 1.2 Statement of the problem

The thermal evolution of neutron stars is a subject of intense research, both theoretically and observationally. The evolution depends very sensitively on the state of dense matter at supra nuclear densities, which essentially controls the neutrino emission. The evolution depends, too, on the structure of the stellar outer layers which control the photon emission. Various internal heating processes and the magnetic field strength and structure will influence the thermal evolution[4]. So, here we focus on the temperature evolution of the NSs.

### Research questions

- In what way does the internal structure of a NS affect its temperature evolution?
- How does a binary star affect the temperature evolution of a NS?

- What is the effect of interstellar dust nearby the NS on its temperature evolution?
- What is the cooling time of a neutron star?
- What parameters do affect the temperature evolution of Neutron stars?
- How do the neutrino and photon emission cooling temperatures be compared?

## 1.3 Objectives

### 1.3.1 General objective

To study the temperature evolution of neutron stars

### 1.3.2 Specific objectives

- To describe the way how internal structure of a NS affect its temperature evolution.
- To discuss how a binary star affect the temperature evolution of a NS.
- To determine the effect of interstellar dust nearby a NS on its temperature evolution.
- To derive cooling time of a neutron star.
- To specify some parameters that affect temperature evolution of NSs.
- To compare neutrino and photon emission cooling temperatures.

## 1.4 Methodology

Mainly, General Relativistic Tolman-Oppenheimer-Volkoff (TOV) equations are used to describe the structure and evolution of Neutron Stars. However, due to more complicated condensation by gravity, the equation of state being considered is not an easy task where further field theory equations are needed to model. The equation of modelings by itself is an ongoing active research area. Here, in our work, we consider cooling of NSs where energy conservation and transformation is used. In the energy transformation process,

cooling of the star via electromagnetic emission in terms of luminosity is used in tracking the evolutionary scenario of the star.

Mathematica 13 is used to generate some numerical data of the theoretical model and in the processing of observational data to analysis the results. Latex is used for document process.

Organization of the work: in chapter two an introduction of the Neutron Star be provided. In chapter three the basic mathematical equations and evolutionary models will be provided. In chapter four the results and discussions be given while the fifth chapter summary and conclusion will be provided.

# Chapter 2

## Introduction to neutron stars

Some massive stars collapse to form neutron stars at the end of their life cycle, as has been both observed and explained theoretically. Under the extreme temperatures and pressures inside neutron stars, the neutrons are normally kept apart by a degeneracy pressure, stabilizing the star and hindering further gravitational collapse. However, it is hypothesized that under even more extreme temperature and pressure, the degeneracy pressure of the neutrons is overcome, and the neutrons are forced to merge and dissolve into their constituent quarks, creating an ultra-dense phase of quark matter based on densely packed quarks. In this state, a new equilibrium is supposed to emerge, as a new degeneracy pressure between the quarks, as well as repulsive electromagnetic forces, will occur and hinder total gravitational collapse[1]. Thermal evolution of neutron stars (NSs) provides such an opportunity. As is well known, a NS is a compact astrophysical object that consists mainly of degenerate neutrons. It is as heavy as the sun, and the total mass is enclosed in a small sphere of radius  $R \sim 10$  km. The mass density reaches  $\sim 10^{12} \frac{kg}{cm^3}$ , and hence NSs offer ultra-dense environments which cannot be achieved on the earth. The study of the NS cooling began in the mid 60's before the first discovery of the pulsar, and has been updated until today along with the development of the theory of micro-physics in the ultra dense environment. There are also a few tens of measurements of surface temperatures, and thus this NS cooling theory can now be tested against the observations[7].

Neutron stars play a unique role in physics and astrophysics. On the one hand, they contain matter under extreme physical conditions, and their theories are based on risky and far extrapolations of what we consider reliable physical theories of the structure of matter tested in laboratory. On the other hand, their observations offer the unique opportunity to test these theories. Moreover, neutron stars are important dramatic personae on the stage of modern astrophysics; they participate in many astronomical phenomena. To calculate neutron star structure, one needs the dependence of the pressure on density, the so called equation of state (EOS), in this huge density range, taking due account of temperature, more than  $10^9$  K in young neutron stars, and magnetic fields, sometimes above  $10^{15}$  G. Neutron stars are observed in all bands of electromagnetic spectrum in our Galaxy and in nearby satellite galaxies (such as the Large Magellanic Cloud and Small Magellanic Cloud). The neutron star astronomy is extending into the Local group of galaxies and even further[8].

All stars evolve and this includes neutron stars. Neutron stars do not evolve in the typical manner of a main sequence star because they are not powered by the fusion process. However, neutron stars do undergo thermal, rotational, and magnetic field evolution. The time scale for these evolutionary processes is rapid compared to those involving main sequence stars. Although, the time scale is quick on an astrophysical scale (millions of years instead of billions of years) it is still observable and is a field of current research. Since most observed pulsars are thought to be quite old, the temperature, magnetic field and rotational properties listed above have evolved from their initial values. The life cycle of a neutron star can be described by its thermal, rotational, and magnetic field evolution. These three quantities are all connected to each other through the observed pulsar radiation. Because we are interested in the neutron star magnetic field generation, it is important to understand the neutron star internal thermal evolution to assist in setting the boundary values in our calculations of the neutron magnetization. Neutron star rotational evolution has direct application in the neutron star electromagnetic field structure and the resulting radiation mechanisms[8].

## 2.1 Neutron star progenitor and formation

Neutron stars are relativistic compact objects formed by the collapsing cores of massive stars at the end of their evolution. The energy released by the collapsing core launches a shock that ejects the outer layers of the progenitor star in a so called supernova explosion. The masses of neutron stars are in the range  $M \simeq 1 - 3M_{\odot}$ . Accurately measured masses in binary pulsars are clustered near  $M \simeq 1.4M_{\odot}$ . The highest measured masses are  $M \simeq 2M_{\odot}$ . There is a firm theoretical upper limit to the mass of neutron stars  $M_{max} \simeq 3.2M_{\odot}$  [9]. Supernova explosions driven by big stars' iron-core collapse are thought to give birth to neutron stars. In this work, the progenitor masses of neutron stars in isolation are considered to be between 8 and 40  $M_{\odot}$ . However, the lower limit could be even higher than 10  $M_{\odot}$ ; Ritossa et al. (1996) found that a 10  $M_{\odot}$  star evolves into a 1.26  $M_{\odot}$  oxygen-neon white dwarf. This would necessitate a 30 percent reduction in the model occurrence rate of supernovae from single stars. If a star is stripped from its hydrogen envelope due to Roche-lobe overflow in a close binary, the lower mass limit for the formation of a neutron star increases to about 12  $M_{\odot}$ . The necessary condition for the formation of a neutron star is the requirement for the helium star remnant of a binary component to have a mass in excess of about (Tutuko Chandrasekhar) Yungelson 1973; Habetts 1985), which enables the formation of a carbon-oxygen core of a Chandrasekhar mass. The upper limit is less well known. Neutron stars are created in the aftermath of the gravitational collapse of the core of a massive star ( $> 8M_{\odot}$ ) at the end of its life, which triggers a Type II supernova explosion. Newly-born neutron stars or proto-neutron stars are rich in leptons, mostly  $e^{-}$  and  $\nu_e$ . The detailed explosion mechanism of Type II supernovae is not understood, but it is probable that neutrinos play a crucial role. One of the most remarkable aspects is that neutrinos become temporarily trapped within the star during collapse. The typical neutrino-matter cross section is  $\sigma \approx 10^{-40} cm^2$  resulting in a mean free pass  $\lambda \approx (\sigma n)^{-1} \approx 10 cm$  where the density is  $n \cong 2$  to  $3n_o$ . This length is much less than the proto-neutron star radius, which exceeds 20 km. The gravitational binding energy released in the collapse of the binding energy released in the collapse of the progenitor star's white dwarf-like core to a neutron star is about  $\frac{3GM}{5R^2} \cong 3 \times 10^{53} erg$



( $G$  is the gravitational constant), which is about 10 percent of its total mass energy  $Mc^2$ . The kinetic energy of the expanding remnant is on the order of  $1 \times 10^{51}$  to  $2 \times 10^{51} \text{ erg}$  and is reduced by a factor of 100. Nearly all the energy is carried off by neutrinos and antineutrinos of all flavors in roughly equal proportions [9]. A neutron star formed in gravitational collapse of a stellar core is initially very hot, with the internal temperature  $\sim 10^{11}$  K [10]. Any main-sequence star with an initial mass of above 8 times the mass of the sun ( $8M_{\odot}$ ) has the potential to produce a neutron star. As the star evolves away from the main sequence, subsequent nuclear burning produces an iron-rich core [11]. When all nuclear fuel in the core has been exhausted, the core must be supported by degeneracy pressure alone. Further deposits of mass from shell burning cause the core to exceed the Chandrasekhar limit. A neutron star is formed with very high rotation speed, and then over a very long period it slows. Neutron stars are known that have rotation periods from about 1.4 ms to 30 s. The neutron star's density also gives it very high surface gravity, with typical values ranging from  $10^{12}$  to  $10^{13} \text{ m/s}^2$  (more than  $10^{11}$  times that of Earth). One measure of such immense gravity is the fact that neutron stars have an escape velocity ranging from 100,000 km/s to 150,000 km/s, that is, from a third to half the speed of light. The neutron star's gravity accelerates in falling matter to tremendous speed.

## 2.2 Structure of neutron stars

The cold equation of state above nuclear density determines the macroscopic properties of neutron stars, which are potentially extractable from astrophysical observation. Unlike white dwarfs, all neutron stars are expected to be described by the same one-parameter equation of state, so multiple neutron star observations can be used to constrain the properties of the single neutron-star EOS. Isolated neutron stars are generally characterized by their mass, radius, and spin frequency. In binary pulsars, the spin-orbit coupling can also give information about the stars' moments of inertia [12].

Neutron star structure and evolution are calculated from the general relativistic cor-

rections to the Newtonian formulation. The Tolman-Oppenheimer-Volkoff (TOV) equations are a set of coupled ODE that constrain a spherically symmetric body structure in equilibrium. The TOV equations can be solved with the assumption of NSs as spherically symmetric objects, and the intended M-R curves can be retrieved[13]. Neutron stars are the last stage of massive stars. It is widely accepted that when the mass of the star is  $(3 - 6)M_{\odot} \leq M \leq (5 - 8)M_{\odot}$  it collapses to a NS. A neutron star is a compact object whose typical mass is  $1.4M_{\odot}$  and typical radius is 10 km. The dominant component of NSs is degenerate neutrons, and their degenerate pressure largely support the NS against the gravity[7]. For sufficiently old stars the redshifted temperature ( $Te^{\phi/c^2}$ ) in the core is uniform, whereas temperature gradients can be appreciable in the envelope. The full set of general relativistic equations can to a very good degree of approximation be reduced to a single equation in the envelope. This equation, which determines the thermal structure of the envelope, can be written as

$$\frac{dT}{dp} = \frac{3}{16} \frac{k}{T^3} \frac{T_s^4}{g_s} \quad (2.1)$$

where T is the temperature, P is the total pressure, k is the total opacity of the stellar matter,  $T_s$  is the effective surface temperature[14]. The static structure of neutron stars is calculated by solving the Einstein equations. We usually assume the spherical symmetry, with which it is called Tolman-OppenheimerVolkof (TOV) equation. The property of matter components is provided by the form of equation of state (EOS). Although there are uncertainties in EOS due to uncertain nuclear interaction, we have qualitative understanding of the large part of the inner structure. Internal structure of a neutron star can be described as consisting of outer crust, inner crust, outer core and inner core . Each of the regions will be briefly described here:[15].

The crust of the neutron star has odd properties. It is very difficult to compress crust material very much, or to move elements of crust up and down, because strong gravity and pressure forces maintain a firm balance. But it is simpler to move parts of the crust horizontally, in which by applying only shear strains to it. In the magnetar, the magnetic field is strong enough to push horizontally and twist the crust. The magnetic field lines

outside the star also get twisted because they are anchored to the crust. The rotational energy of the magnetar quickly decreases after its birth. This crusts twist strengthen the current outside the magnetar and generate X-rays. As a result the magnetic field itself can provide an energy source for potentially observable emissions. This leads the magnetar to dissipate a significant amount of magnetic energy during the first ten thousand years. But in the radio pulsar, the magnetic field is not strong enough (compared to the magnetar) to push and twist the crust. As a result the field doesn't play a significant role to strengthen the current outside the radio pulsar and the magnetic energy is not dissipated very quickly. Pulsar's magnetic field is essentially stable; its main role is passively facilitate the loss of rotational energy. The crust of neutron star has two layers. They are:

### 2.2.1 Outer Crust

Approximately 0.3 – 0.6km thick depending on the particular equation of state, the outer crust is a solid lattice of heavy nuclei (probably in part  $^{56}Fe$ ) and a sea of degenerate electrons. The outer crust consists of a lattice of neutron rich nuclei immersed in a uniform gas of electrons. As we move towards the center of neutron star the increasing pressure due to increase in overlaying matter favors the neutronization (which happened during supernova stage) and consequently increasing the neutron density in the nuclei. Thus the outer crust contains nuclei of different neutron to proton ratio[15]. The ions form a strongly coupled Coulomb system that is solid in most of the envelope but is liquid at the lowest densities. The electron Fermi energy grows with increasing  $\rho$ , and as a consequence, nuclei tend to become richer in neutrons because it is energetically favorable to convert electrons and protons into neutrons (and neutrinos) by electron capture processes. The nuclei can also undergo other transformations, such as pycnonuclear reactions, absorption of neutrons, and emission of neutrons. At the base of the outer crust, neutrons begin to drip out of nuclei, there by producing a neutron gas between nuclei. This occurs at a density  $\rho = \rho_{ND} \simeq 4 \times 10^{11} g/cm^3$ . The thickness of the outer crust is a few hundred meters.

### 2.2.2 Inner Crust

Approximately 0.5 – 0.8km thick, the inner crust begins at the neutron drip density of  $(0.5)\rho_o$  where neutrons leak out of nuclei because the energy required to remove a neutron from a nuclei is zero[16]. The inner crust consists of a solid lattice and a Fermi gas of relativistic, degenerate neutrons that form a superfluid in the  $^1S_o$  pairing state[17]. The crustal superfluid component of the inner crust rotates nearly independently from the remainder of the star, while the crustal lattice is strongly coupled to the core and outer crust[18]. As we continue to move inwards, the density continues to increase until it reaches the neutron drip density ( $4.3 \times 10^{11} gcm^{-3}$ ) at which neutrons drip out of nuclei and we have a region of free neutrons and electrons and nuclei. The nuclei are in equilibrium with free neutrons. The density of free neutrons increases as we move towards the boundary of outer core where density is nearly  $0.5\rho_o$  ( $\rho_o \sim 2.7 \times 10^{14} gcm^{-3}$ ) and mostly free neutrons exist here. The inner crust contains hierarchy of phases of nuclei called the nuclear pasta phases. Also, since the interior temperatures of neutron stars are very small compared to Fermi energy of neutrons, neutrons may form cooper pairs and hence superfluid at critical temperature of about  $10^9 - 10^{10} K$ .

### 2.2.3 Outer core

The outer part of the core is mainly comprised of a neutron superfluid in the  $^3P_2$  paired state[19]. For the density  $\rho \geq 2.8 \times 10^{14} gcm^{-3}$  the nuclei are broken into fluid consisting of neutrons, protons and electrons. This threshold density is called nuclear saturation density, and as a number density, it is  $n_o \simeq 0.16 fm^{-3}$  which is a typical nucleon density inside a nucleus. This region is called (outer) core. Due to the attractive nuclear force, protons form Cooper pair and the core becomes the superconductor. Neutrons also form Cooper pairs, but the pairing type is different from that in the crust; in the crust s-wave interaction is responsible for the pairing, while in the core, p-wave is the dominant channel of attractive force. These pairings affect the heat capacity and neutrino emissivity. Muons are also produced in the high density region in the core where the electron chemical potential exceeds the muon mass. The core comprises a large percentage of the mass of the

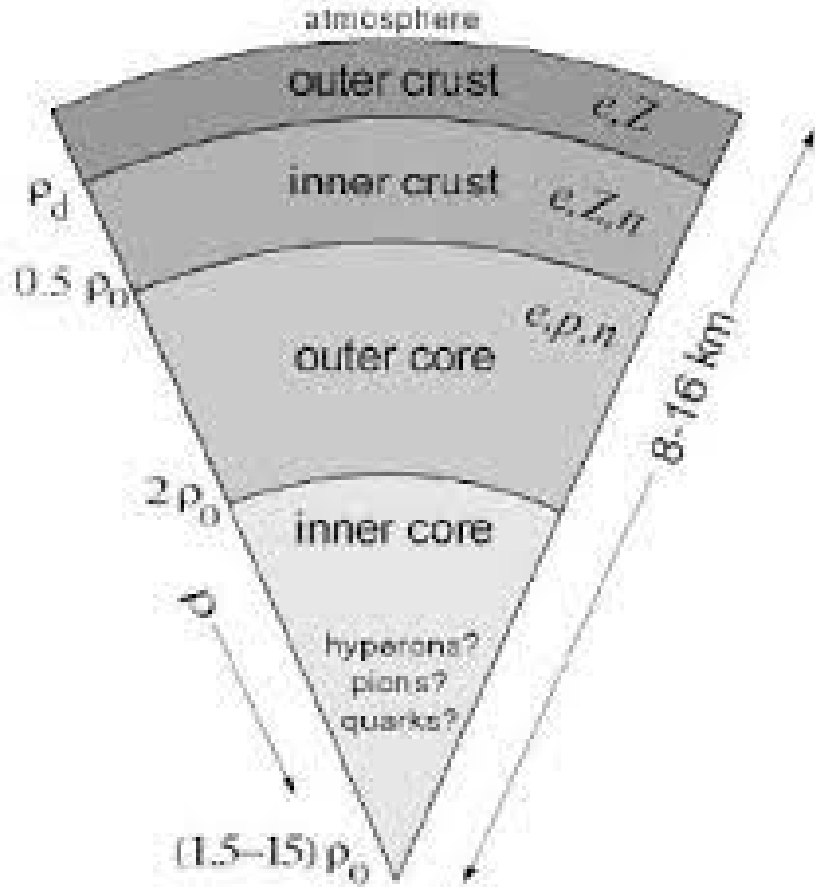


Figure 2.1: neutron star structure

neutron star (depending on the equation of state, up to 99 percent), leading to densities large enough that all nuclei should be dissolved into their constituent nucleons[10].

### 2.2.4 Inner core

The density of the inner core is  $(3-9)\rho_0$ , thus poorly understood but may consist of exotic matter such as quarks, hyperons or kaons. In addition to nucleons and charged leptons (electrons and muons), pion, hyperon or quark matter may appear in the extremely dense region close to the neutron star's center. The condition for these exotic particles to appear is highly uncertain because the EOS for such high density is not well understood[10].

## 2.3 Observation and properties of neutron stars

Neutron stars are observed in all bands of the electromagnetic spectrum: radio, infrared, optical, ultraviolet, X-ray and  $\gamma$ -ray. Information on the properties of neutron stars can be obtained not only from the observation of their electromagnetic radiation but also through the detection of the neutrinos emitted during the supernova explosion that signals the birth of the star. Gravitational waves, originated either from the oscillation modes of neutron stars or during the coalescence of two neutrons stars such as the event GW170817 recently detected by the Advanced LIGO and Advanced VIRGO collaborations offer a new way of observing the objects and constitute a very valuable new source of information. In particular constraints on the nuclear equation of state can be established by comparing the precise shape of the detected gravitational wave signal which depends on the masses of the two neutron stars and on their so called tidal deformabilities with the different model predictions. Multimessenger observations of neutron star mergers can potentially provide detailed information on the properties of the merging objects such as their mass and radii, improve our present understanding of the nucleosynthesis of heavy elements in the Universe and enable test of the theory of gravity and dark matter. Neutron stars can be observed either as isolated objects or forming binary systems together with other neutron stars, white dwarfs or ordinary (main-sequence and red giant) stars. Modern theories of binary evolution predict also the existence of binary systems formed by neutron stars and black holes, although this kind of systems have not been discovered yet. After fifty years of observations we have collected an enormous amount of data on different neutron star observables that include; masses, radii, rotational periods, surface temperatures, gravitational redshifts, quasiperiodic oscillations, magnetic fields, glitches, timing noise and, very recently gravitational waves. In the following lines we shortly review how two of them, masses and radii, are measured. Neutron star masses can be inferred directly from observations of binary systems and likely also from supernova explosions.

Masses: NS binaries orbiting in a Keplerian path can be used to measure their masses . The parameters used for these measurements are the orbital period  $P_{orb}$  , the semimajor

axis projection from the pulsar line of sight  $x = (a_1 \sin i)/c$ , the eccentricity  $e$ , the time of periastron  $T_p$ , and the longitude of the periastron  $L_p$ . Here,  $i$  is the inclination of the orbit, and  $c$  is the speed of light. From Kepler's Third Law, the mass function of the binaries can be written as,

Periods: Most radio pulsars do have a stable pulsation period. Pulsars can be observed in two different classes based on the pulsar timing measurements: normal pulsars with a period of 1 second and those with a period order of milliseconds. The first observed pulsar PSR B1919+21 had a pulsation period of 1.33 s, and a millisecond pulsar was discovered in 1982 with a period of 1.558 ms. observed eight new millisecond pulsars ranging from 2.74 to 8.48 ms in globular clusters. Quasiperiodic pulsations: These pulsations can arise in X-ray binaries from the difference of the rotational frequency of neutron stars with the Keplerian frequency of the innermost stable orbit. Detection of quasiperiodic oscillations might help with the analysis of strong-field general relativity. Magnetic fields: A rapidly rotating neutron star is believed to power strong magnetic fields. A millisecond pulsar can have magnetic field strength on the order of  $10^8$  -  $10^9$  G where the field strength in normal pulsars might go up to  $10^{12}$  G. There are objects with much higher field strength associated with them, and they are mainly referred to as magnetars. The atmosphere of neutron stars depends on magnetic effects, whereas strong fields might kickstart phase transition within the star[13]. A neutron star has a mass of at least 1.1 solar mass ( $M_\odot$ ). The upper limit of mass for a neutron star is called the Tolman–Oppenheimer–Volkoff limit and is generally held to be around  $2.1(M_\odot)$  but a recent estimate puts the upper limit at  $2.16(M_\odot)$ . The maximum observed mass of neutron stars is about  $2.14(M_\odot)$ . The temperature inside a newly formed neutron star is from around  $10^{11}$  to  $10^{12}$  kelvins. However, the huge number of neutrinos it emits carry away so much energy that the temperature of an isolated neutron star falls within a few years to around  $10^6$  kelvins. At this lower temperature, most of the light generated by a neutron star is in X-rays[20].

Neutron stars have overall densities of  $3.7 \times 10^{17}$  to  $5.9 \times 10^{17} \text{ kg m}^{-3}$  ( $2.6 \times 10^{14}$  to  $4.1 \times 10^{14}$  times the density of the Sun), which is comparable to the approximate density of

an atomic nucleus of  $3 \times 10^{17} \text{kgm}^{-3}$ . The neutron star's density varies from about  $1 \times 10^9 \text{kgm}^{-3}$  in the crust increasing with depth—to about  $6 \times 10^{17}$  (denser than an atomic nucleus) deeper inside[20].

For observers, properties of the neutron star are altered by the gravitational red-shift as follows: for the observed luminosity,

$$L_{\infty} = L \left(1 - \frac{r_g}{R}\right) \quad (2.2)$$

the observed effective temperature,

$$T_{s\infty} = T_s \sqrt{1 - \frac{r_g}{R}} \quad (2.3)$$

and the observed stellar radius,

$$R_{\infty} = R \left(1 - \frac{r_g}{R}\right)^{-\frac{1}{2}} \quad (2.4)$$

here  $r_g = 2GM/C^2$  is the gravitational radius[21]. Luminosities, temperatures, and radii discussed in this work are proper, and not redshifted, quantities[22].

A neutron star has a mass of at least 1.1 solar mass ( $M_{\odot}$ ). The upper limit of mass for a neutron star is called the Tolman–Oppenheimer–Volkoff limit and is generally held to be around  $2.1(M_{\odot})$  but a recent estimate puts the upper limit at  $2.16(M_{\odot})$ . The maximum observed mass of neutron stars is about  $2.14(M_{\odot})$ . The temperature inside a newly formed neutron star is from around  $10^{11}$  to  $10^{12}$  kelvins. However, the huge number of neutrinos it emits carry away so much energy that the temperature of an isolated neutron star falls within a few years to around  $10^6$  kelvins. At this lower temperature, most of the light generated by a neutron star is in X-rays[23]. Neutron stars have overall densities of  $3.7 \times 10^{17}$  to  $5.9 \times 10^{17} \text{kgm}^{-3}$  ( $2.6 \times 10^{14}$  to  $4.1 \times 10^{14}$  times the density of the Sun), which is comparable to the approximate density of an atomic nucleus of  $3 \times 10^{17} \text{kgm}^{-3}$ . The neutron star's density varies from about  $1 \times 10^9 \text{kgm}^{-3}$  in the crust increasing with



depth—to about  $6 \times 10^{17}$  (denser than an atomic nucleus) deeper inside[23].

# Chapter 3

## Thermal evolution of neutron stars

The thermal evolution of a neutron star is a complicated phenomena and requires an equation of state describing the nucleonic matter inside the neutron star as presented in earlier chapters. Still Tolman-Oppenheimer-Volkoff theoretical model is the best candidate fitting to observations. Some of the newer models have investigated the neutron star core and they have proposed such ideas as neutron lattices, quark liquid, and hyperon liquids [8].

In this relativistic model, the thermal evolutionary model is further developed by employing energy balance in the process of interaction, thermal heating and emissions. The emission possess two parts: neutrinos from the interiors and photons at the surfaces.

In subsequent sections, the TOV equations and thermal processing theoretical models and some key principles of the energy balance and conversion process be presented.

### 3.1 Tolman-Oppenheimer–Volkoff equations

The details derivations and implications of the TOV equations are given in the classical books [1], [24] and elsewhere. The fundamental hydrostatic equilibrium and energy-momentum-matter transport equation is given as

$$\frac{dp}{dr} = - \left( \rho + \frac{p}{c^2} \right) G \left( \frac{m + 4\pi r^3 p/c^2}{r^2} \right) e^{2\Lambda} \quad (3.1)$$

where  $\rho$  is the mass-energy density,  $G$  is the Newtonian gravitational constant,  $m$  and  $r$  are the gravitational mass and radius of the spherical inner envelope considered as the star respectively,  $e^\Lambda$  is the surface value of the relativistic length correction expressed as the Schwarzschild metric factor.

The gravitational potential  $\Phi$  equation, that determines the source is given as

$$\frac{d\Phi}{dr} = G \left( \frac{m + 4\pi r^3 \rho / c^2}{r^2} \right) e^{2\Lambda} \quad (3.2)$$

The equation for the neutrino luminosity,  $L_\nu$ , is

$$\frac{d(L_\nu e^{2\Phi/c^2})}{dr} = \epsilon_\nu e^{2\Phi/c^2} 4\pi r^2 e^\Lambda \quad (3.3)$$

where  $\epsilon_\nu$  and  $L_\nu$  are, respectively, the neutrino emissivity per unit volume its luminosity.

In non-accreting neutron stars, since there is no burning, the equation of energy conservation can be written as

$$\frac{d(L_\nu e^{2\Phi/c^2})}{dr} = -c_v^2 \frac{dT}{dt} e^\Lambda 4\pi r^2 \quad (3.4)$$

where  $L$  is the net luminosity and is equal to  $L_d + L_\nu$ ,  $c_v$  is the specific heat per unit volume; and  $t$  is the time measured by an observer at  $r = \infty$ , who is at rest with respect to the star. The equation which determines the gravitational mass,  $m$ , enclosed within a sphere of radius  $r$  is

$$\frac{dm}{dr} = 4\pi r^2 \rho \quad (3.5)$$

The quantities  $p$ ,  $\rho$ ,  $T$ ,  $k$ ,  $L_d$ ,  $L_\nu$ ,  $c_v$ , and  $e^\Lambda$  are all local ones, i.e, they are evaluated in a proper reference frame co-moving with the stellar material[7].

## 3.2 Thermal heating and energy balance process

The surface temperatures of neutron stars, according to the standard model, persist over  $10^6 K$  for around  $10^5$  years, making them potentially visible in the x-ray or UV

bands[25].In this part, we will show that for most neutron stars older than a few tens of years, the whole set of general relativistic equations may be reduced to a single equation in the envelope to a very good degree of approximation.This equation, which governs the envelope's thermal structure, can be represented as

$$\frac{dT}{dp} = \frac{3}{16} \frac{k}{T^3} \frac{T_s^4}{g_s} \quad (3.6)$$

where  $T$  is the temperature,  $P$  is the total pressure,  $k$  is the total opacity of the stellar matter,  $T_s$  is the effective surface temperature, and

$$g_s = \frac{GM}{R^2} e^{A_s} \quad (3.7)$$

is the proper surface gravity of the star (i.e, the acceleration due to gravity as measured on the surface). In equation 3.7  $G$  is the Newtonian gravitational constant,  $M$  and  $R$  are the gravitational mass and radius of the star, respectively, and  $e^{A_s}$  As is the surface value of the relativistic length correction factor.

$$e^{A_s} = \left(1 - \frac{2GM}{c^2}\right)^{-\frac{1}{2}} \quad (3.8)$$

$c$  is the velocity of light,  $r$  is the radial coordinate of Schwarzschild, and  $m(r)$  is the gravitational mass contained within a sphere of radius  $r$ . The thermal structure of neutron star envelopes is defined by the two parameters  $g_s$  and  $T_s$  according to equation 3.6. As a result, models of neutron star envelopes may be simply estimated and classified in terms of surface gravity and surface temperature, regardless of the core structure.In fact, we will show that the single parameter  $\frac{T_s^4}{g_s}$  determines the thermal structure to a great extent.

Neutron stars form at extremely high temperatures, similar to  $10^{11}K$ , in the imploding cores of supernova explosions.Various processes of neutrino emission (namely, Urca processes and neutrino bremsstrahlung) radiate much of the original thermal energy away from the interior of the star, leaving a one-day-old neutron star with an internal tempera-

ture of roughly  $10^9 - 10^{10} K$ . The star's interior (densities  $\rho > 10^{10} gcm^{-3}$ ) becomes almost isothermal after  $\sim 100$ yr (typical time of thermal relaxation), and the energy balance of the cooling neutron star is determined by the following equation:

$$C(T_i) \frac{dT_i}{dt_i} = -L_v(T_i) - L_\gamma(T_s) + \sum_{k_i} H_k \quad (3.9)$$

where  $T_i$  and  $T_s$  are the internal and surface temperatures,  $C(T_i)$  is the heat capacity of the neutron star. Neutron star cooling thus means a decrease of thermal energy, which is mainly stored in the stellar core, due to energy loss by neutrinos from the interior ( $L_v = \int Q_v d_v$ ,  $Q_v$  is the neutrino emissivity) plus energy loss by thermal photons from the surface ( $L_\gamma = 4\pi R^2 \sigma T_s^4$ ). The relationship between  $T_s$  and  $T_i$  is determined by the thermal insulation of the outer envelope ( $\rho < 10^{10} gcm^{-3}$ ), where the temperature gradient is formed. The results of model calculations, assuming that the outer envelope is composed of iron, can be fitted with a simple relation. The thermal evolution of a neutron star after the age of  $\sim 10 - 100$  yr, when the neutron star has cooled down to  $T_s = 1.5 - 3 \times 10^6 K$ , can follow two different scenarios, depending on the still poorly known properties of super-dense matter. According to the so-called standard cooling scenario, the temperature decreases gradually, down to  $\sim 0.3 - 1 \times 10^6 K$ , by the end of the neutrino cooling era and then falls down exponentially, becoming lower than  $\sim 0.1 \times 10^6 K$  in  $\sim 10^7$  yr. photon emission from the neutron star surface takes over as the main cooling mechanism. The thermal evolution of a neutron star after the age of  $\sim 10 - 100$ yr, when the neutron star has cooled down to  $T_s = 1.5 - 3 \times 10^6 K$ , can follow two different scenarios, depending on the still poorly known properties of super-dense matter. According to the so-called standard cooling scenario, the temperature decreases gradually, down to  $\sim 0.3 - 1 \times 10^6 K$ , by the end of the neutrino cooling era and then falls down exponentially, becoming lower than  $\sim 0.1 \times 10^6 K$  in  $\sim 10^7$  yr. In this scenario, the main neutrino generation processes are the modified Urca reactions,  $n + N \rightarrow P + N + e + v_e^-$  and  $p + N + e \rightarrow n + N + v_e$ , where N is a nucleon (neutron or proton) needed to conserve momentum of reacting particles. In the accelerated cooling scenarios, associated with

higher central densities (up to  $10^{15} \text{gcm}^{-3}$ ) and/or exotic interior composition, a sharp drop of temperature, down to  $0.3 - 0.5 \times 10^6 K$ , occurs at an age of  $\sim 10 - 100 \text{yr}$ , followed by a more gradual decrease, down to the same  $\sim 0.1 \times 10^6 K$  at  $\sim 10^7 \text{yr}$ . The faster cooling is caused by the direct Urca reactions,  $n \rightarrow p + e + \nu_e^-$  and  $p + e \rightarrow n + \nu_e^-$ , allowed at very high densities.

The thermal evolution of neutron stars is sensitive to the adopted nuclear equation of state (EOS), the stellar mass, the assumed magnetic field strength, appearance of superfluidity, and the possible presence of color-superconducting quark matter. Therefore, theoretical cooling calculations serve as a principal window on the properties of superdense hadronic matter and neutron star structure. The thermal evolution of neutron stars also yields information about such temperature-sensitive properties as transport coefficients, crust solidification, and internal pulsar heating mechanisms. The basic features of the thermal evolution of a neutron star are easily grasped by simply considering the thermal energy conservation equation for the star. Since neutron stars are subject to general relativistic effects, this equation reads

$$\frac{d(e^\Phi E_{th})}{dt} = C_v e^\Phi \frac{\partial T}{\partial t} - \nabla \cdot [e^\Phi k \cdot \nabla (e^{\Phi T})] = -e^{2\Phi} (+L_\nu + L_\gamma - H) \quad (3.10)$$

where  $E_{th}$  is the thermal energy content of the star,  $T$  its internal temperature, and  $C_v$  its total specific heat.  $e^\Phi$ ,  $\Phi$ , being the gravitational potential, is the so-called red-shift factor which reflects the curvature of the space-time inside and outside of the neutron star. At its surface the relativistic corrections to Newtonian physics are about 30%. The energy sinks are the total neutrino luminosity  $L_\nu$ . The thermal evolution model we use for the first time in evolutionary calculations takes advantage of Baym's (1981) observations. As in the full evolution model, the spatial dependence of the temperature throughout the star is followed, but changes in the hydrostatic structure are neglected a single degenerate structure is used at all times. The requirement of degeneracy restricts the validity of the thermal evolution models to ages greater than about 1 month. But attempted observations on any neutron star this young would most likely result from the attention drawn to it by a supernova explosion; the expanding ejecta of the precursor

star would then interfere with any signal from the neutron star itself. Given the thermal evolution equations, one must devise a way to solve them on a computer. We present here a new penta-diagonal method that couples three time levels in a single time step. This scheme is more accurate than the tri diagonal Crank-Nicholson method at late times when the surface luminosity dominates the energy loss from the neutron star. Our model is, like all others reported so far, one-dimensional and spherically symmetric. The basic equations, we present the differencing schemes and other model details used in solving those equations, and we compare the penta-diagonal method with the tri-diagonal. Next, we describe the neutrino emission rates, superfluid treatment, and other microphysics used in the model. We compare our results with the full evolution models of Nomoto and Tsuruta (1987) and suggest reasons for some of the small discrepancies among models that agree well in their gross features. We also present results for neutron star models with gravitational masses  $M = 1.4M_{\odot}$  with and without surface magnetic fields. Enhanced neutrino emission may result from exotic constituents of matter at very high density. As examples of such accelerated cooling mechanisms, we consider neutrino emission from a pion condensate and from free quarks. These two specific models are at best uncertain and are used primarily to show the generic effects of enhanced core neutrino emission. In a core-collapse supernova explosion, a neutron star is born hot, with a temperature of  $\sim 10^{11}K$ . It cools in about a 100 yr due to neutrino emission in the core and heat diffusion in the crust. As a result, the characteristics and thermal condition of the crust are linked to the evolution of the surface temperature. The neutrino emission from the core later drives the temperature inside the core and the crust to equilibrium, as well as the cooling of the entire neutron star. The core's qualities are reflected in the thermal state. After a few tens of thousands of years, the cooling is dominated by photon emission at the surface. As a result, depending on the age of the isolated neutron star whose temperature has been established, the physics in different regions of the star can be constrained. The flux conservation argument is based on the idea from general physics that magnetic field lines cannot be destroyed. Flux conservation is applied because the stellar matter electrical conductivity effectively becomes infinite during the supernova.

This increases the Ohmic diffusion time scale to be larger than the gravitational collapse time; therefore, any particle penetrated by a magnetic field line is frozen' to th at field line . This also explains how the magnetic field becomes connected to the pulsar surface due to the particle-magnetic field coupling. There are however two ideas. The two magnetic field evolution ideas are magnetic field decay and rotational alignment . These two ideas are based on explaining why the pulsar would disappear from sight. In the rotational alignment theory, there is no magnetic field decay but the pulsar magnetic and rotation axes become aligned. This alignment would no longer have the magnetic field axis sweeping our line of sight and the pulsar would turn off'. The magnetic field decay assumes an exponential decay due to the equivalence seen from models of crustal decay and magnetic torque decay. Again these two ideas are still debatable due to the lack of direct information regarding the pulsar magnetic field[8].

The first pulsar magnetic field model can be attributed to Woltjer in 1964 and shortly after Pacini in 1968 . They both suggested a magnetic dipole field which is generated through the magnetic field flux conservation. This magnetic field generation theory is known as the fossil field theory because the new stellar object's magnetic field is a remnant of its precursor. The flux conservation argument is based on the idea from general physics that magnetic field lines cannot be destroyed. Flux conservation is applied because the stellar matter electrical conductivity effectively becomes infinite during the super nova. This increases the Ohmic diffusion time scale to be larger than the gravitational collapse time; therefore, any particle penetrated by a magnetic field line is frozen' to th at field line. This also explains how the magnetic field becomes connected to the pulsar surface due to the particle-magnetic field coupling. A simple calculation of the surface magnetic field strength using the flux conservation theory shows why the theory is so widely accepted.

$$\Phi_i = \Phi_f \tag{3.11}$$

This calculation makes use of the typical pulsar radius and a main sequence star radius to obtain the area ratio[8]. The magnetic field on a neutron star is over one billion times stronger than the field on Earth. This strong magnetic field causes the pulsing effect in



neutron stars. A neutron star's magnetic field is over one billion times greater than the field on Earth. The pulsating effect of neutron stars is caused by this intense magnetic field. A neutron star's magnetic field is a dipole with north and south poles, just like Earth's magnetic field and every other magnetic field known to science. These magnetic poles are not parallel to the neutron star's spin axis. The magnetic field of a neutron star accelerates charged particles above the magnetic poles (NSs). Quite possibly, it serves as a most important unifying agent relating and explaining the observational properties of many diverse classes of NSs (e.g., rotation-powered pulsars, magnetars, isolated neutron stars etc[8].) There are two main types of behaviour associated to magnetic field evolution in strongly magnetised neutron stars: Firstly, there are events caused by changes occurring in short timescales ranging from a fraction of a second up to a few months, variations in their radiative and timing properties. Secondly, there are observations that hint towards a longer-term evolution of the magnetic field on time-scales comparable to the life of a neutron star. The short term variations of strongly magnetised neutron stars indicate that the magnetic field evolves in an impulsive manner. Magnetars are notorious for the bursting and flaring activity which is often accompanied by changes in their timing behaviour. Such changes have the characteristics of torque variations indicating alterations in the structure and strength of their magnetic field which most likely is ultimately the main responsible for their spin-down. The long term evolution, there is a strong correlation between the X-ray luminosity of magnetars and the strength of their magnetic field with the one of that host stronger magnetic fields having higher thermal  $L_x$ . In particular it is possible to fit their KeV part of the electromagnetic spectrum with a black body radiation model implying that part of their X-ray luminosity has a thermal origin. The rapid cooling that should take place during the first few 100 years in a hot newly formed neutron star, according to thermal evolution models in 2004 Yakovlev and Pethick suggests that an extra source of energy must be present to maintain temperatures above than  $5 \times 10^6 K$  for several[22].

Then we provide the differential equations which govern the thermal evolution of a neutron star. We assume the star is spherically symmetric. We do not assume the whole star

is in thermal equilibrium. Hence we also consider the heat transport inside the NS. In the following, we use the unit of  $c = 1$ . The energy conservation determines the temperature evolution as

$$C_v \frac{dT}{dt}(r) e^{\Phi} 4\pi r^2 e^{\Lambda}(r) = -d \frac{(L e^{2\Phi}(r))}{dr} \quad (3.12)$$

where  $C_v$  is the specific heat and  $L$  is the luminosity, energy loss per unit time. Note that the luminosity is red-shifted twice because of the time delay as well as energy red-shift. Neutrinos are produced by the weak interaction, and they are collision-less inside the NS for the temperature of our interest ( $T \leq 10^{10}$  K). Therefore, they are emitted from the whole star [7]. Then the neutrino luminosity  $L_v$  is written by the neutrino emissivity  $Q_v$ , the energy loss per unit time and unit volume, as

$$d \frac{(L_v e^{2\Phi}(r))}{dr} = Q_v e^{2\Phi}(r) 4\pi r^2 e^{\Lambda}(r) \quad (3.13)$$

On the other hand, other particles such as photons and electrons collide with each other, transporting heat from one place to another. The heat transport equation is written as

$$-\lambda \frac{d(T e^{\Phi}(r))}{dr} = \frac{L_d}{4\pi r^2} e^{\Phi}(r) e^{\Lambda}(r) \quad (3.14)$$

where  $\lambda$  is thermal conductivity and  $L_d$  the luminosity due to the thermal radiation and conduction. From the definition of thermal conductivity,  $-\lambda(T e^{\Phi})/d_r$  corresponds to the energy flux per unit area due to the temperature gradient along  $r$  direction. Thus this equation is the definition of  $L_d$ . Since the total luminosity is decomposed as  $L = L_v + L_d$ , Eq.3.12 is written as.

$$c_v \frac{dT(r)}{dr} e^{\Phi}(r) 4\pi r^2 e^{\Lambda}(r) = Q_v e^{\Phi}(r) 4\pi r^2 e^{2\Lambda}(r) - \frac{d(L_d e^{2\Phi}(r))}{dr} \quad (3.15)$$

The calculation of thermal evolution goes as follows

- 1 Solve TOV equation with an input EOS.
- 2 Using the solution of TOV equation, calculate  $c_v$ ,  $\lambda$  and  $Q_v$  with some microphysics input.

3 Solve Eq.3.14 and Eq.3.15 for  $T_r$  and  $L_{d(r)}$  with an appropriate boundary condition[7]. Energy is transported through the star by electron conduction (conduction dominates photon diffusion, except in the outer atmosphere layers, which are calculated separately). The temperature gradient is given by the relativistic equation of radiative transport[22]

$$e^\Phi L = 4\pi r^2 \sqrt{1 - \frac{2Gm}{c^2 r}} d\left(\frac{T e^\Phi}{dr}\right) \quad (3.16)$$

where  $e^\Phi$  is the redshift, L is the luminosity, r is the radius, m is the gravitational mass interior to r,  $\kappa$  is the thermal conductivity, T is the temperature, and the constants have their customary meaning. The luminosity is found from the energy balance equation with the contraction contribution dropped[26].

$$d(Le^{2\Phi}) = \frac{4\pi r^2}{\sqrt{1 - 2Gm/c^2 r}} e^{2\Phi} \left[ -Q_V - ne^{-\Phi} \frac{d}{dt} \left( \frac{\epsilon}{N} \right) \right] \quad (3.17)$$

where n is baryon density,  $Q_V$  is the neutrino emissivity per unit volume, and  $\epsilon$  is the internal energy density per volume. By defining a heat capacity C K, such that  $d\epsilon/dt = C_v dT/dt$  and changing variables to

$$T^\sim = e^\Phi T \quad (3.18)$$

and

$$L^\sim = e^{2\Phi} L \quad (3.19)$$

### 3.3 Neutrino and photon cooling

The basic features of the thermal evolution of a neutron star follow from the equation 3.10. In the neutrino cooling era,  $L_\nu \gg L_\gamma$  we find then

$$\frac{dT}{dt} = \frac{q_\nu}{c_\nu} T^7 \quad (3.20)$$

which has the solution

$$t - t_o = A \left( \frac{1}{T^6} - \frac{1}{T_o^6} \right) \quad (3.21)$$

where  $T_o$  is the initial temperature at  $t_o$ . This gives for  $T \ll T_o$  a behavior  $T \propto t^{-1/6}$ , or a surface temperature  $T_* \propto t^{-1/2}$ . The very slow decay of the temperature is a direct consequence from the high exponent in the neutrino emissivity.

The photon luminosity can be written as

$$L_* = 4\pi R^2 \sigma T_*^6 = S T^{2+4a} \quad (3.22)$$

where the effective temperature  $T_*$  has been converted into the internal temperature  $T$  through an envelope model with a power-law dependence,  $T_* \propto T^{1/2+a}$ .

In the photon cooling era,  $L_\gamma \gg L_\nu$ ,

$$t - t_1 = \frac{B}{4aS} \left( \frac{1}{T^{4a}} - \frac{1}{T_1^{4a}} \right) \quad (3.23)$$

where  $T_1$  is the temperature at  $t_1$ . This last equation shows that the temperature as a function of time is a very steep function in the photon cooling era,  $T \propto t^{-1/4a}$  and therefore  $L_* \propto t_{-2/a}$ , while  $T \propto t^{-1/6}$  in the neutrino cooling era, i.e.  $T_* \propto t^{-1/8}$ , and therefore  $L_* \propto t^{-1/3}$  (see Fig ??). Since  $a \ll 1$ , we see that, during the photon cooling era, the evolution is very sensitive to the nature of the envelope and to changes in the specific heat, as induced, e.g. by nucleon pairing. The transition from neutrino dominated cooling towards photon dominated cooling occurs at an age of about 100,000 years.

# Chapter 4

## Result and discussion

Here, the materials developed in earlier chapters are being used to discuss the temperature cooling of NSs. First we present the implications of the theoretical models and then provide data analysis from observation in different environments. We also discuss on the progress of the study area based on our reviews.

### 4.1 Theoretical results

#### 4.1.1 Neutrino vs photon cooling

Using the equations of chap. 3, section 3.3, we did generate a theoretical numerical data for the cooling of neutrinos and photons as a function of time with simplifying boundary conditions in parameterized forms. The temperature vs time LogLog plots of the cooling curves for neutrino and photon emission at the surface are as shown in figure 4.1

As we observe from the plots, in both particles the temperatures continue to fall down in time, cooling the star. However, neutrino cooling is faster than photon cooling.

The luminosity vs temperature plot of the neutrinos and photons is shown as in figure 4.2. As we observe from the plots the luminosity increases with increasing temperature for both emissions. But, the neutrino emission luminosity is dominant in the beginning while that of the photon becomes dominant later.

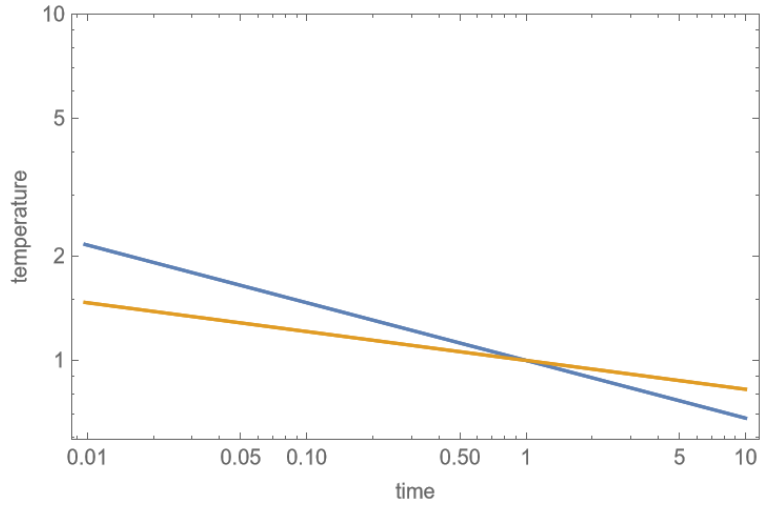


Figure 4.1: The LogLog plots of temperature vs time for neutrino and photon emission for isolated neutron stars. **Blue**: neutrino emission curve. **Orange**: photon emission curve.

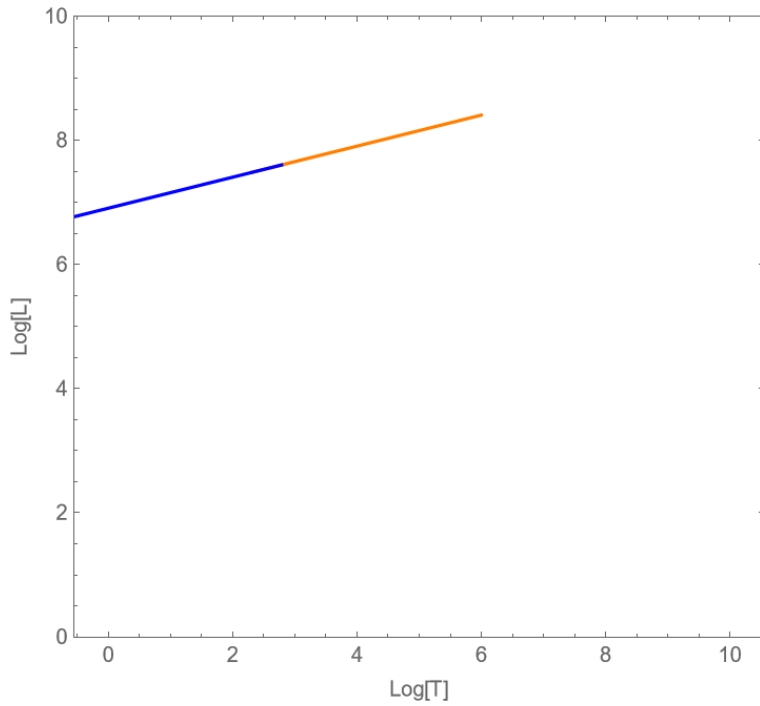


Figure 4.2: The luminosity vs temperature plot of neutrino and photon emissions for isolated neutron stars. **Blue**: neutrino emission. **Orange**: photon emission.

## 4.2 Observational data and their implications

Here, we considered two observational data adopted from [1, page 273]

1. Neutron stars with hydrogen atmospheres and
2. Neutron stars with black-body atmospheres.

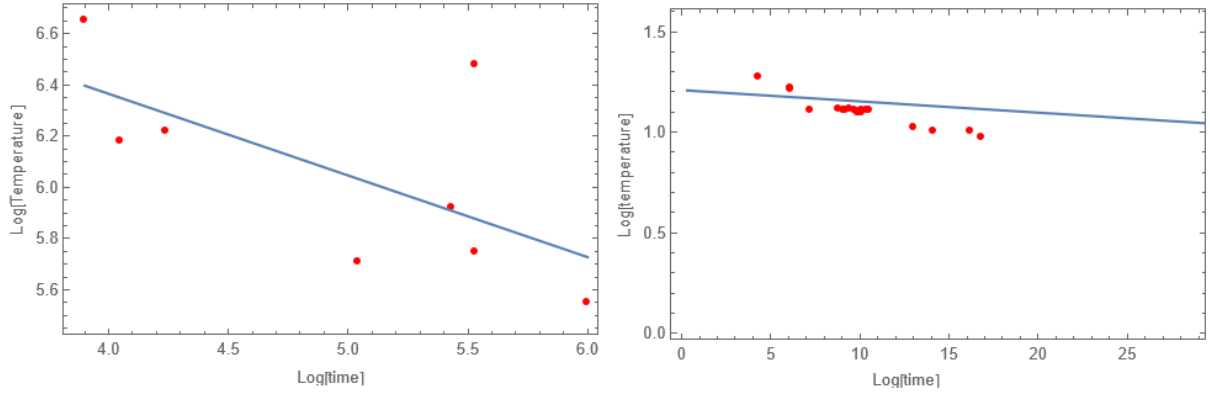


Figure 4.3: Temperature vs age distribution of NSs in black-body and hydrogen atmospheres. **Left:** Environment is in black body atmosphere. **Right:** Environment is in hydrogen atmosphere.

In both data the time - temperature relationships of the distribution indicate that NSs with relative higher ages possess lower temperatures and vice-versa in general trend. The plots of these data are shown as in figure 4.3 However, the cooling time is faster in the black-body atmosphere environment than that of the hydrogen reach atmosphere. This is in a good agreement with theory because as expected in the hydrogen reach atmosphere the NS accretes matter to heat itself decreasing the cooling effect.

The luminosity vs temperature distributions of the NSs in the black-body atmosphere is shown as in figure 4.4. In the plots, the upper luminosity vs temperature is indicated by red color while the lower luminosity of the distribution is indicated in blue. The mean linear fitting is indicated by the dashed green color.

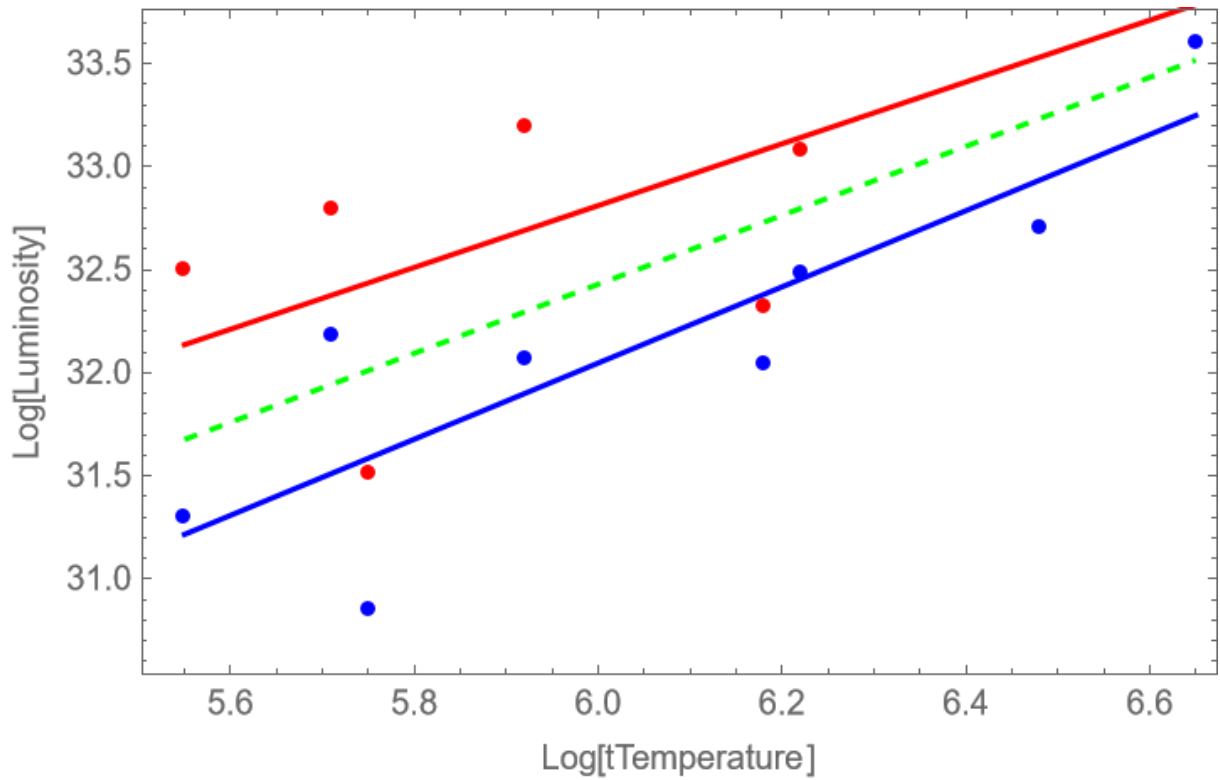


Figure 4.4: Temperature vs age distribution of NSs in black-body and hydrogen atmospheres. **Left:** Environment is in black body atmosphere. **Right:** Environment is in hydrogen atmosphere.



# Chapter 5

## Summary and conclusion

Compact objects are stars who have exhausted all their fuels where the normal gas thermal pressure can no longer support them against gravity collapse. In this sense they are considered as collapsed stars. They are characterized with high density and small size. There are two cases of these end products: stars that can support themselves by quantum degeneracy pressure (WDs and NSs) against gravity from further collapses or a non-degenerate complete collapse (BHs).

The compactness of these stars give them many extreme properties which make them relevant for the high energy astrophysics, emission of X-rays and Gamma-rays. They exhibit peculiar properties that cannot be created in normal laboratories and involve phases of matter that are not yet well understood. Their astrophysical studies are active including their associated relativistic phenomena both theoretically and observationally.

In this thesis we focused on cooling of NSs where energy conservation and transformation, mainly derived from relativistic considerations, is used. In the energy transformation process, cooling of the star via electromagnetic emission in terms of luminosity is used in tracking the evolutionary scenario of the star. Using Mathematica 13 we did the analytical analysis for numerical data generation to compare with observational data. The results are by pattern are similar. However, a detailed analysis with fine data sets and boundary conditions are needed in the future for better analysis and progress.

# References

- [1] Read, J. S. (2008). Neutron stars in compact binary systems: From the equation of state to gravitational radiation. The University of Wisconsin-Milwaukee.
- [2] Pons, J. A., and Viganò, D. (2019). Magnetic, thermal and rotational evolution of isolated neutron stars. *Living Reviews in Computational Astrophysics*, 5(1), 1-65.
- [3] Al-mamun, M. (2021). Neutron Star Structure from Electromagnetic and Gravitational Wave Observations.
- [4] EKŞİ, K. Y. (2016). Neutron stars: compact objects with relativistic gravity. *Turkish Journal of Physics*, 40(2), 127-138.
- [5] Yanagi, K. (2020). Thermal Evolution of Neutron Stars as a Probe of Physics beyond the Standard Model. arXiv preprint arXiv:2003.08199.
- [6] Tsiopelas, S., and Sagun, V. Tsiopelas, S., and Sagun, V. (2020). Neutron star cooling within the equation of state with induced surface tension. *Particles*, 3(4), 693-705.
- [7] Yakovlev, D. G., and Pethick, C. J. (2004). Neutron star cooling. *Annu. Rev. Astron. .*, 42, 169-210.
- [8] Kargaltsev, O., Pavlov, G. G., and Romani, R. W. (2004). Ultraviolet Emission from the Millisecond Pulsar J0437-4715. *The Astrophysical Journal*, 602(1), 327.
- [9] Alpar, M. A., Chau, H. F., Cheng, K. S., and Pines, D. (1994). Postglitch relaxation of the Crab pulsar: Evidence for crust cracking. *The Astrophysical Journal*, 427, L29-L31.

- [10] Steven Weinberg . *Gravitation and Cosmology*Wiley, New York, 1972,Chap. 11.
- [11] Poulter, M., Peel, M. (2011). Physics on Wikipedia. *Physics World*, 24(09), 32.
- [12] Geppert, U. R. (2017). Thermal evolution of neutron stars. *Handbook of Supernovae*, 1353.
- [13] Van Riper, K. A. (1991). Neutron star thermal evolution. *The Astrophysical Journal Supplement Series*, 75, 449-462.
- [14] ThurmanThurman III, H. O. (2004). Neutron star electromagnetic field structure. Old Dominion University.
- [15] Portieres Zwart, SF and Yungelson, Lev R 1998, Formation and evolution of binary neutron stars,pages=173-188
- [16] Haensel, P., Potekhin, A. Y., and Yakovlev, D. G. (Eds.). (2007). *Neutron Stars 1*. New York, NY: Springer New York.
- [17] Imshennik, V. S., and Manukovskii, K. V. (2007). Three-dimensional explosion dynamics of a critical-mass neutron star (in a binary). *Astronomy Letters*, 33(7), 468-480.
- [18] Lattimer, J. M., Pethick, C. J., Prakash, M., and Haensel, P. (1991). Direct URCA process in neutron stars. *Physical review letters*, 66(21), 2701.
- [19] Becker, W. (Ed.). (2009). *Neutron stars and pulsars* (Vol. 357, p. 652). Berlin: Springer.
- [20] Tsuruta, S., and Cameron, A. G. W. (1966). Cooling and detectability of neutron stars. *Canadian Journal of Physics*, 44(8), 1863-1894.
- [21] Pines, D., and Ventura, J. (Eds.). (1991). *Neutron Stars: Theory and Observation, Or, Neutron Stars: an Interdisciplinary Field*. Kluwer Academic Publishers.
- [22] Pines, D., and Alpar, M. A. (1985). Superfluidity in neutron stars. *Nature*, 316(6023), 27-32.

- [23] Anderson, P. W., and Itoh, N. (1975). Pulsar glitches and restlessness as a hard superfluidity phenomenon. *Nature*, 256(5512), 25-27.
- [24] Lattimer, J. M., and Prakash, M. (2004). The physics of neutron stars. *science*, 304(5670), 536-542.
- [25] Han, J. L., Manchester, R. N., Lyne, A. G., Qiao, G. J., and Van Straten, W. (2006). Pulsar rotation measures and the large-scale structure of the galactic magnetic field. *The Astrophysical Journal*, 642(2), 868.
- [26] Bauswein, A., Janka, H. T., and Oechslin, R. (2010). Testing approximations of thermal effects in neutron star merger simulations. *Physical Review D*, 82(8), 084043.
- [27] Gusakov, M. E., Kantor, E. M., and Ofengeim, D. D. (2017). Evolution of the magnetic field in neutron stars. *Physical Review D*, 96(10), 103012.
- [28] Thompson, C., and Duncan, R. C. (2001). The giant flare of 1998 August 27 from SGR 1900+ 14. II. Radiative mechanism and physical constraints on the source. *The Astrophysical Journal*, 561(2), 980.
- [29] Faucher-Giguere, C. A., and Kaspi, V. M. (2006). Birth and evolution of isolated radio pulsars. *The Astrophysical Journal*, 643(1), 332.
- [30] Slavin, J. D., Dwek, E., and Jones, A. P. (2015). Destruction of interstellar dust in evolving supernova remnant shock waves. *The Astrophysical Journal*, 803(1), 7.
- [31] Geleta, M., Bekele, A. (2016). Survey of medium and large-sized mammals in Wacha Protected Forest, Western Ethiopia. *Journal of agricultural science*, 6(3), 71-79.
- [32] Sabine, C. L. (2004). The following resources related to this article are available online at. *science*, 1097403, 305.
- [33] Lattimer, J. M., and Prakash, M. (2004). The physics of neutron stars. *science*, 304(5670), 536-542.

- [34] Slavin, J. D., Dwek, E., and Jones, A. P. (2015). Destruction of interstellar dust in evolving supernova remnant shock waves. *The Astrophysical Journal*, 803(1), 7.
- [35] gudmundsson1983structure, title=Structure of neutron star envelopes, author=Gudmundsson, Einar H and Pethick, CJ and Epstein, Richard I, journal=The Astrophysical Journal, volume=272, pages=286–300, year=1983
- [36] Yakovlev, D. G., Kaminker, A. D., and Levenfish, K. P. (1998). Neutrino emission due to Cooper pairing of nucleons in cooling neutron stars. arXiv preprint astro-ph/9812366.
- [37] Thorne, K. S. (1977). The relativistic equations of stellar structure and evolution. *The Astrophysical Journal*, 212, 825-831.
- [38] van1991neutron title=Neutron star thermal evolution, author=Van Riper, Kenneth A, journal=The Astrophysical Journal Supplement Series, volume=75,
- [39] thurman2004neutron, title=Neutron star electromagnetic field structure, author=Thurman III, Hugh O, year=2004, publisher=Old Dominion University
- [40] taam1983, title=The evolution of a stripped giant-neutron star binary, author=Taam, RONALD E, journal=The Astrophysical Journal, volume=270, pages=694–699, year=1983
- [41] Thorne, K. S. (1977). The relativistic equations of stellar structure and evolution. *The Astrophysical Journal*, 212, 825-831.
- [42] konar2017gravity, title=Gravity defied from potato asteroids to magnetised neutron stars, author=Konar, Sushan, journal=Resonance, volume=22, number=6, pages=597–610, year=2017, publisher=Springer
- [43] Pines, D., and Ventura, J. (Eds.). (1991). *Neutron Stars: Theory and Observation, Or, Neutron Stars: an Interdisciplinary Field*. Kluwer Academic Publishers.
- [44] Potekhin, A. Y., and Chabrier, G. (2021). Crust structure and thermal evolution of neutron stars in soft X-ray transients. *Astronomy and Astrophysics*, 645, A102.

- [45] Viganò, D., Rea, N., Pons, J. A., Perna, R., Aguilera, D. N., and Miralles, J. A. (2013). Unifying the observational diversity of isolated neutron stars via magnetothermal evolution models. *Monthly Notices of the Royal Astronomical Society*, 434(1), 123-141.
- [46] Viganò, D., and Pons, J. A. (2012). Central compact objects and the hidden magnetic field scenario. *Monthly Notices of the Royal Astronomical Society*, 425(4), 2487-2492.
- [47] urpin1995accretion, title=Accretion and evolution of the neutron star magnetic field, author=Urpin, V and Geppert, U, journal=Monthly Notices of the Royal Astronomical Society, volume=275, number=4, pages=1117–1124, year=1995, publisher=Oxford University Press Oxford, UK
- [48] Page, D., Geppert, U., and Zannias, T. (2000). General relativistic treatment of the thermal, magnetic and rotational evolution of neutron stars with crustal magnetic fields. arXiv preprint astro-ph/0005301.
- [49] Thurman III, H. O. (2004). Neutron star electromagnetic field structure. Old Dominion University.
- [50] Camenzind, M. (2007). Compact objects in astrophysics (pp. 1-25). Springer Berlin Heidelberg.
- [51] de Avellar, M. G. B. Diferentes abordagens Composição e ao Ambiente das Estrelas de Nêutrons.
- [52] Chabrier, G., Potekhin, A. Y., and Yakovlev, D. G. (1997). Cooling neutron stars with accreted envelopes. *The Astrophysical Journal*, 477(2), L99.
- [53] Shapiro, S. L., Teukolsky, S. A., Holes, B., Dwarfs, W., and Stars, N. (1983). The physics of compact objects. Wiley, New York, 1983, 119-123.
- [54] Miralles, J. A., Urpin, V., and Konenkov, D. (1998). Joule heating and the thermal evolution of old neutron stars. *The Astrophysical Journal*, 503(1), 368.

[55] Geleta Bekele Title=MAGNETIC FIELD EVOLUTION OF NEUTRON STAR.Year JUNE 2014

[56] Page, D., Geppert, U., and Zannias, T. (2000). General relativistic treatment of the thermal, magnetic and rotational evolution of neutron stars with crustal magnetic fields. arXiv preprint astro-ph/0005301.

# References

- [1] Max Camenzind. *Compact objects in astrophysics*. Springer, 2007.
- [2] José A Pons and Daniele Viganò. Magnetic, thermal and rotational evolution of isolated neutron stars. *Living Reviews in Computational Astrophysics*, 5(1):1–65, 2019.
- [3] Morgane Fortin. *Evolution & dynamics of neutron stars: from microphysics to astrophysics*. PhD thesis, Observatoire de Paris; Polish Academy of Sciences, 2012.
- [4] Milton Ruiz, Stuart L Shapiro, and Antonios Tsokaros. Multimessenger binary mergers containing neutron stars: Gravitational waves, jets, and  $\gamma$ -ray bursts. *Frontiers in Astronomy and Space Sciences*, 8:656907, 2021.
- [5] Marcio Guilherme Bronzato de Avellar. Diferentes abordagens a composição e ao ambiente das estrelas de nêutrons.
- [6] MV Beznogov and DG Yakovlev. Statistical approach to thermal evolution of neutron stars. In *Journal of Physics: Conference Series*, volume 661, page 012002. IOP Publishing, 2015.
- [7] Keisuke Yanagi. Thermal evolution of neutron stars as a probe of physics beyond the standard model. *arXiv preprint arXiv:2003.08199*, 2020.
- [8] Hugh O Thurman III. *Neutron star electromagnetic field structure*. Old Dominion University, 2004.
- [9] KAZIM YAVUZ EKŞİ. Neutron stars: compact objects with relativistic gravity. *Turkish Journal of Physics*, 40(2):127–138, 2016.



- [10] James M Lattimer and Maddappa Prakash. The physics of neutron stars. *science*, 304(5670):536–542, 2004.
- [11] Christopher L Sabine. The following resources related to this article are available online at. *science*, 1097403:305, 2004.
- [12] Jocelyn S Read. *Neutron stars in compact binary systems: From the equation of state to gravitational radiation*. The University of Wisconsin-Milwaukee, 2008.
- [13] Mohammad Al-mamun. Neutron star structure from electromagnetic and gravitational wave observations. 2021.
- [14] Sachiko Tsuruta and AGW Cameron. Cooling and detectability of neutron stars. *Canadian Journal of Physics*, 44(8):1863–1894, 1966.
- [15] Einar H Gudmundsson, CJ Pethick, and Richard I Epstein. Structure of neutron star envelopes. *The Astrophysical Journal*, 272:286–300, 1983.
- [16] David Pines and Joseph Ventura. *Neutron Stars: Theory and Observation, Or, Neutron Stars: an Interdisciplinary Field*. Kluwer Academic Publishers, 1991.
- [17] David Pines and M Ali Alpar. Superfluidity in neutron stars. *Nature*, 316(6023):27–32, 1985.
- [18] PW Anderson and N Itoh. Pulsar glitches and restlessness as a hard superfluidity phenomenon. *Nature*, 256(5512):25–27, 1975.
- [19] Jinlin Lin Han, Richard N Manchester, Andrew G Lyne, Guo Jun Qiao, and W Van Straten. Pulsar rotation measures and the large-scale structure of the galactic magnetic field. *The Astrophysical Journal*, 642(2):868, 2006.
- [20] Martin Poulter and Mike Peel. Physics on wikipedia. *Physics World*, 24(09):32, 2011.
- [21] Michelle Suzanne Ouellette. *Heating and cooling of neutron star crusts*. Michigan State University, 2005.

- [22] Kip S Thorne. The relativistic equations of stellar structure and evolution. *The Astrophysical Journal*, 212:825–831, 1977.
- [23] MA Alpar, HF Chau, KS Cheng, and D Pines. Postglitch relaxation of the crab pulsar: Evidence for crust cracking. *The Astrophysical Journal*, 427:L29–L31, 1994.
- [24] Steven Weinberg. *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*. 1972.
- [25] James M Lattimer, CJ Pethick, Madappa Prakash, and Pawel Haensel. Direct urca process in neutron stars. *Physical review letters*, 66(21):2701, 1991.
- [26] Kenneth A Van Riper. Neutron star thermal evolution. *The Astrophysical Journal Supplement Series*, 75:449–462, 1991.