ASTROPHYSICAL GRAVITATIONAL COLLAPSE

By

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Abstract

In this thesis we reviewed astrophysical collapses relevant to star formation and compact objects. In the case of star formation scenario, we reviewed the agents of molecular cloud collapse where gravity, turbulence, thermal and magnetic pressures play major roles. In the case of stellar collapse to compact objects, we reviewed the general relativistic hydrostatic equation to rediscover the collapse conditions and the mass-radius relationships of the compacts using the Lane-Emden equations. For the analysis of the results we have used the latest Mathematica 13 to plot graphs. Our results are in good agreement with the previous works.

Keys : Astrophysical collapse, Gravity, Molecular cloud, Turbulence, Pressure, Stellar evolution, Compact objects.

Chapter 1

General Introduction

1.1 Background and literature review

Astrophysical collapses are important phenomena in stellar formation and ends. They provide a great deal of information about origin, composition within and the surrounding environment and as well in the test of fundamental physics theories and models. However, it is at its infant stage that need further progress and developments.

Gravitational collapse is the contraction of astronomical system due to self gravitation that tends to pull matter towards its enter. It is a fundamental mechanism for structure formation in the universe to form packs of different systems with hierarchy of condensed structures such as clusters of galaxies, stellar groups, stars, planets and so on.
Among the collapses, galaxy formation and the antecedent evolution is fundamental to cosmology to study the universe (see for example [4, 9, 28]). Stellar formation and its collapse to compact objects are important in fundamental gravitational physics. Astrophysical activities and phenomena observed during collapses used to probe some of the most extreme physical conditions in the Universe and give unique insights into strong gravity, the properties of matter at extreme densities, and energetic particle acceleration.

In stellar formation, stars are being born through the gradual gravitational collapse of a cloud of interstellar matter [20]. The compression caused by the collapse raises temperature until thermonuclear fusion occurs at the center of the star leading to an outward thermal pressure balance with that of gravity (inward pressure). The star then exists in a state of dynamic equilibrium until all its energy sources are exhausted, to halt further fusion. Then, the stellar collapse to reach other equilibrium state with quantum degenerate pressures or completely collapses to singularity called black hole [5, 14].

The gravitational collapse of massive stars and the resulting spacetime singularities are being considered as the natural phenomena where quantum and gravity come together to operate in unison. On a smaller scale, gravitational collapse can trigger star formation, where a local cloud of matter contracts under its own gravity, heats up enormously to cause nuclear burning within, and thus first a proto-star and then a star is born. A massive star, when it runs out of internal nuclear fuel toward the end of its life-cycle, undergoes a continual gravitational collapse.
The evolution of stars depend upon their masses. It is best understood by Hertzsprung-Russell diagram [26]. The movement of a star in this diagram, as it evolves, will tell us how its radius, temperature and luminosity vary in response to the behavior of the core. This informs the final state of the end collapse of stars [30].

The process of star formation takes around a million years from the time the initial gas cloud starts to collapse until the star is created and shines like the Sun. The leftover material from the star's birth is used to create planets and other objects that orbit the central star [6]. The main driver for the collapse of a molecular cloud and formation of a star is clearly gravity and when a molecular cloud reaches a certain critical density, it collapses under its own weight leading to the dissipation of gravitational potential energy. In general the time taken for this collapse to occur can be roughly underestimated by considering the free-fall time which just depends on the density of the collapsing gas, ρ and this corresponds to a scenario with zero pressure [3].

In normal stars, such as the Sun, the inward gravitational pull is held in equilibrium by the thermal pressure of the gas. This thermal pressure will be sufficient to resist the gravitational pull only if the star is hot enough. The star can therefore remain in equilibrium as long as the thermonuclear reactions in its core supply enough heat; that is, as long as the energy released in these reactions compensates for the energy lost by radiation at the surface. In a star that has exhausted its supply of nuclear fuel, the thermal pressure will ultimately disappear, and the star will collapse under its own weight [23].

When a massive star more than about ten solar masses exhausted its internal nuclear fuel, it is believed to enter the stage of a continual gravitational collapse without any final equilibrium state. The star then goes on shrinking in its radius, reaching higher and higher densities. According to the general theory of relativity, the final fate of the ultra-dense object that forms as a result of the collapse could be a black hole in space and time from which not even light rays escape [15].

The terminology for the structure of molecular clouds is not fixed; here the Giant molecular clouds (GMCs) have masses in excess of $10^4 M_\odot$ and contain most of the molecular mass [22].

Molecular clouds have a hierarchical structure that extends from the scale of the cloud down to the thermal Jeans mass in the case of gravitationally bound clouds, and down to much smaller masses for unbound structures. Overdense regions (at a range of scales) within GMCs are termed clumps. Star forming clumps are the massive clumps out of which stellar clusters form, and they are generally gravitationally bound. Cores are the regions out of which individual stars (or small multiple systems like binaries) form, and are necessarily gravitationally bound.

Star formation takes place in the dense cores of GMCs. Measuring the assembly time from the diffuse atomic gas to the dense molecular gas and the collapse time of these molecular clouds until they start forming stars, provides constraints on which physical mechanisms drive the star formation process in galaxies [13].

1.2 Statement of the problem

Astrophysical collapses are important phenomena in stellar formation and ends. Since the birth of General theory of relativity, these phenomena have become important study area in evolutionary scenarios. Astrophysical activities within and the surrounding environment of these collapses have provided a great deal of information about origin, composition and tests of theories. However, it is at its infant stage that need further progress and developments.

Here, we study these phenomena that are related to the following questions:

Research questions

1. What are some of the astrophysical collapses?
2. How molecular clouds collapse to form stars?
3. What parameters determine a star collapse?
4. Is there relationship between the progenitor star and its end product?
5. What is the effect of gravity in formation and end of stellar?
6. What is the minimum size of a molecular cloud used for collapse and fragmentation to form a star?
7. How do thermal and magnetic pressures effect stellar formation and evolutions?

1.3 Objectives of the study

1.3.1 General objective

To study astrophysical gravitational collapse

1.3.2 Specific objectives

- To review kinds of astrophysical collapses.
- To provide conditions how molecular clouds collapse in the formation of stars.
- To describe the parameters that determine a star collapse.
- To give the relationship between the progenitor star and its end product.
- To discuss on the effect of gravity in stellar formation and its end.
- To investigate the minimum size of a molecular cloud used for collapse and fragmentation for star formation.
- To examine the effect of thermal and magnetic pressures in stellar formation and evolutions.

1.3.3 Methodology of the Study

Related literature reviews is used to discuss astrophysical collapses. General relativity based equations like Virial theorem and Lane-Emden equations are used to analyze conditions and end scenarios of evolutionary dynamics of astrophysical systems. Latest Mathematica 13 is used for analytical analysis of the work while Latex is used for document process.

Organization of the work is: in chapter 2 we provide collapse of molecular cloud in stellar formation process. In chapter 3 we give star collapse and compact objects of the formation process. In chapter 4, we give our results and discussions whereas in the final chapter 5 we give our summary and conclusion.

Chapter 2

Molecular Cloud Collapse and Stellar Formations

Molecular clouds collapse to produce the stars we see. The nature of this collapse is uncertain, and two theories for how a molecular cloud collapses dominate the scientific investigations. In the first, when the gas temperature relative to absolute zero ($0K$) the Jeans length of a cloud is smaller than the scale of a cloud and the gravitational force is stronger than the gas pressure then, the cloud shrinks. Second when the temperature of a cloud to drop as the cloud shrinks from the 100K in the cool interstellar medium and in the outer layers of molecular clouds to the 10K, the cloud must collapse if only gas pressure provides support against the force of gravity [1]. The star-forming clouds in the ISM is influenced by gravity and by a wide variety of chemical processes, including formation and destruction of molecules and dust grains (which changes the thermodynamic behavior of the gas) and changes in ionization state (which alter how strongly the gas couples to magnetic fields).

2.1 Agents of molecular cloud

The main agents of molecular cloud collapse are gravitational instability, turbulence and magnetic field.

2.1.1 Gravitational instability

Star formation begins with small density fluctuations in an initially nearly uniform medium that are amplified by gravity in a process called 'gravitational instability'. Jeans studied the growth of plane wave density perturbations in an infinite uniform medium that has a finite pressure but no rotation, magnetic fields, or turbulence, and he showed that short wavelength perturbations are pressure dominated and propagate as sound waves, while perturbations whose wavelength exceeds a critical value called the 'Jeans length' are gravity dominated and do not propagate but grow exponentially. For an isothermal medium with a uniform density ρ and a constant temperature T that is fixed by radiative processes [18].

Here, we will study a typical size where the self-gravity play an important role and form density inhomogeneities the Jean wavelength. Consider a uniform gas with density ρ_0 and pressure p_0 without motion $u_0 = 0$. In this uniform gas distribution, we assume small perturbations. As a result, the distributions of the density, the pressure and the velocity are perturbed from the uniform ones as

$$\rho = \rho_0 + \delta\rho \tag{2.1}$$

$$p = p_0 + \delta p \tag{2.2}$$

$$\vec{v} = \vec{v_0} + \delta\vec{v} = \delta\vec{v} \tag{2.3}$$

where the amplitudes of perturbations are assumed much small, that is, $\dfrac{|\delta\rho|}{\rho_o} < < 1$, $\dfrac{|\delta p|}{p_o} < < 1$ and $\dfrac{|du|}{c_s} < < 1$. We assume the variables changes only in the x direction. In this case the basic equations for isothermal gas are:

$$\frac{\partial\rho}{\partial t} + \frac{\partial\rho v}{\partial x} = 0 \tag{2.4}$$

$$\rho\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{\partial p}{\partial x} + \rho g_x \tag{2.5}$$

$$p = c_s\rho \tag{2.6}$$

where u and g_x represent the x-component of the velocity and that of the gravity, respectively. Using equations 2.1, 2.2, 2.3, and equation 2.4 becomes:

$$\frac{\partial\rho_0 + \partial\rho}{\partial t} + \frac{\partial(\rho_0 + \delta\rho)(v_0 + \delta v)}{\partial x} = 0 \tag{2.7}$$

Noticing that the amplitudes of variables with and without δ are completely different. two equations are obtained from equation 2.7 as

$$\frac{\partial\rho_0}{\partial t} + \frac{\partial\rho_0 v_0}{\partial x} = 0 \tag{2.8}$$

$$\frac{\partial\delta\rho}{\partial t} + \frac{\partial\rho_0 + \delta\rho v_0}{\partial x} = 0 \tag{2.9}$$

where the above is the equation for unperturbed state and the lower describes the relation between the quantities with δ. Equation 2.7 is automatically satisfied by the assumption of uniform distribution. In equation 2.9 the last term is equal to zero. Equation of motion.

$$(\rho_0 + \delta\rho)(\frac{\partial v_0 \delta v}{\partial t} \quad + (v + \delta v)\frac{\partial v_0 \delta v}{\partial x}) = \frac{\partial p_0 \delta p}{\partial t} + (\rho_0 + \delta\rho)\frac{\partial\phi_0 + \delta\phi}{\partial x} \tag{2.10}$$

gives the relationship between the terms containing only one variable with δ as follows:

$$\rho_0\frac{\partial\delta v}{\partial t} = -\frac{\partial x}{\delta p}\rho_0\frac{\partial\delta\phi}{\partial x} \tag{2.11}$$

The perturbations of pressure and density are related with each other as follows: for the isothermal gas

$$\frac{\delta p}{\delta\rho} = (\frac{\partial p}{\partial\rho})_T = \frac{p_0}{\rho_0} = c_s^2 \tag{2.12}$$

and for isentropic gas

$$\frac{\delta p}{\delta\rho} = (\frac{\partial p}{\partial\rho})_{\alpha d} = \gamma\frac{p_0}{\rho_0} = c_s^2 \tag{2.13}$$

2.1.2 Turbulence

Turbulence is a state of violent commotion, agitation, or disturbance, with a turbulent fluid further defined as one in which the velocity at any point fluctuates irregularly in both magnitude and direction. [22].
Turbulence is a multiscale phenomenon in which kinetic energy cascades from large scales to small scales. The bulk of the specific kinetic energy remains at large scales. Turbulence appears to be dynamically important from scales of whole MCs down to cores [2].

The internal random motions in molecular clouds are often referred to as 'turbulence'. Supersonic turbulence may play an important role in structuring molecular clouds, since supersonic motions can generate shocks that produce large density fluctuations. In many cases, self-gravity is roughly balanced by the turbulent motions in molecular clouds, and this suggests that gravity and turbulence are equally important in controlling the structure and evolution of these clouds. Gravity cannot be balanced for long by any kind of turbulence unless the turbulence is continually regenerated by a suitable energy source [18].

Turbulent flows tend to have hierarchical structure which may explain the hierarchical distribution of stars in star forming regions shown by statistical studies of the distribution of neighboring stars in young stellar clusters. Hierarchical clustering seems to be a common feature of all star forming regions. It may be a natural outcome of turbulent fragmentation.

Turbulence driven on large scales and freely decaying turbulence lead to star formation in aggregates and clusters. Decaying turbulence typically leads to the formation of a bound stellar cluster , while aggregates associated with large-scale, coherent, shock fronts often have higher velocity dispersions that result in their complete dispersal. Turbulent flows controlling the formation of clouds and cores differ in the role played by magnetic fields trigger more about the origin of star-formation. Generally, clouds do not become gravitationally bound, and supersonic turbulence will dissipate on roughly the free-fall time scale as collapse of gravitationally bound clouds proceeds. Thermal energy and turbulence are the two leading candidates in which turbulence plays a dual role, both creating overdensities to initiate gravitational contraction or collapse, and countering the effects of gravity in these overdense regions.

2.1.3 Magnetic field

Strong magnetic field

In the strong magnetic field theory, clouds are formed with subcritical masses, $M < M_\phi = \phi/2\pi\sqrt{G}$, where M_ϕ is the critical mass, ϕ is the magnetic flux, and G is the gravitational constant. Hence, the magnetic pressure is sufficiently strong to counteract gravity and prevent gravitational collapse. Because the magnetic field is frozen only into the ionized gas and dust, neutral gas and dust contract gravitationally through the field and the ions, increasing mass in the cloud cores. The magnetic field strength also increases, but more slowly than dense mass. This process is known as (gravity-driven) ambipolar diffusion. When the core mass reaches and exceeds M_ϕ , the core becomes supercritical ($M > M_\phi$), collapses, and forms stars. During the collapse, the magnetic field is dragged inward but cannot become strong enough to halt the collapse.

Weak magnetic field

The weak-field theory of star formation says molecular clouds are intermittent phenomena, with short ($\sim 10^6$ years) lifetimes. Magnetic fields are sufficiently weak that the low-density ISM is supercritical ($M > M_\phi$). Although magnetic pressure cannot stop the collapse, it can dominate turbulent pressure during the late stages of core collapse. Of course, the formation of clouds may lead to $M/M\phi$ varying within cloud complexes, so some volumes in a cloud complex may be subcritical and some supercritical. In addition, the requirement on a region in a molecular cloud that must be satisfied before it can collapse and form a star was first derived by Sir James Jeans [8]. His method was a linear stability analysis performed on the basic hydrodynamic equations, assuming an isothermal gas, including only thermal and gravitational effects.

A simpler method of defining the initial conditions for collapse, which, however, includes all relevant physical effects, is to require that the absolute value of the gravitational energy must exceed the sum of the thermal, turbulent, and magnetic energies. This requirement defines a mass of molecular clouds that is gravitationally bound. For this mass to be as small as a solar mass, the requirement can be satisfied only in the coolest, densest parts of the interstellar medium.

Hence, we assume that star formation occurs mostly in molecular clouds that are in virial equilibrium. To equate their energy content and, henceforth, determine the agents governing the evolution of clouds we introduce the virial theorem and discuss the importance of gravity, thermal pressure, magnetic forces and kinetic energy from turbulent motions. The virial theorem provides a general equation to relate the globally averaged energies of a system.

Furthermore, although gravity is the most leading dynamical quantities that favors star formation by drawing material together, it cannot form alone a star. Therefore, some additional force must hinder the process. Magnetic fields and turbulence are the two leading candidates in which turbulence plays a dual role, both creating over densities to initiate gravitational contraction or collapse, and countering the effects of gravity in these overdense regions, while magnetic fields play a role in the final stage of star formation, both in mediating gas accretion and in launching the bipolar jets due to the conservation of angular momentum that typically announce the birth of a new star. Now to obtain a condition of mechanical equilibrium, assume un magnetized spherical and uniform molecular cloud and a standard notation as the radius R, the volume $V_{cl} = 4\pi R^3/3$ and the mass M, recall the virial equilibrium equation.

$$4\pi R^3(\bar{p} - p_0) - (\frac{3}{5}\frac{GM^2}{R}) = 0 \tag{2.14}$$

$$3MC_s^2 - 4\pi R^3 p_0 - \frac{3}{5}\frac{GM^2}{R} = 0 \tag{2.15}$$

where, $\bar{p} = \mathrm{c}_s^2\ (\frac{3M}{4\pi R^3})$ 2.14, is the mean internal pressure of the spherical molecular cloud, c_s is isothermal sound speed in a fluid and p_0 is the constant external pressure. However, for magnetized molecular clouds, the virial equilibrium is given by,

$$4\pi R^3(\bar{p} - p_0) - (\frac{3}{5}\frac{GM^2}{R}) + \frac{\phi_B^2}{6\pi^2}[\frac{1}{R} - \frac{1}{R_0}] = 0 \tag{2.16}$$

But as $R_0 \longrightarrow \infty$, $\frac{1}{R_0} \approx 0$, Hence

$$4\pi R^3(\bar{p} - p_0) - \frac{3G}{5R}(M^2 - M_\Phi^2) = 0 \tag{2.17}$$

where, $M_\phi = \frac{5\phi_B^2}{18\pi^2 G}$
Now let$[M^2 - M_\phi^2] = M_{eff}^2$. So the above equation 2.17 becomes,

$$4\pi R^3(\bar{p} - p_0) - \frac{3G}{5R}M_{eff}^2 = 0$$

$$\Rightarrow P_{net} = \frac{3G}{20\pi}\frac{M_{eff}^2}{R^4} \tag{2.18}$$

where, $P_{net} = [\bar{p} - p_0]$ is the net thermal pressure that turbulence contribution comes from.

In addition to being turbulent, molecular clouds are also significantly magnetized, and magnetic fields can also be important for the dynamics and evolution of these clouds. If molecular clouds are sufficiently strongly magnetized, their internal motions might be predominantly wavelike, consisting basically of magnetohydrodynamic (MHD) waves such as

Alfven waves; since Alfven waves involve only transverse and non-compressional motions, they might be expected to dissipate more slowly than purely hydrodynamic supersonic turbulence. Wavelike MHD turbulence might then provide a source of pressure that can supplement thermal pressure and help to support molecular clouds for a significant time against gravity [22]. The importance of the magnetic field to cloud structure is determined by the ratio of the mass to the magnetic critical mass M $_\phi$, which is defined by the condition that the magnetic energy must be equal to the gravitational energy, B = |W |, for a cold cloud in magnetostatic equilibrium:

$$M_\phi \equiv c_\phi \frac{\phi}{G^{1/2}} \tag{2.19}$$

where ϕ is the magnetic flux threading the cloud. Magnetic fields alone cannot prevent gravitational collapse in magnetically supercritical clouds (M $>$ M $_\phi$), whereas gravitational collapse is not possible in magnetically subcritical clouds (M $<$ M $_\phi$); keep in mind, however, that M can change as the result of flows along the field, and M $_\phi$ can change owing to ambipolar diffusion. The numerical coefficient c$_\phi$ depends on the internal distribution of density and magnetic fields. A cold cloud with a poloidal field and a constant mass to flux ratio has $c_\phi = 0.17$.

Magnatohydrostatic clouds: In the magnetized medium, the Lorentz force is

$$\begin{aligned} F &= \frac{1}{4\pi}(\nabla \times B) \times B \\ F &= -\frac{1}{8\pi}\nabla B^2 + \frac{1}{4\pi}(B.\nabla)B \end{aligned} \tag{2.20}$$

where the first RHS term is called the magnetic pressure, which has an effect to support the cloud against the self-gravity. The virial analysis is also applicable to the magneto-hydrostatic clouds. The terms related to the magnetic fields are

$$\begin{aligned} M &= \int \frac{B^2}{8\pi} dv + \int_s (r.B)B.nds - \int \frac{B^2}{8\pi} r.nds \\ M &\simeq \int \frac{B^2 - B_0^2}{8\pi} dv \simeq \int \frac{\phi_B^2}{6\pi^2}(\frac{1}{R} - \frac{1}{R_0}) \end{aligned} \tag{2.21}$$

where ϕ_B^2 represents a magnetic flux and it is assumed to be conserved if we change the radius, R, that is $\phi_B^2 = \pi B_o R_o^2 = \pi B R^2$. But, from the condition of mechanical equilibrium in un magnetized cloud the virial theorem will have the form,

$$\frac{1}{2}\frac{d^2 I}{dt^2} = 2(T - T_0) + w$$

where the LHS term (moment of inertia) becomes zero at Virial equilibrium while

$$T = \int (\frac{3}{4}P_{th} + \frac{1}{2}\rho v^2)d = \frac{3}{4}\bar{p}V_{cl}$$

is a term corresponding to the thermal pressure plus turbulent pressure, and

$$T_0 = \int P_{th} r.nds = \frac{3}{4}p_0 V_{cl}$$

comes from a surface pressure, as well as

$$w = -\int \rho r.\nabla\phi dv = -\frac{3}{5}\frac{GM^2}{R}$$

is a gravitational energy. So using these value with magnetic fields we get

$$4\pi R^3[\bar{p} - p_0] - \frac{3}{5}\frac{GM^2}{R} + \frac{\phi_B^2}{6\pi^2} = 0 \tag{2.22}$$

This is the equation of virial equilibrium of a spherical, isothermal, magnetized cloud, where we ignored the term $\frac{\phi_B^2}{6\pi^2 R_0}$. The last two terms are rewritten as

$$\frac{3G}{5R}[M^2 - M_\phi^2]$$

where M_ϕ^2 is defined as $3GM_\phi^2/5 = \phi_B^2/6\pi^2$. This shows the effects of the magnetic fields:

1. B-fields effectively reduce the gravitational mass as $M^2 - M_\phi^2 = M^2 - 5\phi_B^2/(18\pi^2 G)$. This plays a part to support a cloud.

2. However, even a cloud contracts (decreasing its radius from R_0 to R), the ratio of the

gravitational to the magnetic terms keeps constant since these two terms are proportional to R^{-1} . Thus, if the magnetic term does not work initially, the gravitational term continues to predominate over the magnetic term [29].

2.2 Virial theorem and equilibrium

If thermal pressure were the only force opposing gravity, molecular clouds might then be expected to collapse rapidly and efficiently into stars. Most molecular clouds are indeed observed to be forming stars, but they do so only very inefficiently, typically turning only a few percent of their mass into stars before being dispersed. The fact that molecular clouds do not quickly turn most of their mass into stars, despite the strong dominance of gravity over thermal pressure, has long been considered problematic, and has led to the widely held view that additional effects such as magnetic fields or turbulence support these clouds in near-equilibrium against gravity and prevent a rapid collapse. Hence, star formation mostly occur in molecular clouds that are in virial theorem. In this case agents governing the evolution of clouds and the importance of gravity, thermal pressure, magnetic forces and kinetic energy from turbulent motion. This virial theorem provides a general equation to relate the globally average energies of a system [12]. And the simplified result of this equation (virial) is the balance equation:

$$\frac{1}{2}\frac{\partial^2 I}{\partial t^2} = 2T + 2U + W + M \tag{2.23}$$

If the cloud is in virial equilibrium, the net moment of inertia vanishes and the gravitational potential energy W has to be balanced by the sum of kinetic energy T, thermal energy U and magnetic energy M.

2.3 Molecular cloud collapse

Molecular clouds are the initial conditions for star formation, and so any understanding of star formation must necessarily also explain the origin of molecular clouds and a large space cloud filled with gas and dust. It is also found inside the interstellar medium (ISM), as well as molecular clouds are made of a mix of atoms, molecules, and dust which are often of gigantic size as birthplaces of stars. Many molecular clouds are believed to be contracting slowly under self-gravity and stars would eventually form in the central regions. Cooling process may lead to the formation of dense, cold gas clouds within which star formation can occur [25].

A molecular cloud is very cold, only a few degrees above absolute zero, which is the lowest temperature possible, also called 0°K. But, when gas and dust start to collapse in a region within the molecular cloud, it slowly heats up. When matter is squeezed together, the density of the matter will increase and the matter will start to heat up. The outer edge of a collapsing region will have a temperature of around 10°K above absolute zero and the inner region will slowly heat up to around 300°K, which is around room temperature [6].

The main driver for the collapse of a molecular cloud and formation of a star is clearly gravity. When a molecular cloud reaches a certain critical density, it collapses under its own weight leading to the dissipation of gravitational potential energy. Although the collapse may be hindered somewhat by thermal and turbulence pressures within the molecular cloud, in general the time taken for this collapse to occur can be roughly underestimated by considering the free-fall time which just depends on the density of the collapsing gas, ρ. This corresponds to a scenario with zero pressure [3].

$$t_{ff} = \sqrt{\frac{3\pi}{32G_\rho}} \tag{2.24}$$

where t_{ff} is free-fall time.

Star formation in gas that has already evolved to the molecular cloud state. Although this material is already relatively cold and dense, in general it has too much magnetic, rotational, thermal, and turbulent energy to allow collapse into low-mass stars. A number of physical processes, however, tend to dissipate these energies, or allow local collapse in spite of them, on time scales up to 10^7yr. The main effects that need to be considered are heating, cooling, the role of shock waves, magnetic braking of rotation, diffusion of the magnetic field with respect to the gas, and the generation and decay of turbulence.

2.3.1 Orders of collapsing molecular cloud

Dark clouds(clumps) size = 200,000AU, time = 0 $\longrightarrow$ pre-stellar core, size = 10,000AU, time = 10-100 thousand years $\longrightarrow$ Protostar, size = 1000AU, time = up to 10^6 and a proto-star, which means it is at its very first stage of becoming a real star [6]. Stars form from the collapse of clouds of molecular gas and dust in the interstellar medium, primarily in the spiral arms of galaxies. The vast majority of stars form in giant molecular clouds (GMCs), and some in smaller diffuse molecular clouds [19].

2.3.2 Interstellar clouds and hydrostatic equilibrium

By Einstein's Theory of General Relativity, any matter with mass curves space-time, so even the hydrogen and helium gas curved the space-time, so as time elapsed, the atoms came together and started forming large interstellar clouds. These clouds formed patch-like structures in our universe and started to make filaments like structure of gases where denser regions became more dense and rarer regions became more empty. The denser regions (nodes) are shown brighter and rarer regions are shown darker. Due to the action of gravity, the gas will be dragged towards the center. As the cloud gets converted to a spherical ball and starts contracting, the atoms present inside exert an outward force due to the repelling force of the outer electron of each atom, thus creating internal pressure. And at a particular density and radius of the cloud, the inward Gravitational force and outward force due to the internal pressure will be balanced and the cloud is in a state of equilibrium called as 'hydrostatic equilibrium' [27].

2.3.3 Equation of hydrostatic equilibrium

$$dm = 4\pi r^2 \rho(r) dr \Rightarrow \frac{dm}{dr} = 4\pi r^2 \rho(r) \tag{2.25}$$

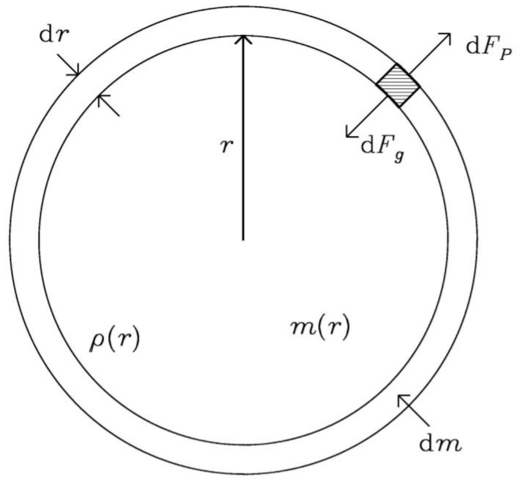

Figure 2.1: spherically symmetric interstellar cloud in hydrostatic equilibrium [27]

where $S = 4\pi r^2$, $dV = S * dr = 4\pi r^2 dr$ and $dm = \rho(r)dV = 4\pi r^2 \rho(r)dr$. It also 'R' is spherical cloud of radius, 'r' is the inner radius of the shell, 'r + dr' is the outer radius, 'dr' is the thickness of the shell, S is the surface area of the shell and (dV) is Volume of the shell. Then the gravitational force exerted on the shell of mass 'dm' by the cloud inside the shell of mass 'm(r)' is ,

$$dF_g = \frac{G_m(r)dm}{r^2} = \frac{G_m(r)}{r^2} * 4\pi r^2 \rho(r)dr \tag{2.26}$$

There will be outward internal pressure, because of the photons which are traveling outward which exert a pressure on the shell which is dP , where, $dP = \frac{dF_P}{S}$, then we get the outward force to be,

$$dF_P = SdP = 4\pi r^2 dP \tag{2.27}$$

When the spherical cloud is in equilibrium, then the net force acting on the shell has to be zero. $\Rightarrow \Sigma F = dF_g + dF_P$. Then substituting the values of dF_g and dF_p from Eq.

2.26 and Eq. 2.27 to the above equation we get,

$$
\begin{aligned}
4\pi G_m(r)\rho(r)dr + 4\pi r^2 dP &= 0 \\
4\pi r^2 dP &= -4\pi G_m(r)\rho(r)dr \\
\frac{dP}{dr} &= -\frac{G_m(r)\rho(r)}{r^2}
\end{aligned} \tag{2.28}
$$

Thus, eq. 2.28 gives the equation for Hydrostatic equilibrium of the cloud. If the mass of the cloud is more than a particular value called as the 'Jean's mass', then the internal pressure of cloud will be insufficient to prevent the collapse cloud under gravity. Jean's mass 'M j ' and Jeans Density, '$\rho_j'$is given to be,

$$M_j = \frac{3k_BTR}{2G_m} \tag{2.29}$$

$$\rho_j = (\frac{3}{4\pi M^2})(\frac{3k_BT}{2G_m}) \tag{2.30}$$

Jean's mass of a cloud depends on the factors like its average temperature (T), radius (R), and average mass of a gas-particle(m) which will be in the order of $\approx 10^4 M_{\odot}$. And Jean's density of the cloud depends on the average temperature, total mass (M), and average mass of the gas-particle.

If any interstellar cloud holds these conditions true, i.e., $M > Mj$ and $\rho > \rho_j$, then the condition of Hydrostatic equilibrium breaks down and the cloud will collapse, it gets compressed for millions of years. The size of the cloud will reduce a lot and its size reduces to $\approx 10^{-4}$ times its initial value. This makes the innermost part, i.e., the center of the cloud, very hot such that the speed of the particles increases to a large value and creates enough amount of internal pressure which balances the further gravitational collapse and reaches a condition of temporary Hydrostatic equilibrium. This object which is in a temporary Hydrostatic equilibrium is called a 'Protostar '.

When the protostar is in temporary hydro-static equilibrium, it gathers mass continuously from the nearby interstellar clouds. Due to an increase in mass the gravitational collapse starts when the protostar gathers enough mass such that its mass exceeds $0.08 M_{\odot}$, then the gravitational pull overcomes the internal pressure of protostar and it starts to collapse. Due to an increase in the external pressure on the center of the protostar, the core temperature becomes very hot, such that it provides sufficient kinetic energy for protons to fuse.

2.3.4 Gravitational potential energy of a spherical cloud

The total gravitational potential (binding) energy of a sphere cloud of uniform density [17] is given by

$$E_g = -\int_0^{M_c} \frac{GM(r)}{r} dm \tag{2.31}$$

where M (r) is the mass contained in radius r . The gravitational potential energy of a system is the energy required to assemble the mass by bringing matter from infinity to the point of interest. Substituting $dm = 4\pi\rho_c r^2 dr$, and $\rho_c = \frac{3M_c}{4\pi R_c^3}$ Equation 2.31 becomes

$$E_g = -\int_0^R \frac{G(\frac{4}{3}\pi\rho r^3)}{r} 4\pi\rho r^2 dr = -\int_0^R \frac{16}{3} G\pi^2\rho^2 r^4 dr \tag{2.32}$$

where Substituting ρ we have:

$$E_g = -\frac{16}{15} G\pi^2\rho^2 R^5 = -\frac{3GM^2}{5R} \tag{2.33}$$

Therefore, a spherical cloud with mass M_c , density ρ_c and radius $R_c$has gravitational energy given by

$$E_g = -c\frac{GM_c^2}{R_c} \tag{2.34}$$

where c is a constant depending on the cloud density. We choose the value of this constant either 3/5 or 1 when we assume spherical uniform density cloud or need to consider uncertainty respectively . From the Virial theorem $E_g + 2U = E_g + 2E_k = 0$, this indicates that gravitational energy and the internal energy are in balance means the MCs neither collapse nor expand. The total energy of the system is related to gravitational potential energy E_g by $E_{tot} = E_g + U = E_g - \frac{1}{2}E_g = \frac{1}{2}E_g$, it implies that the internal energy must increase as the gravitational energy goes to negative. The total energy (E_{tot}) is negative means the molecular cloud is bound and stable. Which must be negative as cloud is bound (i.e. $E_g < 0$), gravitational systems tend to do this very rapidly, a bound system that is not in virial equilibrium will change its configuration very rapidly to get into virial equilibrium.

The internal energies (thermal, kinetic, or magnetic) are positive, and they are assumed to act as supporting agents against collapse. If $E_g + 2U = E_g + 2E_k < 0$, the cloud must be contracting, while $E_g + 2U = E_g + 2E_k > 0$ implies that the E_g cloud is expanding. For the MC to collapse $|\dfrac{E_g}{2}| > E_k$

2.4 Jeans mass

The jeans mass scale is the classic mass scales for the contraction of a gravitating MC. Jeans mass: is minimum or critical mass that is necessary to initiate the spontaneous collapse of the cloud. We believe that stars form when a portion of a molecular cloud collapses gravitationally. Since this collapse is resisted by various things, and there is evidence for many molecular clouds that have not collapsed, it is clear that the collapse initiating star formation occurs only under some circumstances. The Jeans mass depends on the radius of the cloud, its temperature, and the average mass of the particles in the cloud. The Jeans mass is determined when the magnitude of the gravitational potential energy exceeds the magnitude of the gas kinetic energy. It is given by $M_J = (\frac{5KT_c}{G\mu m_H})^{3/2}(\frac{3}{4\pi\rho_c})^{1/2}$ where k is the Boltzmann constant, T_c the temperature, ρ_c the density of the cloud, R_c the radius of the cloud, G the gravitational constant, and $\mu m_H = m$ the average mass of a gas particle. If the cloud's mass exceeds this value, gravity can overcome the gas kinetic energy and initiate collapse [17].

2.5 The released energy during collapse

The gravitational energy released when a cloud of radius R_c collapsed to the surface of a pre-stellar of radius R_{core} [17] is given by

$$\Delta E_g = \frac{GM_c^2}{R_{core}} - \frac{GM_c^2}{R_c} \tag{2.35}$$

$$\Delta E_g = \frac{GM_c^2}{R_{core}} \tag{2.36}$$

Here we assumed $\frac{1}{R_c} << 1$. This implies the energy released is mainly dependent on the core radius. Half of the energy described in Equation 2.36 is according to the Virial theorem, thus we have

$$E_{grad} = \frac{1}{2}\Delta E_g = \frac{GM_c^2}{2R_{core}} \tag{2.37}$$

E_{grad} is read as the gravitational energy radiated away.

2.6 Gravitational energy released per free-fall time

The luminosity due to gravity in-terms of the free fall time [17] as

$$L_{gf} = \frac{E_{grad}}{t_{ff}} \tag{2.38}$$

Thus using equation 3.1, and 2.36, in equation 2.38 we have

$$\begin{aligned} L_{gf} = \frac{GM_c^2}{2R_{core}}(\frac{32G\rho_c}{3\pi})^{1/2} &= \frac{G^{3/2}M_c^2}{2R_{core}}(\frac{32\rho_c}{3\pi})^{1/2} \\ &= 1.8426G^{3/2}M_c^2(\frac{3M}{4\pi R^3})^{1/2} \\ &= 0.9003\frac{G^{3/2}M_c^2}{2R_{core}}(\frac{M_c}{R_c^3})^{1/2} \end{aligned} \tag{2.39}$$

After substituting the value of G and L_{gf} is given by

$$L_{gf} \approx 3.0721x10^{-17}(\frac{M_c^5}{R_c^3 R_{core}^2})^{1/2} J/s \tag{2.40}$$

where M_c and R_c are mass and radius of the cloud respectively, L_{gf} gravitational limunosity, $G \approx 6.67x10^{-11}m^3.Kg^{-1}.s^{-2}$ and $\rho_c = \frac{3M}{4\pi R^3}$ denotes the initial density of cloud.

2.7 Stellar formation

The first step in the process of creating stars is the formation of dense cores within a clump of gas and dust. It is generally thought that all the material for the star comes from the core, the larger structure surrounding the forming star. Eventually, the gravitational force of the infalling gas becomes strong enough to overwhelm the pressure exerted by the cold material that forms the dense cores. The material then undergoes a rapid collapse, and the density of the core increases greatly as a result. During the time a dense core is contracting to become a true star, but before the fusion of protons to produce helium begins, we call the object a protostar [10].

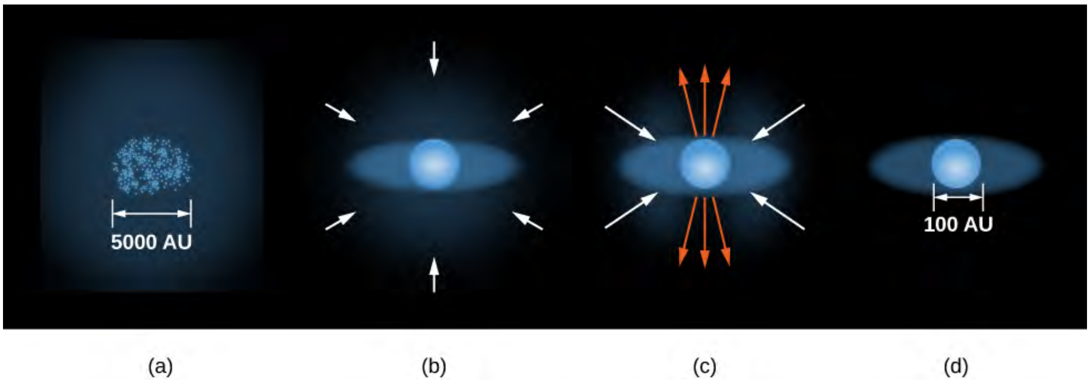

Figure 2.2: Formation of a Star: (a) Dense cores form within a molecular cloud.
(b) A protostar with a surrounding disk of material forms at the center of a dense core, accumulating additional material from the molecular cloud through gravitational attraction.
(c) A stellar wind breaks out but is confined by the disk to flow out along the two poles of the star. (d) Eventually, this wind sweeps away the cloud material and halts the accumulation of additional material, and a newly formed star, surrounded by a disk, becomes observable.

The natural turbulence inside a clump tends to give any portion of it some initial spinning motion (even if it is very slow). As a result, each collapsing core is expected to spin. According to the law of conservation of angular momentum, a rotating body spins more rapidly as it decreases in size. In other words, if the object can turn its material around a smaller circle, it can move that material more quickly. When a core contracts to form a protostar: as it shrinks, its rate of spin increases. But all directions on a spinning sphere are not created equal. As the protostar rotates, it is much easier for material to fall right onto the poles (which spin most slowly) than onto the equator (where material moves around most rapidly). The protostar and disk at this stage are embedded in an envelope of dust and gas from which material is still falling onto the protostar. This dusty envelope blocks visible light, but infrared radiation can get through. As a result, in this phase of its evolution, the protostar itself is emitting infrared radiation and so is observable only in the infrared region of the spectrum. Once almost all of the available material has been accreted and the central protostar has reached nearly its final mass, it is given a special name: it is called a T Tauri star, named after one of the best studied and brightest members of this class of stars, which was discovered in the constellation of Taurus. Only stars with masses less than or similar to the mass of the Sun become T Tauri stars.

Stars form regularly almost everywhere in our Milky Way galaxy. A sure sign of star formation is the presence of luminous hot stars. Theories of star formation are concerned with cloud contraction, cloud fragmentation, formation of protostars, manifestations of protostars with accretion disks up to the onset of nuclear fusion and the emergence of stars as MS stars.

Chapter 3

Stellar Evolution and Collapse

3.1 Stellar evolution

The evolution of stars is best understood by looking at the Hertzsprung–Russell diagram, as well as the evolution of the core of the star is depends upon the mass of the star. Thus, the movement of a star in this diagram, as it evolves, will tell us how its radius, temperature and luminosity vary in response to the behaviour of the core of the star [30].

Depending on their mass, stars evolution implies; for White dwarfs, as the mass increases their radius decreases. Because there is no nuclear fusion occurring in it to produce any outward force. So a white dwarf is stable only because of electron degeneracy pressure which is exerted by electrons against gravity preventing further collapse. But electron degeneracy pressure will overcome gravity only if its core mass is less than $1.4 M_{\odot}$ and original mass is $80 M_J$to $8 M_{\odot}$. But if the white dwarf accretes any mass from nearby material or star, and if its mass exceeds 'Chandrasekhar Limit', then electrons won't be able to push against gravity and then collapse forming a Type I Supernova. A mass of the core is between $1.4 M_{\odot} - 3 M_{\odot}$ and origin mass is $8 M_{\odot}$to $25 M_{\odot}$ and if the supernova remnant core left behind is more than 1.4 M , then the degenerate electron pressure cannot overcome gravity. So, even electrons fuse with protons by a process called 'Electron capture' forming an object which is like one big giant nuclei full of neutrons held together not by the strong force, rather mainly by gravity called as Neutron Star. If the mass of the remnant core of a supernova is more than $3M_{\circledcirc}$ then even the neutron degeneracy pressure cannot resist the inward gravitational pull. So neutron degeneracy pressure breaks after $3M\circledcirc$ also known as the 'Tolman–Oppenheimer–Volkoff' (TOV) limit. After this limit, the neutrons are also compressed by gravity and since there isn't any opposing force, the whole matter content will be compressed to a single point with Zero volume and infinite density and forms a Black hole [27].

Stars are the main astrophysical objects in the universe accessible to observation, and they go through consecutive cycles of evolution from birth to death [19]. Five stages of stellar evolution are described in the below descriptions; (1) Star-Forming Nebula (Gravity rules): In this, a cloud of gas and dust forms many stars, and single star is created when clumps of this material (mostly hydrogen gas) are pulled together by the force of gravity. (2) Birth of the Star (Protostar) gravity rules and fusion begins: As a region of the cloud collapses, gravity pulls the clumps of gas together and the gas in the center becomes hot enough and dense enough to begin fusion and hydrogen atoms inside the clumps smash into each other, combining to create helium and releasing light and heat then star begins to shine, (3) Life of the Star(Main Sequence) Gravity and fusion in balance: The fusion in the core generates an outward force to balance the inward gravitational force from the outer layers, (4) Red Giant (Fusion overtakes gravity): As the core nears the end of its fuel supply, the outer layers continue to push inward, increasing the temperature in the core. This produces a new series of fusion reactions that produce enough outward force to overpower the inward gravitational force and expand the star, (5) Death of low-mass star (Planetary Nebula with White Dwarf) fusion ends and gravity wins: As the core runs out of fuel for fusion, it emits one last push outward, ejecting the star's outer layers, which drift away into space. The core then contracts under its own gravity, forming a white dwarf, and Death of a high-mass Star (Supernova, with Neutron Star or Black Hole) fusion ends and gravity wins: The massive core continues to fuse elements and expands the star so it is even larger. Once the core runs out of fuel, it collapses to form a neutron star. The outer layers then collapse as well. As material falls toward the star's center, it bounces off the core and explodes outward through the star.

This explosion is called a supernova. In the most massive stars, the collapsed core will become a black hole. It may be instructive to show a poster that illustrates the stages of stellar evolution [24].

The life of stars is governed by three important fundamental forces: gravity, the electromagnetic force, and the strong nuclear force. Of these, gravity determines the course of the evolution of a star, i.e., to contract, while the others are, in a way, just modifying the general workings of contraction [7].

3.2 Stellar collapse

When the massive star runs out of its nuclear fuel, the force of gravity takes over and a catastrophic gravitational collapse of the star takes place. The star that lived for millions of years and which stretched to millions of kilometers in size, now collapses catastrophically within a matter of seconds. According to the general theory of relativity, the outcome of such a continual collapse will be a spacetime singularity where all physical quantities such as densities and spacetime curvatures diverge. The fate of a massive star, when it collapses continually under the force of its own gravity, was highlighted by Chandrasekhar. when he pointed out: For a star of small mass the natural white-dwarf stage is an initial step towards complete extinction and a star of large mass cannot pass into the white-dwarf stage, and one is left speculating on other possibilities. By general relativity predicts, under reasonable physical conditions, the gravitationally collapsing massive star must terminate into a spacetime singularity, where the matter energy densities, spacetime curvatures and other physical quantities blow up.

The star collapses to a radius smaller than the horizon,it enters the black hole, finally collapsing to a spacetime singularity with extreme densities that is hidden inside the black hole and invisible to any external observers. For the collapsing star to create a black hole, an event horizon must develop prior to the time of the final singularity formation [16].

The hydrostatic equilibrium equation is obtained via an approximation of the known

Tolmann-Oppenheimer-Volkoff (TOV) equations, derived directly from Einstein's field equations. Assuming a polytropic equation of state, these equations are reduced to the Lane-Emden differential equations given as

$$\frac{1}{\xi^2}\left(\frac{d}{d\xi}\xi^2\frac{d\theta}{d\xi}\right) = -\theta^n \tag{3.1}$$

Chapter 4

Result and Discussion

4.1 Molecular clouds collapse to form stars

The basic principle that the formation of the star will be realized only if the star forming molecular cloud fragmentation be supported against inward force by gravity and thermal pressure are at equilibrium.

In the strong magnetic field theory, clouds are formed with subcritical masses, $M < M_\phi = \phi/2\pi\sqrt{G}$, where M_ϕ is the critical mass, ϕ is the magnetic flux, and G is the gravitational constant. Hence, the magnetic pressure is sufficiently strong to counteract gravity and prevent gravitational collapse. Because the magnetic field is frozen only into the ionized gas and dust, neutral gas and dust contract gravitationally through the field and the ions, increasing mass in the cloud cores.

In the weak magnetic field theory of star formation says molecular clouds are intermittent phenomena, with short ($\sim 10^6$ years) lifetimes. Thus, magnetic fields are sufficiently weak that the low-density ISM is supercritical ($M > M_\phi$).

Magnetic fields and turbulence are the two leading candidates in which turbulence plays a dual role, both creating over densities to initiate gravitational contraction or collapse, and countering the effects of gravity in these overdense regions, while magnetic fields play a role in the final stage of star formation, both in mediating gas accretion and in launching the bipolar jets due to the conservation of angular momentum that typically announce the birth of a new star.

A simpler method of defining the initial conditions for collapse, which, however, includes all relevant physical effects, is to require that the absolute value of the gravitational energy must exceed the sum of the thermal, turbulent, and magnetic energies. This requirement defines a mass of molecular clouds that is gravitationally bound.

Now to obtain a condition of mechanical equilibrium, assume un magnetized spherical and uniform molecular cloud and a standard notation as the radius R, the volume $V_{cl} = 4\pi R^3/3$ and the mass M, recall the virial equilibrium equation.

To simplify the assumed model, let gravitational mass M is constant and a magnetic flux ϕ is conserved. So using the constants values, eqn 2.18 becomes,

$$p_{net} = 1.06 \times 10^{-12} \frac{M^2}{R^4} \tag{4.1}$$

But, the mass of star forming molecular cloud is estimated between of the order of $10^5 M_\odot$ to $20 \times 10^5 M_\odot$ ($M_\odot \sim 10^{36}$ Kg), while the radius is from $20pc$ to $1pc, R \sim 10^{16}$ meter.

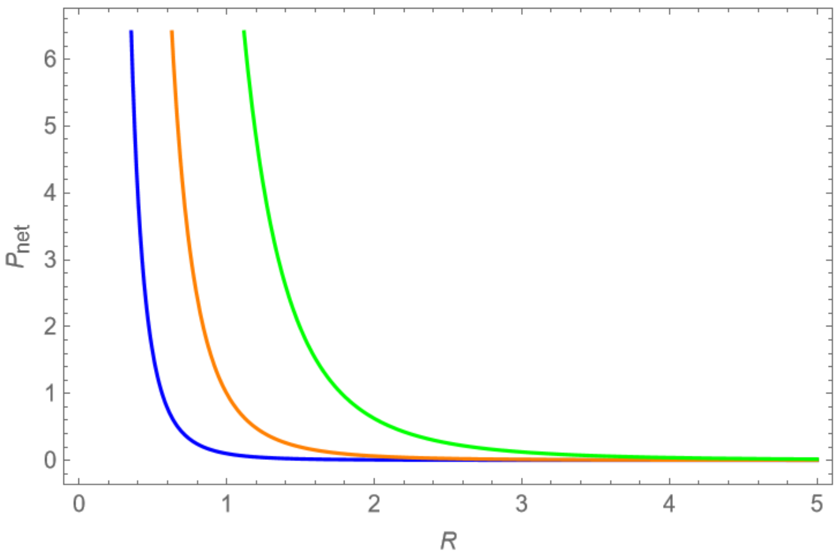

Figure 4.1: The variation of internal pressure vs radial for different masses of molecular cloud

This graph shows that the variation of the mean internal pressure of the molecular cloud with the increase of radius. Also it is observed from the above graph that there is a little difference in the variation of the pressure with radius between different masses of the molecular cloud. Furthermore this implies that the stability of molecular clouds against gravitational collapse is held up by internal turbulent pressure, and it is valid only if there were no perturbations such as shocks passing through the cloud and a mechanism were present to supply turbulence inside the cloud.

However, in the presence of magnetic field(flux is different from zero), the variation of the mean pressure with radius for different masses of molecular clouds has been examined in three different cases:

(i). The case in which $M = M_\phi$ this implies that the effective mass is exactly zero. Where M_ϕ is the mass of the molecular cloud that collapse in the presence of magnetic field, M is the mass of molecular cloud due to gravity and M_{eff} is the net effective mass of molecular cloud. In this case the inward gravitational pressure totaly balanced by the magnetic pressure. In another hand, the internal thermal(turbulence) pressure will be balanced the external thermal pressure. As a result, the molecular cloud becomes at equilibrium and it does not collapse to form stars.

(ii). When $M_\phi^2 < M^2$, Here, the effective mass and the net pressure P_{net} are greater than zero. In magnetically supercritical clouds, since the gravitational mass dominates the magnetic mass term, magnetic fields alone cannot prevent gravitational collapse. So, there must be an additional force to balance against gravity which arises due to the prevailing turbulence in the molecular cloud in such a way, it plays a part to support a molecular cloud that gravitationally collapse to undergo star formation.

(iii). When $M_\phi^2 > M^2$. In this case (magnetically subcritical clouds), P_{net} and M_{eff} become negative. As a result, gravitational collapse is not possible. This indicate that the external pressure exceed the internal pressure which is impossible since P_{net} is the difference between $\bar{p}$ and p_0 and consequently there is no formation of the star. So,it enables the molecular cloud to undergo a contraction. Here, since the magnetic flux is conserved, the ratio of gravitational mass to the magnetic mass term does not change as the cloud collapse. As a result, there is no more contraction of molecular clouds rather than expansion. Consequently, the pressure decreases as the radius increases.

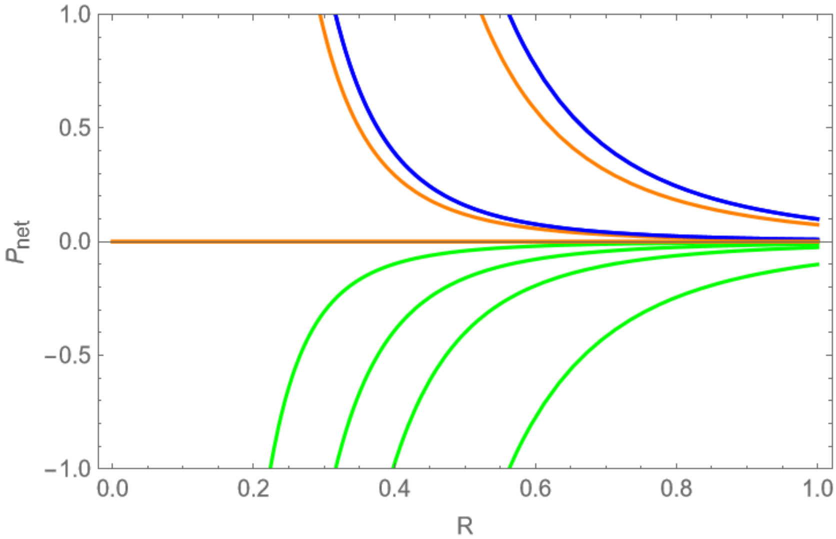

Figure 4.2: The variation of internal pressure vs radius for different mass of molecular cloud in the case when $M\phi < M$ and $M\phi > M$

4.2 Star lifetime

One of the most useful tools enabling astronomers to make predictions of stellar evolution is the Hertzsprung-Russell diagram [11].

Protostars whose mass is more than 0.08 M will become a star which is the main sequence and most of the stars whose core is still fusing Hydrogen (H) into Helium (He). Stars

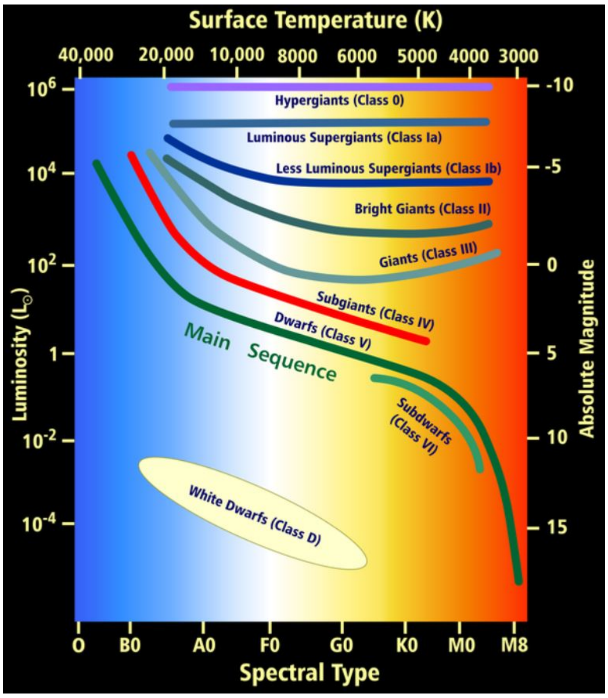

Figure 4.3: Stellar evolution of main sequence on H-R diagram

spend their 90% of life in this stage for billions of years and are governed mainly by the equation of hydrostatic equilibrium where the inward gravitational force is balanced by the outward radiation force.

In the above H-R diagram the stars are represented by the green line in the above HR diagram. In the main sequence, the more luminous stars are more massive, and emitted peak wavelength is in the blue region. The stars whose luminosity is average have average mass and the peak wavelength is in the yellow region. And the fainter stars have very little mass and emitted peak wavelength is in the red region. Astronomers have discovered that although the stars convert hydrogen into helium on the main sequence, their lifetime is dependent on their mass. Also the theory predicts that the luminosity of a star is proportional to the cube of the mass $L \propto M^3$ with agreement of observations data $L \propto$

$M^{3.5}$ and this slope is very nearly what Eddington's theory predicts. This relationship can be considering that the luminosity (L) of a star is tied to its mass (M) by $L = M^{3.5}$. Luminosity is usually given in units of the Sun's luminosity L_o or a luminosity $= 1$, or in the SI unit, the Watt. Mass is also given in solar units where the solar Mass $M_o = 1$. Main sequence lifetimes can be determined with the expression of $T_{ms} = 10^{10}$ years $(\frac{M}{M_0})^{-2.5}$, where T_{ms} is the main sequence lifetime, 10^{10} years is the lifetime of the Sun, M_o is the Sun's mass and M is mass of the star. Clearly, smaller mass stars will live longer while higher mass but their short lifetimes may lead to disappearing as supernovae. Such explosions will provide a shockwave that will compress the surrounding molecular cloud even more and lead to the production of new clusters of stars.

The time τ_H a star spends in the hydrogen burning phase depends on its mass M. This is because the luminosity L of a star (or the total energy radiated per unit time) depends on the mass of the star rather strongly. When mass-luminosity relation $L \propto M^{3.5}$, and $M^{3.5}$ gives a reasonable fit to the observational data on luminosity over the entire mass range. Let E_H be the energy that can be released by fusion of hydrogen. The lifetime of the star in this phase can be written as

$$\tau_H = \frac{E_H}{L} \tag{4.2}$$

where $E_H \propto Mc^2$ and $L \propto M^{3.5}$ then $\tau_H \sim \dfrac{M}{L} \sim M^{-2.5}$ [30].

There are two types of supernova; type I and type II. 97% of all stars in the universe will collapse to form white dwarf stars at the end of their lives and most of the other 3% form type II supernova explosions. A star's helium will typically fuse to form carbon at the end of its life, and then collapse into a white dwarf [21]. After the helium in its core is exhausted, the evolution of a massive star takes a significantly different course from that of lower-mass stars. In a massive star, the weight of the outer layers is sufficient to force the carbon core to contract until it becomes hot enough to fuse carbon into oxygen, neon, and magnesium. This cycle of contraction, heating, and the ignition of another nuclear fuel repeats several more times. After each of the possible nuclear fuels is exhausted, the core contracts again until it reaches a new temperature high enough to fuse still heavier nuclei. The products of carbon fusion can be further converted into silicon, sulfur, calcium, and argon. And these elements, when heated to a still higher temperature, can combine to produce iron.

4.3 Parameters determine the collapse of stars

The fate of a massive star, when it collapses continually under the force of its own gravity, was highlighted by Chandrasekhar. when he pointed out: For a star of small mass the natural white-dwarf stage is an initial step towards complete extinction and a star of large mass (cannot pass into the white-dwarf stage, and one is left speculating on other possibilities.
For stellar masses less than about 1.44 solar masses, the energy from the gravitational collapse is not sufficient to produce the neutrons (protons +electrons) of a neutron star; so the collapse is halted by electron degeneracy to form white dwarfs. This maximum mass for a white dwarf is called the Chandrasekhar limit. Above 1.44 solar masses, the star contracts further forming a neutron star and for masses greater than 2.3 solar masses, even neutron degeneracy can't prevent further collapse and it continues toward the black hole state. In the compact stars collapse of the mass-radius relation is pointed out by chandrasekhar limit and Tolman–Oppenheimer–Volkoff limit. In this case mass-radius relationship of white dwarf is derived by chandrasekhar in $R \propto M^{-1/3}$. i.e., given with the approximation of the equation of hydrostatic equilibrium is given by

$$\frac{P}{R} \propto \frac{M\rho}{R^2} \tag{4.3}$$

where R is the radius of the star and the pressure P at the surface is zero:

$$\frac{dP}{dr} = \frac{(P(r) - P_{surface})}{R} = \frac{P}{R}$$

and using equation of state $P \propto \rho^{5/3}$ and $\rho \propto M/R^3$, equation 4.3 becomes

$$\frac{1}{R}(\frac{M}{R^3})^{5/3} \propto \frac{M}{R^2}(\frac{M}{R^3})$$
$$\frac{M^{5/3}}{R^6} \propto \frac{M^2}{R^5}$$

$$\Rightarrow R = M^{-1/3} \tag{4.4}$$

Thus, the radius is inversely proportional to the cube root of the mass of the white dwarf and a white dwarf with a mass equal to the mass of the Sun will have a radius roughly equal to the radius of the Earth.

The analytic general solution of the mass-radius relationship of the compacts is given as in the plot of figure 4.4. The particular polytropic indices used run from 0 through 5 with increasing level by 0.5 (as can be inferred from the legends or from left to right). As we observe from the plot the larger the radius of the star corresponding to the smaller mass and vice-versa. Accordingly, white white dwarfs are expected to have the lower mass while that of the neutron star is with higher mass.

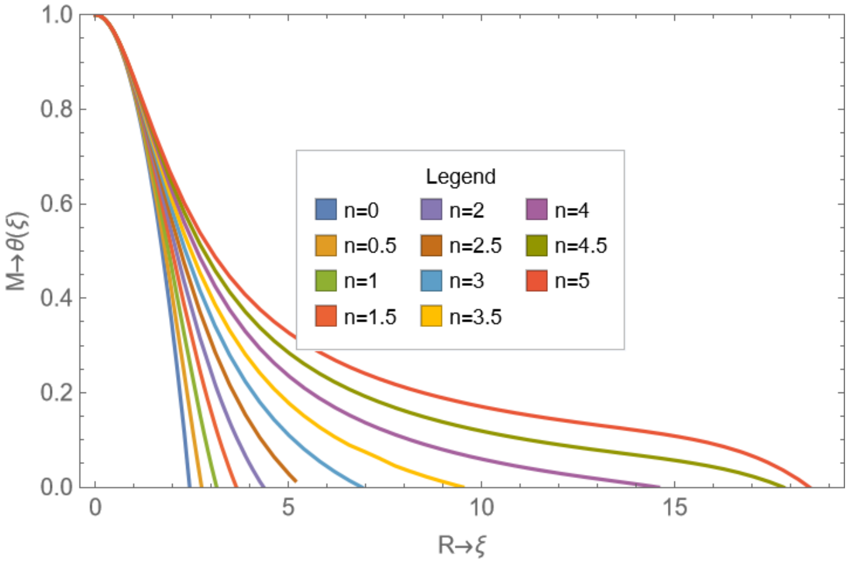

Figure 4.4: An analytical solution of the Mass-Radius relationship of collapsed stars generated from Lane-Emden equation 3.1

4.4 Kinds of star burning will occur at the end product

Depending on their mass, compact object stars are white dwarfs, neutron star and black hole. So a white dwarf is stable only because of electron degeneracy pressure which is exerted by electrons against gravity preventing further collapse. But electron degeneracy pressure will overcome gravity only if its core mass is less than $1.4M_{\odot}$. But if the white dwarf accretes any mass from nearby material or star, and if its mass exceeds 'Chandrasekhar Limit', then electrons won't be able to push against gravity and then collapse forming a Type I Supernova.

A mass of the core is between $1.4M_{\odot} - 3M_{\odot}$ and if the supernova remnant core left behind is more than 1.4 M , then the degenerate electron pressure cannot overcome gravity. So, even electrons fuse with protons by a process called 'Electron capture' forming an object which is like one big giant nuclei full of neutrons held together not by the strong force, rather mainly by gravity called as neutron star.

If the mass of the remnant core of a supernova is more than $3M_{\odot}$ then even the neutron degeneracy pressure cannot resist the inward gravitational pull. So neutron degeneracy pressure breaks after $3M\odot$ also known as the 'Tolman–Oppenheimer–Volkoff' (TOV) limit. After this limit, the neutrons are also compressed by gravity and since there isn't any opposing force, the whole matter content will be compressed to a single point with Zero volume and infinite density and forms a Black hole. Therefor from stellar mass increasing the supernovae explosion undergo in end product of black hole star.

Chapter 5

Conclusion

One of the most important aspects of General Relativity is the formation of spacetime singularities and horizons as the end results of a gravitational collapse of a massive object. In star forming by gravity rules a cloud of gas and dust forms many stars, and single star is created when clumps of this material (mostly hydrogen gas) are pulled together by the force of gravity.

In birth of the Star (Protostar) by gravity rules and fusion gravity pulls the clumps of gas together and hydrogen atoms inside the clumps smash into each other to create helium and releasing light and heat then star begins to shine. In the life of the Star(Main Sequence) by Gravity and fusion balance; the fusion in the core generates an outward force to balance the inward gravitational force from the outer layers.

In the red giant star by fusion overtakes gravity rule, the core nears the end of its fuel supply, the outer layers continue to push inward increasing the temperature in the core and produces a new series of fusion reactions that produce enough outward force to overpower the inward gravitational force and expand the star.

In the death of low-mass star (Planetary Nebula with White Dwarf) by fusion ends and gravity wins ; as the core runs out of fuel for fusion, it emits one last push outward, ejecting the star's outer layers, which drift away into space the core then contracts under its own gravity, forming a white dwarf. In the death of a high-mass Star (Supernova, with Neutron Star or Black Hole) by fusion ends and gravity wins; the massive core continues to fuse elements and expands the star and the core runs out of fuel and collapses to form a neutron star.

As the outer layers then collapse as well, a material falls toward the star's center, it bounces off the core and explodes outward through the star and the explosion is called a supernova. In the most massive stars, the collapsed core will become a black hole. Therefore the star formation to death illustrates the stages of stellar evolution.

When a molecular cloud reaches a certain critical density, it collapses under its own weight leading to the dissipation of gravitational potential energy. At a particular density and radius of the cloud, the inward Gravitational force and outward force due to the internal pressure will be balanced and the cloud is in a state of equilibrium said to be hydrostatic equilibrium.

Gas and dust in space, the most massive reservoirs of interstellar matter and some of the most massive objects in the Milky Way Galaxy are the giant molecular clouds. In the Main sequence lifetimes, smaller mass stars will live longer while higher mass but their short lifetimes may lead to disappearing as supernovae.

In the final work we are going to discuss on the roles (effects) of gravity, turbulence and magnetic field in star formation. We started from the analysis of the Krumholz and McKee [2005], model that comprises all the three components. The basic principle that the formation of the star will be realized only if the star forming molecular cloud fragmentation be supported against inward force by gravity and external pressure are at equilibrium, of course these clouds have to become stars. The internal pressure in the star forming molecular cloud is in a state of variable mean pressure due to the changing density resulted from gravity. However, due to the thermal instability, there is a net outward pressure given by $(\bar{p} - p_0)$. For simplified model we consider that the star forming molecular cloud mass is constant,i.e., we neglect any accretion from the surrounding background. The contribution from magnetic field is also to be analyzed as incorporated in equation 2.22.The approach that we used for analysis is a case by case examination of the affecting parameters in the formation of protostar. Accordingly, in the first case we look at the process in the absence of magnetic field implementing the boundary condition so far we have discussed.For the numerical analysis we consider the observational data in which the mass of star forming molecular cloud is in between 10^5 to 10^6 solar mass, while the radius is in between 1 to 20 parsec.

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