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Research Article

GPS Receiver Position Estimation and DOP Analysis Using a New Form of the Observation Matrix Approximations

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A location sensor is a feature that communicates with a Global Positioning System (GPS) receiver to learn about the status of the current location. This work presents the GPS receiver position estimation and Dilution of Precision (DOP) analysis using a new approximate form of observation matrix which can be used in place of the classic observation matrix that was derived from the Taylor's series. It has been realized that, the approximate observation matrix is numerically stable and provides greater precision in calculating DOP values and estimating the position of a GPS receiver. The experimental results show that the proposed observation matrix provides better precision in DOP analysis and GPS receiver position estimation with a fast convergence rate and improved algorithm stability. Therefore, it can be concluded that the proposed new observation matrix plays a significant role to estimate accurately the location of the GPS receiver position and to enhance all parameters of the DOP.

1. Introduction

The GPS sensors estimation process mathematically depends on the observation matrix which is formed by using the pseudo-range equations at a particular epoch. The observation matrix in use was derived from the first order Taylor's series. Thus, the classical observation matrix has a constant (unity) as the fourth parameter which affects the iteration process of position estimation and DOP computation in terms of precision. Therefore, a direct difference method is used in this paper to improve the order of the observation matrix. A direct difference method was used to the Extended Kalman Filter (EKF) to modify its gain [1, 2]. Thus, EKF was modified and the developed new method was named as modified gain EKF (MGEKF). This method was used to obtain an approximate gain matrix "g" to replace the measurement matrix (h(X)) in the EKF covariance during the measurement update stage. The only distinction between EKF [3] and MGEKF is the covariance matrix in the measurement update stage. The new gain matrix "g" has proved to be effective in SONAR tracking applications [4, 5]. In this article, an attempt is made to obtain an approximate form of observation matrix ($\tilde{H}$) for GPS applications by using the *direct difference method*. Although classical observation matrix (H) is the standard one, how can the *direct difference method* helps to obtain the approximate form of observation matrix $\tilde{H}$ is presented in this article.

The performance of the approximation form of observation matrix is evaluated on analysis of dilution of precision (DOP) [6] and estimation of a GPS receiver position. Among DOP parameters, Geometric Dilution of Precision (GDOP) exhibits the geometric effect on the association of positioning determination error and measurement error. Errors in determining the GPS receiver positions are usually GDOP multiplied by measurement error; in other words, GDOP is the error gain, which means that a smaller GDOP usually results in a larger exact position. So, a smaller GDOP is better. It has been shown that more number of satellites make the GDOP value lesser for accurate position estimation, i.e., more satellites result in a lower GDOP, and fewer satellites generally result in poor GDOP [7] and lead to a corresponding reduction in the positioning error.

Many methods were developed to improve the position accuracy of the GPS based on GDOP [8–11]. One of the common methods is optimal satellite selection [12]. The easiest way is to choose a combination of all satellites, all of which must be taken into account to obtain optimum position accuracy with minimum GDP [13]. The other DOP parameters are Horizontal Dilution of Precision (HDOP), Position Dilution of Precision (PDOP), Time Dilution of Precision (TDOP), and Vertical Dilution of Precision (VDOP). The goal of any GPS algorithms for satellite selection is to reduce the GDOP to enhance positioning accuracy. A minimum GDOP can provide higher positioning accuracy by preventing the effects of poor geometry.

The GDOP computation using H is given by equation (1).

$$GDOP = \sqrt{trace\left(H^T H\right)^{-1}}. \tag{1}$$

Various research works has been presented in the literature which are dealing with the precise computation of the GDOP attempting to enhance the accuracy of the positioning [8, 10, 11]. It has been analyzed that selecting four satellites optimally from more satellites in the immediate vicinity is tedious, time-consuming, and power-consuming. Thus, a greater number of satellites in the immediate vicinity can decrease the GDOP. In case the number of visible satellites is small, then the above said choice is good for providing highly accurate positioning [11].

The GDOP computation formula includes receiver clock bias along with north, east, and up components. In [12–15], the direct difference method is used only for position estimation, but the *receiver clock bias* is not included either in the modified gain matrix 'g' or in the observation matrix, without which it is impossible to obtain the GDOP and the accuracy enhancement in the GPS receiver position estimate. But the proposed $\tilde{H}$ in this article includes the *receiver clock bias*. The *receiver clock bias* parameter is always equal to one for the H, while for the proposed $\tilde{H}$ it is a function of the previous estimate and the current measurement.

To demonstrate the efficiency of the $\tilde{H}$ over H, it is applied to compute GDOP and to estimate GPS receiver position [16–18]. Comparative results show a notable improvement due to the proposed observation matrix. The major contributions of this article are summarized as follows:

(i) An approximate form of observation matrix ($\tilde{H}$) is derived using the *direct difference method*

(ii) Experiments are being conducted with real-time GPS data to show significant performance improvements with the proposed $\tilde{H}$ by analyzing GDOP and GPS receiver position estimates

(iii) Position estimation analysis using EKF and MGEKF with the inclusion of the *receiver clock bias* parameter is also presented

The remaining part of this manuscript are ordered as follows. The derivation of the proposed observation matrix has been discussed in the second section [19–23]. The third section presents experimental results and applications [24–26]. Lastly the conclusion is given under section four.

2. Derivation for Proposed Observation Matrix

Let us consider the true GPS pseudo-range equation (13), for simplicity, by neglecting all other correctable error sources except *receiver clock bias* which is given by,

$$P_{sr}^{i} = \sqrt{\left(x_{sat}^{i} - x_r\right)^2 + \left(y_{sat}^{i} - y_r\right)^2 + \left(z_{sat}^{i} - z_r\right)^2} + Clk_r. \tag{2}$$

In the above equation, (x_r, y_r, z_r) and Clk_r are considered as true 3D position coordinates and the *receiver clock bias*, respectively, and $(x_{sat}^i, y_{sat}^i, z_{sat}^i)$ are $i-th$ satellite coordinates. Now, consider the estimated pseudo-range equation as,

$$\widehat{P}_{sr}^{i} = \sqrt{\left(x_{sat}^{i} - \widehat{x}_r\right)^2 + \left(y_{sat}^{i} - \widehat{y}_r\right)^2 + \left(z_{sat}^{i} - \widehat{z}_r\right)^2} + \widehat{Clk}_r, \tag{3}$$

where $(\widehat{x}_r, \widehat{y}_r, \widehat{z}_r)$ and $\widehat{Clk}_r$ are considered as estimated 3D position coordinates with receiver clock bias. To determine the proposed observation matrix ($\tilde{H}$) for GPS positioning application, it is given by the following format [13].

$$\left[P_{sr}^{i} - \widehat{P}_{sr}^{i}\right] = \tilde{H} \begin{bmatrix} x_r - \widehat{x}_r \\ y_r - \widehat{y}_r \\ z_r - \widehat{z}_r \\ Clk_r - \widehat{Clk}_r \end{bmatrix}. \tag{4}$$

Now, the proposed observation matrix ($\tilde{H}$) is obtained by simplifying the difference between equations (2) and (3), i.e., between true and estimated pseudo-ranges (using the *direct difference method* in [1]). That is,

$$\left[P^i_{sr} - \widehat{P}^i_{sr}\right] = \eta. \tag{5}$$

This approach is done differently, it starts by considering the difference between squares of true and estimated pseudo-range equations (2) and (3), respectively.

$$\begin{aligned}\left(P^i_{sr}\right)^2 - \left(\widehat{P}^i_{sr}\right)^2 & \\ &\left(\sqrt{\left(x^i_{sat} - x_r\right)^2 + \left(y^i_{sat} - y_r\right)^2 + \left(z^i_{sat} - z_r\right)^2} + Clk_r\right)^2 \\ &- \left(\sqrt{\left(x^i_{sat} - \widehat{x}_r\right)^2 + \left(y^i_{sat} - \widehat{y}_r\right)^2 + \left(z^i_{sat} - \widehat{z}_r\right)^2} + C\widehat{lk}_r\right)^2.\end{aligned} \tag{6}$$

This can be written as,

By considering the right-hand side of the equations (6),

$$\begin{aligned}\Delta\delta = &\left(\sqrt{\left(x^i_{sat} - x_r\right)^2 + \left(y^i_{sat} - y_r\right)^2 + \left(z^i_{sat} - z_r\right)^2} + Clk_r\right)^2 \\ &\left(-\sqrt{\left(x^i_{sat} - \widehat{x}_r\right)^2 + \left(y^i_{sat} - \widehat{y}_r\right)^2 + \left(z^i_{sat} - \widehat{z}_r\right)^2} + C\widehat{lk}_r\right)^2.\end{aligned} \tag{7}$$

Now, equation (6) can be written as,

$$\left(P^i_{sr}\right)^2 - \left(\widehat{P}^i_{sr}\right)^2 = \left[P^i_{sr} - \widehat{P}^i_{sr}\right]\left[P^i_{sr} + \widehat{P}^i_{sr}\right] = \Delta\delta, \tag{8}$$

$$\left[P^i_{sr} - \widehat{P}^i_{sr}\right] = \frac{\Delta\delta}{\left[P^i_{sr} + \widehat{P}^i_{sr}\right]}. \tag{9}$$

By solving $\Delta\delta$, the approximate form of equation (9) becomes,

$$\left[P^i_{sr} - \widehat{P}^i_{sr}\right] \cong \left[\frac{x_r + \widehat{x}_r - 2x^i_{sat}}{P^i_{sr} + \widehat{P}^i_{sr}} \;\; \frac{y_r + \widehat{y}_r - 2y^i_{sat}}{P^i_{sr} + \widehat{P}^i_{sr}} \;\; \frac{z_r + \widehat{z}_r - 2z^i_{sat}}{P^i_{sr} + \widehat{P}^i_{sr}} \;\; \frac{2\left(Clk_r C\widehat{lk}_r\right)\left(P^i_{sr} - \widehat{P}^i_{sr}\right)}{\left(Clk_r + C\widehat{lk}_r\right)\left(P^i_{sr} + \widehat{P}^i_{sr}\right)}\right]\begin{bmatrix} x_r - \widehat{x}_r \\ y_r - \widehat{y}_r \\ z_r - \widehat{z}_r \\ Clk_r - C\widehat{lk}_r \end{bmatrix}. \tag{10}$$

$$\left[P^i_{sr} - \widehat{P}^i_{sr}\right] \cong \tilde{H}\left[X - \widehat{X}\right]. \tag{11}$$

Therefore, equation (11) can be written as,

$$\Delta Z = \tilde{H}.\Delta X, \tag{12}$$

where $[P^i_{sr} - \widehat{P}^i_{sr}] = \Delta Z$ is the differential pseudo-range measurement vector, $[X - \widehat{X}] = \Delta X$ is the differential state vector, and $\tilde{H}$ is the proposed observation matrix.

The equation (12) is the same as the standard differential measurement equation $\Delta Z = H.\Delta X$ as given in [13], where ΔZ is the differential pseudo-range measurement vector, ΔX denotes the differential state vector, and H is the classical observation matrix. Comparing both expressions $\tilde{H}$ is similar to H of the standard differential measurement equation. Therefore, when comparing equations (10) and (12), the $\tilde{H}$ is given by:

$$\tilde{H} \cong \left[\frac{x_r + \widehat{x}_r - 2x^i_{sat}}{P^i_{sr} + \widehat{P}^i_{sr}} \;\; \frac{y_r + \widehat{y}_r - 2y^i_{sat}}{P^i_{sr} + \widehat{P}^i_{sr}} \;\; \frac{z_r + \widehat{z}_r - 2z^i_{sat}}{P^i_{sr} + \widehat{P}^i_{sr}} \;\; \frac{2\left(Clk_r C\widehat{lk}_r\right)\left(P^i_{sr} - \widehat{P}^i_{sr}\right)}{\left(Clk_r + C\widehat{lk}_r\right)\left(P^i_{sr} + \widehat{P}^i_{sr}\right)}\right]. \tag{13}$$

For "i" the number of visible satellites, equation (13) can be written as,

$$\tilde{H} \cong \begin{bmatrix} \frac{x_r + \hat{x}_r - 2x_{sat}^1}{P_{sr}^1 + \hat{P}_{sr}^1} & \frac{y_r + \hat{y}_r - 2y_{sat}^1}{P_{sr}^1 + \hat{P}_{sr}^1} & \frac{z_r + \hat{z}_r - 2z_{sat}^1}{P_{sr}^1 + \hat{P}_{sr}^1} & \frac{2\left(Clk_r Cl\hat{k}_r\right)\left(P_{sr}^1 - \hat{P}_{sr}^1\right)}{\left(Clk_r + Cl\hat{k}_r\right)\left(P_{sr}^1 + \hat{P}_{sr}^1\right)} \\ \frac{x_r + \hat{x}_r - 2x_{sat}^2}{P_{sr}^2 + \hat{P}_{sr}^2} & \frac{y_r + \hat{y}_r - 2y_{sat}^2}{P_{sr}^2 + \hat{P}_{sr}^2} & \frac{z_r + \hat{z}_r - 2z_{sat}^2}{P_{sr}^2 + \hat{P}_{sr}^2} & \frac{2\left(Clk_r Cl\hat{k}_r\right)\left(P_{sr}^2 - \hat{P}_{sr}^2\right)}{\left(Clk_r + Cl\hat{k}_r\right)\left(P_{sr}^2 + \hat{P}_{sr}^2\right)} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{x_r + \hat{x}_r - 2x_{sat}^i}{P_{sr}^i + \hat{P}_{sr}^i} & \frac{y_r + \hat{y}_r - 2y_{sat}^i}{P_{sr}^i + \hat{P}_{sr}^i} & \frac{z_r + \hat{z}_r - 2z_{sat}^i}{P_{sr}^i + \hat{P}_{sr}^i} & \frac{2\left(Clk_r Cl\hat{k}_r\right)\left(P_{sr}^i - \hat{P}_{sr}^i\right)}{\left(Clk_r + Cl\hat{k}_r\right)\left(P_{sr}^i + \hat{P}_{sr}^i\right)} \end{bmatrix}. \quad (14)$$

In the above equation, (x_r, y_r, z_r) and Clk_r are considered as true 3D position coordinates and the receiver clock bias, respectively, which are practically unavailable. Hence, these are replaced with the estimated coordinates of the previous iteration in the Least Squares or any other optimization approach [1, 2, 4]. Then, the classical observation matrix H [13] is written as follows.

$$H = \begin{bmatrix} \frac{x_r - x_{sat}^1}{P_{sr}^1} & \frac{y_r - y_{sat}^1}{P_{sr}^1} & \frac{z_r - z_{sat}^1}{P_{sr}^1} & 1 \\ \frac{x_r - x_{sat}^2}{P_{sr}^2} & \frac{y_r - y_{sat}^2}{P_{sr}^2} & \frac{z_r - z_{sat}^2}{P_{sr}^2} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{x_r - x_{sat}^i}{P_{sr}^i} & \frac{y_r - y_{sat}^i}{P_{sr}^i} & \frac{z_r - z_{sat}^i}{P_{sr}^i} & 1 \end{bmatrix}. \quad (15)$$

Thus, the equation (14) shows that all parameters in $\tilde{H}$ are a function of true and estimated values. If true and estimated values of all parameters are equal in equation (14), then the first three parameters of all rows $\tilde{H}$ will be the same as the parameters H in equation (15). But the fourth parameter will be zero, and hence all values of the fourth column are zero, which not only contradicts H but also makes $\tilde{H}$ a singular or degenerate matrix. Thus, $\tilde{H}$ becomes a noninvertible matrix. Therefore, to eliminate $\tilde{H}$ singularity, only the receiver clock bias terms are retained, and the $(P_{sr}^i - \hat{P}_{sr}^i)/(P_{sr}^i + \hat{P}_{sr}^i)$ term is excluded because its value is practically very low and affects the entire term to become zero. Thus, if the true and estimated values are equal, then the fourth term becomes Clk_r which is the *receiver clock bias*, and therefore $\tilde{H}$ is not a singular matrix and can be the invertible matrix. Therefore, the final expression of the proposed approximate form of observation matrix is given by,

$$\tilde{H} \cong \begin{bmatrix} \frac{x_r + \hat{x}_r - 2x_{sat}^1}{P_{sr}^1 + \hat{P}_{sr}^1} & \frac{y_r + \hat{y}_r - 2y_{sat}^1}{P_{sr}^1 + \hat{P}_{sr}^1} & \frac{z_r + \hat{z}_r - 2z_{sat}^1}{P_{sr}^1 + \hat{P}_{sr}^1} & \frac{2\left(Clk_r Cl\hat{k}_r\right)}{\left(Clk_r + Cl\hat{k}_r\right)} \\ \frac{x_r + \hat{x}_r - 2x_{sat}^2}{P_{sr}^2 + \hat{P}_{sr}^2} & \frac{y_r + \hat{y}_r - 2y_{sat}^2}{P_{sr}^2 + \hat{P}_{sr}^2} & \frac{z_r + \hat{z}_r - 2z_{sat}^2}{P_{sr}^2 + \hat{P}_{sr}^2} & \frac{2\left(Clk_r Cl\hat{k}_r\right)}{\left(Clk_r + Cl\hat{k}_r\right)} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{x_r + \hat{x}_r - 2x_{sat}^i}{P_{sr}^i + \hat{P}_{sr}^i} & \frac{y_r + \hat{y}_r - 2y_{sat}^i}{P_{sr}^i + \hat{P}_{sr}^i} & \frac{z_r + \hat{z}_r - 2z_{sat}^i}{P_{sr}^i + \hat{P}_{sr}^i} & \frac{2\left(Clk_r Cl\hat{k}_r\right)}{\left(Clk_r + Cl\hat{k}_r\right)} \end{bmatrix}. \quad (16)$$

Table 1: Comparison of DOP values.

Type of DOP		Time of observation	Due to classical observation matrix H	Due to the proposed observation matrix $\tilde{H}$
GDOP	Maximum	05 : 39 Hrs.	2.281	1.882
	Minimum	15 : 42 Hrs.	1.244	1.146
PDOP	Maximum	05 : 39 Hrs.	1.976	1.882
	Minimum	15 : 42 Hrs.	1.146	1.146
HDOP	Maximum	05 : 39 Hrs.	1.812	1.715
	Minimum	15 : 42 Hrs.	1.054	1.054
VDOP	Maximum	05 : 56 Hrs.	0.797	0.780
	Minimum	00 : 69 Hrs.	0.416	0.416
TDOP	Maximum	05 : 39 Hrs.	1.139	0.032
	Minimum	15 : 42 Hrs.	0.484	0.017

Now, the fourth parameter (*receiver clock bias*) in the last column of the $\tilde{H}$ is a variable unlike in H and its value changes for each iteration which impacts the GDOP calculation for each epoch and also on GPS receiver position estimation. Not only the fourth parameter but also the first three parameters in the $\tilde{H}$ will have an impact on all DOP parameters and the position estimation process. Thus, the computation of all DOP parameters and estimation of GPS receiver/user position by using $\tilde{H}$ leads to increased precision in the results when compared to that of H. The increased precision is due to the higher-order $\tilde{H}$ approximation. This is proved by simulating on real-time GPS data which is presented in the next section. General range values of all DOP parameters are given in [27–29].

3. Experimental Results

The efficiency of the *proposed observation matrix* $\tilde{H}$ is evaluated by computing the GDOP and estimating the position of a GPS receiver/user. The dual-frequency GPS receiver (NovAtel: DL-V3-L1L2) is used to collect real-time data which is installed at the Department of ECE, AUCE (A), Andhra University (Lat.17.73^{0}N/Long.83.31°E), Visakhapatnam region, India.

The efficiency of $\tilde{H}$ is presented in two stages. That is:

(a) DOP computational analysis is to prove there is an improved precision in the DOP parameters calculation

(b) In terms of Root Mean Square (RMS) position error to prove its fast convergence rate and improved precision in the estimated results

3.1. DOP Computation. In the proposed $\tilde{H}$, the fourth parameter is a variable for each iteration. The small change in the fourth parameter has high impact on the next iteration which improves the precision of the estimated parameters. Thus, at the end of final iteration the accuracy of the estimation improves. Table 1 presents the comparison of computed DOP values using H and $\tilde{H}$. Also, comparative results are shown in Figures 1–5. In the legend of the figures, the notation "Existing observation matrix" means "classical observation matrix" (H). DOP parameters least value is generally unity (i.e., 1) if the best of 4 satellite vehicles data is considered. But DOP values are less than one (<1) when multiple satellite vehicle data is considered. In this article, GPS input data is taken from all satellite vehicles which are in view with respect to the GPS receiver. Due to page limitations, see [10, 13, 21–26], and [27] for a detailed explanation of all DOP parameters and their calculation formulas. But, the GDOP calculation formula is given in equation (1).

In Table 1, for the same satellite configuration and time, the obtained DOP values are more precise due to the application of the $\tilde{H}$. Some of these DOP values are less than one which is commonly obtained when a greater number of satellites are visible to a GPS receiver.

As shown in Figure 1, the minimum GDOP obtained using proposed observation matrix ($\tilde{H}$) is 0.831 at 23:36 Hrs. Whereas, the minimum GDOP obtained using classical observation matrix (H) is 1.244 at 15:42 Hrs. The proposed $\tilde{H}$ calculated GDOP values with improved precision for the same number of considered satellites as the H. From the Figure 1, it is clearly observed that at each instant of the 24-hour duration the GDOP values are computed precisely by using $\tilde{H}$. Thus, in case of optimal satellite selection method, the proposed $\tilde{H}$ is very significant for choosing optimal satellites among the available satellites in the vicinity.

As it can be seen from Figure 2, the minimum PDOP obtained using proposed observation matrix ($\tilde{H}$) is 0.832 at 23:39 Hrs. Whereas, the minimum PDOP obtained using classical observation matrix (H) is 1.146 at 15:42 Hrs. The proposed $\tilde{H}$ calculated PDOP values with improved precision for the same number of considered satellites as the H. From the Figure 2, it is clearly observed that at each instant of the 24-hour duration the PDOP values computed precisely by using $\tilde{H}$. Thus, the proposed $\tilde{H}$ is very significant to describe the error caused by the relative position of the GPS satellites.

The minimum HDOP obtained using proposed observation matrix ($\tilde{H}$) is 0.691 at 23:24 Hrs. While, the minimum HDOP obtained using classical observation matrix (H) is

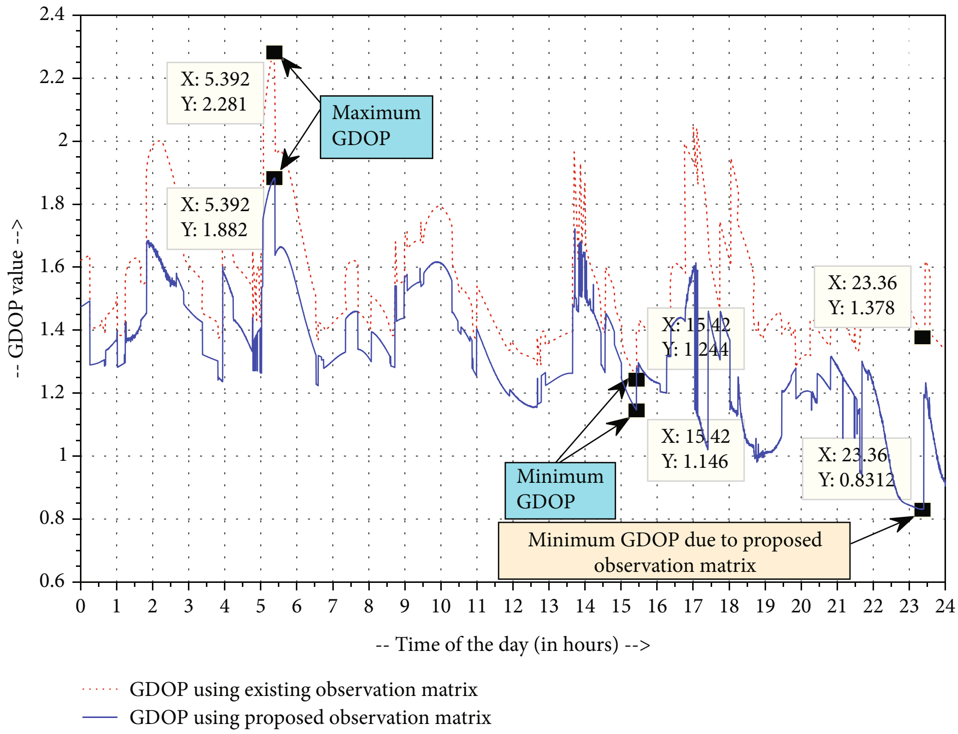

Figure 1: Comparisons of GDOP values.

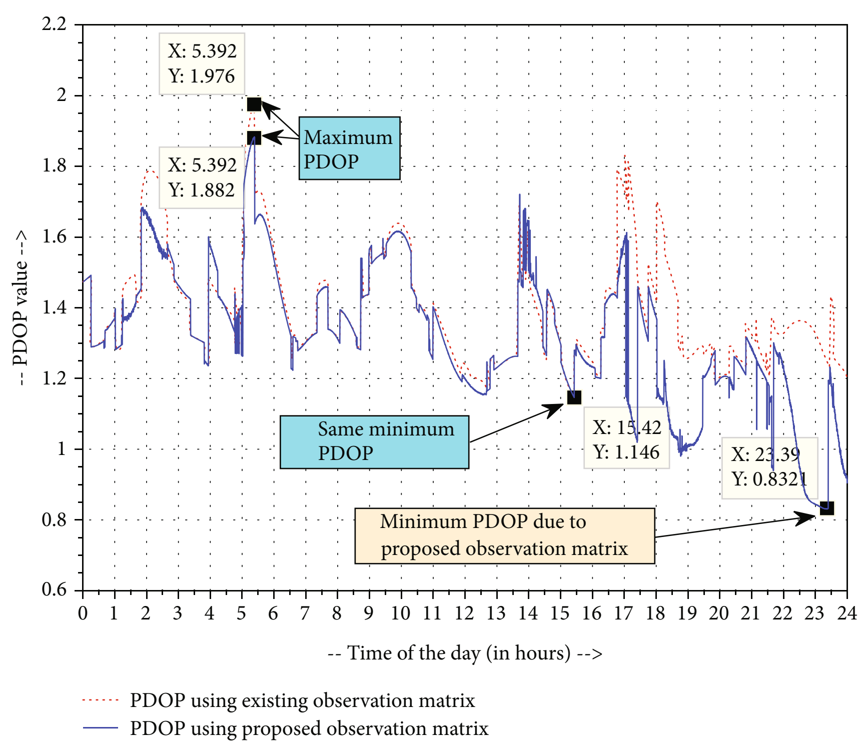

Figure 2: Comparisons of PDOP values.

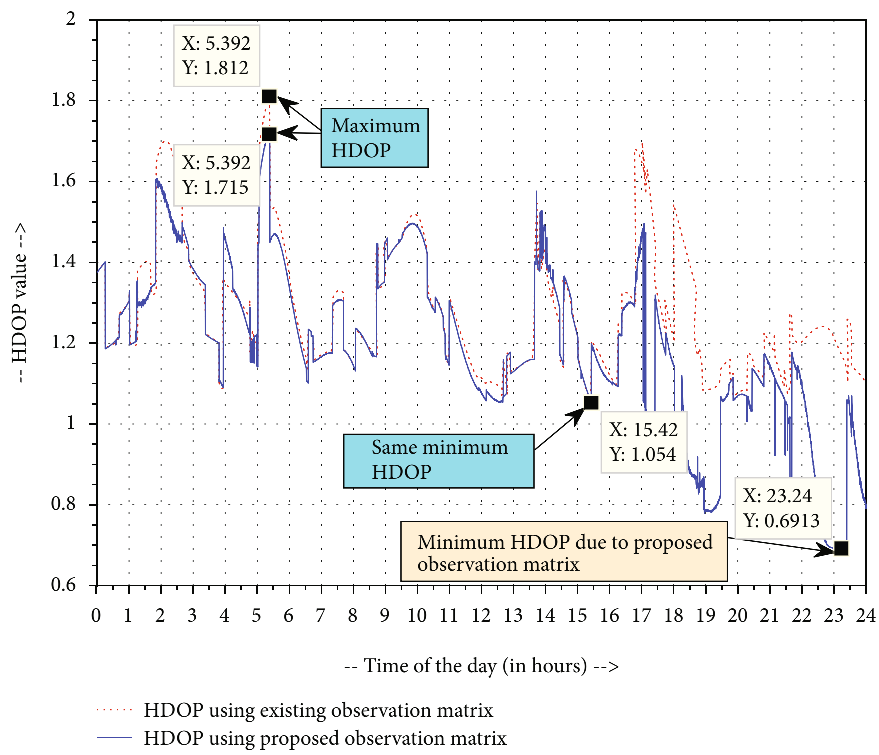

FIGURE 3: Comparisons of HDOP values.

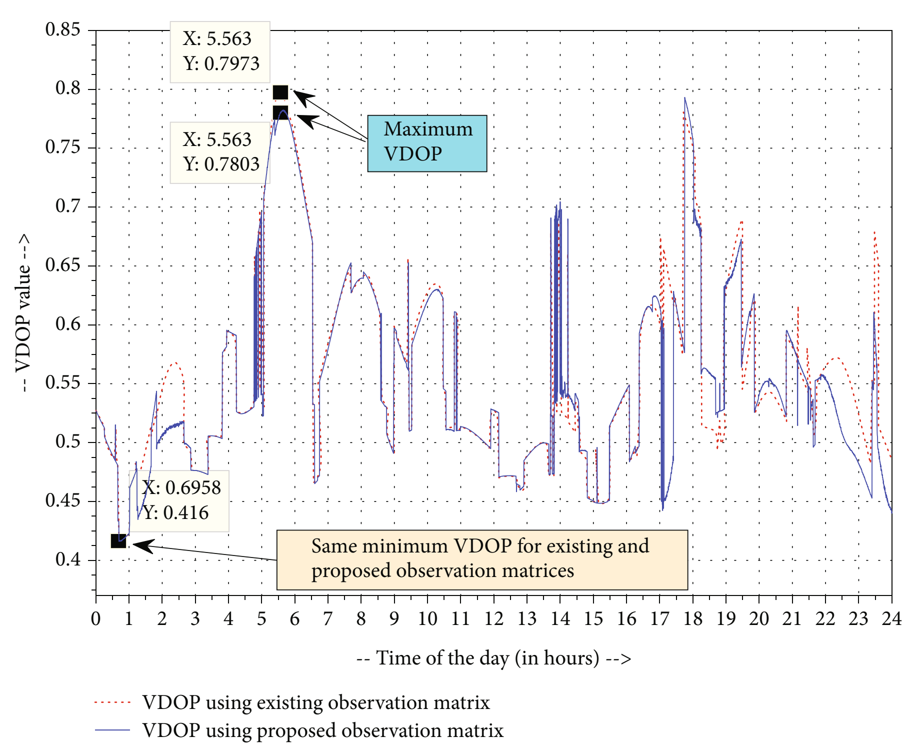

FIGURE 4: Comparisons of VDOP values.

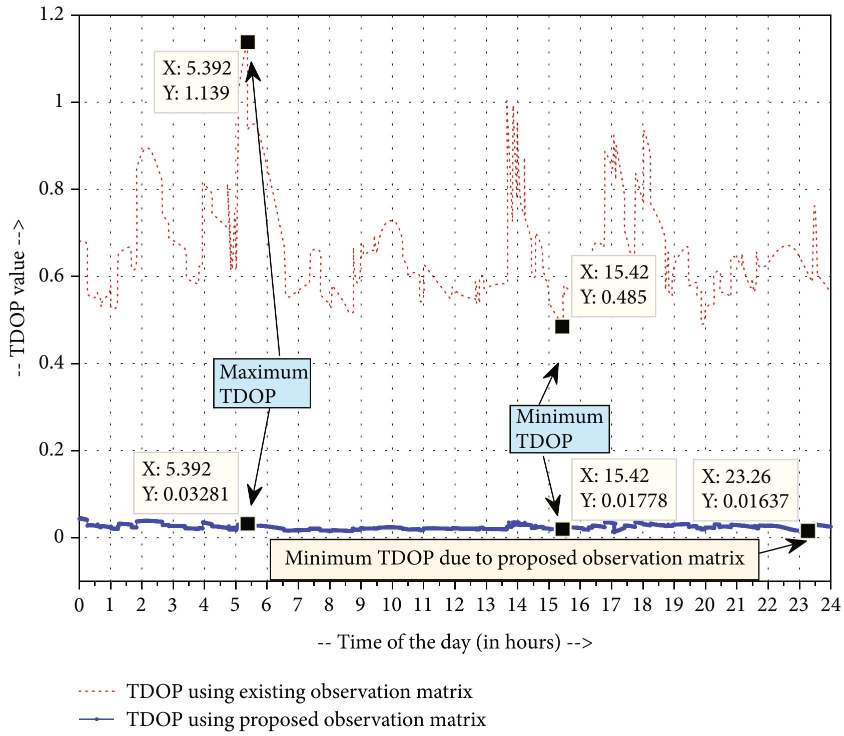

FIGURE 5: Comparisons of TDOP values.

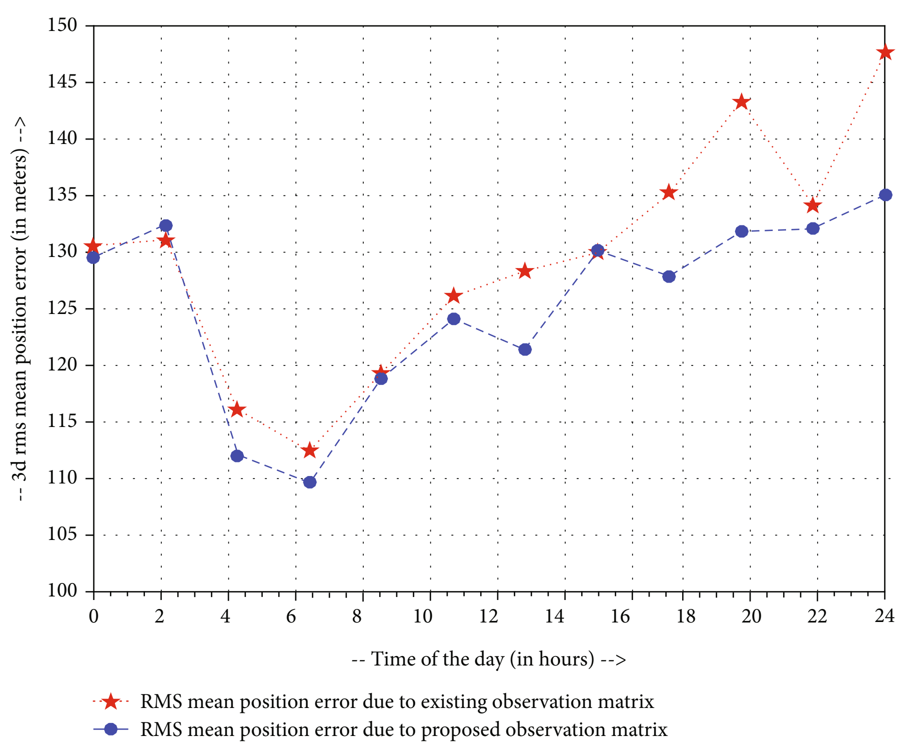

FIGURE 6: Comparisons of RMS mean position error due to LS algorithm.

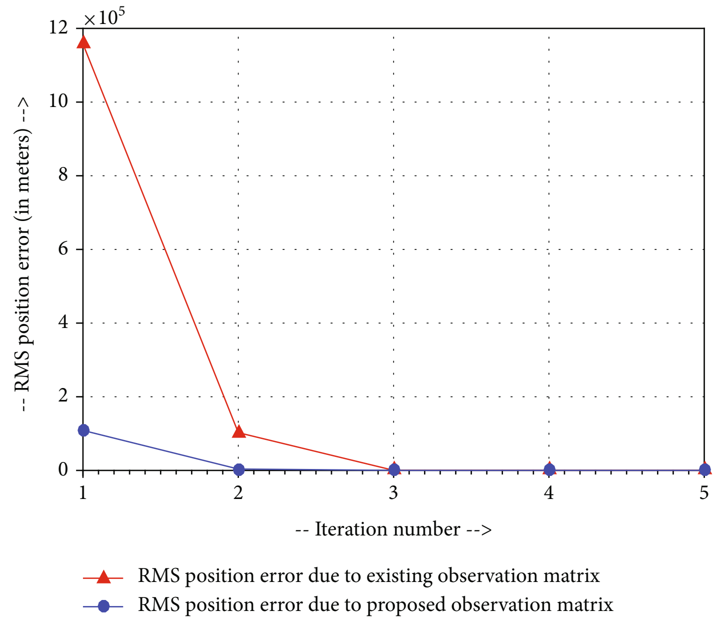

FIGURE 7: Comparisons of the iteration process.

1.054 at 15:42 Hrs (see Figure 3). The proposed $\tilde{H}$ calculated HDOP values with improved precision for the same number of considered satellites as the H. From the Figure 3, it is clearly observed that at each instant of the 24-hour duration the HDOP values computed precisely by using $\tilde{H}$. Thus, the proposed $\tilde{H}$ is very significant to more precisely estimate the accuracy of GPS horizontal (latitude/longitude) position fixes by adjusting the error estimates according to the geometry of the satellites used.

From Figure 4, it is evident that the obtained minimum VDOP using H and $\tilde{H}$ is the same i.e., 0.416 at 00:69 Hrs, i.e., at 00:41 minutes.

The obtained minimum TDOP using proposed observation matrix ($\tilde{H}$) is 0.016 at 23:26 Hrs (see Figure 5). Whereas, the minimum TDOP obtained using classical observation matrix (H) is 0.485 at 15:42 Hrs. The TDOP precision is increased due to the variable expression introduced in the $\tilde{H}$ unlike the constant in H. The variability will be effected for each iteration of each epoch, hence the precision changes which are improved here.

From Table 1 and the graphs (Figures 1–5), it is evident that DOP values due to $\tilde{H}$ are more precise when compared to that of H for the same satellite configuration. Therefore, the $\tilde{H}$ is much useful in the Visakhapatnam region for positioning, navigation, and timing applications.

3.2. GPS Receiver/User Position Estimation. The observation matrices H and $\tilde{H}$ are applied in the least-squares (LS) algorithm to estimate the position of the GPS user/receiver. The comparative results reveal that, the RMS error in position due to the application of $\tilde{H}$ is lesser than the RMS mean position error due H as shown in Figure 6. Thus, $\tilde{H}$ improves the precision in the estimation of 3D position [22, 25, 28–36]. The convergence rate of both proposed and existing observation matrices in terms of the number of iterations is given in Figure 7.

As it can be observed from Figure 7, $\tilde{H}$ converge from the first iteration and converge completely on the second iteration. Whereas, H starts with a large error and converges on the third iteration, and offers a high RMS position error when compared to that of $\tilde{H}$, as shown in Figure 6. Consequently, $\tilde{H}$ offers a fast convergence rate with less RMS position error, which is very useful in real-time applications.

Algorithms MGEKF [1] and EKF [3] are also implemented on the same GPS data, and the corresponding RMS position errors are compared, as shown in Figure 8. From the figure it can be depicted that the RMS error in position due to the EKF algorithm increasing from 08:00 hours onward, while MGEKF results continue to decrease from the start, which shows the stability of the MGEKF algorithm. Thus, the only distinction between EKF and MGEKF is the modified gain function "g", which is the proposed observation matrix $\tilde{H}$ in the MGEKF. Thus, the proposed observation matrix $\tilde{H}$ provides not only fast convergence and stability to the MGEKF algorithm but also a precise result.

An analysis of the results presented in sections 3.1 and 3.2 shows that using $\tilde{H}$ has two advantages. These are: (i) It provides fast convergence, (ii) It improves the precision of DOP calculations and estimation of receiver/user position by reducing the RMS position error. Due to the modified Taylor's series first order parameters in the proposed $\tilde{H}$, the $\tilde{H}$ is numerically stable than the H. Also described that how closely the both H and $\tilde{H}$ are related at an ideal case. From the observation of MGEKF results, the state estimates are found to be much more consistent than from the EKF

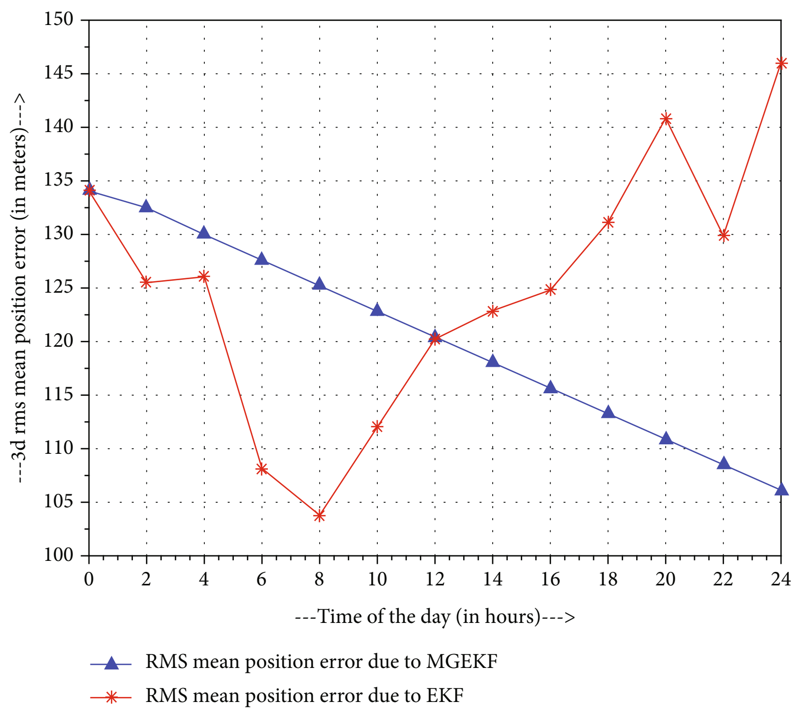

Figure 8: Comparisons of RMS position error due to MGEKF and EKF.

filter. Therefore, the *proposed observation matrix* $\tilde{H}$ can be used in the place of the classical observation matrix H to obtain more precise DOP values which exhibits the geometric effect on the association of positioning determination error and measurement error and improved precision in the estimation of GPS receiver position coordinates. The *proposed observation matrix* $\tilde{H}$ is useful for obtaining a more accurate body trajectory estimate in GNSS applications. It is also useful to identify the best GPS satellites with low DOP values in navigation evaluation applications where optimal satellites selection is applied. This can be useful for vertical guidance (APV) approaches at airports. It can also be used for advanced systems such as GPS Aided GEO Augmented Navigation (GAGAN).

4. Conclusion

A new observation matrix is derived based on the *direct difference method* presented in the works of Galkowski. The *proposed observation matrix* $\tilde{H}$ is a higher-order approximate form which makes the fourth parameter as a variable unlike in the classical observation matrix. Thus, the precision of all DOP parameters improved and also provided a fast convergence rate, stability of the algorithm, and improved precision in GPS receiver position estimation. Hence, the proposed observation matrix is useful for GPS sensors in obtaining a precise location for location sensor-dependent applications, GNSS trajectory estimation, optimal satellites selection process, and also GPS depended augmentation systems. In future works, the present work can be tested for the suitability for Indian Regional Navigation Satellite System (IRNSS) and IRNSS based augmentation systems.

Acronyms

DOP: Dilution of precision
GDOP: Geometric dilution of precision
PDOP: Position dilution of precision
HDOP: Horizontal dilution of precision
VDOP: Vertical dilution of precision
TDOP: Time dilution of precision
GPS: Global positioning system
EKF: Extended Kalman filter
MGEKF: Modified gain extended Kalman filter.

Data Availability

The GPS receiver data used to support the findings of this study are available from the authors upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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