ANISOTROPIC PROBLEMS RELATED TO DARK SECTOR OF THE UNIVERSE

BY

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Contents

List Of Abbreviations

ΛCMD - Λ cold cold dark matter

CDM - Cold dark matter

HDM - Hot dark matter

QCD - Quantum chromodynamics

WDM - Warm dark matter

DM - Dark mater

DE - Dark energy

CMB - Cosmic microwave background

SIDM - Self interacting dark matter

FLRW - Friedmann-Lemiter-Robertson-Walker

COBE - Cosmic Background Explore

CMBR - Cosmic Microwave Background Radiation

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Abstract

This thesis provides a comprehensive review of the ΛCDM model and alternative theories that seek to incorporate anisotropic considerations in cosmology. Through an in-depth analysis, the study examines how deviations from perfect isotropy within the ΛCDM model influence gravitational dynamics and the formation of cosmic structures. Alternative theories such as $f(R)$ gravity, Inflationary Cosmology, and Emergent Universe Models are explored as extensions or modifications to the ΛCDM model, each offering unique insights into the dynamics of the universe. While $f(R)$ gravity and Inflationary Cosmology can be viewed as further developments of the ΛCDM model, Emergent Universe Models face challenges in aligning with observational data and maintaining the successes of General Relativity in local tests. Overall, the thesis presents a valuable exploration of these alternative cosmological theories and their implications for our understanding of the cosmos.

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1 Introduction

1.1 Background

The story of the universe has been told in many ways by different cultures throughout history. In the modern period, we have a more scientific understanding of the universe, but we still lack a comprehensive story that encompasses both the scientific and humanistic aspects of the universe. A new type of narrative is needed that is based on the account of the emergent universe as communicated to us through our observational sciences. This new story will help us to understand ourselves and the universe in a new way [Swimme and Berry, 1992]. The cosmos, with its vast and intricate tapestry, continues to captivate the imagination of scientists as they delve deeper into the mysteries of the universe. The universe is everything that exists, from the smallest subatomic particle to the largest galaxy. It includes all of space and time, and everything in between. The universe is constantly expanding, and it is thought to be about 13.8 billion years old. The universe is made up of a variety of different objects, including stars, planets, galaxies, and black holes. Stars are massive balls of gas that produce light and heat. Planets are smaller objects that orbit stars. Galaxies are large collections of stars, planets, and other objects. Black holes are regions of space where gravity is so strong that nothing, not even light, can escape [Chaboyer, 1998, Ellis et al., 1978]. A significant facet of this exploration involves the investi-

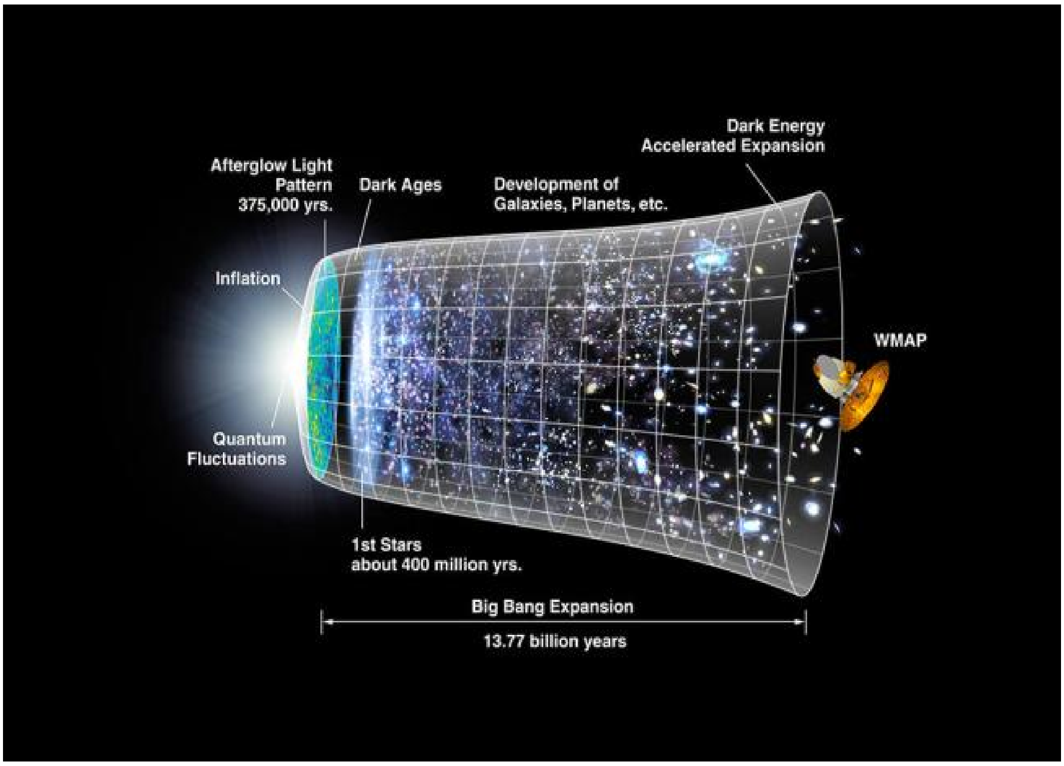

Image source:https://www.cfa.harvard.edu/research/topic/dark-energy-and-dark-matter

Figure 1.1: A cosmic timeline spanning from the Big Bang on the left to the present day on the right. Dark energy will rule the future and drive galaxies ever-further apart, while dark matter will rule the cosmos for a large portion of that time, producing galaxies and galaxy clusters.

gation of the dark sector, a realm that conceals the elusive components of dark matter and dark energy. While much progress has been made in understanding the cosmic landscape, anisotropic phenomena within the dark sector present a challenging and intriguing frontier that demands scholarly attention. Anisotropy, the property of exhibiting directional dependence, introduces a new layer of complexity to our comprehension of the dark sector. This phenomenon manifests in deviations from isotropy, where uniformity is expected. Understanding anisotropic patterns in the dark sector holds the promise of unraveling concealed insights into the fundamental nature of dark matter and dark energy. This study situates itself within the broader context of contemporary astrophysical research, where anisotropic problems related to the dark sector emerge as focal points of inquiry. As emphasized by [Das et al., 2023], the investigation of anisotropy in the distribution and behavior of dark matter and dark energy opens avenues for refining current cosmological models and advancing our understanding of the underlying principles governing the universe. In this context, the present research endeavors to address key questions surrounding anisotropic challenges in the dark sector, exploring their implications for cosmology and astrophysics [Bozorgnia et al., 2013].

The understanding of the universe's evolution and composition has been a central focus of cosmology. Recent advancements in observational astronomy and theoretical physics have unveiled that a significant portion of the universe is composed of enigmatic entities known as dark matter and dark energy. While these components remain elusive, their role in shaping the cosmos is undeniable. However, one of the intriguing challenges that cosmologists grapple with is the anisotropic behavior observed within the dark sector [Zuckerman and Malkan, 1996].

1.2 Contents of the Universe

The modern universe is a vast and complex place, containing billions of galaxies, each with billions of stars. The universe is constantly expanding, and it is thought to be about 13.8 billion years old. The contents of the universe are divided into two main components: Baryonic matter and dark matter. Baryonic matter is the matter that we can be seen and touched, such as atoms and molecules. The observable universe is home to 100 billion galaxies, each containing 100 billion stars and likely a similar number of planets. Galaxies are held together by the gravity of mysterious dark matter. The universe is expanding, and it is doing so at an accelerating pace, driven by dark energy, an even more mysterious form of energy whose gravitational force repels rather than attracts. The overarching theme in the story of our universe is the evolution from the simplicity of the quark soup to the complexity we see today in galaxies, stars, planets, and life. These features emerged one by one over billions of years, guided by the basic laws of physics [Starobinsky, 2000]. Dark matter is a mysterious substance that does not interact with light, and it makes up about 85% of the matter in the universe. The energy flows on the earth are embedded in the energy flows in the universe. A delicate balance among gravitation, nuclear reactions and radiation keep the energy flowing too fast.[1]

image source [https://sciencenotes.org/what-is-dark-matter/#google_vignette]

1.3 Statement of the Problem

The universe model so far is considered with isotropic model at large scale. While locally (today) different observation claims there is anisotropy, which is the behavior refers to anomalies in dark matter and dark energy properties, and distributions, So our consideration is to study the anisotropy related to the dark sector of the universe [Farnes, 2018, Yang et al., 2019, Amirhashchi et al., 2022] and these Concern raises question such as their

[1] https://wmap.gsfc.nasa.gov/universe/uni_matter.html, https://www.cfa.harvard.edu/big-questions/what-universe-made

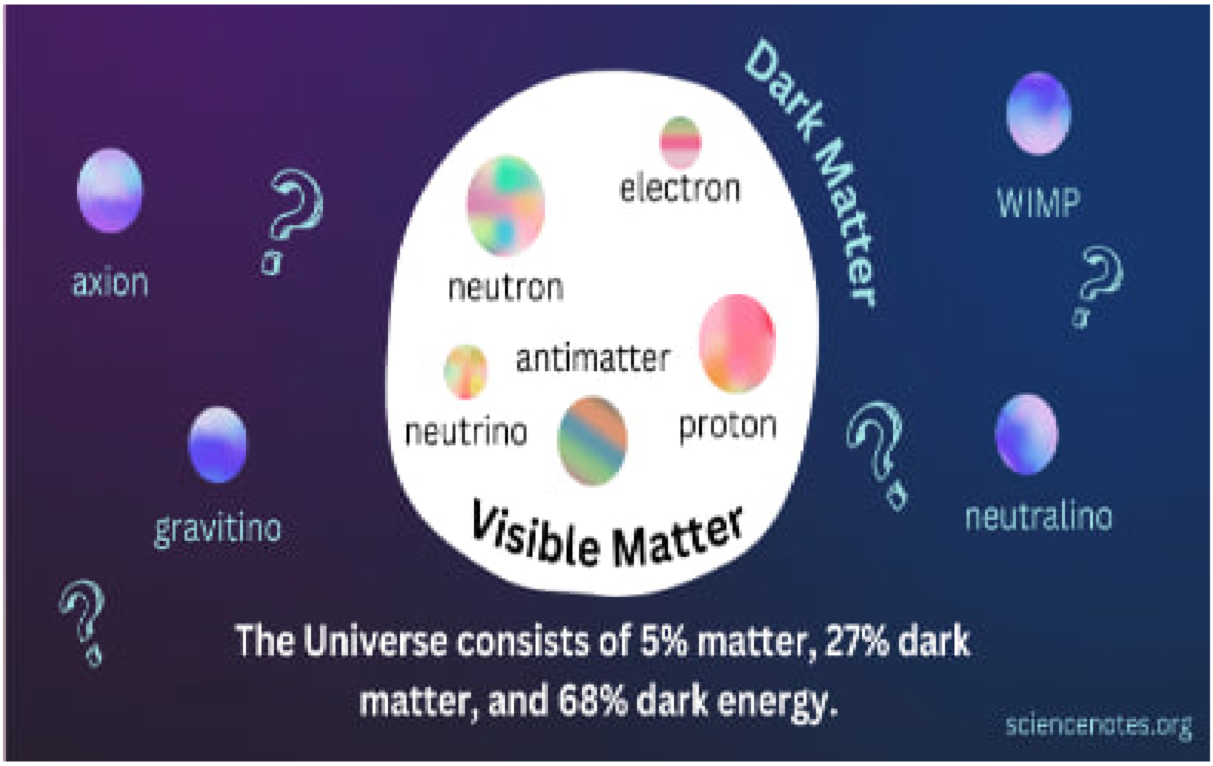

Figure 1.2: Contents of the universe

effect on dark sector, on large scale structure, and their implication for future evolution of the universe .

Research Questions

1. What are the observable effects of anisotropy in the dark sector of the universe?

2. How does anisotropy in the dark sector influence the large-scale structure of the universe?

3. How do the anisotropies affect our current understanding of dark matter and dark energy?

4. What are the implications of anisotropy in the dark sector for the future evolution of the universe?

1.4 Objectives

1.4.1 General objective

To investigate the anisotropic problem related to dark sector of the universe.

1.4.2 Specific objectives

- To investigate the observable effects of anisotropy in the dark sector of the universe
- To analyze how does anisotropy in the dark sector influence the large-scale structure of the universe
- To assess how do these anisotropies affect our current understanding of dark matter and dark energy
- To investigate the implications of anisotropy in the dark sector for the future evolution of the universe.

1.5 Methodology

The literature review is used to identify the existing anisotropic problems observed. Then the standard cosmological model and recent alternative models are used to study the anisotropic problems and problems of the model in studying it. Finally we try to simulate the theoretical models we extracted from the boundary conditions to observational data

Chapter 2

2 The Standard Cosmological model (ΛCDM) and Anisotropic Phenomena

2.1 The Standard Cosmological model: ΛCDM

The majority of research conducted in the 1980s concentrated on cold dark matter with a critical density in matter of approximately 95% CDM and 5% baryons. These materials demonstrated the ability to form galaxies and clusters of galaxies, but there were still issues. For example, the model called for a Hubble constant that was lower than what was preferred by observations, and observations conducted between 1988 and 1990 revealed more large-scale galaxy clustering than was predicted. After COBE discovered CMB anisotropy in 1992, these challenges became more acute, and until the mid-1990s, a number of modified CDM models, including as ΛCDM nd mixed cold and hot dark [Martel and Shapiro, 2003] matter, were actively studied. After the observations of accelerating expansion in 1998, the ΛCDM model emerged as the dominant model. This was swiftly corroborated by additional observations in 2000, the total (matter-energy) density was measured to be close to 100% of critical by the BOOMER and microwave background experiment, while in 2001 the matter density was measured to be near 25% by the 2nd FGRS galaxy redshift survey. The significant difference between these values suggests a positive Λ or dark energy. WMAP's (2003–2010) and Planck's (2013–2015) far more accurate spacecraft measurements of the microwave background have continued to confirm the model and refine the parameter values, most of which are now limited to 1% uncertainty [Tamm, 2015, Martel and Shapiro, 2003].

2.1.1 The FLRW metric

The FLRW metric describes the geometry of the universe in terms of an isotropic and homogeneous space. It is characterized by a line element in terms of coordinates t,r,θ and ϕ. The line element is given by:

$$ds^2 = -dt^2 + \alpha(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 \right) \tag{2.1}$$

where α(t) is the scale factor representing the size of the universe at time t, and k is the curvature constant which can take the values 1, 0, or-1 determining the geometry of the space.

Let's break down the metric components:

1. Time component $ds^2_{tt} = -d^2_t$: This component represents the time interval. It is negative to maintain the signature of the metric.
2. Spatial components:
   - $ds^2_{rr} = \frac{\alpha(t)^2 dr^2}{1-kr^2}$: This term represents the radial distance element. It is modified by the curvature constant k.
   - $ds^2_{\theta\theta} = \alpha(t)^2 r^2 d\theta^2$: This term represents the angular distance element in the θ direction.
   - $ds^2_{\phi\phi} = \alpha(t)^2 r^2 \sin^2\theta d\phi^2$: This term represents the angular distance element in the ϕ direction.

The Friedmann Equation

Now, to solve the Einstein equations for this metric, we need to choose an appropriate energy-momentum tensor. Typically, for a homogeneous and isotropic universe, we use the perfect fluid energy-momentum tensor, given by:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} \tag{2.2}$$

where ρ is the energy density, p is the pressure, and u^μ is the four-velocity of the fluid. or a perfect fluid, u^μ is given by $u^\mu = (1,0,0,0)$ in comoving coordinates. let's derive the Friedmann equations in detail. We start with the Einstein field equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} \tag{2.3}$$

where $G_{\mu\nu}$ is the Einstein tensor, Λ is the cosmological constant, $g_{\mu\nu}$ is the metric tensor, and $T_{\mu\nu}$ is the energy-momentum tensor.
In the FLRW metric, the metric tensor $g_{\mu\nu}$ is given by:

$$ds^2 = -dt^2 + a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right) \tag{2.4}$$

For simplicity, we consider the matter-dominated era where $p = 0$. Therefore, the energy-momentum tensor $T_{\mu\nu}$ simplifies to:

$$T_{\mu\nu} = \rho u_\mu u_\nu \tag{2.5}$$

Now, let's calculate the components of the Einstein tensor $G_{\mu\nu}$ using the FLRW metric. Since

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \tag{2.6}$$

where $Rg_{\mu\nu}$ is the Ricci tensor and R is the Ricci scalar, we need to compute $R_{\mu\nu}$ and Now, let's calculate the components of the Einstein tensor $G_{\mu\nu}$using the FLRW metric. Since $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$, where $Rg_{\mu\nu}$ is the Ricci tensor and R is the Ricci scalar, we need to compute $R_{\mu\nu}$ and R. The non-zero Christoffel symbols for the FLRW metric are:

$$\Gamma^i_{j0} = \Gamma^0_{ji} = \tfrac{\dot{a}}{a} g_{ij} \text{ and } \Gamma^i_{jk} = -a\dot{a} g^i_j$$

where $\dot{a}$ denotes the derivative of the scale factor α(t) with respect to time. Using the Christoffel symbols, we can compute the non-zero components of the Ricci tensor $R_{\mu\nu}$ and the Ricci scalar R.

$$R_{\mu\nu} = \Gamma^\alpha_{\mu\nu,\alpha} - \Gamma^\alpha_{\mu\alpha,\nu} + \Gamma^\beta_{\mu\alpha}\Gamma^\alpha_{\nu\beta} - \Gamma^\alpha_{\mu\nu}\Gamma^\beta_{\alpha\beta} \tag{2.7}$$

then

$$R_{00} = \Gamma^0_{00,\alpha} - \Gamma^0_{0\alpha,0} + \Gamma^\beta_{0\alpha}\Gamma^0_{0\beta} - \Gamma^\alpha_{00}\Gamma^\beta_{\alpha\beta}, \text{ since } \Gamma^\alpha_{00} = 0 \text{ and } \Gamma^\alpha_{0\alpha} = 0 \text{ for all } \alpha$$

then we left with

$$R_{00} = \Gamma^\alpha_{00,0} = \partial_\alpha \Gamma^\alpha_{00} - \partial_0 \Gamma^\alpha_{0\alpha}, \text{ Using the Christoffel symbols we computed}$$
$$\Gamma^0_{\alpha\alpha} = \Gamma^\alpha_{0\alpha} = \tfrac{\dot{a}}{a} g_{\alpha\alpha}, \text{ so}$$

$$R_{00} = \partial_\alpha \left(\tfrac{\dot{a}}{a} g^{\alpha\alpha}\right) - \partial_0 \left(\tfrac{\dot{a}}{a} g^{\alpha\alpha}\right)$$

The only non-zero derivative terms here are $\frac{\partial}{\partial t}\left(\frac{\dot{a}}{a} g^{tt}\right)$ and $\frac{\partial}{\partial t}\left(\frac{\dot{a}}{a} g^{rr}\right)$ the scale factor α(t) depends only on time

$$R_{00} = -3\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right) \tag{2.8}$$

$R_{ij} = \Gamma^\alpha_{ij,\alpha} - \Gamma^\alpha_{i\alpha,j} + \Gamma^\alpha_{i\beta}\Gamma^\beta_{j\alpha} - \Gamma^\alpha_{ij}\Gamma^\beta_{\alpha\beta}$, similarly, as $\Gamma^\alpha_{ij} = 0$,for all i, j, α and we

left with

$R_{ij} = \Gamma^\alpha_{ij,\alpha} = \partial_\alpha \Gamma^\alpha_{ij} - \partial_i \Gamma^k_{i\alpha}$, again using the Christoffel symbols $\Gamma^i_{ij} = -\frac{\dot{a}}{a} g^{\alpha\alpha} g^{ij}$

so,

$R_{ij} = -\frac{\partial}{\partial\alpha}\left(\frac{\dot{a}}{a} g^{\alpha\alpha} g_{ij}\right)$,The only non-zero derivative term here is $\partial_t \frac{a}{\dot{a}} g^{tt} g_{ij}$ since the scale factor α(t) depends only on time, then

$$R_{ij} = g_{ij}\left(2\frac{\ddot{a}}{\dot{a}} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right) \tag{2.9}$$

$$R = -6\left(\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right) \tag{2.10}$$

now using equation (2.3, 2.4) and Substituting the components of $R_{\mu\nu}$ and R and the FLRW metric, we obtain:

$$-3\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right) - \frac{1}{2}\left(-6\left(\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right)\right)g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi\rho u_\mu u_\nu$$

Now, we can write down the Friedmann equations by considering the 00 and ij components:

$$-3\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right) + 3\left(\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right) - 3\Lambda = 8\pi\rho$$

$$-2\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right) + 2\left(\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right) - 2\Lambda = 0$$

Simplifying and rearranging terms, we obtain the Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{2.11}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho + \frac{\Lambda}{3} \tag{2.12}$$

equation (2.11) and (2.12) are called Friedmann's equation and they govern the dynamics of the universe's expansion in the matter-dominated era [Weinberg, 2008, Weinberg, 1972, Peebles, 1993, Friedman, 1922, Stefano et al., 2023]. The Lambda Cold Dark Matter (ΛCDM) model stands as the prevailing cosmological framework, providing a robust explanation for the large-scale structure of the universe. Central to its success is its treatment of isotropy, the assumption that the universe appears the same in all directions, a cornerstone principle in modern cosmology [Stefano et al., 2023, Lusso et al., 2019]. At its core, the ΛCDM model relies on the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, which describes a homogeneous and isotropic universe. This metric is governed by the Einstein field equations of general relativity, where the energy content of the universe dictates its evolution. The energy density contributions in

ΛCDM consist primarily of ordinary matter, dark matter, and dark energy.

2.1.2 Homogeneity and Isotropy in the FLRW Metric:

The FLRW metric describes a spatially homogeneous and isotropic universe. The isotropy assumption implies that the metric should not favor any direction in space; the universe should appear the same in all directions. Mathematically, this translates into the metric being independent of spatial coordinates (r, θ, ϕ).
The isotropic nature of the FLRW metric is explicitly reflected in its form:

$$ds^2 = -c^2 dt^2 + a(t)^2 \left(1 - kr^2\right) dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{2.13}$$

Here, the scale factor $a(t)$ solely depends on time t and not on spatial coordinates, thus ensuring isotropy[2]. The Friedmann equations govern the evolution of the scale factor $a(t)$ and the Hubble parameter H in first equation since $H^2 = \left(\frac{\dot{a}}{a}\right)^2$ in the context of general relativity. These equations describe how the universe expands or contracts over time, driven by the energy content of the universe as they are given in equation (2.11 and 2.12). For the universe to be isotropic, the expansion rate H must be the same in all directions. This requires that the energy density ρ and pressure p are uniform throughout space. Consequently, the scale factor $a(t)$ evolves uniformly with time t in all spatial directions Consequently, the scale factor $a(t)$ evolves uniformly with time t in all spatial directions. The Friedmann equations encapsulate this isotropic expansion by incorporating terms dependent solely on time and not on spatial coordinates. Thus, the mathematical framework of the ΛCDM model, based on the

[2] https://www.physicsforums.com/insights/journey-cosmos-flrw-metric-friedmann-equation/

FLRW metric and the Friedmann equations, inherently ensures isotropy by construction. [Friedman, 1922, Friedman, 1999][3].

2.2 Observational perspective related Isotropy

There is a very high degree of isotropy in the temperature of the Cosmic Microwave Background (CMB), as demonstrated by the FIRAS experiment conducted on the COBE satellites [Smoot et al., 1992, Bennett et al., 2003]. Nevertheless, the experiment's angular resolution was insufficient to draw any meaningful conclusions about the isotropy of the geographic distribution of the minute temperature variations in the CMB. The CMB fluctuations' statistical characteristics ought to be the identical in all directions across the sky, in accordance with the cosmological concept of isotropy. The statistical characteristics of the CMB fluctuations should be the same in all directions across the sky, in accordance with the cosmological concept of isotropy. This has now been tested using the most recent WMAP satellite data.

2.3 Anisotropy through observation

2.3.1 Magnetic monopole through observation

Cosmic radiation searches began much earlier than accelerator searches. The majority of these were conducted in the 1980s as a result of the GUT theories' development, which incorporate the Magnetic Monopoles Solution. Among the most popular methods have been nuclear emulsions, scintillator counters, and tracketch detectors. The searches conducted at the LHC at CERN are the most recent. ATLAS searched for gD massive par-

[3] https://www.physicsforums.com/insights/journey-cosmos-flrw-metric-friedmann-equation/

ticles with masses as high as 2.5 TeV [Patrizii et al., 2019]. The MoEDAL experiment has investigated magnetic monopole masses up to 6 TeV and established upper bounds on the formation of magnetic monopoles with charges up to $5g_D$ [Pinfold, 2019].

2.3.2 Temperature anisotropy through observation

The Planck satellite mission, launched by the European Space Agency (ESA) in 2009, was designed to study the cosmic microwave background (CMB) radiation, the relic radiation from the early universe. The mission aimed to provide high-resolution maps of the CMB temperature and polarization anisotropies, which contain crucial information about the universe's composition, geometry, and evolution.

Planck Data Releases:

Early Release: The first data release, known as the Early Release Compact Source Catalogue, was made public in 2011, containing information about compact sources detected by Planck.

First Release: The initial cosmological results, including temperature maps and basic cosmological parameters, were released in 2013, marking the first major release of Planck data.

Second Release: The second data release in 2015 provided more detailed temperature and polarization maps, along with improved cosmological parameter estimates and additional scientific analyses.

Legacy Release: The final data release, known as the Legacy Release, occurred in 2018, providing the most comprehensive set of Planck data products, including high-quality maps and detailed scientific analyses.

High-Frequency Instrument (HFI): The High-Frequency Instrument

(HFI) aboard the Planck satellite was a sophisticated suite of instruments designed to observe the cosmic microwave background (CMB) radiation with unprecedented sensitivity and resolution. It consisted of 52 bolometric detectors operating at six frequencies spanning the range from 100 GHz to 857 GHz. The HFI played a crucial role in providing high-resolution maps of the CMB temperature anisotropies, as well as polarization measurements, which were essential for understanding the early universe's structure and evolution. The HFI's bolometric detectors were highly sensitive to small fluctuations in the intensity of CMB radiation, allowing for precise measurements of temperature variations across the sky. By operating at multiple frequencies, the HFI was able to characterize the spectral properties of the CMB and distinguish between cosmological signals and foreground contamination from sources such as Galactic dust and synchrotron emission. The high angular resolution of the HFI enabled detailed mapping of temperature fluctuations on small angular scales, revealing fine structures in the CMB that are indicative of primordial density fluctuations and acoustic oscillations in the early universe. These maps provided valuable constraints on cosmological parameters, such as the density of dark matter and baryons, the amplitude of primordial density fluctuations, and the overall geometry of the universe. In addition to temperature measurements, the HFI also performed polarization measurements of the CMB, which provided insights into the universe's magnetic fields, the epoch of reionization, and the nature of primordial gravitational waves. These measurements helped to further refine our understanding of cosmological models and the physical processes that shaped the early universe. Overall, the High-Frequency Instrument on the Planck satellite played a crucial role in advancing our understanding of the cosmic microwave background and its implications for cosmology, providing high-resolution maps of temperature anisotropies and polarization measurements that have greatly enhanced our knowledge of the universe's origins and evolution [Aghanim et al., 2020].

Low-Frequency Instrument (LFI): The Low-Frequency Instrument (LFI) aboard the Planck satellite was designed to complement the observations made by the High-Frequency Instrument (HFI) by focusing on mapping the cosmic microwave background (CMB) temperature anisotropies at lower frequencies with high sensitivity and low noise. The LFI consisted of 22 radiometers that covered frequencies ranging from 30 GHz to 70 GHz. The primary objective of the LFI was to provide accurate measurements of the CMB temperature fluctuations on large angular scales, which are crucial for constraining cosmological parameters such as the density of dark matter and baryons, the Hubble constant, and the overall geometry of the universe. By operating at lower frequencies, the LFI was less susceptible to foreground contamination from sources such as Galactic dust and synchrotron emission, allowing cleaner measurements of the CMB signal. The LFI's radiometers were designed to be highly sensitive to small temperature variations in the CMB, with low noise levels to ensure accurate measurements. This sensitivity, combined with the instrument's wide frequency coverage, enabled the LFI to produce high-quality maps of the CMB temperature anisotropies with unprecedented precision. In addition to its primary goal of mapping the CMB temperature fluctuations, the LFI also contributed to studies of foreground emission from our own Galaxy, such as the distribution of interstellar dust and the properties of the Galactic magnetic field. By characterizing and removing these foreground contaminants, the LFI helped to improve the accuracy of cosmological parameter estimates derived from the Planck data. Overall, the low frequency instrument on the Planck satellite played a vital role in advancing our understanding of the cosmic microwave background and its implications for cosmology, providing high-quality measurements of CMB temperature anisotropies on large angular scales [Aghanim et al., 2020].

2.3.3 Gravitational anisotropy through observation

Gravitational anisotropy characterized by directional variations in gravitational forces, is a key phenomenon shaping the large-scale structure and evolution of the universe. analysis of gravitational anisotropy through various observational techniques, including galaxy surveys, cosmic microwave background (CMB) observations, gravitational lensing studies, pulsar timing arrays, large-scale structure surveys, and gravitational wave detectors [Chen et al., 2022]. Galaxy surveys, particularly from the Sloan Digital Sky Survey (SDSS), reveal significant anisotropies in the spatial distribution of galaxies. The observed clustering patterns indicate the presence of massive structures, such as galaxy clusters and superclusters, exhibiting directional dependence in gravitational forces. This suggests that gravitational anisotropy plays a crucial role in shaping the large-scale structure of the universe, influencing the formation and evolution of cosmic structures over cosmic epochs [Tutusaus et al., 2023].
Gravitational Wave Detectors: By detecting gravitational waves from events such as black hole mergers and neutron star collisions using instruments like the Laser Interferometer Gravitational-Wave Observatory (LIGO), it

is confirmed that the existence of gravitational radiation and investigate anisotropies in the gravitational wave background [Agazie et al., 2023].

2.4 ΛCDM and anisotropy consideration

2.4.1 Introduction

Many cosmological observations [Hamilton, 2014] have questioned the validity of the conventional cosmological model (ΛCDM), which has been in place for the last 20 years. The cosmological principle (isotropy, homogeneity, general relativity, cold dark matter, and baryonic matter), flatness of space, the presence of a cosmological constant, and gaussian-scale invariant matter perturbations produced during inflation are the underlying presumptions of the model. These presumptions underpin the model's precise predictions, which are in line with the great bulk of cosmological evidence. Among these forecasts are the following.

1. The spectrum of the cosmic microwave background (CMB): There is a good match between the angular power spectrum of CMB primordial disturbances [Komatsu et al., 2009] and the ΛCDM predictions. Nonetheless, a few questions about the direction and size of low multipole moments (also known as CMB anomalies) remain unanswered for the mainstream model [Tegmark et al., 2003, Bennett et al., 2011, Hanson and Lewis, 2009].

2. The CMB perturbation statistics: The gaussianity of the standard model [Ade et al., 2014] is predicted by these data [Smith et al., 2009].

3. accelerated expansion: The presence of a cosmological constant is compatible with cosmological measurements that map the recent ac-

celerated expansion rate of the cosmos using standard candles [Type Ia Supernovae (SnIa)] [Amanullah et al., 2010] and standard rulers [Baryon Acoustic Oscillations] [Percival et al., 2007]. In spite of the constantly improving evidence, there is no longer a need for increasingly complex models based on dynamic dark energy or changed gravity. Over the previous ten years, the probability of the cosmological constant vs more complex homogenous models has been steadily rising [Sanchez et al., 2009].

4. Large Scale Structure: ΛCDM [Nesseris and Perivolaropoulos, 2008] (fundamental statistics of galaxies [Trujillo-Gomez et al., 2010], halo power spectrum [Reid et al., 2010]) and observations of large scale structure accord well.

Type Ia supernovae (SNe Ia) measurements show that the expansion of our universe is occurring at an accelerated rate [Riess et al., 1998, Perlmutter et al., 1999]. This result was later confirmed by a number of other observations, including the large scale structure (LSS), the cosmic microwave background (CMB), and the baryonic acoustic oscillations (BAOs). Numerous cosmological models have been put out or examined in the literature in an effort to better understand the dynamics of the cosmos. Nevertheless, of all the cosmological models, ΛCDM has shown to be the most straightforward mathematical model that is generally acknowledged by the cosmological community and is known as the standard cosmological model of cosmology. This concept consists of two primary components: dark energy (DE) in the form of the cosmological constant (Λ) which causes the late time rapid expansion of the Universe, and cold (non-relativistic) dark matter (CDM) which is the explanation for the structure development. A wide variety of scales and epochs are well-fitted by the ΛCDM model [Aghanim et al., 2020, Abbott et al., 2019], which also well predicts the late-time rapid expansion of the universe [Bamba et al., 2012]. Even while the model fits the existing cosmic evidence rather well, there are still a number of theoretical and empirical issues. For example, the precise nature of DE is still unknown, and under the ΛCDM paradigm, DE is considered the most basic version of the cosmological constant, with no firm physical basis. One of the main pillars of ΛCDM cosmology is the Cosmological Principle (CP), which states that the Universe is statistically homogeneous and isotropic in space and matter on enormous scales (100Mpc). The Friedmann Lemaitre-Robertson-Walker (FLRW) space-time metric, in which all three of the metric's spatial orthogonal components are functions of cosmic time (t) solely, describes such a world mathematically. This is the main space-time measure that makes the standard model of cosmology more efficient and wonderfully predictive. On the other hand, observational data indicate that there may be minute variations in CMB tensities coming from different sky orientations [Yeung and Chu, 2022]. Other intriguing empirical evidences have called into doubt the validity of CP. For example, the authors in [Fosalba and Gaztanaga,] discovered significant evidence for a violation of the CP of isotropy after analyzing the temperature anisotropy data from the Planck Legacy. It is probable that this infraction reflects a statistical variation of around 10^{-9}. See also the recent and intriguing review [Aluri et al., 2023], where the authors outline current observational signs for departures from CP predictions while emphasizing contrasts and synergies that really motivate further study in this subject. Homogeneous and anisotropic metrics serve as a mathematical expression for the spatially homogeneous and anisotropic Universe, and the associated models are referred to as Bianchi type models [Watanabe et al., 2009].

2.4.2 Anisotropic Dark Universe

In cosmology, the study of the dark universe that is, the make up and properties of dark matter and dark energy has garnered a lot of interest [De Vaucouleurs, 1970]. The possibility that dark matter and dark energy have anisotropic properties that is, that their characteristics differ depending on where in the cosmos they are found is one of the universe's most fascinating features [Heckler, 1997, Bennett, 2006, 200, 2003, Kofman et al., 1993], and this anisotropic dilemma has been the subject of numerous investigations with the goal of determining any possible consequences for our comprehension of the structure development and evolution of the universe. A number of theoretical models have been put out to account for the apparent anisotropy issue in the dark cosmos. These include various models of dark matter and dark energy, as well as modified gravity theories, such as scalar tensor theories or theories with extra dimensions. These ideas can explain the anisotropy in the dark universe by introducing new physics that goes beyond the standard model. Numerous cosmological probes, including larges cale structure surveys, gravitational lensing, and the cosmic microwave background radiation, have provided observational evidence for the anisotropy problem of the dark universe. These measurements have yielded indications of possible departures from isotropy in the cosmic scale distribution of dark matter and energy. To definitively determine the existence and nature of the anisotropic problem in the dark cosmos, more investigation and analysis are required [200, 2003, Aguirre et al., 2009]. The possible anisotropic character of the dark universe is one of the main obstacles to understanding it. Researchers aim to learn more about the basic characteristics of dark matter and dark energy, as well as how they shaped the structure and evolution of our universe, by examining the anisotropic problem. In order to solve the anisotropy problem of the dark universe, scientists are still using observational evidence along with theoretical models. An important area of cosmology research that aims to comprehend the possible variances and directional dependencies of dark matter and dark energy is the anisotropic problem of the dark cosmos. It calls into question our existing understanding of the homogeneity and isotropy of the cosmos and raises issues about the fundamental nature and behavior of these enigmatic components. Determining the underlying nature of dark matter and dark energy, as well as their influence on the overall structure and evolution of the universe, requires an understanding of and solution to the anisotropy problem of the dark universe. Studies and research on the anisotropic problem of the dark universe are still in progress. To further comprehend the anomalies in space, scientists are examining altered gravity theories, alternate dark matter and dark energy models, and observational data from numerous cosmological missions [Maartens, 2011]. Among the several Bianchi type models, BI cosmological model has attracted more attention due to its fundamental properties: it has more degrees of freedom with respect to FLRW characterized by Lie groups and it recovers isotropic scenario as special case and permits a small amount of anisotropy. This small amount of anisotropy may affect the physical behavior of the universe in early times of evolution [Amirhashchi et al., 2022].

2.4.3 Governing Equation and models

General form of the metric for anisotropic universe model with three orthogonal directions of different scale given by:

$$ds^2 = -dt^2 + A(dx)^2 + B(dy)^2 + C(dz)^2 \tag{2.14}$$

where A, B, and C are functions of cosmic time t only and represent the scale factors along the principal axes x, y and z, respectively. It includes Friedmann–Lemaitre spacetimes. Moreover, the average expansion scale factor is defined as follows: $a = (ABC)^{\frac{1}{3}}$ and the average Hubble parameter: $H = \frac{\dot{a}}{a} = \frac{H_x+H_y+H_z}{3}$, where the corresponding directional Hubble parameters along the principal axes x, y, and z are being defined as $H_x = \frac{\dot{A}}{A}$, $H_y = \frac{\dot{B}}{B}$ and $H_z = \frac{\dot{C}}{C}$ respectively.
The Einstein field equations in GR read as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{2.15}$$

where the left side of the above expression shows the Einstein tensor $G_{\mu\nu}$, the Ricci tensor $R_{\mu\nu}$ the Ricci scalar R, and the metric tensor $g_{\mu\nu}$. Additionally, the right side shows the Newton's gravitational constant G and the energy momentum tensor $T_{\mu\nu}$. Further, for a perfect fluid with energy density ρ and pressure p, $T_{\mu\nu}$ takes the form

$$T_{\nu\mu} = \mathrm{diag}[-\rho, p, p, p] \tag{2.16}$$

Because of the Bianchi identity that was twice contracted $G^{\mu\nu};\nu = 0$ the Einstein field equations (2.15) satisfy the conservation equation

In case of the perfect fluid matter distribution, it reduces to

$$\dot{\rho}+3H(\rho+p)=0 \tag{2.17}$$

where its cosmic time (t) derivative is indicated by the dot. The continuity equation (2.17) gives the development of the energy density ρ_i of a perfect fluid i with pressure p_i, energy density ρ_i, and constant EoS $\omega_i = p_i/\rho_i$. This is as follows:

$$\rho_i = \rho_{i0} a^{-3(1+w_i)} \tag{2.18}$$

where ρ_{i0} denotes the current value of ρ_i, or at $a = a_0 = 1$. In this case, a=$a_0 = 1$ denotes the cosmic scale factor a's current value. From this point forward, the value of every quantity in the current universe is indicated by the subscript 0.

We examine the standard cosmic fluids that make up the universe: pressureless fluid (baryonic and cold dark matter) with an EoS of $\omega_m = p_m/\rho_m = 0$, radiation (photons and neutrinos) with an EoS of $\omega_r = \frac{p_r}{\rho_r} = \frac{1}{3}$, and DE fluid with a constant EoS ω_{Dm}. Further, assuming only gravitational interaction between these energy components,the continuity equation (2.17) being satisfied by each component separately, and in view of (2.18), this gives

$$\rho \equiv \rho_r + \rho_m + \rho_{DE} = \rho_{r0} a^{-4} + \rho_{m0} a^{-3} + \rho_{DE0} a^{-3(1+w_{DE0})} \tag{2.19}$$

Furthermore, the Einstein field equations (2.15) yield the following set of differential equations for the metric (2.14):

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{B}}{B}\frac{\dot{C}}{C}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}=8\pi G\rho \tag{2.20}$$

$$-\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}-\frac{\dot{B}}{B}\frac{\dot{C}}{C}=8\pi G\rho \tag{2.21}$$

$$-\frac{\ddot{A}}{A}-\frac{\ddot{C}}{C}-\frac{\dot{A}}{A}\frac{\dot{C}}{C}=8\pi G\rho \tag{2.22}$$

$$-\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}-\frac{\dot{A}}{A}\frac{\dot{B}}{B}=8\pi G\rho \tag{2.23}$$

The shear scalar can be expressed in terms of the directional Hubble parameters as follows:

$$\sigma^2=\frac{(H_x-H_y)^2+(H_y-H_z)^2+(H_z-H_x)^2}{6} \tag{2.24}$$

It is possible to rewrite equations (2.20)–(2.23) as follows:

$$3H^2-\sigma^2=8\pi G\rho \tag{2.25}$$

$$\dot{H}_x-\dot{H}_y+3H(H_x-H_y)=0 \tag{2.26}$$

$$\dot{H}_y-\dot{H}_z+3H(H_y-H_z)=0 \tag{2.27}$$

$$\dot{H}_z-\dot{H}_x+3H(H_z-H_x=0 \tag{2.28}$$

Equations (2.26–2.28) and the time derivative of σ^2 given in Eq. (2.24) are used to obtain the following shear propagation equation:

$$\dot{\sigma}+3H\sigma=0 \tag{2.29}$$

its integration further leads to

$$\sigma^2=\sigma^2a^{-6} \tag{2.30}$$

Using equations (2.19) and (2.30) into (2.25), we obtain

$$\frac{H^2}{H_0^2} = \Omega_{\sigma 0}a^{-6} + \Omega_{r0}a^{-4} + \Omega_{m0}a^{-3} + \Omega_{de}a^{-3(1+\omega_{de0})} \tag{2.31}$$

where $\Omega_{\sigma 0} = \frac{\sigma_0^2}{3H_0^2}$, $\Omega_{r0} = \frac{8\pi G}{3H_0^2}\rho_{r0}$, $\Omega_{m0} = \frac{8\pi G}{3H_0^2}\rho_{m0}$ and $\Omega_{de} = \frac{8\pi G}{3H_0^2}\rho_{de0}$, respectively denote the expansion anisotropy, radiation, matter, and DE density parameters, satisfying $\Omega_{\sigma 0} + \Omega_{r0} + \Omega_{m0} + \Omega_{de0} = 1$. Here, the expansion anisotropy parameter $\Omega_{\sigma 0} = 0$ is purely a geometric term which is non-negative. Moreover, the spatially flat and homogenous but likely non-isotropic Universe is depicted by the generalized Friedmann equation (2.24), because of the existence of the expansion anisotropy.

In the absence of expansion anisotropy, equation (2.31) reduces to

$$\frac{H^2}{H_0^2} = \Omega_{r0}a^{-4} + \Omega_{m0}a^{-3} + \Omega_{de}a^{-3(1+\omega_{de0})} \tag{2.32}$$

It is an isotropic extension of ΛCDM cosmology

To derive the Hubble parameter $H(z)$ with anisotropy in terms of modified parameter and redshift we start with the modified matter density parameter for the anisotropic ΛCDM model. We'll follow the steps to include anisotropy effects on the matter and dark energy density parameters and then derive the expression for $H(z)$.

Modified Matter Density Parameter; The matter density parameter with anisotropy is given by:

$$\Omega_{m,\text{aniso}} = \Omega_m\left(1 + \sigma_0(1+z)^2\right)$$

Modified Dark Energy Density Parameter; for consistency, we assume the anisotropy affects the dark energy density parameter in a similar way as:

$$\Omega_{\Lambda,\text{aniso}} = \Omega_\Lambda\left(1 + \sigma_0(1+z)^2\right)$$

The general form of the Hubble parameter in a cosmological model is:

$$H(z) = H_0\sqrt{\Omega_{m,\text{aniso}}(1+z)^3 + \Omega_{\Lambda,\text{aniso}}} \tag{2.33}$$

Substitute the anisotropic parameters:

$$H(z) = H_0\sqrt{\left[\Omega_m\left(1+\sigma_0(1+z)^2\right)\right](1+z)^3 + \Omega_\Lambda\left(1+\sigma_0(1+z)^2\right)}$$

then by Expand and simplify the expression:

$$H(z) = H_0\sqrt{\Omega_m(1+\sigma_0(1+z)^2)(1+z)^3 + \Omega_\Lambda(1+\sigma_0(1+z)^2)}$$

$$H(z) = H_0\sqrt{\Omega_m\left[(1+z)^3 + \sigma_0(1+z)^5\right] + \Omega_\Lambda(1+\sigma_0(1+z)^2)}$$

then lets separate terms for clarity:

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_m\sigma_0(1+z)^5 + \Omega_\Lambda + \Omega_\Lambda\sigma_0(1+z)^2}$$

Thus, the Hubble parameter with anisotropy is:

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_m\sigma_0(1+z)^5 + \Omega_\Lambda + \Omega_\Lambda\sigma_0(1+z)^2}$$

The extended ΛCDM model modifies the Hubble parameter by including terms that account for the effects of anisotropy. The key additional terms $\Omega_m\sigma_0(1+z)^5$ and $\Omega_\Lambda\sigma_0(1+z)^2$ represent the influence of anisotropy on the densities of matter and dark energy, respectively. This modification reflects how the expansion rate of the universe is altered when considering anisotropic effects in the dark sector. The Hubble parameter $H(z)$ as a function of redshift z without including anisotropic term is given by:

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda} \tag{2.34}$$

where:

- H_0 is the Hubble constant.
- Ω_m is the matter density parameter.
- Ω_Λ is the dark energy density parameter.

2.4.4 ΛCDM and temperature anisotropy

At its essence, temperature anisotropy denotes the spatial variance in temperature across cosmic scales, serving as a profound indicator of underlying cosmological dynamics. The cosmic microwave background (CMB) radiation, a relic of the early universe, stands as the quintessential canvas upon which temperature anisotropy is meticulously observed and analyzed. Inflationary cosmology provides a theoretical framework for this analysis, predicting specific statistical properties of temperature fluctuations. This allows direct comparisons with observational data. Experiments such as the Wilkinson microwave anisotropy probe and Planck satellite mission have revolutionized our understanding of temperature anisotropy, revealing intricate patterns of temperature fluctuations across multiple scales [Hu and Dodelson, 2002]. Inflationary cosmology provides a theoretical framework for this analysis, predicting specific statistical properties of temperature fluctuations. The small temperature fluctuations observed in the Cosmic Microwave Background (CMBR) across the sky, which are not uniform in all directions, are indicative of inhomogeneities and anisotropies in the universe at the time of recombination. These fluctuations are consistent with the predictions of the ΛCDM (Lambda Cold Dark Matter) model, which posits that the universe is inhomogeneous at smaller scales compared to its statistical homogeneity and isotropy on scales larger than approximately 250 million light years. The observed CMBR fluctuations are believed to be the result of quantum fluctuations in the early universe that were amplified by inflation, a period of rapid expansion that occurred shortly after the Big Bang. Inflation is thought to have stretched and smoothed out the universe, but it also generated small, random fluctuations in the density of matter and energy. These fluctuations later evolved into the large-scale structure of the universe, including galaxies and galaxy clusters.The CMBR temperature fluctuations are measured using instruments such as the Cosmic Background Explorer (COBE), the Wilkinson Microwave Anisotropy Probe (WMAP), and the Planck satellite. These observations have provided strong evidence for the ΛCDM model and have helped to refine our understanding of the universe's evolution [Novosyadlyj et al., 2015, Hinshaw et al., 2003][4].

In order to compare theoretical predictions of temperature anisotropies with observational data, it is convenient to define the angular power spectrum. One can expand temperature anisotropies in terms of multipole components as $\triangle T/T \equiv \sum_{l=1}^{\infty}\sum_{m=-l}^{\infty} a_{lm}Y_{lm}$. The angular power spectrum C_l is defined by the ensemble average of the square of the coefficient a_{lm} as

$$\langle a_{lm}a^*_{lm}\rangle \equiv C_l \delta_{ll'}\delta_{mm'}. \tag{2.35}$$

Accordingly, the ensemble average of square temperature fluctuations is written as

$$\langle |\triangle T/T|^2\rangle = \sum_{ll'mm'} \langle a_{lm}a^*_{lm}\rangle \int dcos\theta \int d\phi Y_{lm}(\theta,\phi)Y^*_{lm}(\theta,\phi) = \sum_{l}^{\infty} \frac{2l+1}{4\pi} C_l \tag{2.36}$$

[4] https://www.science.gov/topicpages/w/wall+temperature+fluctuations

Note here a corresponding angular scale θ for a given is $\theta(degree) = 180/l$ from the above equation the amplitude of temperature fluctuations in logarithmic space can be described as $l(2l+1)C_l/4\pi$. [Sugiyama, 2014]

The cosmic microwave temperature fluctuations from the five years of

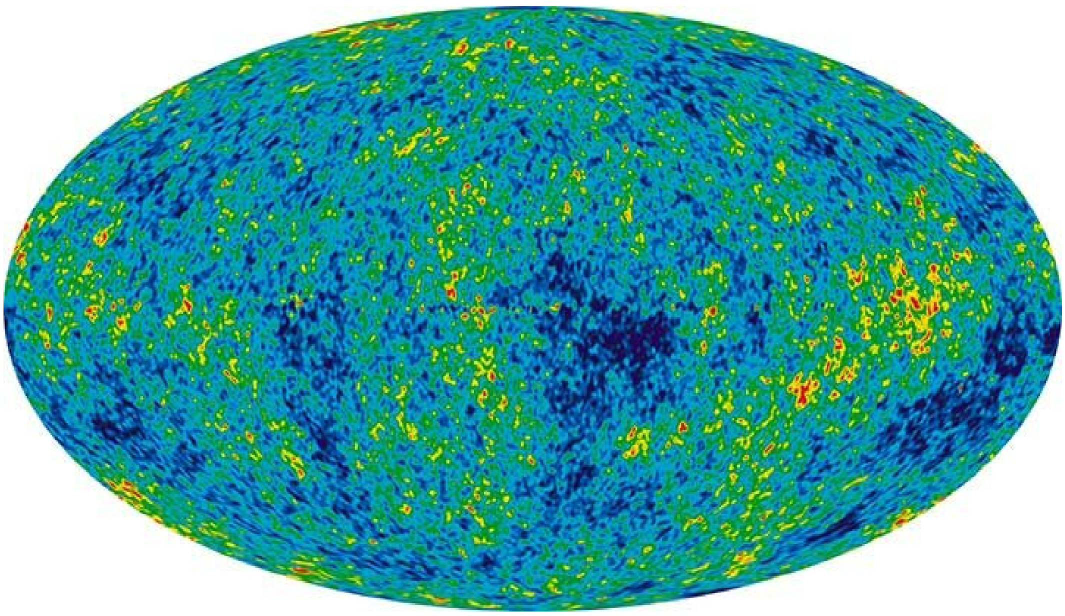

Figure 2.1: Five year WMAP image of background cosmic radiation (2010) [Saha, 2014]

WMAP data are seen throughout the sky in figure (2.1) above. Similar to a weather map, the colors show the minute temperature variations, with an average temperature of 2.725 Kelvin. There is a 0.0002 degree difference in temperature between the blue and red zones, where red region represent warmer zone and blue cregion represents colder zone. The ΛCDM model successfully explains the observed temperature anisotropies in the Cosmic Microwave Background (CMB). These anisotropies are small variations in the CMB temperature across the sky, which provide crucial information about the early universe. The model assumes that the universe was nearly homogeneous and isotropic on large scales in its early stages. Tiny quantum fluctuations in the density of matter and radiation during inflation led to the observed temperature anisotropies in the CMB.[5]

As the universe expanded and cooled, these small density variations grew under the influence of gravity, forming the large-scale structure we see today, such as galaxies and galaxy clusters. The ΛCDM model accurately predicts the statistical properties of the CMB temperature anisotropies, which have been measured with high precision by experiments like COBE, WMAP, and Planck. The model's predictions for the CMB anisotropies are in excellent agreement with observations, making it a cornerstone of modern cosmology. The temperature anisotropies provide strong evidence for the ΛCDM model's key components, such as dark matter and dark energy, and support the idea that the universe began with a hot, dense state known as the Big Bang [Avsajanishvili et al., 2024].

2.4.5 ΛCDM and gravitational anisotropy

The ΛCDM model assumes that the universe is isotropic on large scales, meaning it looks the same in all directions. However, observational results have revealed the presence of gravitational anisotropies, which pose challenges to this assumption. Gravitational anisotropy refers to directional variations or asymmetries in the gravitational field within the universe, manifesting as deviations from isotropy in the distribution of matter and energy. Analyses of the cosmic microwave background (CMB) radiation, such as those conducted by the Planck satellite, have revealed subtle deviations from isotropy in the gravitational field on large scales. These deviations, known as gravitational anisotropies, can be observed through variations in the CMB temperature and polarization across dif-

[5] https://astro.ucla.edu/~wright/CMB-DT.html

ferent regions of the sky [Vagnozzi, 2021]. Observational studies have also found apparent correlations between the plane of the Solar System, the rotation of galaxies, and certain aspects of the CMB, suggesting a violation of the Copernican principle and cosmological principle. he ΛCDM model assumes that the data and interpretation of the CMB are correct. However, the observed gravitational anisotropies challenge this assumption. The model predicts that the universe should be isotropic on large scales, but observational results indicate the presence of directional variations in the gravitational field [Aluri et al., 2023]. These anisotropies are not well accounted for by the ΛCDM model, which is less well constrained on sub-galactic scales [Aluri et al., 2023]. The gravitational interaction between dark matter and dark energy is a major contribution to the present universe [Amirhashchi et al., 2022].

The mathematical expression for the calculation of gravitational anisotropy in the dark sector involves the use of Einstein's field equations, which describe the curvature of spacetime due to the presence of matter and energy. The field equations can be written as:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{2.37}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, R is the scalar curvature, $g_{\mu\nu}$ is the metric tensor, $T_{\mu\nu}$ is the stress-energy tensor, G is the gravitational constant [Redi et al., 2021]. Several studies have been conducted to explore gravitational anisotropy in the dark sector of the universe. for instance [Redi et al., 2021] examined the production of a conformal dark sector in the relativistic regime where the dark sector is approximately scale providing general analytical formulas that depend solely on the central charge of the dark sector. They also assessed the relevance of interactions that can lead to a variety of phenomena including non-perturbative mass gaps, out of equilibrium phase transitions, and cannibalism in the dark sector.

2.4.6 Magnetic Monopole

Remarkably resilient has been the hypothesis that magnetic monopoles steady particles bearing magnetic charges should exist. Dirac presented the first convincing case in 1931 [Dirac, 1931]. He pointed out that all electric charges have to be integer multiples of a fundamental unit in order for electric charge to be quantized if monopoles are present. Nature really exhibits electric charge quantization, and until recently, there was no other explanation for this profound occurrence. Polyakov and 't Hooft discovered that general theories on fundamental interactions lead to the presence of monopoles, which are essential components of any "grand unified" theory of particle physics. This revelation has profound ramifications still being investigated. Hooft and Polyakov showed the need for monopoles in grand unified theories, whereas Dirac had shown how magnetic monopoles were consistent with quantum electrodynamics. In addition, in a given unified model, the monopole's attributes are predictable and calculable. [Preskill, 1984]. monopoles around today must have been produced in the very early universe, when higher energies were available. Thus, the abundance of magnetic monopoles is a cosmological issue. It is anticipated that the cosmos underwent a phase change at a critical temperature T_c; of order the unification mass scale M_x, as it cooled. The whole, grand unified gauge symmetry was restored [Guth and Weinberg, 1981] at temperatures greater than T_c; the scalar field Φ, which serves as an order parameter for the gauge symmetry breakdown, had a vanishing expectation value. However, monopoles are only possible when the gauge symmetry breaks spontaneously, which means that when T was higher than T_c, monopoles were absent. The expected value of Φ activated and monopole manufacturing was made feasible when T dropped below T_c. Monopoles are produced through phase transitions, either second-order or strongly first-order, which involve large fluctuations or super cooling. The abundance of monopoles is not in thermal equilibrium. In a second-order phase transition, the scalar field Φ undergoes random fluctuations, leading to the creation of monopoles and antimonopoles. The quenching process occurs when the scalar field is uncorrelated over distances larger than a certain correlation length [Kibble, 1976].

- Classical Magnetic monopoles

Dirac determined the relationship between the elementary electric charge (e) and the magnetic charge (g) in his 1931 article [Dirac, 1931, Dirac, 1948].

$$g = ng_D = \frac{1}{2}\frac{\hbar c}{e} \sim \frac{137}{2}en \tag{2.38}$$

where n= 1,2,3... and g_D is the dirac unit magnetic charge.

- supermassive magnetic monopoles

According to GUT, magnetic monopoles with magnetic charges of $g = ng_D$ emerge after the phase transition that coincides with the spontaneous splitting of the Grand Unified group into subgroups, including $U(1)$, at a cosmic period of $\sim 10^{-34}$ s after the Big Bang. The mass of GUT MMs is huge $M_M \sim 10^{16} - 10^{17}$ GeVc^{-2}

- Intermediate mass monopoles

In theories with an intermediate mass scale [Huguet and Peter, 2000], MMs with masses of $M_M \sim 10^5 - 10^{13}$ GeVc^{-2} could have been produced at later phase transitions in the Early Universe [Kephart and Shafi, 2001]. Intermediate mass MMs could be accelerated to relativistic velocities by galactic magnetic fields.

2.5 Other models

2.5.1 f(R) Gravity

f(R) gravity is a modification of Einstein's General Relativity that aims to explain the accelerated expansion of the universe without invoking dark energy. In this theory, the Ricci scalar R in the Einstein-Hilbert action is replaced by a general function $f(R)$. This modification allows for a broader range of gravitational dynamics, offering a potential solution to various cosmological and astrophysical challenges [De Felice and Tsujikawa, 2010]

The field equations of f (R) gravity in the metric formalism is given by:

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,f(R) + \int d^4x\,\mathscr{L}_M(g_{\mu\nu}, \Psi_M) \tag{2.39}$$

where $\kappa^2 = 8\pi G$, g is the determinant of the metric $g_{\mu\nu}$, and $\mathscr{L}_M$ is a matter Lagrangian that depends on $g_{\mu\nu}$ and matter fields Ψ_M. The Ricci scalar R is defined by $R = g^{\mu\nu}R_{\mu\nu}$, where the Ricci tensor $R_{\mu\nu}$ is

$$R_{\mu\nu} = R^{\alpha}_{\mu\alpha\nu} = \partial_\lambda\Gamma^\lambda_{\mu\nu} - \partial_\mu\Gamma^\lambda_{\lambda\nu} + \Gamma^\lambda_{\mu\nu}\Gamma^\rho_{\rho\lambda} - \Gamma^\lambda_{\nu\rho}\Gamma^\rho_{\mu\lambda}$$

In the case of the torsion-less metric formalism, the connections $\Gamma^\alpha_{\beta\gamma}$ are the usual metric connec tions defined in terms of the metric tensor $g_{\mu\nu}$, as

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\lambda}\left(\frac{\partial g_{\gamma\lambda}}{\partial x^{\beta}} + \frac{\partial g_{\lambda\beta}}{\partial x^{\gamma}} - \frac{\partial g_{\beta\gamma}}{\partial x^{\lambda}}\right) \tag{2.40}$$

This follows from the metricity relation,$\nabla_{\lambda} g_{\mu\nu} = \frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} - g_{\rho\nu}\Gamma^{\rho}_{\mu\lambda} - g_{\mu\rho}\Gamma^{\rho}_{\nu\lambda} = 0$

2.5.2 Equations of motion in f(R) Gravity

The field equation can be derived by varying the action (2.39) with respect to $g_{\mu\nu}$

$$\Sigma_{\mu\nu} \equiv F(R)R_{\mu\nu}(g) - \frac{1}{2} f(R) g_{\mu\nu} - \nabla_{\mu}\nabla_{\nu} F(R) + g_{\mu\nu}\Box F(R) = \kappa^2 T^{(M)}_{\mu\nu} \tag{2.41}$$

where $F(R) = \frac{\partial f}{\partial R}$. $T^{(M)}_{\mu\nu}$ is the energy-momentum tensor of the matter fields defined by the variational derivative of $\mathscr{L}_M$in terms of $g^{\mu\nu}$

$$T^{(M)}_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta \mathscr{L}_M}{\delta g^{\mu\nu}} \tag{2.42}$$

This satisfies the continuity equation

$$\nabla^{\mu} T^{(M)}_{\mu\nu} = 0 \tag{2.43}$$

as well as $\Sigma_{\mu\nu}$, i.e., $\nabla\Sigma_{\mu\nu} = 0$.[6] The trace of Eq. (2.41) gives;

$$3\Box F(R) + F(R)R - 2f(R) = \kappa^2 T \tag{2.44}$$

where $T = g^{\mu\nu} T^{(M)}_{\mu\nu}$ and $\Box F = \left(\frac{1}{\sqrt{-g}}\right)\partial_{\mu}\left(\sqrt{-g} g^{\mu\nu}\partial_{\nu} F\right)$, In the absence of the cosmological constant, Einstein gravity is equivalent to $f(R) = R$ and

[6] http://www.livingreviews.org/lrr-2010-3

$f(R) = 1$, meaning that the word $\Box F(R)$ in Eq. (2.44) disappears. Since $R = -\kappa^2 T =$ in this instance, the matter (the trace T) immediately determines the Ricci scalar R.

The field equation (2.41) can be written in the following form [De Felice and Tsujikawa, 2010]

$$G_{\mu\nu} = \kappa^2 \left(T^{(M)}_{\mu\nu} + T^{(D)}_{\mu\nu} \right) \tag{2.45}$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ and

$$\kappa^2 T^{(D)}_{\mu\nu} \equiv \frac{1}{2} g_{\mu\nu}(f - R) + \nabla_\mu \nabla_\nu F - g_{\mu\nu} \Box F + (1 - F) R_{\mu\nu} \tag{2.46}$$

since $\nabla G_{\mu\nu} = 0$ and condition in equation (2.43) it follow that

$$\nabla^\mu T^{(D)}_{\mu\nu} = 0 \tag{2.47}$$

2.5.3 f(R) Gravity model and anisotropic problems

An anisotropic consideration of the $f(R)$ gravity model involves studying its effects on anisotropic cosmological solutions. Anisotropy in cosmology refers to the idea that the universe may have different expansion rates along different spatial directions [Loo et al., 2023]. Introducing anisotropy into the $f(R)$ gravity model involves considering a metric that allows for different expansion rates along different spatial directions. One common choice is to employ Bianchi models, which are spatially homogeneous but anisotropic cosmological solutions to the Einstein field equations

the mathematical calculation and how $f(R)$ gravity deals with anisotropic problems is metric that describes the geometry of the universe. In the case of Bianchi models, the line element can be written as:

$$ds^2 = -dt^2 + a_1^2(t)dx^2 + a_2^2(t)dy^2 + a_3^2(t)dz^2 \tag{2.48}$$

where $a_i(t)$ are the scale factors that depend on time and describe the expansion along the x, y, and z directions, respectively.

To solve the field equations for $f(R)$ gravity in the presence of an anisotropic metric, we need to handle the coupled, nonlinear differential equations that govern the dynamics of the scale factors $a_i(t)$. We'll consider $f(R) = R + \alpha R^2$ and use the Bianchi type I metric as an example in which the matric is given by equation(2.48) and The Ricci scalar R for this metric is given by:

$$R = 2\left(\frac{\ddot{a_1}}{a_1} + \frac{\ddot{a_2}}{a_2} + \frac{\ddot{a_3}}{a_3}\right) + \left(\frac{\dot{a_1}\dot{a_2}}{a_1a_2} + \frac{\dot{a_1}\dot{a_3}}{a_1a_3} + \frac{\dot{a_2}\dot{a_3}}{a_2a_3}\right)$$

Field Equations For $f(R) = R + \alpha R^2$, the field equations are:

$$(1+2\alpha R)R_{\mu\nu} - \frac{1}{2}(R+\alpha R^2)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)(1+2\alpha R) = 8\pi G T_{\mu\nu}$$

derivatives and Ansatz; Let's simplify by assuming a perfect fluid with $T_{\mu\nu} = \mathrm{diag}(\rho, -p, -p, -p)$. We need to express the second derivatives of $a_i(t)$ in terms of $a_i(t)$, $\dot{a}_i(t)$, and $\ddot{a}_i(t)$.

for the coupled ODEs From the above expressions, we derive the equations for $\ddot{a}_i$:

$$\ddot{a}_1 = -\frac{\dot{a}_1\dot{a}_2}{a_1a_2} - \frac{\dot{a}_1\dot{a}_3}{a_1a_3} - \alpha R\frac{\dot{a}_1}{a_1}$$
$$\ddot{a}_2 = -\frac{\dot{a}_2\dot{a}_1}{a_2a_1} - \frac{\dot{a}_2\dot{a}_3}{a_2a_3} - \alpha R\frac{\dot{a}_2}{a_2}$$
$$\ddot{a}_3 = -\frac{\dot{a}_3\dot{a}_1}{a_3a_1} - \frac{\dot{a}_3\dot{a}_2}{a_3a_2} - \alpha R\frac{\dot{a}_3}{a_3}$$

Initial Conditions: Using the the following initial conditions:

$$a_1(0) = a_2(0) = a_3(0) = 1$$

$$\dot{a}_1(0) = H_1, \quad \dot{a}_2(0) = H_2, \quad \dot{a}_3(0) = H_3$$

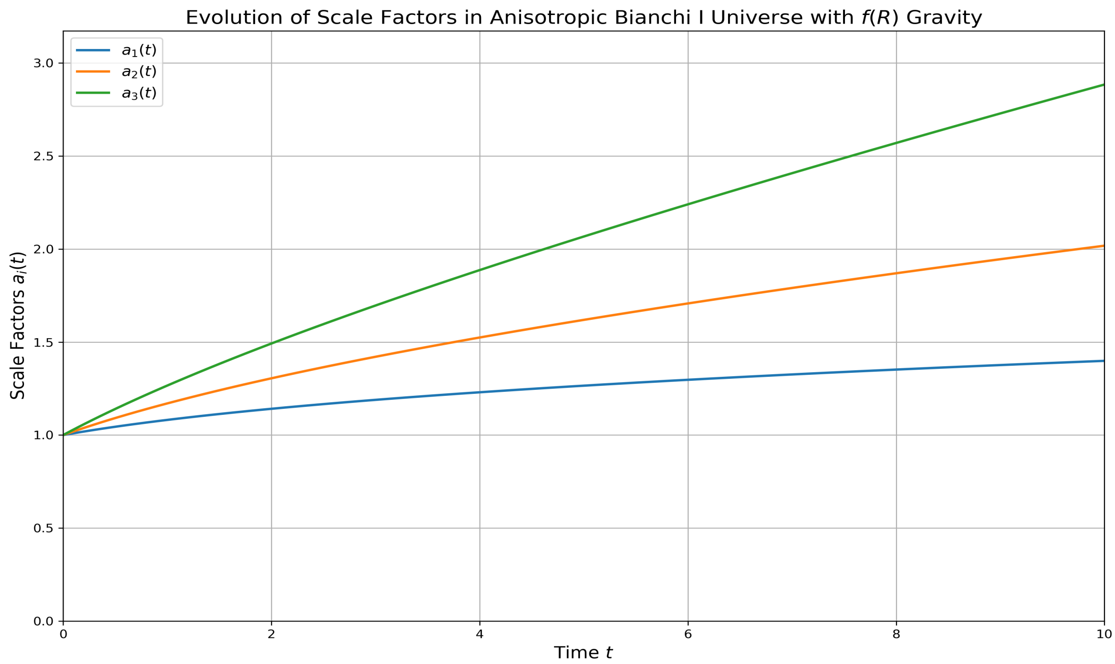

Figure 2.2: Evolution of Scale Factors in Anisotropic Bianchi I Universe with $f(R)$ Gravity

Hubble Parameter in Terms of Redshift for Anisotropic $f(R)$ Gravity can be expressed as:

$$\Omega_{m,\text{aniso}} = \Omega_m \left(1 + \sigma_0 (1+z)^2\right) \tag{2.49}$$

The modified higher-order term parameter is given by:

$$\alpha_{\text{aniso}} = \alpha \left(1 + \sigma_0 (1+z)^2\right) \tag{2.50}$$

Hubble Parameter with Anisotropy in $f(R)$ Gravity:

$$H(z) = H_0 \sqrt{\Omega_{m,\text{aniso}}(1+z)^3 + \alpha_{\text{aniso}}(1+z)^6} \tag{2.51}$$

Simplifying further, we get:

$$H(z) = H_0 \sqrt{\Omega_m (1+z)^3 \left(1+\sigma_0(1+z)^2\right) + \alpha(1+z)^6 \left(1+\sigma_0(1+z)^2\right)} \tag{2.52}$$

The Hubble constant, H_0, is the current value of the Hubble parameter, indicating the present expansion rate of the universe. Matter Density Parameter $\Omega_{m,\text{aniso}}$

$$\Omega_{m,\text{aniso}} = \Omega_m \left(1+\sigma_0(1+z)^2\right)$$

This term represents the matter density in the universe, adjusted for anisotropy. The factor $\left(1+\sigma_0(1+z)^2\right)$ accounts for the variation in density due to anisotropic effects as a function of redshift z.

Higher-Order Term Parameter α_{aniso}

$$\alpha_{\text{aniso}} = \alpha \left(1+\sigma_0(1+z)^2\right)$$

This term represents the contribution of higher-order corrections from $f(R)$ gravity, modified to include anisotropic effects.

Matter Density Contribution $\Omega_{m,\text{aniso}}(1+z)^3$

$$\Omega_{m,\text{aniso}}(1+z)^3 = \Omega_m \left(1+\sigma_0(1+z)^2\right)(1+z)^3$$

This term shows how the matter density evolves with redshift, including the effect of anisotropy. The factor $(1+z)^3$ arises from the scaling of matter density with the expansion of the universe. Higher-Order Term Contribution $\alpha_{\text{aniso}}(1+z)^6$

$$\alpha_{\text{aniso}}(1+z)^6 = \alpha\left(1+\sigma_0(1+z)^2\right)(1+z)^6$$

This term represents the evolution of the higher-order term from $f(R)$ gravity with redshift. The factor $(1+z)^6$ is typical for such terms in modified gravity models, and it is adjusted here for anisotropic effects. In the $f(R)$ gravity model, the Hubble parameter $H(z)$ as a function of redshift z without including anisotropic term or for isotropic universe is given by:

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \alpha(1+z)^6} \tag{2.53}$$

where:

- H_0 is the Hubble constant.
- Ω_m is the matter density parameter.
- α is a parameter that quantifies the contribution of the higher-order curvature terms in $f(R)$ gravity.

Chapter 3

3 Dark sector of the universe and anisotropic consideration

3.1 A comprehensive view of cosmological Dark Side

Understanding the nature of Dark Energy (DE) and Dark Matter (DM) is now one of physics' biggest difficulties. These are basic problems in particle physics and cosmology, and despite a plethora of theories and proposals, no truly suitable theory or model has been found up to this point, leaving DM and DE devoid of a conclusive explanation at the fundamental level [Capozziello and Lambiase, 2013]. In the attempt to explain the huge amount of recent observational data and more important try to preserve the conceptual structure of General Relativity, cosmologists had the necessity to introduce the two new fundamental concepts of Dark Matter and Dark Energy and observations have provided evidence for the presence of dark matter for more than eight decades. Numerous astronomical data points have since been used to test and validate it[7]. The motion of gas and stars in the outer regions of galaxies was observed in the 1970s, and theoretical arguments demonstrated that the local group and disk galaxies require significant dark matter halos for their dynamical stability, building on the early pioneering observations by Fritz Zwicky in 1933 that the velocities of galaxies in the Coma cluster greatly exceeded the expectations based solely on the sum of the individual galaxy masses, thus

[7] https://www.cfa.harvard.edu/research/topic/dark-energy-and-dark-matter

requiring significant additional "dark matter," [Einasto et al., 1974, Ostriker and Peebles, 1973]. The amount and distribution of dark matter on large scales were further revealed by more in-depth observations of galaxy clusters and large-scale structure, which demonstrated that the universe's total mass density is approximately 25% of the critical density [Bahcall and Cen, 1992, Bahcall et al., 1995].The gravitational pull of DM on the universe is the source of all available evidence for it.

3.1.1 Baryonic matter

Common visible (or measurable) matter, such stars, galaxies, and gas, is included in the universe's baryonic contents. However, dark matter and dark energy make up the majority of the universe's matter, which is invisible to the naked eye. The cosmos is composed of dark energy (about 68% of the universe), dark matter (about 26.8%), and visible baryonic matter (which includes gas and stars) (about 5% of the universe), according to the most recent calculations[8].

3.1.2 Dark matter

The dark matter is non baryonic because baryonic matter interacts with itself too strongly to match what we observe. For example, two collections of baryonic matter even gases, unless they are very sparse tend to run into each other, not pass through each other.

The mass-energy content of the universe is composed of 5% ordinary matter, 26.8% dark matter, and 68% dark energy, according to the mainstream lambda-CDM model of cosmology [Clavin and Harrington, 2013, Spergel,

[8] https://en.wikipedia.org/wiki/Universe

2015, Collaboration et al., 2013, Francis, 2013].

Dark matter appears to behave differently. the baryons in the universe of discourse constitute only $\Omega_b \sim 0.045$ of the indispensable density, supported on observations of the deuterium and unusual light element abundances [Pettini and Cooke, 2012],as well as on the precise observations of the cosmic microwave background(CMB) spectrum of density fluctuations and its heavy particle acoustic oscillations [Collaboration et al., 2013, Bennett et al., 2013]. Thus, because the add together mass density of the universe of discourse greatly exceeds the baryonic mass, with $\Omega_b \sim 0.25$ versus $\Omega_b \sim 0.045$, the dark matter has to be non baryonic(with $\Omega_b \sim 0.2$). [Not whole of the baryons in the universe shine; some baryons are"dark,"such as stellar remnants, planets, rocks, etc.;these"dark baryons"are disunite of the $\Omega_b = 0.045$ baryonic budget, not the dark matter.]The Holocene observations of the CMB [Bennett et al., 2013] beautifully support whole of the above findings and play considerably more precise determination of the mass density of both the dark and baryonic matter. There are three classes of elementary particle dark matter candidates, which are called hot, warm, and cold. [Primack, 1997].

Hot dark matter: Hot dark matter (HDM) refers to particles that were still in thermal equilibrium with the rest of the matter in the early universe after the QCD deconfinement transition, which occurred at a temperature of about 10^2 MeV. This means that HDM particles were relativistic, and their distribution was determined by the laws of thermodynamics. HDM particles are typically light, with masses less than about 100 GeV. Some examples of HDM particles include neutrinos, axions, and sterile neutrinos. Neutrinos are the most abundant type of HDM particle, and they make up about 10% of the total mass-energy density of the universe. Axions are hypothetical particles that were proposed to solve the strong problem in quantum chromo dynamics. Sterile neutrinos are hypothetical particles that are not involved in the weak interaction. HDM particles are important for understanding the evolution of the universe. They contribute to the expansion rate of the universe, and they can affect the formation of galaxies and other large-scale structures [Primack and Blumenthal, 1983]. Hot dark matter (HDM) particles have a cosmological number density that is roughly comparable to that of the microwave background photons. This implies that the mass of HDM particles must be less than a few tens of eV. However, free streaming of HDM particles destroys any perturbations smaller than super cluster size, which is about$\sim 10^{15}$ Mpc. This means that HDM particles cannot be responsible for the formation of galaxies and other small-scale structures. As a result, HDM particles are not considered to be a viable candidate for dark matter [Primack and Blumenthal, 1983].

warm dark matter: Warm dark matter (WDM) particles interact much more weakly than neutrinos. They decouple from the rest of the matter in the early universe at a temperature that is higher than the QCD deconfinement temperature. This means that WDM particles are non-relativistic, and their distribution is not determined by the laws of thermodynamics. WDM particles are typically heavier than HDM particles, with masses in the range of 10-100 GeV. WDM particles can survive free streaming, which means that they can form small-scale structures, such as galaxies and galaxy clusters. This makes them a viable candidate for dark matter. Some examples of WDM particles include axions, sterile neutrinos, and gravitinos. Axions are hypothetical particles that were proposed to solve the strong problem in quantum chromodynamics. Sterile neutrinos are hypothetical particles that do not interact with the weak interaction. Gravitinos are hypothetical particles that are the super partners of gravitons.WDM particles are important for understanding the evolution of the universe. They contribute to the expansion rate of the universe, and they can affect the formation of galaxies and other large-scale structures [Primack and Blumenthal, 1983].

cold dark matter: Cold dark matter (CDM) consists of particles that do not interact with the rest of the matter in the universe. This means that CDM particles do not experience free streaming, and they can form large-scale structures, such as galaxies and galaxy clusters. There are two main types of CDM particles: cold Bose condensates and heavy remnants of annihilation or decay. Cold Bose condensates are particles that are bosons, and they have very low masses. They can form a Bose-Einstein condensate, which is a state of matter in which all of the particles are in the same quantum state. This makes them very difficult to detect. Heavy remnants of annihilation or decay are particles that are produced by the annihilation or decay of other particles. They are typically much heavier than cold Bose condensates, and they can be detected through their gravitational effects. CDM particles are important for understanding the evolution of the universe. They contribute to the expansion rate of the universe, and they can affect the formation of galaxies and other large-scale structures [Primack and Blumenthal, 1983].

3.1.3 Dark matter properties

Some key properties of dark matter include:

- General Principles

  The gravitational influence of an arbitrary dark matter component is controlled by its stress-energy tensor $T_{\mu\nu}(x,\eta)$,where $\eta = \int dt/a$ is the conformal time with a(t) as the scale factor normalized to unity today. In general, the symmetric four-tensor can be divided into four classes: the energy density, the isotropic stress or pressure, the momentum density, and the anisotropic stress. The anisotropic stress can be further classified based on how they transform under rotations into three categories: a scalar component, vector components, and tensor components. It is important to note that only the scalar components display gravitational instability [Hu, 1998].

- Non-baryonic

  Dark matter is not made of baryons, which are particles like protons and neutrons that comprise ordinary matter [Trimble, 1987]

- Non-luminous

  It does not emit, reflect, or absorb light, or any other electromagnetic radiation, making it invisible [Trimble, 1987]

- Gravity-interacting

  Dark matter interacts gravitationally with ordinary matter and light Collisionless: Dark matter particles do not interact with each other or other particles via strong or weak forces [Trimble, 1987][9].

3.2 Dark energy

Dark energy is a mysterious force that is causing the expansion of the universe to accelerate. It is thought to make up about 68% of the universe, and it is the most dominant form of energy in the universe. Dark energy is not well understood, but it is thought to be a type of energy that has negative pressure. This means that it pushes space apart, causing the universe to expand faster and faster. The discovery of dark energy was a major surprise to physicists. They had expected that the expansion of the universe would slow down over time, but instead, it is accelerating. This has lead to a number of new theories about the nature of dark energy and its role in the universe. One possibility is that dark energy is a new type of field that permeates the entire universe. This field could be responsible for the acceleration of the universe, but it is not yet known what it is or how it works.

3.2.1 The Cosmological Constant and Vacuum Energy

Einstein introduced the concept of cosmological constant in 1917, which is the most basic illustration of dark energy. When the cosmological constant is present, the Einstein equations take on the following form:

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi G}{c^4} T_{ik} + \lambda g_{ik} \tag{3.1}$$

The cosmological constant (Λ), which Einstein initially included in the left-hand side of his field equations, is now typically moved to the right-

[9] https://sciencenotes.org/what-is-dark-matter/#google_vignette

hand side (RHS) in order to regard it as an effective form of matter. When Λ and pressureless dust (dark matter) are present in a homogeneous and isotropic Friedmann-Robertson-Walker (FRW) universe, the Raychaudhury equation, which is derived from (3.1), becomes

$$\ddot{a} = -\frac{4\pi G}{3} a \rho_m + \frac{\lambda}{3} \tag{3.2}$$

Equation (3.2) can be rewritten in the form of a force law:

$$F = -\frac{GM}{R^2} + \frac{\lambda}{3} R, \quad (R \equiv a) \tag{3.3}$$

This indicates that a repulsive force with a distance-dependent value is generated by the cosmological constant. As (3.2) illustrates, the cosmos may have accelerated due to the repulsive nature of Λ. Despite being first mentioned in physics in 1917, the physical foundation for a cosmological constant was not well understood until the 1960s, when it was realized that zero-point vacuum fluctuations had to have the form $T_{ik} = \Lambda g_{ik}$ in order to preserve Lorenz invariance [Zel'Dovich, 1968]. It turns out that for both bosonic and fermionic fields, the vacuum expectation value of the energy momentum is divergent, which leads to the "cosmological constant problem." In fact, the cosmic constant that is produced by vacuum fluctuations is

$$\frac{\lambda}{8\pi G} = \langle T_{00} \rangle_{\text{vac}} \propto \int_0^\infty (k^2 + m^2) k^2 \, dk \tag{3.4}$$

The vacuum energy has an infinite value since the integral diverges as k^4. The vacuum energy $\langle T_{00} \rangle_{\text{vac}} \sim \frac{c^5}{G^2} \sim 10^{76}\,\text{GeV}^4$ is still incredibly large,

even if one decides to 'regularize' T_{ik} by imposing an ultraviolet cutoff at the Planck scale. This value is 123 orders of magnitude larger than the currently observed $\rho_\Lambda \sim 10^{-47}\,\mathrm{GeV}^4$. A QCD cutoff yields $\Lambda^4_{\mathrm{QCD}} \sim 10^{-3}\,\mathrm{GeV}^4$, which is still forty orders of magnitude bigger than observed, thus a smaller ultraviolet cut-off does not fare any better.
The discovery of supersymmetry in the 1970s raised hopes that a careful balance between bosons and fermions in nature would solve the cosmological constant problem because bosons and fermions (of identical mass) contribute equally but with opposite signs to the vacuum expectation value of physical quantities. The cosmological constant should disappear in the early cosmos but reemerge in later ages when the temperature has fallen below T_{SUSY}, as supersymmetry (if it exists) is destroyed at the low temperatures that the universe is experiencing today. This is obviously an unfavorable situation, and nearly the exact reverse of what is desired, as a very small present value of Λ is consistent with observations, but a big value of Λ at an early time is useful from the perspective of inflation [Sahni, 2002].

The multiverse, an internally growing fractal, is the new cosmological paradigm. The multiverse includes a large number of parts with different coupling constants, masses of fundamental particles, and other natural constants. Our Universe, whose age is about 14 Gyr, is one of them. During this time, the Universe has gone through a number of stages, including inflation, reheating, the radiation-dominated stage, and the matter-dominated stage, and is now in the vacuum-dominated stage. Starting from the redshift $Z \sim 0.7$ the Universe is expanding with acceleration (at larger redshifts, the expansion decelerates). The content of th Universe is also enigmatic [Burdyuzha et al., 2010].

Supernova measurements provided the first observational evidence for the existence of dark energy. Because Type 1A supernovae have a consistent luminosity, they are useful for measuring distances precisely. The cosmos is expanding faster when this distance is compared to the redshift, which indicates how quickly the supernova is retreating [Overbye, 2017, Peebles and Ratra, 2003]. Before this discovery, scientists believed that the universe's expansion would eventually slow down due to the gravitational pull of matter and energy. Several separate lines of evidence supporting the presence of dark energy have been found since the finding of accelerated expansion[10].

3.2.2 Dynamical Models of Dark Energy

Among the forms of matter (dark energy) that could propel an accelerated period in the history of our universe is the cosmological constant. In fact, (3.1) can be broadly applied to

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum_i \left(\rho_i + 3p_i\right) = -\frac{4\pi G}{3}\sum_i \rho_i(1+3w_i) \tag{3.5}$$

where the equation of state represents the sum of all forms of matter in the cosmos.

3.2.3 Nature and Properties of Dark Energy

Dark energy is often associated with the cosmological constant, a term introduced by Albert Einstein in his equations of general relativity. The

[10] https://en.wikipedia.org/wiki/Dark_energy

nature of dark energy is often described as a mysterious negative pressure force that drives the accelerated expansion of the universe. Recent observations, such as those from the Planck satellite, have provided important information about the properties of dark energy, suggesting that it accounts for about 68% of the universe's total energy density. The density of dark energy is around $6 \times 10^{-10} J/m^3$ ($-7 \times 10^{-30} g/cm^3$), which is far smaller than the density of dark matter or regular matter in galaxies. However, because of its uniformity throughout space, it dominates the mass–energy content of the cosmos. [Steinhardt and Turok, 2006][[11]].

3.3 Dark matter and Dark energy interaction

It is true that the anisotropic character of dark energy and dark matter interactions can affect the dynamics of the cosmos. This may cause the cosmos to evolve anisotropic structures, which could have an impact on how matter and energy are distributed in the future. Numerous facets of this interaction have been studied, such as the impact of dark energy anisotropic stress on the matter power spectrum, the coupling between dark energy and dark matter, and the implications of anisotropic stress in interacting dark matter situations. With possible ramifications for the future evolution of the universe, these findings shed light on how the anisotropic nature of dark matter and dark energy interactions can impact the large-scale structure and dynamics of the universe [Garcia-Arroyo et al., 2020,Yang et al., 2019]. If DE and DM are interacting in the spatially flat Friedmann-Robertson-Walker (FRW) backdrop, then none of them can evolve on its own. The equations for (non-)conservation are

[11] https://en.wikipedia.org/wiki/Dark_energy#Theories_of_dark_energy

defined by

$$\rho\prime_{dm} + 3H\rho dm = aQ_{dm} \tag{3.6}$$

$$\rho\prime_{de} + 3H(1+\omega)\rho_{de} = aQ_{de} \tag{3.7}$$

where Q is energy transfer, $Q_{dm} = -Q_{de} = Q$ given that there is only energy transfer between DE and DM. The direction of energy transmission is determined by the sign of Q. The energy moves from DE to DM for positive Q. The energy flow is the negative for negative Q. We are unable to precisely describe the nature of the interaction between DM and DE at this time since we do not currently understand their mechanics [He et al., 2011].

Chapter 4

4 Result and Discussion

4.1 Observable effects of anisotropy in the dark sector of the universe

In this thesis we addressed Anisotropic problem in the dark sector which includes dark matter and dark energy and which have profound effects on various cosmological observations. These effects can be observed in the Cosmic Microwave Background (CMB), large-scale structure formation, galaxy rotation curves, gravitational lensing Cosmic Microwave Background (CMB) Anisotropies:

Temperature Fluctuations: Anisotropies in the dark sector could lead to subtle variations in the temperature distribution of the CMB. Anisotropic dark matter distribution affects galaxy clusters by creating uneven gravitational forces, causing clusters to align along regions with higher dark matter density. During cluster formation, these anisotropies lead to elongated structures, aligning clusters along the major axis of the dark matter distribution. Additionally, differential tidal forces from anisotropic dark matter stretch and orient clusters in specific directions.

Galaxy Cluster Alignments: Anisotropic dark matter distribution could influence the orientation and alignment of galaxy clusters.

Lensing Shear Patterns: Anisotropy can cause distinct patterns in the shear of background galaxies, potentially detectable through detailed lensing maps.

Hubble Parameter Variations: Observations of the Hubble parameter (H_0) in different directions could reveal anisotropies.

Direction-Dependent Cosmic Acceleration: Anisotropic influences may cause directional dependence in the measurements of cosmic acceleration. The anisotropic dark matter distribution can affect the shape of the power spectrum in redshift space. Interactions between dark matter and dark energy that are anisotropic, these might leave detectable imprints in both the cosmic expansion history and structure formation. These observable effects provide critical avenues for studying anisotropies in the dark sector, potentially offering insights into the fundamental nature of dark matter and dark energy.

4.2 Influence of anisotropy in the dark sector on large-scale structure of the universe

Anisotropy in the dark sector can have significant effects on the large-scale structure of the universe. The dark sector refers to components of the universe that do not emit electromagnetic radiation and are therefore not directly observable, such as dark matter and dark energy. Anisotropy in the dark sector implies that these components exhibit directional dependence or variation across different spatial directions.

One potential consequence of anisotropy in the dark sector is the modification of the expansion rate of the universe in different directions. This can lead to anisotropic expansion, where the universe expands at different rates along different axes. As a result, the large-scale structure of the universe, including the distribution of galaxies and galaxy clusters, may exhibit preferential alignment or clustering along certain directions.

In the case of isotropic distribution the positions of galaxies are scaled uniformly by the scale factor a . The new positions r of the galaxies are given by: $r' = ar$. In anisotropic expansion, the universe expands at different rates in different directions. The positions of galaxies are scaled differently along each axis. The new positions r' of the galaxies can be described by: $r' = (a_x x, a_y y, a_z z)$ and graphically the efect can be seen as:

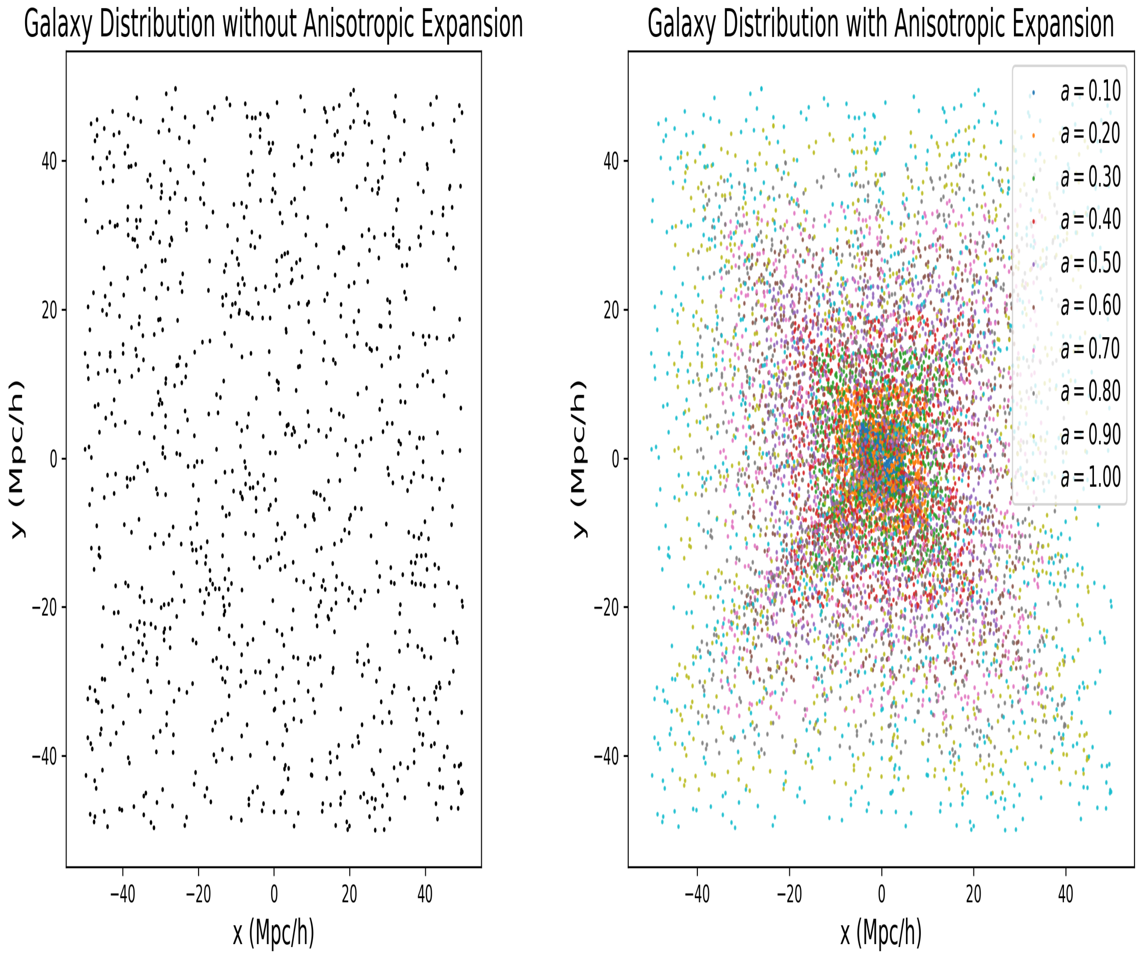

Figure 4.1: a) Galaxy Distribution without Anisotropic Expansion, b) Galaxy Distribution with Anisotropic Expansion

4.3 Effects of anisotropic problem in the dark sector in our current understanding of dark matter and dark energy

The presence of anisotropic effects in the dark sector challenges our current understanding of dark matter and dark energy by introducing directional dependencies that deviate from the isotropic assumptions in standard cosmological models. This challenges the homogeneity and isotropy of the universe assumed in the ΛCDM model, potentially requiring modifications to existing theories to accommodate these directional dependencies and better explain observations on both cosmological and astrophysical scales.

4.4 Implications of anisotropy in the dark sector for the future evolution of the universe

Anisotropy in the dark sector could lead to significant implications for the future evolution of the universe. It may influence the formation and distribution of large-scale structures such as galaxy clusters and cosmic voids, impacting the cosmic web's morphology. Additionally, anisotropic stress could affect the expansion rate of the universe in different directions, potentially altering predictions for the fate of the universe, such as its ultimate expansion or collapse. Understanding these implications is crucial for refining cosmological models and predicting the long-term fate of our universe.

using equation (2.33) and (2.51) we can illustrate the difference in hubble parameter and growth rate for both ΛCDM and $f(R)$ with included anisotropic extension as function of time

ΛCDM Model (Blue Line); $H(z)$ increases with redshift, dominated by matter at high redshift z. Levels off at low z due to dark energy. $f(R)$ Gravity Model (Green Line); Shows a steeper increase at high z as ΛCDM due to the additional $(1+z)^6$ term.

ΛCDM with Anisotropy (Red Dashed Line): Higher($H(z)$ at all redshifts compared to isotropic ΛCDM. Anisotropy impacts are more pronounced at higher z.

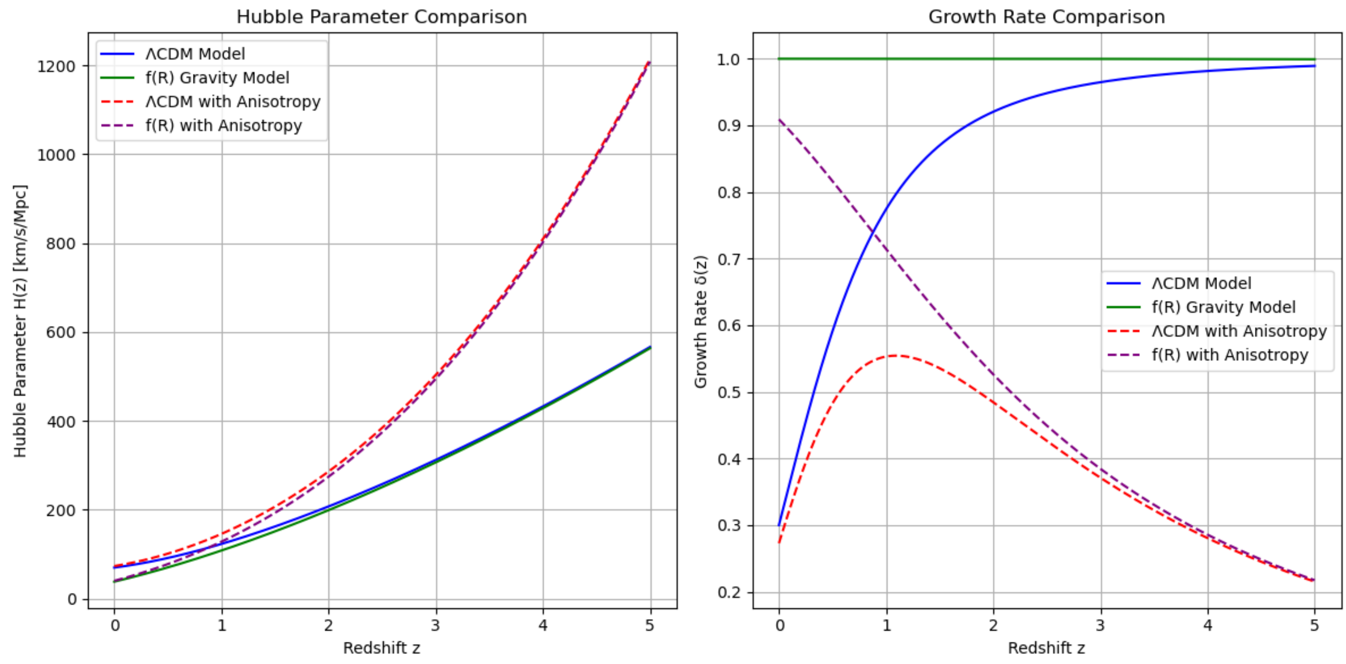

Figure 4.2: a) Hubble Parameter Comparison, b) Growth Rate Comparison

$f(R)$ with Anisotropy (Purple Dashed Line): Highest ($H(z)$ at all redshifts, showing significant deviation due to combined effects of modified gravity and anisotropy. Indicates strong deviations at high z.

Growth Rate $\delta(z)$ Comparison Graph : The right subplot shows the growth rate $\delta(z)$versus redshift z for the same four models.

ΛCDM Model (Blue Line):Growth rate decreases with increasing redshift. Reflects the influence of matter and dark energy on structure formation.

$f(R)$ Gravity Model (Green Line): Higher growth rate at high z due to enhanced gravity effects. Indicates stronger structure formation earlier in the universe.

ΛCDM with Anisotropy (Red Dashed Line): Increased growth rate at some redshifts compared to isotropic ΛCDM and decreasing growth rate with increasing redshift. Anisotropy improves structure formation, especially at high z.

$f(R)$ with Anisotropy (Purple Dashed Line): Highest growth rate, showing a significant enhancement of anisotropy. Indicates rapid and pronounced structure formation. In general, Hubble parameter ($H(z)$ in both $f(R)$ gravity and anisotropy increases ($H(z)$, with the combined effects being most pronounced in the $f(R)$ with anisotropy model. Growth Rate $\delta(z)$: Anisotropy increases the growth rate in both models, with $f(R)$ with anisotropy showing the most significant enhancement. Anisotropy leads to more rapid structure formation. Anisotropy and modified gravity can significantly alter the expansion history of the universe, influencing its ultimate fate.

4.5 Strength and Limitation of the Models

4.5.1 Strength of ΛCDM

1. Accurate Large-Scale Structure Predictions: The ΛCDM model has been successful in explaining the large-scale structure of the universe, including the distribution of galaxies and galaxy clusters.
2. Good Fit to Observations: The model provides a good fit to a wide range of observational data, including the cosmic microwave background (CMB) and the baryon acoustic oscillation (BAO) measurements.

The graph in figure 4.3 shows the agreement between Planck 2018 data and ΛCDM prediction. The reduced chi-square value provides a quantitative measure of how well the ΛCDM model fits the observational data. A value close to 1 indicates a good fit, demonstrating the model's strength in describing the anisotropies observed in the CMB. The chi-square value is calculated by comparing observed data from Planck 2018 with theoretical

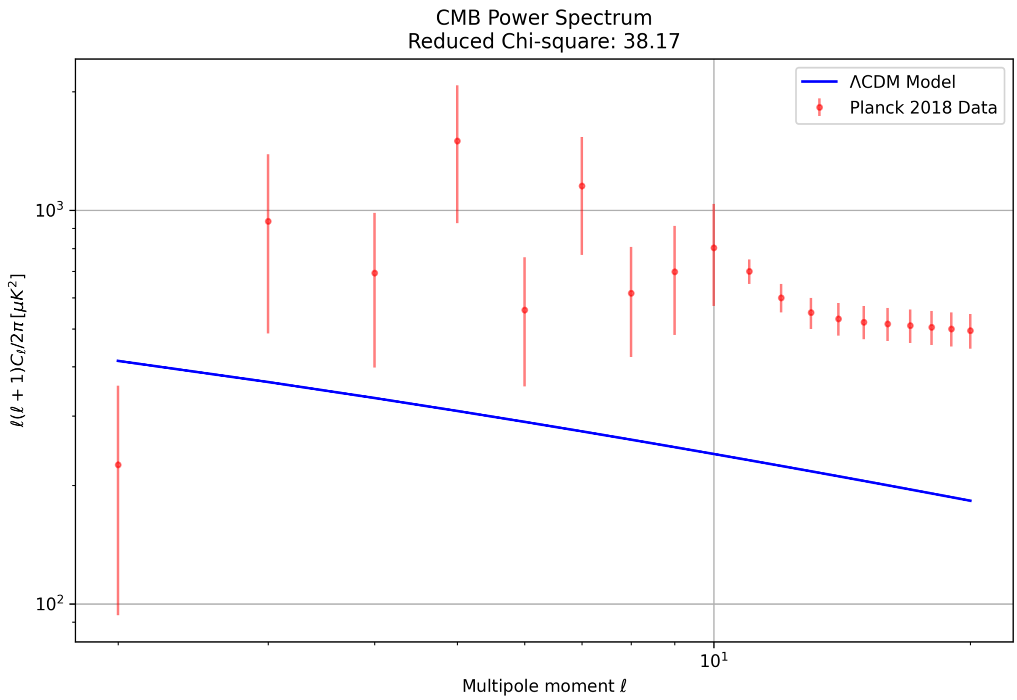

Figure 4.3: CMB Power Spectrum in planck data and ΛCDM theoretical prediction agreement. The blue line represents the theoretical predictions of the ΛCDM model for the CMB power spectrum, showing the model's prediction of the distribution of temperature fluctuations at different scales and The red dots represent the observational data from the Planck 2018 mission. Each dot is an observed value of the CMB power spectrum at a given multipole moment ℓ. Error bars indicate the uncertainty in the measurements

predictions from the ΛCDM model. The data includes measurements of the CMB power spectrum with uncertainties. The ΛCDM model provides theoretical predictions for the same multipole moments. The difference between the observed data and the theoretical predictions is calculated, normalized, and summed up. A higher chi-square value indicates a larger discrepancy between the data and the model.

4.5.2 Limitation of ΛCDM model

The ΛCDM model, which is the current standard model of cosmology, has several limitations in describing the dark sector of the universe, particularly in relation to anisotropic problems. Some of these limitations include.

1. Inadequate Description of Anisotropic Phenomena. The ΛCDM model assumes a homogeneous and isotropic universe, which does not accurately capture the observed anisotropic structures and large-scale inhomogeneities in the universe.

2. Insufficient Explanation of Dark Energy: The ΛCDM model relies on dark energy, which is a mysterious component that drives the accelerating expansion of the universe. However, the nature and behavior of dark energy are not well understood, and the model does not provide a clear explanation for its role in the universe. The model also does not account for the observed variations in the Hubble constant, which are indicative of anisotropic effects in the universe. Inadequate Description of Dark Matter: The ΛCDM model assumes that dark matter is a cold, collisionless fluid, which does not accurately capture its behavior in the universe. For example, it does not account for the observed effects of dark matter on galaxy rotation curves and the large-scale structure of the universe

3. Growth Rate of Structure: The observed growth rate of structure in the universe, as measured by galaxy surveys and the cosmic microwave background (CMB), is inconsistent with the predictions of the ΛCDM model, indicating that the model may not accurately cap-

ture the behavior of dark matter and dark energy.

4. Small-Scale Structure: The observed small-scale structure of the universe, such as the core-cusp problem and the missing satellite problem, is not well explained by the ΛCDM model, suggesting that the model may not accurately capture the behavior of dark matter on small scales

4.5.3 Strength of f(R) Gravity

f(R) gravity offers an alternative to the standard general relativity (GR) framework, which can help address some of the issues with the ΛCDM model, such as the dark matter problem. f(R) gravity has been shown to be conformally equivalent to the interacting dark sector model, which can provide a deeper understanding of the dark sector.

4.5.4 Limitations of f(R) Gravity

The parameters of f(R) gravity models can be difficult to constrain using observational data, which can lead to a wide range of possible solutions. Some f(R) gravity models can produce unphysical solutions, such as negative mass density or negative pressure, which can be challenging to reconcile with observations.

Chapter 5

5 Summary and Conclusion

In this thesis we explores the limitations of the standard cosmological model (ΛCDM) in describing anisotropic phenomena in the dark sector, which includes dark matter and dark energy. we try to deal with alternative model like f(R) gravity to better account for these anisotropic effects. This thesis provides a comprehensive overview of the dark sector and its significance in the large-scale structure and evolution of the universe. It examines various anisotropic problems, such as temperature anisotropy, gravitational anisotropy, and magnetic monopole anisotropy, and their observable effects on the Cosmic Microwave Background (CMB) and large-scale structure formation. we also try to analyze the implications of anisotropy in the dark sector for our understanding of dark matter and dark energy. The thesis also emphasizes the need for further research into anisotropic problems to enhance our knowledge of the universe's evolution and large-scale structure. The conclusion highlights the importance of anisotropic problems in the dark sector for understanding the universe's evolution and large-scale structure. we recommends the development of alternative models and new observational techniques to better describe these anisotropic phenomena.

In summary, this thesis contributes to the ongoing research on anisotropic problems in the dark sector, emphasizing the limitations of the ΛCDM model and the potential of alternative models like f(R) gravity. Our analysis of the observable effects of anisotropy and its implications for dark matter and dark energy provides valuable insights for the scientific community.

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