QUANTUM FLUCTUATION AND THE DARK ENERGY SECTOR CORRELATION PROBLEM IN THE UNIVERSE MODEL

By

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DECLARATION

I hereby declare that this MSc research paper is my original work and has not been presented for a degree in any other university, and that all sources of material used for the paper have been duly acknowledged.

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External Examiner ————————————-

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ABSTRACT

In this thesis, we studied quantum fluctuation and the dark energy sector correlation problem in the universe model. Quantum fluctuations, originating from the inherent uncertainty at the quantum level, and dark energy, hypothesized to drive the universe's accelerated expansion, represent two fundamental yet enigmatic phenomena in modern cosmology. Despite significant advancements in understanding each independently, reconciling their relationship remains a profound challenge. Through a comprehensive review of theoretical frameworks and observational data, this study was investigated the theoretical basis for quantum fluctuations in the universe, and its relations to the dynamics of dark energy; inflationary models, quintessence models, and modified gravity theories address the correlation problem between quantum fluctuations and dark energy. Moreover we analyzed the observational evidence supports or challenges the correlation between quantum fluctuations and dark energy in the cosmic microwave background radiation and large-scale structure of the universe and the implications of the correlation between quantum fluctuations and dark energy for the fundamental nature of dark energy and its role in the evolution of the universe model.

Key Words: Universe model, Dark energy, Quantum fluctuation.

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ACRONYMS

CMB - Cosmic Microwave Background

LSS - Large Scale Structure

IAU - International Astronomical Union

WASP - Wide-Angle Search for Planets

NASA - National Aeronautics and Space Administration

RD - Radiation dominated

MD - Matter dominated

CDM - Cold Dark Matter

BAO - Baryon Acoustic Osillations

SN - Supernovae

DE - Dark Energy

FLRW - Friedman Lemaitre Robertson Walker

1

Introduction

1.1 Background of the Study

Now as we know that the universe was undergoing such a rapid expansion, logically it must have been much smaller in the past. At very early times the universe was so small that the theory that ruled the realm was the theory of the small, Quantum Mechanics. Quantum fluctuation refers to the temporary changes in the amount of energy in a point in space, as described by quantum mechanics. In the context of the universe, quantum fluctuations are believed to have played a crucial role in the early universe's evolution, giving rise to the structures we observe today, such as galaxies and galaxy clusters. A quantum fluctuation is the temporary change in the amount of energy in a point in space, as explained in Werner Heisenberg's uncertainty principle. Now this point is a little tricky but essentially what happens is that the field driving inflation experiences these quantum fluctuations, and everything else at this time is rapidly expanding, these quantum fluctuations are blown up too (2; 3).

In quantum physics, a quantum fluctuation (also known as a vacuum state fluctuation or vacuum fluctuation) is the temporary random change in the amount of energy in a point in space,(4) as prescribed by Werner Heisenberg's uncertainty principle. They are minute random fluctuations in the values of the fields which represent elementary particles, such as electric and magnetic fields which represent the electromagnetic force carried by photons, W and Z fields which carry the weak force, and gluon fields which carry the strong force (5).

Quantum fluctuations of the vacuum are a fundamental phenomenon in nature, characterized by the presence of virtual photons with real impacts and interactions that can be manipulated and observed in experimental quantum systems. These fluctuations are a manifestation of the inherent uncertainty in quantum mechanics. One of the fundamental principles in the study of the quantum vacuum is Heisenberg's Uncertainty Principle. This principle states that it is not possible to simultaneously know with precision both the position and the momentum (a quantity related to movement) of a particle (3; 6).

Dark energy, on the other hand, is a mysterious form of energy that is hypothesized to permeate all of space and is responsible for the accelerated expansion of the universe. Its nature is not well understood, and it is one of the most significant unsolved problems in cosmology. The mystery of dark energy has left scientists scratching their heads for many years. The concept of dark energy is based on the observation that the universe is expanding at an accelerating rate, which can only be explained by the existence of some unknown force that is pushing galaxies apart. There are several theories about what dark energy could be, but none of them have been proven definitively. The most widely accepted theory is that dark energy is a property of space itself, known as the cosmological constant. However, there are other theories that suggest dark energy could be caused by the behavior of exotic particles or even modifications to the laws of gravity (7).

The correlation between quantum fluctuations and dark energy is an area of active research. Some theories suggest that quantum fluctuations in the early universe may have influenced the properties of dark energy, potentially leaving imprints that can be observed today. Understanding this correlation could provide insights into the fundamental nature of both quantum mechanics and dark energy, shedding light on some of the most profound mysteries of the universe (8).

1.2 Statement of Problem

The correlation between quantum fluctuation and the dark energy sector presents a fundamental challenge in modern cosmology. Quantum fluctuations, arising from the inherent uncertainty at the quantum level, are believed to have influenced the evolution of the early universe, giving rise to the structures we observe today. On the other hand, dark energy, an enigmatic form of energy that permeates space and drives the accelerated expansion of the universe, remains poorly understood. Understanding the relationship between quantum fluctuations and dark energy is crucial for unraveling the mysteries of the universe's evolution and structure (9).

However, the correlation between quantum fluctuations and the dark energy sector is not well understood and represents a significant gap in our current understanding of cosmology. Existing theories provide partial explanations but fall short of a comprehensive framework that can fully account for the observed phenomena. Additionally, observational constraints on this correlation are limited, highlighting the need for further research and analysis (10).

Therefore, this study was investigated the correlation between quantum fluctuations and the dark energy sector in the universe model. By examining theoretical frameworks, and conducting observational analyses this research seeks to deepen our understanding of the fundamental nature of quantum fluctuations and dark energy and their role in shaping the universe's evolution.

1.3 Research Questions

1. What is the theoretical basis for quantum fluctuations in the universe, and how do they relate to the dynamics of dark energy?
2. How do inflationary models, quintessence models, and modified gravity theories address the correlation problem between quantum fluctuations and dark energy?
3. What observational evidence supports or challenges the correlation between quantum fluctuations and dark energy in the cosmic microwave background radiation and large-scale structure of the universe?
4. What are the implications of the correlation between quantum fluctuations and dark energy for the fundamental nature of dark energy and its role in the evolution of the universe model?

1.4 Objectives

1.4.1 General objective

- To study quantum fluctuation and the dark energy sector correlation problem in the universe model.

1.4.2 Specific Objectives

1. To study the theoretical basis for quantum fluctuations in the universe, and its relations to the dynamics of dark energy.
2. To identify inflationary models, quintessence models, and modified gravity theories address the correlation problem between quantum fluctuations and dark energy.
3. To analyze the observational evidence supports or challenges the correlation between quantum fluctuations and dark energy in the cosmic microwave background radiation and large-scale structure of the universe.
4. To Investigate the implications of the correlation between quantum fluctuations and dark energy for the fundamental nature of dark energy and its role in the evolution of the universe model.

1.5 Methodology

In this thesis, our research methodology involved examining a spectrum of universe models, including Steady State, Inflationary, Multiverse, Cyclic, and Brane cosmology. Each model was analyzed for its unique theoretical constructs and implications regarding the evolution of the universe. Furthermore, our investigation extended to the scrutiny of observational evidence, particularly addressing the discrediting of the Steady State model in light of the Big Bang theory.

A significant aspect of this study involved probing the correlation between quantum fluctuations and the dark energy sector within the universe model. This was accomplished by delving into theoretical frameworks, and by conducting observational analyses. Through these methods, our research discussed to deepen the comprehension of the intrinsic nature of quantum fluctuations and dark energy, and their consequential impact on the evolution of the universe.

Ultimately, this document endeavors to offer a comprehensive understanding of the discussed cosmological models while also identifying avenues for further research and exploration in the field.

2

Uviverse Model

2.1 Introduction To Uviverse Model

Every model of the universe must include the expansion we observe. Another key element of the models is that the cosmological principle (which we discussed in The Evolution and Distribution of Galaxies) is valid: on the large scale, the universe at any given time is the same everywhere (homogeneous and isotropic). As a result, the expansion rate must be the same everywhere during any epoch of cosmic time. If so, we don't need to think about the entire universe when we think about the expansion, we can just look at any sufficiently large portion of it. (Some models for dark energy would allow the expansion rate to be different in different directions, and scientists are designing experiments to test this idea. However, until such evidence is found, we will assume that the cosmological principle applies throughout the universe (11; 12).

In Galaxies, we hinted that when we think of the expansion of the universe, it is more correct to think of space itself stretching rather than of galaxies moving through static space. Nevertheless, we have since been discussing the red shifts of galaxies as if they resulted from the motion of the galaxies themselves.

Now, however, it is time to finally put such simplistic notions behind us and take a more sophisticated look at the cosmic expansion. Recall from our discussion of Einstein's theory of general relativity that space or, more precisely, space time is not a mere backdrop to the action of the universe, as Newton thought. Rather, it is an active participant affected by and in turn affecting the matter and energy in the universe.

Since the expansion of the universe is the stretching of all space time, all points in the universe are stretching together. Thus, the expansion began everywhere at once. Unfortunately for tourist agencies of the future, there is no location you can visit where the stretching of space began or where we can say that the Big Bang happened.

To describe just how space stretches, we say the cosmic expansion causes the universe to undergo a uniform change in scale over time. By scale we mean, for example, the distance between two clusters of galaxies. It is customary to represent the scale by the factor R; if R doubles, then the distance between the clusters has doubled. Since the universe is expanding at the same rate everywhere, the change in R tells us how much it has expanded (or contracted) at any given time. For a static universe, R would be constant as time passes. In an expanding universe, R increases with time (1).

If it is space that is stretching rather than galaxies moving through space, But now that you are an older and wiser student of cosmology, this view will simply not do.

Before astronomers knew about dark energy or had a good measurement of how much matter exists in the universe, they made speculative models about how the universe might evolve over time. The four possible scenarios are shown in Figure 2.1. In this diagram, time moves forward from the bottom upward, and the scale of space increases by the horizontal circles becoming wider(1).

The yellow square marks the present in all four cases, and for all four, the Hubble constant is equal to the same value at the present time. Time is measured in the vertical direction. The first two universes on the left are ones in which the rate of expansion slows over time. The one on the left will eventually slow, come to a stop and reverse, ending up in a "big crunch," while the one next to it will continue to expand forever, but ever-more slowly as time passes. The "coasting" universe is one that expands at a constant rate given by the Hubble constant throughout all of cosmic time. The accelerating universe on the right will continue to expand faster and faster forever (1). The simplest scenario of an expanding universe would be one in which R increases with time at a constant rate. But you already know that life is not so simple. The universe contains a great deal of mass and its gravity

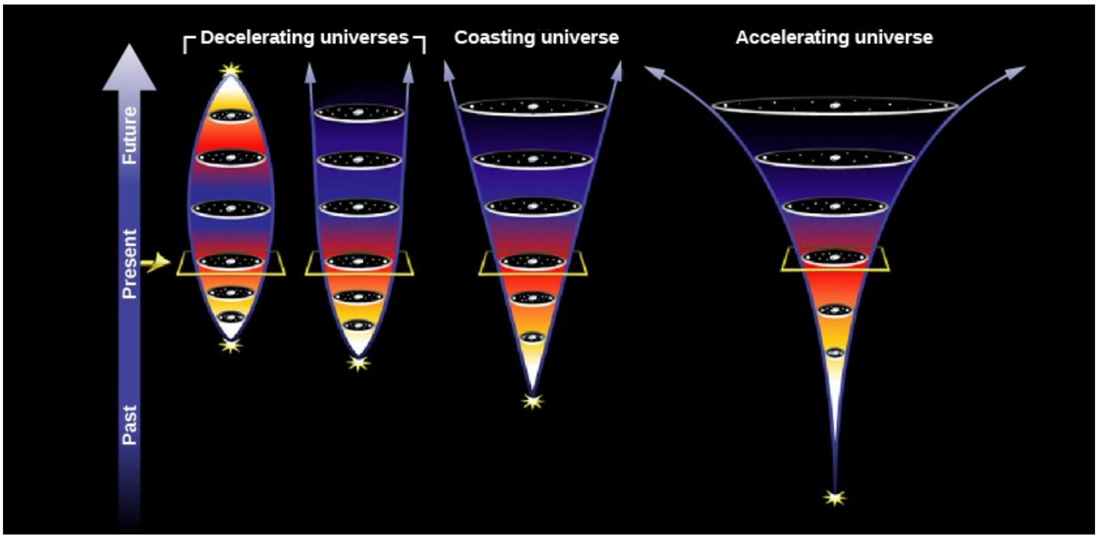

Figure 2.1: Diagram detailing the Four Possible Models of the Universe (1).

decelerates the expansion by a large amount if the universe contains a lot of matter, or by a negligible amount if the universe is nearly empty. Then there is the observed acceleration, which astronomers blame on a kind of dark energy.

Let's first explore the range of possibilities with models for different amounts of mass in the universe and for different contributions by dark energy. In some models as we shall see the universe expands forever. In others, it stops expanding and starts to contract. After looking at the extreme possibilities, we will look at recent observations that allow us to choose the most likely scenario(1).

We should perhaps pause for a minute to note how remarkable it is that we can do this at all. Our understanding of the principles that underlie how the universe works on the large scale and our observations of how the objects in the universe change with time allow us to model the evolution of the entire cosmos these days. It is one of the loftiest achievements of the human mind.

What astronomers look at in practice, to determine the kind of universe we live in, is the average density of the universe. This is the mass of matter (including the equivalent mass of energy) that would be contained in each unit of volume (say, 1 cubic centimeter) if all the stars, galaxies, and other objects were taken apart, atom by atom, and if all those particles, along with the light and other energy, were distributed throughout all of space with absolute uniformity. If the average density is

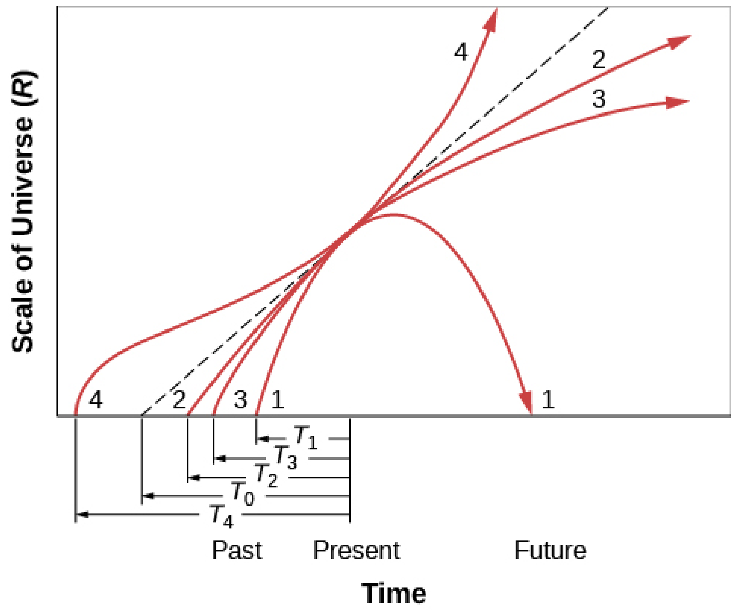

Figure 2.2: Diagram detailing the Models of the Universe (1).

low, there is less mass and less gravity, and the universe will not decelerate very much. It can therefore expand forever. Higher average density, on the other hand, means there is more mass and more gravity and that the stretching of space might slow down enough that the expansion will eventually stop. An extremely high density might even cause the universe to collapse again (1).

For a given rate of expansion, there is a critical density the mass per unit volume that will be just enough to slow the expansion to zero at some time infinitely far in the future. If the actual density is higher than this critical density, then the expansion will ultimately reverse and the universe will begin to contract. If the actual density is lower, then the universe will expand forever.

These various possibilities are illustrated in Figure 2.2. In this graph, one of the most comprehensive in all of science, we chart the development of the scale of space in the cosmos against the passage of time. Time increases to the right, and the scale of the universe, R, increases upward in the figure. Today, at the point marked "present" along the time axis, R is increasing in each model. We know that the galaxies are currently expanding away from each other, no matter which model is right. The same situation holds for a baseball thrown high into the air. While it may eventually fall back down, near the beginning of the throw it moves upward most rapidly.

The various lines moving across the graph correspond to different models of the universe. The straight dashed line corresponds to the empty universe with no deceleration; it intercepts the time axis at a time, T0 (the Hubble time), in the past. This is not a realistic model but gives us a measure to compare other models to. The curves below the dashed line represent models with no dark energy and with varying amounts of deceleration, starting from the Big Bang at shorter times in the past. The curve above the dashed line shows what happens if the expansion is accelerating. Let's take a closer look at the future according to the different models.

This graph plots R, the scale of the universe, against time for various cosmological models. Curve 1 represents a universe where the density is greater than the critical value; this model predicts that the universe will eventually collapse. Curve 2 represents a universe with a density lower than critical; the universe will continue to expand but at an ever slower rate. Curve 3 is a critical density universe; in this universe, the expansion will gradually slow to a stop infinitely far in the future. Curve 4 represents a universe that is accelerating because of the effects of dark energy. The dashed line is for an empty universe, one in which the expansion is not slowed by gravity or accelerated by dark energy. Time is very compressed on this graph.

Big Bang Model: The Big Bang model is one of the most widely accepted cosmological models. It proposes that the universe began as a singularity, a point of infinite density and temperature, around 13.8 billion years ago. The universe then expanded and cooled, leading to the formation of matter, galaxies, and ultimately, the structures we observe today.

Steady State Model: The Steady State model suggests that the universe has no beginning or end and has always existed in a state of constant density. According to this model, new matter is continuously created to maintain the overall density as the universe expands. However, this model is not widely accepted today due to observational evidence supporting the Big Bang theory.

Inflationary Model: The Inflationary model proposes that the early universe underwent a rapid and exponential expansion phase. This period of inflation helped to explain some of the observed large scale structure of the universe and the uniformity of the cosmic microwave background radiation. The Inflationary model is often incorporated into the Big Bang model.

Multiverse Models: Multiverse models suggest the existence of multiple universes, each with its own set of physical laws and constants. These models are often speculative and are proposed to explain the fine-tuning of the universe's fundamental constants and the origin of different physical laws.

Cyclic Models: Cyclic models propose that the universe undergoes an infinite series of cycles, where each cycle begins with a Big Bang, followed by expansion, contraction, and a new Big Bang. These models attempt to explain the observed large-scale structure of the universe and avoid the problem of a singular beginning.

Brane Cosmology: Brane cosmology, or the ”brane world” scenario, suggests that our universe is a four dimensional brane embedded in a higher dimensional space. Gravitational interactions between branes can lead to cosmic phenomena that differ from traditional cosmological models.

2.2 Evolution of the Universe: From Geocentric to Heliocentric Models

The night sky has been a subject of human curiosity from the earliest civilizations on earth. From Babylonians, Egyptians, Greeks, and Indus all had a fascination for the celestial objects and the elite of the intellectuals built theories to explain the miracles of the heavens. Earlier they were accepted to be from the gods, and later the explanation took more logical and scientific forms. However, it was not until the Greeks development that proper theories about the earth and the rotation of the planets emerged. Heliocentric and geocentric are two explanations of the arrangement of the universe, including the solar system. The geocentric model says that the earth is at the center of the cosmos or universe, and the planets, the sun and the moon, and the stars circles around it. The early heliocentric models consider the sun as the center, and the planets revolve around the sun.

2.2.1 Geocentric Model

The most predominant theory of the structure of the universe in the ancient world was the geocentric model. It says that the earth is at the center of the universe, and every other celestial body rotates around the earth. The origin of this theory is obvious; it is the elementary naked eye observation of the movement of the objects in the sky. The path of an object in the sky always seems to be in the same vicinity and repeatedly it rises from east and sets from west approximately at the same points on the horizon. Also, the earth always seems to be stationary or motionless and still. Therefore, the closest conclusion is that these objects move in circles around the earth. Greeks were strong supporters of this theory, especially the great philosophers Aristotle and Ptolemy. After the death of Ptolemy, the theory lasted for more than 2000 years unchallenged.

2.2.2 Heliocentric Model

The concept that the sun is at the center of the universe, also first emerged in Ancient Greece. It was the Greek philosopher Aristarchus of Samos who proposed the theory in 3rd century BC, but was not taken seriously because of the influence of the Aristotelian view of the universe and lack of proof of the theory at that time. It was during the Renaissance era that mathematician and Catholic cleric, Nicholaus Copernicus, developed a mathematical model to explain the motion of the heavenly bodies. In his model, the sun was at the center of the solar system and the planet moved around the sun, including the earth. And the moon was considered to move around the earth. This changed the way of thinking about the universe and differed from the religious beliefs at that time. The major feature of the Copernican theory can be summarized as follows:

1. The motion of the celestial bodies is uniform, eternal, and circular or compounded of several circles.

2. The center of the cosmos is the Sun. Around the Sun, in the order of Mercury, Venus, Earth and Moon, Mars, Jupiter, and Saturn moves in their own orbits and the stars are fixed in the sky.

3. The earth has three motions; daily rotation, annual revolution, and annual tilting off its axis.

4. The retrograde motion or the backward motion of the planets is as explained by the Earth's motion.

5. The distance from the Earth to the Sun is small compared to the distance to the stars.

2.3 Model of the universe based on general relativity

General relativity is the geometric theory of gravitation published by Albert Einstein in 1915 and the current description of gravitation in modern physics. It is the basis of current cosmological models of the universe. General relativity generalizes special relativity and Newton's law of universal gravitation, providing a unified description of gravity as a geometric property of space and time, or space time. In particular, the curvature of space time is directly related to the energy and momentum of whatever matter and radiation are present(13).

The relation is specified by the Einstein field equations, a system of partial differential equations. In general relativity, the distribution of matter and energy determines the geometry of space time, which in turn describes the acceleration of matter. Therefore, solutions of the Einstein field equations describe the evolution of the universe. Combined with measurements of the amount, type, and distribution of matter in the universe, the equations of general relativity describe the evolution of the universe over time(13).

With the assumption of the cosmological principle that the universe is homogeneous and isotropic everywhere, a specific solution of the field equations that describes the universe is the metric tensor called the Friedmann Lemaître Robertson Walker metric,

$$ds^2 = c^2dt^2 + R(t)^2(dr^2 - kr^2 + r^2d\theta^2 + r^2sin^2\theta d\phi^2)ds^2 = c^2dt^2 + R(t)^2(\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2) \tag{2.1}$$

where (r, θ, φ) correspond to a spherical coordinate system. This metric has only two undetermined parameters. An overall dimensionless length scale factor R describes the size scale of the universe as a function of time (an increase in R is the expansion of the universe), (13) and a curvature index k describes the geometry. The index k is defined so that it can take only one of three values: θ, corresponding to flat Euclidean geometry; 1, corresponding to a space of positive curvature; or -1, corresponding to a space of positive or negative curvature. The value of R as a function of time t depends upon k and the cosmological constant Λ(13). The cosmological constant represents the energy density of the vacuum of space and could be related to dark energy. The equation describing how R varies with time is known as the Friedmann equation after its inventor, Alexander Friedmann.(14).

The solutions for R(t) depend on k and Λ, but some qualitative features of such solutions are general. First and most importantly, the length scale R of the universe can remain constant only if the universe is perfectly isotropic with positive curvature (k=1) and has one precise value of density everywhere, as first noted by Albert Einstein(14). However, this equilibrium is unstable: if the density were slightly different from the needed value, at any place, the difference would be amplified over time.

Second, all solutions suggest that there was a gravitational singularity in the past, when R went to zero and matter and energy were infinitely dense. It may seem that this conclusion is uncertain because it is based on the questionable assumptions of perfect homogeneity and isotropy (the cosmological principle) and that only the gravitational interaction is significant. However, the Penrose Hawking singularity theorems show that a singularity should exist for very general conditions. Hence, according to Einstein's field equations, R grew rapidly from an unimaginably hot, dense state that existed immediately following this singularity (when R had a small, finite value); this is the essence of the Big Bang model of the universe. Understanding the singularity of the Big Bang likely requires a quantum theory of gravity, which has not yet been formulated(15).

Third, the curvature index k determines the sign of the curvature of constant-time spatial surfaces(13) averaged over sufficiently large length scales (greater than about a billion light-years). If k=1, the curvature is positive and the universe has a finite volume(16). A universe with positive curvature is often visualized as a three-dimensional sphere embedded in a four-dimensional space. Conversely, if k is zero or negative, the universe has an infinite volume. It may seem counter-intuitive that an infinite and yet infinitely dense universe could be created in a single instant when R = 0, but exactly that is predicted mathematically when k is non positive and the cosmological principle is satisfied. By analogy, an infinite plane has zero curvature but infinite area, whereas an infinite cylinder is finite in one direction and a torus is finite in both. A toroidal universe could behave like a normal universe with periodic boundary conditions.

The ultimate fate of the universe is still unknown because it depends critically on the curvature index k and the cosmological constant Λ. If the universe were sufficiently dense, k would equal +1, meaning that its average curvature throughout is positive and the universe will eventually re collapse in a Big Crunch, possibly starting a new universe in a Big Bounce. Conversely, if the universe were insufficiently dense, k would equal 0 or -1 and the universe would expand forever, cooling off and eventually reaching the Big Freeze and the heat death of the universe. Modern data suggests that the expansion of the universe is accelerating; if this acceleration is sufficiently rapid, the universe may eventually reach a big rip. Observational, the universe appears to be flat (k = 0), with an overall density that is very close to the critical value between re collapse and eternal expansion (16).

In Newtonian theory, gravity is an attractive force between massive objects. However, in General Relativity, gravity is a geometric property of the space time. This theory relates the space time geometry to the distribution of matter and energy of the Universe via the Einstein field equations,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} + \bigwedge g_{\mu\nu} \quad (2.2)$$

where $G\mu\nu$ is the Einstein tensor,$R\mu\nu$ is the Ricci tensor, R is the Ricci scalar, $g\mu\nu$ is the metric tensor, G is the Newton's constant, $T\mu\nu$ is the energy momentum tensor, and $\bigwedge$ is the cosmological constant; the left-hand side of the equation is a function of the metric and the right hand side of the energy. Initially, Einstein added the cosmological constant to reproduce a static Universe (17). However, this constant was later dropped when Hubble discovered that the Universe is expanding. Nowadays, the cosmological constant was re-introduced to allow for an accelerated expansion of the Universe; $\bigwedge$ now represents the vacuum energy of the space time and contributes towards the total energy of the Universe (17).

Under General Relativity, a homogeneous and isotropic Universe can be described by the Friedman Lemaitre Robertson Walker (FLRW) metric:

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) \tag{2.3}$$

where t is the cosmic time, (r, θ , ϕ) are the spatial coordinates in spherical coordinates, a(t) is the scale factor, and k is the spatial curvature constant that describes the Universe as open (k = 1), flat (k = 0), and closed (k = 1). The scale factor a(t) is a dimensionless function of time that encodes the expansion (or contraction) of the Universe and it is defined to be one at the present time. To quantify the change in the scale factor, we define the Hubble parameter,

$$H[t] = \frac{\hat{a}}{a} \tag{2.4}$$

where â is the derivative of a with respect to time. The current value of the Hubble parameter, H0 $\equiv$ H(0), is called the Hubble constant. Using the Einstein field equations (Eq.2.2), we can more generally describe the dynamics of the Universe through Friedman's equations,

$$\frac{\dot{a}}{a}^2 = \frac{8\pi G}{3}\rho - \frac{-k}{a^2} = H^2 \tag{2.5}$$

$$\frac{\dot{a}}{a}^2 = \frac{8\pi G}{3}\rho - \frac{-k}{a^2} = H^2 \tag{2.6}$$

where $\ddot{a}$ is the second derivative of a with respect to time, ρ is the energy density, and P is the pressure; these equations connect the content and the expansion of the Universe. Since the FLRW describes a homogeneous and isotropic Universe, the components of the Universe are perfect fluids, so we write their equation of state as $P = w\rho$. Imposing the conservation of the energy-momentum, we obtain a third relation.

$$\rho + 3\frac{\dot{a}}{a}(\rho + p) = 0 \tag{2.7}$$

It will be useful to go back to our first-order Friedmann Equation in its original form,

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3}Pt - k\frac{a}{c}^2 + \frac{1}{3}\Lambda \tag{2.8}$$

First, we will consider a manifestly non-physical Universe, an open ($k < 0$) one with no matter, no radiation, and no cosmological constant. This just gives the very simple equation

$$\dot{a}^2 = kc^2 \tag{2.9}$$

$$\frac{\dot{a}}{a} = t^{-1} \tag{2.10}$$

This is the "wide open", or Milne Universe. Note that it also corresponds to the far future evolution of any open Universe (k = -1 or Ω k $>$ 0 or $\Omega_{tot} < 1$) in which the matter and radiation densities have diluted so much as to become negligible, as long as there is no cosmological constant $\Omega\Lambda$ = 0.

3

Dark Energy in the Universe Model and Quantum Fluctuation Vacuum Energy

3.1 Dark Energy

In physical cosmology and astronomy, dark energy is an unknown form of energy that affects the universe on the largest scales. Its primary effect is to drive the accelerating expansion of the universe. Assuming that the ΛCDM model of cosmology is correct, dark energy is the dominant component of the universe, contributing 68 of the total energy in the present day observable universe while dark matter and ordinary (baryonic) matter contribute 26 and 5, respectively, and other components such as neutrinos and photons are nearly negligible. Dark energy's density is very low: ($\approx 7 \times 10^{-30} g/cm^3$), much less than the density of ordinary matter or dark matter within galaxies. However, it dominates the universe's mass energy content because it is uniform across space (9; 10; 8). An explanation for why the expansion of the universe is accelerating remains elusive. However, in the present dark-energy era, it dominates the mass energy of the universe because it is uniform across space (18; 19).

Two proposed forms for dark energy are the cosmological constant, a constant energy density filling space homogeneously (20), and scalar fields such as quintessence or moduli, dynamic quantities whose energy density can vary in time and space while still permeating then enough to cause the observed rate of expansion. Contributions from scalar fields that are constant in space are usually also included in the cosmological constant. The cosmological constant can be formulated to be equivalent to vacuum energy. Scalar fields having only a slight amount of spatial inhomogeneity would be difficult to distinguish from a cosmological constant.

Whatever the dark energy turns out to be, we should note that the discovery that the rate of expansion has not been constant since the beginning of the universe complicates the calculation of the age of the universe. Interestingly, the acceleration seems not to have started with the Big Bang. During the first several billion years after the Big Bang, when galaxies were close together, gravity was strong enough to slow the expansion. As galaxies moved farther apart, the effect of gravity weakened. Several billion years after the Big Bang, dark energy took over, and the expansion began to accelerate (figure 3.1) (1).

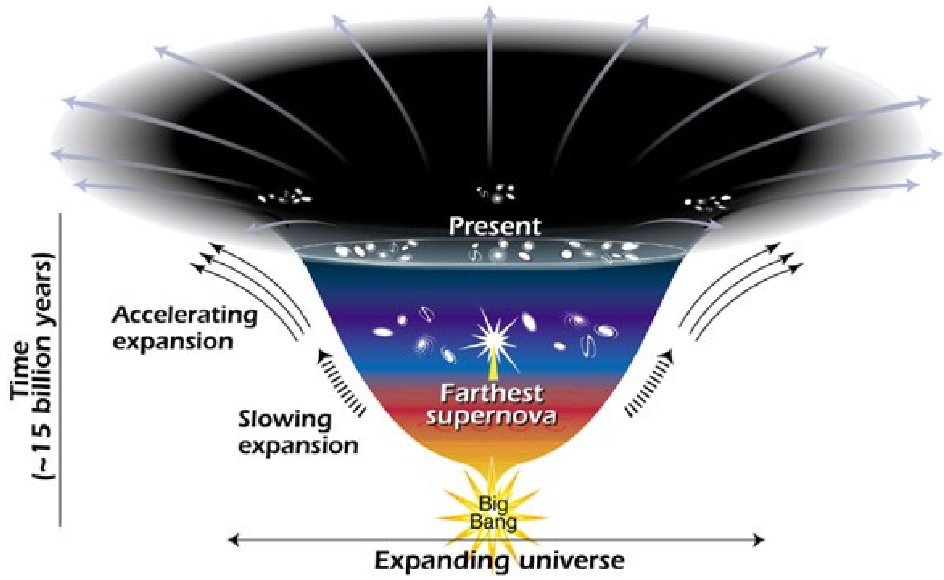

Figure 3.1: Diagram detailing the Dark-Energy (1).

3.2 Dark Energy in the Universe Model

The Standard Cosmological Model, also known as the ΛCDM model, is the prevailing paradigm for understanding the large scale structure and evolution of the universe. It is based on the principles of general relativity and incorporates key concepts from particle physics (21).

Needless to say, the cosmological constant problem is one of the biggest challenges in theoretical physics. Naive estimation leads the cosmological constant to be of the Planck scale $(10^{19}GeV)^4$; if susy breaks at Tev scale, the cosmological constant should be in the order $(1Tev)^4$. The discovery of the current accelerated expansion of the universe causes the problem to be more difficult to solve (22), which implies that the cosmological constant is in the scale $(10^{-3}ev)^4$. There exists a big hierarchy difference between the theoretical estimation and observation value. Since the cosmological constant is related to the vacuum expectation value of some quantum fields and it can be measured only through gravitational experiments. Therefore the cosmological constant problem is essentially a problem in quantum gravity. Although a completely successful quantum theory of gravity is still not yet available, quantum mechanics together with general relativity may shed some lights on this issue(23).

3.3 Observational evidence of dark energy

The existence of dark energy is supported by a number of observations. This includes:- The age of the Universe compared to oldest stars, supernovae observations, Cosmic Microwave Background (CMB), baryon acoustic oscillations (BAO), and large scale structure (LSS). Even before 1998 it was known that in a CDM Universe the cosmic age can be smaller than the age of the oldest stars. Dark energy can account for this discrepancy because its presence can make the cosmic age longer. The first strong evidence for the acceleration of the Universe today came however by measuring the luminosity distance of the type Ia supernovae (SN Ia). The CMB observations are also consistent with the presence of dark energy, although the constraint coming from the CMB alone is not so strong. The measurements of BAO have provided another independent test for the existence of dark energy. The power spectrum of matter distributions also favors a Universe with dark energy rather than the CDM Universe.

3.4 Quantum Fluctuations

Quantum fluctuations are temporary changes in the energy of a quantum system, arising due to the inherent uncertainty in quantum mechanics. According to the principles of quantum mechanics, the state of a system is described by a wave function, which contains information about the probability of finding the system in different states.

Quantum fluctuations arise because, even in a state with the lowest possible energy (the ground state), the uncertainty principle allows for the temporary ”borrowing” of energy, as long as it is returned within a very short period of time. This means that a system can briefly have more energy than its ground state, leading to fluctuations in its energy. These fluctuations are fundamental aspects of quantum mechanics and are responsible for a variety of phenomena, including the Casimir effect (which arises from fluctuations in the quantum vacuum) and the spontaneous emission of light from atoms. In the context of cosmology, quantum fluctuations in the early universe are believed to have played a crucial role in the formation of structures such as galaxies and galaxy clusters, as they provided the initial seeds for the gravitational collapse of matter.

Quantum fluctuations can manifest in various forms depending on the context and the specific physical system under consideration. Some common types of quantum fluctuations include:

Vacuum Fluctuations: These fluctuations occur in the vacuum state of quantum fields, where particles and antiparticles continuously pop into existence and annihilate each other on very short timescales. Vacuum fluctuations contribute to the zero point energy of the vacuum and are responsible for phenomena such as the Casimir effect (7).

Spontaneous Emission: In quantum mechanics, atoms can transition between energy levels by emitting or absorbing photons. The timing and energy of these emissions are subject to quantum fluctuations, leading to uncertainty in the exact moment and properties of the emitted photons(9).

Zero Point Motion: Quantum mechanics predicts that particles such as electrons cannot be at rest but instead exhibit zero-point motion, meaning they continuously undergo tiny fluctuations in position and momentum even at absolute zero temperature. Zero-point energy is a central concept in quantum physics and arises from the interpretation of quantum mechanics of fields and particles. Unlike classical mechanics, where a system in its lowest state would have zero energy, quantum mechanics predicts that even in its lowest energy state (the ground state), a system possesses a finite amount of energy, known as zero-point energy.

Quantum Tunneling: Quantum particles can tunnel through energy barriers that would be classically impassable. This phenomenon arises from the wave-like nature of particles and leads to fluctuations in the probability of finding a particle in a particular region of space (10).

Quantum Field Fluctuations: Quantum fields, which describe the fundamental particles and forces in nature, can undergo fluctuations in their values at each point in space and time. These fluctuations contribute to the dynamics of particle interactions and can lead to the creation and annihilation of particle-antiparticle pairs (8).

These are just a few examples of the types of quantum fluctuations encountered in physics. Quantum mechanics governs the behavior of particles and fields at the smallest scales, and fluctuations are inherent to the probabilistic nature of quantum theory (21).

3.5 Quantum Fluctuations and the Origin of Structure

Now consider the elementary notion of quantum fluctuations. Elementary courses in modern physics illustrate how the uncertainty principle of Heisenberg $\Delta E \Delta t \sim \hbar$ indicates a small fluctuation in the energy of a particle associated with a probability that particle may pass over a potential energy barrier. This quantum effect is well documented in physics such as radioactive decay. This suggests a quantum particle in motion exhibits stochastic deviations from its classical motion. This is seen in the Ehrenfest theorem (24).

$$\frac{d}{dt}\langle|r|\rangle = \frac{1}{m}\langle|p|\rangle \tag{3.1}$$

where both sides of the equation have a sum over all states. The velocity of any particle 10 Quantum Fluctuations of Spacetime is an expectation from a sum over velocities, where most deviate from a classical result. The inference is that if one makes a momentum measurement of the particle there is a probability that the particle is under a momentum fluctuation. A measurement has a probability of finding the momentum of the particle different from the classical result (24).

The unit less number u is defined as the angular separation of the lens and the source. The amplification factor is given in terms of this value. The relevant time scale is called the Einstein time and it's given by the time it takes the lens to traverse an angular distance relative to the source in the sky. For typical microlensing events, an angular distance is on the order of a few (22).

3.6 Quantum Fluctuation Vacuum Energy

A quantum fluctuation (also known as a vacuum state fluctuation or vacuum fluctuation) is the temporary random change in the amount of energy in a point in space, (**?**) as prescribed by Werner Heisenberg's uncertainty principle. They are minute random fluctuations in the values of the fields which represent elementary particles, such as electric and magnetic fields which represent the electromagnetic force carried by photons, W and Z fields which carry the weak force, and gluon fields which carry the strong force (5).

Vacuum fluctuations appear as virtual particles, which are always created in particle antiparticle pairs (5). Since they are created spontaneously without a source of energy, vacuum fluctuations and virtual particles are said to violate the conservation of energy. The disposition of this energy gap is not well-understood; some physicists believe that the energy is transferred to or from the macroscopic environment in the course of the measurement process, while others believe that the observable energy is only conserved "on average"(25). No experiment has been confirmed as definitive evidence of violations of the conservation of energy principle in quantum mechanics, but that doesn't rule out that some newer experiments, as proposed, may find evidence of violations of the conservation of energy principle in quantum mechanics

3.7 Vacuum fluctuations during RD and MD

It is straightforward to repeat the same calculation for a radiation dominated and for a matter dominated era. For the RD epoch we use the modes. Then, recalling that $dt = ad\eta$

$$\phi k = \frac{1}{a}\phi k = -\frac{1}{a^2}[ik + \frac{\dot{a}}{a}]e^{-ik\eta} \tag{3.2}$$

where as usual the dot denotes the derivative with respect to cosmic time t and the prime the derivative with respect to η. Using $\frac{\dot{a}}{a} = \dot{a} = aH(t)$, we get

$$\rho(\Lambda_c) = \frac{1}{4\pi^2}\int_0^{a\Lambda_c}(dkk\frac{k^2}{a^4} + \frac{H^2(t)}{2a^2}) = \frac{\Lambda_c^4}{16\pi^2} + \frac{H^2(t)\Lambda_c^2}{16\pi^2}, \tag{3.3}$$

so the energy density turns out to be identical to, except that now H is replaced by H(t) For the pressure we get

$$p(\Lambda_c) = \frac{\Lambda_c^4}{48\pi^2} + \frac{H^2(t)\Lambda_c^2}{16\pi^2}, \tag{3.4}$$

Thus, once removed the Minkowski term, we remain with

$$\rho bare(\Lambda_c) = \frac{H^2(t)\Lambda_c^2}{16\pi^2}, \tag{3.5}$$

while for the pressure we get $p(\Lambda_c) = \rho bare(\Lambda_c)$. Similarly to what we have discussed in the de Sitter case, this means that the natural value of the renormalized energy density is

$$\rho_{vac}(t) = constant \times \sigma \frac{H^2(t)M^2}{16\pi^2}, \tag{3.6}$$

where M is the scale where quantum gravity sets in, "const" is a numerical constant O(1) and σ=+1. On the other hand, the relation $p(\Lambda_c) = \rho bare(\Lambda_c)$ does not imply that the renormalized energy density and the renor- malized pressure satisfy an equation of state with w =+1. As in the de Sitter case, the finite part in the counterterm for the pressure can be chosen so to reproduce any ob- served value of w, in particular, the value w =-1. The issue of general covariance is however now more subtle, since even the choice w = -1 now leads to

$$\langle|T_{\mu\nu}o\rangle = constant \times \sigma \frac{H^2(t)M^2}{16\pi^2}(-g_{\mu\nu}) \tag{3.7}$$

which,because of the time dependence of$H(t)$, is no longer covariantly conserved. However, general covariance actually only requires that the total energy momentum tensor, including that of matter, radiation, etc., be covariantly conserved. The fact that energy momentum tensor associated to vacuum fluctuations is not separately conserved means that there must be an energy exchange between vacuum fluctuations and other energy sources such as radiation or matter, so that we are actually dealing with an interacting dark energy model.

4

Correlation Between Dark Energy and Quantum Vacuum Fluctuation

4.1 Spatial correlations

Since the origin of dark energy in this model is quantum fluctuations of matter fields during inflation, it is expected that this model produces inhomogeneous dark energy of a predictable form. This was later confirmed by (8; 26), which used the same stochastic formalism to calculate the off-coincidence correlators in this theory (27).

It is shown that the curvature of space time induced by vacuum fluctuations of quantum fields should be proportional to the square of Newton's constant G. This offers a possible explanation for the success of the formula $\rho \sim Gm^6c^2h^{-4}, \rho$ being the dark energy density and m a typical mass of elementary particles. The observed accelerated expansion of the universe is currently assumed to derive from a positive mass density and a negative pressure, constant throughout space and time, which is known as "dark energy". The mass density, ρDE, and the pressure, pDE, are $\rho DE \approx pDE \approx 10^{-26} kg/m^3$ (28).

$$\rho_{DE} \approx -pDE \approx 10^{-26} \frac{Kg}{m^3} \tag{4.1}$$

Many proposals have been made for the origin of dark energy. The most popular is to identify it with the cosmological constant Λ introduced by Einstein in 1917 or, what is equivalent in practice, to assume that it derives from the quantum vacuum. Indeed the equality ρ DE $\approx$ -pDE is appropriate for the vacuum (in Malinowski space, or when the space time curvature is small) because it is invariant under Lorentz transformations.A problem appears however when one attempts to estimate the value of ρ DE . In particular if the dark energy is due to the interplay between quantum mechanics and gravity,it should be either strictly zero or of order Planck's density, that is (28):

$$\rho_{DE} \sim \frac{c^5}{G^2\hbar} \simeq 10^{97}\frac{Kg}{m^3} \tag{4.2}$$

This is about 123 orders of magnitude larger than eq. 4.1. Some explanations have been proposed for the huge value of the ratio between the Planck density, eq.4.2 , and the dark energy density, eq. 4.1. For instance Novello starts proving that, if Λ is a fundamental parameter, to be put on the left side of the Einstein equation (eq. 4.7 below), the space-time in the absence of matter is de sitter and the mass of the graviton is not zero. Then the ratio between eq. 4.2 and eq. 4.1 may correspond to the number of gravitons in the visible universe. On the other hand if Λ has to do with the matter content of the vacuum, it should appear on the right side of the Einstein equation. In this case ρ_{DE} should be related to other fundamental constants. Support to the latter assumption is the fact that a fairly good fit to eq. 4.1 may be obtained from a specific combination of the fundamental constants G, h and c plus a mass parameter m of the order of a typical mass of fundamental particles, that is (10)

$$\rho_{DE} \sim G\frac{m^6c^2}{\hbar^4} \tag{4.3}$$

Actually the observed value, eq.(1) , is obtained if the mass m is

$$m \sim 7.6 \times 10^{-29} kg \tag{4.4}$$

which is about $\frac{1}{20}$ times the proton mass or about 80 times the electron mass. I believe that dark energy is indeed associated to the matter content of the quantum vacuum and the purpose of the present paper is to provide a possible explanation for the estimate eq. 4.3 . Furthermore I propose that dark energy derives from the quantum vacuum fluctuations, an assumption considered recently by Padmanabhan (8), but my approach here is different. It rests upon the interplay between quantum mechanics and general relativity. Thus we shall work within quantized gravity. If we choose an appropriate coordinate system in the universe, with coordinates (28)

$$x \equiv x_1, x_2, x_3, x_4,$$

our aim is to relate the quantum state, $|\phi\rangle$, and the stress energy tensor operator, Tν $\mu(x)$, of quantum fields with the space-time structure.That structure is given by the metric tensor operator, $\hat{g}\mu\nu$. The study of the quantum fields in curved space times and the gravitational back reaction of the fields is a difficult subject. However for the purposes of the present paper it is enough to assume that the expectation value of the stress-energy tensor may be written in the form

$$\langle \Phi | T^{\mu}_{\nu} | \Phi \rangle = (T^{mat})^{\mu}_{\nu} + (T^{vat})^{\mu}_{\nu} \tag{4.5}$$

where $(T^{mat})^{\mu}_{\nu}$ is the stress energy tensor associated to matter,baryonic and possibly dark, plus radiation, and $(T^{vat})^{\mu}_{\nu}$ the tensor due to the vacuum. Both terms may be treated as classical. The latter contribution might be relevant when the space time metric departs substantially from Minkowski´s. This may be the case in the early universe (so explaining "inflation") or near galaxies (so explaining "dark matter"). Both these possibilities have been explored recently using f(R)-gravity (28).

It is convenient to define a stress energy tensor operator,$(\check{T}^{fluct})^{\mu}_{\nu}$,such that may be rewritten

$$\langle vac | (\check{T}^{fluct})^{\mu}_{\nu} | vac \rangle = 0, (\check{T}^{fluct})^{\mu}_{\nu} = \check{T}^{\mu}_{\nu} - \hat{I}(T^{vac})^{\mu}_{\nu} \tag{4.6}$$

Where $\hat{I}$ is the identity operator.the existence of vacuum fluctuations meansthat, although the expectation of$(\check{T}^{fluct})^{\mu}_{\nu}$ is zero, there are correlated flucuations, that is

$$\langle vac | (\check{T}^{fluct}(x))^{\mu}_{\nu} \times (\check{T}^{fluct}(y))^{\sigma}_{\lambda} | vac \rangle \neq 0 \tag{4.7}$$

correlations between quantum fluctuations may produce observable effects. An illustrative example is the (van der Waals) interaction between two molecules at a distance, d, much bigger than the typical size of a molecule. If both molecules possess a permanent electric dipole moment, then at low enough temperature they are oriented so that the molecules attract each other. In fact there is a dipole dipole (negative) interaction energy which scales as d^3. Now let us consider two neutral molecules which do not possess permanent dipole moment (28).

In this case classical physics predicts that there is no electrostatic interaction between them. Quantum theory however predicts an interaction due to the fact that quantum fluctuations give rise to instantaneous dipole moments in both molecules, which are correlated so that the energy is a minimum. This gives rise to an interaction which scales precisely as the square of the coupling parameter above mentioned, that is $(d^{-3})^2 = d^{-6}$. Indeed it is well known that the interaction energy between non polar molecules decreases with the six power of the distance (when retardation effects are negligible) (28).

The general behaviour may be understood via perturbation theory. To first order an average of the quantum fluctuations appears, which is zero (this is similar to. To second order however the perturbation involves the product of two correlated fluctuations, which is not zero (this is similar to. and it gives the interaction to lowest order. The square of the quantum fluctuations involves the coupling constant squared. Classical general relativity predicts that any stress-energy, $T_{\mu\nu}$, produces curvature of the space-time measured by the Ricci tensor, R^{μ}_{ν}, the scaling parameter being Newton constant, G. Indeed this is the essential content of Einstein equation

$$R^{\mu}_{\nu} - \frac{1}{2} g^{\nu}_{\mu} R = 8\pi c^{-2} G T^{\nu}_{\mu} \tag{4.8}$$

From the vacuum expectation of the metric tensor we may derive the Ricci tensor, R^{μ}_{ν} , which measures the curvature induced by the quantum vacuum fluctuations. Thus I may write schematically

$$R^{\mu}_{\nu} \sim G^2 \times \langle quantumcorrelation \rangle \tag{4.9}$$

$$\langle quantumcorrelation \rangle \sim \frac{m^6}{\hbar^4} \tag{4.10}$$

Quantum vacuum fluctuations possess spherical symmetry, that is depend only on the radial coordinate, r, and the time,t,

$$d\hat{s}^2 = \hat{A}(r,t)dr^2 + \hat{I}r^2(d\Theta^2 + \sin^2 \Theta d\phi^2) - \hat{B}(r,t)dt^2. \tag{4.11}$$

$$\hat{A}(r,t) = (1 - \frac{2Gm(r,t)}{r})^{-1}, m(r,t) \equiv 4\pi \int_0^r \rho(x,t) x^2 dx. \tag{4.12}$$

$$\hat{B}(r,t) = \exp[2G \int_0^r \frac{m(x,t) + 4\pi x^3 p(x,t)}{x^2 - 2Gm(x,t)x} dx] \tag{4.13}$$

where $\rho \equiv T^t_t$is the mass density and $p \equiv T^r_r$ the radial pressure. In our model the transverse pressure, $q \equiv T^{\theta}_{\theta} = T^{\psi}_{\psi}$, may be different from the radial pressure, but it plays no role in the following. In quantized gravity $\rho(r,t) \equiv T^t_t$ and $p(r,t) \equiv -T^t_t$ at different places may not commute. Therefore getting the quantized counterparts of eqs.(4.14) is not trivial. The problem is easier if we work to second order in Newton´s constant, G, which is enough for the purposes of this paper. Thus I will write the first eqs.(4.14) in the form

$$\hat{A}(r,t) = 1 + \frac{2Gm}{r}\frac{4G^2m^2}{r^2} + 0(G^3) \tag{4.14}$$

Similarly the second eq.(4.14) may be written

$$\hat{B}(r,t) = 1 + 2G\int_0^r [x^{-2}m(x) + 4\pi x p(x)]dx + 4G^2\int_0^r [x^{-3}m(x)^2 + 4(x)p(x)]dx$$
$$+ 2G^2[\int_0^r [x^{-2}m(x) + 4\pi x p(x)]dx]^2 + 0(G^3) \tag{4.15}$$

Now I propose that the quantized counterparts of these equations are obtained with the replacements $\rho(r,t) \rightarrow \rho(r,t)$,$p(r,t) \rightarrow p(r,t)$plus the substitution of symmetrically ordered products of operators for the usual products of the corresponding classical quantities. Thus for instance

$$p(r_1,t_1)p(r_2,t_2) \rightarrow p(r_1,t_1)p(r_2,t_2) + p(r_2,t_2)p(r_1,t_1) \tag{4.16}$$

After that it is straight forward to get the desired vacuum expectations, that is

$$\langle vac|\hat{A}(r,t)|vac\rangle \approx 1 + \langle vac|\frac{2G}{r}m(r) + \frac{4G^2}{r^2}m(r)^2|vac\rangle$$
$$m(r) \equiv \int_0^r \rho(x)4\pi x^2 dx \tag{4.17}$$

4.2 Cosmological Implications

4.2.1 Vacuum fluctuations and the dominant component of dark energy

The results of the previous section show that, with the subtraction that we advocate, zero-point vacuum fluctuations do not contribute to the cosmological constant since their energy density. Vacuum fluctuations refer to the spontaneous creation and annihilation of particle-antiparticle pairs in empty space, as predicted by quantum mechanics. These fluctuations are a fundamental aspect of quantum field theory and have measurable effects on physical phenomena (29).

The dominant component of dark energy is believed to be a cosmological constant, represented by the energy density of empty space. This constant exerts a repulsive force, driving the accelerated expansion of the universe. Despite its name, dark energy remains mysterious, comprising roughly of the universe's total energy density. Its nature is a subject of ongoing research in cosmology (29).

5

Result and Discussion

5.1 Theoretical basis for quantum fluctuations in the universe, and relation to the dynamics of dark energy

Quantum fluctuations are inherent fluctuations in the energy of a quantum mechanical system due to the uncertainty principle. In the context of the universe, these fluctuations are believed to have originated during the early stages of the universe, particularly during cosmic inflation, a period of rapid expansion shortly after the Big Bang.

During inflation, the universe underwent exponential expansion, which would have amplified any quantum fluctuations that existed at that time. These fluctuations are thought to have left imprints on the cosmic microwave background radiation, which we can observe today in the form of tiny temperature variations across the sky.

The dynamics of dark energy, on the other hand, are related to the accelerated expansion of the universe observed in recent cosmological measurements. Dark energy is a hypothetical form of energy that permeates all of space and is causing this accelerated expansion. The exact nature of dark energy is still unknown, but it is often modeled as a cosmological constant in the equations of general relativity.

The theoretical basis for the relationship between quantum fluctuations and dark energy lies in the idea that the vacuum energy associated with quantum fluctuations could potentially contribute to the energy density of the universe and act as a form of dark energy. However, the actual amount of energy predicted by quantum field theory is many orders of magnitude larger than the observed dark energy density, leading to what is known as the cosmological constant problem.

Despite this discrepancy, the connection between quantum fluctuations and dark energy remains an active area of research, as it may provide insights into the nature of both phenomena and their role in the evolution of the universe. As the smallest brick of vacuum, the Planckon is a quantum radiation standing wave in the sphere with the radius r_P , volume v_P , period T_P , mass m_P , and energy $\frac{\in_P}{den}$ as follows

$$r_P = (\frac{\hbar G}{c^3})^{\frac{1}{2}} \approx GT_P, v_P = \frac{4\pi}{3} rP^3, T_P = (\frac{\hbar G}{c^5})^{\frac{1}{2}} \tag{5.1}$$

$$m_P = \frac{1}{2}(\frac{\hbar c}{G})^{\frac{1}{2}} = \frac{\hbar}{2cr_P}, \in_P = m_P c^2 = \frac{\hbar c}{2r_P}, \tag{5.2}$$

where $T_P \sim 10^{-43}$s,$r_P = cT_P \sim 10^{-33}$cm,$\in_P \sim 10^{19}$GeV,$m_P \sim 10^{-5}$g,and the gravity constant G=6.67×10^{-8}erg.$\frac{cm}{g^2}$. Based on the Planckon densely piled vacuum model, the vacuum zero energy density is just the Planckon zero energy density (the Planckon energy P also corresponds to the cutoff of vacuum quantum fluctuation energy):

$$\rho_{vac} = \rho_p = \frac{\in_P}{v_P} = m_P c^2/(\frac{4\pi}{3} r_P^3) = \frac{3c^4}{8} \times \frac{1}{r_P^2} = \frac{K}{(ct_p)^2} \tag{5.3}$$

$$K = \frac{3c^4}{8}, t_p = \sqrt{\frac{3c^2}{8\pi G \rho_v ac}}. \tag{5.4}$$

According to the above assumptions, the time evolution of the isotropic and homogeneous universe obeys the Einstein-Friedmann equations [3]:
The expanding equation:

$$(\frac{\acute{R}}{R})^2 = \frac{8\rho}{3c^2} R = \Lambda = \lambda^2, \Lambda = \frac{8\pi G\rho}{3c^2}, \lambda = \sqrt{\frac{8\pi G\rho}{3c^2}} \tag{5.5}$$

The accelerating equation:

$$\frac{\acute{R}}{R} = -\frac{8\pi G\rho}{3c^2}(\rho + 3P) \tag{5.6}$$

According to the exact moden cosmology, the universe energy density contains the components of radiation ρ_r , cold matter ρ_m , dark matter $\rho_d m$, and dark energy$\rho_d e$. The equation of state is as follows:

$$p_i = \omega_i \times \rho_i, \rho = \sum_i \rho_i, p = \sum_i p_i = \sum_i \omega_i \times \rho_i = \sum_i \omega_i x_i \rho = \omega\rho \tag{5.7}$$

$$\omega(t) = \sum_i \omega_i(t), x_i(t) = \rho_i(t)/\rho(t), \sum_i x_i(t) = 1, i = r, m, dm, de \tag{5.8}$$

$$\omega_m = \omega_d m = 0, \omega_r = \frac{1}{3}, \omega_d e = -1, -1 \leq \omega(t) \leq \frac{1}{3} \tag{5.9}$$

Einstein-Friedmann equations, which show that during the evolution of the universe, the energy density components of ρ_i (t), x_i (t), and the coefficient ω_i (t) of the equations of state vary in their own intervals and make the total energy density $\rho(t)$ vary in time continuously.

5.2 Addressing the Correlation Problem between Quantum Fluctuations and Dark Energy

Inflationary models, quintessence models, and modified gravity theories offer different approaches to address the correlation problem between quantum fluctuations and dark energy.

5.2.1 Inflationary Models

Inflationary models propose a rapid expansion of the universe during the first fraction of a second after the Big Bang. Inflationary models can address the correlation problem by suggesting that quantum fluctuations during inflation seeded the large-scale structures observed in the universe today. These fluctuations would have been stretched to cosmological scales during inflation, providing a mechanism for the origin of the density perturbations that evolved into galaxies and galaxy clusters. However, inflationary models typically do not directly address the nature of dark energy or its correlation with quantum fluctuations.

5.2.2 Quintessence Models

Quintessence is a hypothetical form of dark energy that varies with time and space, in contrast to the cosmological constant, which has a constant energy density. Quintessence models attempt to explain the accelerated expansion of the universe by introducing a scalar field that slowly evolves over time. These models can potentially address the correlation problem by linking the dynamics of the scalar field to quantum fluctuations. Quantum fluctuations of the scalar field could contribute to the energy density of the universe, leading to the observed acceleration. However, quintessence models require fine-tuning of the scalar field potential to match observational data, similar to the cosmological constant problem.

Caldwell et al. (30) named quintessence a canonical scalar field φ with a potential V (φ) responsible for the late-time cosmic acceleration. Unlike the cosmological constant, the equation of state of quintessence dynamically changes with time. The cosmological dynamics for quintessence in the presence of matter and radiation has a long history already in the 1980s the cosmological consequences for such a system had been discussed by a number of authors. The cosmological evolution can be easily understood by a dynamical system approach. The energy density of quintessence does not need to be very small with respect to radiation or matter in the early Universe unlike the cosmological constant scenario. The existence of the so-called "tracker" field [39] is important to alleviate then coincidence problem of dark energy. The tracker fields correspond to attractor-like solutions in which the field energy density tracks the background fluid density for a wide range of initial conditions. Quintessence is based on scalar field models using a canonical field with a slowly varying potential. It is known however that scalar fields with non-canonical kinetic terms often appear in the context of particle physics. The action for such models is in general given by:

$$S = \int d^4x\sqrt{-g}[\frac{1}{2k^2}R + P(\phi, X)] + S_m \quad (5.10)$$

where $P(\varphi, X)$ is a function in terms of a scalar field φ and its kinetic energy $X = -\frac{1}{2}g_{\mu\nu}\partial\mu\varphi\partial\nu\ \varphi \equiv -(\frac{1}{2})(\triangle\varphi)^2$.

5.2.3 Modified Gravity Models

Modified gravity theories propose modifications to the equations of general relativity to explain the accelerated expansion of the universe without invoking dark energy. These theories often involve adding new gravitational fields or modifying the behavior of gravity on large scales. Some modified gravity theories, such as scalar-tensor theories, can be related to quintessence models, as they involve scalar fields that can contribute to the cosmic acceleration. These theories can potentially address the correlation problem by providing alternative explanations for the observed acceleration that are not directly tied to quantum fluctuations.

The next approach is based on modified gravity models in which the Einstein tensor $G_{\mu\nu}$ on the l.h.s. of the Einstein equations is modified. It is however important to realize that within General Relativity this division is mostly a practical way to classify the variety of dark energy models but does not carry a fundamental meaning. One can always write down Einstein's equations in the standard form $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ by absorbing in $T_{\mu\nu}$ all the gravity modifications that one conventionally puts on the l.h.s.. In other words, one can define a covariantly conserved energy-momentum tensor that equals the Einstein tensor. There is no way, within General Relativity, i.e. by using only gravitational interactions, to distinguish modified matter from modified gravity. At first-order in perturbation theory, for instance, one can define the equation of state and the sound speed of a dark energy field so that it reproduces any modified gravity model. Of course, from the viewpoint of quantum field theory the situation is different and the field content of the two classes are in general different and in principle distinguishable.

One of the simplest modified gravity models is the so-called f(R) gravity in which the 4-dimensional action is given by some general function f(R) of the Ricci scalar R:

$$S = \frac{1}{2k^2}\int d^4x\sqrt{-g}f(R) + S_m(g_{\mu\nu}, \psi_m) \tag{5.11}$$

where as usual $k^2 = 8\pi G, and S_m$ is a matter action with matter fields ψ_m. Here G is a bare gravitational constant: we will see that the observed value will in general be different. The matter fields in Sm obey standard conservation equations and therefore the metric $g_{\mu\nu}$ corresponds to the physical frame (which here is the Jordan frame).

Overall, while these theoretical frameworks offer different approaches to understanding the correlation between quantum fluctuations and dark energy, none of them provide a definitive solution to the problem.

$$H = H_0\sqrt{\frac{\Omega_{r0}}{a^4} + \frac{\Omega_{m0}}{a^3} + \Omega_{\Lambda,\text{model}}}$$

$$\Omega_{m,t} = \frac{\Omega_{m0}}{a^3} \div \left(\frac{\Omega_{r0}}{a^4} + \frac{\Omega_{m0}}{a^3} + \Omega_{\Lambda,\text{model}}\right)$$

$$\Omega_{r,t} = \frac{\Omega_{r0}}{a^4} \div \left(\frac{\Omega_{r0}}{a^4} + \frac{\Omega_{m0}}{a^3} + \Omega_{\Lambda,\text{model}}\right)$$

$$\Omega_{\Lambda,t} = \frac{\Omega_{\Lambda,\text{model}}}{\left(\frac{\Omega_{r0}}{a^4} + \frac{\Omega_{m0}}{a^3} + \Omega_{\Lambda,\text{model}}\right)}$$

5.3 Observational evidence for the correlation between quantum fluctuations and dark energy in the CMB radiation and LSS of the universe

Observational evidence for the correlation between quantum fluctuations and dark energy can be seen in the cosmic microwave background (CMB) radiation and the large-scale structure of the universe:

Cosmic Microwave Background (CMB) Radiation: The CMB is the remnant radiation from the Big Bang and provides a snapshot of the universe when it was just 380,000 years old. Tiny temperature fluctuations in the CMB, known as temperature anisotropies, are believed to be caused by quantum fluctuations in the early universe. These quantum fluctuations were stretched to cosmological scales during inflation, leading to the observed patterns in the CMB. The detailed analysis of these temperature anisotropies, particularly their statistical properties, provides strong evidence for the inflationary paradigm and the role of quantum fluctuations in seeding the large-scale structure of the universe.

Large Scale Structure of the Universe: The distribution of galaxies and galaxy clusters in the universe also reflects the imprint of quantum fluctuations. The clustering of matter on large scales is a direct consequence of the gravitational collapse of density perturbations seeded by quantum fluctuations. The statistical properties of the large scale structure, such as the power spectrum of galaxy clustering, can be compared with predictions from cosmological models that include inflation and quantum fluctuations. Observations of the large scale structure, therefore, provide indirect evidence for the correlation between quantum fluctuations and the evolution of the universe.

The correlation between quantum fluctuations and dark energy is supported by observational evidence from the CMB radiation and the large scale structure of the universe. These observations provide a crucial link between the microscopic quantum world and the large scale structure of the cosmos, shedding light on the fundamental processes that have shaped the universe's evolution.

5.4 The implications of the correlation between quantum fluctuations and dark energy

The correlation between quantum fluctuations and dark energy has profound implications for our understanding of the fundamental nature of dark energy and its role in the evolution of the universe model. Here are some key implications:

Quantum Origins of Dark Energy: The correlation suggests that dark energy may have originated from quantum fluctuations in the early universe. This challenges traditional views of dark energy as a constant, smooth component of the universe and implies that its properties may be inherently quantum mechanical in nature.

Dynamic Nature of Dark Energy: If dark energy is indeed related to quantum fluctuations, it may exhibit a more dynamic behavior than previously thought. Quantum effects could cause dark energy to vary over time and space, leading to observable changes in the expansion rate of the universe.

Connection to Fundamental Physics: Understanding the correlation could provide insights into the connection between dark energy and fundamental physics, such as quantum field theory and theories of quantum gravity. It may reveal new symmetries or interactions that govern the behavior of dark energy.

Cosmic Structure Formation: Quantum fluctuations that gave rise to dark energy in the early universe may have also seeded the initial density perturbations that led to the formation of cosmic structures, such as galaxies and galaxy clusters. This implies a deep connection between the quantum realm and the large scale structure of the universe.

Resolution of the Cosmological Constant Problem: The correlation could help resolve the long standing cosmological constant problem, which is the discrepancy between the predicted and observed energy density of dark energy. Understanding the quantum origins of dark energy may provide new insights into how to reconcile these discrepancies.

Overall, the correlation between quantum fluctuations and dark energy represents a fascinating intersection of cosmology, quantum physics, and fundamental theory. It opens up new avenues for exploring the nature of dark energy and its role in shaping the universe on the largest scales.

quantum vacuum fluctuations necessarily give rise to a curvature of space- time which roughly agrees with that attributed to dark energy. However a rigorous derivation would be required, which is not possible in the absence of a quantum gravity theory.

$$\text{Dark Energy Density (Correlated)}: \quad \rho_{\text{DE,corr}}(t) = 0.7 \times \left(\frac{t}{t_0}\right)$$

$$\text{Dark Energy Density (Uncorrelated)}: \quad \rho_{\text{DE,uncorr}}(t) = 0.7$$

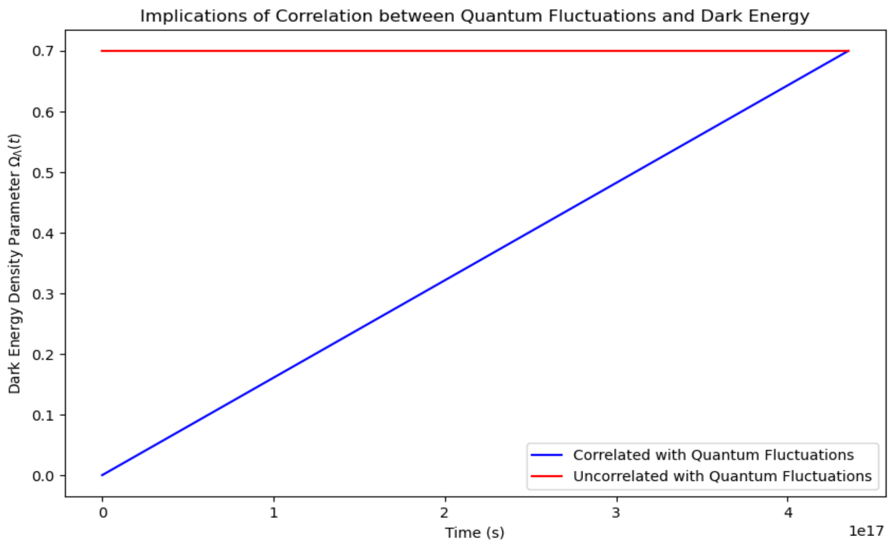

Figure 5.1: Implications of Correlation between Quantum Fluctuations and Dark Energy

The implications of the correlation between quantum fluctuations and dark energy for the fundamental nature of dark energy and its role in the evolution of the universe. Let's break down the explanation: Dark Energy Density Evolution Equations: Two functions are defined to model the evolution of the dark energy density parameter over cosmic time for two different scenarios:

Correlated with Quantum Fluctuations: In this scenario, the dark energy density parameter is assumed to increase linearly with time, influenced by quantum fluctuations. This represents a dynamic dark energy component that evolves with the expansion of the universe, potentially influenced by quantum effects. Uncorrelated with Quantum Fluctuations: Here, the dark energy density parameter remains constant over time, unaffected by quantum fluctuations. This represents a static dark energy component that does not change with the expansion of the universe and is independent of quantum effects.

6

Summary and Conclusion

6.1 Conclusion

After thorough investigation into the correlation problem between quantum fluctuations and the dark energy sector within the universe model, our research concludes that while significant progress has been made in understanding both phenomena individually, reconciling their interrelation remains a formidable challenge. Quantum fluctuations, stemming from the inherent uncertainty at the quantum level, are well established as influential forces in shaping the universe evolution. On the other hand, dark energy, believed to drive the universe's accelerated expansion, presents a profound mystery, with its nature and origin still elusive. Our analysis suggests that any correlation between these two aspects of the universe model is likely subtle and intricate, requiring innovative theoretical frameworks and observational insights to unravel. Further interdisciplinary collaborations between quantum physicists, cosmologists, and observational astronomers are essential to deepen our comprehension of these fundamental aspects of the cosmos.

One aspect of the cosmological constant problem, or more generally of the problem of understanding the origin of dark energy, is to understand why zero point fluctuations of quantum fields do not produce an energy density of the order of M^4, where M is the scale of the UV cutoff of the quantum field theory, despite the fact that this seems to be the natural value suggested by quantum field theory. We have proposed that the solutions to this long-standing puzzle has a purely classical origin, and is related to the correct definition of energy in classical general relativity.

We have applied this subtraction procedure to a FRW space-time with Hubble parameterH_t and we have found that the remaining energy density, after renormalization, has a natural value proportional to $M^2 H_t^2$ (and a sign that could in principle be either positive or negative, just as in the Casimir effect). For $M \simeq M_P l$ this gives an energy density just of the order of the critical density of the Universe. As we have discussed, however, such an energy density has a time dependence that is not compatible with present observations, if we identify it with the dark energy component with $\Omega_\Lambda \approx 0.7$ which in the standard ΛCDM cosmology is responsible for the observed acceleration of the Universe. It is however possible that it represents a new form of dark energy, whose normalized energy density today,Ω_Z, is smaller than Ω_Λ. Values of $\Omega_z \leq$ a few $\times 10^{-3}$ are compatible with the observations that we have discussed, but could give observable effects in more detailed studies that make use of the specific signature of zero-point fluctuations, namely, an energy density with a time dependence proportional to H_t^2, as well as in future more accurate cosmological observations.

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